Saving and interest rates in Japan:Why they have fallen and why they will remain low.

R.Anton Braun*
University of Tokyo

Daisuke Ikeda
University of Tokyo

Douglas H. Joines
University of Southern California

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*Corresponding author: Faculity of Economics University of Tokyo, Hongo Sanchome, Bunkyo Ku, Tokyo, Japan. email:toni@e.u-tokyo.ac.jp

Abstract

This paper quantifies the role of alternative shocks in accounting for the recent declines in Japanese saving rates and interest rates and provides some projections about their future course. We consider four distinct sources of variation in saving rates and real interest rates: changes in birth rates, changes in survival rates, changes in technology and changes in uninsurable labor risk. The emprical relevance of these factors is explored using a computable dynamic OLG model. We find that the combined effects of demographics and slower total factor productivity successfully explain both the levels and the magnitudes of the declines in the saving rate and the after-tax real interest rate during the 1990s. Model simulations indicate that the Japanese savings puzzle is over.

1 Introduction

One of the biggest distinctions between Japanese and U.S. households is that Japanese save more. As recently as 1990 the gap between the personal saving rate in Japan and the U.S. exceeded 8%. This gap has spawned a large body of research that has documented these differences and tried to account for the Japanese savings puzzle (see e.g. Hayashi (1997) or Horioka(1990) for a reviews of this literature). Recently this gap has been narrowing. In 2002 the gap had fallen to less than 2% leading some to predict that the Japanese savings rate is about to fall below the United States saving rate of 4%. Associated with this decline in the Japanese personal saving rate has been a concurrent decline in the after-tax real interest rate from 6% in 1990 to 4% in 2000.

This paper quantifies the role of alternative shocks in accounting for the recent declines in Japanese saving rates and interest rates and provides some projections about their future course. We start from the premise of the life-cycle hypothesis of Modigliani and Brumberg (1954). This choice is motivated by recent findings of Hayashi (1995) and Horioka et.al. (2000). Hayashi estimates Engel curves for Japanese households and finds that they are inconsistent with the hypothesis that bequest motives are important. Horioka et.al. (2000) argue, more generally, that survey evidence of Japanese households is much more consistent with the life-cycle hypothesis than the alternatives of altruistic or dynastic households. Under the lifecycle hypothesis households savings patterns will vary with age. With the further assumption of overlapping generations, demographic changes such as the aging of a babyboom can have important implications for aggregate savings rates. In order to measure the magnitude of these effects we assume that households live for 65 years. Households are assumed to interact in perfectly competitive markets in a closed general equilibrium economy. Japan is one of the largest economies in the planet in terms of per capita GDP. And we think it reasonable to assume that real interest rates are determined in the domestic market.

We consider four distinct sources of variation in saving rates and real interest rates: changes in birth rates, changes in survival rates, changes in technology and changes in uninsurable labor risk. The interaction of birth rates and survival rates jointly determine the distribution of different age groups in the total population at any point of time. As birth rates and survival rates vary we can model the effects of a babyboom on the population distribution and thus on saving rates and interest rates.

In the past decade unemployment rates have risen from 2.2% in 1990 to

5.5% in 2003. From the perspective of Japanese households this represents a big increase in unemployment risk. If this risk is largely uninsurable then households will respond to it by increasing their demand for savings. In addition, the general equilibrium effects described in Aiyagari (1994) real interest rates will also fall.

Finally, Hayashi and Prescott (2002) have documented the important role of the productivity slow-down in explaining Japanese real interest rates in the 1990s. In addition, to creating intertemporal substitution effects, persistent slow growth in total factor productivity also induces wealth effects that can reduce saving rates.

We calibrate our model to Japanese data and conduct two types of computational experiments. First, we perform a comparative steady-state analysis with a 1990 benchmark. Our goal here is to get a preliminary assessment of the quantitative role of the four factors in explaining the decline in the saving rate and after-tax real interest rate during the 1990s.

From Hayashi and Prescott (2002) we know that slower total factor productivity growth can explain much of the fall in real interest rates in the 1990s. Their model produces about a 1.5% decline in the after-tax real interest rate where as in the data interest rates fall by about 2%. Our model produces a 3.8% decline in the after-tax real interest rate across steady-states. This prediction of the model can be decomposed into 4 parts- total factor productivity, unemployment risks, birth rates and survival rates. When we perform this decomposition we find that declines in the birth rate and total factor productivity have the largest effects. A 1% decline in the birth rate reduces the after-tax real interest rate by 1%. A 2% decline in total factor productivity also reduces the after-tax real interest rate by about 1%. The magnitude of the other two factors effects is smaller. An increase in unemployment risk that raises the mean unemployment rate from 2% to 5% only reduces the after-tax real interest rate by 0.5%. And a change in survival rates that captures the measured increase in life-expectancy patterns between 1990 and 2000 produces only a 0.3% decline in the after-tax real interest rate.

The model also produces a decline in the personal saving rate. Under our baseline specification the model predicts that steady-state decline in the saving rate of 4% . However, the model fails to reproduce the high level of the saving rate in 1990. In our dataset the measured saving rate is 14% whereas the model predicts a steay-state value of only 8.8%.

A limitation of the steady-state analysis is that it can take a long time to transit from one steady-state to another. What we would really like to know is whether the factors we model can individually or jointly reproduce the measured declines in saving rates and interest rates over a ten year period. To address this question we conduct a dynamic analysis. We abstract from unemployment risk because it is quantitatively small based on the steady-state analysis. Abstracting from unemployment risk also substantially reduces the computational burden. In computing these dynamic simulations we calibrate the demographics to projections made by the National Institute of Population and Social Security Research (IPSS) and posit a gradual recovery of total factor productivity growth from 0.3% in 2000 to 2% in 2010. Under this baseline parameterization we find that the combined effects of demographics and slower total factor productivity successfully explain both the levels and the magnitudes of the declines in the saving rate and the after-tax real interest rate during the 1990s.

We also document our model's projections for the future time path of the saving rate and the after-tax real interest rate. These simulations indicate that the Japanese savings puzzle is over. In future years the Japanese personal saving rate falls to about 5% and then gradually rises to 8%. Projections for the after-tax real interest rate suggest that interest rates will only exhibit a small rise of about 50 basis points over the next 20 years. These forecasts are predicated on a recovery in total factor productivity growth. If instead total factor productivity growth remains low both the saving rate and the after-tax real interest rate fall to about 1%.

Our results are related to recent work by Hayashi and Prescott (2002). They find that slower growth in total factor productivity during the 1990s can also reproduce the measured decline in the real interest rate. One difference between their explanation and ours is implications for the future. Their theory predicts that the saving rate and the real interest rate will recover to historical levels if total factor productivity growth recovers. Our theory when used in conjunction with current demographic projections implies that the saving rate and the real interest rate will remain below their historical averages even if total factor productivity growth recovers. Our work is also related to research by Hayashi et.al.(1988) and Rios Riull (2002), Hayashi et.al.(1988) found that the rapid economic growth, demographics and housing market imperfections can explain the high Japanese savings puzzle. They also produce the long-run projections that show the saving rate declines in future years. Rios-Rull(2002) found that permanent shocks to demographics have big implications for Spanish saving rates and interest rates.

The remainder of the paper is divided into five sections. In section 2 we describe the model economy. Section 3 reports the calibration of the model and the steady-state analysis, section 4 reports results of the dynamic analysis and section 5 contains our conclusions.

2 Model

2.1 Demographic Structure

This economy evolves in discrete time. We will index time by t where $t \in \{..., -2, -1, 0, +1, +2, ...\}$. Households can live at most J periods and J cohorts of households are alive in any period t. They experience mortality risk in each period of their lifetime. The dynamics of demographics are governed by the first-order Markov process:

$$\boldsymbol{\mu}_{t+1} = \begin{bmatrix} (1+n_{1,t})\psi_{1,t} & 0 & 0 & \dots & 0 \\ \psi_{2,t} & 0 & 0 & \dots & 0 \\ 0 & \psi_{3,t} & 0 & \dots & 0 \\ \multicolumn{5}{c}{\dotfill} \\ 0 & 0 & 0 & \psi_{J,t} & 0 \end{bmatrix} \boldsymbol{\mu}_t \equiv \Gamma_t \boldsymbol{\mu}_t \tag{1}$$

where $\boldsymbol{\mu}_t$ is a $J \times 1$ vector that describes the population of each cohort in period t, $\psi_{j,t}$ is the conditional probability of a household in cohort $j-1$ surviving from period t to period $t+1$, and $n_{1,t}$ is the birth rate in period t. The aggregate population in period t, denoted by N_t, is given by:

$$N_t = \sum_{j=1}^{J} \mu_{j,t} \tag{2}$$

where $\mu_{j,t}$ is a j-th element of $\boldsymbol{\mu}_t$. The population growth rate is then given by: $n_t = N_{t+1}/N_t$. The unconditional probability of surviving from birth in period $t-j+1$ to age j in period t is:

$$\pi_{j,t} = \psi_{j,t}\pi_{j-1,t-1} \tag{3}$$

where $\pi_{1,t-j+1} = 1$ for all t.

2.2 Firms' Problem

Firms combine capital and labor using a Cobb-Douglas constant returns to scale prodution function:

$$Y_t = A_t K_t^{\alpha}(h_t N_t)^{1-\alpha} \tag{4}$$

where Y_t is the output which can be used either for consumption or investment, K_t is the capital stock, h_t is the average efficiency of labor input and

A_t is total factor productivity. Total factor productivity grows at the rate $\gamma_t = A_{t+1}^{1/(1-\alpha)}/A_t^{1/(1-\alpha)}$. We will assume that the the market for goods and the markets for the two factor inputs are competitive . Then labor and capital inputs are chosen according to:

$$r_t = \alpha A_t K_t^{\alpha-1}(h_t N_t)^{1-\alpha} \tag{5}$$

$$w_t = (1-\alpha) A_t K_t^{\alpha}(h_t N_t)^{-\alpha} \tag{6}$$

where r_t is the rental rate on capital and w_t is the wage rate.

2.3 Households' Problem

All households have one adult but may have one or more children. The utility function for a household born in period t is given by:

$$U_t = \sum_{j=1}^{J} \beta^{j-1} \pi_{j,t+j-1} u(c_{j,t+j-1}/\eta_j) \tag{7}$$

where β is the preference discount rate, $c_{j,t+j-1}$ is total household consumption for a household of age j in period $t+j-1$, and η_j is the scale of a household of age j. This specification of preferences makes it possible for the size of a household to vary with the age of the adult member but imposes the restriction that the distribution of household size, $[\eta_1, ..., \eta_J]$, remain fixed over time.

Households inelastically supply their endowment of one unit of time but are subject to non-diversifiable employment risk. The employment risk follows a two state Markov chain. This Markov chain takes on the value of $\bar{s}_1 = 1$ for employed households and $\bar{s}_2 = 0$ for unemployed households. The transition matrix is given by:

$$P = \begin{bmatrix} 1-\nu & \nu \\ 1-\nu & \nu \end{bmatrix} \tag{8}$$

where the (k,l) element of the matrix P, $P(k,l)$, is $prob(s_{j+1} = \bar{s}_l | s_j = \bar{s}_k)$ and ν is the unemployment rate. Employed workers of age j receive labor income that consists of an efficiency weighted wage rate $w_t \varepsilon_j s$ in period t.The efficiency index ε_j is assumed to drop to zero, $\varepsilon_j = 0$, for all $j \geq J_r$, and $s \in \{\bar{s}_1, \bar{s}_2\}$ where J_r is the retirement age. The budget constraint for a household of age-j in period t is:

$$c_{j,t} + a_{j,t} \leq R_t a_{j-1,t-1} + w_t \varepsilon_j s_j + m w_t \varepsilon_j (1 - s_j) + \xi_t - \theta_{j,t} \tag{9}$$

where $a_{j,t}$ are purchases of assets in period t, $\theta_{j,t}$ are taxes on capital income imposed by the government, $mw_t\varepsilon_j$ is the value of unemployment benefits received by an umployed worker and ξ_t is a lump sum government transfer. Households are also subject to a borrowing constraint that rules out negative holdings of assets: $a_{j,t} \geq 0$.

Taxes imposed by the government are given by

$$\theta_{j,t} = \tau_a(R_t - 1)a_{j-1,t-1} \tag{10}$$

where τ_a is the tax rate on capital income.

2.4 Household's Decision Rules

Suppose that a household's asset holdings take on a finite number of values $a \in \{0, ..., \bar{a}\}$. Then we can summarize the situation of an age-j household in period t, with the state variable, $x_{j,t}$. The individual state consists of asset holdings $a_{j-1,t-1}$ and employment status: $x_{j,t} = \{a_{j-1,t-1}, s_j\}$. The period t wealth distribution describes the measure of households in each individual state:$\lambda_j(x_{j,t})$, $j = 1, ...J$.Then the aggregate state of the economy, denoted X_t, is composed of the aggregate capital stock, K_t, total factor productivity, A_t, the wealth distribution, $\boldsymbol{\lambda}_t$,and the population distribution,$\boldsymbol{\mu}_t$,: $X_t = \{K_t, A_t, \boldsymbol{\lambda}_t, \boldsymbol{\mu}_t\}$. Finally, define the government policy rule in period t as Ψ_i. It will be convenient when solving the household's problem to assume that households know the entire future path of government policies: $\mathbf{\Psi}^t = \{\Psi_i\}_{i=t}^{\infty}$ and the entire future path of total factor productivity. With these various definitions and assumptions in hand, we can now state Bellman's equation for a typical household:

$$\begin{aligned} &V_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t) \qquad (11) \\ &= \max\{u(c_{j,t}/\eta_j) + \beta\psi_{j+1} \sum_{s_{j+1}} V_{j+1}(x_{j+1,t+1}; X_{t+1}, \mathbf{\Psi}^{t+1}) P(s_{j+1}, s_j)\} \end{aligned}$$

subject to

$$c_{j,t} + a_{j,t} \leq R(X_t)a_{j-1,t-1} + w(X_t)\varepsilon_j s_j - \theta_{j,t} + mw(X_t)\varepsilon_j(1 - s_j) + \xi_t \tag{12}$$

$$a_{j,t} \geq 0, \quad c_{j,t} \geq 0 \tag{13}$$

$$\boldsymbol{\mu}_{t+1} = \Gamma_t \boldsymbol{\mu}_t \tag{14}$$

and the law of motion of the aggregate wealth distribution and the law of motion for the aggregate capital stock given by $K_{t+1} = K(X_t)$. Since households die at the end of period J, $V_{J+1,t} = 0$ for all t. A solution to the household's problem consists of a sequence of value functions: $\{V_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t)\}_{j=1}^{J}$

for all t, and policy functions:$\{a_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t), c_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t)\}_{j=1}^J$ for all t. The law of motion for the wealth distribution is computed using forward recursion on the following sum:

$$\lambda_{j+1}(x_{j+1,t+1}) = \sum_s \sum_{a_{j-1,t-1} \in \Lambda_{j,t}} P(s_j, s_{j+1}) \lambda_j(x_{j,t}) \tag{15}$$

where the set $\Lambda_{j,t}$ is $\{a_{j-1,t-1} | a_{j,t} \in a_{j,t}(x_{j,t}, X_t; \mathbf{\Psi}^t)\}$. The recursion starts from the initial conditions of a newly-born household with zero assets:

$$\begin{aligned} \lambda_0(0, \bar{s}_1) =& 1 - \nu \\ \lambda_0(0, \bar{s}_2) =& \nu \end{aligned} \tag{16}$$

and otherwise $\lambda_0(\cdot) = 0$.

2.5 Government

The government raises revenue by taxing capital income at the rate τ_a. It receives additional revenue by imposing a 100% tax on all accidental bequests. Total accidental bequests in period $t+1$ are:

$$B_{t+1} = \sum_j \sum_a \sum_s (1 - \psi_{j+1,t+1}) R(X_{t+1}) a_{j,t}(x_{j,t}) \lambda_j(x_{j,t}) \mu_{j,t} \tag{17}$$

and total tax revenue from capital income taxes is:

$$T_t = \sum_j \sum_a \sum_s \theta_{j,t}(x_{j,t}) \lambda_j(x_{j,t}) \mu_{j,t} \tag{18}$$

Note that $\theta_{j,t}$ depends on $x_{j,t}$ since it is a function of $a_{j-1,t-1}$ by (10). Total government expenditure is the sum of unemployment insurance benefits and lamp-sum transfers:

$$G_t = \sum_j \sum_a m w(X_t) \varepsilon_j \lambda_j(x_{j,t} | s_j = \bar{s}_2) \mu_{j,t} + \sum_j \xi_t \mu_{j,t} \tag{19}$$

Assume that government expenditure in period t equals period t government revenue or that the government budget constraint is balanced in each period: $B_t + T_t = G_t$. Then lump-sum transfers are:

$$\xi_t = (B_t + T_t - \sum_j \sum_a m w(X_t) \varepsilon_j \lambda_j(x_{j,t} | s_j = \bar{s}_2) \mu_{j,t}) / N_t \tag{20}$$

2.6 Recursive Competitive Equilibrium

Given this description of the economy we can now define a recursive competitive equilibrium.

Definition 1: Recursive Competitive Equilibrium

Given: government policy rules $\{\mathbf{\Psi}^t\}_t$, a law of motion for population $\{\Gamma_t\}_t$, a law of motion for unemployment P; a recursive competitive equilibrium is a set of value functions $\{V_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t\}_{j=1}^J$ for all t, policy functions $\{a_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t), c_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t\}_{j=1}^J$ for all t, a wealth distribution $\boldsymbol{\lambda}_t$, factor prices $\{w(X_t), r(X_t)\}$ for all t, a law of motion for aggregate capital $K_{t+1} = K(X_t)$ and a function for the average efficiency of labor input $h_t = h(X_t)$ such that:

- Given the functions of factor prices $\{w(X_t), R(X_t)\}$ and the law of motion for aggregate capital $K(X_t)$ and the function for average efficiecny of labor input $h(X_t)$, the set of household's policy functions $\{a_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t), c_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t)\}$ solve the household's dynamic program (11).
- The factor prices are competitively determined so that (5) and (6) hold and $R_t = r_t + 1 - \delta$.
- The commodity market clears:

  $$Y_t = C_t + I_t$$

  where $C_t = \sum_j \sum_a \sum_s c_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t)\lambda_j(x_{j,t})\mu_{j,t}$ is the aggregate consumption and I_t is the aggregate investment.
- The laws of motion for aggregate capital and average efficiency of labor input are given by:

  $$K(X_t) = \sum_j \sum_a \sum_s a_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t)\lambda_j(x_{j,t})\mu_{j,t}$$

  $$h(X_t) = \sum_j^{J_r - 1} \sum_a \epsilon_j \lambda(x_{j,t} | s_j = \bar{s}_1)\mu_{j,t}/N_t$$

  where $N_t = \sum_{j=1}^J \mu_{j,t}$.
- The measure of household $\boldsymbol{\lambda}_t$ is generated by (15).

- The government budget constraint is satisfied and balanced in each period
$$B_t + T_t = G_t$$

In section 3 we will perform a comparative steady-state analysis. This analysis will be based on the notion of a stationary recursive competitive equilibrium. Before we can define a stationary recursive competitive equilibrium we need to define some of the building blocks.

Definition 2: Stationary population distribution

Suppose that the birth rate and the conditional survival probabilities are constant over time: $n_{1,t} = n_1$ for all t and $\psi_{j,t} = \psi_j$ for all t and j. Then a stationary population distribution, $\boldsymbol{\mu}_t^*$, satisfies $\boldsymbol{\mu}_{t+1}^* = \Gamma^* \boldsymbol{\mu}_t$ and $\boldsymbol{\mu}_{t+1}^* = (1+n_1) \cdot \boldsymbol{\mu}_t^*$ where

$$\Gamma^* = \begin{bmatrix} (1+n_1)\psi_1 & 0 & 0 & \dots & 0 \\ \psi_2 & 0 & 0 & \dots & 0 \\ 0 & \psi_3 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \psi_J & 0 \end{bmatrix}$$

A stationary population distribution has two desirable properties. First, cohort shares in total population are constant over time: $\mu_{j,t+1}/N_{t+1} = \mu_{j,t}/N_t$ for all t. Second, the aggregate population growth rate is time invariant: $n_t = N_{t+1}/N_t = n_1$ for all t. This allows us to convert the growth economy into a stationary economy using the following transformations:

$$\tilde{c}_{j,t} = \frac{c_{j,t}}{A_t^{1/(1-\alpha)}}, \quad \tilde{a}_{j,t} = \frac{a_{j,t}}{A_t^{1/(1-\alpha)}}$$

Other per-capita variables in the household budget constraint are transformed in same way. Aggregate variables in period t are transformed by dividing by $A_t^{1/(1-\alpha)} N_t$.

Definition 3: Stationary recursive competitive equilibrium

Suppose the population distribution is stationary and the growth rate of total factor productivity is constant over time :$\gamma_t = \gamma^*$ for all t. Then a stationary recursive competitive equilibrium is a recursive competitive equilibrium that satisfies:

$$\tilde{c}_{j,t} = \tilde{c}_j^*, \quad \tilde{a}_{j,t} = \tilde{a}_j^*$$

for all t and j, i.e., the factor prices are constant over time: $\{r_t, \tilde{w}_t\} = \{r^*, \tilde{w}^*\}$ for all t where $\tilde{w} = w/A^{1/(1-\alpha)}$.

This completes the description of the model.

3 Comparative Steady-state Analysis

In this section we report results based on comparisons across steady-states. We assume that each household has one adult member. New households are formed when individuals reach the age of 21 and households die off no later than at the end of the 85th year of life of the head of household. Steady-state comparisons provide a basis for assessing the relative importance of demographics, total factor productivity and employment risk in the long-run.

The model is calibrated to Japanese data. The starting point for our calibration is Hayashi and Prescott (2002). Following Hayashi and Prescott (2002), the capital share parameter, α, is set to 0.36; the depreciation rate on capital, δ, is set to 0.089; and the capital tax rate, τ_a,is fixed at 0.48. We assume further that the utility function is isoelastic: $u(c) = \frac{c_t^{1-\sigma}}{1-\sigma}$ and choose, σ, the risk aversion coefficent to be 2. The preference discount factor is chosen to reproduce the 1990 Japanese after-tax real interest rate of 6%. This yields a value of $\beta = 0.97$. The profile of efficiency wages is constructed from Japanese data on employment, wages, and weekly hours following the methodology described in Hansen (1993). [1] Total factor productivity is assumed to grow at a constant rate of 2% per annum.The transition matrix governing employment status is calibrated to match the 1990 unemployment rate of 2%. The birth rate is initially set to a baseline value of 1% and mortality risk for each generation is calibrated to Japanese 1990 survival rates. Finally, the family scale is currently configured to match U.S. data as reported in Cubeddu and Rios-Rull (1996).

Row 1 of Table 1 reports results for the baseline 1990 steady-state. Row 2 shows a second steady-state that is designed to reflect the situation of the Japanese economy as of 2000. This latter calibration assumes an unemployment rate of 5%, total factor productivity growth of 0.29%, a birth rate of 0% and 2000 survival rates.[2]

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[1] See the data appendix for more detail.

[2] For total factor productivity we took the average value for the 1990's. Total factor productivity exibits relatively large fluctuations on an annual basis. Taking the average value for the 1990's smooths out these fluctuations.

Consider first the interest rate implications of the model. In Japanese data the after-tax real interest rate was 6% in 1990 and 3.9% in 2000. Comparing the first two rows we see that the model predicts that the after-tax interest rate falls to a level of 2.2% in the new steady-state.

Table 1

Simulation	(n_1, ν, γ, ψ)	(K/Y)	$\hat{r}$	s
1990 baseline	$(.01, .02, 1.02, \psi^{1990})$	1.76	6.01%	8.77%
2000 baseline	$(.00, .05, 1.0029, \psi^{2000})$	2.73	2.23%	4.83%
Analysis of 2000 baseline				
Low birth rate	$(.00, .02, 1.02, \psi^{1990})$	1.95	4.96%	7.77%
High unemployment risk	$(.01, .05, 1.02, \psi^{1990})$	1.83	5.60%	9.05%
Low TFP	$(.01, .02, 1.0029, \psi^{1990})$	2.29	3.54%	7.09%
2000 mortality	$(.01, .02, 1.02, \psi^{2000})$	1.81	5.69%	8.67%

Note: $\hat{r}$ is the after tax real interest rate, denoted by $(1-\tau_a)(r-\delta)$, the s is the personal saving rate defined by $1 - C/Y^d$ where Y^d is an aggregate disposable income and the ψ^t means the time-t survival rates $\{\psi_{j,t}\}_{j=1}^{J}$.

Rows three through six provide information on the contribution of each of the four factors in isolation. According to these results a lower birth rate and lower total factor productivity growth are the biggest contributing factors. A 1% decline in the birth rate produces a deline of 1% in the real interest rate. The reason for this result can be seen in Figure 1. Figure 1 reports the distribution of households by age for the 1990 baseline (row 1) and the low birth rate simulation (row 3) and the associated distributions of asset holdings by age. Observe first, that a lower birth rate increases the measure of households that are close to retirement age. Since these households also have high wealth, the low birth rate wealth distribution has a higher peak than the high birth rate distribution. With a lower fraction of young workers aggregate labor input is also lower. Both of these effects raise the capital output ratio and lower the after-tax interest rate.

Slower growth in total factor productivity also increases the capital output ratio and thus lowers the after-tax real interest rate. As Hayashi and Prescott (2002) have previously observed this effect is quantitatively big during the 1990s. In our simulation a 2% decline in total factor productivity lowers the steady-state after-tax real interest rate by 2.5%.

The effects of higher unemployment risk and higher survival rates are much smaller. A rise in the unemployment rate from 2% to 5% only reduces the after-tax real interest rate by 0.4%. And higher survival probabilities only lower the after-tax real interest rate by 0.3%.

Consider next the model's implications for saving rates. In Japanese data the personal saving rate was 14% in 1990 and 7% in 2000. The model also predicts a decline in savings. [3] The magnitude of the decline, however, is only half as big as in the data. Moroever, the model fails to predict the level of the saving rate in 1990. The two biggest contributing factors are once again lower birth rates and lower total factor productivity growth. Lower birth rates contribute about 1% to the decline in the saving rate and lower total factor productivity growth contributes about 1.7%.

Overall, the comparative steady-state analysis finds that both demographics and total factor productivity are quantitatively important determinants of saving rates and real interest rates. Unemployment risks appear to be much less important. There are several limitations of the steady-state analysis. The first is timing. It is not clear whether a dynamic simulation would produce declines of the right magnitudes over a ten year horizon. A second issue is that it is hard to assert that the Japanese economy was resting at a steady-state in 1990. We turn next to conduct a dynamic analysis that overcomes these limitations.

4 Dynamic Analysis

In this section we report simulation results that model the transitional dynamics. Modeling the transition allows us to describe the full dynamic path for equilibrium quantities and prices in response to changes in fertility and total factor productivity. Our simulations suggest that some big changes in savings rates and the after-tax real interest rate are in store for the Japanese economy over the next 15 years.

The results in Section 3 indicate that unemployment risk only has a small impact on saving and interest rates. For this reason and also to ease the computational burden of calculating the transitional dynamics we abstract from unemployment risk in this section. Omitting unemployment risk affects savings demand and thus the equilibrium after-tax real interest rate. Consequently, for these simulations β is set to 0.98. This insures that we continue to match the 1990 after-tax real interest rate which is the initial period in our simulations.

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[3] Our measure of the personal savings rate is the ratio of saving to disposable income from the System of National Accounts (SNA). We define saving and disposable income as in Iiduka (2003).

In order to conduct dynamic simulations it is also necessary to specify the entire future time path for the demographic variables and total factor productivity. The demographics are pinned down by two sequences: a sequence of birth rates: $n_{1,t}$ for all future t, and the sequence of age specific survival probabilities: $\{\psi_{j,t}\}_{j=1}^{J}$ for all future t. Our baseline parameterization is calculated in the following way. Considering first the demographic variables, for the period 1990 through 2000 we use actual data on birth rates and survival probabilities. This data is described in more detail in the data appendix. For the period 2000-2050, we chose these two sequences in a way that matches the forecasts produced by the National Institute for Population and Social Security Research (IPSS). Surprisingly, we found that we could do a good job of reproducing their forecasts if we hold fixed the $\{\psi_{j,t}\}_{j=1}^{J}$ at their average value for the 1990-2000 period and set the birth rate to its 2000 value of -0.1%. Figure 2 shows the projections by IPSS and the projections implied by our parameterization. From these figures it is clear that our parameterization does a reasonable job of reproducing the IPSS projections. The most notable difference is that our projections for the fraction of the populations between 66-85 is lower than those of the IPSS from about 2020 on. For the period beyond 2050 we continue to hold the $\{\psi_{j,t}\}_{j=1}^{J}$ fixed and gradually let the birth rate rise to zero over a 15 year period and then hold it constant at this value thereafter. Figure 3 shows the implications of our baseline demographic assumptions for the time path of total population and the time path of fractions of different age groups in total population. This parameterization implies that the Japanese population falls by about 50% over the next 100 years.

For total factor productivity we use data from Hayashi and Prescott (2002) for the period 1990 through 2000. Between 2001 and 2010, we assumed a gradual increase in total factor productivity growth to 2% over a 10 year period. Thereafter total factor productivity growth is assumed to grow at a constrant rate of 2%.

To calculate the transitional dynamics we assume that in 1990 the economy is in a particular type of steady-state. [4] One day households and firms wake up only to discover that their assumptions about future variables are wrong and instead that they are going to be subject to the sequence of total factor productivities and demographics described above. The simulations then follow households responses to these sequences of exogenous variables

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[4] This steadystate is somewhat special in that household's believe that the current age distribution will be the same forever. This is the same assumption made by Rios-Rull (2002) and Nishiyama (2002).

until the economy settles down in a new steady-state.

Figures 5(a) and (b) report our baseline results. The upper panel of each Figure shows the response of the after-tax real interest rate and the lower panel shows the response of the personal saving rate which is the ratio of saving to disposable income calculated using SNA data. The only difference between Figure 5(a) and 5(b) is the dating. Figure 5(a) shows long-run patterns and Figure 5(b) focuses on the 1990s.

Consider first, the 1990s. The model successfully reproduces the data patterns in both savings and the real interest rate. In terms of saving, the model now successfully captures the initial magnitude of the saving rate in 1990. When households wake up to the new news in 1990 they discover that the future looks pretty different. Now they see over 10 years of low total factor productivity growth and the future demographic patterns reported in Figure 3. The average response to this news is to save more. Over time, this also implies lower market interest rates. It is interesting that the two gaps between the models predictions for savings and the actual data correspond to what are known to be one time events. The model fails to predict households' response to the increase in the value added tax from 3% to 5% in 1997. Moreover, the model fails to predict the sharp decline in savings in 2000 and 2001 as the Teigaku savings deposits mature. If we use Iiduka(2003)'s adjustment for this latter event, the model's predicted saving in 2001 lies midway between the actual and the adjusted saving measures.

Given the success of our model in reproducing the 1990s it is also interesting to explore its implications for the future. In this regard,the single most important fact about Japanese saving in the post World War II period has been its magnitude. Our results indicate Japanese saving puzzle is a historical artifact. The results reported in Figure 5 predict that Japan's saving rate will never exceed 10% again. Saving rates fall to a low of about 5% in 2018 and then gradually rise to about 7% by the year 2100. It is worth pointing out that this pattern is driven by persistent but transient shocks to demographics and total factor productivity. Below we will show that the declines in savings and interest rates are even larger if these shocks are assumed to be permanent.

Hayashi et.al.(1988) also provide long-run projections for saving rates. Their model predicts a 10% decline in the saving rate between 2000 and 2030. In comparison to our results the decline in the saving rate starts about 10 year later but the overall magnitude of the change is about the same.

The baseline results in Figure 5 also point to further albeit more moderate declines in the after-tax real interest rate in future years. Between 2003 and 2018 the after-tax real interest rate is projected to decline by another 60 basis points. This decline is, however, small relative to the declines that occured during the 1990s. Currently it is estimated that Japanese life insurance companies are experiencing negative spreads on as much as 70% of all outstanding life insurance policies (see Nikkei Weekly August 25, 2003). Our projections suggest that it would be a mistake to bet on higher future real interest rates resolving this problem.

What is the contribution of total factor productivity and demographic factors to these results? In order to answer this question we report two other simulations in Figures 6 and 7. Figure 6 reports results that isolate the effects of demographics by holding fixed the growth rate of TFP at 2% in all periods. Figure 7 holds the demographic pattern fixed at its 1990 level and assumes that TFP varies as in the baseline parameterization. [5] Considering first Figure 6, we observe that abstracting from total factor productivity affects the timing of the declines in the after-tax interest rate and the saving rate but doesn't affect the peak responses of these variables. Here the saving rate falls to a minimum value of 6% in 2050 and the after-tax real interest rate falls to a level of about 3% in 2040.

From Figure 7 we see that in isolation, declining total factor productivity produces sharp declines in the saving rate and the after-tax real interest rate in the 1990s but these responses quickly damp as total factor productivity returns to 2% in subsequent periods. The magnitude of these declines is also quite a bit smaller than those reported in the baseline parameterization. Here the saving rate never falls below 10% and the after-tax real interest rate only declines to 4.7% in 2002 before rising thereafter.

Overall, these results indicate that small changes in birth rates produce very persistent responses in the saving rate and the after-tax real interest rate. Shocks to total factor productivity, in contrast, have big contemporaneous effects but, don't produce much internal propagation in the model.

The savings patterns in our baseline results are quite different from the steady-state results reported in Section 3. In the comparative steady-state analysis we found that the saving rates were low and that there were only very small changes in the saving rate across steady-states. The saving rate in the initial steady-state of Figure 5 is only 11% but jumps up to 15% once the transition starts. There are two reasons for these differences. First,

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[5] As the case in calculating the initial steady-state of the dynamic analysis, the fixed demographics in its 1990 level is a special assumption. We conduct this simulation just because we want to isolate the effect of the total factor productivity. And this simulation's environment is much alike real business cycle model in the sense that the demographic structure does not change.

as described above when households receive their time zero suprise it affects their demand for savings. Second, the initial steady-state used for the transition analysis assumes that the population distribution remains fixed in all periods at its 1990 level. More generally, a steady-state equilibrium as defined in Section 2 would iterate on the population distribution until it settles down to a stationary distribution. This is not done here and we believe this makes sense for the transition analysis because it means that when households wake up in the morning of 1990 and find that the future path of demographics and total factor productivity is now going to change, at least, the actual population distribution is consistent with what they expected.

Underlying the baseline results is an assumption that the current patterns in total factor productivity and demographics are persistent but transient departures from their previous values. TFP gradually recovers to 2% annualized growth between 2000 and 2010 and the birth rate gradually returns to 0% after 2050. We next turn to describe the properties of our model when the current low growth rate of TFP and the current birth rate are assumed to be permanent. Figure 8 reports results for a simulation in which TFP growth is assumed to remain permanently at its 1990's average value of 0.29% after 2000 while the birth rate is set to its baseline values. The assumption of permanently low total factor productivity growth is also maintained by Hayashi and Prescott (2002).

Consider first the real interest rate. This plot has three noteworthy features. First, observe that the recovery of real interest rates after 2004 that occurs under the baseline parameterization in Figure 5 is predicated on a recovery of total factory productivity growth. If instead total factory productivity growth remains low, the real interest rate continues to decline until about 2040. Second, permanently low productivity growth implies that the bottom of the trough is much lower than under the baseline scenario. Here the real interest rate falls to about 1.2% whereas in the baseline specification it only falls to about 3%. Third, the new steady-state interest rate is also lower. The final steady-state value in Figure 8 of 2.2% is over 2% lower than the baseline specification.

We see similar patterns in the saving rate. The decline in the saving rate is much larger now falling to about 1% as compared to 5% for the baseline specification. In addition, the new steady-state saving rate is lower: 5% here as compared to 8% for the baseline specification. Overall, these results indicate that the maintained hypothesis under the baseline specification of a recovery in TFP growth to 2%, is playing an important role in attenuating the declines in the saving rate and the interest rate due to demographic changes after 2000.

Figure 9 reports results for a simulation in which the birth rate remains permanently at its 2000 level of -0.1% after 2000 and TFP follows is baseline trajectory. Between 2000 and 2050 the patterns here are very similar to those reported in the baseline specification. This is because both simulations assume a negative -0.1% birth rate during this period. The main difference between this simulation and the baseline is that the the baseline specification implies a recovery in the real interest rate and the saving rate after 2050 wheras this specification shows no recovery. The saving rate remains stuck at about 6% and the real interest rate doesn't rise much above 3%.

5 Conclusion

In this paper we have shown that the measured declines in saving rates and real interest rates during the 1990s are consistent with the predictions of theory. Both low total factor productivity growth and the life cycle hypothesis play important roles in accounting for these facts. Our theory also has sharp implications for the future evolution of saving rates and interest rates. It provides a quantitative confirmation of previous claims that the Japanese savings puzzle is over. According to our theory, Japanese savings rates will never again exceed 10%. Moreover, these projections are reasonalby robust. The population distribution, which is the key determinant of savings, only changes gradually over time in a highly predictable way. Thus, even when we posit a robust recovery in total factor productivity, saving rates remain low by historical standards.

In future work we plan to investigate the ability of our theory to account for the previously high saving rates that Japan experienced in the 1970s. We also plan to generalize our specification to allow for a household labor supply decision. One property of our model is that wages are rising. With endogenous labor supply, participation rates of family members is likely to increase. This increase in family labor supply has interesting implications for, among other things, the sustainability of the Japanese social security system. Another issue we plan to address is bequest motives. Inheritance tax rates in Japan are high and very progressive. We plan to introduce a bequest motive into our model and investigate the implications of changes in inheritance taxes on bequests.

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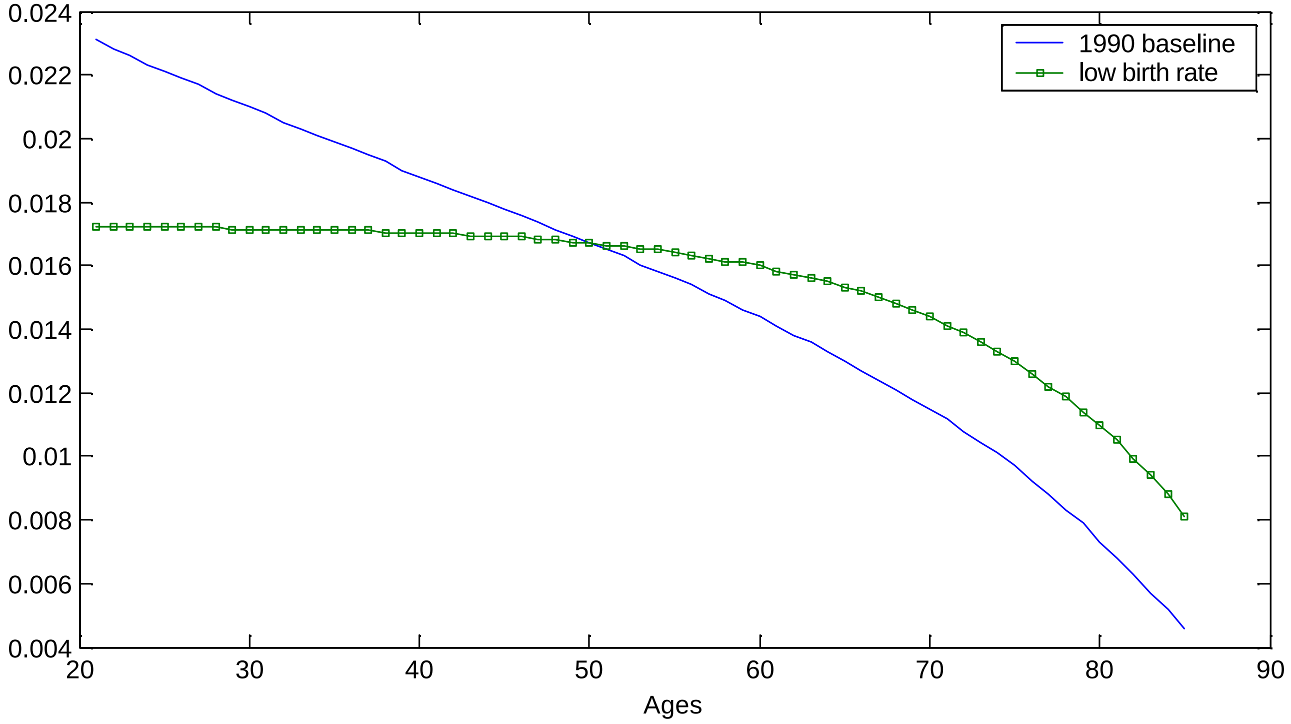
Figure 1

The distribution of households by age



The households asset distribution by age

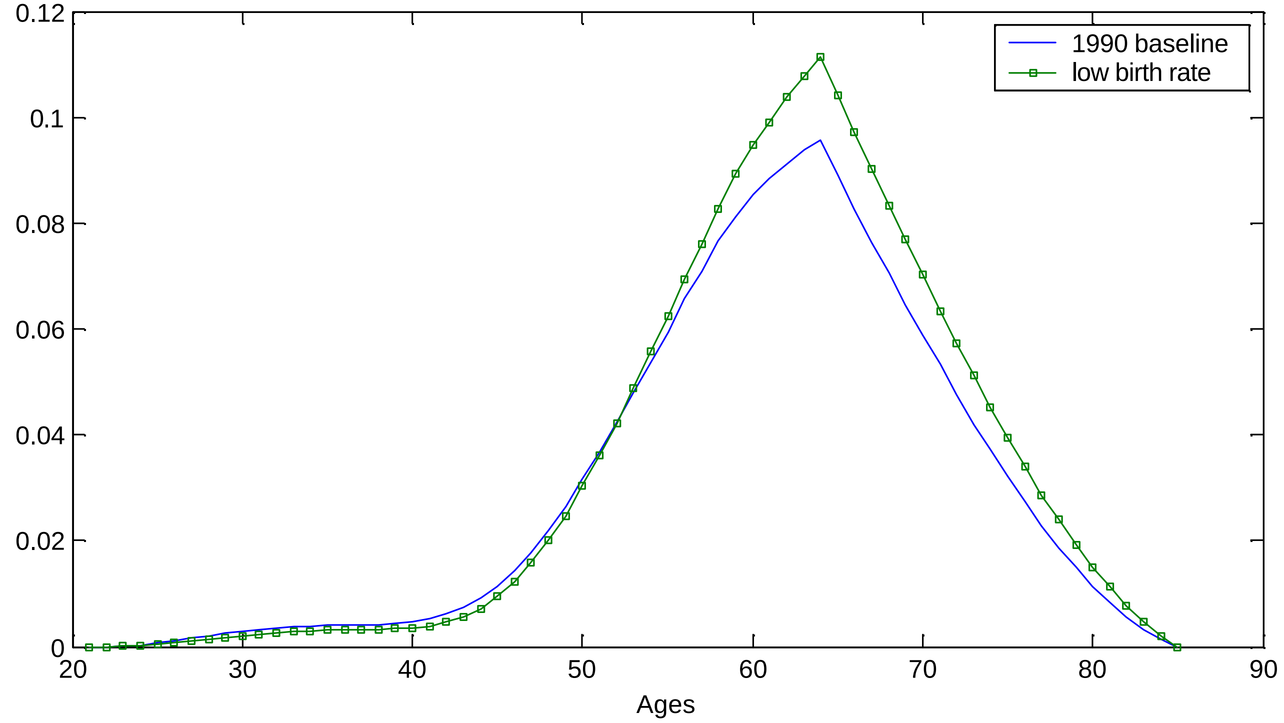
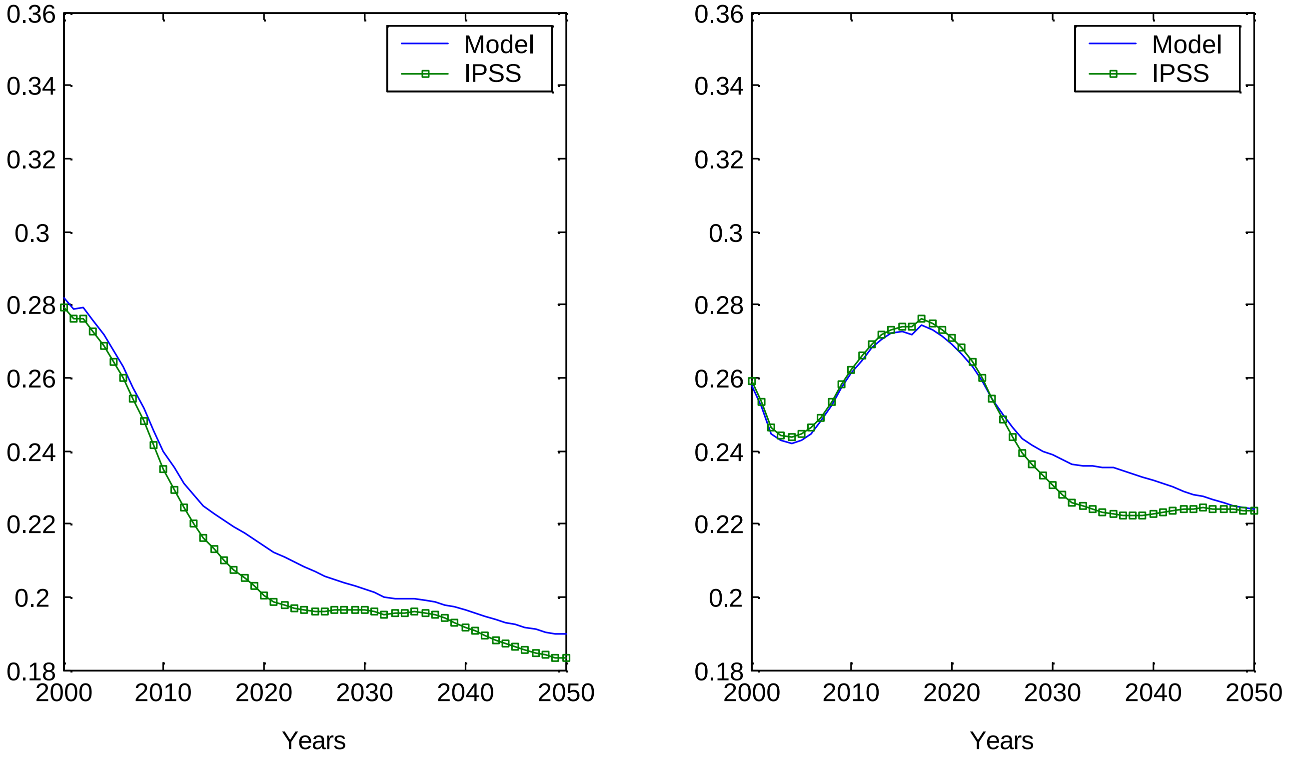
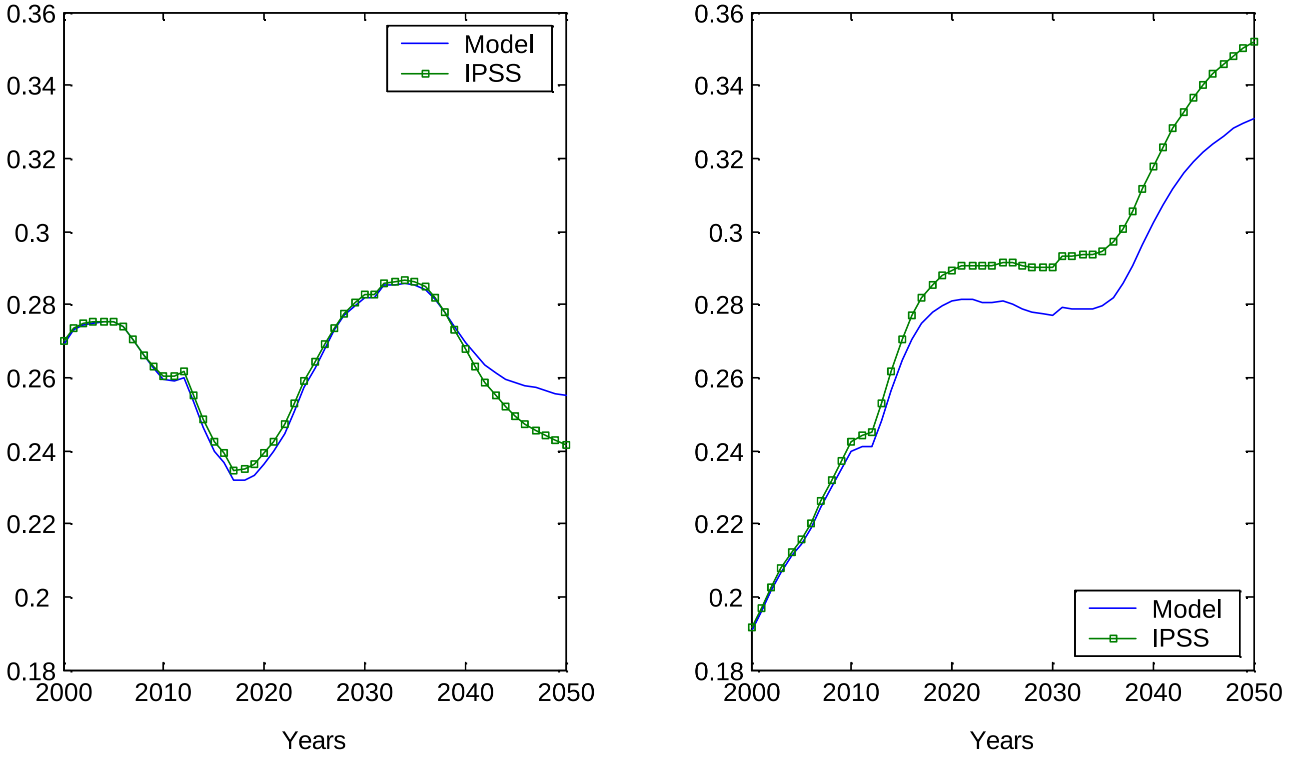


Figure 2 : Demographic Comparison



The fraction of age cohort 21-35 to total population

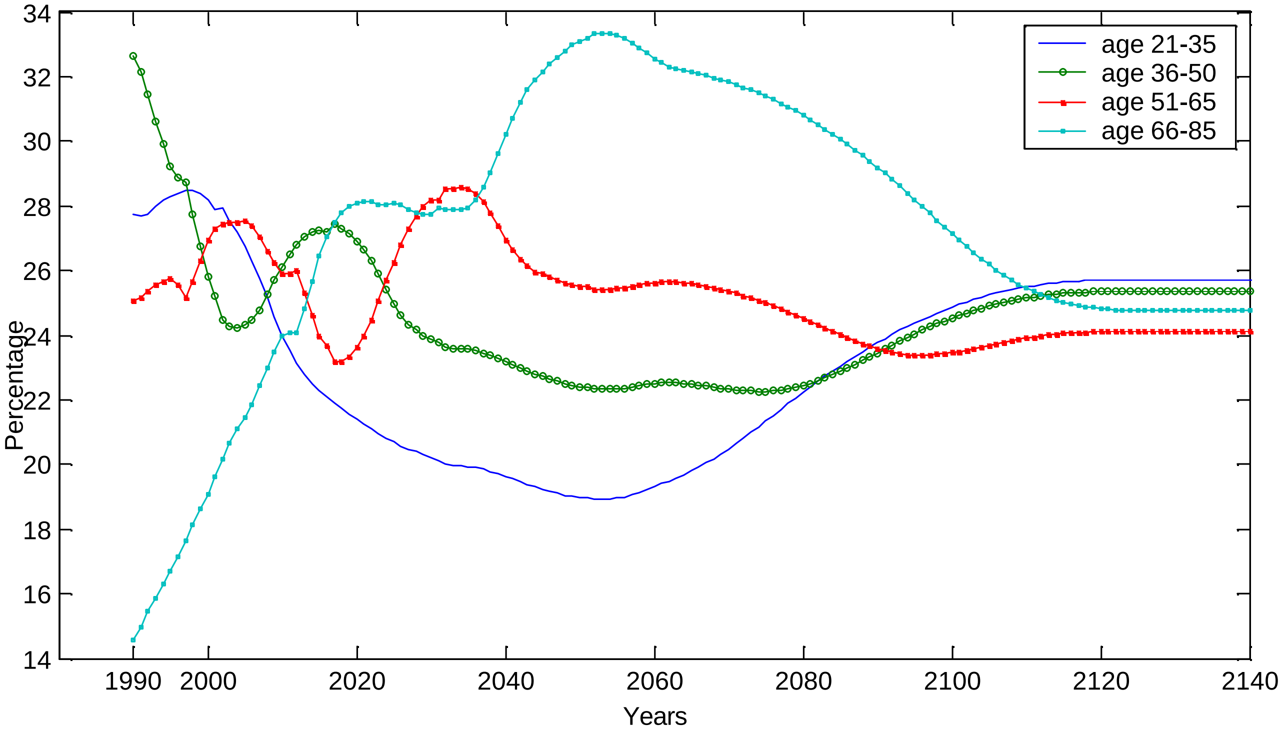
The fraction of age cohort 36-50 to total population



The fraction of age cohort 51-65 to total population

The fraction of age cohort 66-85 to total population

Figure 3
Baseline
The evolution of the demographic structure



The evolution of the total population

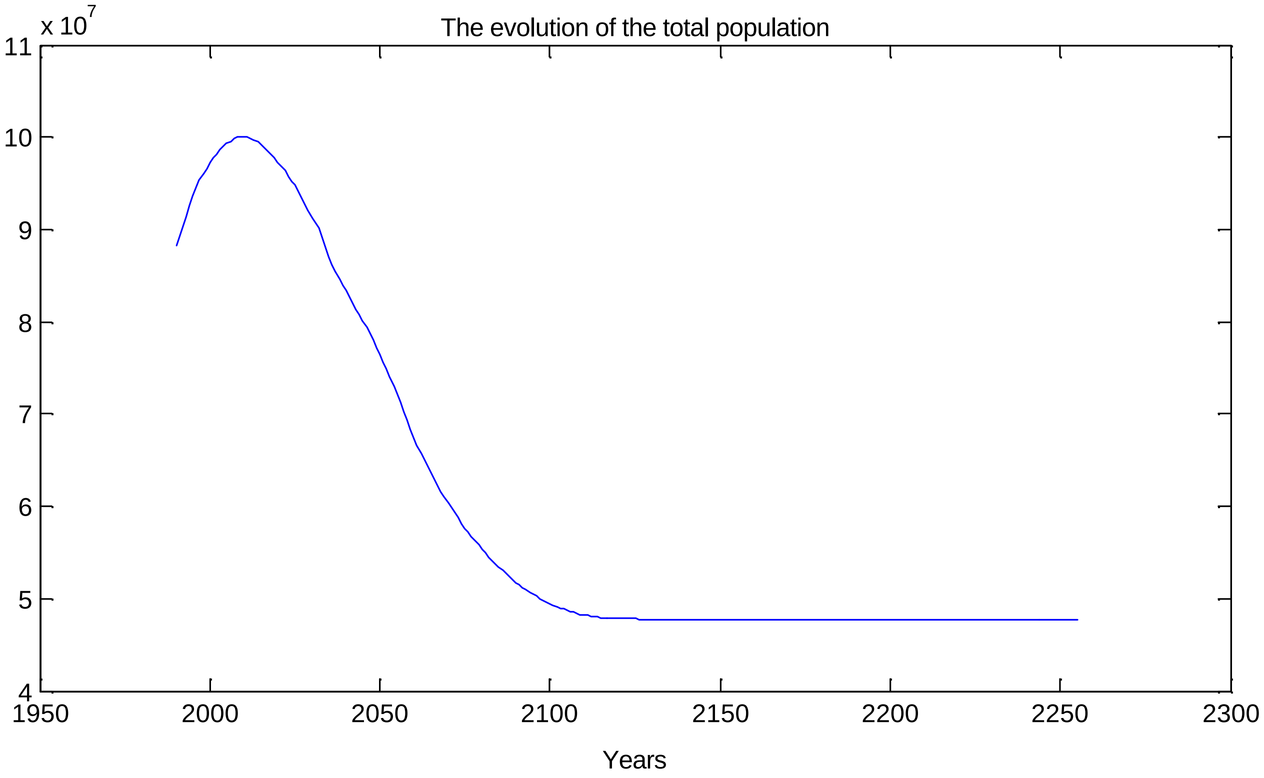
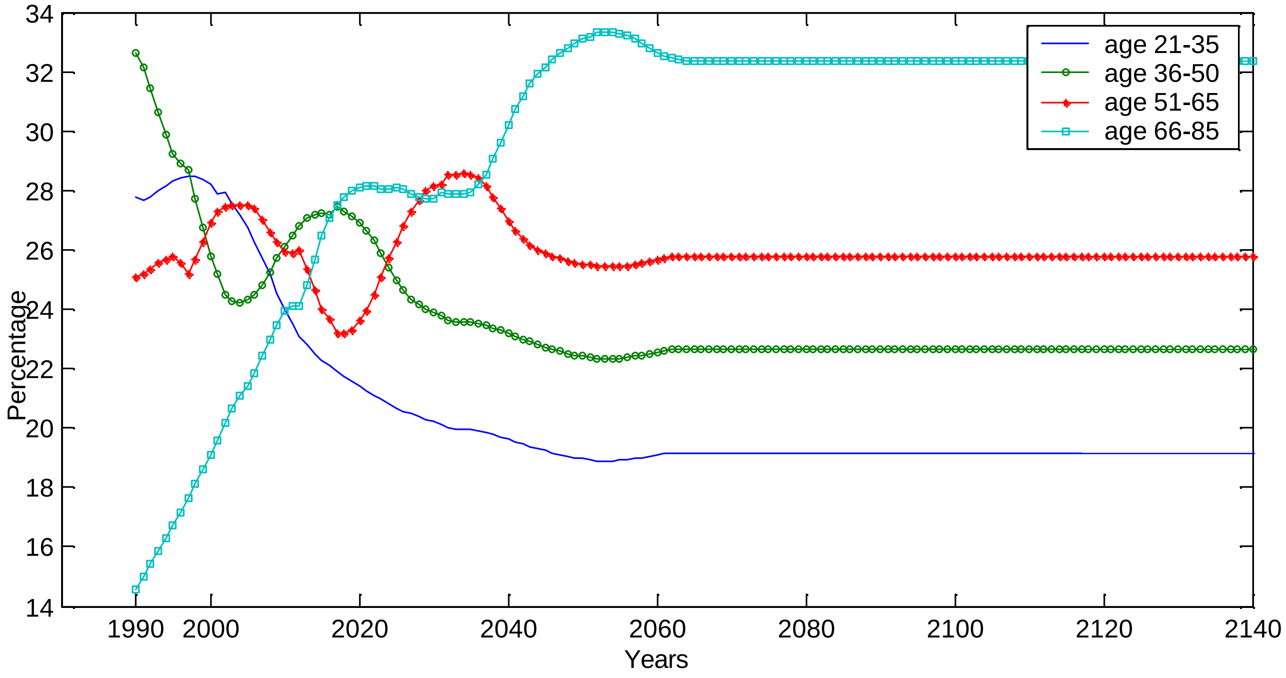


Figure 4
Permanently low birth rate
The evolution of the demographic structure



The evolution of the total population

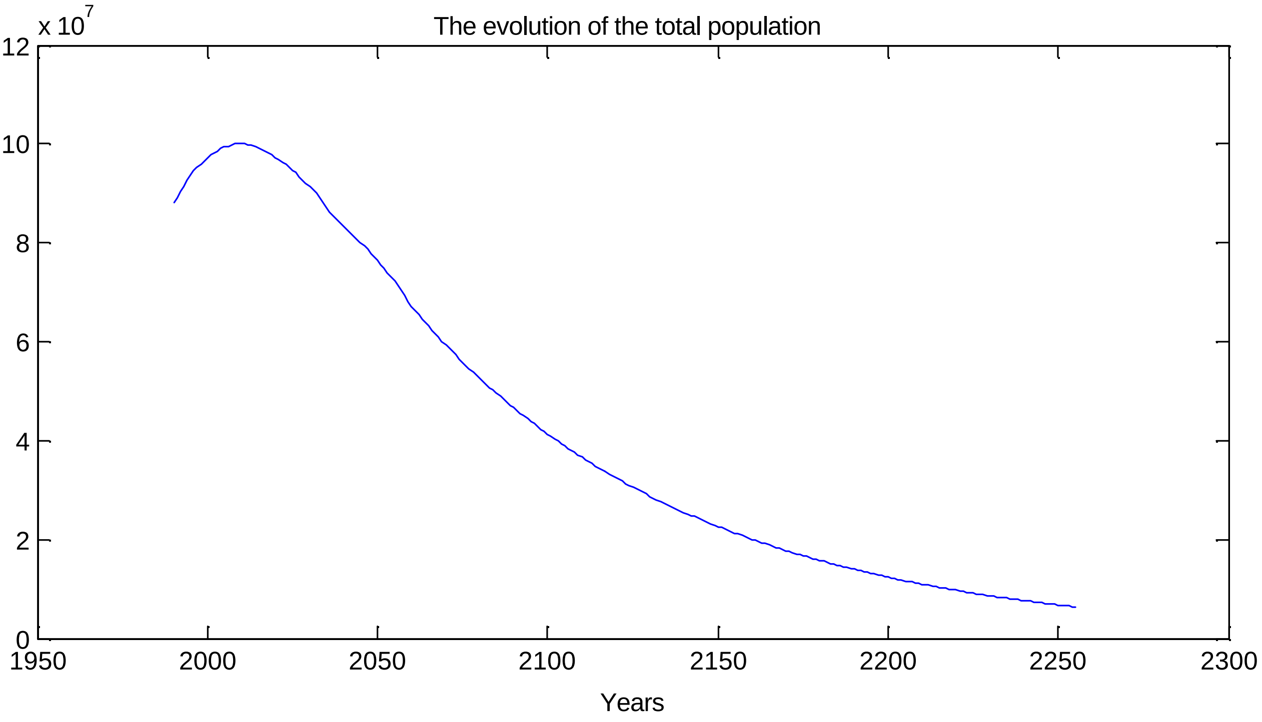
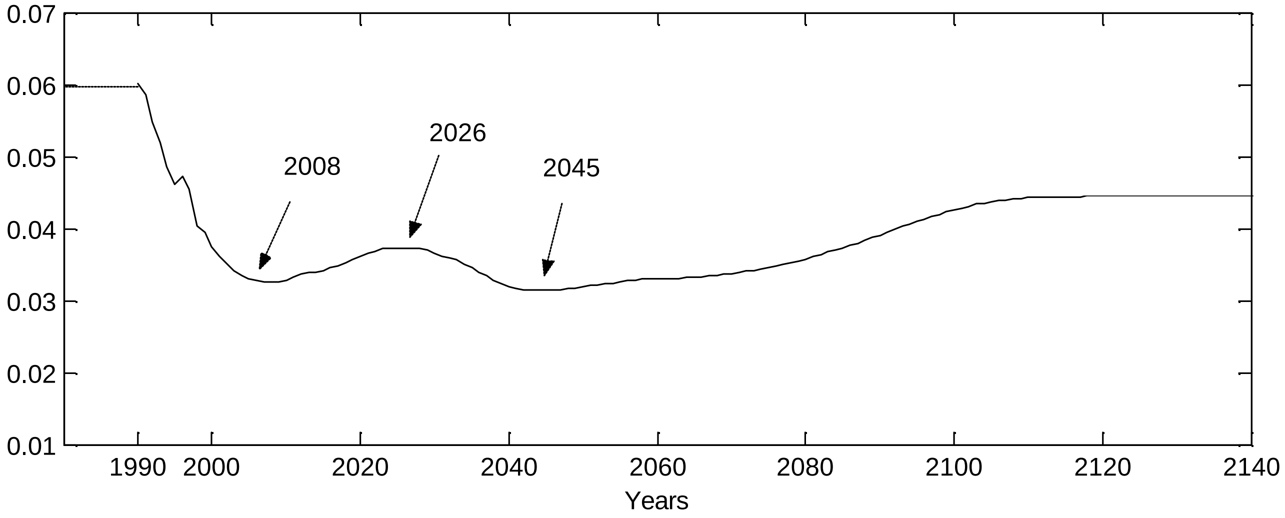
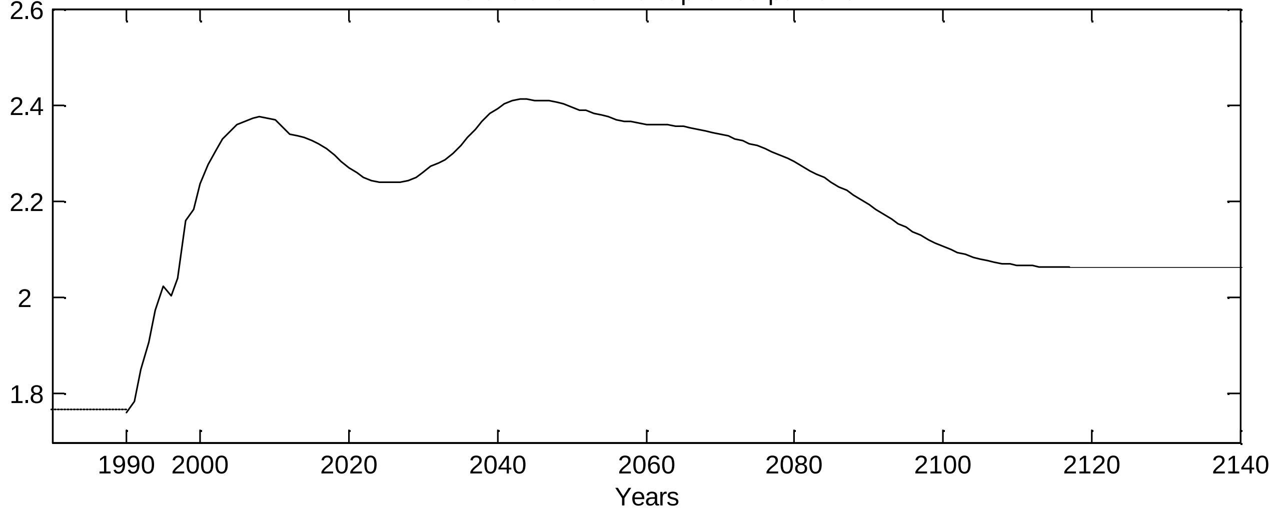


Figure 5 (a)
Baseline

The evolution of the after-tax real interest rate



The evolution of the capital-output ratio



The evolution of the personal saving rate

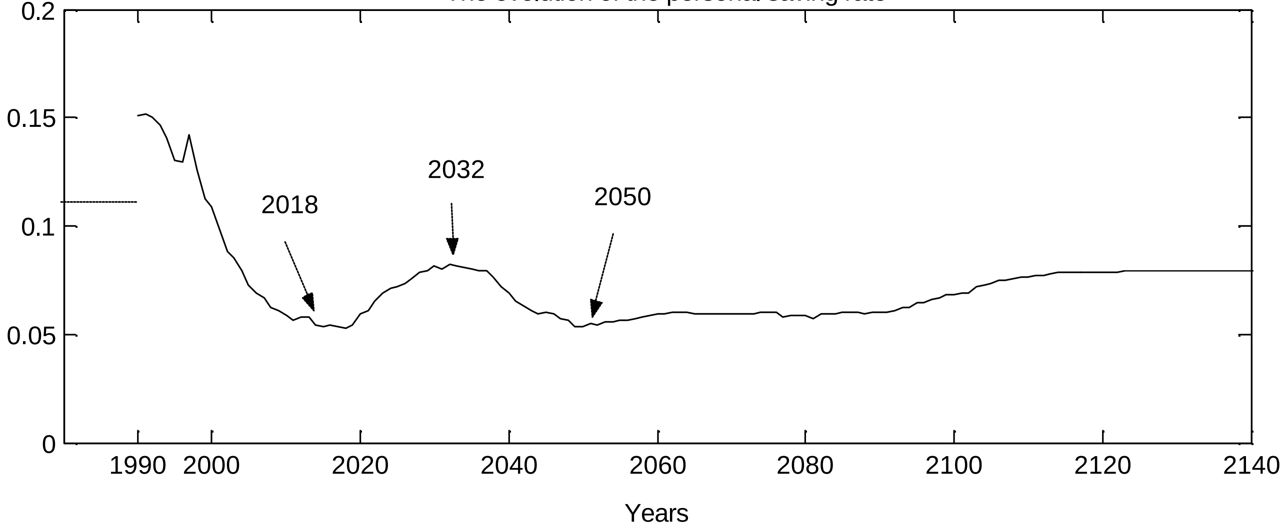
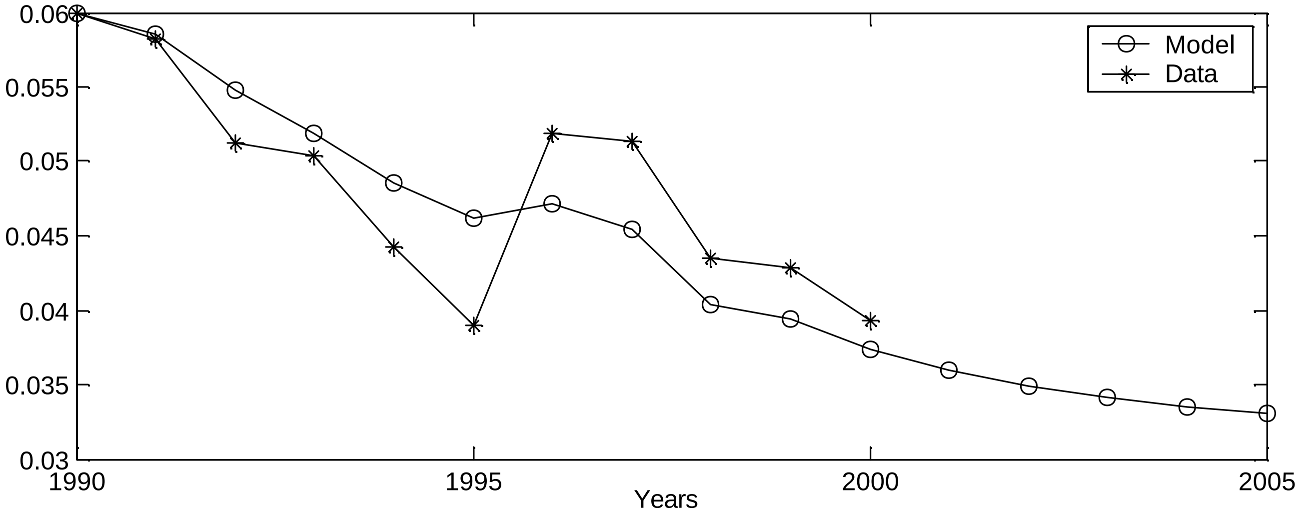


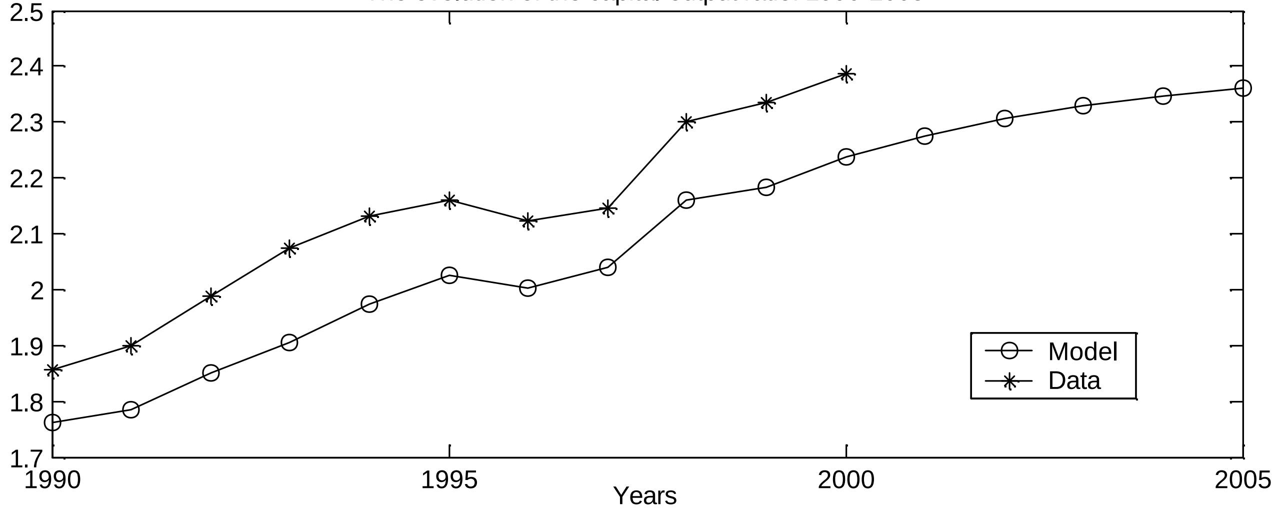
Figure 5 (b)

Baseline

The evolution of the after-tax real interest rate: 1990-2005



The evolution of the capital-output ratio: 1990-2005



The evolution of the personal saving rate: 1990-2005

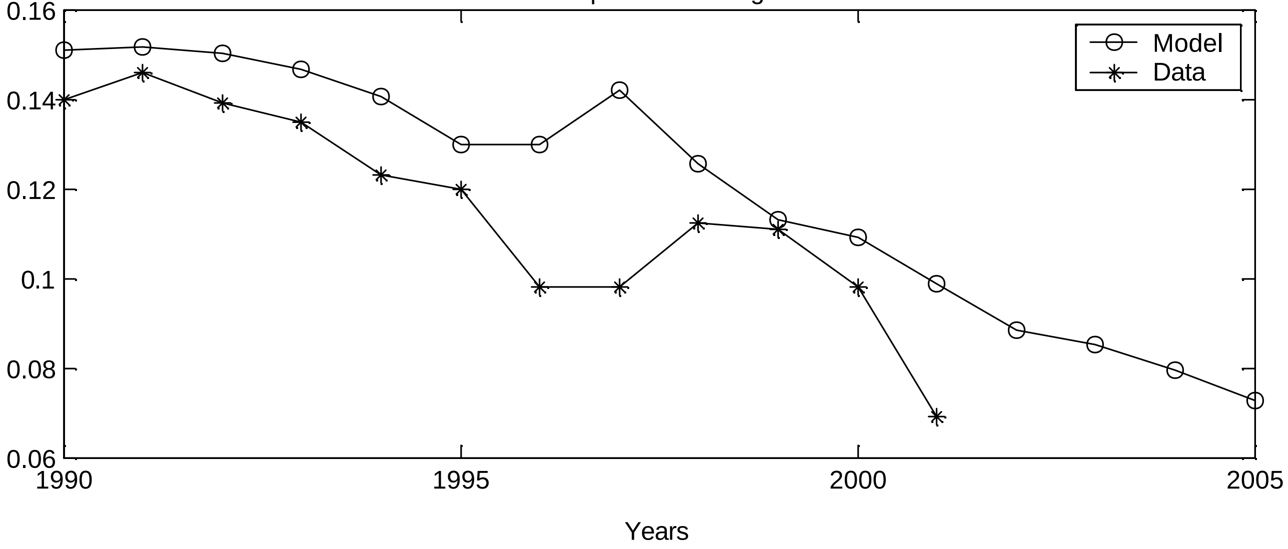
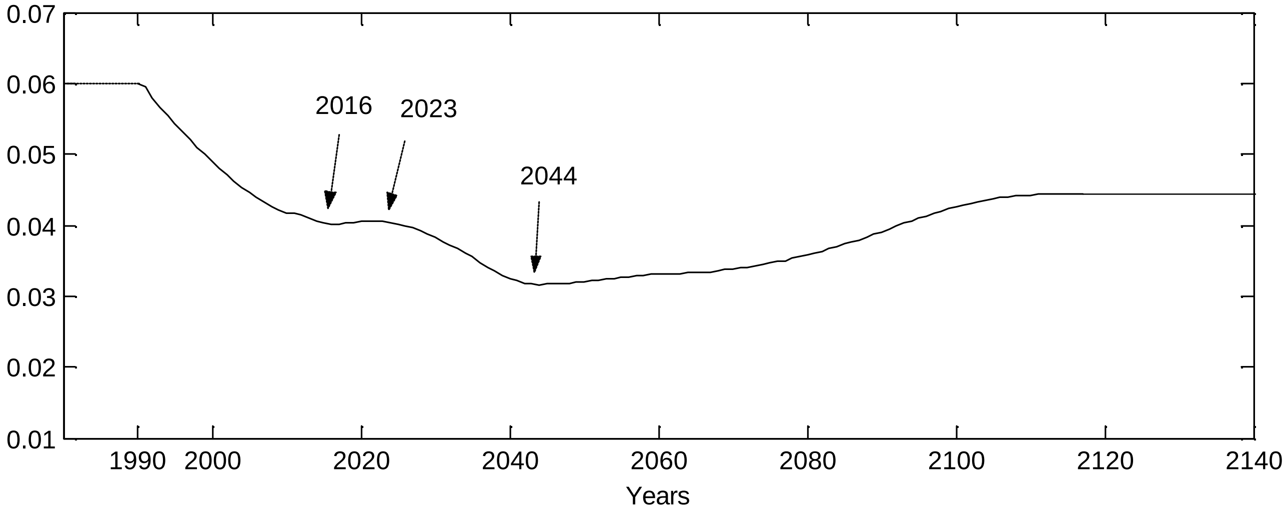


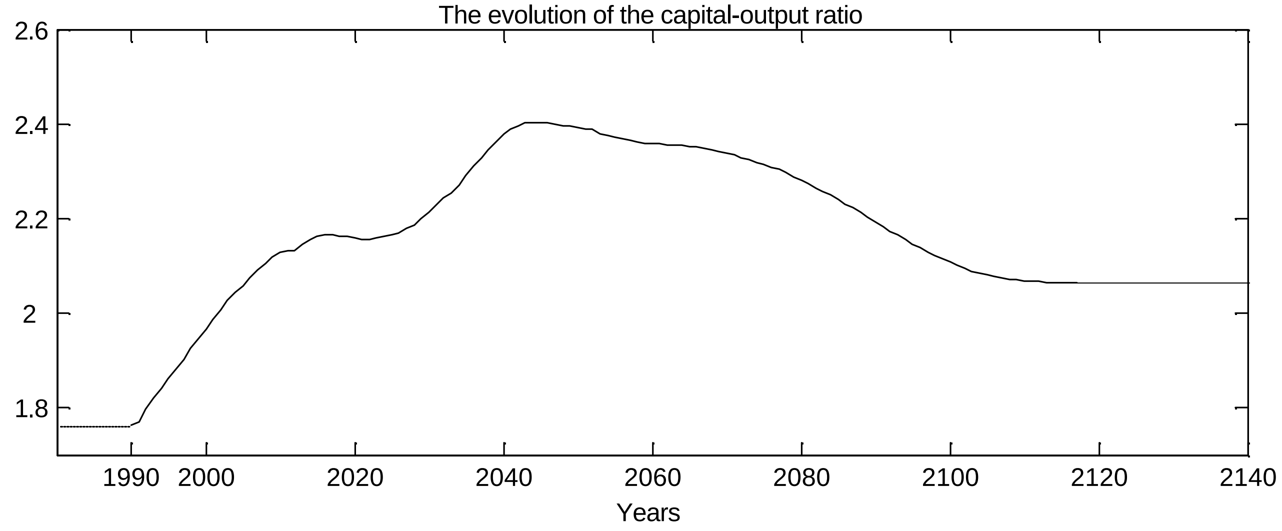
Figure 6 (a)

Baseline demographics & 2% TFP growth rate

The evolution of the after-tax real interest rate



The evolution of the capital-output ratio



The evolution of the personal saving rate

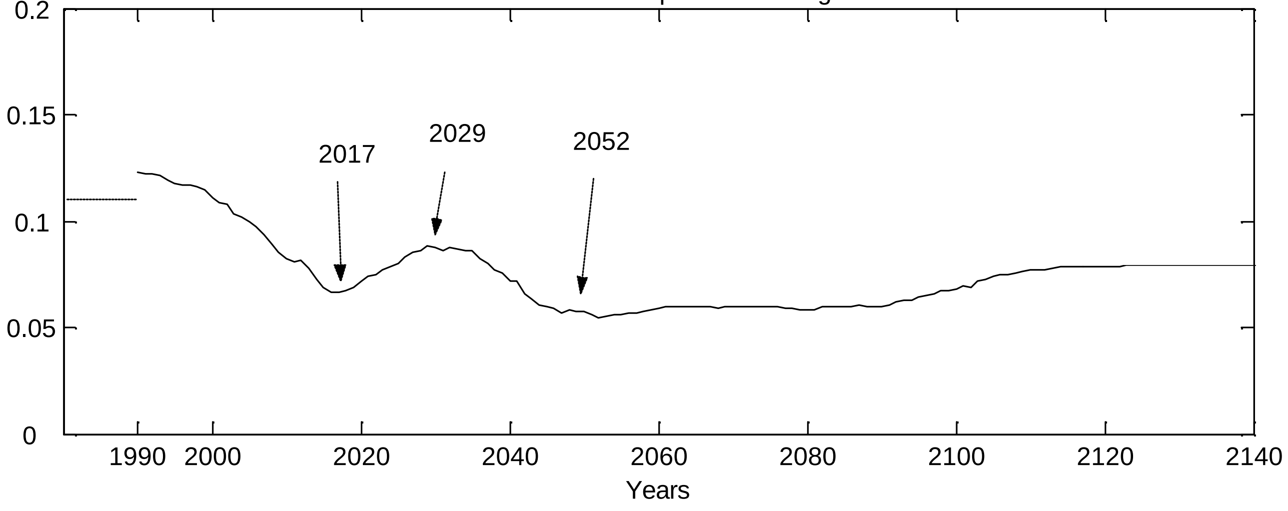
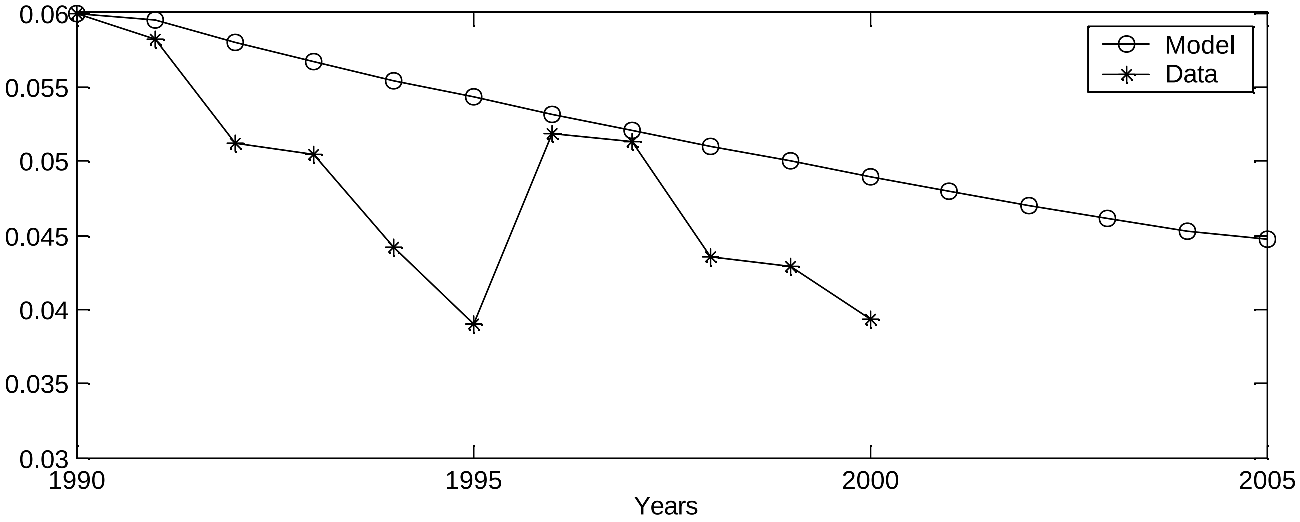
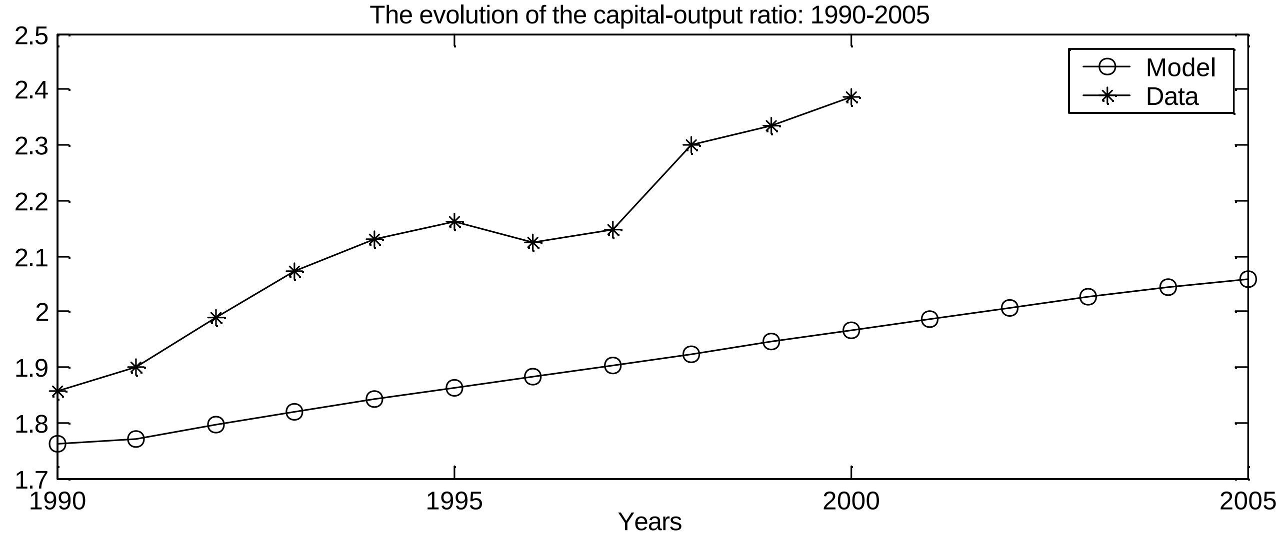


Figure 6 (b)
Baseline demographics & 2% TFP growth rate

The evolution of the after-tax real interest rate: 1990-2005



The evolution of the capital-output ratio: 1990-2005



The evolution of the personal saving rate: 1990-2005

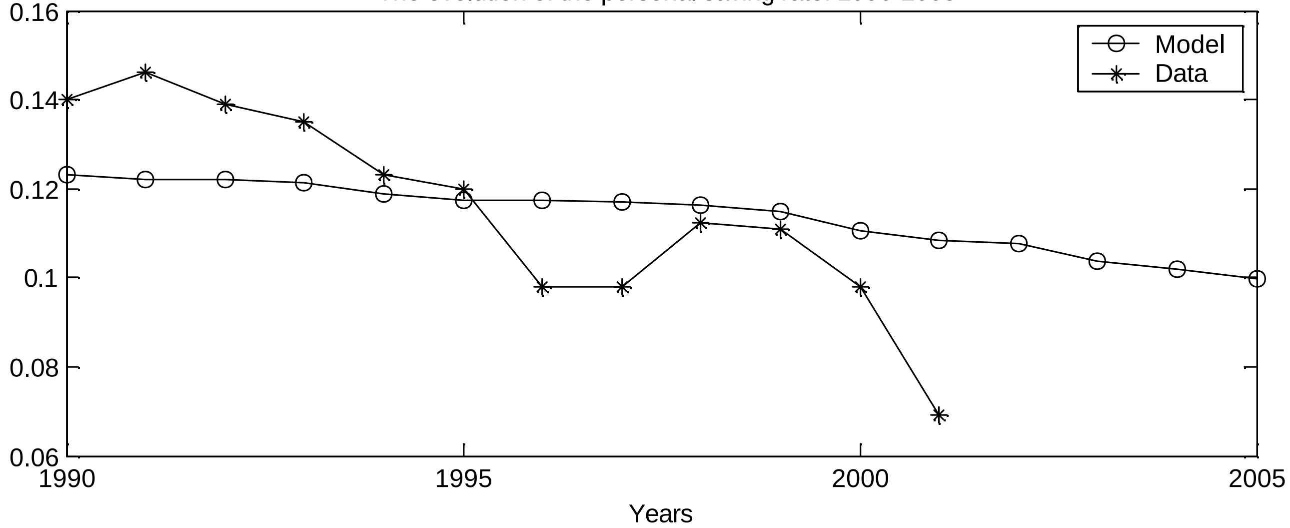
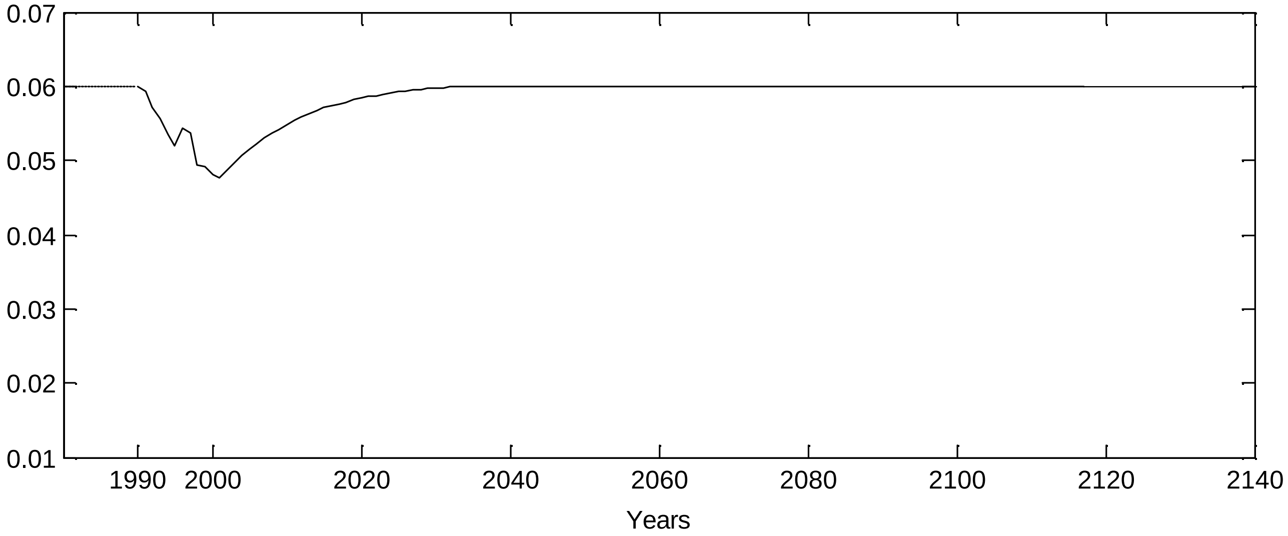
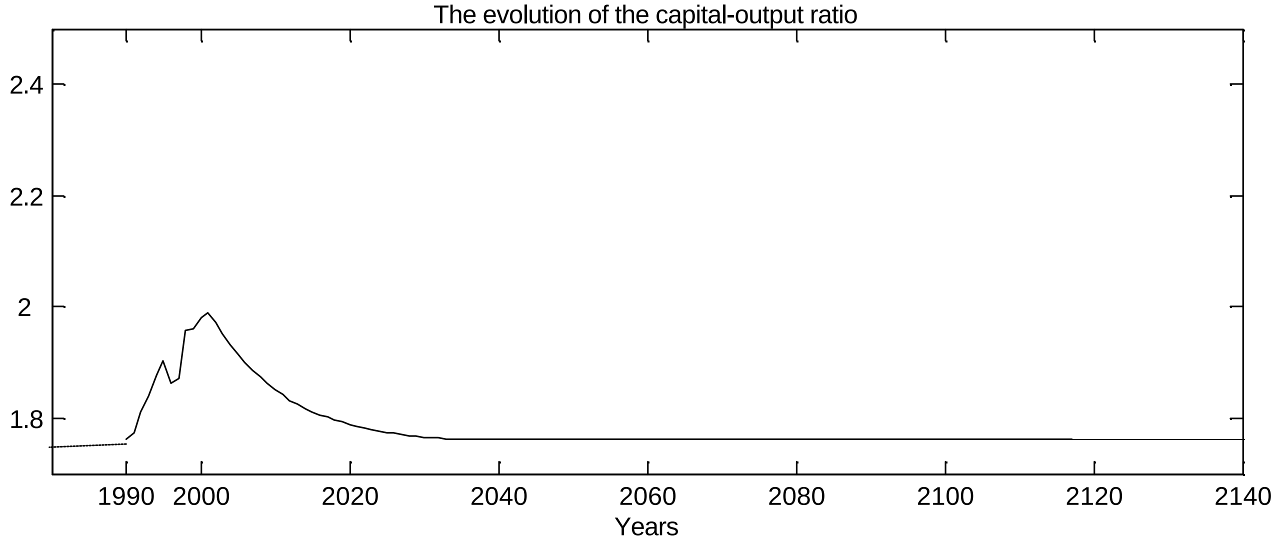


Figure 7 (a)
Baseline TFP growth rate& 1990 demographics

The evolution of the after-tax real interest rate



The evolution of the capital-output ratio



The evolution of the personal saving rate

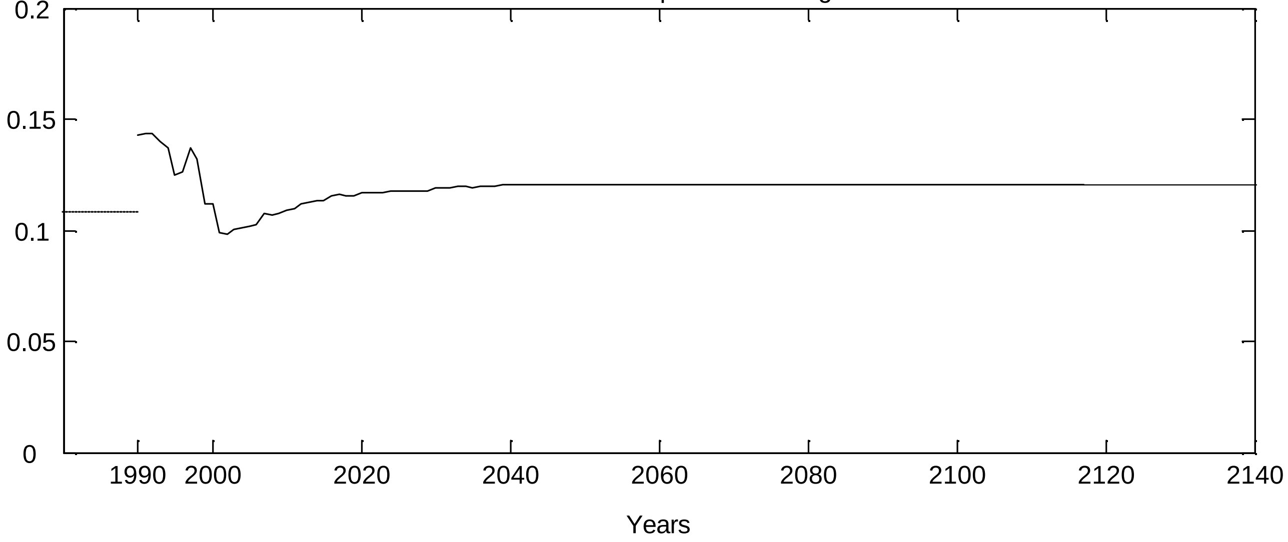


Figure 7 (b)

Baseline TFP growth rate & 1990 demographics

The evolution of the after-tax real interest rate: 1990-2005

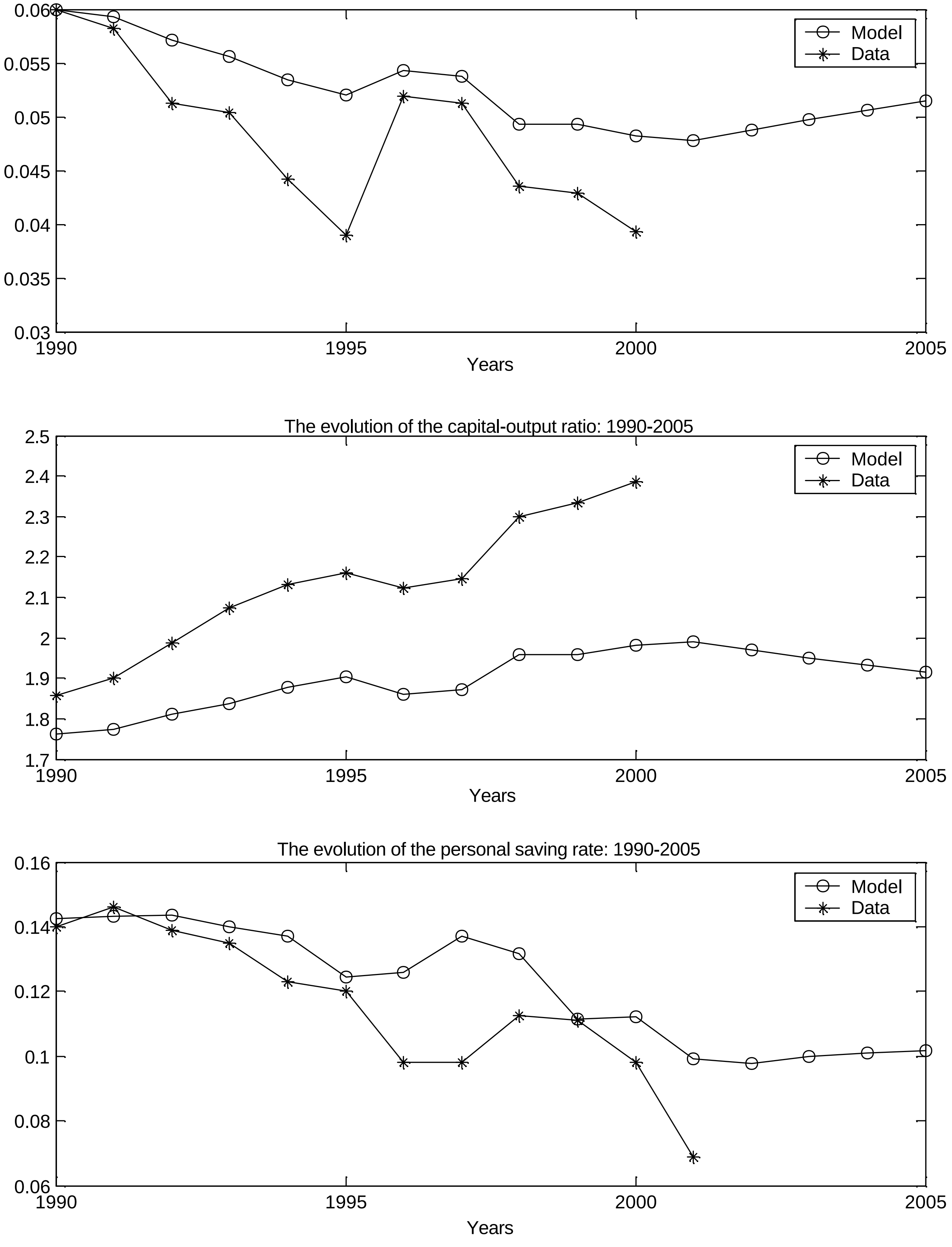
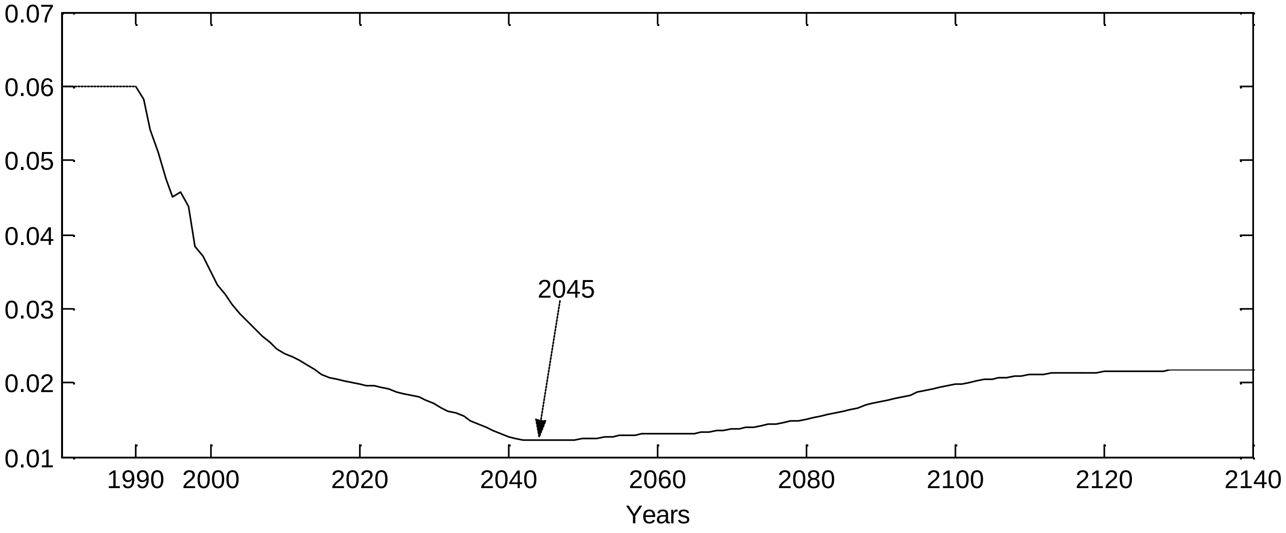


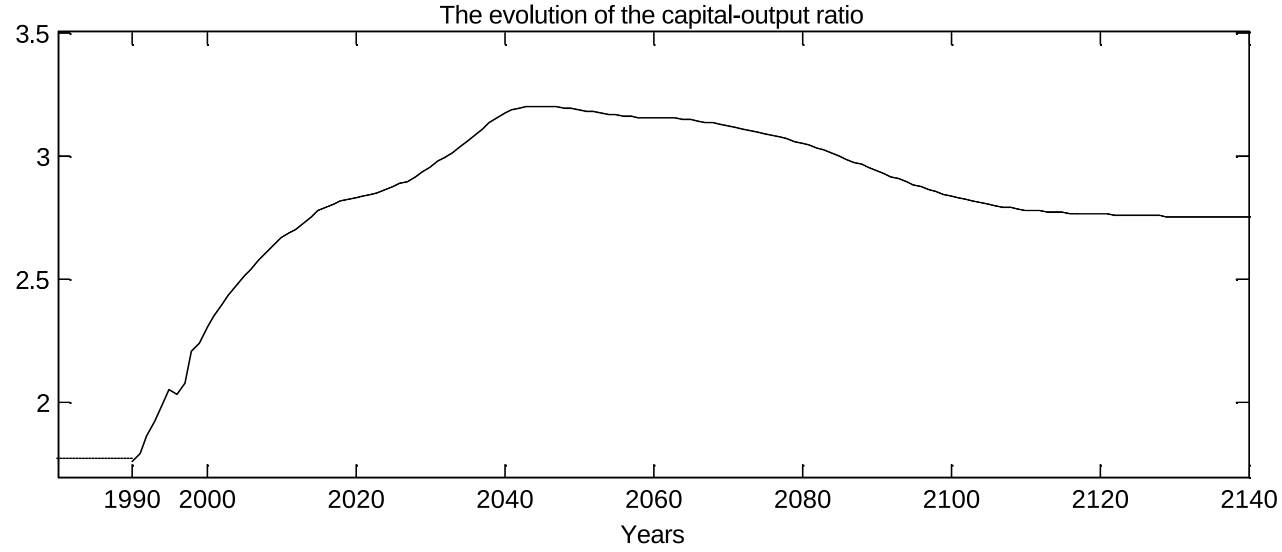
Figure 8 (a)

Baseline demographics & pemanently low TFP growth rate

The evolution of the after-tax real interest rate



The evolution of the capital-output ratio



The evolution of the personal saving rate

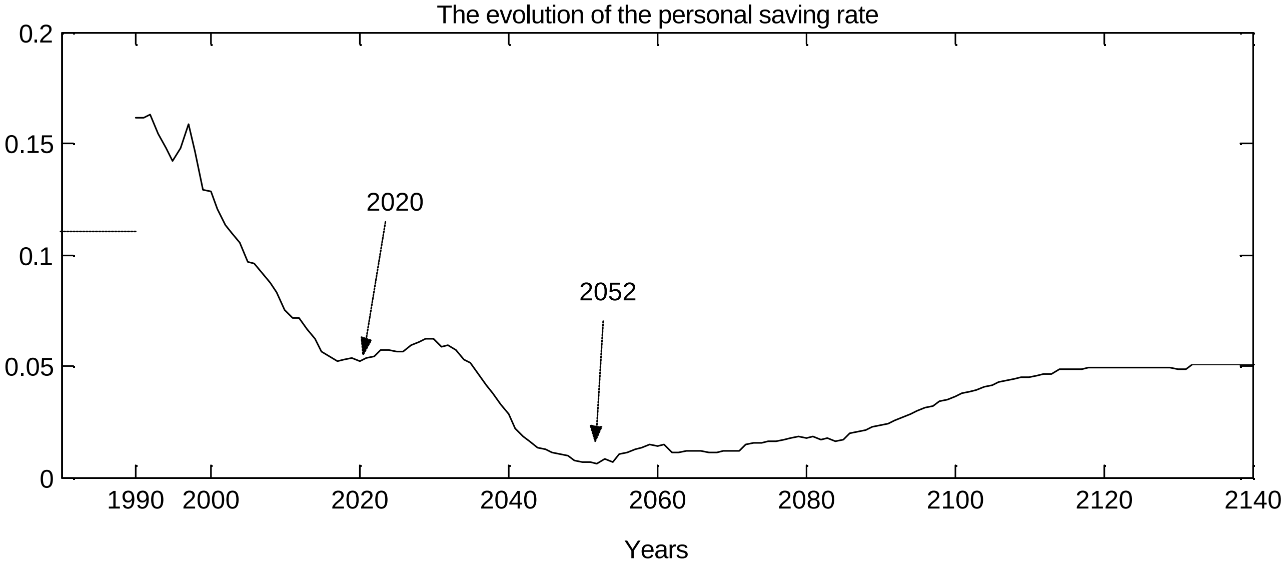
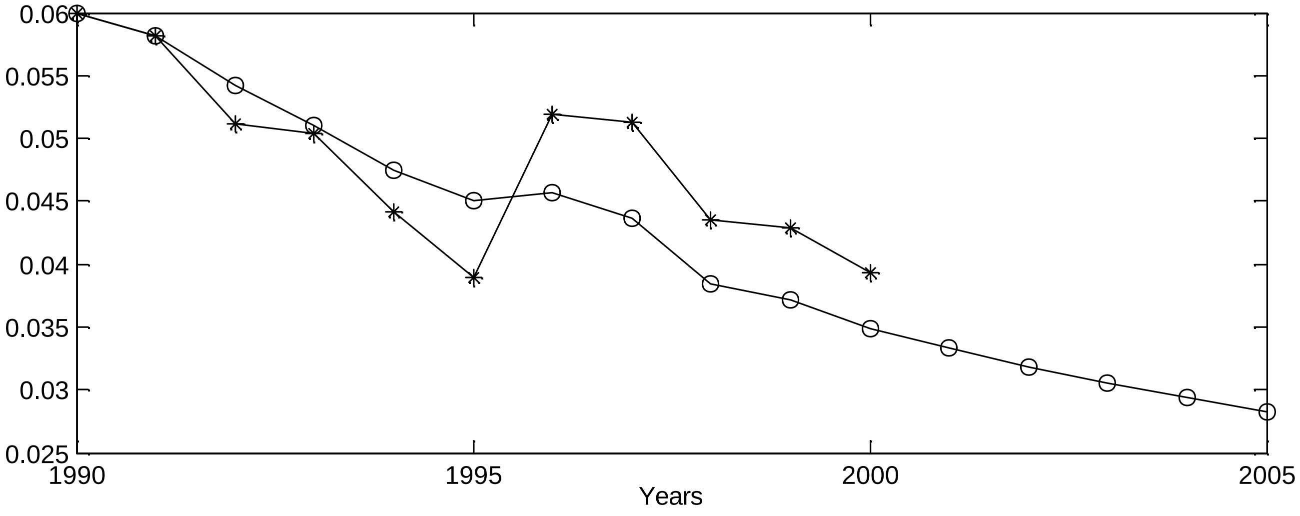


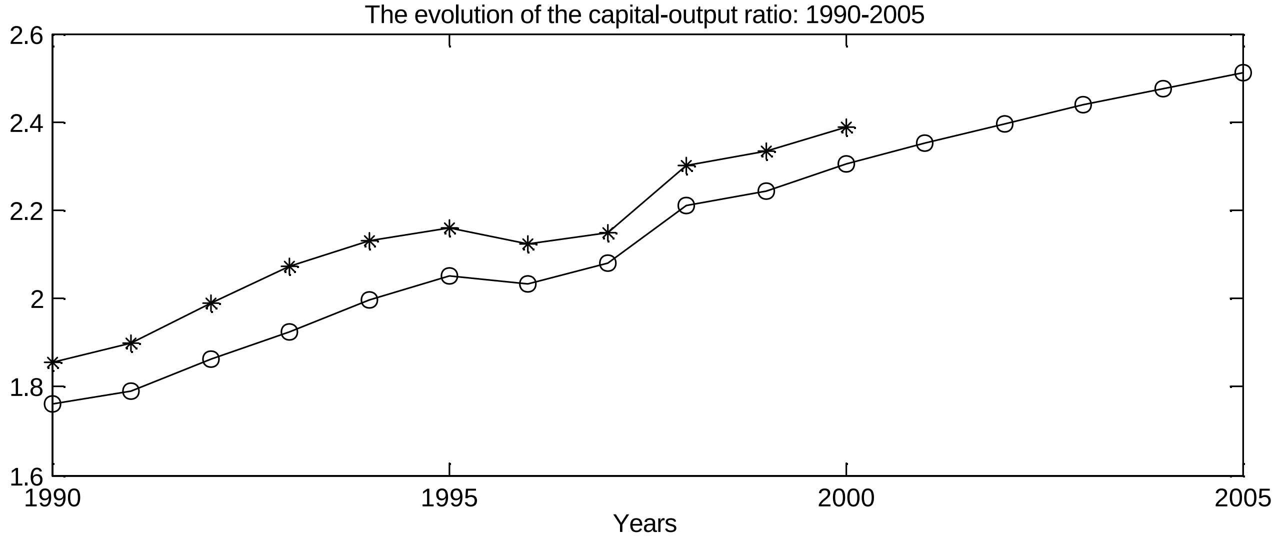
Figure 8 (b)

Baseline demographics & permanently low TFP growth rate

The evolution of the after-tax real interest rate: 1990-2005



The evolution of the capital-output ratio: 1990-2005



The evolution of the personal saving rate: 1990-2005

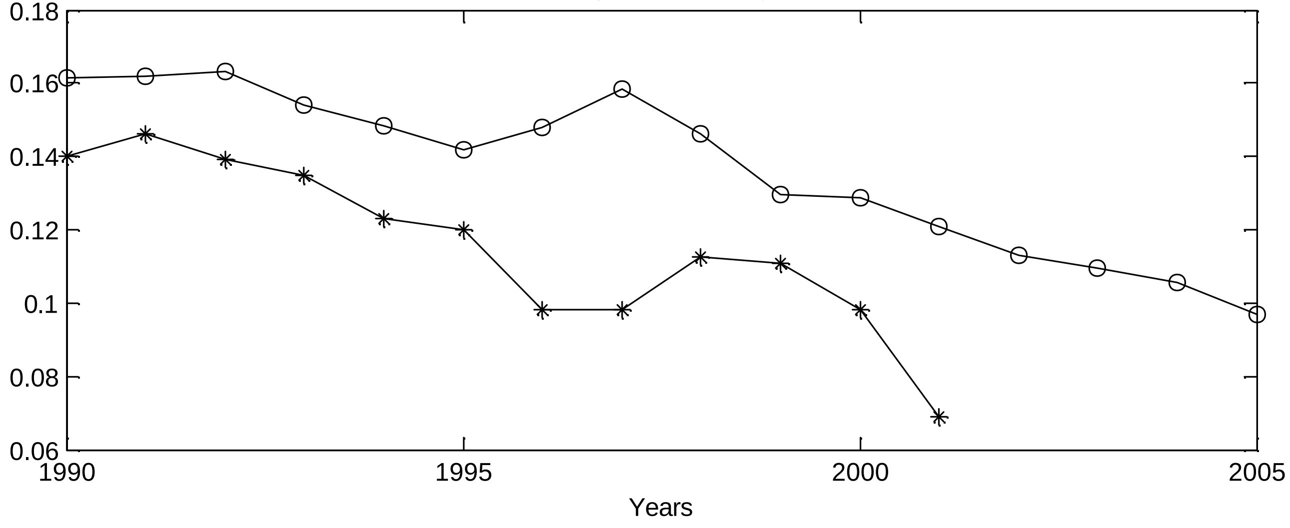
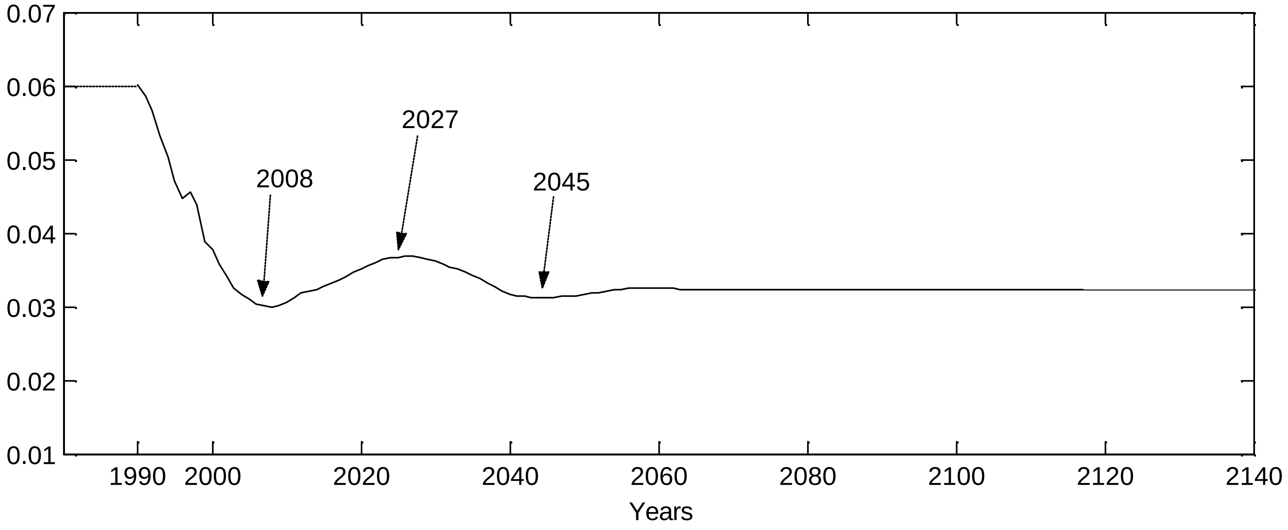


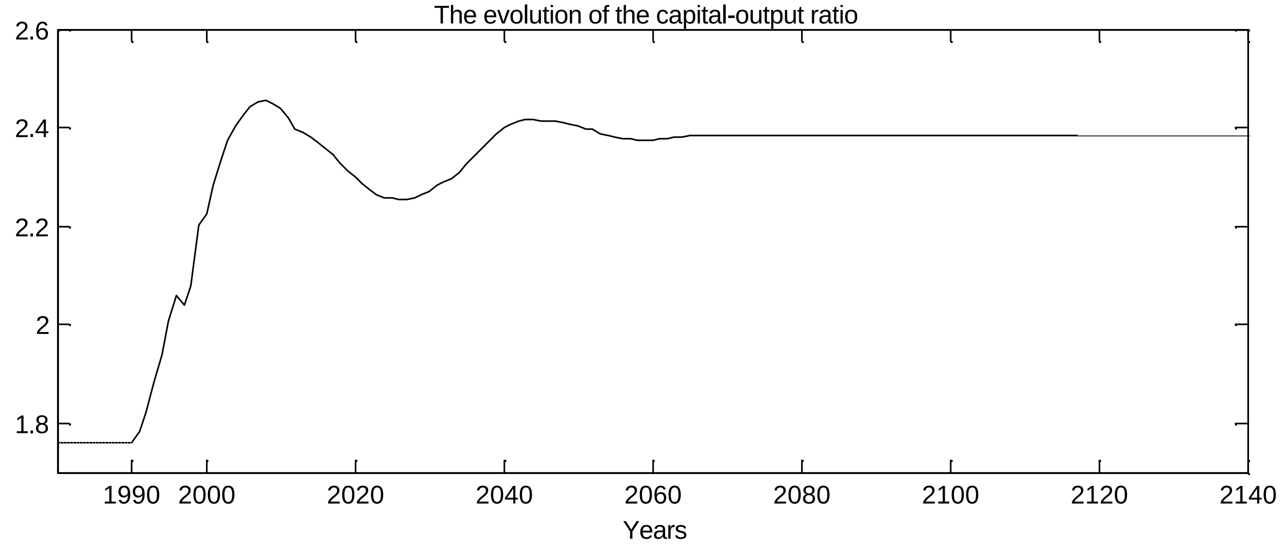
Figure 9 (a)

Baseline TFP growth rate & permanently low birth rate

The evolution of the after-tax real interest rate



The evolution of the capital-output ratio



The evolution of the personal saving rate

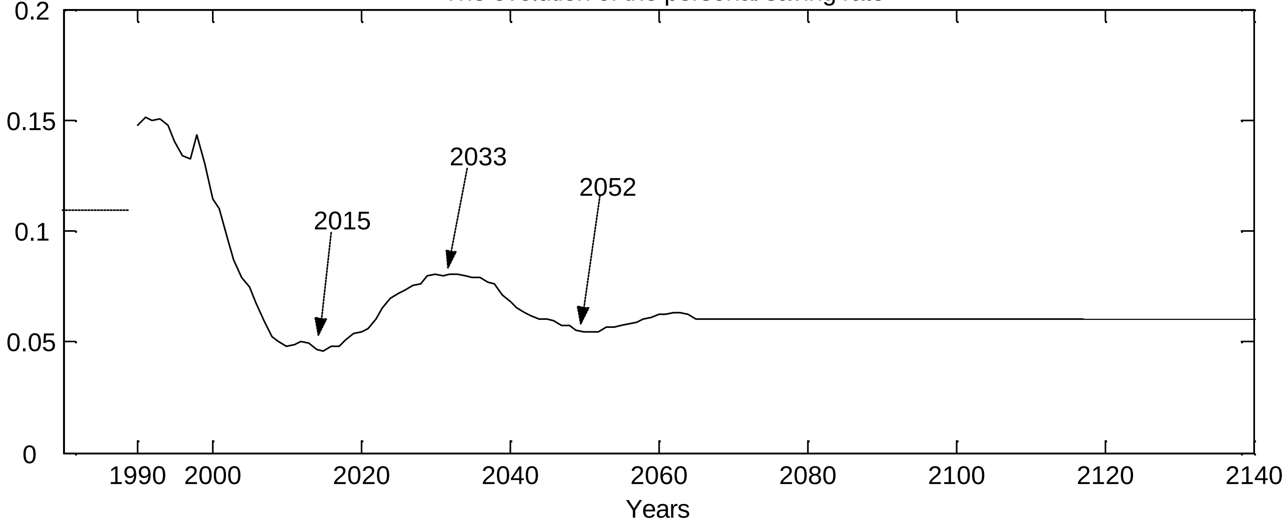
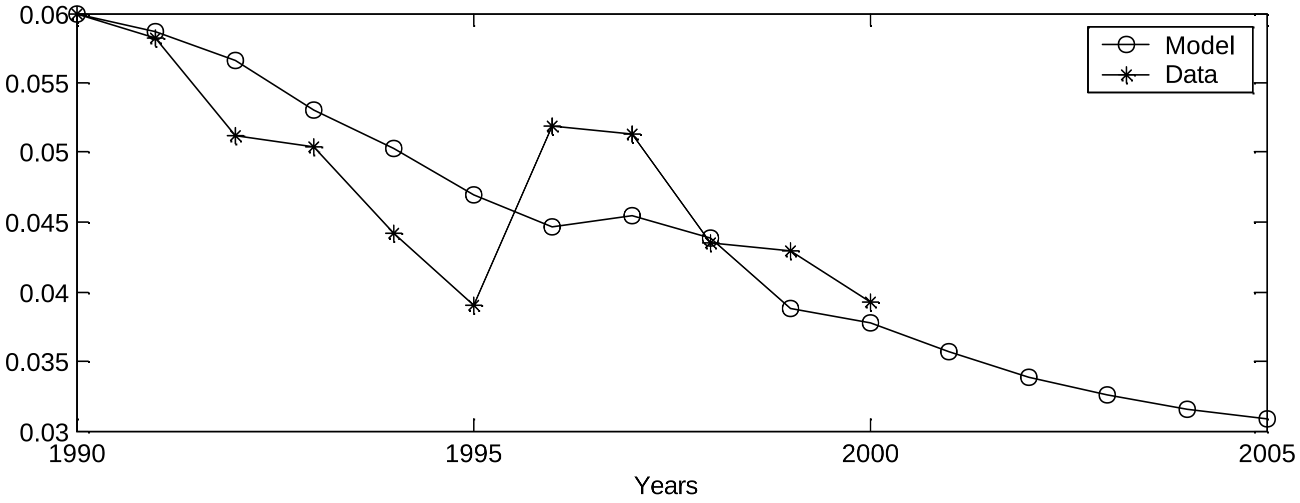


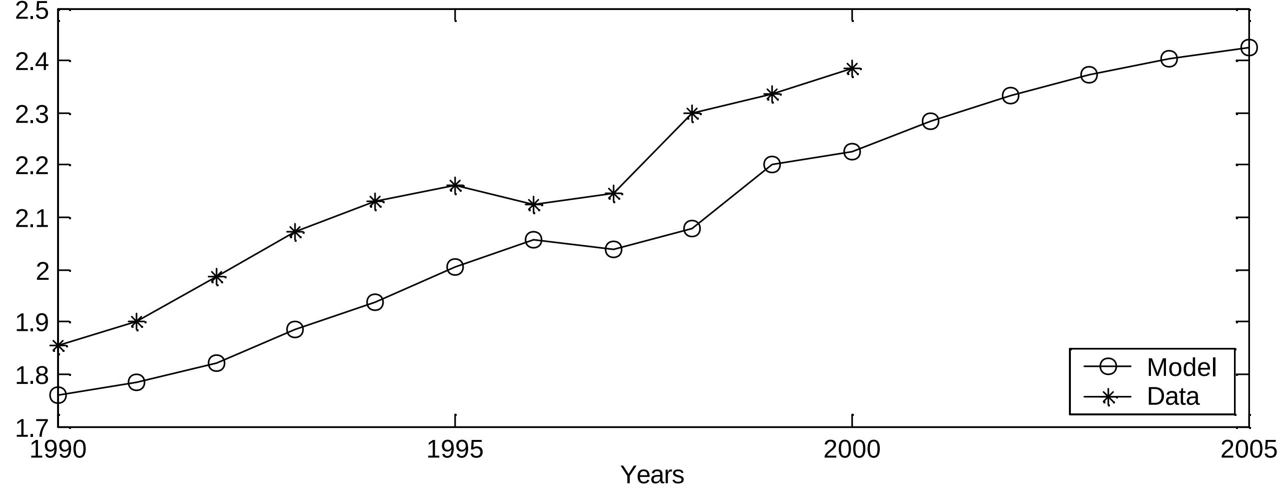
FIgure 9 (b)

Baseline TFP growth rate & permanently low birth rate

The evolution of the after-tax real interest rate: 1990-2005



The evolution of the capital-output ratio: 1990-2005



The evolution of the personal saving rate: 1990-2005