Currency crisis triggers: sunspots or thresholds?*

Bernardo Guimaraes†

September 2007

Abstract

If currency crises are triggered when the currency overvaluation hits a threshold, the expected magnitude of a devaluation, conditional on its occurrence, is substantially different from the unconditional expected currency overvaluation. That is not true if currency crises are triggered by sunspots. Therefore, implications for the behaviour of the probability and the expected magnitude of a devaluation depend on what triggers currency crises. Those two variables are not observable but can be estimated using data on exchange rate options. This paper identifies the probability and expected magnitude of a devaluation of Brazilian *Real* in the period leading up to the end of the Brazilian pegged exchange rate regime and contrasts the estimates to the predictions from a simple model of currency crises under different assumptions about the trigger. The empirical findings favour thresholds and learning over sunspots.

KEYWORDS: Currency crises, sunspots, exchange rate, options.

JEL CLASSIFICATION: F3, G1

1 Introduction

What triggers currency crises? This is an old question, tackled by a somewhat large literature comprising several models that may be divided into two main camps. In one class of models, there are multiple equilibria and so-called sunspot events that move the economy to a new equilibrium and trigger currency crises.[1] In the other camp, a crisis is triggered when an economy goes beyond a certain threshold.[2]

Ideally, evaluation of these different theories would be guided by empirical evidence, but that branch of the literature has not clearly favoured either camp. This is, at least in part, due to two key obstacles for empirical work on currency crises. The first is that

*This paper is partly based on Chapter 2 of my PhD dissertation at Yale University. I am very grateful to Stephen Morris, my advisor, and Francesco Caselli for their insights. I also thank my colleagues at LSE and several seminar participants for their comments, BM&F for the data and Nathan Foley-Fisher for able research assistance financed by STICERD.

†London School of Economics, Department of Economics. Email: b.guimaraes@lse.ac.uk

[1] Some important contributions in this camp are Obstfeld (1986), Obstfeld (1996) and Jeanne (1997). For a survey, see Rangvid (2001).

[2] The so called first generation models, like Krugman (1979) and Flood and Garber (1984), and the global game models, as Morris and Shin (1998), are in this camp.

currency crises are rare events, so data explicitly relating to them are relatively scarce. The second obstacle is that we don't really understand the exchange rate.[3]

The paucity of data on currency crises is an obstacle that may be overcome using financial price data, which are abundant and reflect expectations about currency devaluations. Thus analysing agents' expectations through movements in financial prices is a good way to understand crises and that is the strategy pursued in this paper.

The second problem is harder to overcome. The inability of models to explain adequately the behaviour of the exchange rate implies that it is difficult to get reasonable estimates for the rate which would prevail if the government were to abandon the exchange rate peg at a given moment — I will call it the shadow exchange rate.[4] That is the key latent variable in currency crises because it is the major determinant of the profits/losses agents will make if they decide to attack the currency and because the deviation of the shadow from the pegged exchange rate is the most important factor determining the cost to the government of maintaining the peg. The absence of a good model for the shadow exchange rate makes it quite difficult to empirically assess the nexus between the shadow rate and currency crises, and hence to test the theories about currency crisis triggers.

This paper presents an alternative procedure for determining the currency crises triggers, where evaluation is based on the behaviour of the probability and expected magnitude of a devaluation. A key insight is that when crises are triggered by currency overvaluation crossing a threshold, the expected magnitude of a devaluation, conditional on its occurrence, is equal to the threshold value, which may differ substantially from the unconditional expected currency overvaluation. On the other hand, if crises are triggered by sunspots, uncorrelated with the economic variables that determine the exchange rate in a floating regime, then the expected magnitude of a devaluation conditional on its occurrence is similar to the unconditional expected currency overvaluation.

The procedure followed in this paper is first to use options data to identify the behaviour of the probability and expected magnitude of a devaluation (conditional on occurrence) and subsequently compare it to the predictions of a simple model of currency crises under different assumptions about the trigger. The Brazilian case is particularly suitable for study because there are relatively good data on exchange rate options between the Brazilian Real and the US Dollar.[5]

In the model that best matches the data, the peg will be abandoned when the cur-

[3] Obstfeld and Rogoff (2000) define the exchange rate disconnect puzzle as "the exceedingly weak relationship (except perhaps in the long run) between the exchange rate and virtually any macroeconomic aggregates".

[4] This definition is different from that of Flood and Garber (1984), where the shadow exchange rate is the floating rate that would prevail if agents were to purchase all the government's reserves at a given moment.

[5] European exchange rate options were regularly traded at São Paulo Futures Exchange (BM&F).

rency overvaluation hits a certain threshold, but agents don't know what the threshold is. Besides explaining the determinants of the probability and expected magnitude of a devaluation, the model presents an intuitive rationale for why moderate movements in the shadow exchange rate sometimes generate large increases in the probability of a devaluation and sometimes don't.

The probability and expected magnitude of a devaluation cannot be observed but may be identified using a statistical approach based on a no-arbitrage condition and option data. Options provide information about the probability density of the exchange rate at different points, so it is possible to disentangle the "thickness of the tail of the distribution" (probability of a devaluation) and the "distance from the tail to the center" (the expected magnitude of a devaluation).

To give a simple intuition for identification, suppose the price of an asset tomorrow will be 1 with probability $1-p$ and 3 with probability p. In a risk-neutral world, a call option with strike price 1 costs $2p$, a call option with strike price 2 costs p. If the probability of a devaluation (p) increases, both options get more expensive but the ratio of their prices remains equal to 2. If the magnitude of the devaluation increases from 3 to 4, the option with strike price 1 will cost $3p$, a call option with strike price 2 will cost $2p$ — the ratio changes.

The empirical results unveil completely different patterns for the probability and expected magnitude of a devaluation (conditional on its occurrence). The probability was volatile and mostly driven by contagion from external crises, as the Asian and Russian crises triggered by far the greatest increases in the probability that the peg would be abandoned. In contrast, the expected magnitude was stable and entirely unaffected by the Russian episode. A good theoretical model should predict these patterns for the probability and expected magnitude of a devaluation.

In addition, these data suggest that the Asian and Russian crises negatively impacted the Brazilian shadow exchange rate. They explicitly show that the crises coincided with both the greatest increases in the risk of a devaluation in Brazil and the largest depreciations of other Latin American currencies, like the Mexican Peso. Since the crises were fairly exogenous to the Latin American economies, it is natural to assume that if the Brazilian currency were allowed to float, it would also have depreciated. A good model should be consistent with such increases in the shadow exchange rate on those dates.

Financial data has previously been used to obtain estimates of the risk of a devaluation. In the literature, the usual procedure is to regress the estimates on macroeconomic fundamentals, which typically fails to find clear connections between those fundamentals and currency crises.[6] Such failure has sometimes been taken as support for models with multiple equilibria, but may simply reflect the fact that macroeconomic variables are poor explanatory variables for the exchange rate.[7]

The second stage of the paper compares the results of the empirical analysis to the predictions of the alternative models of currency crises. This necessitates a study of the behaviour of the probability and magnitude under each alternative set of assumptions. The simple framework developed in this paper permits the study of these alternatives in a single model, where both the prevailing exchange rate and the shadow rate follow stochastic processes. Implications for the behaviour of probability and expected magnitude of a devaluation crucially depend on what triggers currency crises.

The implications of the sunspot model are at odds with the empirical findings. If currency crises are triggered by sunspots, the probability of a devaluation depends little on the currency overvaluation, as it is mostly affected by sunspots, exogenous random variables. Therefore, the expected magnitude of a devaluation conditional on its occurrence is similar to the unconditional expected currency overvaluation. Thus the expected magnitude follows closely the current overvaluation. So, were sunspots the triggers of currency crises, the shocks to the Brazilian shadow exchange rate should have had important effects on the expected magnitude of a devaluation.

The predictions of the model when a devaluation is triggered by currency overvaluation crossing a constant and known threshold are consistent with some, but not all, empirical findings. If the currency devaluation is triggered when the currency overvaluation hits a constant and known threshold, the magnitude of a devaluation equals that threshold. The expected magnitude of a devaluation conditional on its occurrence is therefore different from the unconditional expected currency overvaluation. Movements in the shadow exchange rate only affect the probability of a devaluation as they move the economy closer to or farther from the threshold. This model is consistent with the different impacts of the shadow rate in probability and magnitude. However, it also yields excessively high values for the probability and implies no movement in the expected magnitude of a devaluation.

If instead the devaluation is triggered by currency overvaluation crossing an unknown

[6] Rose and Svensson (1994) state that "it is difficult to find economically meaningful relationships between realignment expectations and macroeconomic variables". Campa and Chang (1998) and Campa, Chang and Refalo (2002) report that "macroeconomic variables are largely unable to explain intertemporal movements in realignment risk". See also Campa and Chang (1996) and Malz (1996) for studies of the credibility of the ERM using data on options.

[7] A few contributions to the study of currency crises have attempted to connect theoretical models and empirical work. Blanco and Garber (1986) generate predictions on expectations about the recurrent devaluation of the Mexican Peso using a variation on the monetary model of Flood and Garber (1984). In one of the few explicit tests for sunspots, Jeanne (1997) performs a likelihood ratio test for the existence of multiple equilibria in the French Franc crisis. He finds some inexplicable shifts between different equilibria.

threshold, and agents are learning about it, then all the predictions are consistent with the empirical findings. The results are identical to the case of a known threshold, with two additions. First, the expected magnitude of a devaluation increases when the currency overvaluation goes above its previous maximum, as it indicates to the agents that the threshold is higher than they previously thought. Second, the probability of a devaluation does not get too high because of uncertainty about the threshold.

In the model, a shock to the exchange rate which puts it near to a point where the peg might be abandoned leads to a high probability of devaluation, implying markets are nervous. But if the peg is maintained, markets learn that the threshold for abandoning the peg is higher than the level just reached, thus leading to a relatively lower probability of devaluation at any given lower position. Markets become relatively more tranquil at such point, but the expected magnitude of a devaluation increases.

Last, the model provides a way to back out the shadow exchange rate from the option data. The environment of the model is sufficiently simple that an option can be priced and used to estimate the path of the currency overvaluation. The implied path is consistent with large but not massive increases in the shadow rate following the crises in Asia and Russia.

A brief comparison between the results of this analysis and the theoretical literature on currency crises is deferred to the concluding section. Before that, Section 2 contains the empirical identification of the probability and expected magnitude of a devaluation. Section 3 shows the implications for those variables from different models, and Section 4 presents estimates of the shadow exchange rate using the model with an unknown threshold.

2 The probability and magnitude of a devaluation

The Brazilian crawling peg was instituted in March 1995 as part of a plan intended to counter the persistent inflation experienced by the economy. Under the peg, the exchange rate could float inside a mini-band that was less than 1% wide. The mini-band was readjusted by about 0.6% each month, distributed over 5 to 7 smaller changes. The objective was to stabilise and sustain the exchange rate, but the sustainability was questionable given the large current account deficits that suggested the Brazilian *Real* was overvalued.

As Figure 1 shows, interest rates increased by 20 and 15 percentage points due to the Asian (end of 1997) and Russian crises (from August 1998) respectively. This is a testament to the impact these events had on the credibility of the peg — which was finally

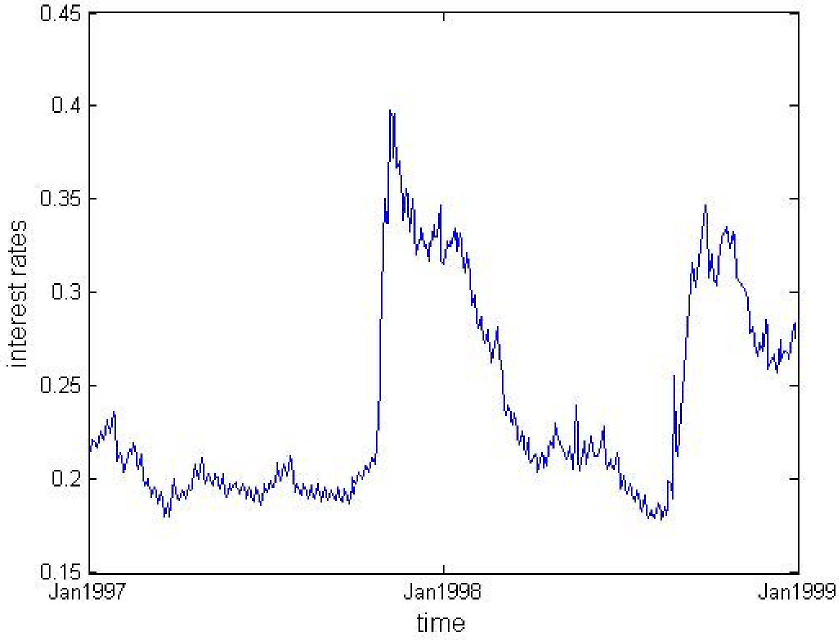

Figure 1: Interest Rates

abandoned in January 1999. The remainder of this section identifies the probability and expected magnitude of a devaluation of the *Real* from January 1997 to January 1999.

2.1 Empirical identification

The exchange rate risk in a pegged regime depends on the probability that the peg will be abandoned and on the expected size of a consequent currency devaluation. The forward premium is roughly the product of these two variables and may be estimated through some relatively simple calculations. However, observing the forward premium alone does not permit individual identification of the probability of a devaluation and its expected magnitude: a forward premium of 3% a year may refer to an expected devaluation of 30% with probability 10% a year, or an expected devaluation of 5% with probability 60% a year, and so on.

Options are a richer source of data because they provide information about the probability density of the exchange rate at different points, which allows identification of the probability and expected magnitude of a devaluation. Extracting information on the risks of discrete price jumps from data on options is not a novelty. It was the approach taken by Bates (1991) to estimate Merton's (1976) model when testing whether the stock market crash of 1987 had been expected.

However, it is unusual for the empirical literature to identify the probabilities and expected magnitudes of jumps. That is because, in general, the risk of discrete jumps co-exist with the regular disturbances. Subtle changes in options prices correspond to significant changes in the probability distribution of the future value of the asset, so it is difficult to obtain accurate estimates for both the probability and magnitude of a jump.[8] Identification in the case of the Brazilian pegged regime is relatively easier because the volatility of the exchange rate before the crisis was very small, so most of the information contained in option prices relates to the risk of a change in regime. Indeed, the vast majority of options in the sample was worth zero at maturity, which means they were all about the risk of a large devaluation.

Campa *et al* (2002) estimated the credibility of the Brazilian exchange regime using a non-parametric method. Provided the data were very accurate, their method would obtain the risk neutral densities without any further assumptions. However, in Brazil the lack of liquidity in the market for options leads to substantial noise in the option prices and a purely non-parametric approach cannot be applied. If their methodology were used to construct daily risk-neutral densities, 58% of the days in the sample of this paper would generate probability density functions with some negative values. A better alternative when faced with such noisy data is to use an adequate model to help identify the probability and expected magnitude of a devaluation.

2.2 The asset pricing model

In this paper, a simple asset pricing model is used to estimate the probability and expected magnitude of a devaluation. The model replicates the main features of the Brazilian crawling peg in a simple way.

Denote by S the exchange rate and s its logarithm. Initially, the exchange rate follows a standard Brownian motion with low volatility:[9]

$$ds = \mu_1 dt + \sigma_1 dX$$

In January 1999, the Brazilian exchange rate was allowed to float, and a discrete devaluation took place. Accordingly, in this simple model, the pegged regime may be

[8] For example, Bates (1991) estimates the probability and expected size of a jump in the US stock market before the crash of 1987 and finds significant risk of a negative adjustment in parts of his sample. The estimators of the probability and expected magnitude of the jump are very inaccurate but negatively correlated, so estimators of the *product* of them (roughly the price of the risk of a jump) are more precise.

[9] Such a formulation does not correspond to a mini-band regime, but it serves as a good approximation for the short run path of the Brazilian exchange rate under a maintained peg.

abandoned at any time. It is convenient to translate the probability of a regime switch into a hazard rate, and the simplest way to do it is to assume that the above process can be interrupted by a Poisson event with hazard rate λ that leads to a discrete jump in the exchange rate and to a new diffusion process, is assumed to last forever.

The simplest way to model the magnitude of the jump is to assume it is a constant (k):

$$\frac{S^{after}}{S^{before}} = (1+k)$$

The floating regime is described by a Brownian motion with drift and much higher volatility:

$$ds = \mu_2 dt + \sigma_2 dX$$

It is easy to extend the model to incorporate a log-normal jump, and the formula for the price of an option is similar. However, as the standard deviation of the jump plays a role similar to σ_2, it is not possible to get significant estimates for both with the available data for options on Brazilian Real.

The formulae and estimations in this paper consider risk-neutral agents.[10] A call option gives its owner the right to purchase one unit of foreign currency at strike price X. As explained in Appendix A.1, the price of a European call with maturity at time T is:

$$C^{mod} = e^{-\lambda T} BS\left(Se^{(-q-\lambda k)T}, T; X, r, \sigma_1^2\right) + \int_0^T \lambda e^{-\lambda t} BS\left(Se^{-qT-\lambda kt}(1+k), T; X, r, \frac{(\sigma_1^2 t + \sigma_2^2 (T-t))}{T}\right) dt \qquad (1)$$

where r is the domestic interest rate, q is the interest rate denominated in foreign currency, X is the strike price, S is the spot exchange rate and $BS(S, T; X, r, \sigma^2)$ denotes the Black and Scholes price of a call option. The first term of Equation 1 represents the value of the option if the peg is not abandoned at time T. The integrand of the second term is the option price given a devaluation at time t (multiplied by its probability density function).

[10] The formula would also be valid for risk averse agents if the risk of a jump were diversifiable and uncorrelated with the market as in Merton (1976). In this case, it would be possible to get an instantaneous zero-beta portfolio and the price of an option would not depend on any individual preferences. In particular, options would have the same value as in a risk-neutral world.

If the risk of a change in the exchange regime is systematic and cannot be diversified, there is no way to get a riskless portfolio and a price independent of agents' risk aversion. Then, using additional assumptions about individuals' preferences and the correlation between their wealth and the underlying assets, it is possible to get richer theoretical models as in Bates (1991, 1996). However, the empirical results would depend on those assumptions. If agents are risk-neutral, observable financial prices are sufficient for the estimations.

The parameter λ reflects the "thickness of the tail of the distribution" and k corresponds to the "distance between the tail and the center of the distribution". Intuitively, the estimated changes in the expected magnitude of a devaluation are due to changes in the ratio between the jump component of the prices of options with different strike prices. The estimated changes in probability reflect changes in the jump component of the prices of options without changes in their ratios.

To help illustrate identification, consider the following example. For some standard parameter values,[11] options with different λ's and k's, and strike prices equal to 1020 and 1100 are priced as shown in Table 1.

Table 1: Example for identification

λ	k	C(X=1020)	C(X=1100)	ratio
0.10	0.10	0.90	0.28	3.19
0.20	0.10	1.79	0.56	3.21
0.10	0.20	1.86	1.10	1.70

If $\lambda = 0.1$ and $k = 0.1$, an option with strike 1020 is worth 3.2 options with strike 1100. If $k = 0.1$ and $\lambda = 0.2$, both options get roughly twice as expensive because the probability of the option having a positive value at maturity double while the probability density of the asset conditional on the occurrence of a jump has not changed. Therefore, the ratio between the option prices remains almost unchanged. On the other hand, if $\lambda = 0.1$ and $k = 0.2$, the higher magnitude of a devaluation translates into a higher expected value of $S - X$ conditional on a $S > X$. Crucially, this increase is more pronounced in the case of the option with higher strike price (higher X) and the option-price ratio falls to 1.7. That is the key for identifying the probability and the expected size of a currency devaluation.

2.3 Data and estimation

The observed price of a call option (C^{obs}) is assumed to be equal to the model price (C^{mod}) plus an error term:

$$C^{obs} = C^{mod}(S, r, q, T; X, k, \lambda, \sigma_1, \sigma_2) + \epsilon \tag{2}$$

where ϵ is a mean-zero error term, independent of the observable variables. The parameters of Equation 2 were estimated by non-linear least squares.

[11] $S = 1000$, $T = 0.1$ year, $r = 0.2$/year, $q = 0.1$/year, $\sigma_1 = 0.01$/year and $\sigma_2 = 0.25$/year.

To estimate the parameters of Equation 2, the following data are required: domestic interest rates denominated in domestic and foreign currency; spot exchange rate; and option prices. Interest rate and exchange rate data are available from very liquid markets.[12] Unfortunately, the option market is much less liquid and, since there is no reliable record of the time each option was traded, the price of the last trade for every option must be used.[13] The available data refer to trades realised at potentially different times. Especially in periods when the markets were nervous, this may introduce large measurement error in the dependent variable, as discussed in Appendix B.[14]

The options are European calls, the underlying asset is the US Dollar and the contracts are to be paid in Brazilian *Real*. 75% of the options in the sample were traded less than 45 days before maturity, so the obtained estimates are measures of expectations about the peg in the short run. Options traded too close to maturity (less than 10 days) were discarded, as they contain little information about implicit distributions and their prices are not much greater than the bid-ask spread. In addition, transactions in at least four strike classes with the same maturity were required for each day. Finally, some questionable observations of a few far out-of-the-money option classes were excluded. In the end, there were 3,587 observations in the sample corresponding to 474 days and 25 months. Appendix B provides more details on the data.

λ and k are constants in the model but in the estimations they are allowed to vary over time. This is a potential source of inconsistency, however some Monte Carlo experiments presented in Appendix A.2 show that, for at least some diffusion processes of λ, such a procedure yields reasonable estimates. This is hardly surprising, as prices of European calls do not depend on the particular paths of the hazard rate and magnitude of jump but on the probability distribution of the exchange rate at the maturity date. Indeed, the estimation of different λ's and k's is the standard procedure in the literature (see, for example, Bates (1991, 1996) and Jondeau and Rockinger (2000)). In the empirical work, λ and k are either estimated for each of the 695 sets of options of a certain maturity traded in a given day or constrained to be constant during each of the 25 months.

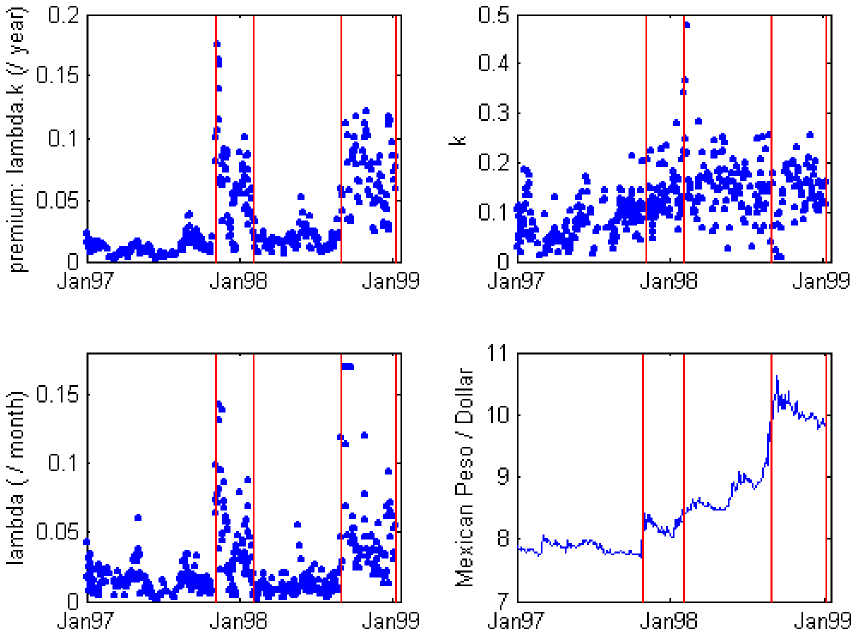

Figure 2: Daily Estimates

2.4 Results

Figure 2 shows the results when λ and k are allowed to vary across dates and maturity dates, assuming $\sigma_1 = 0.75\%$ per year and $\sigma_2 = 25.5\%$ per year.[15] Among the 695 $\{\lambda, k\}$ estimated, 442 pairs have a t-statistic higher than 2 for both estimates. Figure 2 shows just the results for those 442 'significant' days. The estimates of λ higher than 0.17 are plotted as if they were equal to 0.17 (five cases yield significant estimates of λ between 0.25 and 0.40).[16] The vertical lines mark the periods in which the 'devaluation premium' is high.

The top left graph shows the devaluation premium, λk. Unsurprisingly, it resembles Figure 1: the two major shocks in the series follow the Asian and the Russian crises.[17] The options allow us to disentangle and determine the relative importance of the two

[12] All the data are from contracts traded at São Paulo Futures Exchange (BM&F).

[13] In theory, options were traded at the exchange. In practice, options were traded over the counter and then registered at BM&F.

[14] It is possible to interpret the error term in Equation 2 as measurement error in the dependent variable.

[15] 25.5% is the standard deviation of the observed daily changes in the exchange rate from 1/19/99 to 12/31/99

[16] Nothing substantial changes in the Figures if the lower bound for t-statistics and the censorship limit is altered.

[17] Actually, the interest rate rise is greater than the increase in the devaluation premium due to the additional increase in the risk of default.

key components of the forward premium. A dramatic increase in the probability of a devaluation follows both crises, and is the predominant cause of the rise in the devaluation premium. The expected magnitude appears lower in 1997 than in 1998, but shows no sign of being affected by the foreign crises.

Figure 3 presents the estimates when λ and k are constrained to be constant within each month, whilst maintaining the assumption that $\sigma_1 = 0.75\%$ per year and $\sigma_2 = 25.5\%$ per year. It should be noted that, if there are substantial variations in the probability and expected magnitude during a month, it is not clear how mixing different option dates will impact the estimates. Nonetheless, it is a useful exercise to help understand the daily estimates.

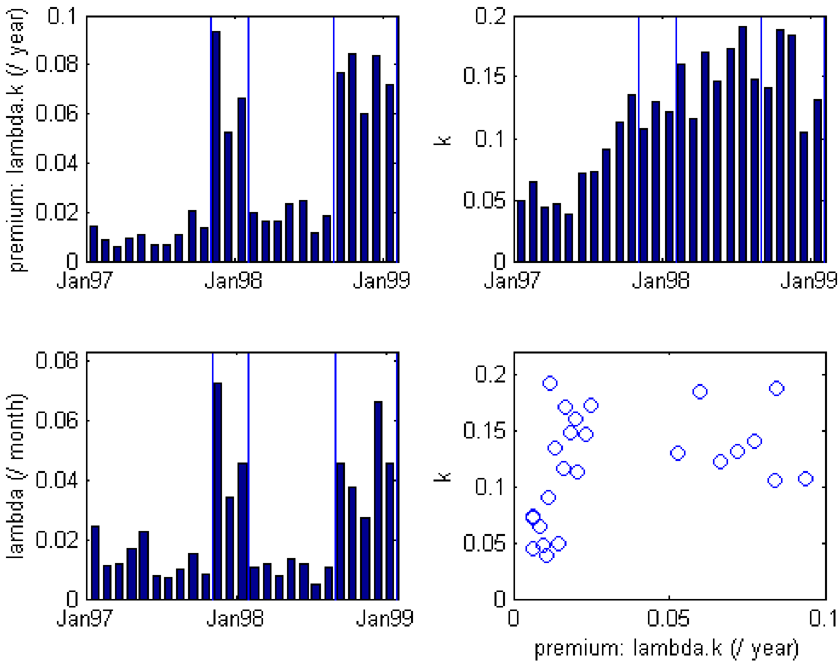

Figure 3: Monthly estimates

Figures 2 and 3 portray expectations about the Brazilian pegged regime from January 1997 to January 1999. At the end of October 1997, the Asian crisis strongly affected the credibility of the *Real*. The probability of a devaluation reached its peak in November 1997 but had returned to previous levels by February 1998. It remained low until August, when Russia defaulted on its debt, upon which it sharply rose and remained around 5% per month until January 1999, when the peg was removed. The parameter k increased at some point in 1997 and remained roughly stable around 15% after the Asian crisis until the end of the pegged regime. In 1998, virtually all changes in the devaluation premium were due to movements in the probability of a devaluation; the Russian crisis appears to have had no effect on the expected magnitude.

Both the Asian and Russian crises strongly affected the probability of a devaluation but had little or no effect on its expected magnitude. More generally, fluctuations in the devaluation premium are largely explained by movements in λ alone. The correlation between λ and λk is 92%, while the correlation between k and λk is only 37%.

Table 2 shows the value of estimates and standard errors in the monthly exercise. The lowest pseudo-t-statistic is 2.96 and the average pseudo-t-statistic is 7.6.

Interestingly, as shown in Figure 2, the greatest jumps in the Mexican exchange rate coincided with the largest movements in the probability of a devaluation in Brazil. Like Brazil, Mexico had large current account deficits by that time and few direct links with Russia, Korea or Hong Kong, but its currency was floating. It is reasonable to expect that the Brazilian "shadow exchange rate" and the Mexican floating rate would respond to the Asian and Russian crises in similar ways: had the Brazilian currency been floating, it would have depreciated.[18]

The monthly probability of a devaluation was almost always below 10% and remained around 5% from September 1998 until January 1999. Even as the regime break approached, the estimates of λ did not increase sharply.[19] Indeed, Brazilian interest rates were decreasing (from 2.93% per month in October 1998 to 2.38% per month in December 1998), the government entered into an arrangement with the IMF towards the end of 1998, and some macroeconomic reports were pointing to an increase in the credibility of the currency by December 1998.[20]

The results also show that agents underestimated the size of the jump: while the expected depreciation is never greater than 20%, the observed devaluation was as high as 60%. Some comments on the discrepancy between the expected and the observed devaluation are in Appendix C.

[18] Actually, the crises of 1997-8 negatively affected all the main Latin American synchronically floating currencies. The Chilean *Peso*, despite the good economic performance of Chile, was adversely hit by the Asian crisis. The Colombian *Peso* lost 10% of its value in the month following the Russian default (its average monthly devaluation over the period was 2%).

[19] There are estimates for λ until 01/08/99, 3 business days before the jump. Options get slightly more expensive 1 or 2 days before the devaluation.

[20] For example, the December 1998 economic analysis bulletin of IPEA (the Brazilian institute for research in applied economics) states that '(...) the pressure on the exchange rate got milder, and now a speculative attack is less likely to occur', IPEA (1998, in Portuguese), page 6.

Table 2: Monthly estimates of k and λ

	k		λ (year^{-1})	
Jan-1997	0.0491	(0.0042)	0.2929	(0.0166)
Feb-1997	0.0646	(0.0136)	0.1307	(0.0196)
Mar-1997	0.0442	(0.0060)	0.1378	(0.0099)
Apr-1997	0.0471	(0.0092)	0.2060	(0.0245)
May-1997	0.0389	(0.0055)	0.2702	(0.0229)
Jun-1997	0.0715	(0.0112)	0.0914	(0.0119)
Jul-1997	0.0735	(0.0132)	0.0876	(0.0135)
Ago-1997	0.0904	(0.0239)	0.1212	(0.0324)
Sep-1997	0.1135	(0.0122)	0.1812	(0.0233)
Oct-1997	0.1350	(0.0401)	0.0987	(0.0334)
Nov-1997	0.1076	(0.0131)	0.8690	(0.1252)
Dec-1997	0.1299	(0.0154)	0.4054	(0.0586)
Jan-1998	0.1216	(0.0121)	0.5472	(0.0667)
Feb-1998	0.1606	(0.0205)	0.1247	(0.0193)
Mar-1998	0.1154	(0.0112)	0.1411	(0.0148)
Apr-1998	0.1707	(0.0138)	0.0961	(0.0090)
May-1998	0.1464	(0.0254)	0.1589	(0.0316)
Jun-1998	0.1723	(0.0164)	0.1436	(0.0153)
Jul-1998	0.1913	(0.0280)	0.0612	(0.0097)
Ago-1998	0.1481	(0.0330)	0.1252	(0.0303)
Sep-1998	0.1411	(0.0217)	0.5461	(0.0935)
Oct-1998	0.1877	(0.0216)	0.4509	(0.0600)
Nov-1998	0.1845	(0.0274)	0.3251	(0.0554)
Dec-1998	0.1051	(0.0157)	0.7966	(0.1362)
Jan-1999	0.1319	(0.0149)	0.5431	(0.0733)

Standard deviations in parentheses.

3 Theoretical models

In order to analyse the behaviour of the probability and expected magnitude of a devaluation, this section presents a simple model of currency crises and analyses its implications under different assumptions for the currency crises trigger.

In Section 2, the probability and expected magnitude of a devaluation are exogenous variables to be estimated. Here, they will be obtained endogenously. The primitives of the model are the path of the exchange rate and the currency overvaluation, as well as the mechanism that triggers currency crises. This imposes restrictions on the behaviour of the probability and expected magnitude of a devaluation, which may be inconsistent with the results presented in Section 2.

The exchange rate process before the peg is abandoned is identical to that of the previous section:

$$ds = \mu_1 dt + \sigma_1 dX_1$$

Currency overvaluation in logs is denoted by θ and follows a similar stochastic process:[21]

$$d\theta = \mu_\theta dt + \sigma_\theta dX_\theta$$

All variables and parameters are observed by the agents.

Denoting by ϕ the shadow exchange rate, i.e. the exchange rate if the government decided to abandon the peg:

$$\phi = s + \theta$$

Thus, when the pegged regime is abandoned, the de facto magnitude of the devaluation will equal to θ — the exchange rate jump from s to ϕ. However, the expected magnitude of a devaluation *conditional on its occurrence* will depend on what triggers the currency crisis.

3.1 Sunspot model

The defining feature of currency crises models with multiple equilibria is that the occurrence of a devaluation does not depend strongly on economic fundamentals. The mechanism is the following: if everyone expects the peg to be kept and refrains from attacking the currency, the government is able to keep the peg, but if all agents expect

[21] Assuming some kind of slow mean reversion would not qualitatively change the results.

it to be abandoned and decide to attack, the government is forced to let the currency devalue. So expectations are self-fulfilling. The fate of the peg depends on what agents do, which in turn depends on sunspot variables, disconnected from the economy.

In Obstfeld (1996), in the multiple-equilibrium region, there is no link between the probability of a devaluation and economic fundamentals. In Jeanne (1997), fundamental variables do have an impact on the probability of a devaluation, but they are very small: a switch from the "good" to the "bad" equilibrium — which is assumed to be disconnected from economic variables — has substantially higher impact on the likelihood of a devaluation than *any* change in fundamentals, unless it takes the economy away from the multiple-equilibrium region.

That defining characteristic of a sunspot model can be translated to this framework in the following way: if $\theta > 0$, a currency devaluation occurs with an exogenous probability p, that may be time varying, and might obey any kind of stochastic process.

According to this model, by assumption, the probability of a devaluation should not depend on θ, as p depends mostly on sunspots, uncorrelated with θ.

As the probability of a devaluation is disconnected from θ, the expected magnitude of a devaluation conditional on its occurrence at time t is equal to the expected value of θ, conditional on $\theta > 0$. That is because conditioning on a currency devaluation yields no extra information about θ, so:

$$E(magn_t) = E(\theta_t|\theta > 0)$$

If θ is significantly larger than 0, then $E(\theta_t|\theta > 0)$ is approximately equal to $E(\theta_t)$. The expected magnitude of a devaluation is very close to the unconditional expected value of θ at time t, which is solely determined by the parameters of the stochastic process and the current θ.

In this model, a negative shock to the shadow exchange rate should have no important impact on the probability of a devaluation but should strongly affect its expected magnitude.

$$\frac{\partial Emagn}{\partial \theta} > 0$$

This prediction is at odds with the behaviour of the expected magnitude of a devaluation following the Asian and Russian crises. While the overvaluation of Brazilian currency is expected to have increased during those episodes, the expected magnitude hasn't been affected at all by the Russian default and hasn't changed much with the Asian crisis.

This model takes the sunspot idea to the extreme, by assuming that the probability of a devaluation does not depend at all on θ. But even if movements in θ have some impact on the probability of a devaluation, as long as the impact is not so strong, the expected magnitude should be significantly affected.

Some mean reversion in the path of θ changes the expected value but doesn't change the conclusion: shocks to θ should strongly affect the expected magnitude of a devaluation, conditional on its occurrence in 1 or 2 months.

3.2 Threshold model

Some models in the literature predict that the peg will be abandoned whenever some fundamental variable hits a threshold. In this framework, that would mean that the government will abandon the peg whenever the currency overvaluation hits a threshold, θ^*. For this subsection, it is assumed that the threshold is deterministic and known to all agents in the economy. This leads immediately to:

$$E(magn) = \theta^*$$

which is independent of θ.

The important difference here is that the expected magnitude of a devaluation conditional on its occurrence is substantially different from the unconditional expectation of θ. The devaluation occurs when θ crosses θ^* and conditioning the expected value of θ on that information makes the whole difference.

As θ moves up, closer to θ^*, the probability that θ will hit the threshold in the next few weeks increases, which is why we get:

$$\frac{\partial prob}{\partial \theta} > 0 \quad , \quad \frac{\partial Emagn}{\partial \theta} = 0$$

This model thus predicts that the increases in the Brazilian currency overvaluation following the Asian and Russian crises should affect the probability of a devaluation but not its expected magnitude.

Those implications match the movements of probability and expected magnitude in 1998 and do a reasonably good job in 1997: the Asian crisis produced strong impacts on the probability of a devaluation and had only weak effects on its magnitude.

The possibility of discrete jumps in the shadow exchange rate would weaken this result: the expected magnitude of a devaluation would then be somewhat affected by the possibility of jumps. But if the frequency of the jumps is small, the effect would not be very large. Thus the probability of a devaluation should be strongly affected by such jumps.

However, the model leaves unexplained the increases in the expected magnitude of a devaluation in 1997.

Moreover, quantitatively, the model does not do well. Given the characteristics of the Brownian motion,

$$\lim_{\theta \to \theta^*} prob = 1$$

that is, the probability of a devaluation gets arbitrarily large as θ approaches the threshold. This is much more than the 5% or 10% a month observed in the data.

3.3 Threshold model with uncertainty

The problems with the threshold model vanish if the threshold is uncertain, that is, if the government will abandon the peg when currency overvaluation crosses a threshold, θ^*, unknown to the agents.

Denote the maximum value that θ has achieved up to time t by $\theta^{\min}$. We know that $\theta^* > \theta^{\min}$, otherwise the peg would have been abandoned before. Agents have common uncertainty about θ^*, $g(\theta^*|\theta^{\min})$.

The probability of a devaluation before time τ is then:

$$prob = \int_{\theta^{\min}}^{\infty} g(\theta^*|\theta^{\min}).preach(\theta^*).d\theta^*$$

where $preach(\theta^*)$ is the probability that θ^* will be reached before time τ. As long as $\theta < \theta^{\min}$, it can also be written as:

$$\begin{aligned} prob &= \int_{\theta^{\min}}^{\infty} g(\theta^*|\theta^{\min}).preach(\theta^{\min}).preach(\theta^*|\theta^{\min}).d\theta^* \\ &= preach(\theta^{\min}).\int_{\theta^{\min}}^{\infty} g(\theta^*|\theta^{\min}).preach(\theta^*|\theta^{\min}).d\theta^* \end{aligned}$$

where $preach(\theta^{\min})$ is the probability that $\theta^{\min}$ will be reached before time τ and $preach(\theta^*|\theta^{\min})$ is the probability that θ^* will be reached before τ conditional on $\theta^{\min}$ being reached before τ. The second equality arises because $preach(\theta^{\min})$ is independent of θ^*.

Provided θ keeps below $\theta^{\min}$:

$$\left.\frac{\partial prob}{\partial \theta}\right|_{\theta < \theta^{\min}} = \frac{\partial preach(\theta^{\min})}{\partial \theta} \int_{\theta^{\min}}^{\infty} g(\theta^*|\theta^{\min}).preach(\theta^*|\theta^{\min}).d\theta^* > 0$$

As long as $\theta < \theta^{\min}$, increases in θ drive the economy closer to the region where the devaluation is possible, increasing the probability of a devaluation.

The expected magnitude of a devaluation conditional on its occurrence up to time τ

is:

$$\begin{aligned}
E(magn_t) &= \frac{\int_{\theta^{\min}}^{\infty} \theta^*.g(\theta^*|\theta^{\min}).preach(\theta^*).d\theta^*}{\int_{\theta^{\min}}^{\infty} g(\theta^*|\theta^{\min}).preach(\theta^*).d\theta^*} \\
&= \frac{\int_{\theta^{\min}}^{\infty} \theta^*.g(\theta^*|\theta^{\min}).preach(\theta^{\min}).preach(\theta^*|\theta^{\min}).d\theta^*}{\int_{\theta^{\min}}^{\infty} g(\theta^*|\theta^{\min}).preach(\theta^{\min}).preach(\theta^*|\theta^{\min}).d\theta^*} \\
&= \frac{\int_{\theta^{\min}}^{\infty} \theta^*.g(\theta^*|\theta^{\min}).preach(\theta^*|\theta^{\min}).d\theta^*}{\int_{\theta^{\min}}^{\infty} g(\theta^*|\theta^{\min}).preach(\theta^*|\theta^{\min}).d\theta^*}
\end{aligned}$$

which is independent of θ.

Provided θ remains below $\theta^{\min}$, movements in θ do not affect the expected magnitude of a devaluation:

$$\left.\frac{\partial Emagn}{\partial \theta}\right|_{\theta<\theta^{\min}} = 0$$

The intuition is that while $\theta < \theta^{\min}$, increases in θ provide no extra information about θ^*, causing therefore no impact on the expected magnitude of a devaluation. As in the case with a known threshold, the unconditional expected currency overvaluation is different from the expected magnitude of a devaluation conditional on its occurrence.

On the other hand, when θ is at $\theta^{\min}$, upward movements in θ increase $\theta^{\min}$ and the expected magnitude of a devaluation conditional on its occurrence is thus affected by movements in θ. It can be shown (and it is intuitive) that the expected magnitude of a devaluation is increasing in $\theta^{\min}$. When $\theta = \theta^{\min}$, the probability of a devaluation is at its highest since the last time θ reached $\theta^{\min}$ and the expected magnitude increases with any shock to θ.

In the model, agents do not have any information about θ^* besides $\theta^{\min}$ and its distribution, so they just update their beliefs about θ^* when θ reaches $\theta^{\min}$. One could think that, in reality, agents could access other sources of information about θ^*. However, it is actually difficult for the government to communicate its commitment to the peg because the incentive to assert the peg will only be abandoned in dramatic circunstances (abnormally high values of θ) is always present, as the devaluation premium depends negatively on the expected value of θ^*, which can be interpreted as a higher perceived commitment to the peg. A higher $\theta^{\min}$ leads to a lower forward premium because it lowers the probability of a devaluation and increases its expected magnitude, but the effect on the probability dominates.

Given the uncertainty about θ, the probability of a devaluation does not need to get as high as in the case with a known threshold. The next section examines the quantitative implications of the model.

4 Empirical estimation of the model

The model with the uncertain threshold is simple enough that it is possible to price an option in that environment. So, using the data on option prices, we can back out the path of θ. This exercises serves for two purposes: (i) to check whether the simple model can generate values for the probability and expected magnitude of the devaluation consistent with the data, under reasonable assumptions on parameters; and (ii) to examine the path of the shadow exchange rate implied by the model and the option data.

Some simplifying assumption ought to be made in order to facilitate the estimation process. I assume that dX_1 and dX_θ are independent, so:

$$d\phi = (\mu_\theta + \mu_1)dt + \sqrt{\sigma_\theta^2 + \sigma_1^2}dX_\phi$$

and the distribution of θ^*, $g(\theta^*|\theta^{\min})$, is exponential

$$g(\theta^*|\theta^{\min}) = \delta e^{-\delta(\theta^* - \theta^{\min})}$$

Given the exponential distribution, the probability of a devaluation depends only on $\theta - \theta^{\min}$. When $\theta = \theta^{\min}$, the probability is at its maximum, independent of $\theta^{\min}$.

Denote by θ_t and $\theta_t^{\min}$ the values of θ and $\theta^{\min}$ at date t.

In a risk-neutral world, the price of the option with strike price X and maturity at date τ is equal to:

$$c = \int_{\theta^{\min}}^{\infty} \delta e^{-\delta\left(\theta^* - \theta^{\min}\right)} c_2(\theta^*) d\theta^*$$

where $c_2(\theta^*)$ is the price of a call option conditional on a given value of θ^*.

$c_2(\theta^*)$ is given by:

$$c_2(\theta^*) = \int_t^{\tau} c_1(\theta^*, T)h(\theta^*, T)dT + \left(1 - \int_t^{\tau} h(\theta^*, T)dT\right) BS\left(S^{-q\tau - prob.E(magn)}, \tau; X, r, \sigma_1^2\right)$$

where $c_1(\theta^*, T)$ is the price of a call option conditional on θ^* being reached at time T, $h(\theta^*, T)$ is the probability density that θ^* will be reached at time T, BS is the Black-Scholes price of an option, $prob$ and $E(magn)$ are the probability and expected magnitude of a devaluation, given by the formulae presented at the last section.

Last, $c_1(\theta^*, T)$ is worth:

$$c_1(\theta^*, T) = e^{-r(\tau - t)} \int_X^{\infty} \left(e^{\theta_\tau} - X\right) f(\theta_\tau|\theta_T = \theta^*) d\theta_\tau$$

where $f(\theta_\tau|\theta_T = \theta^*)$ is the probability density of θ_τ conditional on $\theta_T = \theta^*$.

The densities f and h depend on the diffusion process of θ. Thus, the option price can be calculated as a function of θ_t, $\theta_t^{\min}$, the other parameters $(\mu_1, \sigma_1^2, \mu_\theta, \sigma_\theta^2, \delta)$ and observables (S, X, r, q, τ).

The parameters $(\mu_1, \sigma_1^2, \mu_\theta, \sigma_\theta^2, \delta)$ are calibrated. The values of μ_1 and σ_1 are taken from the crawling mini-band regime, in the period of January-1997 to January-1999. I set $\mu_\theta = 0$ and $\sigma_\theta = 10\%/\text{year}$. $\delta = 6$, which implies that the probability of a devaluation if $\theta \in [\theta^{\min}, 1.1 \times \theta^{\min}]$ is 45% and the probability of a devaluation if $\theta \in [\theta^{\min}, 1.2 \times \theta^{\min}]$ is 70%.

I estimate a value of θ_t for every day and $\theta_0^{\min}$. Then, $\theta_t^{\min} = \max\left\{\theta_{t-1}^{\min}, \theta_t\right\}$. The values of θ_t are estimated sequentially, for the sake of simplicity. The results are shown in Figure 4.

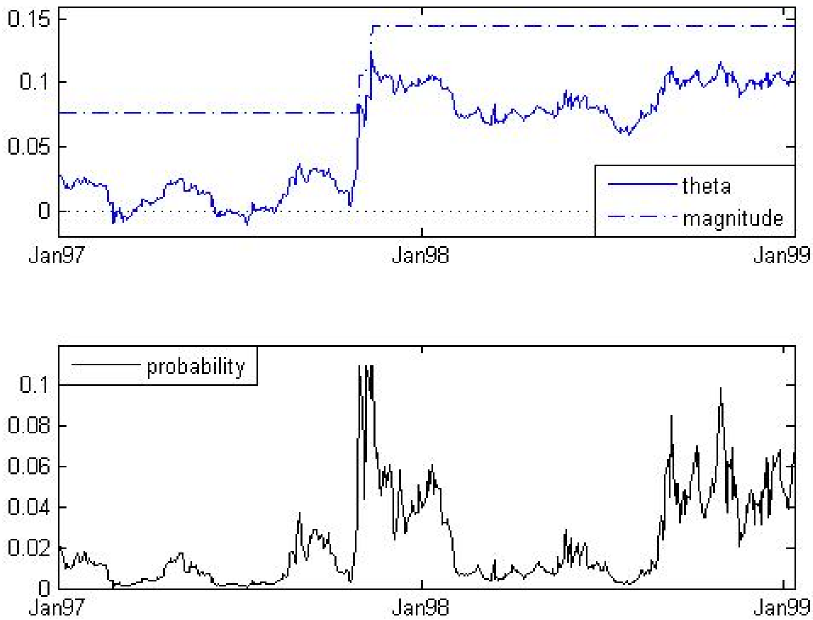

Figure 4: Path of θ, probability and expected magnitude

The top graph shows θ_t and the expected magnitude of a devaluation conditional on its occurrence in a month. The latter is equal to $\theta^{\min}$ plus a constant, that depends on δ, the parameters of the stochastic processes, and the time span. The value of θ_t at the end of 1998 is close to the peak reached at the end of 1997. The bottom graph shows the probability.

The path of the probability of a devaluation is very similar to that obtained in Section

2. The model thus generates sensible orders of magnitude for the probability of a devaluation. Also consistent with the results obtained in Section 2, the expected magnitude of a devaluation conditional on its occurrence increases following the Asian crises and stays constant from then on.

The results tell the following story: in 1997, fluctuations in θ have small or moderate impacts on the probability of a devaluation until the end of October 1997, when a large shock to the currency overvaluation occurs: θ increases by around 10%. Then, the current value of θ_t has surpassed $\theta_{t-1}^{\min}$ and reached the region where an immediate devaluation is possible. The probability of a change in regime is very high, agents are uncertain whether the government will let the currency float or not. However, despite the very high interest rates resulting from the high risk of a devaluation, the government keeps the peg. Agents learn, $\theta^{\min}$ increases, so the expected θ^* and hence the expected magnitude of a devaluation is higher than before the crisis.[22]

By the end of February 1998, the probability of a devaluation is back to low levels. The currency overvaluation, θ, has decreased a bit, and although much higher than before the crisis, it is far enough from $\theta^{\min}$ to yield a low probability of a devaluation in the short run.

The currency overvaluation still decreases more in 1998 until the first signs of trouble from Russia come. Then, it takes only a 5% increase in the shadow exchange rate to drive θ very close to $\theta^{\min}$ and trigger a massive increase in the probability of a devaluation. The expected magnitude of a devaluation does not change — as θ does not go beyond $\theta^{\min}$ this time — but any sharp movement in θ may throw it beyond $\theta^{\min}$ and, perhaps, trigger a devaluation.

5 Concluding Remarks

The Asian crisis in 1997 and the Russian crisis in 1998 have shaken financial markets across the world. This paper shows that their negative impact on the Brazilian economy was reflected in the probability of a devaluation, not in the expected magnitude. It offers the following explanation: by driving the shadow exchange rate to a region where agents were unsure about whether the peg would be kept, those shocks increased the odds of a devaluation. The defence of the *Real* by the Brazilian government following the Asian crisis convinced the agents that the threshold for abandoning the peg was higher, which increased the expected magnitude of a devaluation, conditional on its occurrence, but

[22] Similar learning effects after a “fire test” are present in other papers (e.g., Chari and Kehoe, 2003).

allowed for a decrease in the probability when things got slightly better. The subsequent negative shock in the second half of 1998 drove the exchange rate close to the threshold again, and once more the probability of a devaluation soared. According to the threshold model with uncertainty, a shift of the Brazilian shadow exchange rate of 10% with the Asian crises and 5% with the Russian crises can explain the massive increases in the probability of a devaluation.

It is important to connect the models in this paper with those presented in the literature.

The first generation models of currency crises (Krugman, (1979), Flood and Garber (1984)) predicted that a currency attack would occur when fundamentals crossed a fundamental threshold. Some recent dynamic models of currency crises based on that framework also yield fundamental thresholds for a currency devaluation (Guimaraes (2006), Broner (2007)), although others lead to different implications (e.g., Pastine (2002)).

Obstfeld and Rogoff (1995) defend the idea that the costs and benefits for the government to keep the peg are the key determinants for the fate of the exchange rate regime, as countries usually have enough reserves to sustain a currency peg. If the government is always able to keep the peg when it wants, but may decide to leave it if the overvaluation is 'high enough', there is a threshold — θ^* in this paper. If agents don't know how high is 'high enough', then this can be written as the model in this paper.

Obstfeld (1986, 1996), among many others, developed a multiple-equilibria explanation for the puzzling behaviour of markets with respect to currency crises. Assuming that the costs and benefits for the government to maintain the peg depend on what agents do, Obstfeld (1986, 1996) get a coordination game between the agents and, with complete information, multiple equilibria. In the particular case of Brazil, the multiple equilibria story does not seem to be playing a rule.

Morris and Shin (1998, 1999) added incomplete information to the coordination game and obtained a unique equilibrium in which a currency crisis occurs if economic fundamentals go beyond a threshold. Adding uncertainty about the government costs and benefits to Morris and Shin (1999) would lead to a model with an uncertain threshold, with implications similar to the model in this paper.

It has been argued that currency pegs seem to alternate between nervous periods (when they are subject to attacks) and tranquil periods (when they are not). Some recent theoretical papers propose explanations for such puzzling market behaviour exploiting the subtleties of the information flow (Angeletos *et al* (2007), Chamley (2003), Chari and Kehoe (2003)). In the simple model presented in this paper, moderate shifts in the shadow exchange rate may have small or very large impacts on the risk of a devaluation, depending on the distance between the shadow exchange rate (θ^*) and the point at which the peg may be abandoned ($\theta^{\min}$). That is a simpler explanation for the distinct market reactions to changes in economic fundamentals. The models based on information processing yield interesting insights, but their complexity by far exceeds what can be tested today.

References

[1] Angeletos, George-Marios; Hellwig, Christian and Pavan, Alessandro, 2007, "Dynamic Global Games of Regime Change: Learning, Multiplicity and Timing of Attacks", *Econometrica*, forthcoming.

[2] Bates, David S., 1991, "The crash of '87: Was it expected? The Evidence from Options Markets", *Journal of Finance* 46, 1009-1044.

[3] Bates, David S., 1996, "Jumps and stochastic volatility: Exchange rate processes implicit in Deutsche Mark options", *Review of Financial Studies* 9, 69-108.

[4] Blanco, Herminio and Garber, Peter, 1986, "Recurrent devaluation and speculative attacks on the Mexican Peso", *Journal of Political Economy* 94, 148-166.

[5] Broner, Fernando, 2007, "Discrete devaluations and multiple equilibria in a first generation model of currency crises", *Journal of Monetary Economics*, forthcoming.

[6] Campa, Jose M., Chang, P. H. Kevin, 1996, "Arbitrage-based tests of target zone credibility: evidence from ERM crossrate options", *American Economic Review* 86, 726–740.

[7] Campa, Jose M., Chang, P. H. Kevin, 1998, "ERM realignment risk and its economic determinants as reflected in cross-rate options", *Economic Journal* 108, 1046– 1066.

[8] Campa, José M.; Chang, P. H. Kevin and Refalo, James F., 2002, "An options-based analysis of emerging market exchange rate expectations: Brazil's Real plan, 1994 - 1997", *Journal of Development Economics* 69, 227-253.

[9] Chamley, Christophe, 2003, "Dynamic speculative attacks", *American Economic Review* 93, 603-621.

[10] Chari, V. V. and Kehoe, Patrick, 2003, "Hot money", *Journal of Political Economy* 111, 1262-1292.

[11] Flood, Robert and Garber, Peter, 1984, "Collapsing exchange-rate regimes: some linear examples", *Journal of International Economics* 17, 1-13.

[12] Guimaraes, Bernardo, 2006, "Dynamics of currency crises with asset market frictions", *Journal of International Economics* 68, 141-158.

[13] IPEA, 1998, Carta de Conjuntura 85, december/98.

[14] Jeanne, Olivier, 1997, "Are currency crises self-fulfilling? A test", *Journal of International Economics* 43, 263-286.

[15] Jondeau, Eric and Rockinger, Michael, 2000, "Reading the smile: the message conveyed by methods which infer risk neutral densities", *Journal of International Money and Finance* 19, 885-915.

[16] Krugman, Paul, 1979, "A model of balance-of-payments crises", *Journal of Money, Credit and Banking* 11, 311-325.

[17] Malz, Allan, 1996, "Using option prices to estimate realignment probabilities in the European Monetary System: the case of sterling-mark", *Journal of International Money and Finance* 15, 717-748.

[18] Merton, Robert C., 1976, "Option pricing when underlying stock returns are discontinuous", *Journal of Financial Economics* 3, 125-144.

[19] Morris, Stephen and Shin, Hyun S., 1998, "Unique equilibrium in a model of self-fulfilling currency attacks", *American Economic Review* 88, 587-597.

[20] Morris, Stephen and Shin, Hyun S., 1999, "A theory of the onset of currency attacks", in "Asian financial crisis: causes, contagion and consequences", edited by Agenor, Vines and Weber.

[21] Obstfeld, Maurice, 1986, "Rational and self-fulfilling balance of payment crises", *American Economic Review* 76, 72-81.

[22] Obstfeld, Maurice, 1996, "Models of currency crises with self-fulfilling features", *European Economic Review* 40, 1037-1047.

[23] Obstfeld, Maurice and Rogoff, Kenneth, 1995, "The mirage of fixed exchange rates", *Journal of Economic Perspectives* 9, n4, 73-96.

[24] Obstfeld, Maurice and Rogoff, Kenneth, 2000, "The Six Major Puzzles in International Macroeconomics: Is there a commom cause?", *NBER Macroeconomics Annual* 15.

[25] Pastine, Ivan, 2002, "Speculation and the decision to abandon a fixed exchange rate regime", *Journal of International Economics* 57, 197-229.

[26] Rangvid, Jesper, 2001, "Second generation models of currency crises", *Journal of Economic Surveys* 15, 613-646.

[27] Revista Época, January 18th, 1999.

[28] Rose, Andrew and Svensson, Lars, 1994, "European exchange rate credibility before the fall", *European Economic Review* 38, 1185-1216.

A The asset pricing model

A.1 Formula for the price of a call

This section provides an intuitive explanation of equation 1, that is:

$$C^{mod} = e^{-\lambda T} BS\left(Se^{(-q-\lambda k)T}, T; X, r, \sigma_1^2\right) +$$
$$\int_0^T \lambda e^{-\lambda t} BS\left(Se^{-qT-\lambda kt}(1+k), T; X, r, \frac{(\sigma_1^2 t + \sigma_2^2(T-t))}{T}\right) dt$$

where $BS\left(S, T; X, r, \sigma^2\right)$ denotes the Black-Scholes price of a European call option if the underlying asset follows a Brownian motion with a drift $\left(\frac{dS}{S} = \mu.dt + \sigma.dX\right)$, r is the interest rate, X is the strike price, S is the spot exchange rate and T is the time to maturity.

The price of an exchange rate option with the above characteristics is:

$$BS\left(S.e^{-qT}, T; X, r, \sigma^2\right) \tag{3}$$

where q is the interest rate denominated in foreign currency.

The first term of equation 1 is the value of the option if there is no devaluation until time T. This happens with probability $e^{-\lambda T}$. Conditional on that, the value of a call option is given by:

$$BS\left(Se^{(-q-\lambda k)T}, T; X, r, \sigma_1^2\right)$$

which is equation 3 with the spot exchange rate S multiplied by $e^{-\lambda kT}$. This term accounts for the devaluation premium — the instantaneous expected return on domestic currency equals its return conditional on no devaluation *minus* λk.

The probability density of a devaluation at time t is $\lambda e^{-\lambda t}$. Conditional on that, the value of a call option is:

$$BS\left(Se^{-qT-\lambda kt}(1+k), T; X, r, \frac{(\sigma_1^2 t + \sigma_2^2(T-t))}{T}\right)$$

The exchange rate in this case is distributed as if it followed a regular Brownian motion starting from $Se^{-qT-\lambda kt}(1+k)$ and with volatility $\frac{(\sigma_1^2 t+\sigma_2^2(T-t))}{T}$. The spot exchange rate needs to be corrected by the jump (multiplied by $(1+k)$) and by the devaluation premium up to time t (multiplied by $e^{-\lambda kt}$). The volatility is just a weighted average of the variances in the 2 regimes.

The second term of equation 1 integrates the products of prices and probability densities.

A.2 Theoretical option price if λ varies

Although the model assumes a fixed hazard rate λ, our estimations do not impose such constraint. So, how different would theoretical option prices be if λ was allowed to vary?

The answer may depend on the underlying process for λ. Monte Carlo simulations were used to approximate option prices for a particular case, when the hazard rate λ behaves according to the following equation:

$$d\log(\lambda) = \sigma_\lambda . dX$$

The table below shows the prices of options with 0.2 year to maturity for different σ_λ's but same expected λ after 0.1 year [23].

$E(\lambda \mid t=0.10)$	$\sigma_\lambda = 0$	$\sigma_\lambda = 0.5$
0.15	3.499 (0.008)	3.487 (0.010)
0.20	4.602 (0.009)	4.600 (0.008)
0.25	5.651 (0.009)	5.664 (0.009)

The lack of sensitivity to σ_λ is not due to little volatility. If $\sigma_\lambda = 0.5$ and $\lambda(t=0) = 0.1975$, $E(\lambda|t = 0.10) = 0.20$ but the 95% confidence interval for $\lambda(t = 0.20)$ is wide: $[0.127, 0.306]$ — λ varies significantly in the 0.2-year period.

The results show that, at least for this particular case, changes in the standard deviation of the diffusion process for λ have no impact on option prices. This example seems

[23] Some simplifications were made to reduce computations cost of this exercise, so all prices are probably slightly overestimated. The parameters used were: $\sigma_1 = .01, \sigma_2 = .10; k = .20, S = 1000, X = 1100, \tau = .20, r = .22, q = .11$.

to confirm our intuition that the estimates of λ obtained in this work should be close to what agents perceived as an average hazard rate.

B The Data

Table 3 shows the data for the last week in October-1997 and the first week in November-1997. All information refers to contracts with maturity on the last day of November. The data contains 695 rows like the 10 presented in table 3.

Table 3: A subset of the data

	X									
Day	1113	1115	1120	1150	1170	1200	F	S	τ	DI
10/27		2.25	2.20	1.40	1.50	1.30	1115.8	1102.7	32	97958
10/28		3.50	3.50	1.40	2.00	2.10	1116.9	1106.4	31	97841
10/29		3.00	4.50		2.00	2.20	1118.2	1102.4	30	97746
10/30	12.00	11.00	12.50	5.00	5.00	5.00	1125.8	1106.3	29	97473
10/31	11.00	7.00	11.00	4.00	4.50	3.30	1124.9	1103.1	28	97056
11/03		7.00	8.00	5.51	4.50	3.00	1121.6	1103.0	25	97123
11/04	5.50	5.50	6.40	3.51	3.49	2.00	1116.9	1104.1	24	97338
11/05	4.30	3.50	3.25	2.60	1.75	2.00	1118.1	1104.1	23	97402
11/06	8.00	7.00	6.00	4.30	2.70	3.00	1118.4	1106.9	22	97541
11/07	13.70	10.50	11.00	7.50	8.00	8.00	1123.5	1108.2	21	97392

The first column shows the trading day. Columns 2 to 7 show the prices of options with strike price shown in the first line of the table: for example, on 10/27, options that give its holder the right to buy US$1000 for BR$1115 were traded at price BR$2.25. F denotes the future exchange rate: on 10/27, US$1000 on the last day of November were priced at BR$1115.80. S is the spot exchange rate: on 10/27, US$1000 cost BR$1102.70. τ in this table is just the number of days until maturity and DI is an interest rate derivative contract: on 10/27, BR$100,000 on the first day of December were worth BR$97,958. The information on future contracts of interest rate and exchange rate allows us to calculate interest rates denominated in domestic and foreign currency.

The peg was not abandoned in November-1997, so on the maturity date of those options, the exchange rate was BR$1109 for US$1000 and all options shown at Table 3 were worth 0.

Option prices present huge daily variations, which suggests that large intra-day fluctuations may also occur. As the data refers to options traded in potentially different times, this may bring severe measurement error to the dependent variable of equation 2. In an extreme example, on 10/31/97, the price of a call with strike 1115 (Reais/US$1000) and maturity 12/01/97 is 7.00 and a call with strike 1120 and same maturity costs 11.00. The sum of the absolute measurement error is therefore greater than 4.00! There are plenty of examples like this, less dramatic though.

The price of a call option must be (weakly) convex as function of the strike price, otherwise there are arbitrage opportunities (see Campa *et al* (2002)). Violation of such properties are evidence of noise in the data on options, probably due to trades being realised at different times, and generate probability density functions with negative values if the methodology of Campa *et al* (2002)) is applied. In our sample, convexity is violated in 58% of the 695 'days' in our sample, and two thirds of those 695 days consist of only 4 or 5 points.

C Expected and observed jump size

On 02/01/99, the first maturity day of options after the devaluation, the exchange rate was at 1.983 R$/US$, 63.7% higher than 3 weeks before. According to our estimates, agents were expecting a substantially smaller jump.[24] Actually, this belief is confirmed by the exchange rate path right after the devaluation. Table 4 shows the spot exchange rates in January-99 — Future rates display the same pattern. On January 13^{th}, the end of the pegged regime was announced and the Central Bank tried to impose a new upper bound of fluctuation, at R$1.32/US$.[25] Two days later, the new-born band was abandoned and the exchange rate started to float. On the 15^{th}, even though Brazilian *Real* had lost this first battle, the US Dollar was still just 21% more expensive than before the jump. The spot rate would go up gradually and increase every single day until the end of the month.

The behaviour of the exchange rate in the very short run after the devaluation is interesting: there seems to be a clear and very strong upward trend for the price of the US Dollar, suggesting either that bad news for Brazilian economy was arriving every single day or that the market took a couple of weeks to update its more optimistic prior. A look at the newspapers of January-1999 favours the latter explanation.[26]

[24] Malz (1996) estimates the expected devaluation of the Sterling Pound in 1992, conditional on its occurrence, and finds that the expected jump was much smaller than the observed depreciation of 12.5%, which suggests that the market was also surprised by the extent of the Sterling devaluation.

[25] That would mean a devaluation of around 9%.

[26] For example, the magazine Epoca published on 01/18/99, when the devaluation was already above 20%, brought Finance Minister Pedro Malan arguing that Brazilian currency overvaluation was slightly *lower than 10%* — he cited studies from institutions such as Morgan, Lloyds, IMF and Goldman Sachs that confirmed his opinion. He dismissed the estimations of an overvaluation of "20%, 25%, 30% and *even* 40%" as based on some "simplistic calculations". On that day, 40% sounded

Table 4: Spot exchange rate in January-99

Day	Exchange Rate	Jump
1/11/99	1.211	
1/12/99	1.211	
1/13/99	1.319	8.9%
1/14/99	1.319	8.9%
1/15/99	1.466	21.0%
1/18/99	1.538	27.0%
1/19/99	1.558	28.6%
1/20/99	1.574	29.9%
1/21/99	1.660	37.0%
1/22/99	1.705	40.7%
1/25/99	1.761	45.3%
1/26/99	1.877	54.9%
1/27/99	1.889	55.9%
1/28/99	1.921	58.5%
1/29/99	1.983	63.7%

like a bad joke. Reality proved to be different.