Industrial Policy in Economic Development: Information and Coordination

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VERY PRELIMINARY AND INCOMPLETE

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Abstract

Intermediate input producers have private information about their own productivity. When intermediate inputs are complements in production, entry decisions may be inefficient. Then, even though the government has less knowledge about productivity than the market, the government may be able to increase production by limiting sectors of production (and thus reducing coordination problems). Government policy will be most effective when capital is scarce, when technologies are known, and when production technologies are complex.

There are few questions in economics as important as the causes of the

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tremendous differences in income per capita across countries and the related question of whether there are any policies which can promote growth in poor countries. One of the more frequently used set of policies, broadly referred to as industrial policy, advocates the government actively shutting down, requiring costly permits or taxing certain sectors of the economy while encouraging other sectors to produce. Industrial policy has been widely implemented as a tool for development in countries ranging from South Korea (Amsden, 1992; Amsden, 2000; Wade, 1990), Japan (Wade, 1990), and France (Eichengreen, 2006) to Brazil (Evans, 1995) and India (Chibber, 2006). Advocates of industrial policy claim that it is difficult if not impossible to find examples of countries which have developped without industrial policy (Stiglitz, 1999). Opponents of industrial policy, on the other hand, claim that countries which used industrial policy and grew would have grown even faster in industrial policy's absence (Krueger, 1992; Summers quoted in Rodrik, 2004).

For all the policy debate, including among economists, over industrial policy, there have been no formal models from which to analyze the costs and benefits of sectoral intervention. There was a related debate in the 1940s and 1950s about balanced and unbalanced growth. Rosenstein-Rodan (1943) advocated state intervention to coordinate balanced investments across sectors. He advocated "big push" or "balanced growth" policies, arguing for demand side externalities from investment in modern sectors. Development, he argued, should be balanced across modern sectors. This demand externality argument was formalized by Murphy, Shleifer, and Vishny (1989).

Hirschman (1958) criticized balanced growth development strategies, suggesting that following a sectorally broad development path would not allow for rich enough development of intermediate inputs. He argued for more laissez-faire policies because he thought that the market would develop networks of complementary intermediate inputs where necessary. Our paper formalizes Hirschman's idea in a growth model with imperfect common knowledge about productivity.

Our paper builds on a recent literature on the importance of developing a rich set of complementary intermediate inputs for growth (Blanchard and Kremer, 1997; Ciccone, 2002; Kremer, 1993). Recently, Jones has calibrated a growth model of complementary inputs and found that, due to strong complementarities and high shares of intermediate inputs in output, differences in input quality across countries can explain the entirety of the 50-fold gap between developing and developed countries. Coordinating input production through industrial policy, therefore, can be an empirically important channel for growth.

Our results are similar to Bolton and Farrell (1990) who consider a similar tradeoff between centralized decision-making by a less informed planner and uncoordinated decentralized decision making in a market. They consider alternative producers with different cost structures of a given good. The costs of centralized decision making is that the less informed government may pick a less efficient firm to produce. The costs of using the market are that firms may inefficiently wait to see if their potential competitors produce and also that there is some chance of duplication.

When intermediate inputs are complements in production and entry costs are private information, entry decisions will generically be inefficient. There is a benefit from coordination. Narrow entry in a broad range of sectors is less efficient than concentrated entry in a much small number of sectors. However, government knows less about the distribution of productivity across sectors than the private market itself. In the absence of better information, the government can help coordinate by pursuing the coarse policy of shutting down sectors. In the presence of strong complementarities, this will raise output though at the cost of intermediate input productivity.Government policy will be most effective when capital is scarce, when technologies are known, and when production technologies are complex.

Methodologically, we borrow from the literature on growth with product variety as in Grossman and Helpman (1991) and Melitz (2003). Our results that the benefits of industrial policy may decline with growth are due to our assumption of a fixed number of sectors. Eventually, as in Acemoglu and Zilibotti (2000), the number of sectors (markets) increase and the coordination failure problem dissipates.

In section one, we intuitively discuss the main tradeoff between better information provided by the market and better coordination provided by the government. In section two, we describe the model setup. In section three, we present the model when capital is abundant. In section four, we consider the model on the transition path. In particular, we consider the role for industrial policy along the transition path. In section five, we compare uniform taxation policy to industrial policy. In section six, we solve the social planner's problem. In section seven, we allow for international trade first in output goods and then in inputs. Finally, in section eight, we conclude.

1 Example of the Mechanism

In this section, we will illustrate the logic of our argument with the aid of two simple extreme example.[1] Suppose there are two sectors: computers and cars, where each sector requires two intermediate inputs. For computers, the two inputs are the shell and the CPU. For cars, they are the body and the motor. In each sector, inputs are complements. A computer can not be produced without a CPU and a car will not run without a body. Capital is only sufficient to cover the cost of producing two inputs. The cost of production is private information to each input producer. Since firms compete for the scarce capital, the two firms facing the best productivity realizations will make the highest bid to secure the necessary capital. Suppose the ranking of the productivities is

$$\text{body}>\text{CPU}>\text{motor}>\text{shell}$$

Then, the market would select the car body and the CPU, the two most productive intermediate input and no final output would be produced. On the other hand, if government shut down one of the two sectors, say cars, then the shell and the CPU could both enter in which case even though the gov-

[1] In this section we focus on extreme cases. However, the model is more general and allow for any degree of substitutability between inputs.

ernment's input selection is less productive than the market and even though the government's selection of sector is the least productive of the two possible ones, productivity is higher than under the market where no output is produced. Note, though, that if the ranking among the realizations had been

body>motor>CPU>shell

then the market would have performed better in the absence of industrial policy than in the presence of it. Industrial policy trades off having lower average productivity of inputs than the market (which selects the most productive inputs) for having more final output or having final output with a higher probability. Now suppose there is enough capital so that all four intermediate inputs can produce. Then, industrial policy will only limit entry and will not help coordinate. This is our story for how, in the case of input complementarity, industrial policy can be useful and why its usefulness can dissipate with development.

Now we turn to the case of substitutes. Suppose our two sectors are now textiles and beer. The two inputs for textiles are wool and cotton and the two inputs for beer are oats and wheat. The inputs now are perfect substitutes. Either wool or cotton can be used to produce textiles and either oats or wheat are needed to produce beer but both are not necessary. Suppose our ranking of input productivities is now:

wool>cotton>wheat>oats

The market outcome with capital sufficient to produce only two intermediate inputs will then lead to two inputs and thus only one output: textiles. If the government shut down sectors, even if it shut down the most productive intermediate inputs: wool and wheat, it would still allow for two final goods, both textiles and beer. Now the problem is too much entry within sectors and not enough across sectors as opposed to the case of complements when the problem was too much entry across sectors and not enough within sector mobilization. Again, if the ranking had been

wool>wheat>cotton>oats

then the market would have selected two intermediate inputs capable of producing two different sectors of final output. In this case, industrial policy trades off more efficient input selection by the market with greater coordination and potentially greater product diversity by the government. Once again, these benefits dwindle when the capital stock grows and all four intermediates are capable of entering. We now turn to a model of this basic intuition.

2 Model environment

2.1 Preferences

The economy is populated by a continuum of identical infinitely lived households whose preferences are parameterized by a time-separable logarithmic utility. More formally;

$$U = \int_0^\infty \log\left(c_t\right) \cdot e^{-\rho t} dt,$$

where $\rho > 0$ is the discount factor. The only productive asset is physical capital which is assumed, for simplicity, not to depreciate. The representative household owns ownership claims on capital and a balanced portfolio of shares of all firms in the economy.

2.2 Final Output Production

There is a unique consumption good, produced by a competitive representative firm. The production of final goods uses a continuum one of differentiated intermediate goods as inputs, each produced by a different industry. The technology features constant elasticity of substitution between intermediate goods;

$$Y = \left[\int_0^1 x(\omega)^{\delta} \, d\omega \right]^{\frac{1}{\delta}}, \tag{1}$$

where ω indexes intermediate industries, and we assume $0 < \delta < 1$, i.e., intermediate goods are gross substitutes.[2]

The demand for intermediate goods derived from the profit-maximizing choice of final producers is

$$x(\omega) = \frac{E}{P} \left(\frac{p(\omega)}{P} \right)^{-\frac{1}{1-\delta}}, \tag{2}$$

$E \equiv \int_{\omega \in \Omega} p(\omega) \, x(\omega) \, d\omega$ denotes the total expenditure, which is normalized to unity, i.e., $E = 1$. P is the price of the final good and is given by:

$$P = \left[\int_{\omega \in \Omega} p(\omega)^{-\frac{\delta}{1-\delta}} \, d\omega \right]^{-\frac{1-\delta}{\delta}}. \tag{3}$$

[2] We can alternatively interpret each $x(\omega)$ as a final good, and assume that consumers have CES preferences over the differentiated final goods.

2.3 Intermediate Production

The production of intermediate goods uses I differentiated basic inputs that are specific to each industry. For instance, the technology in industry ω is described by the following production function

$$x(\omega) = \left[\sum_{i=0}^{I} \frac{1}{I} \tilde{x}_i(\omega)^{\xi} \, d\omega \right]^{\frac{1}{\xi}} \tag{4}$$

where $\xi > 0$, implying that basic inputs are gross substitutes in the production of intermediate goods.[3]

Each basic input is produced by one firm using capital as the only productive factor. Production of basic inputs requires both fixed (overhead) and variable costs. More formally, the technology to produce the basic input i in sector ω is described by following cost function

$$TC_i(\omega) = q \left[s_i + \frac{\tilde{x}_i(\omega)}{\phi} \right]$$

where $\tilde{x}_i(\omega)$ denotes the production level, q is the rental rate of capital, and s_i is an entry cost that is sunk before any production takes place. Entry costs are heterogenous across firms, and are assumed to be drawn from a uniform distribution over the support $[0, \bar{s}]$. Realizations that are i.i.d. across industries and over time. Marginal costs of production are instead identical across all producers of basic inputs. Thus, fixed (sunk) costs are only source of heterogeneity across firms.

[3] Our analysis can be extended to the case of $\xi \leq 0$, i.e., firms produce gross complements. However, in this case there would exist another equilibrium with no entry and zero production (see discussion below).

3 Equilibrium Definition

The focal point of the analysis is the entry and production game among basic input producers. These producers face a two-stage decision problem. In the first stage, they observe their own technology (which is private information), and decide simultaneously and without coordination whether to enter. In the second stage, the $I_n(\omega) \leq I$ firms that have entered industry ω act as an integrated team to produce and sell the intermediate good, $x(\omega)$. More formally, the team sets the production of each basic input, $\tilde{x}_i(\omega)$, and the price at which the intermediate good is sold, $p(\omega)$, so as to maximize the joint profit, subject to the demand constraint, (2). The resulting revenue (net of production costs) is shared equally by all firms in the team. The assumption that basic input producers collude is for simplicity. It avoids complications associated with double marginalization that are orthogonal to the focus of our analysis. The assumption that profits are shared equally is natural since the marginal cost is the same for all firms, and although firms have paid different entry costs, these are sunk at the production stage.

The entry game is a game of incomplete information since the realization of technology is private information. We focus on symmetric Markov Perfect Equilibrium, where firms can condition their strategy only on pay-off relevant state variables, rather than on the entire history of the game. Since the aggregate capital stock is a sufficient statistic for the determination of the equilibrium, firms will only condition their strategy on K.

Definition 1 *A Markov Perfect Equilibrium (MPE) at t is a set of intermediate good price and output functions, $p(\omega, I_n(\omega); K)$ and $x(\omega, I_n(\omega); K)$ for $\omega \in [0,1]$, basic input output functions, $\tilde{x}_i(\omega, I_n(\omega); K)$ for $i \in \{1, 2, ..., I\}$ and $\omega \in [0,1]$, final good price and output functions, $P(K)$ and $Y(K)$, a rental rate of capital function, $q(K)$, and an entry policy function, $H : [0, \bar{s}] \times R^+ \to \{0,1\}$ (where $H(s_i; K) = 0$ stands for "no entry" and $H(s_i; K) = 1$ stands for "entry") such that, given K:*

1. *[ENTRY] Entry decisions in the first stage of the game maximize the expected profit of the firms producing basic inputs (where no entry yields zero profit), conditional on the own realization of the fixed cost, s_i, and the assumption that all other producers follow the equilibrium strategy, $H(s_i; K)$.*

2. *[PRODUCTION] Production and ¨[when relevant] price-setting decisions maximize the profits of all active firms subject to the relevant technology and demand constraints, conditional on the number of firms producing basic inputs that have entered in the first stage, $I_n(\omega)$. Moreover, the markets for capital, basic inputs, intermediate goods and final good clear.*

We characterize the MPE by backward induction. In the second stage each intermediate good firm (consisting of a team of basic input producers, as explained above) sets prices and production so as to maximize profits subject to the demand in the own industry and with rational expectations about the aggregate equilibrium prices. Note that in this stage all firms decide under perfect information. In the first stage, each basic input producer observes its technology (but not that of other firms) and decides entry optimally.

After solving the MPE of the production sector, we determine the equilibrium law of motion of K from the intertemporal consumption-savings decisions of households, as in standard growth models.

Definition 2 *A dynamic equilibrium is a pair of trajectories* $\{C_t, K_t\}_{t\in[0,\infty]}$ *and associated equilibrium rental rate of capital,* $q(K_t)$ *and final-good output and price functions,* $Y(K_t)$ *and* $P(K_t)$ *, such that, given* K_0

1. *For all* K_t *the equilibrium functions* $q(K_t)$ *,* $Y(K_t)$ *and* $P(K_t)$ *are consistent with the Markov Perfect Equilibrium in Definition 1.*

2. *Atomistic households maximize their present discounted utilities subject to a period budget constraint and a no-Ponzi game condition.*

4 Markov Perfect Equilibrium

4.1 Production stage game

In the production stage game, firms in industry ω observe the number of firms that have entered in the first stage of the game ($I_n(\omega) \leq I$).[4] Industry-level

[4] Our notation suggests that firms only observe the number of firms that have entered in their own industry. This is without loss of generality. Since there is a continuum of industries, the equilibrium distribution of entry decisions across industries is deterministic. Knowing how many firms have entered in each specific industry (expect for the own industry) is irrelevant information for the firms, due to the simmetry of demands and technologies. Therefore the

price and production strategies can therefore be conditioned on both K and $I_n(\omega)$. Since all firms have the same marginal costs, and firms produce differentiated inputs, profit maximization requires that all firms that have entered set production at the same level, to be denoted as $\tilde{x}(\omega, I_n(\omega); K)$. Using (4), we can rewrite the technology of each team as

$$x(\omega, I_n(\omega); K) = \left(\frac{I_n(\omega)}{I}\right)^{\frac{1}{\xi}} \tilde{x}(\omega, I_n(\omega); K), \tag{5}$$

Let $q(K)$ denote the equilibrium rental rate on capital. Then, the marginal cost of production of the intermediate good ω is $I_n(\omega)(I_n(\omega)/I)^{-\frac{1}{\xi}} q(K)/\phi$. Since the demand for intermediate goods is isoelastic (see equation (2)), profit-maximization implies that the price of intermediate goods is a mark-up over the marginal cost,

$$p(\omega, I_n(\omega); K) = p(I_n; K) = \left(\frac{I_n}{I}\right)^{-\frac{1-\xi}{\xi}} \frac{Iq(K)}{\delta\phi}, \tag{6}$$

and their production level equals

$$x(\omega, I_n(\omega); K) = x(I_n; K) = \frac{1}{P(K)}\left(\left(\frac{I_n}{I}\right)^{-\frac{1-\xi}{\xi}} \frac{Iq(K)}{P(K)\delta\phi}\right)^{-\frac{1}{1-\delta}}, \tag{7}$$

where $P(K)$ is the equilibrium price of the final good and we can drop the index ω due to symmetry.

Note that $p(I_n; K)$ and $x(I_n; K)$ are decreasing and increasing in I_n, respectively. The more entry, the higher the industry-level productivity. This results in larger production and a lower price. The effect of I_n on productivity and

results would be identical if we assumed that firms observe entry in all industry, but the notation would be more cumbersome.

output depends on the degree of substitutability between the basic inputs. The limit case in which $\xi \to 0$ is especially interesting. In this case, the intermediate good technology becomes a symmetric Cobb-Douglas in which all I inputs are essential. Thus, the marginal cost of producing the intermediate good tends to infinity whenever $I_n < I$. Thus, if $I_n < I$, $p(I_n; K) \to \infty$, then $x(I_n; K) \to 0$.

4.2 Entry stage game

In the first stage of the game the I producers of basic inputs in each intermediate industry make a simultaneous entry decision assuming that all other firms in the economy follow the equilibrium entry strategy. To charcaterize the symmetric MPE equilibrium, we proceed as follows:

1. We guess the form of the equilibrium entry policy (threshold rule).
2. Conditional on the guess, we characterize the set of equilibrium price and output functions.
3. Given the equilibrium price and output functions, we verify that it is individually rational for firms to follow a threshold rule that has the guessed form.
4. We solve explicitly for the equilibrium policy correspondence by solving a fixed-point problem.

We guess that $H(s_i;K)$ has the following threshold property:

$$H(s_i;K) = \begin{cases} 1 & \text{if} \quad s_i \leq s^*(K) \\ \\ 0 & \text{if} \quad s > s^*(K) \end{cases}. \tag{8}$$

Note that the equilibrium threshold cost, $s^* = s^*(K)$, only depends on the level of aggregate capital, consistent with the focus on MPE.

The expected value of entry for a firm with the realization s_i which assumes that all other firms follow the equilibrium policy, (8), can be written as

$$\pi^e(s_i; s^*(K), K) = \sum_{I_n=0}^{I-1} \binom{I-1}{I_n} \left(\frac{s^*(K)}{\bar{s}}\right)^{I_n} \left(1 - \frac{s^*(K)}{\bar{s}}\right)^{I-1-I_n} \frac{\Pi(I_n+1;K)}{I_n+1} - q(K)\, s_i, \tag{9}$$

where $\Pi(I_n;K)/I_n$ denotes the net cash-flow (revenue minus variable cost) accruing to each active producer of basic inputs when I_n firms are active in the industry.[5] To understand expression (9), note that, conditional on the threshold entry policy, we have I independent Bernoulli trials in each industry, each with probability of success (entry) $s^*/\bar{s}$. Thus, the probability that the number of active firms in an industry equals I_n is $\binom{I}{I_n}(s^*/\bar{s})^{I_n}(1-s^*/\bar{s})^{I-I_n}$, where, as usual, $\binom{I}{I_n} \equiv I!/((I-I_n)! \cdot I_n!)$. As the right hand side of (9) is computed under the assumption that firm i enters, the set of possible events includes up to $I-1$ other firms entering as well. Thus, the summation runs from 0 to $I-1$. For the same reason, if I_n of the "other" firms enter, there are I_n+1 active firms in the industry, and the net cash flow accruing to each firm is therefore

[5] When we refer to profits, we include the fixed cost. Thus, profit equals net cash flow minus fixed cost.

$\Pi\left(I_n+1;K\right)/I_n+1.$

In order for the guess (8) to be verified, we must prove the existence of $s^*\left(K\right)$ such that $\pi^e\left(s_i;s^*\left(K\right),K\right)\geq 0$ for all $s_i\leq s^*\left(K\right)$ and $\pi^e\left(s_i;s^*\left(K\right),K\right)<0$ for all $s_i\leq s^*\left(K\right)$. However, $\pi^e\left(s_i;s^*\left(K\right),K\right)$ depends on both $\Pi\left(I_n;K\right)/I_n$ and $q\left(K\right)$. Therefore, in order to make progress, we must first characterize these equilibrium expression conditional on the guess.

We will proceed through a number of steps. First, we state the following useful mathematical result.

Lemma 3 *Suppose* $0<a<\infty$ *and* $0<b<\infty$*. The function*

$$\Gamma\left(x\right)=\sum_{I_n=0}^{I}\binom{I}{I_n}\left(x\right)^{I_n}\left(1-x\right)^{I-I_n}\left(\frac{I_n}{I}\right)^{\frac{a}{b}}$$

has the following properties: (i) $\Gamma\left(0\right)=0$*, (ii)* $\Gamma'\left(x\right)>0$*, (iii)* $\Gamma''\left(x\right)\gtreqless 0\Leftrightarrow a\lesseqgtr b$.

Proof. See Saez Marti and Sjögren (2007). ■

Second, we note that, since there is a continuum of identical industries, and realizations are iid across industries, the law of large numbers implies that the probability to observe I_n active firms in a particular industry is also the relative frequency of industries with I_n active firms in the economy. This observation allows us to solve for the final good price and output functions, $P\left(K\right)$ and $Y\left(K\right)$.

Lemma 4 *Conditional on the equilibrium entry policy (8), the equilibrium final good price and output functions are given by*

$$Y\left(K\right)=\frac{1}{P\left(K\right)}=\frac{\delta\phi}{Iq\left(K\right)}n(s^*\left(K\right))^{\frac{1-\delta}{\delta}},\tag{10}$$

where

$$n\left(s^*\left(K\right)\right)=\left(\sum_{I_n=0}^{I}\frac{I!}{\left(I-I_n\right)!\cdot I_n!}\left(\frac{s^*\left(K\right)}{\bar{s}}\right)^{I_n}\left(1-\frac{s^*\left(K\right)}{\bar{s}}\right)^{I-I_n}\left(\frac{I_n}{I}\right)^{\frac{\delta(1-\xi)}{\xi(1-\delta)}}\right) \tag{11}$$

with the properties that (i) $n'\left(s^*\left(K\right)\right)>0$ *and (ii)* $n''(s^*\left(K\right))\gtreqless 0$ *if and only if* $\delta\lesseqgtr\xi$.

Proof. Using the definition of price index, (3), and the law of large numbers, we obtain

$$\begin{aligned} P\left(K\right) &= \left[\int_{\omega\in\Omega}p\left(\omega,I_n;K\right)^{-\frac{\delta}{1-\delta}}d\omega\right]^{-\frac{1-\delta}{\delta}} \\ &= \left(\sum_{I_n=0}^{I}\binom{I}{I_n}\left(\frac{s^*}{\bar{s}}\right)^{I_n}\left(1-\frac{s^*}{\bar{s}}\right)^{I-I_n}p\left(I_n;K\right)^{-\frac{\delta}{1-\delta}}\right)^{-\frac{1-\delta}{\delta}} \\ &= \frac{q\left(K\right)I}{\delta\phi}n\left(s^*\right)^{-\frac{1-\delta}{\delta}} \end{aligned}$$

where $n\left(s^*\right)$ is defined in equation (11) and the third equality follows from (6). That $Y\left(K\right)=1P\left(K\right)$ follows from the choice of the price normalization. That $n'\left(s^*\left(K\right)\right)>0$ follows from standard algebra. That $n''(s^*\left(K\right))\gtreqless 0$ if and only if $\delta\lesseqgtr\xi$ follows from Lemma 3. ■

Finally, using (6) and (7), we can characterize the industry equilibrium.

Lemma 5 *Conditional on the equilibrium entry policy (8), (i) the equilibrium intermediate good price functions,* $p\left(I_n;K\right)$, *are as in (6), (ii) the equilibrium intermediate good output functions are given by*

$$x\left(I_n;K\right)=\frac{\delta\phi}{Iq\left(K\right)n\left(s^*\left(K\right)\right)}\left(\frac{I_n}{I}\right)^{\frac{1-\xi}{\xi(1-\delta)}}, \tag{12}$$

where $n\left(s^*\left(K\right)\right)$ *is given by equation (11), (iii) the cash-flow accruing to each* *active basic input producer is*

$$\frac{\Pi\left(I_n;K\right)}{I_n}=\left(\frac{1-\delta}{I\cdot n\left(s^*\left(K\right)\right)}\right)\left(\frac{I_n}{I}\right)^{\frac{\delta-\xi}{\xi(1-\delta)}} \tag{13}$$

Proof. The expression of $x\left(I_n;K\right)$ follows immediately from substituting the expression of P(K) given by (10) into (7). To obtain (13) observe that

$$\begin{aligned}\Pi\left(I_n;K\right)&=p\left(I_n;K\right)x\left(I_n;K\right)-\frac{Iq\left(K\right)}{\phi}\left(\frac{I_n}{I}\right)^{-\frac{1-\xi}{\xi}}x\left(I_n;K\right).\\&=\left(\frac{1-\delta}{\delta}\right)\frac{Iq\left(K\right)}{\phi}\left(\frac{I_n}{I}\right)^{-\frac{1-\xi}{\xi}}x\left(I_n;K\right)\\&=\left(1-\delta\right)\left(\frac{I}{\delta\phi}\frac{q\left(K\right)}{P\left(K\right)}\right)^{-\frac{\delta}{1-\delta}}\left(\frac{I_n}{I}\right)^{\frac{\delta(1-\xi)}{\xi(1-\delta)}}\\&=\left(\frac{1-\delta}{n\left(s^*\right)}\right)\left(\frac{I_n}{I}\right)^{\frac{\delta(1-\xi)}{\xi(1-\delta)}}.\end{aligned}$$

Hence,

$$\frac{\Pi\left(I_n;K\right)}{I_n}=\frac{1}{I}\left(\frac{1-\delta}{n\left(s^*\left(K\right)\right)}\right)\left(\frac{I_n}{I}\right)^{\frac{\delta(1-\xi)}{\xi(1-\delta)}-1},$$

concluding the proof of the Lemma. ■

A corollary of Lemma 5 is that whether the profit per firm increase or decrease with I_n depend on the extent of complementarity within and between industries. If $\xi<\delta$ ($\xi>\delta$), profits per firm increase (decrease) with I_n. Consequently, if $\xi<\delta$ ($\xi>\delta$) entry decisions are strategic complements (substitutes).

The analysis of the limit case of Cobb-Douglas ($\xi\to 0$) is especially revealing. In this case, $n(s^*\left(K\right))\to\left(s^*\left(K\right)/\bar{s}\right)^I$ yields the number of industries in which all I firms enter. Since all inputs are essential, $n(s^*\left(K\right))$ is also the number of productive industries. Production in the remaining $1-n(s^*\left(K\right))$ industries is infinitesimal (more formally, $x\left(I_n;K\right)\to 0$ unless $I_n=I$). Nevertheless, low-productivity industries absorb capital due to the entry costs paid by firms that entered in the first stage based on the expectation of positive profits, but then found the industry productivity to be too low to produce. The sunk investments of these firms is the social waste due to miscoordination. Miscoordination extends to the general case: when firms make their entry decisions, they ignore the productivity that will obtain in their industry. Although all firms which enter produce, some industries have low productivity, and should have attracted no investments in the first best.

Third, Lemma 5 allows us to simplify equation (9) and to obtain an expression for $s^*(K)$ that depends only on the unknown function $q(K)$ and on parameters.

Lemma 6 *The expected profit of firm i at the entry stage can be expressed as*

$$\pi^e(s_i; s^*(K), K) = \left(\frac{1-\delta}{I}\right)\left(\frac{s^*(K)}{\bar{s}}\right)^{-1} - q(K)\, s_i. \tag{14}$$

As long as $s^*(K) < \bar{s}$, in a symmetric MPE $\pi^e(s^*(K); s^*(K), K) = 0$. Or, equivalently,

$$\frac{1-\delta}{I}\left(\frac{s^*(K)}{\bar{s}}\right)^{-1} = q(K)\, s^*(K) \tag{15}$$

Proof. Consider the expression of $\pi^e(s_i; s^*(K))$ in equation (9). First, we use (13) to eliminate $\Pi(I_n+1)/I_n+1$, noting that:

$$\sum_{I_n=0}^{I-1}\binom{I-1}{I_n}\left(\frac{s^*(K)}{\bar{s}}\right)^{I_n}\left(1-\frac{s^*(K)}{\bar{s}}\right)^{I-1-I_n}\frac{\Pi(I_n+1)}{I_n+1}$$

$$= \left(\frac{1-\delta}{I\cdot n(s^*(K))}\right)\sum_{I_n=0}^{I-1}\binom{I-1}{I_n}\left(\frac{s^*(K)}{\bar{s}}\right)^{I_n}\left(1-\frac{s^*(K)}{\bar{s}}\right)^{I-1-I_n}\left(\frac{I_n+1}{I}\right)^{\frac{\delta-\xi}{\xi(1-\delta)}}$$

Next, we note that

$$\sum_{I_n=0}^{I-1}\frac{(I-1)!}{((I-1)-I_n)!\cdot I_n!}\left(\frac{s^*(K)}{\bar{s}}\right)^{I_n}\left(1-\frac{s^*(K)}{\bar{s}}\right)^{I-1-I_n}\left(\frac{I_n+1}{I}\right)^{\frac{\delta-\xi}{\xi(1-\delta)}}$$

$$= \sum_{I_n=1}^{I} \frac{(I-1)!}{(I-I_n)! \cdot (I_n-1)!} \left(\frac{s^*(K)}{\bar{s}}\right)^{I_n-1} \left(1-\frac{s^*(K)}{\bar{s}}\right)^{I-I_n} \left(\frac{I_n}{I}\right)^{\frac{\delta-\xi}{\xi(1-\delta)}}$$

$$= \left(\frac{s^*}{\bar{s}}\right)^{-1} \sum_{I_n=0}^{I} \frac{I!}{(I-I_n)! \cdot I_n!} \left(\frac{s^*(K)}{\bar{s}}\right)^{I_n} \left(1-\frac{s^*(K)}{\bar{s}}\right)^{I-I_n} \left(\frac{I_n}{I}\right)^{\frac{\delta(1-\xi)}{\xi(1-\delta)}+1}$$

$$= \left(\frac{s^*(K)}{\bar{s}}\right)^{-1} n\left(s^*(K)\right)$$

Hence,

$$\pi^e\left(s_i; s^*(K), K\right) = \left(\frac{1-\delta}{I \cdot n\left(s^*(K)\right)}\right) \left(\frac{s^*(K)}{\bar{s}}\right)^{-1} n\left(s^*(K)\right) - q(K)\, s_i.$$

Setting $\pi^e\left(s_i; s^*(K), K\right) = 0$ and rearranging terms yields (15). This concludes the proof of the Lemma. ■

Finally, we close the model by using the market-clearing condition for capital. Recall that capital is used to cover both fixed and variable costs. Fixed costs are paid by all firms receiving a draw $s \leq s^*$. Variable costs depend, in addition, on the number of firms that are active in each industry.

Lemma 7 *The market-clearing condition for capital can be expressed as*

$$\frac{I}{2}\left(s^*(K)\right)^2 + \frac{\delta}{q(K)} = K. \tag{16}$$

Proof. The market clearing condition for capital is

$$I \int_0^{s^*(K)} s ds + \sum_{I_n=0}^{I} \binom{I}{I_n} \left(\frac{s^*(K)}{\bar{s}}\right)^{I_n} \left(1-\frac{s^*(K)}{\bar{s}}\right)^{I-I_n} K_V\left(I_n; K\right) = K. \tag{17}$$

where the first and second terms on the left hand side of (17) are, respectively, the total entry cost and total variable cost, having defined $K_V\left(I_n; K\right) = \left(\left(\frac{I_n}{I}\right)^{-\frac{1-\xi}{\xi}} \frac{I}{\phi}\right) x\left(I_n; K\right)$ as the variable cost for intermediate production for an industry using I_n basic inputs. Using (12) to eliminate $x(I_n; K)$ we can rewrite $K_V(I_n; K)$ as

$$K_V(I_n; K) = \frac{\delta}{q(K)\, n(s^*(K))} \left(\frac{I_n}{I}\right)^{\frac{\delta(1-\xi)}{\xi(1-\delta)}} \tag{18}$$

Finally, substituting in $K_V(I_n; K)$ as given by (18) into (17), and using the definition of $n(s^*(K))$ as given by (11) yields (16). ■

The zero-profit threshold and capital market-clearing conditions, (15)-(16), determine the equilibrium function $s^*(K)$ and $q(K)$, as long as $s^*(K) < \bar{s}$. Solving the two equations, we obtain

$$q(K) = \frac{\bar{s}(1-\delta) + 2\delta}{2K}, \tag{19}$$

$$s^*(K) = \left(\frac{2}{I} \frac{\bar{s}(1-\delta) K}{\bar{s}(1-\delta) + 2\delta}\right)^{\frac{1}{2}}, \tag{20}$$

which in turn imply – using (11) –

$$\begin{aligned} n(s^*(K)) &= \sum_{I_n=0}^{I} \frac{I!}{(I-I_n)! \cdot I_n!} \left(\frac{2}{I} \frac{(1-\delta) K}{\bar{s}(1-\delta) + 2\delta}\right)^{\frac{I_n}{2}} \\ &\quad \cdot \left(1 - \left(\frac{2}{I} \frac{(1-\delta) K}{\bar{s}(1-\delta) + 2\delta}\right)^{\frac{1}{2}}\right)^{I-I_n} \left(\frac{I_n}{I}\right)^{\frac{\delta(1-\xi)}{\xi(1-\delta)}} \\ &\equiv n(K) \end{aligned} \tag{21}$$

The upper bound of K consistent with $s^*(K) < \bar{s}$ and $n(K) < 1$ is

$$\bar{K} = \frac{\bar{s}I}{2} \frac{\bar{s}(1-\delta) + 2\delta}{1-\delta} \tag{22}$$

When $K > \bar{K}$ all firms enter, $n(K) = 1$, and equation (15) ceases to hold. The market-clearing condition, (16), continues to hold as long as one sets

$s^*(K) = \bar{s}$. Then, the equilibrium rental rate of capital is given by $q(K)|_{K \geq \bar{K}} = \delta / \left(K - I\bar{s}^2/2\right)$. Finally, standard algebra shows that, $Y(K) = 1/P(K) = \phi(K - I\bar{s}/2)/I$.

We summarize the characterization of the MPE in the following Proposition.

Proposition 8 *If $K < \bar{K}$, then, the unique symmetric MPE is given by the entry function*

$$H(s_i;K) = \begin{cases} 1 & \text{if} \quad s_i \leq \left(\frac{2}{I}\frac{\bar{s}(1-\delta)K}{\bar{s}(1-\delta)+2\delta}\right)^{\frac{1}{2}} \\ \\ 0 & \text{if} \quad s > \left(\frac{2}{I}\frac{\bar{s}(1-\delta)K}{\bar{s}(1-\delta)+2\delta}\right)^{\frac{1}{2}} \end{cases},$$

the industry-level price and output functions

$$p(\omega, I_n(\omega); K) = p(I_n; K) = \left(\frac{I_n}{I}\right)^{-\frac{1-\xi}{\xi}} \frac{I(\bar{s}(1-\delta)+2\delta)}{2\delta\phi K},$$

$$x(\omega, I_n(\omega); K) = x(I_n; K) = \frac{2\delta\phi K}{I(\bar{s}(1-\delta)+2\delta)}\left(\frac{I_n}{I}\right)^{\frac{1-\xi}{\xi(1-\delta)}},$$

the final-good price and output function

$$Y(K) = \frac{1}{P(K)} = \frac{I(\bar{s}(1-\delta)+2\delta)}{2\delta\phi K} n(s^*)^{-\frac{1-\delta}{\delta}},$$

and the equilibrium rental rate function $q(K)$ given by (19), where $n(K)$ is given by (21) and $\bar{K}$ is given by (22).

If $K \geq \bar{K}$, then, the unique symmetric MPE is given by the equilibrium entry policy function $H(s_i;K) = 1$ for all s_i, implying $I_n(\omega) = I$ for all $\omega \in [0,1]$. The industry-level equilibrium price and output functions are then $p(K) = I/\left(\phi\left(K - I\frac{\bar{s}^2}{2}\right)\right)$ and $x(K) = \phi\left(K - I\frac{\bar{s}^2}{2}\right)/I$, respectively, the equilibrium final-good price and output function satisfy $Y(K) = 1/P(K) = \phi\left(K - I\bar{s}^2/2\right)/I$, and the equilibrium rental rate is $q(K) = \delta/\left(K - I\frac{\bar{s}^2}{2}\right)$.

5 Dynamic Equilibrium

In this section, we characterize the dynamic equilibrium. The representative household's problem can be written as:

$$\max_{\{c_t,k_t\}_{t\in[0,\infty]}} U = \int_0^\infty \log(c_t)\cdot e^{-\rho t}dt$$

$$\text{subject to}$$

$$P(K_t)\dot{k}_t = q(K_t)k_t - P(K_t)(c_t - d_t),$$

$k_0 > 0$ and the transversality condition, $\lim_{T\to 0} k_T c_T^{-1}\exp(-\delta T) = 0$. Here, d_t represents the dividends paid by a fund consisting of a balanced portfolio of all firms in the economy (although some individual firms pay negative dividends, $d_t > 0$). Lower cases denote household-level variables, while upper cases denote aggregate variables that atomistic households take as parametric.

The solution of this problem yields a standard Euler equation, $\dot{c}_t/c = q(K_t)/P(K_t) - \rho$. Substituting in the equilibrium expression of $q(K_t)$ and $P(K_t)$ given by Proposition 8 and aggregating over households, we obtain

$$\dot{C}_t = \begin{cases} \left(\frac{\phi\delta}{I}n(K_t)^{\frac{1-\delta}{\delta}} - \rho\right)C_t & \text{if} \quad K_t < \bar{K} \\ \\ \left(\frac{\phi\delta}{I} - \rho\right)C_t & \text{if} \quad K_t \geq \bar{K} \end{cases}. \tag{23}$$

where $n(K_t)$ is given by (21) and $\bar{K}$ is given by (22). Recall that $n'(K_t) > 0$ for $K_t < \bar{K}$.

To find the equilibrium law of motion of capital, note that aggregating individual budget constraints yields

$$\dot{K}_t = Y(K_t) - C_t = \frac{\delta\phi}{Iq(K)}n(K_t)^{\frac{1-\delta}{\delta}} - C_t.$$

Substituting in the equilibrium function $q(K)$, and recalling that $n(K_t) = 1$ as $K_t \geq \bar{K}$ yields

$$\dot{K}_t = \begin{cases} \frac{\phi\delta}{I}\frac{2}{\bar{s}(1-\delta)+2\delta} n(K_t)^{\frac{1-\delta}{\delta}} K_t - C_t & \text{if} \quad K_t < \bar{K} \\ \\ \frac{\phi}{I}K - \phi\bar{s}^2 - C_t & \text{if} \quad K_t \geq \bar{K} \end{cases} \tag{24}$$

The model features increasing returns to investments as long as $K_t < \bar{K}$. Intuitively, this is due to the reduction in the extent of miscoordination that occurs as capital accumulates. When capital attains the threshold $\bar{K}$, coordination ceases to be a problem. Thereafter, the return to private saving becomes constant, although there are still technological increasing returns due to the overhead costs. Asymptotically, the model features AK dynamics, similar to Gali and Zilibotti (1995), with a constant growth rate of capital, output and consumption,

$$\begin{aligned} \lim_{K\to\infty} \frac{\dot{C}}{C} &= \lim_{K\to\infty} \frac{\dot{K}}{K} = \lim_{K\to\infty} \frac{\dot{Y}}{Y} = \frac{\delta\phi}{I} - \rho \\ \lim_{K\to\infty} \frac{C}{K} &= \rho + \frac{\phi}{I}(1-\delta). \end{aligned}$$

Under the assumption that $\rho < \delta\phi/I$, the equilibrium features self-sustained growth in the long run. It is straightforward to check that the transversality condition is satisfied.

However, low productivity induced by miscoordination at early stages of development can generate poverty traps. To analyze this possibility, define (C_ρ, K_ρ) as a pair of numbers such that

$$\rho = \frac{\phi\delta}{I} n(K_\rho)^{\frac{1-\delta}{\delta}} \text{ and } C_\rho = \frac{2\rho K_\rho}{\bar{s}(1-\delta)+2\delta}.$$

It is easy to check that (C_ρ, K_ρ) defines a fixed point of the dynamic system (23)-(24), i.e., a steady state. Linearization around (C_ρ, K_ρ) shows that the steady state is asymptotically unstable, namely, (C_ρ, K_ρ) is the $\alpha-$limit of the local dynamics. If we assume, as in Gali and Zilibotti (1995), that households have also at their disposal a small endowment (in addition to the output of the production sector), it is easy to show that there are equilibrium dynamics originating from the source (C_ρ, K_ρ) that converge to a steady state in which capital tends to zero and agents only consume the endowment.[6] In this poverty trap there is no industrial activity. Note that the inability of the decentralized economy to achieve a higher productivity of capital is the cause of this persistent underdevelopment.

6 Industrial policy

We now consider two types of industrial policy. Balanced growth policy uniformly across sectors limits the number of intermediate inputs allowed to produce in the sector. We denote this limit by: I_b. Unbalanced growth policy shuts down a percentage $\gamma < 1$ of sectors. It is critical that the government, not having private information over the productivity of individual firms can only randomly shut down intermediate inputs within or sector and can only randomly pick the sectors. The problem is similar to above. However the market clearing condition is modified as follows

[6] WE WILL ADD AN APPENDIX WHERE WE ANALYZE THE DYNAMICS MORE FORMALLY. THE INTERESTED READER CAN SEE GALI AND ZILIBOTTI (1995).

$$\gamma I_b \int_0^{s^*} s ds + \frac{\delta}{q} = K$$

that can be solved to yield:

$$s^* = \sqrt{\frac{2}{\gamma I_b}\left(K - \frac{\delta}{q}\right)}.$$

So, conditional on q being fixed, decreasing γ (i.e., stronger industrial policy) increases s^*, namely the average quality of firms is lower.

The equation of n is also modified since only firms in selected industries can open.

$$\begin{aligned}
n(K) &= \gamma \sum_{I_n=0}^{I_b} \frac{I_b!}{(I_b - I_n)! \cdot I_n!} \left(\left(\frac{2(1-\delta)}{\bar{s}(1-\delta) + 2\delta} \frac{K}{\gamma I_b} \right)^{\frac{1}{2}} \right)^{I_n} \\
&\quad \cdot \left(1 - \left(\frac{2(1-\delta)}{\bar{s}(1-\delta) + 2\delta} \frac{K}{\gamma I_b} \right)^{\frac{1}{2}} \right)^{I_b - I_n} \left(\frac{I_n}{I} \right)^{\frac{\delta(1-\xi)}{\xi(1-\delta)}} \\
&\quad \gamma \left(\sum_{I_n=0}^{I_b} \frac{I_b!}{(I_b - I_n)! \cdot I_n!} \left(\left(\frac{2(1-\delta)}{\bar{s}(1-\delta) + 2\delta} \frac{K}{\gamma I_b} \right)^{\frac{1}{2}} \right)^{I_n} \left(1 - \left(\frac{2(1-\delta)}{\bar{s}(1-\delta) + 2\delta} \frac{K}{\gamma I_b} \right)^{\frac{1}{2}} \right)^{I_b - I_n} \left(\frac{I_n}{I} \right)^{\frac{\delta(1-\xi}{\xi(1-\delta}} \right. \\
&= \gamma \left(\sum_{I_n=0}^{I_b} \frac{I_b!}{(I_b - I_n)! \cdot I_n!} \gamma^{-\frac{I_n}{2}} \left(\left(\frac{2(1-\delta)}{\bar{s}(1-\delta) + 2\delta} \frac{K}{I_b} \right)^{\frac{1}{2}} \right)^{I_n} \left(1 - \gamma^{-\frac{1}{2}} \left(\frac{2(1-\delta)}{\bar{s}(1-\delta) + 2\delta} \frac{K}{I_b} \right)^{\frac{1}{2}} \right)^{I_b - I_n} \left(\frac{I}{I} \right. \right.
\end{aligned}$$

In the case of perfect substitutes $(\xi = 1)$, we get a simple expression for $n(K)$:

$$n(K) = \gamma \left[1 - \left(1 - \gamma^{-\frac{1}{2}} \left(\frac{2}{I_b \bar{s}} \frac{\bar{s}(1-\delta) K}{\bar{s}(1-\delta) + 2\delta} \right)^{\frac{1}{2}} \right)^{I_b} \right]$$

Note that $n(K)$ is decreasing in I_b (since $\frac{dn(K)}{dI_b} = -\gamma \left(\frac{1}{2} \left(1 - \ln \frac{s^*}{\bar{s}} \right) \left(\sqrt{\frac{A}{x}} \right)^x \right) < 0$).

7 International Trade

Suppose two identical countries. To simplify the analysis, we let $\xi \to 0$. There is trade in final goods. After the first stage, firms observe where entry has occurred, and can decide whether to produce and for which market. Exporting to the foreign market entails an additional small fixed cost (that we assume to be infinitesimal for simplicity). When two firms (one in Home and one in Foreign) have entered, if they both enter a market, they compete a la Bertrand and their profits fall to zero. Thus, both firms anticipate this and neither enters the foreign market. This is for simplicity (we should try to deal with the case in which there is competition, interesting but more complicated).

We have now three range of goods:

- n^{H*} goods that are produced by local firms, where $n^{H*} = \left(\frac{s^*}{\bar{s}}\right)^I$
  - of them $n^{H*} * n^{F*}$ are produced in both countries, so local firms only serve the local market
  - the remaining $n^{H*}\left(1 - n^{F*}\right)$ are produced only by local firms, so local firms serve both markets
- the range $[n^{H*}, n^{H*} + \left(1 - n^{H*}\right) * n^{F*}]$ is produced by foreign firms and imported into the home country.

In a symmetric equilibrium, $n^{H*} = n^{F*}$, thus, we have the ranges

- $[0, n^{H*}\left(1 - n^{H*}\right)]$, $[n^{H*}\left(1 - n^{H*}\right), n^{H*}]$, $[n^{H*}, n^*\left(2 - n^*\right)]$, $[n^*\left(2 - n^*\right), 1]$ that are respectively produced at H and exported, produced at H and not exported, imported from F, and not available anywhere.

Equilibrium yields:

- For all $\omega \in [0, n^* (2 - n^*)]$

$$p(\omega) = \frac{Iq}{\delta\phi} = p$$

and

$$x^H(\omega) = x^F(\omega) = \frac{1}{P}\left(\frac{Iq}{P\delta\phi}\right)^{-\frac{1}{1-\delta}} = x,$$

where the superscript refers to where the good is consumed (not where it is produced).

Then

$$P = \left[\int_0^{n^*(2-n^*)} \left(\frac{Iq}{\delta\phi}\right)^{-\frac{\delta}{1-\delta}} d\omega\right]^{-\frac{1-\delta}{\delta}} = \frac{Iq}{\delta\phi}\left(n^*(2-n^*)\right)^{-\frac{1-\delta}{\delta}}. \qquad (25)$$

Consider, next, consumption.

$$C^H = \left[\int_0^{n^*(2-n^*)} \left(x^H(\omega)^\delta\right) d\omega\right]^{\frac{1}{\delta}} = \left(n^*(2-n^*)\right)^{\frac{1}{\delta}} \frac{1}{P}\left(\frac{Iq}{P\delta\phi}\right)^{-\frac{1}{1-\delta}} = \left(n^*(2-n^*)\right)^{\frac{1-\delta}{\delta}} \frac{\delta\phi}{Iq}.$$

Next, we check the normalization:

$$\begin{aligned} PY^H &= \int_0^{n^*} p(\omega)\, x^H(\omega)\, d\omega + \int_0^{n^*(1-n^*)} p(\omega)\, x^F(\omega)\, d\omega \\ &= n^*(2-n^*)\, px \\ &= n^*(2-n^*)\left(\frac{Iq}{P\delta\phi}\right)^{-\frac{\delta}{1-\delta}} \\ &= 1 \end{aligned}$$

where the last equality follows from (25).

Consider the market-clearing condition for capital:

$$\begin{aligned} K &= I\left(\frac{(s^*)^2}{2} + \frac{1}{\phi}\left(n^* x^H(\omega) + n^*(1-n^*)\, x^F(\omega)\right)\right) \\ &= I\left(\frac{(s^*)^2}{2} + \frac{1}{\phi} n^*(2-n^*)\, x\right) \\ &= I\left(\frac{(s^*)^2}{2} + \frac{1}{\phi} n^*(2-n^*)\frac{1}{P}\left(\frac{Iq}{P\delta\phi}\right)^{-\frac{1}{1-\delta}}\right) \\ &= I\frac{(s^*)^2}{2} + \frac{\delta}{q}, \end{aligned}$$

leading to (16), i.e.,

$$s^* = \min\left\{\left(\frac{2}{I}\left(K - \frac{\delta}{q}\right)\right)^{\frac{1}{2}}, \bar{s}\right\}$$

Consider next the zero-profit condition. We first define the cash flow per team member when a good is only marketed in the home country (probability $(n^*)^2$). This yields

$$\begin{aligned} \frac{\Pi^H}{I} &= \left(\frac{1-\delta}{\delta}\right)\frac{q}{\phi}x \\ &= \left(\frac{1-\delta}{\delta}\right)\frac{q}{\phi}\frac{1}{P}\left(\frac{Iq}{P\delta\phi}\right)^{-\frac{1}{1-\delta}} \\ &= \frac{1-\delta}{I\left(n^*(2-n^*)\right)} \end{aligned}$$

Then, the profit when a good is both marketed in the home country and exported (probability $n^*(1-n^*)$). This is just equal to $\frac{\Pi^{H+E}}{I} = 2\frac{\Pi^H}{I}$ since the two markets are identical and there are no trade costs. Thus, the expected profit for a firm facing the fixed cost s_i is

$$\begin{aligned}
\pi^e(s_i;s^*) &= \left(\frac{s^*}{\bar{s}}\right)^{I-1}\left(n^*\frac{\Pi^H}{I}+(1-n^*)\frac{\Pi^{H+E}}{I}\right)-qs_i \\
&= \left(\frac{s^*}{\bar{s}}\right)^{-1}n^*\left(n^*\frac{\Pi^H}{I}+(1-n^*)\frac{\Pi^{H+E}}{I}\right)-qs_i \\
&= \left(\frac{s^*}{\bar{s}}\right)^{-1}n^*(2-n^*)\frac{1-\delta}{I(n^*(2-n^*))}-qs_i \\
&= \frac{1-\delta}{I}\left(\frac{s^*}{\bar{s}}\right)^{-1}-qs_i
\end{aligned}$$

Hence, we have the same exact zero-profit condition as in the no-trade case, see (15):

$$\left(\frac{1-\delta}{I}\right)\left(\frac{s^*}{\bar{s}}\right)^{-1}=qs^*.$$

As long as $s^*<\bar{s}$, we obtain, as in the closed-economy case,

$$\begin{aligned}
q(K) &= \frac{\bar{s}(1-\delta)+2\delta}{2K}, \\
s^*(K) &= \left(\frac{2}{I}\frac{\bar{s}(1-\delta)K}{\bar{s}(1-\delta)+2\delta}\right)^{\frac{1}{2}}.
\end{aligned}$$

Hence, n^* is the same.

In summary:

- Capital is shared between fixed and variable costs EXACTLY as in the closed economy.
- However, P is lower, and this result in a lower consumption of each variety, x.
- Yet, consumers are happy as they can consume more varieties ($n^*(2-n^*)>n^*$). Thus, Y is higher.

Consider industrial policy. There are two types of industrial policy, coordinated and uncoordinated. With coordinated industrial policy, there is an additional gain that is given from the fact that countries can specialize on different goods. Under coordinated policy, the countries would expand γ until $\gamma = 1/2$, and then growth would be extensive. Coordinated industrial policy is equivalent to perfect integration plus doing the optimal industrial policy for the union. Uncoordinated industrial policy is less powerful, but still better than no industrial policy.

7.1 With competition

Now, we assume that when two firms are active they compete a la Bertrand. In this case, each serve its own market but the price equals the marginal cost. There are two sets of prices

- For $\omega \in [0, n^* (1 - n^*)]$

$$p(\omega) = \frac{Iq}{\delta\phi} = p$$

$$x = \frac{1}{P}\left(\frac{Iq}{P\delta\phi}\right)^{-\frac{1}{1-\delta}}$$

- For $\omega \in [n^* (1 - n^*), n^* (2 - n^*)]$

$$p(\omega) = \frac{Iq}{\phi} = p^c = \delta p$$

$$x^c = \frac{1}{P}\left(\frac{Iq}{P\phi}\right)^{-\frac{1}{1-\delta}} = \delta^{-\frac{1}{1-\delta}} x$$

Then

$$
\begin{aligned}
P &= \left[\int_0^{n^*(1-n^*)} \left(\frac{Iq}{\delta\phi}\right)^{-\frac{\delta}{1-\delta}} d\omega + \int_{n^*(1-n^*)}^{n^*} \left(\frac{Iq}{\phi}\right)^{-\frac{\delta}{1-\delta}} d\omega + \int_{n^*}^{n^*(2-n^*)} \left(\frac{Iq}{\delta\phi}\right)^{-\frac{\delta}{1-\delta}} d\omega\right]^{-\frac{1-\delta}{\delta}} \\
&= \frac{Iq}{\phi\delta}\left[2n^*\left(1-n^*\right) + \left(n^*\right)^2 \delta^{-\frac{\delta}{1-\delta}}\right]^{-\frac{1-\delta}{\delta}}.
\end{aligned}
$$

Consider, next, consumption.

$$
\begin{aligned}
C^H &= \left(\int_0^{n^*(1-n^*)} x^\delta d\omega + \int_{n^*(1-n^*)}^{n^*} \left(\delta^{-\frac{1}{1-\delta}} x\right)^\delta d\omega + \int_{n^*}^{n^*(2-n^*)} x^\delta d\omega\right)^{\frac{1}{\delta}} \\
&= \left(2n^*\left(1-n^*\right) x^\delta + \left(n^*\right)^2 \left(\delta^{-\frac{1}{1-\delta}} x\right)^\delta\right)^{\frac{1}{\delta}} \\
&= \frac{1}{P}\left(\frac{Iq}{P\delta\phi}\right)^{-\frac{1}{1-\delta}} \left(2n^*\left(1-n^*\right) + \left(n^*\right)^2 \delta^{-\frac{\delta}{1-\delta}}\right)^{\frac{1}{\delta}} \\
&= \left(2n^*\left(1-n^*\right) + \left(n^*\right)^2 \delta^{-\frac{\delta}{1-\delta}}\right)^{\frac{1-\delta}{\delta}} \frac{\delta\phi}{Iq}.
\end{aligned}
$$

Consider the market-clearing condition for capital:

$$
\begin{aligned}
K &= I\left(\frac{\left(s^*\right)^2}{2} + \frac{1}{\phi}\left(2n^*\left(1-n^*\right) x + \left(n^*\right)^2 \delta^{-\frac{1}{1-\delta}} x\right)\right) \\
&= I\left(\frac{\left(s^*\right)^2}{2} + \frac{x}{\phi}\left(2n^*\left(1-n^*\right) + \left(n^*\right)^2 \delta^{-\frac{1}{1-\delta}}\right)\right) \\
&= I\frac{\left(s^*\right)^2}{2} + \frac{\delta}{q}.
\end{aligned}
$$

leading to (16), i.e.,

$$
s^* = \min\left\{\left(\frac{2}{I}\left(K - \frac{\delta}{q}\right)\right)^{\frac{1}{2}}, \bar{s}\right\}
$$

Consider next the zero-profit condition. We first define the cash flow per team member when a good is marketed in both countries at monopoly price (probability $n^*(1-n^*)$). This yields

$$\begin{aligned}\frac{\Pi^{H+E}}{I} &= 2\left(\frac{1-\delta}{\delta}\right)\frac{q}{\phi}x \\ &= 2\left(\frac{1-\delta}{\delta}\right)\frac{q}{\phi}\frac{1}{P}\left(\frac{Iq}{P\delta\phi}\right)^{-\frac{1}{1-\delta}} \\ &= 2\frac{1-\delta}{I\left(2n^*(1-n^*)+(n^*)^2\,\delta^{-\frac{1}{1-\delta}}\right)}\end{aligned}$$

We can ignore goods sold competitively as these generate zero profits. Thus, the expected profit for a firm facing the fixed cost s_i is

$$\begin{aligned}\pi^e(s_i;s^*) &= \left(\frac{s^*}{\bar{s}}\right)^{I-1}(1-n^*)\frac{\Pi^{H+E}}{I}-qs_i \\ &= \left(\frac{s^*}{\bar{s}}\right)^{-1}\frac{1-\delta}{I}\frac{1}{1+\frac{n^*}{1-n^*}\frac{\delta^{-\frac{1}{1-\delta}}}{2}}-qs_i \\ &= \left(\frac{s^*}{\bar{s}}\right)^{-1}\frac{1-\delta}{I}\frac{2\delta^{\frac{1}{1-\delta}}}{2\delta^{\frac{1}{1-\delta}}+\frac{n^*}{1-n^*}}-qs_i\end{aligned}$$

Now, we have a different (more complex) zero-profit condition:

$$\begin{aligned}\left(\frac{s^*}{\bar{s}}\right)^{-1}\frac{1-\delta}{I}\frac{2\delta^{\frac{1}{1-\delta}}}{2\delta^{\frac{1}{1-\delta}}+\frac{n^*}{1-n^*}} &= qs^* \\ \frac{1-\delta}{\bar{s}qI}\frac{2\delta^{\frac{1}{1-\delta}}}{2\delta^{\frac{1}{1-\delta}}+\frac{\left(\frac{s^*}{\bar{s}}\right)^I}{1-\left(\frac{s^*}{\bar{s}}\right)^I}} &= \left(\frac{s^*}{\bar{s}}\right)^2 \\ \frac{1-\delta}{\bar{s}qI}2\delta^{\frac{1}{1-\delta}} &= \left(\frac{s^*}{\bar{s}}\right)^2\left(2\delta^{\frac{1}{1-\delta}}+\frac{\left(\frac{s^*}{\bar{s}}\right)^I}{1-\left(\frac{s^*}{\bar{s}}\right)^I}\right)\end{aligned}$$

$$s^* = \min\left\{\left(\frac{2}{I}\left(K-\frac{\delta}{q}\right)\right)^{\frac{1}{2}},\bar{s}\right\}$$

where, recall $n^* = \left(\frac{s^*}{\bar{s}}\right)^I$.

Consider now the industrial policy

$$\begin{aligned}\gamma I\frac{(s^*)^2}{2}+\frac{\delta}{q} &= K \\ n^* &= \gamma\left(\frac{s^*}{\bar{s}}\right)^I\end{aligned}$$

or

$$
\begin{aligned}
s^* &= \min\left\{\left(\frac{2}{\gamma I}\left(K-\frac{\delta}{q}\right)\right)^{\frac{1}{2}}, \bar{s}\right\} \\
n^* &= \gamma\left(\frac{s^*}{\bar{s}}\right)^I
\end{aligned}
$$

while the equation

$$
\frac{1-\delta}{\bar{s}qI}2\delta^{\frac{1}{1-\delta}} = \left(\frac{s^*}{\bar{s}}\right)^2\left(2\delta^{\frac{1}{1-\delta}} + \frac{\left(\frac{s^*}{\bar{s}}\right)^I}{1-\left(\frac{s^*}{\bar{s}}\right)^I}\right)
$$

is exactly the same.

7.2 Trade in inputs

Suppose two identical countries. To simplify the analysis, we let $\xi \to 0$. There is trade in intermediate goods. Teams can be formed between local and foreign intermediate producers. If the same intermediate good is produced by two firms, one local and one foreign, they split the money. A team sells to both markets.

In this case, the probability that a "successful" team is formed is $n^* = \left(1-\left(1-\frac{s^*}{\bar{s}}\right)^2\right)^I$. Here, $\left(1-\frac{s^*}{\bar{s}}\right)$ is the probability that a given input in a particular country does not enter, and thus $\left(1-\frac{s^*}{\bar{s}}\right)^2$ is the probability that neither the H nor the F potential producers enter. Therefore, $1-\left(1-\frac{s^*}{\bar{s}}\right)^2$ is the probability that at least one of the two potential producers enter, and this to the power of I is the probability that a team is formed.

For a producer contemplating entry, the expected profit is

$$
\pi(s) = \left(1-\left(1-\frac{s^*}{\bar{s}}\right)^2\right)^{I-1}\left(\frac{s^*}{\bar{s}}\frac{\Pi^{BOTH}}{I} + \left(1-\frac{s^*}{\bar{s}}\right)\frac{\Pi^{ALONE}}{I}\right) - qs_i
$$

where Π^{BOTH} is the cash-flow conditional on the foreign producer of the same input entering. Clearly

$$2\frac{\Pi^{BOTH}}{I} = \frac{\Pi^{ALONE}}{I}$$

Equilibrium yields:

- For all $\omega \in [0, n^*]$

  $$p(\omega) = \frac{Iq}{\delta\phi} = p$$

  and

  $$x(\omega) = \frac{1}{P}\left(\frac{Iq}{P\delta\phi}\right)^{-\frac{1}{1-\delta}} = x,$$

  where the superscript refers to where the good is consumed (not where it is produced).

Then

$$P = \left[\int_0^{n^*}\left(\frac{Iq}{\delta\phi}\right)^{-\frac{\delta}{1-\delta}} d\omega\right]^{-\frac{1-\delta}{\delta}} = \frac{Iq}{\delta\phi}(n^*)^{-\frac{1-\delta}{\delta}}.$$

Consider, next, consumption.

$$C^H = \left[\int_0^{n^*}\left(x^\delta\right) d\omega\right]^{\frac{1}{\delta}} = (n^*)^{\frac{1}{\delta}}\frac{1}{P}\left(\frac{Iq}{P\delta\phi}\right)^{-\frac{1}{1-\delta}} = (n^*)^{\frac{1-\delta}{\delta}}\frac{\delta\phi}{Iq}.$$

Consider the market-clearing condition for capital:

$$\begin{aligned}
K &= I\left(\frac{(s^*)^2}{2} + \frac{n^*}{\phi}\left(\frac{s^*}{\bar{s}}x + \left(1 - \frac{s^*}{\bar{s}}\right)2x\right)\right) \\
&= I\left(\frac{(s^*)^2}{2} + \frac{n^*}{\phi}\left(2 - \frac{s^*}{\bar{s}}\right)x\right) \\
&= I\left(\frac{(s^*)^2}{2} + \frac{n^*}{\phi}\left(2 - \frac{s^*}{\bar{s}}\right)\frac{1}{P}\left(\frac{Iq}{P\delta\phi}\right)^{-\frac{1}{1-\delta}}\right) \\
&= I\frac{(s^*)^2}{2} + \frac{\delta}{q}\left(2 - \frac{s^*}{\bar{s}}\right)
\end{aligned}$$

leading to

$$K = I\frac{(x)^2}{2} + \frac{\delta}{q}\left(2 - \frac{x}{\bar{s}}\right)$$

$$s^* = \min\left\{\frac{\delta + \sqrt{\delta^2 + (2Kq - 4\delta)\, q\bar{s}^2 I}}{q\bar{s}I}, \bar{s}\right\}$$

Consider next the zero-profit condition.

$$\begin{aligned}
\pi^e(s_i; s^*) &= \left(1 - \left(1 - \frac{s^*}{\bar{s}}\right)^2\right)^{I-1}\left(2 - \frac{s^*}{\bar{s}}\right)\frac{\Pi^{BOTH}}{I} - qs_i \\
&= \left(1 - \left(1 - \frac{s^*}{\bar{s}}\right)^2\right)^{I-1}\left(2 - \frac{s^*}{\bar{s}}\right)\left(\frac{1-\delta}{\delta}\right)\frac{q}{\phi}x - qs_i \\
&= \left(1 - \left(1 - \frac{s^*}{\bar{s}}\right)^2\right)^{-1}\left(2 - \frac{s^*}{\bar{s}}\right)\left(\frac{1-\delta}{I}\right) - qs_i
\end{aligned}$$

since

$$\frac{\Pi^{BOTH}}{I} = \frac{2}{2}\left(\frac{1-\delta}{\delta}\right)\frac{q}{\phi}x$$

Zero profit condition

$$\begin{aligned}
\left(1 - \left(1 - \frac{s^*}{\bar{s}}\right)^2\right)^{-1}\left(2 - \frac{s^*}{\bar{s}}\right)\left(\frac{1-\delta}{I}\right) &= qs^* \\
s^* &= \sqrt{\frac{\bar{s}(1-\delta)}{qI}}
\end{aligned}$$

The solution is given by the equations

$$\begin{aligned}
s^* &= \frac{\delta + \sqrt{\delta^2 + (2Kq - 4\delta)\, q\bar{s}^2 I}}{q\bar{s}I} \\
s^* &= \sqrt{\frac{\bar{s}(1-\delta)}{qI}}
\end{aligned}$$

$$\begin{aligned}
s^* &= \frac{\delta + \sqrt{\delta^2 + \left(2K\frac{\bar{s}(1-\delta)}{(s^*)^2 I} - 4\delta\right)\frac{\bar{s}(1-\delta)}{(s^*)^2 I}\bar{s}^2 I}}{\frac{\bar{s}(1-\delta)}{(s^*)^2 I}\bar{s}I} \\
q &= \frac{\bar{s}(1-\delta)}{(s^*)^2 I}
\end{aligned}$$

8 Conclusion

We have presented a model which we think captures the main features of the costs and benefits of industrial policy intervention. The benefits of shutting down certain sectors and encouraging production in other sectors comes from greater coordination across intermediate input producers. The costs come from government's inability to pick winners, which results in a lower average productivity of intermediate input producers. Internationally, industrial policy increases exports.

Our model generates testable implications. Industrial policy should raise the average productivity of final output producers at the expense of intermediate input producers. Also, the variance of productivitiy of of intermediate input producers should go up. Empirical resarch, in the spirit of Jones (2008), testing the the predictions of our model would be an important addition.

Our current model is tractible because it makes simplifying assumptions. We assume that average productivity is distributed i.i.d. over time and across firms. This allows us to restrict ourselves to Markov strategies. However, presumably, we miss some of the dynamics of industrial policy in this simplification. In particular, we do not allow for more productive sectors to coordinate over time through social learning. This would translate our costs of using the market into delay costs and would render our costs of using industrial policy potentially more persistent.

One potential problem with industrial policy is that incumbent firms may use their profits to influence future government industrial policy, which could raise the cost of industrial policy. In practice, industrial policy has, where succesfully implemented, often relied upon observable measures such as export revenues. Our model would have no role for such incentive systems. This is because all firms are equally productive on the margin. This is done for tractibility; however, investigating models which allowed performance incentives as a way to deal with political economy problems would be interesting.

We have compared three alternatives: the free market, uniform tax policies, and industrial policy. We have shown that government implemented industrial policy dominates the other two when complementarities are strong. However, there are many possible institutional arrangements to coordinate actions better. Also, there are many potential actors which can coordinate activities. In many of the countries that used industrial policy, the banking sector played a large allocative role in conjunction with government. Also, vertical integration could potentially allow for better coordination of actions, potentially at the cost of the most efficient inputs being used. Hopefully, this model can be used as a tool for looking not only at industrial policy and development but also at the boundaries of the firm and the organization of production.

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