# DSGE Models in a Data-Rich Environment[*]

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November 15, 2005

PRELIMINARY RESULTS

## Abstract

Standard practice for the estimation of dynamic stochastic general equilibrium (DSGE) models maintains the assumption that economic variables are properly measured by a single indicator, and that all relevant information for the estimation is summarized by a small number of data series. However, recent empirical research on factor models has shown that information contained in large data sets is relevant for the evolution of important macroeconomic series. This suggests that conventional model estimates and inference based on estimated DSGE models are likely to be distorted. In this paper, we propose an empirical framework for the estimation of DSGE models that exploits the relevant information from a data-rich environment. This framework provides an interpretation of all information contained in a large data set, and in particular of the latent factors, through the lenses of a DSGE model. The estimation involves Bayesian Markov-Chain Monte-Carlo (MCMC) methods extended so that the estimates can, in some cases, inherit the properties of classical maximum likelihood estimation. We apply this estimation approach to a state-of-the-art DSGE monetary model. Treating theoretical concepts of the model — such as output, inflation and employment — as partially observed, we show that the information from a large set of macroeconomic indicators is important for accurate estimation of the model. It also allows us to improve the forecasts of important economic variables.

*JEL Classification:* E52, E3, C32

*Keywords*: DSGE models; model estimation; measurement error; large data sets; factor models; forecasting; MCMC techniques; Bayesian estimation.

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# 1 Introduction

Recent macroeconomic research has devoted considerable efforts to the development and estimation of dynamic stochastic general equilibrium (DSGE) models that are internally consistent, and based on first principles.[1] Some recent micro-founded DSGE models, which involve numerous frictions and various types of shocks, appear to replicate the data in important dimensions (see, e.g., Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2003, 2004), Altig, Christiano, Eichenbaum, and Lindé (2003)). For instance, Smets and Wouters (2004) report that a DSGE model with a wide range of shocks fits the data well and performs well in terms of out-of-sample forecasts. Motivated by these promising results, such models are now increasingly perceived as a valuable input to policy making.[2]

In estimating these models, researchers have so far maintained the assumption that *all* relevant information for the estimation is adequately summarized by a relatively small number (i.e., between three and seven) of data series.[3] If the model is well specified and its key variables are directly observed both by the agents of the model and the econometrician, this approach can certainly be justified. In fact, in that case, the small set of model variables contains all the information relevant for estimation.

This is at odds, however, with the fact that central banks and financial market participants monitor and analyze literally hundreds of data series. Moreover, there is growing empirical evidence suggesting that a large set of macroeconomic variables may in fact be crucial to properly capture the economy's dynamics. In a macroeconomic forecasting context, Stock and Watson (1999, 2002) and Forni, Hallin, Lippi and Reichlin (2000) among others find that factors estimated from large

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[1] Examples are Christiano (1988), Altuğ (1989), Bencivenga (1992), Christiano and Eichenbaum (1992), McGrattan (1994), Leeper and Sims (1994), Hall (1996), Anderson, Hansen, McGrattan and Sargent (1996), McGrattan, Rogerson and Wright (1997), Ireland (1997, 2001, 2004), Rotemberg and Woodford (1997), DeJong, Ingram and Whiteman (2000), Schorfheide (2000), Kim (2000), Otrok (2001), Christiano, Eichenbaum and Evans (2005), Amato and Laubach (2003), Boivin and Giannoni (2003), Ruge-Murcia (2003), Edge, Laubach, and Williams (2003), Smets and Wouters (2003, 2004), Altig, Christiano, Eichenbaum and Lindé (2003), Rabanal and Rubio-Ramírez (2003), Giannoni and Woodford (2004), Fernández-Villaverde and Rubio-Ramírez (2004), Justiniano and Preston (2004), Adolfson, Laséen, Lindé and Villani (2005), Juillard, Karam, Laxton and Pesenti (2005), Levin, Onatski, Williams and Williams (2005).

[2] For instance, the Bank of Canada is "completing the development of a new projection model— a sticky-price dynamic stochastic general-equilibrium (DSGE) model of the Canadian economy" (see http://www.bankofcanada.ca/en/fellowship/highlights_res.htm).

[3] The only exception we are aware of is Adolfson et al. (2005) which estimates a relatively large model of a small open economy using fifteen data series. Leeper and Sims (1994) estimated their model with three and ten data series, but report that they were unable to obtain a reasonable fit with ten series.

data sets of macroeconomic variables lead to considerable improvements over small scale VAR models.[4] Bernanke and Boivin (2003) and Giannone, Reichlin and Sala (2004) show that this large information set appears to matter empirically to properly model monetary policy. Bernanke, Boivin and Eliasz (2005) argue that inference based on small-scale VARs, by omitting relevant information, may be importantly distorted. For instance, they provide evidence in favor of Sims' (1992) explanation of the "price puzzle," — i.e., the finding based on small-scaled VARs that prices tend to increase following a monetary policy tightening — according to which conventional VARs do not account for the information that the central bank has about future inflation. Bernanke, Boivin and Eliasz (2005) show that the information from a large set of indicators is indeed important to properly identify the monetary transmission mechanism. These empirical models with large data sets remain however largely non-structural. This limits our ability to determine the source of economic fluctuations, to perform counterfactual experiments, or to analyze optimal policy.

Why would such information be relevant in the context of available DSGE models? If the model of the economy is well specified and all theoretical concepts are directly observed by the agents and the econometrician, there is no scope for using additional data series in the estimation of DSGE models. However, exploiting the information included in these additional series can be important if, as we assume here, some of the key concepts of the model are imperfectly observed or if the data are informative about some exogenous shocks or other state variables. There are at least three reasons to justify such an assumption.

At a first level, measurement error is likely to affect many conventional measures of economic concepts.[5] For instance, while real GDP is the most comprehensive measure of economic activity available, its initial releases are sometimes subject to important revisions, and the "final" releases are not exempt from revisions or even subsequent changes in conceptual definition.[6] For employ-

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[4]Stock and Watson (1999), comparing a wide range of inflation forecasting exercises, found that their best-performing forecast involves a composite index of aggregate activity based on 168 individual activity measures. They argue that the forecasting gains from using this index are economically large and statistically significant over the 1970-1996 sample period. Similar evidence has been found in Forni et al. (2001), Stock and Watson (2002), Bernanke and Boivin (2003) and Boivin and Ng (2003), among others.

[5]Examples include the revision of NIPA accounting involving chained-type adjustments, various efforts made by statistical agencies in capturing quality adjustments in products, as well as work by the Boskin commission seeking to estimate biases in inflation measures.

[6]During the last recession and up until January 2003, GDP data showed negative GDP growth only in the third quarter of 2001. The January 2003 revisions, however, suggest that GDP growth was already negative in the third quarter of 2000, as well as in the first three quarters of 2001. Fixler and Grimm (2003) show that final quarterly

ment, the systematic discrepancies between its two main measures — one obtained from the establishment survey and the other from the population survey — which have received a lot of attention in the aftermath of the 2001 recession,[7] underscore the fact that employment is imperfectly measured.[8] Aggregate prices are also notoriously difficult to measure. One of the most commonly used measure, the Consumer Price Index (CPI), has undergone various changes in methodology since the 1996 Boskin commission, to mitigate important shortcomings. But recent research emphasizes that the current CPI might still be subject to important biases, stemming for instance, from the difficulty of measuring quality improvements or properly adjusting for outlet substitution.[9]

At a second level, there may be conceptual differences between the model variables and the data used to measure them. Whether the difference between the economic concept and the data series used to measure it reflects measurement error or conceptual differences, the concept relevant to macroeconomists might never be directly observed. It is convenient to think of measured indicators as the sum of two components: a component related to the true, latent, economic concept, and some indicator-specific component representing measurement error or conceptual differences with the concept. For model estimations, economists usually select a priori one data series to measure a particular economic variable, and assume implicitly that all other series have no additional informational content. However, once one acknowledges that the data provides only an imperfect indicator of the concept, it is plausible to think that other data series carry useful additional information. For instance, while data on hours worked based on the survey of establishments provides a natural indicator of labor input, data on hours worked form the household survey may provide another noisy indicator. Given each indicator-specific idiosyncrasy, properly exploiting the information from several indicators — rather than from a single one — should help to better separate hours worked (the common component) from the indicator-specific "measurement error." [10] Similarly for "inflation," which is generally closely associated with the growth rate of the GDP deflator or CPI,

estimates of real GDP tend to overstate declines in economic activity and understate the beginnings of recoveries.

[7]See, e.g., Bernanke (2004).

[8]The BLS actually reports standard errors for the employment measures based on both surveys in the Employment Report. The non-farm payroll employment number, being based on a larger sample, is statistically more precise. But it is also subject to biases, such as the double-counting of jobs.

[9]See Hausman (2003), Hausman and Leibtag (2004) and Bils (2004).

[10]In the same spirit, Prescott (1986) used these two indicators to calibrate the labor elasticity of output in his RBC model.

many indicators such as those based on the personal consumption expenditures (PCE) deflator, the core CPI or core-PCE deflator may provide important additional information.[11] Multiple, potentially informative, indicators also exist for output, consumption, investment, real wages. Viewed in this light, existing estimations of DSGE models appear to be based implicitly on an arbitrary choice of data.

A third reason why more information might be important is that some exogenous shocks or other state variables might be partially observed. One example is the productivity shock underlying many DSGE models. In existing estimations, it is treated as completely latent, which amounts to assuming implicitly that no observable measure contains independent information about this shock. But measures of labor productivity, oil prices, or commodity prices may all be correlated with total factor productivity, and thus serve as noisy indicators containing independent information.[12] In principle, since this could be the case for all exogenous shocks, many more indicators could carry important information for the estimation.

Failing to account empirically for the imprecise link between theoretical concepts and observable macroeconomic data can invalidate model estimation and the assessment of whether a particular theory fits the facts. Following Sargent (1989), this has led some researchers to recognize explicitly the presence of measurement error in their empirical framework.[13] However, even when they allow for measurement error, all existing studies that estimate structural models, to our knowledge, are based on at most a *single* (and sometimes arbitrary), observable time series corresponding to each variable of the model. That is, whether or not one considers measurement error in the model estimation, it is typically assumed that a small number of data series contain *all* available information about concepts of the model such as output and inflation. One critique of this treatment of measurement errors, based on a handful of observed data series, is that it does not impose enough structure on the measurement error process, thus giving the estimation too much freedom.

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[11] One could image macroeconomic models to be sufficiently detailed so as to specify a separate role for, e.g., each of the available price indices (such as the CPI, core-CPI, PCE deflator, GDP deflator, and so on). In practice, however, this disctinction is rarely made so that researchers pick a particular price index in a more or less arbitrary way.

[12] This is in part the rationale for the inclusion of commodity prices in VARs to "fix" the price puzzle (see Sims (1992)).

[13] See, e.g., Altuğ (1989), McGrattan (1994), Anderson, Hansen, McGrattan and Sargent (1996), McGrattan, Rogerson and Wright (1997), Schorfheide (2000), Rabanal and Rubio-Ramírez (2003) and Fernández-Villaverde and Rubio-Ramírez (2004). Another practical motivation for adding measurement error is to avoid the stochastic singularity problem that arises when there are fewer theoretical shocks than observable series.

In addition, proper estimation in such a context may force researchers to choose either to ignore measurement error in some of the variables or to constrain a priori the number of exogenous disturbances. Wrongly omitting measurement error, or structural shocks may however lead to distorted results.

In this paper, we propose a general empirical framework to estimate DSGE models that exploits the information from a potentially large panel of data series in a systematic fashion. We relax the common assumption that theoretical concepts are properly measured by a *single* data series, and instead treat them as unobserved common factors for which observed data series are merely imperfect *indicators*. We also include information from indicators that potentially have an unknown relationship with the state variables of the model. This framework provides an interpretation of all information contained in a large data set through the lenses of a DSGE model.

Our framework shares important similarities with the non-structural dynamic factor models of Stock and Watson (1999, 2002), Forni et al. (2000), Bernanke, Boivin and Eliasz (2005) and Giannone, Reichlin and Sala (2004). However a key difference is that we impose the full structure of the DSGE model on the transition equation of the latent factors. This has the added benefit of allowing us to provide a very clear economic interpretation of all estimated latent factors: in our setup, these factors correspond to state variables of the model (i.e., predetermined variables or shocks). In contrast, in empirical studies of factor models, the latent factors do not have a clear interpretation, since they are identified only "up to a rotation." Taken as a whole, the set of these factors spans the space of common components in the data, but each factor does not uniquely characterize a common component.

The estimation involves Markov-Chain Monte-Carlo (MCMC) techniques which deal effectively with the dimensionality problem by working with marginal densities and avoiding gradient methods. Because of the large dimension of models in a data-rich environment, direct estimation by maximum likelihood (ML) is usually infeasible in practice. The specific algorithm we use is based on the MCMCMLE framework of Jacquier, Johannes, and Polson (2004). This approach nests as a special case, the Bayesian MCMC implementation that has been widely used recently,[14] but inherits the properties of ML estimation in other cases.

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[14]See, e.g., Geweke (1999), Schorfheide (2000), Smets and Wouters (2003, 2004), Levin et al. (2005).

The proposed empirical framework has several advantages. First, as a consequence of the factor structure, the latent model concepts and the series-specific component (or "measurement errors") can be consistently identified from the cross-section of macroeconomic indicators, and not exclusively from the dynamic structure implied by the DSGE model. Consequently, unlike in the standard treatment described above, allowing for measurement error does not necessarily help the model fitting the data. It also implies that we can allow for a large number of imperfectly measured concepts without restricting in any way the number of structural shocks that can be identified within the model. Rather than taking a stance on whether "measurement errors" *or* structural shocks should be part of the model, we can remain agnostic and determine empirically their relative importance. A by-product is an empirical assessment of the information content of each indicator. Second, we can exploit the information from indicators that are not directly and unambiguously linked to a specific concept of the model. If the additional information considered is relevant, it should make our estimation more efficient. This is particularly important for forecasting exercises, and for determining more precisely the state of the economy. Third, by imposing the structure of a DSGE model on a dynamic factor models, we endow the estimated factors with a clear economic interpretation, which is typically absent in the macroeconomic applications of dynamic factor models. Fourth, our empirical structural model has predictions for all series included in the data set. It is thus possible to document to response of any variables to any *structural* shocks. This also provides a more stringent test on the "reasonableness" of the estimated model. Finally, the estimation strategy provides a natural way to document the sensitivity of the results to the priors.

We apply our estimation procedure to a state-of-the-art DSGE model based on microeconomic foundations. The model is taken from Smets and Wouters (2004), which builds on the model of Christiano, Eichenbaum and Evans (2005). One important finding is that we are able to considerably improve the forecasts of important economic indicators of inflation, consumption, output, and interest rates, by considering information from a larger data set in our model estimation, and by relaxing the link between some indicators and the model's concepts. Our results suggest that the additional information provided by the data-rich environment is highly relevant for the model estimation. Estimates of important model parameters such as the elasticity of intertemporal substitution or the degree of indexation to past inflation, as well as estimated variances of exogenous

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shocks differ importantly depending on the assumed link between theory and data. This arises even though the estimated latent variables display patterns generally consistent with the variables typically assumed to be observed. The different estimates also imply widely diverging conclusions about the sources of economic fluctuations.

The rest of the paper is structured as follows. Section 2 lays down the formal setup for an arbitrary linear(ized) DSGE model. It explains how we relate the structural model to the large data set, and discusses implications of the setup for a canonical real business cycle (RBC) model. The section then proceeds with a description of the general estimation methodology. Detailed information about the estimation is left in an appendix. Section 3, presents an application of our approach in the context of a state-of-the-art DSGE model, the model of Smets and Wouters (2004). It then provides the estimation results and provides results regarding the source of business cycles fluctuations. Section 4 concludes.

## 2 Data-Rich Environment

We now present a formal framework that merges a general class of dynamic general equilibrium models with a data-rich empirical model. We then discuss the implications of this framework, both in general terms and in the context of a canonical RBC model.

### 2.1 General Framework

Let us consider a general linear (or linearized) rational expectations model of the form

$$AE_t \left[ \begin{array}{c} z_{t+1} \\ Z_{t+1} \end{array} \right] = B \left[ \begin{array}{c} z_t \\ Z_t \end{array} \right] + Cs_t \tag{1}$$

$$s_t = Ms_{t-1} + \varepsilon_t \tag{2}$$

where $E_t[x] \equiv E[x|\mathcal{I}_t]$ denotes the expectation of some variable $x$ conditional on the information set $\mathcal{I}_t$ available at date $t$, $z_t$ is a vector of non-predetermined endogenous variables, $Z_t$ is a vector containing predetermined endogenous variables or lagged exogenous variables (i.e., satisfying $E_t Z_{t+1} = Z_{t+1}$), $s_t$ is a vector of exogenous variables following the process (2), $\varepsilon_t$ is a vector

of mean-zero unforecastable exogenous disturbances (such that $E_t \varepsilon_{t+j} = 0$ for all $j > 0$) with a diagonal variance-covariance matrix $Q$, and $A$, $B$, $C$ and $M$ are conformable matrices of coefficients. Below, we will consider examples of structural dynamic general equilibrium models based on microeconomic foundations that can be cast in the form (1)–(2). Models with additional lags, lagged expectations, or expectations of variables father in the future can be written as in (1) by expanding the vectors $z_t$ and $Z_t$ appropriately. We assume that the information set in period $t$ is $\mathcal{I}_t = \{z_\tau, Z_{\tau+1}, s_\tau, \varepsilon_\tau, \text{ for } \tau \leq t; A, B, C, Q\}$ so that all agents considered in the model are assumed to know the model, its parameters, and the realizations of all variables determined in the present and past.[15] We can solve the model using standard numerical techniques,[16] and express the solution as

$$z_t = DS_t \tag{3}$$

$$S_t = GS_{t-1} + H\varepsilon_t, \tag{4}$$

where

$$S_t \equiv \begin{bmatrix} Z_t \\ s_t \end{bmatrix}$$

is the state vector and the matrices $D, G, H$ are function of the underlying model's structural parameters.

In many applications, the system (1) contains identities and $Z_t$ includes redundant variables such as lags of variables in $z_t$. We will be interested in a subset $F_t$ of the variables in $z_t, S_t$ (all known at date $t$), which refers only to variables characterizing the economy in period $t$. The $(n_F \times 1)$ vector $F_t$ will typically include endogenous variables of interest for which indicators are observable.

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[15] This can be generalized in various ways at the expense of complications for the estimation problem described below. One can for instance assume that some or all of the agents in the model also face imperfect information about the state of the economy, and thus need to solve a filtering problem (see, e.g., Pearlman, Currie, and Levine, 1986, Svensson and Woodford, 2003, 2004) that may or may not be the same as the one of the econometrician described below. The model could still be written in the form (1), except that the vectors $z_t, Z_t$, and $s_t$ would include also estimates on the part of agents of the respective variables. We leave an analysis of imperfect information on the part of economic agents for future work.

[16] See, e.g., Blanchard and Kahn (1980), King and Watson (1998), Klein (1997), McCallum (1998), Sims (2000).

Specifically, we define

$$F_t \equiv F \begin{bmatrix} z_t \\ S_t \end{bmatrix}$$

where $F$ is a matrix that selects the appropriate elements of the vector $[z_t', S_t']'$. Given (3), we can rewrite the variables of interest as a linear combination of the state vector

$$F_t = \Phi S_t, \tag{5}$$

where

$$\Phi \equiv F \begin{bmatrix} D \\ I \end{bmatrix} \tag{6}$$

is entirely determined by the model parameters and the selection of variables in $F_t$. The evolution of $F_t$ is given by (4)–(6).

In order to estimate the model we consider $n_X$ observable macroeconomic variables collected in a vector $X_t$. We collect in a $n_{XF} \times 1$ subvector $X_{F,t} = \left[ x_{F,t}^1, ..., x_{F,t}^{n_{XF}} \right]'$ the indicators of the variables of interest $F_t = \left[ f_t^1, ..., f_t^{n_F} \right]'$, where $n_{XF} \geq n_F$, and assume that the observed indicators relate to the variables of the model according to

$$x_{F,t}^i = c^i + \lambda_F^i f_t^j + e_{F,t}^i \tag{7}$$

for $i = 1, ..n_{XF}$, $j = 1, ...n_F$, where for each $i$, $c^i$ is a constant, $\lambda_F^i$ is a coefficient, and $e_{F,t}^i$ denotes a mean-zero indicator-specific component, which may be viewed as representing measurement error or conceptual differences between the theoretical concept $f_t^j$ and the respective indicator $x_{F,t}^i$. We assume that these indicator-specific components are potentially serially correlated, but that they are uncorrelated across indicators. The set of equations (7) can be rewritten in matrix form as

$$X_{F,t} = \Lambda_F F_t + e_{F,t}, \tag{8}$$

where $e_{F,t}$ is a $n_{XF} \times 1$ vector of mean-zero indicator-specific and potentially serially correlated components, and $\Lambda_F$ is an $(n_{XF} \times n_F)$ matrix of coefficients, and we omit the vector of constants

to simplify the notation. As each element of $X_{F,t}$ is supposed to be an indicator of one of the elements of $F_t$, each row of the matrix $\Lambda_F$ will have at most one nonzero element. However, to the extent that each variable in $F_t$ can be imperfectly measured by many indicators, each column of $\Lambda_F$ can have many nonzero elements.

The observation equation (8) is appropriate in the case that several observable indicators relate directly to the same variable of interest, and that each of the indicator-specific components is uncorrelated with that of other indicators. For instance, if inflation based on the GDP deflator and the CPI correspond to the same concept of inflation in the model, then one may want to include both indicators in $X_{F,t}$. However, if these indicators refer actually to different concepts, then at least one of them should not be included in $X_{F,t}$. Such an indicator, even though it does not relate directly to any variable in $F_t$ should still depend on the evolution of the state vector $S_t$. In fact, to the extent that the theoretical model is true, a potentially very large number of indicators observed should depend on the state vector $S_t$. In that case, it is still possible to consider the information provided by such indicators in the model estimation. To do so, we assume that the remaining data series of $X_t$ which do not correspond to any particular variable of $F_t$ are collected in a $n_{XS} \times 1$ vector $X_{S,t}$ are related to the state vector according to

$$X_{S,t} = \Lambda_S S_t + e_{S,t}, \tag{9}$$

where $e_{S,t}$ is a $n_{XS} \times 1$ vector of mean-zero components that are not related to the model's state vector, and $\Lambda_S$ is an $(n_{XS} \times n_S)$ matrix of coefficients. Equation (9) allows all indicators not associated with any particular variable of the model to potentially provide information about the state vector $S_t$. We propose to capture the information from the data in $X_{S,t}$ in a non-structural way, letting the weights in $\Lambda_S$ be determined by the data.

While the weights $\Lambda_F$ relating the variables of interest to their indicators can be interpreted as structural — i.e., policy invariant — the weights $\Lambda_S$ and $\Gamma$ relating the state vector to all other indicators do not need to be so.[17] Even though (9) may not be reliable to determine the effects of alternative policies on the variables in $X_{S,t}$, information about these variables can be very useful

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[17]In fact the weights $\Lambda_S$ mix the weights that the variables in $X_{S,t}$ would attribute to their theoretical counterpart, with the coefficients that relate these theoretical concepts to the state vector $S_t$.

for the estimation of the state vector and model parameters under *historical* policy. Once the state vector and model parameters are correctly estimated — using the information provided by (9) — counterfactual exercises can legitimately be performed for all variables $F_t$, $S_t$, $X_{F,t}$, without using (9) any more.

Combining (8)–(9) and using (5), we obtain the *observation equation*

$$X_t = \Lambda S_t + e_t \tag{10}$$

where

$$X_t \equiv \begin{bmatrix} X_{F,t} \\ X_{S,t} \end{bmatrix}, \qquad e_t \equiv \begin{bmatrix} e_{F,t} \\ e_{S,t} \end{bmatrix}, \qquad \Lambda \equiv \begin{bmatrix} \Lambda_F \Phi \\ \Lambda_S \end{bmatrix}.$$

While the indicator-specific components $e_{F,t}$ are supposed to be uncorrelated across indicators, we allow the vector $e_{S,t}$ to be correlated across indicators, as we may want to include in the vector $X_{S,t}$ indicators that are driven by some common factors which are not included in the model's vector of state variables. This could happen for instance if several indicators included in $X_{S,t}$ are part of a same category of indicators — e.g., asset prices or commodity prices —, but that their theoretical counterpart is not fully fleshed out in the model. Again, it may be useful to consider such indicators in the estimation, as they may be informative about the state of the model economy, $S_t$. Specifically, we assume that the component of these indicators which is not correlated with the model's state vector has the following factor structure

$$e_{S,t} = \Gamma S_{e,t} + \tilde{e}_{S,t} \tag{11}$$

where $\tilde{e}_{S,t}$ is a $n_{XS} \times 1$ vector of mean-zero indicator-specific (i.e., uncorrelated across indicators) and potentially serially correlated components, and $S_{e,t}$ is a vector of common components in the set of indicators $X_{S,t}$, which are uncorrelated with the model's state vector $S_t$. We furthermore let the series-specific components be serially correlated, so that

$$e_{F,t} = \Psi_F e_{F,t-1} + v_{F,t} \tag{12}$$

$$\tilde{e}_{S,t} = \Psi_S \tilde{e}_{S,t-1} + v_{S,t} \tag{13}$$

where the vectors $v_{F,t}$ and $v_{S,t}$ are assumed to be normally distributed with mean zero and variance $R_F$ and $R_S$, respectively, and where the matrices $R_F, R_S$ and $\Psi_F, \Psi_S$ are assumed to be diagonal.

Our empirical model consists of the transition equation (4) fully determined by the underlying DSGE model, the selection equation (5), and the observation equation (10)-(13) which relates the model's theoretical concepts to the data. It contains as an important special case the measurement error framework proposed by Sargent (1989). In the latter framework, each variable in $F_t$ corresponds to a unique observable indicator in $X_{F,t}$, so that the observation equation reduces to $X_t = F_t + e_t = \Phi S_t + e_t$. In this case $n_{XS} = 0$, $\Lambda_F = I_{n_F}$, $\Lambda = \Phi$. A further trivial special case is one in which model variables are assumed to be directly measured, so that the observation equation reduces to $X_t = F_t = \Phi S_t$.

The key innovation here is to generalize Sargent (1989)'s framework to the case where the vector of observables, $X_t$, may be much larger than the vector $F_t$ of variables in the model, i.e. $n_X >> n_F$, and that their exact relationship, summarized by $\Lambda$, may be partially unknown. The interpretation is that this large number of macroeconomic variables are noisy indicators of model concepts and thus share some common sources of fluctuations. This implies an observation equation with a factor structure similar to the one assumed in the recent non-structural empirical literature which uses a large panel of macroeconomic indicators. However, an important difference with this literature is that, in the present framework, the evolution of the unobserved common components obeys the structure of a DSGE model.

The use of large information sets provides our framework with two important advantages over the existing implementation of DSGE model estimation. First, as the latent variables and the measurement can be identified from the cross-section of macroeconomic indicators, it allows one to identify a much richer pattern of "measurement errors," even in the presence of many structural shocks. This reduces the risk of biased estimation. Second, it has the potential to yield a more efficient estimation procedure. To illustrate these points, consider the following special case of the framework presented above. Suppose that, according to theory, a variable of interest, $f_t$, satisfies

$$f_t = \rho f_{t-1} + \eta_t, \tag{14}$$

where $|\rho| < 1$ and the exogenous disturbance $\eta_t$ is iid.[18] Suppose moreover that we observe an indicator $x_{1t}$ of $f_t$. In the case that $x_{1t}$ constitutes a perfect measure of $f_t$, i.e., that the observation equation (10) is trivially $x_{1t} = f_t$, the variable of interest $f_t$ is known, and the parameter $\rho$ can easily be estimated by OLS or maximum likelihood. Suppose instead that $x_{1t}$ is a noisy indicator of $f_t$ and that the observation equation takes the form

$$x_{1t} = f_t + e_{1t} \tag{15}$$

where $e_{1t}$ is iid.[19] In the case that $\rho \neq 0$, standard techniques such as proposed Sargent (1989) can be applied to estimate $\hat{f}_t$ and disentangle it from the "measurement error," using the Kalman filter. For this to work, however, we need the stochastic process of $f_t$ to be different from the one that drives the measurement error. In contrast, when $\rho = 0$, standard techniques cannot be applied to recover the variable of interest $f_t$, as $x_{1t} = \eta_t + e_{1t}$ is the sum of two variables with the same stochastic process.[20] However, if one or more additional indicators

$$x_{it} = f_t + e_{it} \tag{16}$$

for $i = 2, .., n_X$ are available, then it is possible to estimate $f_t$ even if it is serially uncorrelated. In fact, $f_t$ is a common factor that can be identified through the cross section, on the basis the observation equations (15)–(16), while the dynamic model (14) is used for identification of the shocks $\eta_t$.

More generally, when no more than one indicator is used for any concept of the model — i.e., when $n_X = n_F$, as in existing implementations — both the structural shocks and the unobserved variables have to be identified entirely from the restricted dynamics of the DSGE model, summarized by equations (4)–(5). In that case, having more structural shocks in the model limits the number of independent sources of measurement errors that can be contemplated and it is difficult to formally test whether the resulting model is properly identified or not. Typically, researchers avoid these

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[18] This is a special case of (4)–(5), where $f_t = F_t = S_t$, $\varepsilon_t = \eta_t$, $\Phi = 1$, $G = \rho$ and $H = 1$.

[19] This is a special case of (10) where $X_t = x_{1t}$, $\Lambda_F = 1$, $\Lambda = \Phi = 1$, and $e_t = e_{1t}$.

[20] The likelihood function in this case involves the sum of the variances of $\eta_t$ and $e_{1t}$, so that each variance cannot be identified separately.

problems by assuming either no measurement error or few structural shocks. But as argued in the introduction, measurement error or conceptual differences between the measured indicators and the theoretical variables might be quite prevalent, and if so, ignoring them would lead to biased inference.

In contrast, one key feature of factor models with multiple indicators is that the factors can be identified by the cross-section of macroeconomic indicators alone. This implies that in our framework with a factor structure, the large number $(n_X >> n_F)$ of indicators provides enough restrictions to identify the latent variables, and hence the "measurement errors," from the observation equation (10). As a result, we can allow for a large amount of measurement errors without restricting in any way the number of structural shocks that can be identified in the model. Rather than taking a stance on which source of variations should be part of the model, we can remain agnostic and determine empirically their importance.

Even when the factors can be identified solely from the model dynamics, as in Sargent (1989), considering the information from the large data set provides another important advantage, namely efficiency of the factor estimation. A key property of factor models is that the variances of the factor estimates are of order $1/n_X$ where $n_X$ is again the number of indicators in $X_t$. A consistent estimate of the factors can thus be obtained as $n_X \longrightarrow \infty$ (see Forni et al. (2000), and Stock and Watson (2002), Bai and Ng (2004).) This suggests that exploiting information from a large number of macroeconomic indicators can reduce considerably the uncertainty in the estimated latent variables, which in turn implies a more efficient estimation of model parameters. Estimation efficiency is then important, in particular for forecasting exercises and policy analysis, as forecasting performance is directly related to precision in model estimates.

## 2.2   An Illustrative Example: A Simple RBC Model

To clarify how the empirical framework just discussed can be applied to the estimation of a DSGE model, we first discuss a simple example, the canonical RBC model (see, e.g., King, Plosser and Rebelo (1988)). This model allows us also to relate to much of the literature on estimated DSGE models which has often considered variants of the basic RBC model. In section 3, we estimate a more elaborate model that adds numerous frictions to a RBC model of this kind. In the basic RBC

model considered here, households maximize their lifetime utility which depends on consumption, $c_t$, and leisure, $1 - l_t$,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(c_t\right) + v \log\left(1 - l_t\right)\right], \qquad 0 < \beta < 1, \; v > 0 \tag{17}$$

subject to the following restrictions

$$e^{a_t} k_t^{1-\alpha} l_t^{\alpha} = c_t + k_{t+1} - (1 - \delta) k_t, \qquad 0 < \alpha < 1, \; 0 < \delta < 1 \tag{18}$$

$$a_t = \rho a_{t-1} + v_t, \qquad 0 < \rho < 1 \tag{19}$$

where the exogenous productivity shock $a_t$ follows a mean-zero AR(1) process. Equation (18) indicates that output, which is generated using the capital stock $k_t$ (chosen at date $t - 1$), hours worked, $l_t$, and total factor productivity, $a_t$, is the sum of private consumption and gross investment. Solving this household problem yields a set of first-order necessary conditions which, together with (18) and a transversality condition, characterize the equilibrium evolution of the variables $c_t$, $l_t$, and $k_t$, for given exogenous disturbances and an initial value of the capital stock. As is well known, this model admits a unique deterministic steady state in which all endogenous variables remain constant. As a closed-form solution does generally not exist, the model is commonly log-linearized around the steady state. The model's approximate dynamics around the steady state can be written as

$$\begin{bmatrix} \hat{l}_t \\ \hat{c}_t \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ a_t \end{bmatrix} \tag{20}$$

$$\begin{bmatrix} \hat{k}_t \\ a_t \end{bmatrix} = \begin{bmatrix} g & h\rho \\ 0 & \rho \end{bmatrix} \begin{bmatrix} \hat{k}_{t-1} \\ a_{t-1} \end{bmatrix} + \begin{bmatrix} h \\ 1 \end{bmatrix} v_t \tag{21}$$

which is of the form (3)–(4) with $z_t = \left[\hat{l}, \hat{c}_t\right]'$, $S_t = \left[\hat{k}_t, a_t\right]'$, and the matrices $D, G, H$ are function only of the model parameters. Here, the circumflex denotes percent deviations from the steady state (e.g., $\hat{c}_t \equiv \log\left(c_t/\bar{c}\right)$).

To illustrate the richness of our empirical framework, we consider several variants of the *obser-*

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*vation* equation (10).

**No measurement error.** A common approach to the estimation of DSGE models is to suppose that we have perfect indicators of the variables of interest. In the case that an indicator $X_t = hours_{1t}$ (e.g., based on the establishment survey) is viewed as measuring perfectly the concept $\hat{l}_t$, we may write the selection equation (5) as $F_t = \hat{l}_t = [d_{11}, \ d_{12}] S_t$, so that the observation equation (10) reduces to

$$hours_{1t} = \hat{l}_t = d_{11}\hat{k}_t + d_{12}a_t. \tag{22}$$

In this case, estimation of the model (20)–(21) with the above observation equation would attribute all variations in the indicator $hours_{1t}$ to the only source of exogenous fluctuations, the productivity shock.[21]

**Standard treatment of measurement error.** In the case that $hours_{1t}$ is considered as a noisy indicator of hours, the observation needs to be augmented with a measurement error term,

$$hours_{1t} = \hat{l}_t + e_{1t} = d_{11}\hat{k}_t + d_{12}a_t + e_{1t},$$

and the standard approach proposed by Sargent (1989) is commonly applied.[22] According to this approach, the restrictions of the dynamic model and the Kalman filter are used to estimate the unobserved variables $\hat{l}_t, \hat{k}_t, a_t$ and the measurement error. However, as illustrated in the previous simple example, such an approach may have trouble disentangling the structural disturbances — the innovations to $a_t$ — from the measurement error, $e_t$, and thus may not be able to identify the latent variable of interest, $\hat{l}_t$. Unfortunately, it is difficult to test in practice whether or not the latent variables and the model parameters are actually identified.

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[21] Given that the RBC model considered here has only one source of exogenous fluctuations, using more than one observable series would result in the model rejection, in the absence of measurement error. In fact, as Ingram et al. (1994) point out, since the number of exogenous disturbances is smaller than the number of endogenous variables, one can find particular combinations of endogenous variables that are deterministic, so that their variance-covariance matrix is singular. The model is said to be stochastically singular in this case. As this is not true in the data, the model is sure to be rejected.

[22] See, e.g., Altuğ (1989), McGrattan (1994), Hall (1996), McGrattan, Rogerson and Wright (1997), and Ireland (1999).

**An alternative treatment of noisy indicators: Using multiple indicators of given concepts.** Once one recognizes that the data often contains *noisy indicators* of the concepts that one seeks to measure, there is scope for using additional indicators to get better estimates of the model's parameters and concepts. This can be done generally and systematically in our empirical framework. It suffices to include all relevant indicators in the vector $X_t$, and to let them be related to the respective concepts in $F_t$. For instance, while the establishment survey may provide a good indicator of the concept of hours worked ($hours_{1t}$), it is likely to include measurement error that is uncorrelated with measurement error in the hours as implied by the household survey ($hours_{2t}$). Accounting for the information contained in these two measured series may thus help us get a better estimate of the concept of hours worked and the model parameters.[23] The observation (10) then takes the form

$$
\begin{bmatrix} hours_{1t} \\ hours_{2t} \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda \end{bmatrix} \hat{l}_t + e_t = \begin{bmatrix} d_{11} & d_{12} \\ \lambda d_{11} & \lambda d_{12} \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ a_t \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}.
$$

All indicators of hours are thus assumed to have one common factor, $\hat{l}_t$, on which they "load" with a particular weight. We typically normalize one of the loading coefficients to 1 so as to normalize the scale of the fluctuations in the latent variable $\hat{l}_t$ to be of the same order of magnitude as the respective indicator, but leave the other loading coefficient free to be estimated, in case the fluctuations in the second indicator are of a different magnitude. By exploiting both the cross-sectional and the time series characteristics of the data, and noting that the latent variables are assumed to generate the common variation in both indicators, while the "measurement error" is specific to each series, we can more easily estimate the the latent variables here than in the standard treatment of measurement error. The observation equation mention here only exploits information about employment, but one could easily augment it with indicators of other variables such as consumption.

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[23] In the same spirit, Prescott (1986) used these two indicators of hours worked to calibrate the labor elasticity of output in his RBC model.

**Using information to estimate the state vector through an unknown link.** So far, we have assumed that $\Lambda_S$ is a zero matrix. We have thus implicitly assumed, as do current estimations of DSGE models, that the data series in $X_{S,t}$, which do not measure any specific variable of the vector $F_t$ — here, hours worked — do not contain any additional information about the remaining latent variables.[24] However, if the theoretical model is true, all economic data series should at least partly determined by the state vector. Data on stock prices, commodity prices, oil prices, monetary aggregates and so on could thus be informative about the current state of the economy, even though the model does not explicitly specify model such concepts. Exploiting their information content should result in a more efficient estimation. In our simple example, if oil prices ($Poil_t$) are systematically related to the state vector of the model economy, we can augment our observation equation as follows

$$
\begin{bmatrix} hours_{1t} \\ hours_{2t} \\ Poil_t \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ \lambda d_{11} & \lambda d_{12} \\ \lambda_{S1} & \lambda_{S2} \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ a_t \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix}
$$

where the new parameters $\lambda_{S1}$ and $\lambda_{S2}$ are to be estimated. It is important to note that by expanding the vector $X_t$ of indicators we are not facilitating the model's ability to fit the data. To the contrary, given the factor structure, the more indicators we have in $X_t$, the more we require the state variables (here $\hat{k}_t$ and $a_t$) to explain the common components in the data series, while at the same time satisfying their law of motion given by (21).

## 2.3 Estimation Procedure

We now discuss the general procedure for the estimation of the parameters and the latent variables (in $z_t, Z_t, s_t$) of the structural model (1). This model results in an equilibrium characterized by (3)–(5). We suppose that the observation equation takes the form (10)-(??), where we allow $X_t$ to potentially contain a rich set of macroeconomic indicators, and where $\Lambda$ involves possibly few *a priori* restrictions. Doing so obviously comes at a cost. The high-dimensionality of the problem,

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[24]Several studies, including Christiano (1988), Altuğ (1989) and McGrattan (1994), assume that the capital stock is observed, so that it would be in $F_t$. They however assume that other variables are latent. McGrattan (1994), for instance, using a more elaborate variant of the RBC model presented here assumes that output, investment, government purchases, hours of work, the capital stock and various tax rates are observed, while housing starts and past hours (weighted) are assumed to be latent.

and the presence of unobserved variables, considerably increase the computational burden of the estimation. In particular, methods that rely on explicitly maximizing the likelihood function or the posterior distribution appear impractical (see Bernanke, Boivin and Eliasz (2005)).

To circumvent this problem, we consider a variant of a Markov Chain Monte Carlo (MCMC) algorithm.[25] There are two key general features of these simulation-based techniques that help us in the present context. First, rather than working with the likelihood or posterior directly, these methods approximate the likelihood with empirical densities, thus avoiding gradient methods. Second, by exploiting the Clifford-Hammersley theorem, these methods sample iteratively from a complete set of conditional densities, rather than from the joint density of the parameters and the latent variables. This is particularly useful when the likelihood is not known in closed form, as it is the case in our application. Moreover, by judiciously choosing the break up of the joint likelihood or posterior distribution into the set of conditional densities, the algorithm deals effectively with the high dimensionality of the estimation problem.

The particular variant we consider is the MCMCMLE approach recently proposed by Jacquier, Johannes and Polson (2004). From a mechanical point of view, the distinguishing feature of this approach, compared to the Bayesian MCMC approach considered so far in the macro literature, is to rely on a data augmentation technique, that consists of sampling, at each iteration, $J$ independent copies of the unobservable state variables. When $J$ is equal to one, so that a single sequence of the unobservable state variable is drawn, the estimation corresponds to the standard Bayesian MCMC approach. But for $J$ greater than one, the importance of the likelihood is increased relative to the prior. Jacquier, Johannes and Polson (2004) show that the resulting chain of parameter draws converges to the finite-sample MLE estimate and has all the usual asymptotic properties of MLE. Incidentally, a particularly nice feature of this approach is that it allows us to compare the results for different values of $J$, and thus to assess the importance of the priors in our results.

Like in existing Bayesian implementations of the MCMC algorithm, the structural parameters of equation (1) are drawn using a Metropolis step, since their distribution conditional on the unobservable state variables and the parameters of equations (3)–(4) are not known in closed form.

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[25]See Johannes and Polson (2004) for a survey of these methods and Geweke (1999). Recent applications to the estimation of DSGE models include Schorfheide (2000), Smets and Wouters (2003, 2004), Rabanal and Rubio-Ramírez (2003), Fernández-Villaverde and Rubio-Ramírez (2004) and Justiano and Preston (2004).

The unobservable states are drawn using Carter and Kohn (1994) forward-backward algorithm. The remaining parameters are drawn directly from their known conditional distributions. The precise description of the algorithm is provided in Appendix A.

# 3    Application: Estimating a DSGE Model

## 3.1    Model

We now apply the data-rich environment just described to a state-of-the-art DSGE model based on microeconomic foundations. The model that we consider is taken from Smets and Wouters (2004). It builds on the canonical RBC model presented in the previous section, as well as Rotemberg and Woodford (1997), Christiano, Eichenbaum and Evans (2005) and others, by adding various frictions and allowing for ten different types of exogenous disturbances.[26] The canonical RBC model can be viewed as a special case of the Smets and Wouters (2004) model in the case of no frictions. The Smets and Wouters model has received much attention recently, in part because of its success in fitting actual data, both in the U.S. and in the Euro area (see Smets and Wouters, 2003, 2004). As Smets and Wouters (2004) report, this micro-based model performs also surprisingly well in terms of out-of-sample predictions, in some cases outperforming standard VAR and Bayesian VAR models.

A derivation of the non-linear model from first principles can be found in Smets and Wouters (2004). Here, we merely summarize the important aggregate and log-linearized equilibrium conditions of the model. The model involves optimizing households that consume goods and services, supply specialized labor on a monopolistically competitive labor market, rent capital services to firms, and decide how much capital to accumulate. Firms choose the desired level of labor and capital inputs, and supply differentiated products on a monopolistically competitive goods market. Prices and wages are re-optimized at random intervals as in the Calvo (1983) model. When they are

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[26]While Kydland and Prescott (1982) and Prescott (1986) emphasized the importance of technology shocks as a cause of business cycles fluctuations, many subsequent estimations of RBC models found evidence for important additional shocks. Altuğ (1989) finds that a single index can explain the variability in the series she considers, but the behavior of hours worked is not well explained when she identifies this index with a technology shock. Christiano and Eichenbaum (1992) find that allowing for government consumption shocks and productivity shocks impoves the fit of the model; Bencivenga (1992) emphasizes the role of preference shocks, McGrattan (1994) the role of fiscal disturbances, Ingram et al. (1994) focus on shocks to the depreciation rate of capital and to the level of production, and Ireland (1997), Kim (2000), Schorfheide (2000) add monetary shocks. See also DeJong et al. (2000).

not re-optimized, prices and wages are partially indexed to past inflation rates and to the central bank's inflation target.

More precisely, the model assumes that there exists a continuum of households who derive utility from consumption and leisure. The utility function is non separable in consumption and leisure as in King, Plosser and Rebelo (1988), to allow for a steady state growth path driven by labor-augmenting technological progress, and involves consumption in excess of an external, time-varying habit stock. While households may be heterogenous regarding their wage profile and hours worked, there exists a complete set of state-contingent securities which allows households to pool their risks, so that they all make the same consumption and investment decisions. The Euler equation for optimal consumption decisions log-linearized around the deterministic steady state with constant growth and zero inflation is given by

$$
\begin{aligned}
C_t \;=\; & \frac{h}{1+h}C_{t-1} + \frac{1}{1+h}E_tC_{t+1} + \frac{\sigma_c-1}{\sigma_c\left(1+\lambda_w\right)\left(1+h\right)}\left(L_t - E_tL_{t+1}\right) \\
& -\frac{(1-h)}{(1+h)\,\sigma_c}\left(i_t - E_t\pi_{t+1}\right) + \varepsilon_t^b
\end{aligned}
\tag{23}
$$

where $C_t$ and $L_t$ represent percent deviations of consumption and hours worked from their respective steady state, $i_t$ denotes deviations of the quarterly nominal interest rate from its steady-state level, and $\pi_t$ is quarterly inflation. The parameter $h \in (0,1)$ measures the degree of habit formation and $\sigma_c > 0$ indicates the curvature of the utility function with respect to consumption, and $(1 + \lambda_w)$ is the steady-state markup of the real wage due to market power on the labor market. In the absence of habit formation, (23) states that consumption depends negatively on the ex-ante real interest rate with a coefficient $\sigma_c^{-1}$ (corresponding to the elasticity of intertemporal substitution) and positively on expected future consumption. When $h > 0$, current consumption is also higher the higher past consumption. When $\sigma_c > 1$, hours worked and consumption are complementary. Finally the exogenous disturbance $\varepsilon_t^b$ is a preference shocks that multiplies the entire utility function and that is assumed to follow and AR(1) process with degree of serial correlation $\rho_b$.

On the labor market, households are assumed to re-optimize their wages given the demand for their labor services, with a probability $1 - \xi_w$. When choosing their optimal wage they take into account the probability that wages will not be re-optimized for some periods. Whenever they

cannot re-optimize their wages, they index them to a weighted average of lagged inflation and the central bank's inflation objective $\bar{\pi}_t$. The degree of indexation to lagged inflation is $\gamma_w \in (0,1)$. Optimal wage setting by households results in the following aggregate linearized equation for the real wage

$$
\begin{aligned}
w_t \;=\; & \frac{\beta}{1+\beta}E_t w_{t+1} + \frac{1}{1+\beta}w_{t-1} + \frac{\beta}{1+\beta}\left(E_t\pi_{t+1} - \bar{\pi}_t\right) - \frac{1+\beta\gamma_w}{1+\beta}\left(\pi_t - \bar{\pi}_t\right) + \frac{\gamma_w}{1+\beta}\left(\pi_{t-1} - \bar{\pi}_{t-1}\right) \\
& -\frac{\lambda_w\left(1-\beta\xi_w\right)\left(1-\xi_w\right)}{\left(1+\beta\right)\left(\lambda_w + \left(1+\lambda_w\right)\sigma_L\right)\xi_w}\left[w_t - \sigma_L L_t - \frac{\sigma_c}{1-h}\left(C_t - hC_{t-1}\right) + \varepsilon_t^L\right] + \eta_t^w
\end{aligned}
\tag{24}
$$

where $w_t$ is the percent deviation of the real wage from the steady state path, $\varepsilon_t^L$ is a shock to the disutility of labor, which follows an AR(1) process with degree of serial correlation $\rho_L$, and $\eta_t^w$ is an iid shock to the wage mark-up. The parameter $\beta \in (0,1)$ is the subjective discount factor, $\sigma_L^{-1} > 0$ is the elasticity of work effort with respect to the real wage. The term in square brackets corresponds to the gap between the actual real wage and the real wage that would prevail in the case of flexible prices and flexible wages. A positive gap tends to reduce the actual real wage, and the effect is stronger the smaller the degree of wage rigidity, $\xi_w$, the lower demand elasticity for specialized labor, $(1 + \lambda_w)/\lambda_w$, and the higher the elasticity of labor supply with respect to the real wage, $\sigma_L^{-1}$.

Households choose the capital stock which they rent to firms. To increase the supply of capital services, they can either invest in future capital, or increase the utilization rate of installed capital. Investment in capital takes one period to be installed and involves adjustment costs which assumed to be function of the change in investment, as in Christiano, Eichenbaum and Evans (2005). The relative efficiency of investment goods is also assumed to be affected by an exogenous shock $\varepsilon_t^I$ which follows an AR(1) process with degree of serial correlation $\rho_I$. The log-linearized Euler equation for optimal investment is given by

$$
I_t = \frac{1}{1+\beta}I_{t-1} + \frac{\beta}{1+\beta}E_t I_{t+1} + \frac{1/\varphi}{1+\beta}\left(Q_t + \varepsilon_t^I\right)
\tag{25}
$$

where $I_t$ denotes real investment and $Q$ is the real value of capital, in percent deviations from

steady state, and $\varphi$ is a measure of adjustment costs. The real value of capital follows in turn

$$Q_t = -\left(i_t - E_t \pi_{t+1}\right) + \frac{1-\tau}{1-\tau+\bar{r}^k} E_t Q_{t+1} + \frac{\bar{r}^k}{1-\tau+\bar{r}^k} E_t r_{t+1}^k + \eta_t^Q, \qquad (26)$$

so that the real value of capital relates negatively on the ex-ante real rate of interest, and positively on the expected future real value of capital and the expected future rental rate of capital $r_t^k$. The mean rental rate of capital $\bar{r}^k$ and the depreciation rate of capital, $\tau$, are assumed to satisfy $\beta = 1/\left(1-\tau+\bar{r}^k\right)$. The exogenous shock $\eta_t^Q$, assumed to be iid, is meant as a shortcut for changes in the external finance premium. The capital accumulation equation then involves both the flow of investment, and its relative efficiency

$$K_t = (1-\tau) K_{t-1} + \tau I_{t-1} + \tau \varepsilon_{t-1}^I. \qquad (27)$$

There is a continuum of firms that hire aggregates of labor and capital (adjusted for effective utilization) as inputs, combine them using a Cobb-Douglas production function with constant returns to scale, and a capital share $\alpha \in (0,1)$, and supply a differentiated intermediate good on a monopolistically competitive market. In producing their goods, all intermediate firms face a fix cost and a common stationary technology shock, $\varepsilon_t^a$, assumed to be AR(1) with degree of serial correlation $\rho_a$, and labor augmenting technological progress growing at a constant rate. Intermediate goods are then aggregated into a single final good used for consumption or investment. Minimizing the firms' cost of production results in the linearized demand for labor

$$L_t = -w_t + (1+\psi) r_t^k + K_{t-1}. \qquad (28)$$

This implies that for a given stock of capital, the labor demand depends negatively on the real wage and positively on the capital stock and the rental rate of capital, where $\psi > 0$ is the inverse of the elasticity of the capital utilization cost function.

Similarly to households on the labor market, firms are assumed to re-optimize their prices given the demand for their goods, with a probability $1 - \xi_p$. When they cannot re-optimize their prices, they index them to a weighted average of lagged inflation and the central bank's inflation objective

$\bar{\pi}_t$, with a degree of indexation to lagged inflation $\gamma_p \in (0,1)$. Optimal price setting by firms results in the following aggregate linearized equation for inflation

$$\begin{aligned}
\pi_t - \bar{\pi}_t &= \frac{\beta}{1 + \beta\gamma_p}\left(E_t\pi_{t+1} - \bar{\pi}_t\right) + \frac{\gamma_p}{1 + \beta\gamma_p}\left(\pi_{t-1} - \bar{\pi}_t\right) \\
&\quad + \frac{\left(1 - \beta\xi_p\right)\left(1 - \xi_p\right)}{\left(1 + \beta\gamma_p\right)\xi_p}\left[\alpha r_t^k + (1-\alpha)w_t - \varepsilon_t^a\right] + \eta_t^p.
\end{aligned} \tag{29}$$

As in the canonical New Keynesian supply equation, actual inflation depends on expected future inflation and on the marginal cost, here represented by the expression in brackets. The marginal cost depends in turn on the real rental rate of capital, the real wage, the productivity shock. Here it is the deviation of inflation from the central bank's inflation objective rather then the level of inflation that enters the equation, due to the assumed price indexing. To the extent that prices are indexed to lagged inflation, the lagged inflation also affects current inflation. The exogenous shock $\eta_t^p$ is assumed to be iid and refers to exogenous fluctuations in the price mark-up.

The linearized goods market equilibrium condition can then be written as

$$\begin{aligned}
Y_t &= \left(1 - \tau k_y - g_y\right)C_t + \varepsilon_t^G + \tau k_y I_t + \bar{r}^k k_y \psi r_t^k \tag{30} \\
&= \phi\left[\varepsilon_t^a + \alpha K_{t-1} + \alpha\psi r_t^k + (1-\alpha)L_t\right] \tag{31}
\end{aligned}$$

where $k_y$ is the steady-state capital-output ratio, $g_y$ is the steady-state government spending-output ratio, and $\phi$ is one plus the share of fixed cost in production, and $\psi$ is again the inverse of the elasticity of the capital utilization cost function. Government spending (in percent deviation from steady state, times $g_y$), $\varepsilon_t^G$, is assumed to evolve exogenously and to follow and AR(1) process with serial correlation $\rho_G$. While the first equation corresponds to the aggregate demand side for output, the second equation results from aggregate production.[27]

The model is closed with a specification of an empirical monetary policy reaction function.

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[27] In the first equation, we corrected the equilibrium condition indicated in Smets and Wouters (2004), adding the term $\bar{r}^k k_y \psi r_t^k$ as shown in Onatski and Williams (2004).

Here, we assume that monetary policy follows the generalized Taylor rule

$$
\begin{aligned}
i_t &= \bar{\pi}_t + (1 - \rho) \left[ r_{\pi 0} \left( \pi_t - \bar{\pi}_t \right) + r_{\pi 1} \left( \pi_{t-1} - \bar{\pi}_{t-1} \right) + r_{y0} Y_t + r_{y1} Y_{t-1} \right] \\
&\quad + \rho \left( i_{t-1} - \bar{\pi}_{t-1} \right) + \eta_t^i \tag{32}
\end{aligned}
$$

where $\eta_t^i$ is an iid monetary policy shock. The specification considered here differs slightly from the one in Smets and Wouters (2004) in terms of the response to output. While we suppose that the central bank responds to actual output fluctuations (in deviations from the steady-state trend), Smets and Wouters (2004) assume that the central bank responds to deviations of output from the output that would obtain in the case of flexible prices and flexible wages.[28]

The model is thus summarized by the ten equations (23)–(32). It involves ten endogenous variables $Y_t$, $C_t$, $I_t$, $L_t$, $K_t$, $Q_t$, $r_t^k$, $\pi_t$, $w_t$, $i_t$, and ten exogenous disturbances, six of them auto-correlated ($\varepsilon_t^a$, $\varepsilon_t^b$, $\varepsilon_t^G$, $\varepsilon_t^L$, $\varepsilon_t^I$, $\bar{\pi}_t$) and four iid ($\eta_t^Q, \eta_t^w, \eta_t^p, \eta_t^i$). The system can then be written as in (1), and can be solved using numerical techniques to obtain a solution of the form (3)–(4), where $z_t$ is a vector of endogenous non-predetermined variables, $Z_t$ contains predetermined endogenous variables as well as lagged exogenous variables, and $\varepsilon_t$ is the vector of innovations to the 10 shocks. In the estimation, we will use indicators of the following vector of variables of interest

$$
F_t = [i_t, Y_t, C_t, I_t, \pi_t, w_t, L_t]'.
$$

This vector is related to the state vector

$$
S_t = \left[ i_{t-1}, Y_{t-1}, C_{t-1}, I_{t-1}, \pi_{t-1}, w_{t-1}, K_{t-1}, \varepsilon_t^a, \varepsilon_t^b, \varepsilon_t^G, \varepsilon_t^L, \varepsilon_t^I, \bar{\pi}_t, \eta_t^Q, \eta_t^p, \eta_t^w, \eta_t^i, \varepsilon_{t-1}^I, \bar{\pi}_{t-1} \right]'
$$

through (5)-(6). This state vector follows a law of motion of the form (4).

---

[28]Their "output gap" may be considered more appropriate as it corresponds to the welfare relevant output gap, in the context of this model. It however differs substantially from empirical measures of "output gap" or the CBO's measure. In addition, their measure of output gap requires the specification of a significantly larger model, as the flexible-price, flexible-wage counterpart to the equations mentioned above need to be adjoined to the model, to determine the flexible-price, flexible-wage level of output.

## 3.2 Implementation of the Estimation

We now proceed with the model estimation. We consider several cases each involving different restrictions on the observation equation (10). The first case refers to the standard estimation with a small set of data series, assuming that there is no measurement error. This case effectively attempts to replicate the results of Smets and Wouters (2004). In other cases, we apply the proposed empirical framework using a larger data set, and consider various restrictions on the matrix $\Lambda$ and the vector of series-specific components $e_t$.

### 3.2.1 Prior distributions of the parameters

In all estimations, we assume the same prior distributions as in Smets and Wouters (2004). The prior distributions are summarized in Table 1 and are discussed in more details in Smets and Wouters (2004). As in Smets and Wouters (2004), five of the structural parameters are calibrated, as they are difficult to estimate from percent deviations from the steady state. The discount rate $\beta$ is set at 0.99, the quarterly depreciation rate $\tau$ is set at 0.025, the share of consumption $(1 - \tau k_y - g_y)$ and investment $(\tau k_y)$ are set at 0.65 and 0.17, which implicitly define $g_y$ and $k_y$. The capital-income share in the production function, $\alpha$ is set at 0.24. One difference with respect to Smets and Wouters (2004) involves the parameters of the policy rule which we assume takes the form of a generalized Taylor rule. The (long-run) response of the (annualized) federal funds rate to (annualized) inflation is assumed to be normally distributed with a mean of 1.5 and a variance of 0.5, and the response to detrended output is assumed to have a mean of 0.5 and variance of 0.2. The degree of inertia in monetary policy, or the response to the lagged interest rate is beta distributed with a mean of 0.75 a standard deviation of 1.

### 3.2.2 Data

Smets and Wouters (2004) estimate their model using quarterly U.S. data starting in 1957. For comparability with their results, we also use as long a sample as possible. However, because we use a much larger data set which includes series that start in 1964, our sample extends from 1964:1 to 2002:3. Our large data set contains 99 macroeconomic indicators broadly divided into

11 categories.[29] Details are provided in Appendix B. Seven data series, included in a vector $X_{F1,t}$ are however worth emphasizing as they are used to normalize the seven concepts of the model included in the vector $F_t$. They are the series used by Smets and Wouters (2004) for the estimation of their model. For each of these series, we normalize the corresponding weights in $\Lambda$ to 1. Output $(Y_t)$ is normalized to real GDP. Consumption $(C_t)$ and investment $(I_t)$ are normalized respectively to personal consumption expenditures and fixed private domestic investment.[30] The labor input $(L_t)$ corresponds to hours worked per person.[31] All preceding series are expressed in per capita terms by dividing with the population over 16. The real wage $(w_t)$ is normalized with the hourly compensation for the nonfarm business sector, divided by the GDP deflator. We express all these series in natural logs and remove a linear trend, so that they are expressed in percentage deviations from the trend, consistently with the model concepts. Inflation $(\pi_t)$ is measured as the quarterly percentage change in the GDP deflator. The nominal interest rate $(i_t)$ is the Federal funds rate. Both inflation and the interest rate are demeaned, to be consistent with the model's concepts. Smets and Wouters (2004) assume that in steady-state, the above series are all growing at the rate of labor-augmenting technological progress, and they estimate their model imposing the common trend. This assumption is unfortunately rejected by the data (see Del Negro, Schorfheide, Smets and Wouters, 2004). To circumvent this issue, we detrend all series before the model estimation, so that the model parameters are estimated on the basis of deviations from the steady state.

### 3.2.3 Alternative specifications of the observation equation: Five cases

To assess the importance of adding more information, we consider five different specifications of the empirical model, assuming different sets of restrictions on the observation equation, i.e., different assumptions on the link between the model and the data.

- Case A corresponds to the standard estimation with a small set of data series, assuming

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[29]This data set is an updated version of the data set used in Benanke, Boivin and Eliasz (2005) but discards the series that were discontinued as well as interest-rate spreads. It includes however the series used by Smets and Wouters (2004). The categories are: real output, compensation and wages, employment and hours, consumption, investment, stock prices, exchange rates, interest rates, money and credit, prices and some miscellaneous indicators.

[30]The nominal series for consumption and investment are deflated with the GDP deflator, as in Altig, Christiano, Eichenbaum and Linde (2003), and Smets and Wouters (2004).

[31]As in Smets and Wouters (2004), average hours of the nonfarm business sector are multiplied with the civilian employment to account for the limited coverage of the nonfarm business sector, compared to GDP.

that there is no measurement error. This case effectively attempts to replicate the results of Smets and Wouters (2004).[32] The seven key model variables included in $F_t$ are assumed to be perfectly observed, and only the associated time series mentioned above — included in $X_{F1,t}$ — are used in the estimation. In terms of our general notation, the *observation* equation (10) reduces to $X_{F1,t} = F_t = \Phi S_t$.

In all other cases, we maintain the assumption that the nominal interest rate is perfectly observed, and that the six other indicators collected in $X_{F1,t}$ are closely related to the concepts of the model, $F_t$, but that they may include a series-specific component (or measurement error) that is unrelated to the actual economic concept.

- Case B uses the same small set of data series as in case A, but allows for measurement error (except for the interest rate). The observation equation is thus $X_{F1,t} = F_t + e_{F1,t}$ $= \Phi S_t + e_{F1,t}$ where the first element of $e_{F1,t}$ is set equal to zero. The estimation corresponds to the procedure proposed in Sargent (1989), where the restrictions of the dynamic model are used to estimate the latent variables in $F_t$.

As discussed in Section 2, case B is likely to be affected by identification problems due to the difficulty to disentangle the structural disturbances $\varepsilon_t$ from the measurement errors $e_{F1,t}$, in the face of a large number of shocks and measurement errors. Such problems can be addressed by considering a larger data set which can more easily identify the latent variables of interest by separating the series-specific components from the common factors, $F_t$.

- In Case C, we add several other indicators of the imperfectly measured variables. Besides the seven primary indicators described above and included in $X_{F1,t}$, we add thirteen new indicators related to the variables in $F_t$, and which we collect in a vector $X_{F2,t}$. These additional indicators are selected on the grounds that they cannot be a priori rejected as indicators of the variables of interest.[33] The observation equation for this second set of indicators is of

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[32] As mentioned above, our estimation differs slighlty from the baseline case of Smets and Wouters (2004) for the following reasons: We consider a slightly different policy rule, and we detrend the data before estimating the model parameters instead of estimating a common trend (the growth rate of technology) with the rest of the model.

[33] For consumption, the new indicator is real personal consumption expenditures excluding food and energy, for investment we add real fixed investment (i.e., omitting inventory investment), for inflation we add the indicators

the form $X_{F2,t} = \Lambda_{F2}F_t + e_{F2,t}$, where each element of $e_{F2,t}$ is allowed to follow an AR(1) process. The matrix $\Lambda_{F2}$, which we estimate, is restricted to have as many nonzero elements per column as there are new indicators in $X_{F2,t}$ of the corresponding variable in $F_t$. It has however no more than one nonzero element per row as each indicator is assumed to load on only one variable.

As pointed out in section 2, the case C may be appropriate if the new indicators $X_{F2,t}$ selected relate directly to the variable of interest $F_t$. However if some of these indicators correspond to different concepts than the ones considered in the model, they may distort the estimation. To address this potential problem we consider the following case.

- In Case D, we use the same variables as in case Case C. This case however generalizes Case C by leaving the loading matrix for the thirteen new indicators as totally unrestricted. The idea underlying this case is that the additional indicators considered should be related to the state vector $S_t$, but are not necessarily linked to a single variable of interest contained in $F_t$. The observation equation for the new variables can now be written as $X_{S,t} = \Lambda_S S_t + e_{S,t}$, where each element of $e_{S,t}$ is assumed to follow and AR(1) process and the loading matrix $\Lambda_S$ is left unrestricted and is estimated. This setup provides a flexible way to exploit the information in the expanded data set about the state of the economy.

- Finally Case E exploits the information from our entire data set. While the seven primary indicators contained in $X_{F1,t}$ remain linked to the model's concepts as in Case B, all other indicators are assumed to be related to the state vector $S_t$ in a nonstructural way, and all of the weights of the corresponding matrix $\Lambda_S$ are estimated. Again the motivation for such a specification comes from the fact that if the theoretical model is true, all data series are should be related at least in part to the state vector. Accounting for the information that these series provide should thus result in a more efficient estimation.

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based on the deflator for personal consumption expenditures, the CPI, and the CPI less food and energy. For the real wage, we add indicators based on compensation of employees, personal income, personal income less transfer payments, average hourly earnings in manufacturing and in construction.

## 3.3  Empirical Results

We now describe briefly the empirical results for cases A to E. We focus on the discussion of the estimation results for the standard Bayesian estimation (i.e., $J = 1$) and also report the results for MCMCMLE estimation with $J = 5$. The fact that the results are similar in both cases suggests that the priors on the model parameters play little role for our main conclusions. [Results for the case E when $J = 5$ are incomplete and will be provided in a later version].

In case B, we assume the measurement error, $e_{F1,t}$, to be serially uncorrelated, as this restrictions is necessary to identify the model parameters. Whenever we let $e_{F1,t}$ to be serially correlated, we obtain estimated model parameters that are perfectly aligned with the prior distributions, suggesting that the data is uninformative, i.e., that the parameters are unidentified. As conjectured in Section 2, this highlights the fact that for models with a large number of structural shocks, and using a small set of observable variables, the extent to which measurement error can be allowed is severely limited. For comparison with this standard approach (case B), we assume that the measurement error in the primary indicators is also iid in cases C, D, and E (even though we can relax this assumption in these cases, and still be able to identify the model parameters). We however allow the measurement errors of the secondary indicators ($e_{F2,t}$ or $e_{S,t}$) to be serially correlated. By restricting measurement errors to be iid, we may understate the amount of measurement error in the primary indicators $X_{F1,t}$ as the deviations between these indicators and the estimated latent factor are by assumption idiosyncratic. This guarantees that the departures from the standard setups (cases A and B) are relatively small. Nonetheless, as we show below, even for such small departures, there are important benefits from exploiting information from a richer data set.

### 3.3.1  Series-specific components vs. variations explained by the model's state vector

Table 2 reports the fraction of the variance of the seven key time series explained by the model's vector of state variables, the remaining fraction being due to the series-specific component. In all cases, the variance of the Federal funds rate is fully explained by the model, as this rate is assumed throughout to provide a perfect measure of the short-term interest rate. Similarly, the fraction of the variance explained by the model is 1 by assumption, for all seven key variables in case A.

Once "measurement error" is allowed for, i.e., in cases B, C, D and E, we find some evidence of a series-specific component for all relevant series, but most notably for the GDP deflator, as a measure of inflation. In fact, we find that at least 17% of the variations in the quarterly percentage changes in GDP deflator are due to some idiosyncratic characteristics of the series (when $J = 1$). As a result, not accounting for measurement error in inflation or in the real wage, as in case A, may lead to distorted results. This may potentially bias *all* parameter estimates of the model. We find little evidence of measurement error for the other variables when we focus only on the seven primary time series (case B). However, when we consider additional indicators (cases C, D, E), we find that between 6% and 34% of fluctuations in the real wage series are due to the series-specific component.[34]

Figures 1 and 2 reports the time series of the six variables measured with error in cases B, C, D and E. The solid lines plot the data, while other lines plot the estimated corresponding latent variables on the respective cases. (The 10%-90% standard error bands are not plotted but would be so tight that they would be almost indistinguishable from the other series) Overall, these plots confirm the results found in Table 2. First, even in case E where we exploit the information from a large number of indicators in an unrestricted way, the estimated latent variables display behavior very similar to the variables of reference. As we expect from the results in Table 2, the estimated inflation measure shows the largest discrepancies with its variable of reference, the GDP deflator. As Figure 2 reveals, though, while most of the high frequency fluctuations in the GDP deflator are attributed to "measurement error," the lower frequency fluctuations in the estimated latent inflation are similar to those of the measured series.

Based on these results, it is important to stress that the relative performance of the various cases that we document below will thus be driven by arguably small and reasonable departures from the standard case.

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[34] Again, as mentioned above, the relatively small amount of "measurement error" found in series other than GDP inflation may be partly due to the assumption of iid measurement errors for these indicators.

### 3.3.2 Forecasting performance

So far, we have provided some evidence of "measurement error" and have argued that it is quantitatively important at least in the inflation series. To the extent that there is measurement error, it is important to consider it in the model estimation. Failing to account for it could distort all estimated model parameters. However, once one acknowledges the importance of noise in the data, one still has the choice between different specifications of the observation equation, such as in cases B, C, D, and E. As discussed above, estimates of these key variables are similar regardless of the case considered (see Figures 1 and 2). Nevertheless, these cases may perform differently in forecasting various economic variables. One way to evaluate these specifications is thus to compare their performance in forecasting the key variables of interest.

For each case, using the model estimated over the entire sample, we forecast each variable for various horizons (1 to 8 quarters ahead) for the 1990:1-2003:2 period.[35] Figure 4 reports the percentage gain in forecasting performance for the cases B, C, D and E, relative to the benchmark case A. This percentage gain is measured by the percentage reduction in the root mean squared error (RMSE) of forecasting relative to the RMSE of case A. For reference, we also report the forecasting performance of a standard VAR involving the seven key variables,[36] relative to the one of case A.

Figure 4 reveals several key points. First, the forecasting performance of the DSGE model in case A (i.e., without measurement error, as in Smets and Wouters, 2004), is comparable and in some cases better than the one of the VAR. This model provides better forecasts than the VAR for interest rates, inflation after three quarters, consumption at long horizons, and employment and investment at short horizons (this can be seen by the fact that the solid line lies below 0 in these cases.) This is consistent with the finding of Smets and Wouters (2004) according to which their model is able to outperform VAR forecasts in some cases.

Second, once we allow for measurement error (case B) and we use standard techniques as in Sargent (1989), the forecasting performance of the model is overall worse than in case A. In fact, allowing for measurement error in all series but the federal funds rate results in poorer forecasts

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[35] Because of the time involved in the estimation of each case, we did not re-estimate the model at each period, as is often done. The end of sample is in fact 2003:2 minus the forecasting horizon considered.

[36] The lag lenght of the VAR, selected by BIC, is one.

except for inflation. For output, consumption, hours worked, and the real wage, the RMSEs of forecasts are around 50% (or more) higher than they are in the case that no measurement error is assumed (case A). Even one to two quarters-ahead forecasts of the Federal funds rate yield RMSEs of about 40% higher than in case A.

Third, we find that exploiting the information from a larger data set can lead to important forecasting improvements. The best overall forecasting performance for horizons less than four quarters is achieved by case D which again allows for an idiosyncratic component in each series, but estimates the model based on a larger set of indicators (the seven primary indicators plus 13 new indicators) in an unconstrained way. This specification achieves a forecasting performance of between 25% and 45% higher than case A for real consumption. It also provides the best forecasts of the Federal funds rate at all horizons, and the best short-run forecasts of inflation, with a forecasting performance of about 35% better than in case A in the first couple of quarters. The fact that in case D, the short term forecasts of inflation (as measured by the GDP deflator) are so good is quite remarkable, given that fluctuations in the GDP deflator have been found to contain important measurement error. For real GDP, real investment, hours worked and the real wage, the forecasting performance of case D is about the same as in case A.

Case E, which contains many more data series than case D does not seem to perform better than case D in terms of forecasting, except for consumption. The lower-right panel of Figure 5 reports an overall measure of forecasting performance based on the log determinant of the matrix of RMSEs. It is clear from that figure that case D dominates all other cases reported including the VAR, at short horizons.

Finally the case C, which also allows for measurement error but provides a tighter link between the model variables and the 13 additional indicators, does generally not forecast as well as case D, except for output. The ability of case C to forecast the seven benchmark indicators is generally better than in case B, especially for real GDP, real consumption, hours worked, the real wage, and better over short horizons for the Federal funds rate. Nevertheless the forecasting performance of case C is poor for inflation.[37] The fact that Case D performs overall better than case C suggests

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[37] The fact that in case C, the forecasts of inflation as measured by the GDP deflator are worse than in other cases should however not be too surprising. In fact, the latent inflation concept corresponds to the common component to all indicators of inflation, and thus differs from the inflation based on the GDP deflator, given that CPI inflation and

that some of the restrictions imposed on the matrix $\Lambda_{F2}$ in case C may be too restrictive. As a result, our preferred specification with measurement error is case D.

Figure 6 reports very similar forecasting performances for cases A, B, C, D in the case of the MCMCMLE estimation with $J = 5$. As mentioned above, the fact that the results are very similar in both cases suggests that the priors on the model parameters play little role for our main conclusions.

Additional information could help forecasting either because it provides better estimates of the model parameters or because it provides more accurate estimates of the true state of the economy. To disentangle between these two possible explanations, we compute the forecasting performance of a VAR in which the 7 macroeconomic series used by Smets and Wouters (2004) are replaced with the estimates of the corresponding latent variables obtained in case D, i.e., exploiting information from the larger data set. One way to interpret this VAR is to view it as a factor-augmented VAR in the sense of Bernanke, Boivin and Eliasz (2005), where the factors have been estimated using the structure of the DSGE model. Figure 7 reports the forecasting performance of this VAR (denoted by D-VAR), along with the forecasting performance of the DSGE model in case D, and the standard VAR with series from Smets and Wouters (2004). As indicated in Figure 7, the D-VAR performs even better than the other forecasting models, achieving an forecasting performance of about 15% higher than case A at all horizons considered. The fact that the forecasting performance of the D-VAR is higher than the one of case D suggests that once the state of the economy has been properly estimated, the structure of the DSGE model does not help in improving the forecasting performance. One can thus infer from this that the improvement in forecasts comes primarily from the use of more information in the estimation of the model and the state of the economy. It is important to stress, however, that the model is necessary for the estimation and the identification of the state of the economy.

Overall, this forecasting exercise leads to some important implications. First, as in Smets and Wouters (2004), the results suggest that it is possible to obtain a good forecasting performance from a fully-specified structural model, compared to a VAR. Second it shows that even though

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other indicators behave somewhat differently. As a result, even if the model in case C is able to forecast accurately true inflation, there is no guarantee that it ought to forecast the inflation rate based on the GDP deflator, given that about 25% of the fluctuations in the GDP deflator are due to measurement error (see Table 2).

measurement error is estimated to be small for most variables it can have an important influence on the forecasting performance of the model. Third, simply allowing for measurement error as in Case B is not enough. There is a clear benefit from exploiting more information. Overall these results support our conjecture that there is a scope to exploit more information in the estimation of DSGE models.

An important remaining question is whether these alternative empirical models, and Case D in particular, lead to different conclusions about the structure of the economy and the source of business cycles than one obtains in the absence of measurement error.

### 3.3.3 Implications of model estimates

Table 1 reports the parameter estimates (the median of the posterior distribution) together with the estimated standard errors. Overall, the estimated structural parameters remain relatively unchanged in all cases considered.[38] There are however some notable exceptions. In case D, the curvature of the utility function (the inverse of the elasticity of intertemporal substitution in the case of no habit persistence), $\sigma_c$, is sensibly higher than in the other cases, suggesting that once more measurement error is accounted for, changes in the real interest rate have a smaller effect on consumption. Such a parameter is certainly crucial in particular to assess the effects of monetary policy changes on consumption. Moreover, the estimate of the inverse of the labor supply elasticity, $\sigma_L$, is smaller in case D than in case A, suggesting that once measurement is accounted for, a change in the real wage has a larger effect on labor supply. In addition, the estimate of the degree of price indexing to lagged inflation, $\gamma_p$ is significantly smaller in cases D and E than in other cases, including the case in which measurement error is not allowed.

The most important way in which case D (and case E) differs from the other cases is that the estimated variances of the exogenous shocks tend to be much smaller, in particular than in case A. This can be seen further in Figures 3 and 4 which plot the estimated time series for the state variables such as the capital stock and the exogenous shocks. Again, case D distinguishes itself from the other cases (especially from case A) by displaying substantially smaller fluctuations in

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[38]The estimated parameters for case A are also similar to those reported in Smets and Wouters (2004) except for some variances of shocks, which are due to differences in normalizations of the shocks.

exogenous shocks, in particular in the investment shock $\varepsilon_t^I$, the equity premium shock $\eta_t^Q$, the price markup shock $\eta_t^p$, and wage markup shock $\eta_t^w$. Such a finding is crucial to assess importance of particular shocks in driving business cycle fluctuations. Moreover, it has important implications concerning the number of structural shocks that are required in DSGE models. In particular, while Smets and Wouters (2004) argue that 10 shocks are required, our best performing forecasting model, case D, suggests that there might in fact be too many shocks in the model. For instance, while Table 3 reports that cost-push shocks to prices are estimated to be responsible for 28% of the variance in inflation and 22% of the variance in real wages in case A (which is consistent with the findings of Smets and Wouters, 2004), these shocks have insignificant effects in case D, when measurement error is accounted for. An intuitive explanation for these results is that these additional shocks might be needed in case A to capture the high frequency movements in inflation which are identified in case D as measurement error.

One notable exception, however, is the monetary policy shock, which appears to be more important in data-rich environments (cases D and E) than in cases A and B. While monetary policy shocks contribute little to the variance of the variables in case A (except for the nominal interest rate), they have a much bigger effect once we account for measurement error (case D). In the latter case, 17% of the variance in output and 19% of the variance in consumption is due to monetary policy shocks, eight quarters following the shock.

The state variables displayed in Figures 3 and 4 reveal broadly similar patterns in cases A, B, C, D and E. They however show some important quantitative differences that are worth noting. For example, while the estimated government expenditure shocks are roughly the same for cases that allow for measurement error, they differ substantially when no measurement error is allowed for (case A).

Finally, Table 1 shows that most standard error of parameter estimates are much smaller for the cases D and E than in other cases. As expected, using information from a larger data set in an unconstrained way allows us to increase the efficiency of the estimation.

# 4 Conclusion

Recent DSGE models have achieved important successes in terms of their ability to fit the data and to forecast. As a result, such models are given more attention for policy analysis. Despite the sophistication of these models, existing empirical applications have maintained one important assumption: that a small number of data series is sufficient to estimate the model. This is however at odds both with the fact that market participants and central banks monitor a large number of data series to assess the state of the economy, and with the growing evidence from empirical factor models according to which a large set of macroeconomic variables may in fact be needed to characterize the evolution of the economy.

In this paper, we have proposed a general framework that exploits the information from a data-rich environment for the estimation of a general class of DSGE models. In this framework, measurement error or the fact that many indicators contain a series-specific component that is unrelated to the economic concept that they are supposed to measure provides a scope for using additional indicators in the empirical model. We apply this estimation strategy to a state-of-the-art DGSE model that has been recognized for its empirical success.

Our results suggest that important forecasting improvements can be achieved by exploiting more information in the estimation of the model. Estimating a DSGE model in a data-rich environment is useful for the identification and estimation of the state of the economy. A proper estimation of the state of the economy appears in turn to be useful to improve forecasts of important macroeconomic variables. Moreover, we show that the inference drawn from the estimated model depends crucially on whether additional information is exploited or not. In particular, by comparing the standard implementation of the estimation with few observable variables with our best performing specification (case D), we reach very different conclusions about the sources of business cycles fluctuations.

The results in this paper open the way to many interesting avenues for future research, which we are pursuing. First, while the results reported provide an important scope for using more information in the estimation of DSGE models, more work needs to be done to determine how to optimally choose the variables to include in the empirical model. We have proposed one specification

(case D) that is successful in terms of forecasting. But other specifications within the general framework proposed here may perform even better.

Second, a real-time implementation of the proposed empirical framework should be of interest to central bankers, to the extent that it would allow them to process a large amount of information in real-time, in a systematic fashion, through the lenses of fully-specified structural model. This would also allow to forecast a large number of economic indicators, conditional on a variety of scenarii about any given structural disturbances of the model. A particularly attractive feature of this framework, which we believe is crucial for policy considerations, is that it facilitates the interpretation of observed economic developments through the use of a structural model.

Finally, many researchers have recently given attention to the development of optimal policy rules or optimal target criteria for the conduct of monetary policy, in the context of DSGE models. Such optimal rules often involve forecasts of important macroeconomic variables over the next few quarters.[39] Improved forecasts obtained through better estimates of the state of the economy, using a rich data set, should thus be a key ingredient for such optimal target criteria. This may help making the tools available for the conduct of optimal monetary policy more attractive to policymakers by incorporating their concern for the developments in a large number of data series.

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[39]Giannoni and Woodford (2003, 2004), for instance, characterize such optimal policy rules and target criteria for simpler models than the one presented here. They show that the most important forecasts needed as inputs for the implementation of monetary policy are over short horizons.

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# A    Appendix: MCMCMLE Algorithm (Preliminary)

This appendix describes our implementation of the MCMCMLE algorithm of Jacquier, Johannes and Polson (2004).

The general class of rational expectation models we consider can be represented as follows:

$$AE_t \begin{bmatrix} z_{t+1} \\ Z_{t+1} \end{bmatrix} = B \begin{bmatrix} z_t \\ Z_t \end{bmatrix} + Cs_t \tag{33}$$

$$s_t = Ms_{t-1} + \varepsilon_t \tag{34}$$

and the general form of the solution is:

$$z_t = DS_t \tag{35}$$

$$S_t = GS_{t-1} + H\varepsilon_t, \tag{36}$$

where $S_t \equiv [Z_t', s_t']'$ is the state vector, and the matrices $D$, $G$, and $H$ are non-linear functions of the parameters in matrices $A$, $B$, $C$ and $M$, obtained through numerical solutions techniques. As explained in the text, the variables of interest, collected in the vector $F_t$ constitute a linear combination of the state variables

$$F_t = \Phi S_t,$$

and the matrix $\Phi$ depends on the model parameters and the selection of variables in $F_t$. The measurement equation is given by

$$X_t = \Lambda S_t + e_t, \tag{37}$$

where

$$\Lambda \equiv \begin{bmatrix} \Lambda_F \Phi \\ \Lambda_S \end{bmatrix}.$$

and

$$e_t = \Psi e_{t-1} + v_t.$$

Equations (36) and (37) form a state-space representation of the solution of the model. The vectors

$\varepsilon_t$ and $v_t$ are assumed to be normally distributed with mean zero and variance $Q$ and $R$ respectively. The matrices $R$ and $\Psi$ are diagonal.

The goal is to estimate jointly the structural parameters of the theoretical model $\Theta_M = \{A, B, C, M, Q\}$, the measurement equation parameters $\Lambda$, $\Psi$ and $R$, and the unobserved variables of $\{S_t\}_{t=1}^T$. Let $\Theta = \{\Theta_M, \Lambda, \Psi, R\}$, $\tilde{S}_T = (S_1, S_2, ..., S_T)$ and $\tilde{X}_T = (X_1, X_2, ..., X_T)$. Our problem consists of characterizing the marginal likelihood of $\Theta$, defined as:

$$\mathfrak{L}_T(\Theta) = \int p(\tilde{X}_T|\tilde{S}_T, \Theta)p\left(\tilde{S}_T|\Theta\right) d\tilde{S}_T.$$

The maximum likelihood estimate is the set of parameter values, $\tilde{\Theta}$, that maximizes $\mathfrak{L}_T(\Theta)$. Given the high dimensionality of the problem and the need to integrate out the unobservable states, directly maximizing $\mathfrak{L}_T(\Theta)$ is difficult and impractical for some of models such as the one that we consider in this paper. Instead, the estimation approach that we consider provides an empirical approximation to these densities, and integrates out the states $\tilde{S}_T$ using Monte Carlo techniques. Moreover, by judiciously breaking up $\mathfrak{L}_T(\Theta)$ into the product of conditional densities, and sampling iteratively from the complete set of conditional densities, it effectively deals with the high dimensionality of the problem. As Jacquier, Johannes and Polson (2004) show, empirical distribution of the parameters resulting from the algorithm, that we describe in more details below, converges to the distribution of the maximum likelihood estimates, $\tilde{\Theta}$.

More specifically, provided with an initial value of the parameters, $\Theta^{(0)}$, the algorithm proceeds iteratively as follows. First, $J$ copies of $\tilde{S}_T$ are drawn from

$$p\left(\tilde{S}_T|\Theta^{(0)}, \tilde{X}_T\right) = p(\tilde{X}_T|\tilde{S}_T, \Theta^{(0)})p(\tilde{S}_T|\Theta^{(0)}).$$

Let $\bar{S}_T^{(J,1)} = \left(\tilde{S}_T^{(1,1)}, \tilde{S}_T^{(2,1)}, ..., \tilde{S}_T^{(J,1)}\right)$ denote the collection of these $J$ copies. Treating these draws as extra information, $\Theta^{(1)}$ is drawn from

$$\pi_J\left(\Theta^{(1)}|\bar{S}_T^{(J,1)}, \tilde{X}_T\right) \propto \prod_{j=1}^{J} p(\tilde{X}_T|\tilde{S}_T^{(j,1)}, \Theta^{(0)})p(\tilde{S}_T^{(j,1)}|\Theta^{(0)})p(\Theta^{(0)})$$

where $p(\Theta)$ is the subjective prior distribution on the parameter. This completes the first iteration.

This is repeated for a large number of iterations. Once the algorithm has converged, the empirical distribution of the subsequent draws converges, as the number of iterations gets large, to the distribution of the maximum likelihood estimator. Estimates of $\Theta$ and $\tilde{S}_T$ can be obtained as the mean, median or mode of the converged draws.

In the special case where $J$ is equal to 1, the algorithm collapses to the standard MCMC algorithm. In that case, the marginal distribution of $\Theta$ is given by:

$$\pi_J(\Theta) = \int p(\tilde{X}_T|\tilde{S}_T,\Theta)p(\tilde{S}_T|\Theta)d\tilde{S}_T p(\Theta)$$

For $J$ greater than 1, the algorithm augments the conditioning information set with more draws of the unobserved variables. In that case, the marginal distribution of $\Theta$ is given by

$$
\begin{aligned}
\pi_J(\Theta) &= \left\{ \prod_{j=1}^{J} \int p(\tilde{X}_T|\tilde{S}_T^j,\Theta)p(\tilde{S}_T^j|\Theta)d\tilde{S}_T^j \right\} p(\Theta) \\
&= \mathcal{L}_T(\Theta)^J p(\Theta)
\end{aligned}
$$

Intuitively, this has the effect of increasing the importance of the likelihood relative to the prior in the posterior distribution. By comparing results from different $J$, we are thus able to document the influence of the prior on the results.

We now provide more details on each step of the algorithm.

## A.1 Step 1: Drawing from the conditional distribution $p\left(\tilde{S}_T|\Theta,\tilde{X}_T\right)$

We use the forward-backward algorithm of Carter and Kohn (1994). As in Nelson and Kim (1999, p. 191), the conditional distribution of the whole history of factors can be expressed as the product of conditional distributions of factors at each date $t$ as follows:

$$p\left(\tilde{S}_T|\Theta,\tilde{X}_T\right) = p\left(S_T|\Theta,\tilde{X}_T\right)\prod_{t=1}^{T-1} p\left(S_t|S_{t+1},\Theta,\tilde{X}_T\right)$$

This relies on the Markov property of $S_t$, which implies that $p\left(S_t|S_{t+1},S_{t+2},...,S_T,\Theta,\tilde{X}_T\right) = p\left(S_t|S_{t+1},\Theta,\tilde{X}_T\right)$.

Because the state-space model (36)–(37) is linear and Gaussian, we have

$$S_T | \Theta, \tilde{X}_T \quad \sim \quad N\left(S_{T|T}, P_{T|T}\right)$$

$$S_t | S_{t+1}, \Theta, \tilde{X}_T \quad \sim \quad N\left(S_{t|t,S_{t+1}}, P_{t|t,S_{t+1}}\right)$$

where

$$S_{T|T} \quad = \quad E\left(S_T | \Theta, \tilde{X}_T\right) \tag{38}$$

$$P_{T|T} \quad = \quad Cov(S_T | \Theta, \tilde{X}_T)$$

$$S_{t|t,S_{t+1}} \quad = \quad E\left(S_t | S_{t+1}, \Theta, \tilde{X}_T\right)$$

$$P_{t|t,S_{t+1}} \quad = \quad Cov\left(S_t | S_{t+1}, \Theta, \tilde{X}_T\right)$$

where the notation $S_{t|t}$ refers to the expectation of $S_t$ conditional on information dated $t$ or earlier. To obtain these, we first calculate $S_{t|t}$ and $P_{t|t}$, $t = 1, 2, ..., T$, by Kalman filter, conditional on $\Theta$ and the data through period $t$, $\tilde{X}_t$, with starting values of zeros for the factors and the identity matrix for the covariance matrix (Hamilton, 1994). The last iteration of the filter yields $S_{T|T}$ and $P_{T|T}$, which together with the first line of (38) allows us to draw a value for $S_T$. Treating this drawn value as extra information, we can move "backwards in time" through the sample, using the Kalman filter to obtain updated values of $S_{T-1|T-1,F_T}$ and $P_{T-1|T-1,F_T}$; drawing a value of $S_{T-1}$ using the third line of (38); and continuing in similar manner to draw values for $S_t$, $t = T - 2, T - 3, ...1$. This constitutes one draw of $\tilde{S}_T$. To generate the $J$ independent copies of $\tilde{S}_T$, $\bar{S}_T^{(J)}$, the same steps are repeated $J$ times.[40]

## A.2  Step 2: Drawing from the conditional distribution $\pi_J\left(\Lambda, \Psi, R | \Theta_M, \bar{S}_T^{(J)}, \tilde{X}_T\right)$

Conditional on the observed data and the estimated factors from the previous iteration, a new iteration is begun by drawing a new value of the parameters. With known factors, (37) amounts to

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[40]The Kalman filter is implemented to handle the serial correlation in $e_t$. In particular, in the Kalman filter iterations, the observation equation is rearranged as:

$$X_t^* = (\Lambda F - \Psi \Lambda) S_{t-1} + \Lambda u_t + v_t$$

with $X_t^* = X_t - \Psi X_{t-1}$.

a set of regressions with autoregressive errors. We can thus apply the algorithm proposed by Chib (1993).

This conditional model is non-linear in the parameters. However, since conditional on $\Lambda$ or $\Psi$, the model is linear, we can characterize this distribution through a complete set of conditional distributions that are linear in the parameters. More precisely, we assume that *a priori* $\Lambda$ and $R$ are independent of $\Psi$. Conditional on $\Psi$ and since $R$ is diagonal, we can apply OLS equation by equation to obtain $\hat{\Lambda}_k^{(j)}$ and $\hat{v}_k^{(j)}$. We define $X_{k,t}^* = X_{k,t} - \Psi_{kk} X_{k,t-1}$ and $S_{k,t}^{*(j)} = S_t^{(j)} - \Psi_{kk} S_{t-1}^{(j)}$, set $R_{kl} = 0$, $k \neq l$, and assume a proper (conjugate) but diffuse Inverse-Gamma $(3, 0.001)$ prior for $R_{kk}$. Standard Bayesian results (see Bauwens, Lubrano and Richard, 1999, p. 58) deliver posterior of the form:

$$R_{kk} | \tilde{X}_T, \bar{S}_T^{(J)}, \Psi \sim iG\left(\bar{R}_{kk}, J \times T + 0.001\right)$$

where $\bar{R}_{kk} = 3 + \sum_{j=1}^J \hat{v}_k^{(j)\prime} \hat{v}_k^{(j)} + \sum_{j=1}^J \hat{\Lambda}_k^{(j)\prime} \left(\tilde{S}_{k,T}^{*(j)\prime} \tilde{S}_{k,T}^{*(j)}\right)^{-1} \hat{\Lambda}_k^{(j)} - \bar{\Lambda}_k' \bar{M}_k^{-1} \bar{\Lambda}_k$. Here $M_0^{-1}$ denotes variance parameter in the prior on the coefficients of the $k$-th equation, $\Lambda_k$, which, conditional on the drawn value of $R_{kk}$, is $N(0, R_{kk} M_0^{-1})$. We set $M_0 = I$. We draw values for $\Lambda_k$ from the posterior $N\left(\bar{\Lambda}_k, R_{kk} \bar{M}_k^{-1}\right)$, where $\bar{\Lambda}_k = \bar{M}_k^{-1} \left(\sum_{j=1}^J \tilde{S}_{k,T}^{*(j)\prime} \tilde{S}_{k,T}^{*(j)} \hat{\Lambda}_k^{(j)}\right)$ and $\bar{M}_k^{-1} = M_0 + \sum_{j=1}^J \tilde{S}_{k,T}^{*(j)\prime} \tilde{S}_{k,T}^{*(j)}$.

Conditional on $\Lambda$ and since $R$ is diagonal, we can apply OLS equation by equation to obtain $\hat{\Psi}_{kk}^{(j)}$. Letting $e_k^{(j)}$ be the vector whose elements are given by $e_{k,t}^{(j)} = X_{k,t} - \Lambda_k S_t^{(j)}$, and its lagged version $e_{k,-1}^{(j)}$, and assuming that the prior on $\Psi_{kk}$ is $N(0,1)$, the posterior distribution of $\Psi_{kk}$ is $N(\bar{\bar{\Psi}}_{kk}, \bar{N}_k^{-1})$ where $\bar{\bar{\Psi}}_{kk} = \bar{N}_k^{-1} \left(R_{kk}^{-1} \sum_{j=1}^J e_{k,-1}^{(j)\prime} e_{k,-1}^{(j)} \hat{\Psi}_{kk}^{(j)}\right)$ and $\bar{N}_k = \left(1 + R_{kk}^{-1} \sum_{j=1}^J e_{k,-1}^{(j)\prime} e_{k,-1}^{(j)}\right)$.

## A.3 Step 3: Drawing from the conditional distribution $\pi_J\left(\Theta_M | \Lambda, \Psi, R, \bar{S}_T^{(J)}, \tilde{X}_T\right)$

The elements of the matrices $\Theta_M$ are individually drawn from a proposal scalar Student $t$-distribution, with mean centered around the previous draws of the parameters, i.e. the corresponding elements of $\Theta_M^{(g)}$, and a variance calibrated to yield appropriate acceptance rates.[41] Let $\Theta_M^*$ be the resulting draws. Based on the solution of the model obtained from this last draw, the following ratio is

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[41] See Johannes and Polson (2004) for practical recommendations on the choice of the proposal density and the desired acceptance rate.

computed:

$$r = \frac{\pi_J\left(\Theta_M^* | \Lambda, R, \bar{S}_T^{(J)}, \tilde{X}_T\right)}{\pi_J\left(\Theta_M^{(g)} | \Lambda, R, \bar{S}_T^{(J)}, \tilde{X}_T\right)}$$

With probability $\min(1, r)$, $\Theta_M^{(g+1)} = \Theta_M^*$, and otherwise $\Theta_M^{(g+1)} = \Theta_M^{(g)}$.

Steps 1 to 3 are repeated for each iteration $g$. Inference is based on the distribution of $\left\{\Theta^{(g)}\right\}_{g=b}^{G}$, where $b$ is large enough to guarantee convergence of the algorithm. As noted, the empirical distribution from the sampling procedure should well approximate the joint posterior or normalized joint likelihood, and for $J > 1$, the distribution of the MLE estimator, $\tilde{\Theta}$. Calculating medians and quantiles of $\{\Theta\}_{g=b}^{G}$ provides estimates of the model parameters and the associated confidence regions.

## Appendix B - Data Description

All series were taken from DRI/McGraw Hill Basic Economics Database or directly from the Bureau of Labor Statistics. The format is: series number; transformation code and series description as appears in the database. The transformation codes are: 1 – no transformation; 2 – Detrended log per capita; 3 – detrended logarithm level 4 – logarithm; 5 – first difference of logarithm; 6 – Adjustement specific to average hours and hourly earnings; 0 – variable not used in the estimation (only used for transforming other variables). A * indicate a series that is deflated with the GDP deflator (series #85).

## Real Output

| | | |
|---|---|---|
| 1 | 2 | NIA REAL GROSS DOMESTIC PRODUCT (CHAINED-2000) - UNITED STATES |
| 2 | 2 | Real Gross Domestic Product by Major Type of Product - Goods, Billions of Chained (2000) Dollars , SAAR |
| 3 | 2 | Real Gross Domestic Product by Major Type of Product - Services, Billions of Chained (2000) Dollars , SAAR |
| 4 | 2 | Real Gross Domestic Product by Major Type of Product - Structures, Billions of Chained (2000) Dollars , SAAR |
| 5 | 2 | INDUSTRIAL PRODUCTION INDEX -  PRODUCTS, TOTAL |
| 6 | 2 | INDUSTRIAL PRODUCTION  INDEX -  FINAL PRODUCTS |
| 7 | 2 | INDUSTRIAL PRODUCTION INDEX -  CONSUMER GOODS |
| 8 | 2 | INDUSTRIAL PRODUCTION INDEX -  DURABLE CONSUMER GOODS |
| 9 | 2 | INDUSTRIAL PRODUCTION INDEX -  NONDURABLE CONSUMER GOODS |
| 10 | 2 | INDUSTRIAL PRODUCTION INDEX -  BUSINESS EQUIPMENT |
| 11 | 2 | INDUSTRIAL PRODUCTION INDEX -  MATERIALS |
| 12 | 2 | INDUSTRIAL PRODUCTION INDEX -  DURABLE GOODS MATERIALS |
| 13 | 2 | INDUSTRIAL PRODUCTION INDEX -  NONDURABLE GOODS MATERIALS |
| 14 | 2 | INDUSTRIAL PRODUCTION INDEX -  TOTAL INDEX |
| 15 | 1 | CAPACITY UTILIZATION -  MANUFACTURING (SIC) |
| 16 | 1 | PURCHASING MANAGERS' INDEX (SA) |
| 17 | 1 | NAPM PRODUCTION INDEX (PERCENT) |

## Income, Compensation and Wages

| | | |
|---|---|---|
| 18 | 2* | NIA NOMINAL TOTAL COMPENSATION OF EMPLOYEES - UNITED STATES |
| 19 | 2 | PERSONAL INCOME CHAINED 2000 DOLLARS (BCI) |
| 20 | 2 | PERSONAL INCOME LESS TRANSFER PAYMENTS (CHAINED) (#51) (BIL 92$,SAAR) |
| 21 | 6* | AVERAGE HOURLY EARNINGS, PRODUCTION WORKERS: MANUFACTURING, |
| 22 | 6* | AVERAGE HOURLY EARNINGS, PRODUCTION WORKERS: CONSTRUCTION, |

## Employment and Hours

| | | |
|---|---|---|
| 23 | 6 | AVERAGE WEEKLY HOURS OF PRODUCTION WORKERS: Total private (Table B-1) |
| 24 | 6 | AVERAGE WEEKLY HOURS, PRODUCTION WORKERS: MANUFACTURING, |
| 25 | 6 | AVERAGE WEEKLY OVERTIME, PRODUCTION WORKERS: DURABLE GOODS, |
| 26 | 2 | INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS (1967=100;SA) |
| 27 | 2 | EMPLOYMENT: RATIO; HELP-WANTED ADS:NO. UNEMPLOYED CLF |
| 28 | 1 | UNEMPLOYMENT RATE: ALL WORKERS, 16 YEARS & OVER (%,SA) |
| 29 | 1 | UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS (SA) |
| 30 | 2 | UNEMPLOY.BY DURATION: PERSONS UNEMPL.LESS THAN 5 WKS (THOUS.,SA) |
| 31 | 2 | UNEMPLOY.BY DURATION: PERSONS UNEMPL.5 TO 14 WKS (THOUS.,SA) |
| 32 | 2 | UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 WKS + (THOUS.,SA) |
| 33 | 2 | UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 TO 26 WKS (THOUS.,SA) |
| 34 | 2 | EMPLOYEES, NONFARM - TOTAL NONFARM |
| 35 | 2 | EMPLOYEES, NONFARM - TOTAL PRIVATE |
| 36 | 2 | EMPLOYEES, NONFARM - GOODS-PRODUCING |
| 37 | 2 | EMPLOYEES, NONFARM - MINING |
| 38 | 2 | EMPLOYEES, NONFARM - CONSTRUCTION |
| 39 | 2 | EMPLOYEES, NONFARM - MFG |
| 40 | 2 | EMPLOYEES, NONFARM - DURABLE GOODS |
| 41 | 2 | EMPLOYEES, NONFARM - NONDURABLE GOODS |
| 42 | 2 | EMPLOYEES, NONFARM - SERVICE-PROVIDING |
| 43 | 2 | EMPLOYEES, NONFARM - TRADE, TRANSPORT, UTILITIES |
| 44 | 2 | EMPLOYEES, NONFARM - WHOLESALE TRADE |

| 45 | 2 EMPLOYEES, NONFARM - RETAIL TRADE |
| 46 | 2 EMPLOYEES, NONFARM - GOVERNMENT |

## Consumption

| 47 | 2 REAL PERSONAL CONSUMPTION EXPENDITURES (CHAINED-2000) - UNITED STATES (NIPA) |
| 48 | 2 REAL PERSONAL CONSUMPTION EXPENDITURES (Index 2000=100):  Durable goods (NIPA Table 2.3.3) |
| 49 | 2 Nondurable goods |
| 50 | 2 Services |

## Investment,  Housing Starts and New Orders

| 51 | 2* Gross Private Domestic Investment, Billions of Dollars , SAAR |
| 52 | 2* Gross Private Domestic Investment - Fixed Investment, Billions of Dollars , SAAR |
| 53 | 2* Gross Private Domestic Investment - Fixed Nonresidential , Billions of Dollars , SAAR |
| 54 | 4 HOUSING STARTS:NONFARM(1947-58);TOTAL FARM&NONFARM(1959-)(THOUS.,SA |
| 55 | 1 NAPM INVENTORIES INDEX (PERCENT) |
| 56 | 1 NAPM NEW ORDERS INDEX (PERCENT) |
| 57 | 1 NAPM VENDOR DELIVERIES INDEX (PERCENT) |
| 58 | 2 NEW ORDERS (NET) - CONSUMER GOODS & MATERIALS, 1996 DOLLARS (BCI) |

## Stock Prices

| 59 | 5 S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10) |
| 60 | 5 S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (1941-43=10) |
| 61 | 1 S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD (% PER ANNUM) |
| 62 | 1 S&P'S COMPOSITE COMMON STOCK: PRICE-EARNINGS RATIO (%,NSA) |

## Exchange Rates

| 63 | 5 FOREIGN EXCHANGE RATE: SWITZERLAND (SWISS FRANC PER U.S.$) |
| 64 | 5 FOREIGN EXCHANGE RATE: JAPAN (YEN PER U.S.$) |
| 65 | 5 FOREIGN EXCHANGE RATE: UNITED KINGDOM (CENTS PER POUND) |
| 66 | 5 FOREIGN EXCHANGE RATE: CANADA (CANADIAN $ PER U.S.$) |

## Interest Rates

| 67 | 1 INTEREST RATE: FEDERAL FUNDS (EFFECTIVE) (% PER ANNUM,NSA) |
| 68 | 1 INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,3-MO.(% PER ANN,NSA) |
| 69 | 1 INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,6-MO.(% PER ANN,NSA) |
| 70 | 1 INTEREST RATE: U.S.TREASURY CONST MATURITIES,1-YR.(% PER ANN,NSA) |
| 71 | 1 INTEREST RATE: U.S.TREASURY CONST MATURITIES,5-YR.(% PER ANN,NSA) |
| 72 | 1 INTEREST RATE: U.S.TREASURY CONST MATURITIES,10-YR.(% PER ANN,NSA) |
| 73 | 1 BOND YIELD: MOODY'S AAA CORPORATE (% PER ANNUM) |
| 74 | 1 BOND YIELD: MOODY'S BAA CORPORATE (% PER ANNUM) |

## Money and Credit Quantity Aggregates

| 75 | 2 MONEY STOCK: M1(CURR,TRAV.CKS,DEM DEP,OTHER CK'ABLE DEP)(BIL$,SA) |
| 76 | 2 MONEY STOCK:M2(M1+O'NITE RPS,EURO$,G/P&B/D MMMFS&SAV&SM TIME DEP(BIL$, |
| 77 | 2 MONEY STOCK: M3(M2+LG TIME DEP,TERM RP'S&INST ONLY MMMFS)(BIL$,SA) |
| 78 | 2 MONEY SUPPLY - M2 IN 1996 DOLLARS (BCI) |
| 79 | 2 MONETARY BASE, ADJ FOR RESERVE REQUIREMENT CHANGES(MIL$,SA) |
| 80 | 2 DEPOSITORY INST RESERVES:TOTAL,ADJ FOR RESERVE REQ CHGS(MIL$,SA) |
| 81 | 2 DEPOSITORY INST RESERVES:NONBORROWED,ADJ RES REQ CHGS(MIL$,SA) |
| 82 | 2 COMMERCIAL & INDUSTRIAL LOANS OUSTANDING IN 1996 DOLLARS (BCI) |
| 83 | 1 WKLY RP LG COM'L BANKS:NET CHANGE COM'L & INDUS LOANS(BIL$,SAAR) |
| 84 | 2 CONSUMER CREDIT OUTSTANDING - NONREVOLVING(G19) |

## Prices

| 85 | 5 NIA PRICE DEFLATOR - GROSS DOMESTIC PRODUCT - UNITED STATES |
| 86 | 5 NIA PRICE DEFLATOR - PRIVATE CONSUMPTION EXPENDITURE - UNITED STATES |
| 87 | 5 Gross Private Domestic Investment, Price Deflators (2000=100) , SAAR |
| 88 | 5 Personal Consumption Expenditures - Durable Goods, Price Index (2000=100) , SAAR |
| 89 | 5 Personal Consumption Expenditures - Nondurable Goods, Price Index (2000=100) , SAAR |
| 90 | 5 Personal Consumption Expenditures - Services, Price Index (2000=100) , SAAR |
| 91 | 5 CPI-U: ALL ITEMS LESS MIDICAL CARE (82-84=100,SA) |
| 92 | 5 CPI-U: ALL ITEMS LESS FOOD (82-84=100,SA) |
| 93 | 5 CPI-U: SERVICES (82-84=100,SA) |

94          5 CPI-U: DURABLES (82-84=100,SA)
95          1 NAPM COMMODITY PRICES INDEX (PERCENT)

## Miscellaneous

96          1 U. OF MICH. INDEX OF CONSUMER EXPECTATIONS(BCD-83)
97          3 COMPOSITE CYCLICAL INDICATOR (1996) - LEADING - UNITED STATES
98          3 COMPOSITE CYCLICAL INDICATOR (1996) - LAGGING - UNITED STATES
99          3 COMPOSITE CYCLICAL INDICATOR (1996) - COINCIDENT - UNITED STATES

Table 1a: Parameter Estimates, $J = 1$ (Median of posterior distribution)

| | Prior Distribution | | | Case A | Case B | Case C | Case D | Case E |
|---|---|---|---|---|---|---|---|---|
| | Type | Mean | St.Err. | | | | | |
| $\varphi$ | Normal | 4 | 1.5 | 6.36 | 6.15 | 3.98 | 3.28 | 12.53 |
| | | | | ( 0.96) | ( 1.03) | ( 1.50) | ( 0.85) | ( 0.96) |
| $\sigma_c$ | Normal | 1 | 0.375 | 1.59 | 1.57 | 1.00 | 2.06 | 2.17 |
| | | | | ( 0.17) | ( 0.15) | ( 0.37) | ( 0.11) | ( 0.06) |
| $h$ | Beta | 0.7 | 1.5 | 0.70 | 0.70 | 0.70 | 0.70 | 0.69 |
| | | | | ( 0.00) | ( 0.00) | ( 0.01) | ( 0.00) | ( 0.00) |
| $\sigma_L$ | Normal | 2 | 1.5 | 1.98 | 1.62 | 2.02 | 0.84 | 1.67 |
| | | | | ( 0.69) | ( 0.60) | ( 0.72) | ( 0.29) | ( 0.09) |
| $\phi$ | Normal | 1.25 | 1.5 | 1.68 | 1.46 | 1.25 | 1.56 | 1.65 |
| | | | | ( 0.07) | ( 0.06) | ( 0.12) | ( 0.04) | ( 0.03) |
| $1/\psi$ | Normal | 0.2 | 1.5 | 0.31 | 0.29 | 0.19 | 0.29 | 0.47 |
| | | | | ( 0.06) | ( 0.06) | ( 0.06) | ( 0.05) | ( 0.04) |
| $\xi_\omega$ | Beta | 0.75 | 1.5 | 0.80 | 0.77 | 0.75 | 0.71 | 0.69 |
| | | | | ( 0.04) | ( 0.03) | ( 0.05) | ( 0.03) | ( 0.01) |
| $\xi_p$ | Beta | 0.75 | 1.5 | 0.87 | 0.88 | 0.74 | 0.82 | 0.85 |
| | | | | ( 0.01) | ( 0.01) | ( 0.05) | ( 0.02) | ( 0.01) |
| $\gamma_\omega$ | Beta | 0.5 | 1.5 | 0.39 | 0.53 | 0.50 | 0.50 | 0.29 |
| | | | | ( 0.11) | ( 0.14) | ( 0.14) | ( 0.16) | ( 0.12) |
| $\gamma_p$ | Beta | 0.5 | 1.5 | 0.59 | 0.73 | 0.49 | 0.10 | 0.21 |
| | | | | ( 0.09) | ( 0.09) | ( 0.15) | ( 0.03) | ( 0.06) |
| $\rho$ | Beta | 0.75 | 1.5 | 0.79 | 0.73 | 0.75 | 0.86 | 0.86 |
| | | | | ( 0.02) | ( 0.03) | ( 0.09) | ( 0.00) | ( 0.00) |
| $r_{\pi 0}$ | Normal | 1.5 | 0.5 | 1.79 | 1.86 | 1.80 | 1.87 | 1.68 |
| | | | | ( 0.09) | ( 0.09) | ( 0.09) | ( 0.02) | ( 0.00) |
| $r_{\pi 1}$ | Normal | 0 | 0.2 | -0.25 | -0.25 | -0.30 | -0.30 | -0.28 |
| | | | | ( 0.09) | ( 0.09) | ( 0.09) | ( 0.04) | ( 0.02) |
| $r_{y0}$ | Normal | 0.5 | 0.2 | 0.20 | 0.18 | 0.19 | 0.28 | 0.29 |
| | | | | ( 0.03) | ( 0.03) | ( 0.04) | ( 0.00) | ( 0.00) |
| $r_{y1}$ | Normal | 0 | 0.2 | -0.16 | -0.17 | -0.06 | -0.23 | -0.22 |
| | | | | ( 0.03) | ( 0.03) | ( 0.04) | ( 0.00) | ( 0.00) |

Table 1a (Continued): Parameter Estimates, $J = 1$ (Median of posterior distribution)

| | Prior Distribution | | | Case A | Case B | Case C | Case D | Case E |
|---|---|---|---|---|---|---|---|---|
| | Type | Mean | St.Err. | | | | | |
| $\rho_a$ | Beta | 0.85 | 1.5 | 0.93 | 0.96 | 0.86 | 0.99 | 0.99 |
| | | | | ( 0.03) | ( 0.01) | ( 0.10) | ( 0.00) | ( 0.00) |
| $\rho_b$ | Beta | 0.85 | 1.5 | 0.39 | 0.53 | 0.85 | 0.54 | 0.56 |
| | | | | ( 0.07) | ( 0.07) | ( 0.10) | ( 0.05) | ( 0.01) |
| $\rho_G$ | Beta | 0.85 | 1.5 | 0.95 | 0.97 | 0.86 | 0.99 | 0.99 |
| | | | | ( 0.02) | ( 0.00) | ( 0.10) | ( 0.00) | ( 0.00) |
| $\rho_L$ | Beta | 0.85 | 1.5 | 0.95 | 0.89 | 0.85 | 0.98 | 0.99 |
| | | | | ( 0.02) | ( 0.03) | ( 0.09) | ( 0.00) | ( 0.00) |
| $\rho_I$ | Beta | 0.85 | 1.5 | 0.71 | 0.75 | 0.85 | 0.85 | 0.64 |
| | | | | ( 0.07) | ( 0.06) | ( 0.09) | ( 0.03) | ( 0.01) |
| $\sigma_a^2$ | invGam | 0.25 | 1.5 | 0.19 | 0.08 | 0.31 | 0.10 | 0.07 |
| | | | | ( 0.02) | ( 0.01) | ( 0.55) | ( 0.01) | ( 0.01) |
| $\sigma_b^2$ | invGam | 0.25 | 1.5 | 0.04 | 0.01 | 0.01 | 0.01 | 0.01 |
| | | | | ( 0.01) | ( 0.00) | ( 0.10) | ( 0.00) | ( 0.00) |
| $\sigma_G^2$ | invGam | 0.25 | 1.5 | 0.33 | 0.12 | 0.23 | 0.12 | 0.11 |
| | | | | ( 0.03) | ( 0.02) | ( 0.13) | ( 0.02) | ( 0.02) |
| $\sigma_L^2$ | invGam | 0.25 | 1.5 | 26.63 | 45.51 | 32.93 | 7.83 | 7.93 |
| | | | | ( 13.77) | ( 29.47) | ( 13.09) | ( 2.35) | ( 2.28) |
| $\sigma_I^2$ | invGam | 0.25 | 1.5 | 20.37 | 13.72 | 8.29 | 6.88 | 84.96 |
| | | | | ( 14.13) | ( 10.69) | ( 4.13) | ( 3.15) | ( 14.36) |
| $\sigma_i^2$ | invGam | 0.25 | 1.5 | 0.00 | 0.00 | 0.03 | 0.02 | 0.01 |
| | | | | ( 0.00) | ( 0.00) | ( 0.01) | ( 0.00) | ( 0.00) |
| $\sigma_Q^2$ | invGam | 0.25 | 1.5 | 29.69 | 14.61 | 14.35 | 0.43 | 0.63 |
| | | | | ( 34.95) | ( 16.67) | ( 12.20) | ( 0.48) | ( 0.23) |
| $\sigma_p^2$ | invGam | 0.25 | 1.5 | 0.02 | 0.01 | 0.04 | 0.00 | 0.02 |
| | | | | ( 0.00) | ( 0.00) | ( 0.02) | ( 0.00) | ( 0.00) |
| $\sigma_\omega^2$ | invGam | 0.25 | 1.5 | 0.06 | 0.03 | 0.09 | 0.00 | 0.00 |
| | | | | ( 0.01) | ( 0.01) | ( 0.04) | ( 0.00) | ( 0.00) |
| $\sigma_\pi^2$ | invGam | 0.05 | 1.5 | 0.06 | 0.05 | 0.05 | 0.03 | 0.05 |
| | | | | ( 0.00) | ( 0.00) | ( 0.02) | ( 0.00) | ( 0.00) |

Results are based on 20000 replications. Standard errors are reported in ().

Table 1b: Parameter Estimates, $J = 5$ (Median of posterior distribution)

| | Prior Distribution | | | Case A | Case B | Case C | Case D | Case E |
|---|---|---|---|---|---|---|---|---|
| | Type | Mean | St.Err. | | | | | |
| $\varphi$ | Normal | 4 | 1.5 | 6.36 | 8.74 | 4.13 | 5.83 | $-$ |
| | | | | ( 0.96) | ( 1.02) | ( 1.49) | ( 0.78) | $(-)$ |
| $\sigma_c$ | Normal | 1 | 0.375 | 1.59 | 1.97 | 1.00 | 1.84 | $-$ |
| | | | | ( 0.17) | ( 0.12) | ( 0.37) | ( 0.07) | $(-)$ |
| $h$ | Beta | 0.7 | 1.5 | 0.70 | 0.69 | 0.69 | 0.69 | $-$ |
| | | | | ( 0.00) | ( 0.01) | ( 0.00) | ( 0.01) | $(-)$ |
| $\sigma_L$ | Normal | 2 | 1.5 | 1.98 | 0.07 | 1.95 | 1.60 | $-$ |
| | | | | ( 0.69) | ( 0.11) | ( 0.72) | ( 0.02) | $(-)$ |
| $\phi$ | Normal | 1.25 | 1.5 | 1.68 | 1.88 | 1.24 | 1.82 | $-$ |
| | | | | ( 0.07) | ( 0.05) | ( 0.12) | ( 0.05) | $(-)$ |
| $1/\psi$ | Normal | 0.2 | 1.5 | 0.31 | 0.41 | 0.19 | 0.33 | $-$ |
| | | | | ( 0.06) | ( 0.05) | ( 0.06) | ( 0.04) | $(-)$ |
| $\xi_\omega$ | Beta | 0.75 | 1.5 | 0.80 | 0.71 | 0.74 | 0.81 | $-$ |
| | | | | ( 0.04) | ( 0.03) | ( 0.05) | ( 0.00) | $(-)$ |
| $\xi_p$ | Beta | 0.75 | 1.5 | 0.87 | 0.89 | 0.74 | 0.86 | $-$ |
| | | | | ( 0.01) | ( 0.00) | ( 0.04) | ( 0.00) | $(-)$ |
| $\gamma_\omega$ | Beta | 0.5 | 1.5 | 0.39 | 0.76 | 0.51 | 0.60 | $-$ |
| | | | | ( 0.11) | ( 0.10) | ( 0.15) | ( 0.15) | $(-)$ |
| $\gamma_p$ | Beta | 0.5 | 1.5 | 0.59 | 0.84 | 0.49 | 0.20 | $-$ |
| | | | | ( 0.09) | ( 0.03) | ( 0.14) | ( 0.06) | $(-)$ |
| $\rho$ | Beta | 0.75 | 1.5 | 0.79 | 0.74 | 0.76 | 0.87 | $-$ |
| | | | | ( 0.02) | ( 0.01) | ( 0.09) | ( 0.00) | $(-)$ |
| $r_{\pi 0}$ | Normal | 1.5 | 0.5 | 1.79 | 2.10 | 1.80 | 1.68 | $-$ |
| | | | | ( 0.09) | ( 0.08) | ( 0.10) | ( 0.00) | $(-)$ |
| $r_{\pi 1}$ | Normal | 0 | 0.2 | -0.25 | -0.08 | -0.30 | -0.21 | $-$ |
| | | | | ( 0.09) | ( 0.08) | ( 0.10) | ( 0.01) | $(-)$ |
| $r_{y 0}$ | Normal | 0.5 | 0.2 | 0.20 | 0.33 | 0.18 | 0.29 | $-$ |
| | | | | ( 0.03) | ( 0.03) | ( 0.04) | ( 0.00) | $(-)$ |
| $r_{y 1}$ | Normal | 0 | 0.2 | -0.16 | -0.26 | -0.05 | -0.23 | $-$ |
| | | | | ( 0.03) | ( 0.03) | ( 0.05) | ( 0.00) | $(-)$ |

Table 1b (Continued): Parameter Estimates, $J = 5$ (Median of posterior distribution)

| | Prior Distribution | | | Case A | Case B | Case C | Case D | Case E |
|---|---|---|---|---|---|---|---|---|
| | Type | Mean | St.Err. | | | | | |
| $\rho_a$ | Beta | 0.85 | 1.5 | 0.93 | 0.99 | 0.85 | 0.99 | – |
| | | | | ( 0.03) | ( 0.00) | ( 0.09) | ( 0.00) | (–) |
| $\rho_b$ | Beta | 0.85 | 1.5 | 0.39 | 0.31 | 0.84 | 0.47 | – |
| | | | | ( 0.07) | ( 0.03) | ( 0.10) | ( 0.02) | (–) |
| $\rho_G$ | Beta | 0.85 | 1.5 | 0.95 | 0.97 | 0.85 | 0.99 | – |
| | | | | ( 0.02) | ( 0.00) | ( 0.10) | ( 0.00) | (–) |
| $\rho_L$ | Beta | 0.85 | 1.5 | 0.95 | 0.94 | 0.87 | 0.98 | – |
| | | | | ( 0.02) | ( 0.01) | ( 0.08) | ( 0.00) | (–) |
| $\rho_I$ | Beta | 0.85 | 1.5 | 0.71 | 0.66 | 0.86 | 0.82 | – |
| | | | | ( 0.07) | ( 0.03) | ( 0.09) | ( 0.01) | (–) |
| $\sigma_a^2$ | invGam | 0.25 | 1.5 | 0.19 | 0.22 | 0.51 | 0.12 | – |
| | | | | ( 0.02) | ( 0.01) | ( 0.53) | ( 0.01) | (–) |
| $\sigma_b^2$ | invGam | 0.25 | 1.5 | 0.04 | 0.05 | 0.02 | 0.02 | – |
| | | | | ( 0.01) | ( 0.00) | ( 0.07) | ( 0.00) | (–) |
| $\sigma_G^2$ | invGam | 0.25 | 1.5 | 0.33 | 0.18 | 0.26 | 0.17 | – |
| | | | | ( 0.03) | ( 0.01) | ( 0.12) | ( 0.03) | (–) |
| $\sigma_L^2$ | invGam | 0.25 | 1.5 | 26.63 | 6.55 | 25.86 | 9.68 | – |
| | | | | ( 13.77) | ( 1.18) | ( 5.15) | ( 2.49) | (–) |
| $\sigma_I^2$ | invGam | 0.25 | 1.5 | 20.37 | 46.04 | 8.40 | 12.93 | – |
| | | | | ( 14.13) | ( 15.41) | ( 4.27) | ( 3.33) | (–) |
| $\sigma_i^2$ | invGam | 0.25 | 1.5 | 0.00 | 0.00 | 0.03 | 0.02 | – |
| | | | | ( 0.00) | ( 0.00) | ( 0.01) | ( 0.00) | (–) |
| $\sigma_Q^2$ | invGam | 0.25 | 1.5 | 29.69 | 14.86 | 28.75 | 15.40 | – |
| | | | | ( 34.95) | ( 22.80) | ( 9.36) | ( 7.90) | (–) |
| $\sigma_p^2$ | invGam | 0.25 | 1.5 | 0.02 | 0.00 | 0.04 | 0.01 | – |
| | | | | ( 0.00) | ( 0.00) | ( 0.02) | ( 0.00) | (–) |
| $\sigma_\omega^2$ | invGam | 0.25 | 1.5 | 0.06 | 0.06 | 0.20 | 0.07 | – |
| | | | | ( 0.01) | ( 0.01) | ( 0.04) | ( 0.01) | (–) |
| $\sigma_\pi^2$ | invGam | 0.05 | 1.5 | 0.06 | 0.05 | 0.05 | 0.04 | – |
| | | | | ( 0.00) | ( 0.00) | ( 0.02) | ( 0.00) | (–) |

Results are based on 8000 replications. Standard errors are reported in ().

Table 2: Fraction of variance of key time series explained by the model

|  | Case A | Case B | Case C | Case D | Case E |
|---|---|---|---|---|---|
| $J = 1$ | | | | | |
| Fed funds rate (120) | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Real GDP (1) | 1.00 | 0.98 | 0.98 | 0.98 | 0.97 |
| Real Consumption (49) | 1.00 | 0.98 | 0.99 | 0.98 | 0.98 |
| Real Investment (74) | 1.00 | 0.99 | 0.99 | 0.99 | 0.97 |
| GDP defl. inflation (145) | 1.00 | 0.80 | 0.74 | 0.77 | 0.83 |
| Real wage (18) | 1.00 | 0.98 | 0.94 | 0.87 | 0.66 |
| Hours worked (23) | 1.00 | 0.98 | 0.98 | 0.99 | 0.98 |
| $J = 5$ | | | | | |
| Fed funds rate (120) | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Real GDP (1) | 1.00 | 0.99 | 0.99 | 0.99 | – |
| Real Consumption (49) | 1.00 | 0.99 | 0.99 | 0.99 | – |
| Real Investment (74) | 1.00 | 0.99 | 0.99 | 0.99 | – |
| GDP defl. inflation (145) | 1.00 | 0.88 | 0.79 | 0.90 | – |
| Real wage (18) | 1.00 | 0.99 | 0.96 | 0.98 | – |
| Hours worked (23) | 1.00 | 0.99 | 0.99 | 0.99 | – |

Table 3: Variance decompositions for the 8-quarter horizon ($J = 1$)

| Shock | $i$ | $Y$ | $C$ | $I$ | $\pi$ | $w$ | $L$ | $K$ |
|---|---|---|---|---|---|---|---|---|
| Case A | | | | | | | | |
| Productivity | 0.12 | 0.04 | 0.01 | 0.11 | 0.17 | 0.10 | 0.20 | 0.02 |
| Preference | 0.01 | 0.09 | 0.32 | 0.00 | 0.00 | 0.01 | 0.07 | 0.00 |
| Govt. Expenditures | 0.01 | 0.15 | 0.00 | 0.04 | 0.00 | 0.00 | 0.13 | 0.01 |
| Labor supply | 0.17 | 0.26 | 0.40 | 0.10 | 0.32 | 0.26 | 0.28 | 0.02 |
| Investment | 0.07 | 0.27 | 0.03 | 0.66 | 0.02 | 0.06 | 0.18 | 0.91 |
| Inflation target | 0.12 | 0.00 | 0.00 | 0.00 | 0.15 | 0.00 | 0.00 | 0.00 |
| Equity premium | 0.00 | 0.01 | 0.00 | 0.02 | 0.00 | 0.00 | 0.00 | 0.01 |
| Cost-push prices | 0.08 | 0.03 | 0.03 | 0.00 | 0.28 | 0.23 | 0.01 | 0.00 |
| Cost-push wages | 0.01 | 0.01 | 0.02 | 0.00 | 0.02 | 0.28 | 0.01 | 0.00 |
| Monetary policy | 0.36 | 0.11 | 0.15 | 0.02 | 0.02 | 0.03 | 0.08 | 0.01 |
| Case B | | | | | | | | |
| Productivity | 0.04 | 0.06 | 0.04 | 0.07 | 0.06 | 0.11 | 0.07 | 0.01 |
| Preference | 0.02 | 0.07 | 0.27 | 0.00 | 0.01 | 0.03 | 0.06 | 0.00 |
| Govt. Expenditures | 0.01 | 0.08 | 0.00 | 0.01 | 0.00 | 0.00 | 0.08 | 0.00 |
| Labor supply | 0.11 | 0.34 | 0.48 | 0.17 | 0.19 | 0.44 | 0.44 | 0.04 |
| Investment | 0.19 | 0.31 | 0.03 | 0.69 | 0.14 | 0.12 | 0.24 | 0.91 |
| Inflation target | 0.16 | 0.00 | 0.00 | 0.00 | 0.30 | 0.00 | 0.00 | 0.00 |
| Equity premium | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| Cost-push prices | 0.09 | 0.03 | 0.03 | 0.00 | 0.21 | 0.13 | 0.01 | 0.00 |
| Cost-push wages | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.10 | 0.01 | 0.00 |
| Monetary policy | 0.33 | 0.06 | 0.09 | 0.01 | 0.02 | 0.03 | 0.05 | 0.00 |
| Case C | | | | | | | | |
| Productivity | 0.08 | 0.10 | 0.01 | 0.02 | 0.12 | 0.26 | 0.13 | 0.01 |
| Preference | 0.32 | 0.29 | 0.84 | 0.43 | 0.18 | 0.12 | 0.25 | 0.23 |
| Govt. Expenditures | 0.00 | 0.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 0.00 |
| Labor supply | 0.13 | 0.21 | 0.02 | 0.04 | 0.21 | 0.17 | 0.24 | 0.02 |
| Investment | 0.05 | 0.22 | 0.07 | 0.48 | 0.01 | 0.00 | 0.19 | 0.71 |
| Inflation target | 0.27 | 0.00 | 0.00 | 0.00 | 0.33 | 0.00 | 0.00 | 0.00 |
| Equity premium | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Cost-push prices | 0.02 | 0.01 | 0.00 | 0.00 | 0.08 | 0.10 | 0.00 | 0.00 |
| Cost-push wages | 0.02 | 0.03 | 0.00 | 0.00 | 0.04 | 0.30 | 0.04 | 0.00 |
| Monetary policy | 0.07 | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 | 0.04 | 0.00 |

Table 3 (Continued): Variance decompositions for the 8-quarter horizon ($J = 1$)

| Shock | $i$ | $Y$ | $C$ | $I$ | $\pi$ | $w$ | $L$ | $K$ |
|-------|-----|-----|-----|-----|-------|-----|-----|-----|
| | | | | Case D | | | | |
| Productivity | 0.01 | 0.03 | 0.08 | 0.00 | 0.03 | 0.24 | 0.17 | 0.00 |
| Preference | 0.01 | 0.05 | 0.22 | 0.02 | 0.00 | 0.06 | 0.03 | 0.00 |
| Govt. Expenditures | 0.00 | 0.07 | 0.02 | 0.00 | 0.02 | 0.00 | 0.08 | 0.00 |
| Labor supply | 0.11 | 0.19 | 0.40 | 0.03 | 0.23 | 0.21 | 0.24 | 0.00 |
| Investment | 0.19 | 0.47 | 0.06 | 0.88 | 0.11 | 0.23 | 0.34 | 0.96 |
| Inflation target | 0.53 | 0.00 | 0.00 | 0.00 | 0.48 | 0.00 | 0.00 | 0.00 |
| Equity premium | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Cost-push prices | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 |
| Cost-push wages | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 |
| Monetary policy | 0.12 | 0.17 | 0.19 | 0.04 | 0.09 | 0.20 | 0.11 | 0.02 |
| | | | | Case E | | | | |
| Productivity | 0.03 | 0.02 | 0.05 | 0.00 | 0.07 | 0.10 | 0.23 | 0.00 |
| Preference | 0.02 | 0.12 | 0.30 | 0.01 | 0.01 | 0.10 | 0.09 | 0.00 |
| Govt. Expenditures | 0.01 | 0.08 | 0.04 | 0.00 | 0.03 | 0.00 | 0.09 | 0.00 |
| Labor supply | 0.21 | 0.12 | 0.25 | 0.00 | 0.41 | 0.25 | 0.18 | 0.00 |
| Investment | 0.08 | 0.42 | 0.10 | 0.93 | 0.03 | 0.24 | 0.24 | 0.99 |
| Inflation target | 0.32 | 0.00 | 0.00 | 0.00 | 0.27 | 0.00 | 0.00 | 0.00 |
| Equity premium | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Cost-push prices | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 0.03 | 0.00 | 0.00 |
| Cost-push wages | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Monetary policy | 0.29 | 0.21 | 0.22 | 0.03 | 0.10 | 0.24 | 0.13 | 0.00 |
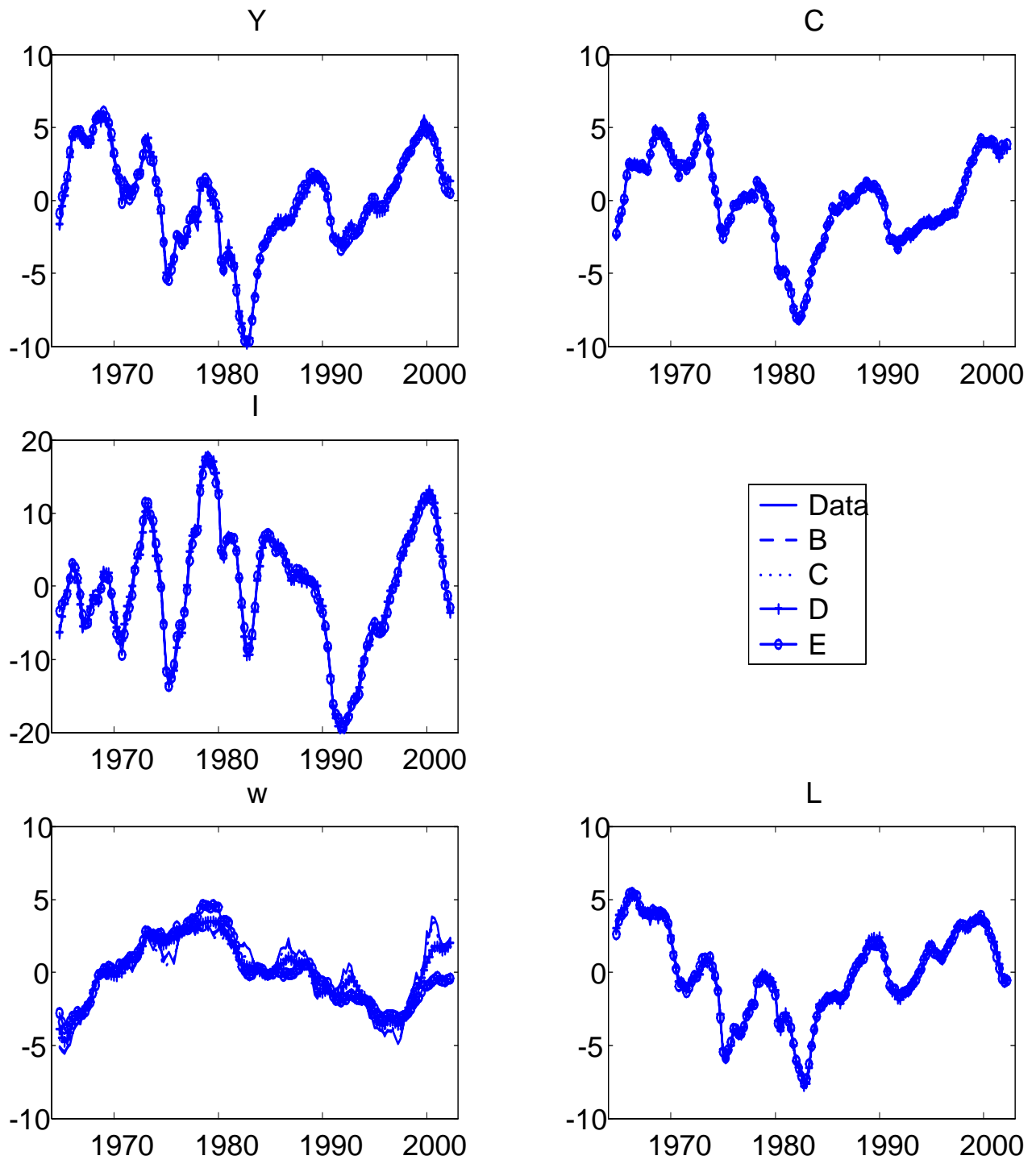
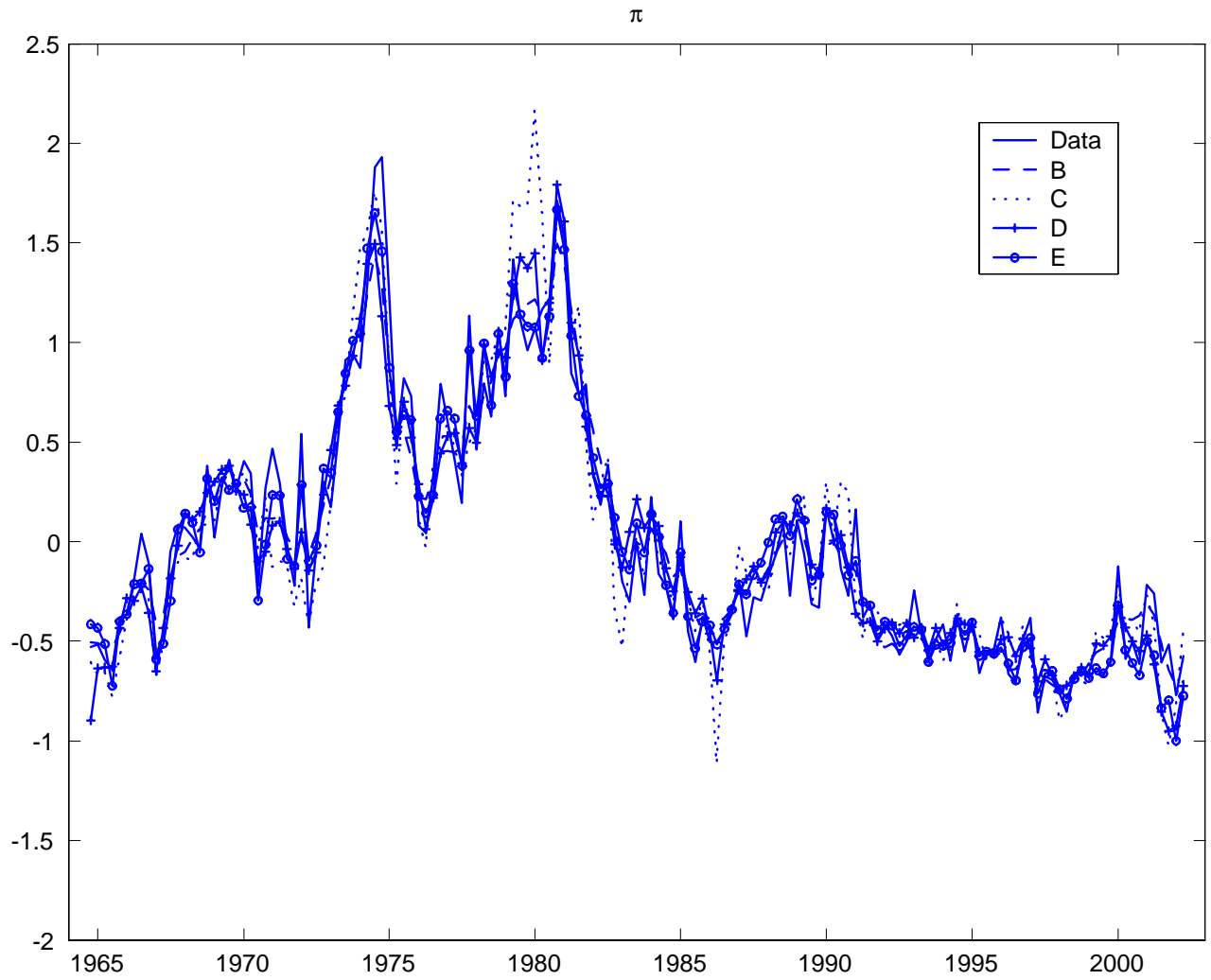Figure 1: Estimated endogenous variables ($J = 1$).
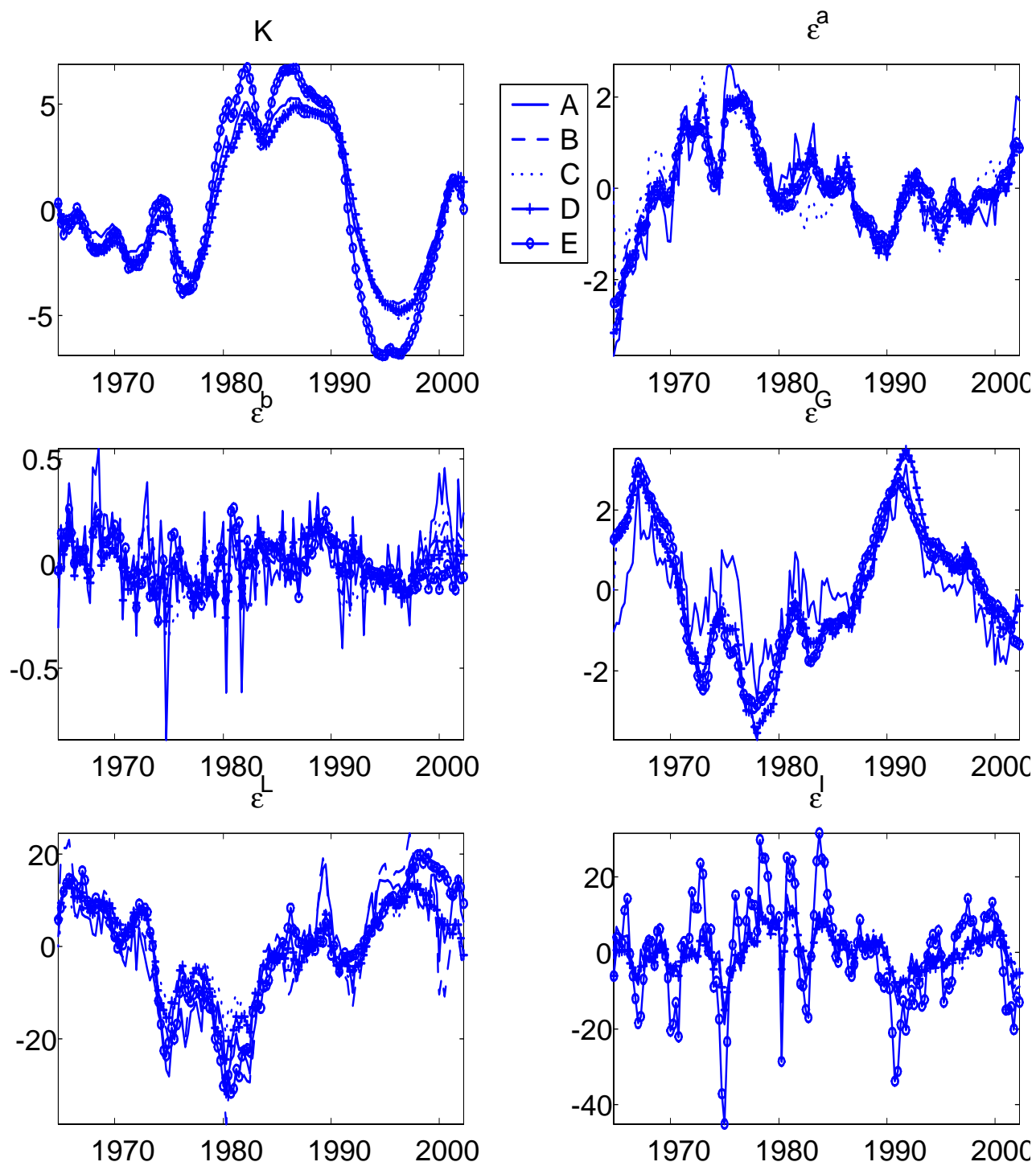
Figure 2: Estimated inflation $(J = 1)$.

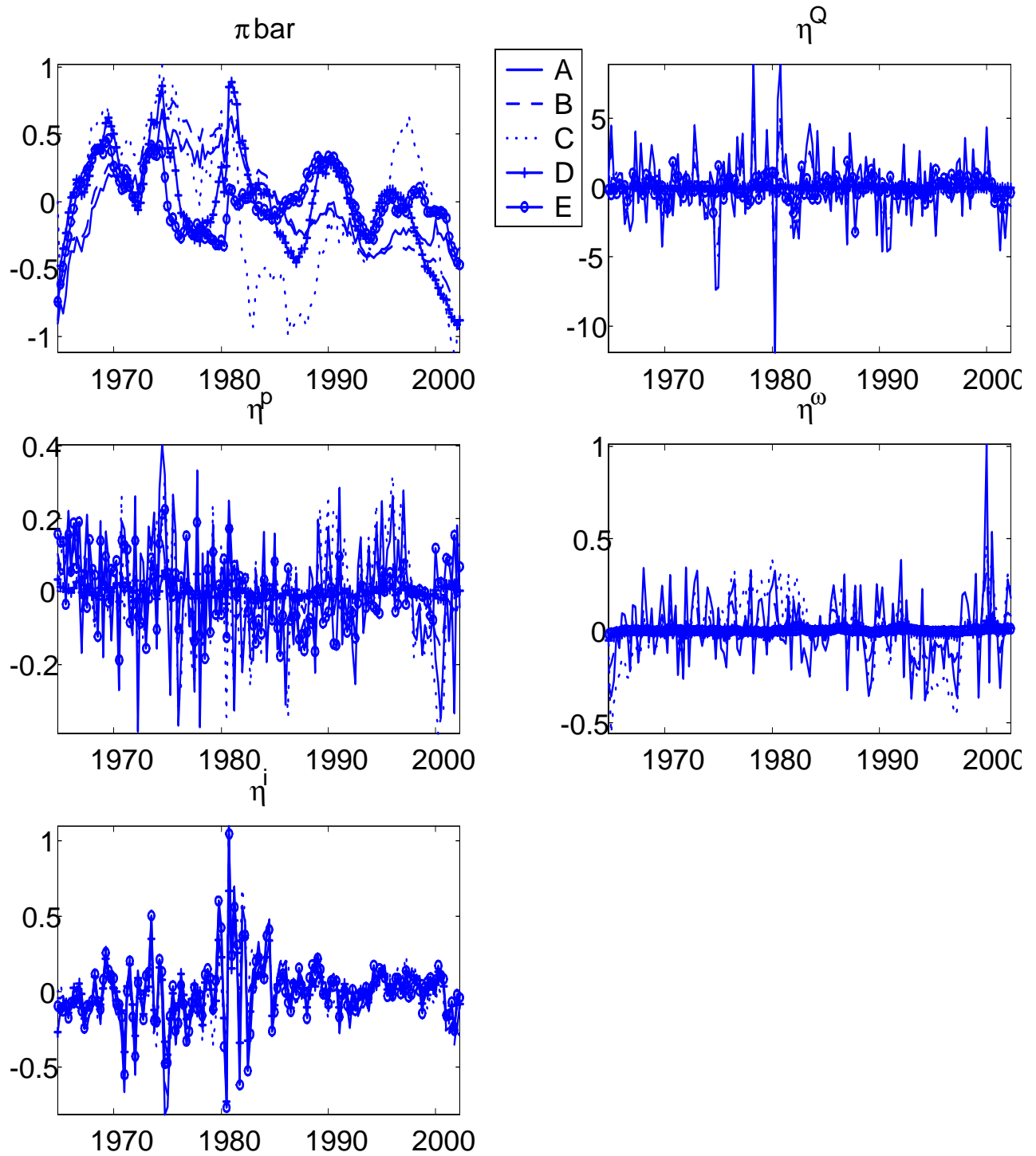Figure 3: Estimated time series of capital $(K)$ and exogenous shocks $(J = 1)$.

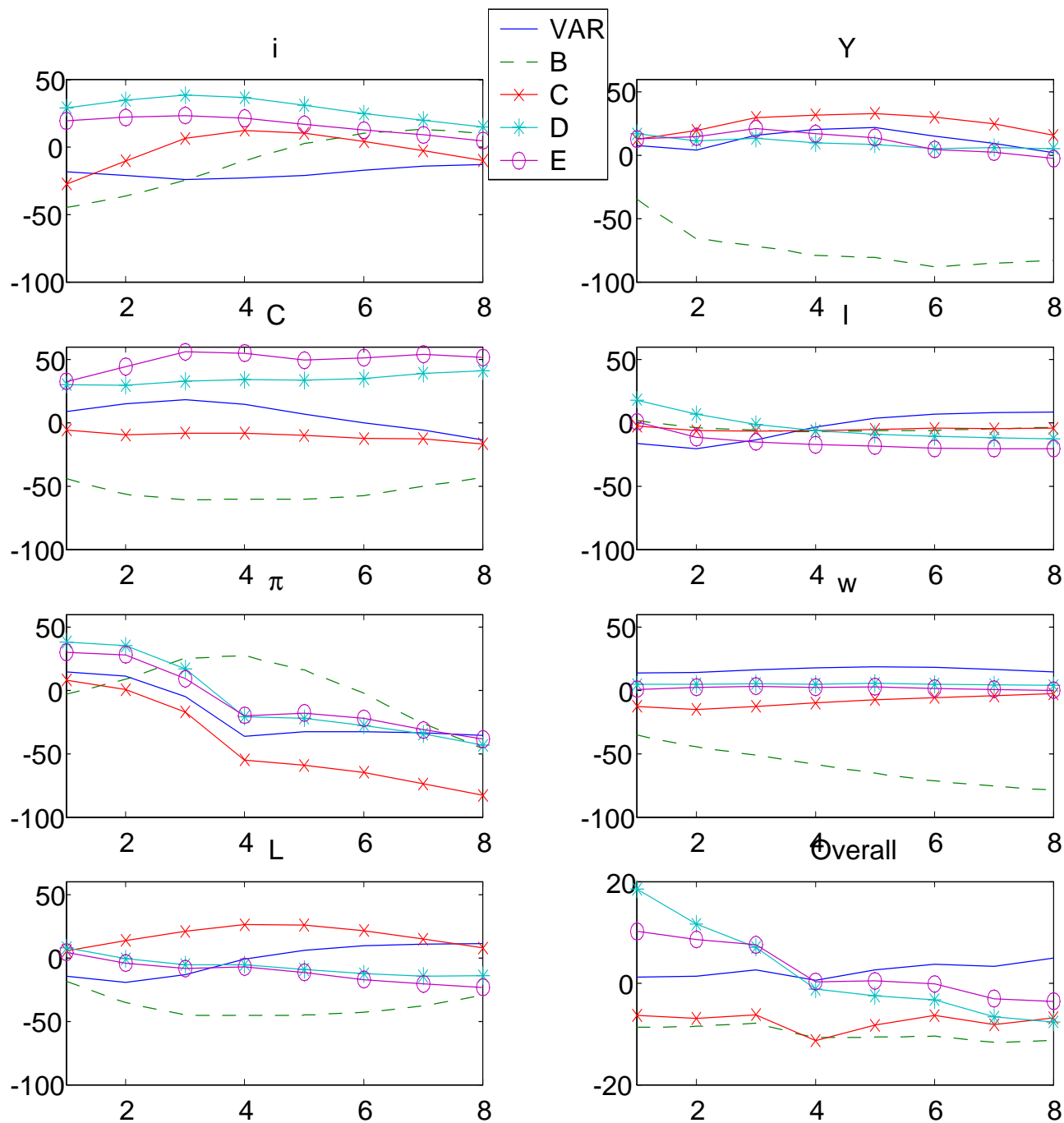Figure 4: Estimated time series of exogenous shocks (continued, $J = 1$).

Figure 5: Percentage gain in forecasting performance relative to case A for different forcasting horizons ($J = 1$). Percentage gain is measured by the percentage reduction in the root mean squared errors (RMSE) of forecasting relative to RMSE of case A. The overall measure corresponds to the log determinant of the forecast-error covariance matrix. The percentage gain in the overall measure is 100 times the difference in overall measure (improvement in forecast performance relative to case A) divided by the number of variables and divided by 2 to convert the variance to standard errors.
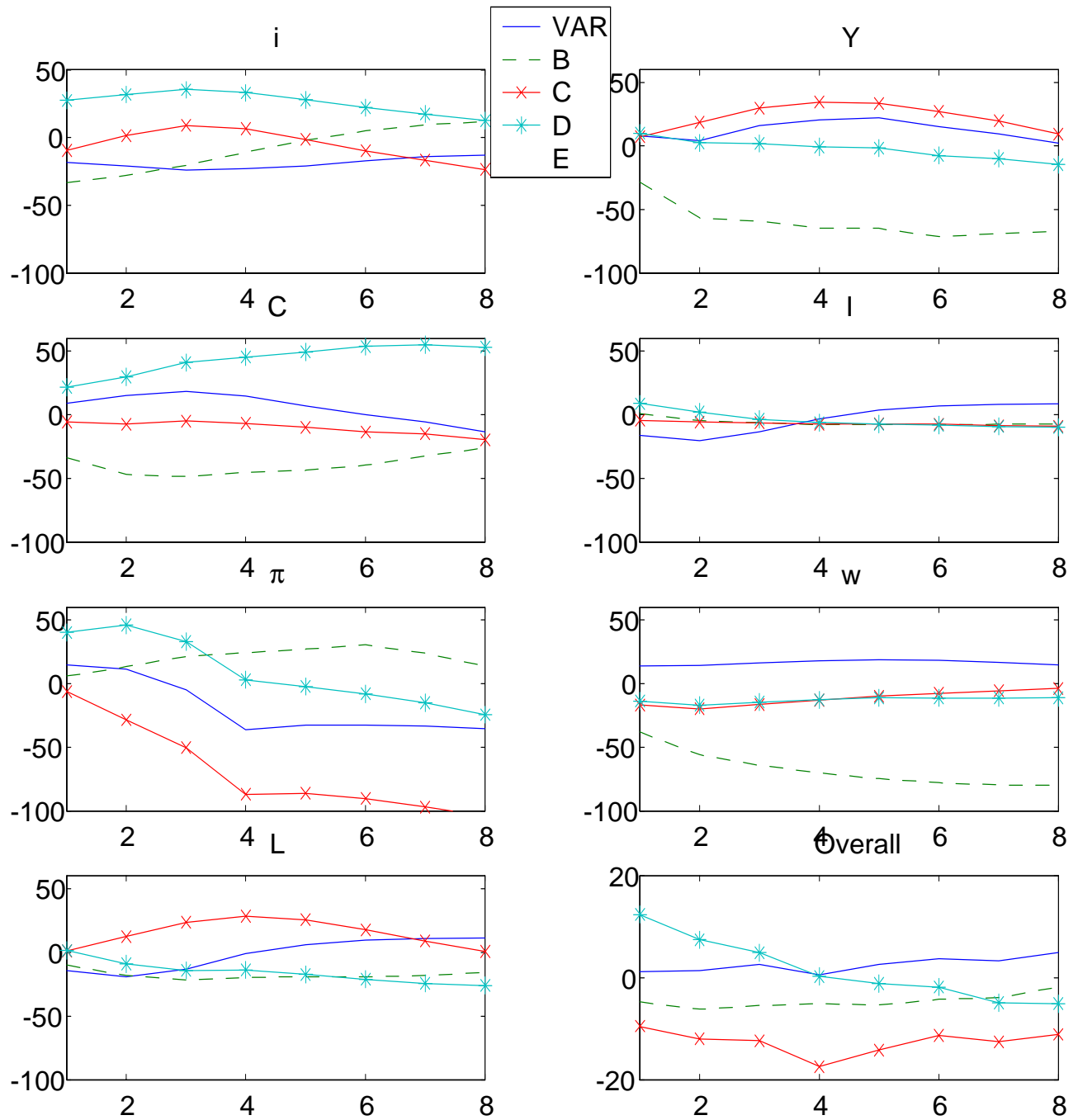
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Figure 6: Percentage gain in forecasting performance relative to case A for different forcasting horizons ($J = 5$). See Figure 5 for explanations.
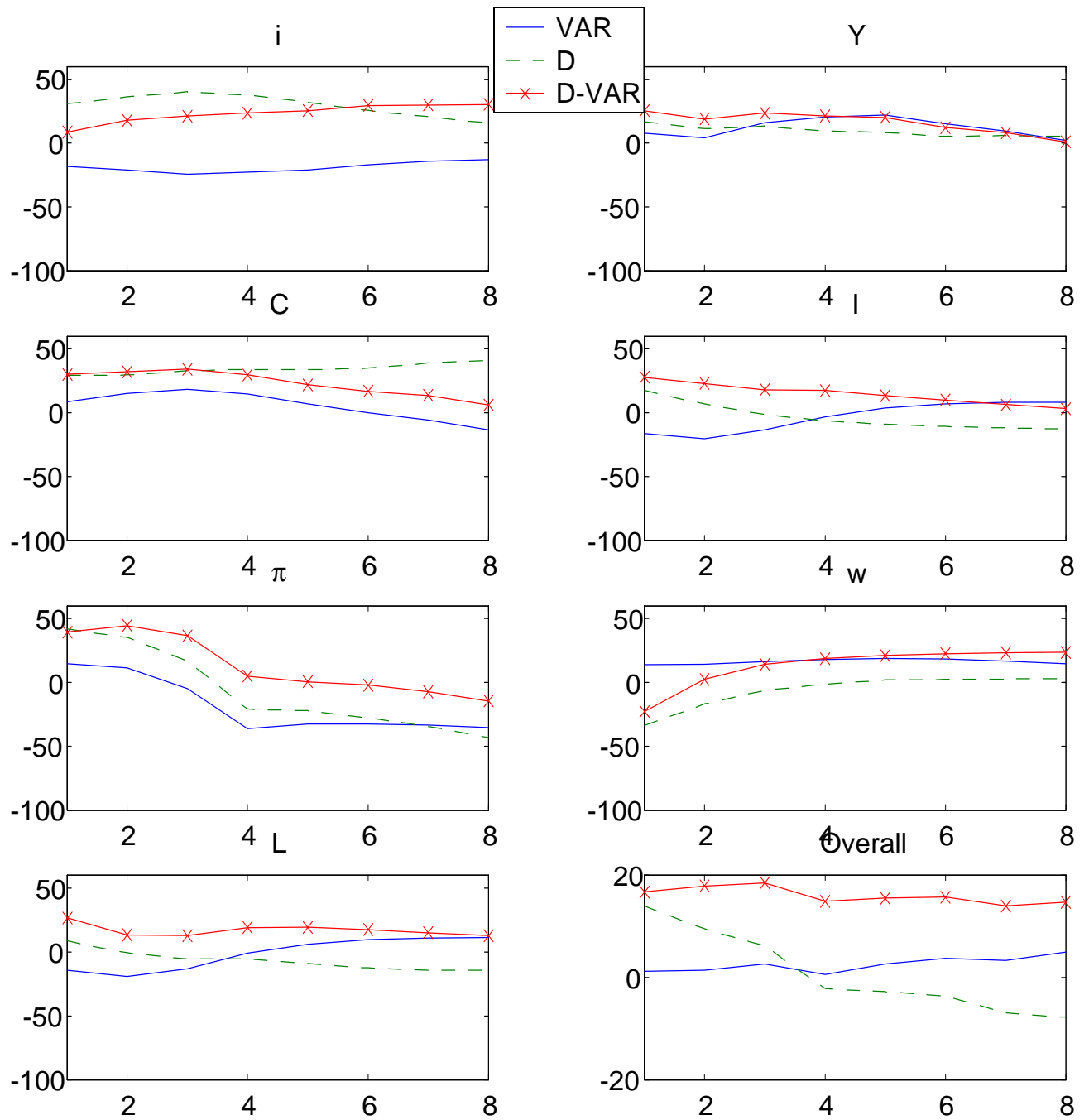
Figure 7: Percentage gain in forecasting performance relative to case A for different forcasting horizons ($J = 1$). See Figure 5 for explanations.