

The Social Contract with Endogenous Sentiments*

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Abstract

We present a model in which an individual's sentiments toward others are determined endogenously on the basis of how they perform relative to the societal average. This, in turn, affects the individual's own behavior and hence other agents' sentiments toward her. We focus on stationary patterns of utility interdependence in a production economy with redistributive taxation. There are two types of stationary equilibria: one in which all agents conform to the societal norm, and a second involving social stratification on the basis of productivity into two or three groups. The tax structure, in that it affects behavior which in turn affects sentiments, plays a crucial role in determining which type of equilibrium occurs and its characteristics as well as the extent of altruism and social cohesion in society. We then show that both types of social contract can be sustained as a political equilibrium. In the cohesive equilibrium with high redistribution, sentiments will be such that a majority of individuals will support high taxation, while in the clustered society with low redistribution the majority of voters will be in favor of keeping taxes low.

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1 Introduction

In spite of the strong similarities in their "fundamentals", the US and Europe show remarkable differences in their "social contract". Accordingly with Alesina et al. (2000) the share of welfare transfers over GDP in 2000 was 11 percent in the US and 18 percent in Europe and the share of total government spending for the same year (excluding interest payments) was 30 percent and 45 percent, respectively. But this is not the only channel through which Europe has built a more redistributive society than the US. Income taxes are more progressive, education and health are publicly provided, and the labor market is much more regulated. These interventions have produced significant differences in the working of the labor market. The unemployment rate is much higher in Europe and the unemployment spells longer. Even in the hours worked there are marked differences across the Atlantic. Alesina et al (2001) find significant differences not only in the average number of hours worked, but also on its dispersion across the population. The mean and the dispersion are larger in the US. Finally, and not independently of the previous features, sociologists and economists have identified a process of a disappearing middle class in the context of U.S. society.¹

Why such seemingly similar societies have ended up with so different social contracts remains an open, challenging question. To add to the puzzle, as pointed out by Bénabou (2000), lower redistribution comes together with higher pre-tax income inequality, while the standard politico-economic argument would predict the opposite: the higher the inequality the more the median voter would benefit from redistribution. Why poor American voters do not press for higher redistribution?

The different arguments put forward are all based on the role played by the different income mobility in the two societies. In a highly mobile society, as the American is supposed to be, poor people would be willing to accept less redistribution in order not to cut the future benefits in case they move up along the income ladder. Bénabou and Ok (2001) develop a model in which the poor face upward mobility prospects and show that under plausible assumptions poor people would vote for moderate redistribution. Picketty (1995) argues that the attitude towards redistribution depends

¹See, for example, Kosters and Ross (1988), Horrigan and Haugen (1988), and Duncan, Smeeding and Rodgers (1991). Wolfson (1994) and Esteban and Ray (1994) mention this phenomenon as a motivation for the concept of "polarization" of a distribution.

on the beliefs held by voters on the role of effort relative to luck in the determination of one's income. The larger the role of randomness the more individuals would vote for greater insurance. Whether this accords with evidence or is a mere belief it is debatable. But, in any case, the World Value survey provides solid evidence that while the majority of Europeans believe that income differences are a matter of luck, only a minority of Americans believe so.²

The problem with this explanation is that this belief does not seem to be substantiated by facts. There is no conclusive evidence that there is more income mobility in the US than in Europe. Yet, attitudes within the US and the European samples are stubbornly different. Accordingly with the results reported by Alesina et al. (2000), the attitudes of the US poor seem totally unaffected by inequality, while the European poor are positively averse to inequality. Further, 60 percent of the Americans interviewed (versus 30 percent of Europeans) believe that poor are lazy. These differences are particularly striking because it is the soft European welfare state the one that could be found guilty of inducing lazy people to remain in (alleviated) poverty. It remains an open question to explain where these negative attitudes towards the poor come from and why they are so much more negative in the country with the lowest degree of redistribution.

Clearly, there is a "cognitive dissonance" in the American society between the belief in the existence of a significant income mobility and the hard evidence that no major difference can be identified relative to the European standards. Bénabou and Tirole (2002) face the question squarely and propose an explanation for the persistence of such beliefs. In a nutshell, optimism about the prospects motivates individuals to work harder and, in the end, obtain a better income. These individuals may be inclined to vote for moderate taxation to keep the desired incentives. Further, if there is little redistribution, the consequences of insufficient motivation are harder and hence individuals would have an even greater incentives in keeping their beliefs highly optimistic.

This paper presents an alternative explanation for the differences between the two social contracts. We propose a simple and plausible mechanism of endogenous formation of individual sentiments for each other based

²See Alesina et al (2000).

on observed behavior.³ The sentiments towards others materialize into assigning some weight to their well-being. Our behavioral assumption is that individuals modify their concern for other people on the basis of how they behave relative to a standard set forth. Specifically, they compare the observed effort contributed by each individual to the societal mean. Thus, for example, if people work 40 hours per week on average, then those who work more are perceived as being industrious and those who work less are perceived as being lazy. Consequently, individuals will increase their esteem for the former and reduce it for the latter. Since the degree of redistribution will influence individual labor supply, it is the specific social contract that determines whether in the steady state equilibrium society forms a cohesive group with all supplying the same amount of labor or if it splinters into clusters with some agents excluded from consideration because their labor supply is below average. We find that indeed there are two types of stationary equilibria. In one all individuals conform to the standard of behavior and supply precisely the mean level of effort. Here, there is no social exclusion and altruism is inversely related to income. This equilibrium is attainable only if there is sufficient redistribution relative to the degree of inequality in individual productivities. In the other type of equilibrium society becomes stratified into two or three clusters: one group of highly productive “winners” who work more than the average number of hours and earn the full admiration of everyone, a second group of low productivity “losers” who work less than the mean and earn no esteem from others, and possibly a third group consisting of those with intermediate productivities who supply exactly the mean number of labor hours and may garner a range of esteem levels.

After analyzing how the social contract influences individual attitudes we examine the type of social contract these individuals would vote for. We show that both types of social contract can be sustained as a political equilibrium. In the cohesive equilibrium with high redistribution and no dispersion in individual labor supply, sentiments will be such that a majority of individuals will support high taxation, while in the clustered society with low redistribution and large dispersion in labor supply, the majority of voters will be in favor of keeping taxes low because the equilibrium sentiments entail admiration for the highly productive individuals and disregard

³Based on Esteban and Kranich (2002a), where we develop a general model of endogenous determination of individual sentiments.

for those performing below the mean.

In sum, our paper offers a simple theory of individual sentiments that has the virtue of determining simultaneously individual attitudes with each other, the pattern of labour supply, the distribution of pre-tax income and the degree of redistribution that would be chosen by majority voting.

In addition to offering a new view of the relationship between institutions and economic outcomes, our model yields further implications that accord well with observation and empirical evidence on individual behavior and the role of heterogeneity. There are numerous examples of rewards that are partially or entirely based on group – rather than individual – performance, thus involving some type of interpersonal redistribution within the team. Examples range from professional services – lawyers or architects – as examined in Kandel and Lazear (1992) to Japanese fishermen, studied by Platteau and Seki (2000). Standard theory would predict high levels of free-riding. Yet, in practice this type of reward is seen as stimulating high performance under some circumstances. Specifically, Platteau and Seki (2000, p.32) find that if the group is not too heterogeneous to begin with, a reward system based on pooling is self-reinforcing. If, on the contrary, a group is too heterogeneous initially, then it will progressively unravel. Recent empirical work by Nalbantian and Schotter (1997) is also in keeping with the view that profit sharing might have a positive role. By introducing an external effect on others, a profit-sharing mechanism induces the development of within-group altruism which in turn increases output. This, too, is consistent with our findings that the more homogenous the population (in terms of productivities), the more likely a common effort equilibrium is to occur.

The paper is organized as follows. In the next section we present the model. In Section 3 we explain the process by which sentiments change as the result of agents' behavior. Section 4 examines the existence and basic properties of stationary equilibria. In Section 5 we discuss the political equilibrium in taxation. Finally, Section 6 contains several concluding remarks and directions for future research.

2 The model

We consider an n -agent production economy in which agents have different abilities and each contributes labor to the production of a single consump-

tion good.⁴ Let $N = \{1, \dots, n\}$ denote the set of individuals. Agents are endowed with time, and each derives *direct* utility from consumption and leisure. However, their well-being also depends on their extended or *social* utility derived from the direct utility experienced by others.

We assume all agents have the same direct preferences represented by the utility function

$$u = u(c, L),$$

where c denotes the consumption good and L denotes labor.

In order to permit an explicit characterization of individual behavior we shall restrict our attention to the following specification:

$$u(c, L) = c - \frac{1}{1 + \gamma} L^{1+\gamma}, \quad \gamma > 0. \quad (1)$$

The social utility of individual i is given by

$$U_i = \sum_{j=1}^n \alpha_i^j u_j, \quad (2)$$

where α_i^j measures the sympathy or concern felt by individual i toward individual j . We assume $\alpha_i^j \leq \alpha_i^i$, for all i, j , and we normalize sentiments by taking $\alpha_i^i = 1$, for $i = 1, \dots, n$. We also assume $\alpha_i^j \geq 0$ for all i, j thus excluding malevolence.⁵ For notational simplicity, we write $\boldsymbol{\alpha}^i = (\alpha_i^1, \dots, \alpha_i^n)$, and we denote the entire $n \times n$ matrix of coefficients by $\boldsymbol{\alpha}$.⁶

Individuals differ in their productivity β_i , with $\sum_{j=1}^n \beta_j = n$, so that the effective labor supply by individual i is $\beta_i L_i$. The average productivity $\bar{\beta}$ will be equal to unity.

⁴Our description of the economy is similar to that in Sen (1966) and Ray and Ueda (1996). However, we depart from Sen in that we endogenize the extent of individual concern for others. This issue is addressed in the following section.

⁵The critical assumption for our results is that α_j^i is bounded below, even if this is an arbitrary negative number. We take the bound to be zero for convenience.

⁶Generally, bold letters denote vectors.

Output is linear in effective labor⁷

$$Y = \sum_{i=1}^n \beta_i L_i. \quad (3)$$

Without loss of generality, we will assume that the agents are ordered such that $\beta_1 \leq \beta_2 \leq \dots \leq \beta_n$.

One of the aims of this paper is to investigate the interplay between institutions and individual altruistic sentiments. A question we shall try to answer is whether some policies are more conducive to social cohesion while others may precipitate social clustering or fragmentation. In particular, we shall focus on the role of redistributive taxation in generating such outcomes.

Thus, we suppose labor income is taxed at a given rate $\tau \in [0, 1]$ and the proceedings are redistributed equally among all agents⁸. Hence, individual after-tax disposable income is

$$y_i^d = (1 - \tau)w_i L_i + \frac{\tau \sum_{h=1}^n w_h L_h}{n} = c_i, \quad (4)$$

so that after-tax income is entirely consumed. We shall use y_i to denote pre-tax income, $y_i = w_i L_i$.

Given the wage vector \mathbf{w} , the tax rate τ , and the altruism coefficients α^i , the choice problem facing individual i consists of selecting the labor supply L_i to maximize $U_i(\mathbf{L})$, subject to the budget constraint(4).

This problem can be rewritten as

$$\max_{L_i} \sum_{j=1}^n \alpha_i^j \left[(1 - \tau)w_j L_j + \frac{\tau \sum_{h=1}^n w_h L_h}{n} - \frac{L_j^{1+\gamma}}{1 + \gamma} \right]. \quad (5)$$

⁷In an earlier version of this paper, Esteban and Kranich (2002b), we examined the case of a CES production function and obtained that the degree of complementarity between the different types of productivity had no effect on the qualitative results obtained.

⁸This is exactly equivalent to the manner in which rewards within a team depend partly on own performance and partly on the joint effort.

Since U_i is concave, the solution to (5) is interior and is given by

$$L_i^\gamma = w_i \left[(1 - \tau) + \tau \frac{\alpha_i}{n} \right], \quad (6)$$

where $\alpha_i \equiv \sum_{j=1}^n \alpha_i^j$ is the total altruism felt by i . Equation (6) describes the optimal behavior of an individual when facing the parameters $\langle \mathbf{w}, \tau, \boldsymbol{\alpha}_i \rangle$.

The first order condition (6) tells us that the marginal disutility of effort L_i^γ should be equated to its total marginal return, private $(1 - \tau) w_i$ plus social $\alpha_i \frac{\tau w_i}{n}$.

Note that γ is the inverse of the wage elasticity of labor supply. As γ becomes large labor supply is less elastic and in the limit it becomes rigid.

Definition 1 *Given the technology, parametrized by $\boldsymbol{\beta} = (\beta_1, \dots, \beta_n)$, as well as τ and $\boldsymbol{\alpha}$, an equilibrium consists of n -vectors \mathbf{L} and \mathbf{w} such that for all $i \in N$, L_i satisfies (6) and $w_i = \beta_i$.*

It follows that the equilibrium labor supply will be

$$L_i = \left[\beta_i \left((1 - \tau) + \tau \frac{\alpha_i}{n} \right) \right]^{\frac{1}{\gamma}}. \quad (7)$$

3 The dynamics of individual sentiments

We now turn to the issue of the endogenous determination of individual sentiments. Here, individual sentiments are reflected in the coefficient matrix $\boldsymbol{\alpha}$. As indicated above, we do not attempt to explain where those sentiments come from initially but simply how they evolve in response to the observed behavior of others, that is, how agents modify their sentiments. We then focus on stationary patterns of interdependence.

The key element of our model, and that which sets it apart, is the assumption that each individual formulates a standard of behavior for others. In our case, such a standard consists of an expected labor supply. Then if individual j 's actual labor supply exceeds i 's standard for j , then i increases its esteem for j . Conversely, if j supplies less than the standard, then i lowers its esteem. If j exactly conforms to i 's standard, then no adjustment occurs.

Two questions arise: (1) how do agents set standards for others? and (2) how do they revise their sentiments in the event other agents fail to conform to their standards?

Regarding the latter, rather than postulate a particular updating procedure, we simply require, as stated previously, that i 's esteem for j changes in accordance with the difference between j 's actual labor supply and i 's standard for j . Formally, treating time as a discrete variable, let $L_j(t)$ be j 's actual labor supply at time t , $L_j^i(t)$ be the amount of labor i thinks j should contribute at t , and $\alpha_i^j(t)$ be the esteem i feels for j at t . Then we require only that $\alpha_i^j(t+1) \geq \alpha_i^j(t)$ as $L_j(t) \geq L_j^i(t)$. Alternatively, we write

$$\alpha_i^j(t+1) = g_i(\alpha_i^j(t), L_j(t) - L_j^i(t)), \quad (8)$$

where g_i is an arbitrary function that is nondecreasing in both arguments, bounded above by 1, bounded below by 0, and $g_i(\alpha, 0) = \alpha$, and we assume g_i is given.

Definition 2 *Then given $\langle \beta, \tau \rangle$, a stationary equilibrium is a triple $(\alpha, \mathbf{L}, \mathbf{w})$ such that (\mathbf{L}, \mathbf{w}) is an equilibrium with respect to $\langle \beta, \tau, \alpha \rangle$, as defined above, and $g_i(\alpha_i^j, L_j - L_j^i) = \alpha_i^j$, for all $i, j, i \neq j$.*

Turning to the issue of how agents set standards, in Esteban and Kranich (2002a) we explore various formulations. Here, we concentrate on a simple and plausible rule, namely, that each agent takes the mean behavior as the societal norm and judges other agents' actions accordingly. For example, if the norm is to work 40 hours per week, then one might measure laziness or industriousness relative to this standard. Thus, anyone supplying labor in excess of the mean garners additional respect, while anyone contributing less loses respect. Notice that in this case all agents revise their coefficients uniformly, that is, all revise their concern for, say, j in the same direction (with the exception of j herself). Stating this formally, let $\bar{L}(t)$ denote the mean of $\{L_1(t), \dots, L_n(t)\}$. According to the *mean standard of behavior*, $L_j^i(t) = \bar{L}(t)$, for all $i, j = 1, \dots, n, i \neq j$.

Note that the proposed ethical norm has a "calvinist" flavor in that it implicitly posits that personal circumstances should be no excuse for falling short of the norm. Think for instance of the widespread view that people working in education are not true hard workers because of the long vacation periods.

Society wide we cannot have a direct observation on the individual behavior of most of the population. We have however views on categories of the population: taxi drivers, farmers, CEOs and so on. The development of concern for others just described can be conceived as referring to broad categories of individuals. Also, in large societies, individuals do not strategically choose their behavior to create an impression on others because the effect they can have on the social image of their own category is seen as negligible.

In the next section, we study the existence and properties of steady-state equilibria under the mean standard of behavior.

4 Stationary equilibrium sentiments

In the model we have described, labor supply and output depend on individual sentiments of concern for each other and these sentiments depend in turn on the observed labor supply. We wish to examine the stationary equilibria of this dynamic adjustment process.

There are two types of stationary solutions.

One corresponds to the case in which everyone conforms to the average behavior. In this case, whatever is the matrix α of coefficients necessary to support such an equilibrium, it will clearly be stationary since no agent will have reason to modify her esteem for any other agent. Since all the agents conform to the behavioral norm, in equilibrium all the agents will supply the same amount of labor effort. We shall call this equilibrium "cohesive" because all individuals will end up following a common pattern of behavior.

In the second type of stationary equilibria, which we shall call "clustered", society is divided into well-defined social groups: one set of individuals who conspicuously work more than the average, another set who work less than the mean, and possibly a third set who work precisely the average. We denote these groups respectively by A , B and M , as they work above, below or at the societal mean, respectively. Therefore,

$$i \in A \iff L_i > L, \quad i \in B \iff L_i < L, \quad \text{and} \quad i \in M \iff L_i = L = \frac{1}{n} \sum_j L_j. \quad (9)$$

This situation can become a stationary equilibrium when individual esteem has already reached its maximum for the hard-workers and its minimum for

the low performers. In this equilibrium a set of individuals will be admired by everyone while another set will earn no consideration at all, thus carrying a social stigma. Formally,

Definition 3 Given $\langle \beta, \tau \rangle$,

(i) a cohesive stationary equilibrium is a triple $(\alpha, \mathbf{L}, \mathbf{w})$ such that (\mathbf{L}, \mathbf{w}) is an equilibrium with respect to $\langle \beta, \tau, \alpha \rangle$, as defined above, $g_i(\alpha_i^j, 0) = \alpha_i^j$, for all i, j , $i \neq j$ and $L_i(t) = \bar{L}(t)$, for all $i = 1, \dots, n$, and

(ii) a clustered stationary equilibrium is a triple $(\alpha, \mathbf{L}, \mathbf{w})$ such that (\mathbf{L}, \mathbf{w}) is an equilibrium with respect to $\langle \beta, \tau, \alpha \rangle$, as defined above, the sets A and B as defined by (9) are non-empty, and $\alpha_i^j = 1$ for all $j \in A$, $\alpha_i^j = 0$ for all $j \in B$, $\alpha_i^j \in (0, 1)$ for all $j \in M$, for all $i \in N$.

We now proceed to the analysis of the two types of stationary equilibria.

4.1 Cohesive, common-effort equilibria

In this case $L_i = L$ for $i = 1, \dots, n$ and $y = \frac{Y}{n} = L$. In view of (7) we observe that the determinant of individual labor supply is total altruism, $\alpha_i = \sum_j \alpha_i^j$ (together with the wage rate), independently of how this total altruism is distributed over the population.

Let us choose an arbitrary common effort, L . Using this value of L in (7) we obtain the level of (total) altruism α_i that renders L the optimal individual choice for each productivity level β_i :

$$\alpha_i = -\frac{(1-\tau)n}{\tau} + \frac{nL^\gamma}{\tau} \frac{1}{\beta_i}. \quad (10)$$

For each L , this defines a relationship between α and β which is depicted in Figure 1. Note that $\alpha \in [1, n]$ and $\beta \in [\beta_1, \beta_n]$.

We, thus have obtained the following property of cohesive equilibria:

Proposition 4 *In any cohesive equilibrium individual total altruism is inversely related to productivity and income.*

Figure1.

Note, that this derived negative relation between income and altruism is endogenous. The intuition of this result is as follows. In a cohesive equilibrium with common effort —and because preferences are additively separable— the marginal direct utility cost of effort is the same for all individuals, regardless of their personal productivity. In equilibrium, all individuals have to receive the same marginal reward. Therefore, highly productive individuals with high private marginal returns have to have low social returns, while the low productive individuals need to make up their return with a highh social marginal motivation.

Can there such a cohesive stationary equilibrium exist? In order to see that such an equilibrium exists, we need only show that there exists some L such that the corresponding values for α are indeed feasible, i.e. $\alpha_i \in [1, n]$ for all i .

Given L and β , the maximum degree of altruism corresponds to the individual with the lowest productivity β_1 , and the smallest altruism to the largest productivity β_n . Since $\alpha_i \in [1, n]$, feasibility requires that

$$\alpha_1 = -\frac{(1-\tau)n}{\tau} + \frac{nL^\gamma}{\tau} \frac{1}{\beta_1} \leq n, \quad (11)$$

and

$$\alpha_n = -\frac{(1-\tau)n}{\tau} + \frac{nL^\gamma}{\tau} \frac{1}{\beta_n} \geq 1. \quad (12)$$

Putting the two restrictions together, one can readily obtain the following Proposition.

Proposition 5 *Let $\langle \beta, \tau \rangle$ be given. A common effort equilibrium $(\alpha, \mathbf{L}, \mathbf{w})$ exists if and only if*

$$\frac{n}{n-1} \frac{\beta_n - \beta_1}{\beta_n} \leq \tau \leq 1. \quad (13)$$

According to (13), an equal effort stationary equilibrium cannot be achieved in all economies. The ratio on the left hand side is the *relative spread* of the distribution of productivities over the population. The relative spread is a (crude) measure of inequality within a distribution. Thus, the inequality in the “fundamentals” of the economy sets a lower bound on the degree of redistribution necessary to support equal effort equilibria.

This result tells us that the type of sentiments necessary to support a cohesive equilibrium require a minimum degree of redistribution. The intuition for this result is straightforward. In a society where individuals require to supplement their private, direct incentives with social incentives in order to contribute the same level of effort, it is indispensable that the social contract permits an effective impact of one's effort on the well-being of the people one cares about. Notice that this result *reverses* the standard causality argument: here we need redistributive institutions to generate fellow-feeling sentiments.

A second implication of this result is that a productivity shock which increases the spread of individual productivities may drive an economy out of the cohesive equilibria with the corresponding change in individual sentiments.

Let us finally point out that, from discussion above, it is obvious that whenever equilibria exist, there will be many.

Proposition 6 *Suppose (13) holds. Then, for every L satisfying*

$$\beta_n \left[(1 - \tau) + \frac{\tau}{n} \right] \leq L^\gamma \leq \beta_1, \quad (14)$$

there exists a matrix α for which the economy is in a stationary equilibrium.

To see this, simply take conditions (11) and (12) and find the maximum and the minimum (resp.) values of L which render them equalities.

A diagrammatic representation of the different equilibria is depicted in Figure 2. For a given level of effort, L , the range of equilibrium values of α consistent with L is determined by (10) evaluated at $\beta = \beta_1$ and $\beta = \beta_n$. The minimum level of effort, L_1 , corresponds to the case in which the most productive individual is completely egoistic, i.e. when the curve defined by (10) passes through $(\beta_n, 1)$. The maximum effort, L_n , is obtained when the least productive individual is fully altruistic, i.e. when the curve passes through (β_1, n) .

Figure 2.

Notice that this continuum of equilibria can be parametrized by the degree of altruism of any particular individual. That is, given β_i , one can derive from (10) the range of α_i consistent with each L that might occur in equilibrium. Which equilibrium will prevail will depend on the initial conditions and the assumed adjustment process for sentiments. In any case, the higher the degree of final altruism, the larger the aggregate output.

4.2 Clustered equilibria

Let us now turn to the second type of equilibria in which society is partitioned into clusters with differentiated behavior. There can be at most three clusters consisting of those agents who supply above average labor, *A-types*, those who supply below average, *B-types*, and possibly a group who supply exactly the mean number of labor hours, *M-types*. In such an equilibrium all those individuals supplying effort above the mean will earn the (maximal) esteem of everyone in society, while those performing below will merit no concern by anyone other than themselves. Because the degree of esteem is bounded above and below, individual sentiments with respect to those in *A* or *B* will reach stationary values on the boundary. At the same time, those in *M* conform to the standard of behavior and might garner a range of esteem levels. Whether a middle group will exist or not will depend on the individual productivities as well as on the degree of redistribution. We focus our discussion on the case of a two-cluster equilibrium and then provide the key insights for the case of three-cluster equilibria.

In a two-cluster equilibrium, all members of group *A* will be admired by everyone and to the maximal extent. Members of group *B* will be admired by no one other than themselves. Let a and b denote the respective cardinalities of the two groups. Then, for every $i \in A$ we have that $\alpha_i = a$, and for every $i \in B$ we have $\alpha_i = a + 1$.

Writing the corresponding labor supply functions (7), and using (3), we have

$$L_i^A = \left[\beta_i \left(1 - \tau + \tau \frac{a}{n} \right) \right]^{\frac{1}{\gamma}}, \quad (15)$$

and

$$L_i^B = \left[\beta_i \left(1 - \tau + \tau \frac{a+1}{n} \right) \right]^{\frac{1}{\gamma}}, \quad (16)$$

where L_i^A and L_i^B denote the labor supply by the i^{th} member of group *A* and *B*, respectively.

The two labor supply curves are depicted in Figure 3, where for diagrammatic convenience we use the transformation $(L_i^k)^\gamma$, $k = A, B$. The transformed labor supply is a linear function of β passing through the origin. Total altruism by the low types is higher than among the high types.

Hence, the slope of the (transformed) labor supply by the B types will be steeper than for the A types. For any particular β , the B types would choose to supply strictly more effort than the A types. Therefore, to support a two-cluster equilibrium in which the B types supply less effort than the A types, we need that there exists a threshold productivity level, $\tilde{\beta}$, such that $(n - a)$ agents have productivity less than $\tilde{\beta}$ and supply effort between c and d along the type B supply curve (such as L_i^B), while a agents have productivity greater than $\tilde{\beta}$ and supply labor between e and f along the type A curve (such as L_i^A). Moreover, it must be the case that the mean labor supply (indicated by \bar{L} in the figure) vertically separate the two chords.

FIGURE 3.

It is immediate that the existence of such an equilibrium will critically depend on the shape of the distribution of productivities.

In the same fashion as for the common effort equilibrium, the existence of equilibrium jointly depends on τ and the shape of the distribution of β . Unfortunately, we cannot give here an explicit, closed form characterization of the entire class of economies containing two-cluster equilibria. Yet, we can prove the following result concerning the existence of taxes for given distribution of productivities, and the existence of a vector β for given taxes, such that a two-cluster equilibrium exists.⁹

Proposition 7 *For any (unequal) distribution of productivities β there is always τ small enough for which a two-cluster equilibrium exists. Further, for any $\tau \in [0, 1)$ there exists a distribution of productivities such that a two-cluster equilibrium exists.*

Three remarks are in order. The first one concerns individual behavior in a two-cluster equilibrium. In spite of the fact that the population is clustered into two classes, this is generally compatible with a rich heterogeneity of individual behavior within each class. Indeed, in this equilibrium the work time supplied is strictly increasing with individual productivity.

Our second remark, wishes to point out that for arbitrary $\tau > 0$ a two-cluster stationary equilibrium may or may not exist, depending on τ and the distribution of β . However, consider the special case of a perfectly bipolar distribution of productivities, e.g. unskilled and skilled labor, with $\frac{n}{2}$

⁹See Appendix.

individuals with productivity β_A and $\frac{n}{2}$ with productivity β_B , with $\beta_A > \beta_B$.¹⁰ Then it can be readily computed that if

$$\frac{\beta_A - \beta_B}{\beta_B} \geq \frac{2\tau}{n(2 - \tau)}, \quad (17)$$

then a two-cluster equilibrium.

The third remark is that condition (17) depends on n . The larger is n , the less restrictive is the inequality and hence the more likely is a given (bi-polar) β to satisfy the restriction. In the (lower) limit case, when $n = 2$, condition (17) requires that,

$$\tau \leq 2 \frac{\beta_A - \beta_B}{\beta_A}. \quad (18)$$

Inequality (18) is complementary to condition (13) for the existence of a common effort equilibrium. For $n = 2$, we either have a common effort equilibrium or a two-cluster equilibrium depending on the degree of redistribution.

Finally, let us briefly refer to the case of three-cluster equilibria. Again, individuals are classified into three groups, A , B and M , depending on the quantity of their labor supply relative to the societal average. We use a and b to denote the cardinalities of the first two groups, and we let m denote the cardinality of the third group, which we refer to as the “middle class.” Recall that each member of M supplies precisely the average labor hours.

Individual choice of labor effort is given by (7), where what matters is the total esteem felt, α_i , and not how this esteem is distributed. Because of the nature of this equilibrium, the esteem earned by each member of groups A and B from every other member of the economy is 1 and 0, respectively. Only the members of the middle group may experience varying degrees of esteem.

The relative size of the middle class has obvious implications for the heterogeneity of observed behavior. As the middle class shrinks, the set of individuals conforming to a common pattern of labor supply shrinks too. In the limit, as the middle class vanishes all individuals deviate and the hours worked will be as disperse as the individual productivities.

¹⁰Here, we implicitly assume n is even. If n were odd, then it would be sufficient to assign $\frac{n-1}{2}$ agents productivity β_A and $\frac{n-1}{2} + 1$ productivity β_B .

Let us consider the least and the most productive members of the middle class with productivities β_{b+1} and β_{b+m} , respectively. For a three-cluster equilibrium the following result can be proven.¹¹

Proposition 8 *A necessary condition for the existence of a three-cluster equilibrium is that*

$$\frac{\beta_{b+m} - \beta_{b+1}}{\beta_{b+m}} \leq \frac{(m-1)\tau}{n - \tau b}. \quad (19)$$

Notice that (19) generalizes restriction (13), which is necessary and sufficient for the existence of a common effort equilibrium. Indeed, this corresponds to the case in which $m = n$ and $b = 0$. For a given size m and productivities of the middle class this result tells us that a minimum degree of redistribution is necessary to support this three-cluster structure as an equilibrium.

Also, (19) sets an implicit upper bound on the size of m . Let us start by observing that as $\tau \rightarrow 0$ the *RHS* $\rightarrow 0$ and hence the support of the productivities of the eventual members of the middle class vanishes.

Our second observation is that, since $m \leq n - 2$ and $b \geq 1$, we have

$$\frac{(m-1)\tau}{n - \tau b} \leq \frac{(n-3)\tau}{n - \tau}.$$

Therefore, substituting in (19), a *necessary condition for the existence* of a three-cluster equilibrium is that

$$\frac{\beta_{b+m} - \beta_{b+1}}{\beta_{b+m}} \leq \frac{(n-3)\tau}{n - \tau}. \quad (20)$$

Clearly, the lower is τ , the smaller the permissible support for the distribution of productivities of the middle class and hence the smaller that class will be.

Proposition 9 *In a three-cluster equilibrium the upper bound on the size of the middle group m increases with the degree of redistribution.*

¹¹See Esteban and Kranich (2002b).

5 Voting over taxes

In most of the literature on (endogenous) income taxation, taxes rightly depend on individual preferences. The more altruistic the agents, the more redistribution we would expect. As described in the Introduction, our argument is that the relationship goes the other way as well. Different redistributive regimes elicit different behavior which, in turn, engender different levels of concern among the participants. Thus, the institutional structure affects the type of society that develops. The question arises as to whether individuals equipped with the sentiments generated in each type of society would democratically choose the tax rate that would support it as an equilibrium.

Following the steps of Roberts (1977) and Meltzer and Richard (1981), we examine whether there are equilibria with taxes that would be supported by a majority of the population. We shall now show that the same economy can have multiple politico-economic equilibria. High redistribution gives rise to such sentiments and (equal) labor supply that a majority of voters would favor such high tax rate, while low redistribution generates the sentiments and high variance labor supply that makes a majority to support low taxation.

Specifically, we shall show that for every common effort equilibrium (with the corresponding tax rate) there are individual preferences for which a majority of individuals would vote for an increase in taxation. On the other hand, we shall show that for every two-cluster equilibria there are preferences for which a majority of individuals would vote for a reduction in the tax rate. It follows that societies with similar structural characteristics, depending on the initial conditions, may end up having radically different social contracts, individuals values and social structures.

As in Meltzer and Richard (1981) we assume that individuals know the labor supply function of the other individuals and take it into account when computing the effects of a change in taxation.

Differentiating extended utility of individual i with respect to t we obtain

$$\frac{dU_i}{dt} = \sum_k \alpha_i^k \frac{du(c_k, l_k)}{dt}, \quad (21)$$

where $u(c_k, l_k)$ is the direct utility of individual k .

Expression (21) tells us that the marginal valuation of a tax increase by any individual i will be a weighted average of the direct marginal impacts on every individual in society. The terms $\frac{du(c_k, l_k)}{dt}$ will be the same for every individual i . What makes the valuation of a given tax change different across individuals is that they might have different sentiments with respect to their fellow citizens and apply different weights, i.e. different α_i vectors.

Notice that now the allocation of the total altruism α_i over the population is essential. We shall thus need being more specific on individual sentiments. So far, our assumption on the ethical norm followed has been strictly "calvinist": personal circumstances are no excuse. While still keeping with this ethical rule, we shall add a mild "catholic" supplement: when we observe two individuals contributing the same effort, we develop a higher appreciation for the least productive one. This implies that if $l_k = l_h$ and $\beta_k > \beta_h$, then $\alpha_i^k < \alpha_i^h$.

Let us thus focus on the terms $\frac{du(c_k, l_k)}{dt}$. Differentiating k 's direct utility with respect to t we obtain (after using (7))

$$\frac{du(c_k, l_k)}{dt} = -\beta_k l_k - \beta_k \frac{t\alpha_k}{n} \frac{\partial l_k}{\partial t} + \frac{dr}{dt}. \quad (22)$$

After some manipulation,¹² we can obtain

$$\frac{du(c_k, l_k)}{dt} = y - y_k + \frac{(l_k - (1-t)\beta_k l_k^{1-\gamma})(\beta_k - l_k^\gamma)}{t\gamma} - t \sum_j \frac{\beta_j l_j^{1-\gamma}(\beta_j - l_j^\gamma)}{n t\gamma}. \quad (26)$$

Note that, in view of (7), l_k^γ is independent of γ and that $\lim_{\gamma \rightarrow \infty} l_k = 1$, $\lim_{\gamma \rightarrow \infty} y_k = \beta_k$ and $\lim_{\gamma \rightarrow \infty} y = 1$.

Therefore, as the labor supply elasticity decreases (i.e. γ increases) we find that

$$\lim_{\gamma \rightarrow \infty} \frac{du(c_k, l_k)}{dt} = 1 - \beta_k. \quad (27)$$

For γ sufficiently large, an increase in the tax rate will produce an increase or decrease in the direct utility of an individual k as this individual's productivity is below or above the mean. Therefore, the direct marginal valuation of an increase in t will be decreasing with income.

¹²Notice that

$$\frac{dr}{dt} = y + t \frac{dy}{dt} = y + t \sum_j \frac{\beta_j}{n} \frac{\partial l_j}{\partial t}, \quad (23)$$

where $y = \frac{Y}{n}$.

Differentiating (7) with respect to t we obtain (after using again (7) to eliminate α_i)

$$\frac{\partial l_i}{\partial t} = -\frac{\beta_i(n - \alpha_i)l_i^{1-\gamma}}{\gamma n} = -\frac{l_i^{1-\gamma}(\beta_i - l_i^\gamma)}{t\gamma}. \quad (24)$$

Hence, using (7) once more, we can write

$$\frac{du(c_k, l_k)}{dt} = y - y_k + \frac{(l_k - (1-t)\beta_k l_k^{1-\gamma})(\beta_k - l_k^\gamma)}{t\gamma} + t \frac{dy}{dt}. \quad (25)$$

From (7) we can deduce that $\beta_k \geq l_k^\gamma$ and that $l_k \geq (1-t)\beta_k l_k^{1-\gamma}$.

Using (24) we have

$$t \frac{dy}{dt} = t \sum_j \frac{\beta_j}{n} \frac{\partial l_j}{\partial t} = -t \sum_j \frac{\beta_j l_j^{1-\gamma}(\beta_j - l_j^\gamma)}{n t\gamma}$$

It follows that

$$\lim_{\gamma \rightarrow \infty} \frac{dU_i}{dt} = \sum_k \alpha_i^k (1 - \beta_k) = \alpha_i - \sum_k \alpha_i^k \beta_k. \quad (28)$$

Let us now define the vector $\tilde{\alpha}_i$ such that $\tilde{\alpha}_i^k = \alpha_i^k$ for all $k \neq i$ and $\tilde{\alpha}_i^i$ being an arbitrary number such that $\tilde{\alpha}_i^i \in (0, 1)$. Note that (with some abuse in notation), using $\tilde{\alpha}_i$ to denote $\tilde{\alpha}_i = \sum_k \tilde{\alpha}_i^k$, we have that

$$\alpha_i = \tilde{\alpha}_i - \tilde{\alpha}_i^i + 1$$

We can now rewrite the expression above as

$$\lim_{\gamma \rightarrow \infty} \frac{dU_i}{dt} = 1 - \tilde{\alpha}_i^i + \tilde{\alpha}_i - (1 - \tilde{\alpha}_i^i) \beta_i - \sum_k \tilde{\alpha}_i^k \beta_k = \quad (29)$$

$$= (1 - \tilde{\alpha}_i^i) (1 - \beta_i) + \tilde{\alpha}_i \sum_k \left(\frac{1}{n} - \frac{\tilde{\alpha}_i^k}{\tilde{\alpha}_i} \right) \beta_k. \quad (30)$$

We are now ready for the analysis of majority voting under the different types of equilibria.

VOTING IN THE COHESIVE EQUILIBRIUM: We now focus on the equal effort equilibria. In this case, because of our additional assumption on sentiments, all individuals will allocate their sentiments for the others inversely related to their productivity. Observe now that

$$\sum_k \left(\frac{1}{n} - \frac{\tilde{\alpha}_i^k}{\tilde{\alpha}_i} \right) \beta_k \geq 0$$

for all vectors $\tilde{\alpha}_i$ with weights that are decreasing with productivity, i.e. such that $\tilde{\alpha}_i^{k+1} \leq \tilde{\alpha}_i^k$ for all k and that this is implied by α_i^k being decreasing in k , excluding $k = i$. It follows that for all the individuals with productivity below the mean, $\beta_k < 1$, an increase in the tax rate will increase their extended utility for γ sufficiently large. Therefore, in all common effort equilibria, and for all productivity distributions in which the median is below the mean, there is γ sufficiently large that there is a majority of individuals supporting an increase in the tax rate.

To complete the argument, we just need to mention that it can be shown¹³ that if $\gamma = 1$ for any equilibrium tax t individuals would unanimously support a reduction in taxation. Therefore, for any equilibrium tax t there exists γ for which the median voter would support it.

¹³See Esteban (2002)

VOTING IN THE CLUSTERED EQUILIBRIUM: Let us now turn to the case of a two-cluster equilibrium. Using, (28) and the α_i corresponding to this type of equilibrium we have that

$$\lim_{\gamma \rightarrow \infty} \frac{dU_i^A}{dt} = a - \beta_i - \sum_{k \in A} \beta_k, \text{ for all } i \in A \text{ and} \quad (31)$$

$$\lim_{\gamma \rightarrow \infty} \frac{dU_i^B}{dt} = 1 + a - \beta_i - \sum_{k \in A} \beta_k, \text{ for all } i \in B. \quad (32)$$

Clearly, the marginal utility of the individuals in the set B will be larger than the ones in the set A . Focusing on the marginal utility of the former, we observe that it depends on the size of grup A . The size of the group A is determined in equilibrium. Yet, it can be readily verified that that for all sizes of A $\lim_{\gamma \rightarrow \infty} \frac{dU_i^B}{dt} < 0$ for all $i \in B$ and *a fortiori* for all $i \in A$.¹⁴

Therefore, in any two-cluster equilibrium, for γ large enough, there is always a majority of individuals that would prefer taxes be reduced.

We can formally present our result in the following Proposition.

Proposition 10 *For any equilibrium, either cohesive or clustered, and any tax rate compatible with it, there are preferences with a sufficiently low wage elasticity of labor supply so that a majority of individuals would vote for the equilibrium tax rate.*

Let us start the discussion of this result by stressing that any tax rate compatible with either of the two types of equilibria can become a politico-economic equilibrium for appropriate preferences, i.e. for appropriate wage

¹⁴We start by noting that if A has one member only, it can be readily verified that $\frac{\beta_n + \beta_1}{2} > 1$ is sufficient for $\lim_{\gamma \rightarrow \infty} \frac{dU_i^B}{dt}$ be negative for all i . As the A group increases one unit its size, from a to $a+1$, the value of the marginal utility increases by $1 - \beta_{n-a-1}$. Therefore, as a gets larger $\lim_{\gamma \rightarrow \infty} \frac{dU_i^B}{dt}$ becomes smaller as long as $\beta_{n-a-1} > 1$, i.e. $n - a - 1 > \mu$. Beyond this point, $\lim_{\gamma \rightarrow \infty} \frac{dU_i^B}{dt}$ increases with any new addition to the set A . Let us thus evaluate this derivative at its largest value possible. This is when $a = n - 1$. Notice that in this case $\sum_{k \in A} \beta_k = n - \beta_1$. Substituting we find

$$\lim_{\gamma \rightarrow \infty} \frac{dU_i^B}{dt} = -(\beta_i - \beta_1) < 0, \text{ for all } i \in B.$$

elasticities of labor supply. In order to obtain intuition for this result, observe that individuals with extended preferences are equivalent to planners solving for the optimal income tax using a Social Welfare Function additive in the individual utilities and using the α_i^j as weights. Therefore, we are checking whether at particular tax rate there will be a majority of SWF with a non-negative marginal valuation of a tax increase. The two type of equilibria restrict the kind of weightings that these SWF can have. However, this restriction is not very sharp. If the elasticity of labor supply is large, the effects of a change in the tax rate will vary significantly across individuals. In that case, the different weightings compatible with one particular type of steady state equilibrium can give rise to overall aggregate valuations of either sign. However, if the elasticity is low, the incidence of the tax will be smaller and hence the use of alternative weightings will not have such profound effects on the aggregate valuation. Our result thus says that, if the elasticity is not too large, we can find a majority of weightings α_i for which the aggregate weighted marginal valuation is non-negative for the cohesive equilibria and non-positive for the clustered equilibria.

6 Concluding remarks

In the Introduction we have selected a number of features that neatly differentiate the US with respect to Europe. In sum, these features are as follows. In the US we observe vis-a-vis Europe a smaller size of the government, higher pre-tax income inequality, less redistribution via taxes and transfers, larger dispersion in labor supply, a thinner middle class and individual attitudes displaying little concern for inequality (particularly among the poor) and a widely shared belief that poor are lazy.

Rather than treat agents' preferences as given, we have presented a model in which they are susceptible to endogenous influences; specifically, other agents' behavior affects our feelings for them. This allows scope for the institutional setting to affect the economic outcome by influencing behavior and hence agents' sentiments and, at the same time this institutional setting be chosen by the individuals that have developed such sentiments.

How does our model fare with respect to this evidence? Our results show that there are two types of equilibria possible, depending on the degree of redistribution (and the size of the government). With high degrees of redistribution we have no dispersion in the individual labor supply and altruism

will be negatively correlated with income, but showing no discrimination of the basis of personal characteristics such as productivity. The degree of redistribution necessary to sustain this equilibrium critically depends on the dispersion of individual productivities. At the other extreme, associated with low progressivity in the income tax, we have a high dispersion in labor supply, a thinner middle class (depending on the degree of redistribution), admiration for the high productivity individuals and complete disregard for the low productivity people who do not manage to supply enough effort to deserve esteem. Even the poor will develop esteem for the hard-working, high-productivity types rather than for the rest of low productivity fellow types.

It is the institutional environment what shapes individual consciousness and behavior. But would the individuals with such consciousness choose this environment? Yes, we show that indeed the sentiments generated by each type of society lead a majority of individuals to vote for the social contract they are in. This seems to provide a fresh view on the nature of the “social contract” in different societies studied most notably by Bénabou and Alesina.

Our model provides an explanation as to why similar societies get installed in significantly different social contracts on the basis of a politico-economic model that displays multiple equilibria. But, we have not given any argument on how to explain whether one country will find itself in one or the other type of equilibrium. Nevertheless, there are some possible explanations suggested by the model.

There is the standard argument of imputing to *history* (read “initial conditions”) the responsibility for our present fate. But, the model suggests additional and more interesting possible causes. In the first place, as we have already emphasized, the elasticity of labor supply plays a key role on whether or not a majority supporting the cohesive equilibrium will obtain. Therefore, differences in the elasticity of labor supply could explain why the US and Europe have taken different roads. A second explanation focus on the role of the distribution of the individual productivities. A productivity shock with the effect of increasing the spread of the distribution would make the cohesive equilibrium unsustainable. Such an economy would be driven towards the clustered equilibrium and the individuals would find it desirable to reduce the progressivity of the tax schedule. This would be easier if this economy had a degree of redistribution close to the threshold level for a cohesive equilibrium to exist. Finally, there is a third explanation

also based on the distribution of individual productivities. We have already mentioned that one of the differences between the US and Europe is that education is public in the latter. It seems reasonable to expect that a system of public education will produce a labor force with a smaller spread in productivities than a private system. Under a private system parents may find it optimal to invest more in the education of their most apt children rather than overinvesting on the least capable. Therefore, we would have that highly redistributive societies (financing public education) would have a lower spread in the skill of their labor force and this, in turn, would make it more likely that a cohesive politico-economic equilibrium could be sustained.

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8 Appendix

We shall now characterize the conditions for the existence of a two-cluster equilibrium.

Proposition 11 *A two-cluster equilibrium exists if there is an integer a , $1 \leq a \leq n - 1$, such that (36a) is satisfied. Further, for any (unequal) distribution of productivities β there is always τ small enough for which a two-cluster equilibrium exists, and for any $\tau \in [0, 1)$ there exists a distribution of productivities such that a two-cluster equilibrium exists.*

Using (15) and (16) in (3), we obtain the equilibrium output for given a . That is,

$$Y = \sum_{i=1}^{n-a} \beta_i^{\frac{1+\gamma}{\gamma}} \left(1 - \tau + \tau \frac{a+1}{n}\right)^{\frac{1}{\gamma}} + \sum_{i=n-a+1}^n \beta_i^{\frac{1+\gamma}{\gamma}} \left(1 - \tau + \tau \frac{a}{n}\right)^{\frac{1}{\gamma}}. \quad (33)$$

Average labor supply is then

$$\bar{L} = \frac{1}{n} \left\{ \sum_{i=1}^{n-a} \beta_i^{\frac{1}{\gamma}} \left(1 - \tau + \tau \frac{a+1}{n}\right)^{\frac{1}{\gamma}} + \sum_{i=n-a+1}^n \beta_i^{\frac{1}{\gamma}} \left(1 - \tau + \tau \frac{a}{n}\right)^{\frac{1}{\gamma}} \right\}. \quad (34)$$

Therefore, a two-cluster equilibrium will exist if there is an a such that

$$L_{n-a}^B < \bar{L} < L_{n-a+1}^A. \quad (35)$$

This condition will be satisfied if and only if there exists a such that,

$$[\beta_{n-a}T(a, \tau)]^{\frac{1}{\gamma}} < \frac{\sum_{i=1}^{n-a} [\beta_i T(a, \tau)]^{\frac{1}{\gamma}} + \sum_{i=n-a+1}^n \beta_i^{\frac{1}{\gamma}}}{n} < \beta_{n-a+1}^{\frac{1}{\gamma}}, \quad (36a)$$

where

$$T(a, \tau) \equiv \left(\frac{1 - \tau + \tau \frac{a+1}{n}}{1 - \tau + \tau \frac{a}{n}} \right) > 1, \text{ for } \tau > 0. \quad (37)$$

We thus have demonstrated the following proposition.

To prove that for any given unequal distribution of productivities there always exists τ small enough for which a two-cluster equilibrium exists, observe that $T(a, \tau) \rightarrow 1$ as $\tau \rightarrow 0$. Therefore, as $\tau \rightarrow 0$, (36a) becomes

$$\beta_{n-a}^{\frac{1}{\gamma}} < \sum_{i=1}^n \frac{1}{n} \beta_i^{\frac{1}{\gamma}} < \beta_{n-a+1}^{\frac{1}{\gamma}}.$$

It is immediate that as long as $\beta_i \neq \beta_{i+1}$ for some i , then such an a always generically exists. ■