The quantitative effects of monetary aggregate targeting in a zero interest rate environment: results from Japan

R.Anton Braun*
University of Tokyo

Takemasa Oda
Bank of Japan

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Abstract

What are the effects of monetary aggregate targeting in a zero nominal interest rate environment? This paper develops a computable overlapping generations model to answer this question. In our model there are two sources of real balance effects: finite lifespans and borrowing constraints. Steady-state results reveal an asymmetry in the welfare costs of alternative monetary policies. A monetary aggregate targeting policy that is too tight has large and negative effects on welfare. A loose monetary aggregate targeting policy has much smaller effects on welfare. A dynamic analysis using data from Japan finds that the "quantitative easing" policy reduced deflationary pressure. Although the effects of this policy on GNP growth are small, there are important distributional effects. The biggest beneficiaries of this policy are the young who experience an easing in borrowing constraints and the old who benefit from lower taxes and higher interest rates. Individuals in other age groups experience consumption losses.

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*Corresponding author: Faculty of Economics, University of Tokyo, e-mail: toni@e.u-tokyo.ac.jp. The views expressed in the paper are those of the authors and not those of the Bank of Japan.

1 Introduction

In April 1999 the Japanese call rate dropped to 0.01 percent. It remained at or about this level with the exception of a short period in 2000 until March 2006. With the nominal interest rate at effectively zero, the question arose as to what further actions the Bank of Japan could take to stimulate the Japanese economy of if not that at least quicken the end of deflation. In March 19, 2001 the Bank of Japan initiated a quantitative easing policy that targeted the level of excess reserves.[1] This policy was pursued until the call rate was raised to 0.25 percent in March 2006.

The objective of this paper is to use an economic model to assess the effects of quantitative easing on the price level and economic activity. When the nominal interest rate is zero money and short-term government bonds are perfect substitutes. In the infinite horizon models that are commonly used for analyzing the effects of monetary policy, a change in the timing of total government liabilities (including money) and lump-sum taxes has no real effects on economic activity. Krugman (1998) refers to this phenomenon as a liquidity trap. Ireland (2005) shows that allowing for positive population growth in a Blanchard (1985) model with infinite lived overlapping generations breaks Ricardian equivalence and induces a real balance effect. Auerbach and Obstfeld (2005) find that large open-market purchases of bonds can counteract deflationary price tendencies and lower the real value of government debt if households expect that the nominal interest rate will eventually rise above zero. Lower government debt reduces the need to tax and this raises household welfare.

We consider the effects of monetary aggregate targeting in a computable overlapping generations (OG) model. We choose this model because it produces real balance effects and its realizations can readily be compared with Japanese macroeconomic outcomes. The model period is a year, households are active for 80 years and they can save by accumulating capital, money or bonds. Savings patterns vary by age and there are active loan markets.

There are two factors that produce real balance effects in our model. First, households are finite lived and the timing of government borrowing can affect their present value tax liabilities and induce wealth effects. Second, we impose borrowing constraints that rule out uncollateralized lending. Lowering taxes increases disposable income and increases current consumption of households who are experiencing binding borrowing constraints.

A steady-state analysis indicates that a zero nominal interest rate is a good monetary policy that maximizes average welfare. However, the risks of small mistakes are asymmetric. As the growth rate of money is lowered from its optimal value welfare declines sharply. Welfare also declines if money growth is too high. However, the welfare losses associated with too rapid money growth are low. The reason for this asymmetry is due to the fact that once the nominal interest rate reaches zero money is a direct competitor with capital. In this situation lowering the growth rate of money further increases the real return

[1]Formally, the Bank of Japan set a target on ”current account balances” that greatly exceeded required reserves.

on holding money and this directly crowds out the capital stock. On the other hand, when the growth rate of money is too high money is dominated in rate of return and households keep their holdings of money low. Households' efforts to economize on their holdings of money limits the crowding out effect of money on capital. This results provides a rationale for why a central bank might want to increase monetary supply when the nominal interest rate is zero.

We also conduct a dynamic analysis that compares our baseline specification with a counterfactual with no quantitative easing. Our dynamic analysis identifies the following effects of quantitative easing. Quantitative easing reduces deflationary pressure and thus reduces the number of periods that the nominal interest rate is zero when compared to a counterfactual with no quantitative easing. In our counter-factual the nominal interest rate is zero for 12 years as compared to 9 years under the baseline specification.

Temporarily high government debt and high interest rates each act to relax borrowing constraints. Higher debt implies that taxes are lower today and this allows those facing binding borrowing constraints to consume more. A higher real interest rate induces an intertemporal substitution effect that also reduces demand for today's consumption. These effects are most pronounced when we allow for age specific labor productivity. With an increasing wage profile young individuals benefit.

The higher real interest rate associated with quantitative easing also benefits retirees. Their savings now have a higher return. Retirees experience another benefit. Their higher mortality risk allows them to escape higher future taxes.

A higher interest rate is also associated with a lower wage rate and this lowers consumption for individuals who are working. This has the biggest effect on middle aged workers who are close to the peak of lifetime labor efficiency.

In our simulations both the benefits and costs of quantitative easing are concentrated among the old. Consumption of retirees rises by as much as 2.3 percent between 2001 and 2005. Workers who have the highest labor productivity experience losses of as high as 1.3 percent. For younger workers the benefits of relaxed borrowing constraints are largely off set by lower wages and the consumption gains are small.

We also compare the baseline specification with two other counterfactuals that are designed to assess the timing and duration of the quantitative easing policy. The counterfactual with longer quantitative easing exhibits a longer period of deflation, a larger real interest rate response and lower output than the baseline scenario. The counterfactual with earlier quantitative easing has the most interesting effects. In early periods there is more deflation but deflation ends earlier than the baseline scenario. This scenario also exhibits higher real wages, consumption and output than the baseline scenario. A comparison of these two scenarios with the baseline also reveals that the effects of higher monetary growth on economic activity differ depending on whether the initial situation is one with a positive nominal interest rate or an initial situation is a zero nominal interest rate.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 explains how we parametrize the model. Section 4 contains our results and we conclude in Section 5.

2 The Model

We consider an economy that involves in discrete time. The structure of the real side of the economy is similar to economy considered by Braun, Ikeda and Joines (2007). Their model reproduces some of the principal macroeconomic facts of the Japanese economy between 1961 and 2002.

2.1 Demographics

Agents are born and become active at age 21. The growth rate of 21 year old individuals, n_1 is assumed to be constant in each period. Agents are subject to mortality risk in each period. If we let $N_{j,t}$ be the number of households of age j in period t, the dynamics of population are governed by a first-order Markov process:

$$\mathbf{N}_{t+1} = \begin{bmatrix} (1+n_1) & 0 & 0 & \dots & 0 \\ \psi_1 & 0 & 0 & \dots & 0 \\ 0 & \psi_2 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \psi_{J-1} & 0 \end{bmatrix} \mathbf{N}_t \equiv \mathbf{\Gamma} \mathbf{N}_t, \tag{1}$$

where $\mathbf{N}_t$ is a $J \times 1$ vector that describes the population of each cohort in period t, ψ_j is the conditional probability that a household of age j survives to the next period and ψ_J is implicitly assumed to be zero. The aggregate population in period t, denoted by N_t, is given by

$$N_t = \sum_{j=1}^{J} N_{j,t}. \tag{2}$$

The population growth rate is then given by $n = N_{t+1}/N_t$. The unconditional probability of surviving from birth in period $t-j+1$ to age $j>1$ in period t is:

$$\xi_j = \psi_{j-1}\xi_{j-1} \tag{3}$$

where $\xi_{1,t} = 1$ for all t.

2.2 Problem for a household born into cohort s

Households are born with zero assets and retire at age 65. The maximum life-span of an individual is $J = 100$ years. Money is introduced by assuming that households receive utility from two goods and assuming that one of the goods, which we refer to as the cash good, c_{1t}, is subject to a cash in advance constraint as in Lucas and Stokey (1987). The credit good, c_{2t}, may be purchased with cash or on credit. Households also value leisure, l_{jt}. Given these definitions expected present value utility of a household belonging to cohort j is:

$$\sum_{j=s}^{J+s} \beta^{j-s} \xi_j u(c^s_{1jt}, c^s_{2jt}, l^s_{jt}). \tag{4}$$

The specific functional form of preferences we will consider are:

$$u(c^s_{1jt}, c^s_{2jt}, l^s_{jt}) = \gamma \ln(c^s_{1jt}) + (1-\gamma)\ln(c^s_{2jt}) + \alpha \ln(l^s_{jt}). \tag{5}$$

This choice of preferences is consistent with balanced growth.[2] A household of age s in period t, who works h^s_t hours receives nominal earnings of $P_t w_t \varepsilon_j h^s_{jt}$. In this expression P_t is the price level, w_t is the wage rate, and ε_j is an age specific efficiency. A household can save by accumulating cash, M^s_{t+1}, bonds B^s_{t+1}, or capital k^s_{t+1}.

At the start of each period households visit a financial market where claims from the previous period are settled. Households also receive a lump-sum transfer from the government T^s_{jt}, and adjust their holdings of money and bonds. Total holdings of assets are restricted by the following borrowing constraint:

$$k^s_{j,t+1} + B^s_{j,t+1} + M^s_{j,t+1} \geq 0. \tag{6}$$

This borrowing constraint rules out uncollateralized borrowing.

After the financial market closes households separate into a worker and shopper. The shopper's purchases of the cash good and investment goods in any period are subject to the following cash in advance constraint:

$$\begin{aligned} \frac{B^s_{j,t+1}}{1+R_t} + P_t[k^s_{j,t+1} - k^s_{j-1,t}] + P_t c^s_{1jt} \leq \\ M^s_{j-1,t} + T^s_{jt} + B^s_{j-1,t} + P_t(1-\tau)(r_t - \delta)k_{j-1,t} \end{aligned} \tag{7}$$

where δ is the depreciation rate on capital and τ is a tax on capital income.[3]The household's overall budget constraint is given by:

$$\begin{aligned} M^s_{j-1,t} + T^s_{jt} + B^s_{j-1,t} + P_t w_t \varepsilon_j h^s_{jt} + P_t(1-\tau)(r_t - \delta)k^s_{j-1,t} \geq \\ P_t(c^s_{1jt} + c^s_{2jt}) + \frac{B^s_{j,t+1}}{1+R_t} + M^s_{j,t+1} + P_t[k^s_{j,t+1} - k^s_{j-1,t}]. \end{aligned} \tag{8}$$

[2]More generally preferences of the form: $\ln([\gamma(c^s_{1jt})^\sigma + (1-\gamma)(c^s_{1jt})^\sigma]^{1/\sigma}) + \alpha \ln(1-n_t)$ are also consistent with balanced growth.

[3]It is more common to assume that investment is not subject to the cash in advance constraint. However, the expressions for the first order conditions are a bit more convenient using this formulation and since we are considering low inflation environments the distinction between this formulation and one that treats capital as a credit good should be small.

Given these definitions the problem for a household born into cohort j is to choose the sequence $\{c^s_{1t}, c^s_{2t}, h^s_t, M^s_{t+1}, B^s_{t+1}, k^s_{t+1}\}^J_{t=s}$ that maximizes (4) subject to (7),(8), and (6). Some important household first order necessary conditions are:

$$\xi_j \gamma / c^s_{1jt} = P_t(\mu_t + \lambda_t) \tag{9}$$

$$\xi_j (1-\gamma)/c^s_{2jt} = P_t \lambda_t \tag{10}$$

$$\xi_j \frac{\alpha}{1-h^s_{jt}} = \lambda_t P_t w_t \varepsilon_j \tag{11}$$

$$\beta(\mu_{t+1} + \lambda_{t+1})/P_{t+1} + \phi_t = (\mu_t + \lambda_t)/\{P_t(1+R_t)\} \tag{12}$$

$$\beta(\lambda_{t+1} + \mu_{t+1})[1 + (1-\tau)(r_{t+1} - \delta)] + \phi_t = (\lambda_t + \mu_t) \tag{13}$$

$$\beta\left(\mu_{t+1} + \lambda_{t+1}\right)/P_{t+1} + \phi_t = \lambda_t / P_t \tag{14}$$

plus the CIA constraint, household budget constraint and the borrowing constraint.

The above expressions can be rearranged to yield the following restrictions on market clearing:

$$\frac{\alpha}{\gamma} \frac{c^s_{1jt}}{1-h^s_{jt}} = w_t \varepsilon_j / (1+R_t) \tag{15}$$

$$\frac{\alpha}{1-\gamma} \frac{c^s_{2jt}}{1-h^s_{jt}} = w_t \varepsilon_j \tag{16}$$

$$\beta[1 + (1-\tau_{t+1})(r_{t+1} - \delta)]\xi_{j+1}\gamma / c^s_{1j,t+1} = \xi_j \gamma / c^s_{1jt} - \phi_t \tag{17}$$

$$(1+R_t)/(1+\pi_{t+1}) = 1 + (1-\tau)(r_{t+1} - \delta) \tag{18}$$

$$\phi_t(M^s_{j,t+1} + k^s_{j,t+1} + B^s_{j,t+1}) = 0, \quad \phi_t \geq 0 \tag{19}$$

$$\mu_t \left\{ \begin{array}{c} M^s_{j-1,t} + T^s_{jt} + B^s_{j-1,t} - \frac{B^s_{j,t+1}}{1+R_t} + (1-\tau)(r_t - \delta)k_{j-1,t} + \\ P_t[k^s_{j-1,t} - k^s_{j,t+1}] - P_t c^s_{1jt} \end{array} \right\} = 0, \quad \mu_t \geq 0 \tag{20}$$

$$\begin{aligned} M^s_{j-1,t} + T^s_{jt} + B^s_{j-1,t} + P_t w_t \varepsilon_j h^s_{jt} + P_t(1-\tau)(r_t - \delta)k^s_{j-1,t} = \\ P_t(c^s_{1t} + c^s_{2t}) + \frac{B^s_{j,t+1}}{1+R_t} + M^s_{j,t+1} + P_t[k^s_{j,t+1} - k^s_{j-1,t}]. \end{aligned} \tag{21}$$

2.3 The Firm's Problem

Firms produce consumption goods with a constant return to scale production technology. In each period t, firms choose labor, H_t, and capital, K_t, to maximize

$$A_t K^\theta_t H^{1-\theta}_t - w_t H_t - r_t K_t, \tag{22}$$

where w_t is the real wage, r_t is the real rental rate on capital, A_t evolves according to

$$A_{t+1} = g_t A_t, g_t > 0.$$

2.4 The Government and aggregate feasibility constraints

The government issues bonds, money and raises revenue through a tax on asset income. Government revenue is used to finance government purchases and lump-sum transfers:

$$P_t G_t + \sum_{j=1}^{J} N_{jt} T_{jt} = \frac{B_{t+1}}{1+R_t} - B_t + M_{t+1} - M_t + P_t \tau (r_t - \delta) K_t \tag{23}$$

The government expands (nominal) money supply at the rate σ_t by making lump-sum transfers to all households alive in a given period according to:

$$M_{t+1} = (1 + \sigma_t) M_t.$$

We don't formally model a social security system. Instead we will assume that accidental bequests are lump-sum transferred back to surviving members of the same cohort.

The aggregate resource constraint for this economy is:

$$A_t K_t^{\theta} H_t^{1-\theta} = \sum_{j=1}^{J} N_{jt}(c_{1jt}^s + c_{2jt}^s) + K_{t+1} - (1-\delta) K_t + G_t \tag{24}$$

2.5 Competitive Equilibrium

Definition *Competitive Equilibrium*
Given an initial population wealth distribution, $\{M_{0j}, k_{0j}, B_{0j}\}_{j=1}^{J}$, a sequence of technologies, $\{A_t\}_{t=0}^{\infty}$, and government policies, $\{\tau, M_{t+1}, B_{t+1}, G_t, \{T_{jt}^s\}_{j=1}^{J}\}_{t=0}^{\infty}$, a competitive equilibrium is a price system $\{r_t, P_t, R_t, w_t\}_{t=0}^{\infty}$ and a sequence of allocations $\{c_{jt}^s, h_{jt}^s, k_{j,t+1}^s, M_{j,t+1}^s\}_{t=0}^{\infty}$ that solves the household problem, the firms problem and satisfies the following market clearing/feasibility conditions:

$$K_{t+1} = \sum_{j=1}^{J} N_{j,t} k^s_{j,t+1} \tag{25}$$

$$H_t = \sum_{j=1}^{J} N_{jt} h^s_{jt} \tag{26}$$

$$M_{t+1} = \sum_{j=1}^{J} N_{jt} M^s_{j,t+1} \tag{27}$$

$$A_t K_t^{\theta} H_t^{1-\theta} = \sum_{j=1}^{J} N_{jt}(c^s_{1jt} + c^s_{2jt}) + K_{t+1} - (1-\delta_t)K_t + G_t. \tag{28}$$

When solving the model we will specify an initial population wealth distribution and a terminal steady-state and then solve for the transitional dynamics. We thus define a steady-state equilibrium next.

Definition *Balanced Growth Equilibrium*
Suppose that technology grows at the constant rate: $g_t = g$, and that money supply grows at a constant rate: $\sigma_t = \sigma$, and the output shares of government purchases, and government debt are constant Then a balanced growth equilibrium is a competitive equilibrium in which the real wage rate grows at the rate of output, the real interest and nominal interest rates are constant and the output shares of capital and consumption are constant.

2.6 Computation of the equilibrium.

Before we compute the equilibrium we transform the economy. This is done using the transformations:

$$\begin{gathered} \hat{K}_t = \frac{K_t}{N_t A_t^{1/(1-\theta)}}, \hat{C}_t = \frac{C_t}{N_t A_t^{1/(1-\theta)}}, \hat{B}_t = \frac{B_t}{P_{t-1} N_t A_t^{1/(1-\theta)}}, \\ \hat{M}_t = \frac{M_t}{P_{t-1} N_t A_t^{1/(1-\theta)}}, \hat{T}_t = T_t/P_t, \hat{H}_t = H_t/N_t, \hat{w}_t = \frac{w_t}{A_t^{1/(1-\theta)}}. \end{gathered} \tag{29}$$

We first describe computation of the steady-state equilibrium. We are interested in considering situations where the nominal interest rate is positive and also in situations where it is zero. In the later situation the cash in advance constraint (7) ceases to bind and the steady-state conditions are different. When $R > 0$we start by guessing the aggregate values of hours, capital real balances and lump-sum transfers $(\hat{H}_0, \hat{K}_0, \hat{M}_0, \hat{T}_0)$. Given these objects we can derive the wage and rental rates $\widetilde{w}_0, r_0$ and solve the household's problem. (Note that the inflation rate can be derived from real balances using the following equation:

$$(1+\pi) = \frac{(1+\sigma)}{(1+n)(1+g_{TFP})} \tag{30}$$

where $1 + g_{TFP} = A_t^{1/(1-\theta)}/A_{t-1}^{1/(1-\theta)}$. When $R > 0$, the solution to the household's problem uniquely determines individual demand for real balances: $\hat{M}_0^{d,s}$ for each cohort $s = \{1, ...J\}$ and labor supply for each cohort $\hat{H}^s$. However, the household's problem only determines the sum of saving in the form of capital and bonds. We denote this sum as $\hat{S}_0^s$.

Given solutions to each cohort's optimization problem we then sum over households to derive aggregate assets supplied by households: $\hat{S}_0^{'}$, aggregate labor supply: $\hat{H}_0^{'}$ and aggregate demand for real balances: $\hat{M}_0^{'}$ Given these objects we can solve for the capital stock using the fact that the stock of government bonds is exogenous and: $\hat{S}' - \hat{B} = \hat{K}'$. Then using the initial guesses of the wage rate and rental rate we can update transfers using the steady-state version of the government budget constraint:

$$\left\{\frac{(1+g_{TFP})(1+n)}{1+R} - \frac{1}{1+\pi}\right\}\hat{B} + \left\{(1+g_{TFP})(1+n) - \frac{1}{1+\pi}\right\}\hat{M}' \\ +\tau(r-\delta)\hat{K}' = \hat{G} + \hat{T}'. \tag{31}$$

Finally, we update our guess of capital, labor, real balances and government transfers by taking a weighted average of the initial guess plus the new values derived from household optimization:

$$\hat{K}_1 = \lambda\hat{K}_0^{'} + (1-\lambda)\hat{K}_0 \tag{32}$$
$$\hat{H}_1 = \lambda\hat{H}_0^{'} + (1-\lambda)\hat{H}_0 \tag{33}$$
$$\hat{T}_1 = \hat{T}_0' \tag{34}$$
$$\hat{M}_1 = \lambda\hat{M}_0' + (1-\lambda)\hat{M}_0 \tag{35}$$

When $R = 0$, the household problem only pins down household supply of *total* assets which now consists of the sum of real balances, capital and bonds: $\hat{S}_0^{'} = \hat{K}_0^{'} + \hat{B} + \hat{M}_0^{'}$. In this case we derive real balances and the capital in the following way. First, we use the fact that:

$$(1+\pi_0) = \frac{(1+\sigma)}{(1+n)(1+g_{TFP})} \tag{36}$$

to pin down the inflation rate. Then we use

$$(1+\pi_0) = (1+r_0)^{-1} \tag{37}$$

to pin down the real interest rate. Given the real interest rate we derive a new guess of the capital stock, $\hat{K}_0^{'}$, from aggregate labor supply plus the marginal product pricing relationship:

$$r_0 = (1-\tau)\left\{\theta\left(\hat{K}_0^{'}/\hat{H}_0^{'}\right)^{\theta-1} - \delta\right\} \tag{38}$$

Then we derive real balances from the saving identity: $\hat{S}_0^{'} - \hat{K}_0' - \hat{B} = \hat{M}_0^{'}$. The updating of the guess proceeds in the same way as before.

When solving for the dynamic transition we proceed in an analogous way. The main distinction is that we now guess and update sequences of the form: $(\hat{H}_{i,t}, \hat{K}_{i,t}, \hat{M}_{i,t}, \hat{T}_{i,t})$ where i denotes the i^{th} iterate and t indexes time.

3 Model Parameterization

The strategy for calibrating the model is similar to the strategy used in Braun, Joines and Ikeda (2007). The preference discount rate β is calibrated to reproduce the average capital output ratio between 1984 and 2000. This results in a value of 0.97. The leisure weight in preferences, α is set to reproduce the value of labor input in the Japanese economy between 1984 and 2000. This yields $\alpha = 2.5$. The capital share parameter is set to 0.362 which is the average value of capital's share of GNP between 1984 and 2000. The depreciation rate calibrated in the same way is 0.085. The average tax rate on asset income over the same period is 0.46. The labor tax rate is set to zero. This assumption is also maintained by Hayashi and Prescott (2002) who argue that principal tax wedge in Japan is a high tax on capital income. With this choice, the remainder of the calibration turns out to be very similar to what one finds in U.S. data. We assume a constant population growth rate of 1 percent per year. We set the share weight on cash goods, $\gamma = 0.07$. This choice reproduces the ratio of real balances of monetary base to GNP which averaged 0.08 between 1984 and 1994.

4 Steady-state Analysis

Here we report results from a comparative steadystate analysis. This analysis provides intuition about the workings of our model. We will document an asymmetry between the welfare cost of inflation and the welfare cost of deflation that provides a rationale for expanding money supply when the nominal interest rate is zero.

Table 1 reports the steady-state properties of our model for alternative settings of the growth rate of money. These results allow for age specific variation in the efficiency of work effort and assume that the population growth rate is 1 percent, the growth rate of TFP is 1.9 percent, the share of government purchases in output is 0.144 and the government debt ratio is 0.22. These correspond to the average value of these variables in Japanese data over the 1984 to 2000 sample period. Table 1 has several noteworthy features. First, observe that there are a range of monetary policies that implement a zero nominal interest rate in our economy. Interestingly, the welfare maximizing choice occurs when the nominal interest rate reaches zero and is associated with a growth rate of money that declines at a rate of 1.43 percent per year. If the Friedman Rule is defined as a monetary policy that sets the nominal interest rate to zero as in Chari, Christiano and Kehoe (1991), then the Friedman Rule is the optimal (steady-state) monetary policy in our economy too.

Bhattacharya, Haslag and Russell (2005) consider the optimality of the

Friedman rule in a 2 period overlapping generations model and find that it is not optimal in their setting. The reason for this is that in their model young households have low initial wealth and yet must pay a lumpsum tax to finance contraction of the money supply when the growth rate of money is negative. We allow agents to borrow against their first period labor earnings and this mitigates the negative effect of lumpsum taxation on the youngest households.

One of the most noteworthy features of Table 1 is an asymmetry in the welfare costs of alternative growth rates of money. The welfare loss associated with large growth rates of money (e.g. 7 percent) is modest. However, lowering the growth rate of money below -1.43 percent has much larger effects on welfare. For instance, steadystate welfare when money growth is -1.6 percent is about the same as steadystate welfare when the growth rate of money is 7 percent per annum!

This asymmetry in the welfare cost of inflation and to much deflation reflects the fact that monetary policy affects real economic activity in a different way when the nominal interest rate is zero. When the average growth rate of money is higher than the optimal level, monetary policy acts as a tax on labor supply and capital. Households act to limit their holdings of cash and this limits the incidence of this tax. This can readily be seen in Figure 1. Higher growth rates of money are associated with lower consumption of cash goods. However, cash goods only constitute 7 percent of total consumption under the Friedman rule. The effect on the capital output ratio and thus the real interest rate is also modest when the growth rate of money exceeds -1.43 percent.

To understand why the welfare losses increase rapidly when the growth rate of money is too low, recall that when the nominal interest rate is zero the cash in advance constraint ceases to bind and money and capital earn the same real return. As the steadystate growth rate of money is lowered from the welfare maximizing level, the inflation rate falls and this increases the real return on money. Holdings of private capital must then fall in order to insure that the capital stock continues to earn the same return as money. There is also a second channel operating here. A lower growth rate of money is also associated with higher lump-sum taxes which is costly to households who are borrowing constrained. Table 1 indicates that the combination of these two mechanisms produces a sharp decline in welfare when money supply contracts at a more rapid rate than 1.43 percent per annum.[4]

This asymmetry has implications for the conduct of monetary policy. Suppose we assume that the monetary authority knows the model but that there is uncertainty about the values of the model parameters including the long run average values or growth rates of the exogenous variables. To be specific lets suppose that the policy maker estimates the growth rate of TFP is 4 percent rather than 2 percent. An estimate of this magnitude would arise if the policy maker were to estimate the growth rate of TFP using Japanese data from 1960 to 1990. In this scenario welfare is maximized when the growth rate of money

[4]The asymmetry we are documenting here would be even larger if investment was treated as a credit good. If investment is a credit good the welfare cost of 7 percent inflation is smaller. However, the welfare cost of too much deflation remains essentially unchanged.

is -0.61 percent and also falls rapidly if the money supply is contracted more rapidly. Setting the growth rate of money to -1.43 percent, which is the *optimal* monetary policy in Table 1, induces very large welfare losses. This property of the model provides a rationale for a monetary authority to pursue an expansionary monetary policy when it finds itself in a zero interest rate environment. The welfare costs associated with too much monetary expansion are much smaller than the welfare costs of a monetary policy that is too tight.

And important limitation of the steady-state analysis is that it is difficult to produce an empirically plausible calibrated specification of the model with a steadystate in which a zero nominal interest rate is associated with deflation and a positive growth rate of money. Thus it is difficult to use a comparative steadystate analysis to understand Japan's experience from the mid 1990s to 2006 when the growth rate of money was positive and yet there was a protracted period of deflation. We turn next to describe the results from a dynamic analysis that reproduces these outcomes.

5 A Dynamic Analysis of "Quantitative Easing"

Japan is an interesting case for considering the effects of monetary aggregate targeting in a zero interest rate environment. In Japan slower real economic growth during the 1990s was associated with a steady decline in the uncollateralized call rate on overnight loans from 7.4 percent in 1990 to 0.06 percent in 1999. The nominal interest rate remained at effectively zero (except for a brief interlude in 2000) until 2006. Once the nominal interest rate reached zero policy makers considered a variety of options for using monetary policy to stimulate the economy. The outcome of these deliberations was the "Quantitative Easing"policy that was adopted on March 19, 2001. This policy which targeted the level of bank deposits at the Bank of Japan was effectively an excess reserve targeting policy. The Bank of Japan announced an end to the quantitative easing policy in March 9, 2006. But, it kept the call rate at zero until July 14, 2006 at which point the call rate was increased to 0.25 percent.

We investigate the quantitative effects of this policy using dynamic perfect foresight simulations. Chen, Imrohoroglu and Imrohoroglu (2007) and Braun, Ikeda and Joines (2007) have previously found that computable general equilibrium models that allow for variation in TFP and demographics can account for some of the principal movements in real economic activity in Japan from 1960 through 2002. Here we abstract from demographic variation and model only variation in TFP and government debt. Our government debt series is taken from Braun, Joines and Ikeda (2007). They construct a government debt series following the methodology of Broda and Weinstein (2005). The initial period of our simulation is taken to be 1984. The initial wealth distribution is taken from the terminal steady-state but is rescaled to reproduce the capital stock in Japanese data in 1984. We set that the initial values of the nominal interest rate, government purchases and government bonds to their values in Japanese data in 1984. The terminal nominal interest rate is 5.9 percent, terminal government debt is 22% of output, terminal government purchases are 14.4 percent of output and terminal TFP growth is 1.9 percent.

We are interested in reproducing variations in the nominal interest rate and monetary base during the period 1986-2006. There are two issues that arise in doing this. First, when the nominal interest rate is zero the composition of government liabilities is indeterminate. Open market operations that exchange money for bonds have no real effects when the nominal interest rate is zero. Monetary policies that alter the total amount of outstanding government debt do have real effects.[5] However, it is hard to ascertain directly what fraction of quantitative easing should be interpreted as having altered the amount of outstanding government debt. First, we treated the sequences of government debt and the nominal interest rate as exogenous and solved for the equilibrium under this assumption.

The resulting sequence of real balances and lump-sum transfers does a good job of reproducing the path of inverse M0 velocity (the ratio of M0/P to GNP) in the period up to 1997. In the period after that though the model understated this ratio. Our model has the property that when the nominal interest rate is zero the composition of government liabilities is indeterminate. Next we used this property of the model to adjust the composition of government liabilities so that we reproduce the actual trajectory of M0/P to GNP during the period 1997-2006.

The resulting trajectories for M0/P to GNP for the model and data are reported in Figure 1 for the specification with no age-specific labor efficiencies.[6] The same figure also reports plots of the capital output ratio, the deviation of output from a 1.9 percent trend and the inflation rate as measured by the growth rate of the GNP price deflator. The general fit of the model is reasonably good. The model reproduces the increase in the capital-output ratio and the decline in output relative to trend that Japan experienced after 1990. However, the model understates the average value of the inflation rate in Japanese data. Broda and Weinstein (2007) argue that problems in price measurement induce an upward bias of about 2 percentage points in the Japanese inflation rate. If we subtract 2 percent from the actual data, the model reproduces the overall level of the inflation rate and also some of its principal movements between 1986 and 2005.

Next we turn to evaluate the effects of quantitative easing by comparing the baseline simulation with counterfactual scenarios. The *no quantitative easing* scenario assumes that the ratio of real balances of M0 to GNP rises at the rate of 2 percent per year between 2000 and 2006. The *longer quantitative easing*

[5] To understand how monetary policy in a zero interest rate environment might be perceived as affecting the total stock of outstanding government debt, consider the Japanese call market for overnight loans. In Japan during the period where the nominal interest rate was zero the Bank of Japan was the primary lender in the call market. The interest rate was so low that private banks couldn't cover even moderate origination costs of e.g. 5000 yen associated with making an overnight loan of e.g. 200,000,000 yen when the annual interest rate was 0.01 percent.

[6] The simulations with age specific labor efficiences take much longer to simulate and have not yet been completed.

scenario allows the ratio of real balances to output to rise to 0.24 in 2011. The *earlier quantitative easing* scenario assumes that quantitative easing is started in 1995 instead of 2002. Figure 2 shows the trajectory of inverse M0 velocity for each of these scenarios.

Consider next Table 2 which summarizes the properties of the inflation rate and nominal interest rate and real interest rate under each scenario. A comparison of the baseline scenario with the no quantitative easing scenario reveals two effects of quantitative easing. The expectation of quantitative easing reduces deflation during the 1990s and reduces the total number of periods that the nominal interest rate is zero. Expectations are clearly playing an important role. The baseline specification shows higher nominal interest rates and higher average inflation rates between 1991 and 2000 which is before quantitative easing was undertaken. After 2000 the two policies are very similar. Although the inflation rate movements are large and of primary importance for the path of the nominal interest rate there are also some small but discernible effects on the real interest rate. The real interest rate is higher under quantitative easing in all sub-periods. The scenario with longer quantitative easing shows similar effects of expectations. However, it produces more deflation than the baseline scenario after 2000. A consequence of this is that the nominal interest rate is zero for a much longer period of time. The effects on the real interest rate are also more pronounced. With the nominal interest rate at its lower bound the real interest rate has to adjust. The result is a higher real interest than the baseline model in the final four sub-periods. Earlier quantitative easing, induces more deflation during the 1990s but less deflation after 2001 as compared to the baseline scenario. Here the interest rate effects move in the opposite direction. The value of the real interest rate in this scenario is lower than the baseline in all but the first sub-sample.

Table 3 reports simulation results for quantity variables. Quantitative easing depresses output when compared with the no quantitative easing scenario and longer quantitative easing depresses output more. Interestingly, earlier quantitative easing produces higher output than any of the other three scenarios between 1991 and 2005. We saw in Table 2 that this scenario also has lower real interest rates and lower average inflation rates in the earlier sub-periods. Inflation also acts as a tax on labor and capital. From the steadystate analysis we can see that lower steady inflation rates are associated with lower monetary base growth, a lower real interest rate, higher wages and higher output. These same mechanisms are operating in the dynamic simulations. The early quantitative easing scenario exhibits lower average monetary based growth than the baseline scenario from 1991-2006 and this accounts for the fact that the earlier quantitative easing scenario has lower average inflation rates, higher output and lower real interest rates than the baseline scenario in the earlier sub-periods.

The dynamic effects are quite different though once the nominal interest rate is zero. This can most readily be observed by comparing the baseline with the longer quantitative easing scenario. The longer quantitative easing scenario exhibits higher money growth and real balances after 2001, a higher real interest rate and lower output. In the steady-state analysis above we saw that once the nominal interest rate is zero higher real balances are net government debt and thus crowd out private capital.

To further explore the nature of this crowding out effect Table 4 reports the ratio of real balances to output and the capital output ratio for the four scenarios. Before discussing these results it should be pointed that in the dynamic analysis we are limiting attention to transitory changes in monetary policy. The ratio of real balances to output and the debt output ratios are the same in both the initial and terminal steady-states in all four scenarios. The results in Tables 2 and 4 indicate that this is an important distinction. Comparing the baseline scenario with the longer quantitative easing scenario, we see from Table 2 that longer quantitative easing produces more deflation. The reason for this can be seen in Table 4. Longer quantitative easing increases real balances and temporarily increases total government debt. Temporarily higher government debt increases the real interest rate and crowds out private capital. This is why the longer quantitative easing simulation exhibits lower capital output ratios, higher real interest rates, lower inflation and lower output than the baseline scenario after 2001. In other words, starting from a situation with zero nominal interest rates, the anticipated inflation effects of temporarily higher money growth are dominated by the fiscal effects of monetary policy on total government debt.

Next we turn to consider the distributional effects of quantitative easing. The distributional effects vary depending on whether productivity has an age specific element to it. In the presence of age-specific earnings young agents face binding borrowing constraints They would like to shift consumption forward from future periods when their income will be high but are unable to collateralize their future high human capital. We first report results for the case where earnings are independent of age and then indicate how the answer changes when we allow labor productivity to vary by age. The model solves much more quickly when the earnings profile is flat. However, we few cohorts face binding borrowing constraints. Quantitative easing temporarily lowers taxes and this, in principal, can relax borrowing constraints. It also raises the real interest rate which reduces the incentive to consume today. These effects can be seen in Table 5 which reports lumpsum transfers, the real interest rate and the average number of constrained cohorts for each sub-period. Notice that the baseline scenario has lower average lumpsum taxes when compared with either the non quantitative easing scenario or the early quantitative easing scenario between 2001 and 2005. The interest rate is also higher. However, as can be seen in Table 5 there is no interesting variation in the number of cohorts that are constrained during the period of quantitative easing. In fact, the most pronounced effect of quantitative easing is to tighten borrowing constraints from 2006 on as lumpsum taxes rise. We have not completed the simulations of the case with age specific labor productivities using the current calibration of the model. But, we have simulated this specification using a slightly different calibration. with age specific earnings there is more interesting variation in borrowing constraints over time. Under quantitative easing the number of periods that agents are borrowing constrained falls from 17 in 2000 to 12 in 2003. For purposes of comparison, in a simulation with no quantitative easing simulation young individuals are borrowing constrained for the first 17 to 19 years of life during the 2000 to 2006 period.

The effects of quantitative easing on consumption vary significantly with the age of the individual. Quantitative easing benefits older individuals most. For retirees a higher real interest rate increases the value of their saving and consumption increases. Moreover, older retirees enjoy the benefits of lower taxes and pass away before taxes rise. The magnitude of these increases can be substantial. For the simulation with a flat wage profile all individuals aged 78 and older as of 2000 experience higher consumption under quantitative easing between 2001 and 2006 than the no quantitative easing scenario. Individuals who are aged 85 and over experience average consumption gains in excess of 1 percent per year. The biggest losers are individuals in their late thirties and early forties they experience consumption declines of about 0.45 percent. With age specific earnings the gains increase to about 2 percent for retirees and the losses increase to about 1 percent of consumption. The reason for these consumption responses is that working individuals experience lower wages and since labor income is a bigger share of total income the negative effects of lower wages dominate the benefits of a higher real interest rate. Although it is difficult to perform welfare comparisons in this type of dynamic setting, the early quantitative delivers the highest wages between 2001 and 2006 and this in turn produces higher average consumption for all households aged 81 and less. The magnitude of the average increase in consumption is large and exceeds 2 percent for individuals aged 32 to 76.

6 Conclusion

We have developed a model that is consistent with the facts from Japan. Our model reproduces some of principal movements in the real economy between 1986 and 2006. Our model also reproduces the evolution of nominal variables and in particular nominal interest rates of zero against a background of very rapid growth in the monetary base. Our results suggest that lower growth of monetary base would have been associated with more deflation in the 1990s and a longer period of zero nominal interest rates. Lengthening the period of quantitative easing lowers inflation after 2006 and also lengthens the period when nominal interest rates are zero.

We have found that quantitative easing was not effective policy for stabilizing output between 2001 and 2005. Output under quantitative easing was nearly the same as in a counter-factual with no quantitative easing. Quantitative easing did have important distributional effects. Young individuals experience some mild benefits due to relaxed borrowing constraints.

The most important effects of quantitative easing are for older individuals. Workers near the peak of their life-time earnings efficiencies experienced consumption losses due to lower wages and a higher tax burden. Older retirees experience substantial consumption gains since they on average don't survive long enough to face higher taxes.

In future work we plan to relax our current assumption that the government budget constraint is met by altering lump-sum taxes and instead make the more realistic assumption that a distortionary tax is adjusted instead. This will likely introduce stronger non-neutralities. Our model generates borrowing and lending in equilibrium. It is consequently a good framework for modeling financial intermediation and central bank lending. In future work we plan to pursue these extensions.

References

[1] Auerbach, Alan J., Maurice Obstfeld (2005) "The Case for Open-Market Purchases in a Liquidity Trap," *American Economic Review* vol. 95 (1), pages 110-137, March.

[2] Bhattacharya, Joydeep, Joseph Haslag, and Steven Russell (2005) "The role of money in two alternative models: When is the Friedman rule optimal, and why?," *Journal of Monetary Economics*, vol. 52 (8), pages 1401-1433, November.

[3] Blanchard, Olivier J. (1985) "Debt, Deficits, and Finite Horizons" *Journal of Political Economy*, vol. 93, pages 223-247, April.

[4] Braun, R. Anton, Daisuke Ikeda and Douglas H. Joines (2007) "The Saving Rate in Japan: Why it Has Fallen and Why it Will Remain Low." forthcoming *International Economic Review*.

[5] Broda, Christian and David E. Weinstein (2005) "Happy News from the Dismal Science: Reassessing the Japanese Fiscal Policy and Sustainability, in Takatoshi Ito, Hugh Patrick and David E. Weinstein eds. "Reviving Japan's Economy" MIT Press.

[6] Broda, Christian and David E. Weinstein (2007) "Defining Price Stability in Japan: A View from America." *Monetary and Economic Studies,* vol. 25 (S-1), pages 169-206, December.

[7] Chari. V.V., Lawrence J. Christiano and Patrick J. Kehoe (1991) "Optimal Fiscal and Monetary Policy: Some Recent Results." *Journal of Money, Credit and Banking* vol. 23 (3) pages 519-539, August.

[8] Chen, Kaiji, Ayse Imrohoroglu and Selo Imrohoroglu (2007) "The Japanese Saving Rate." *American Economic Review*, vol 96 (5) pages 1850-1858, December.

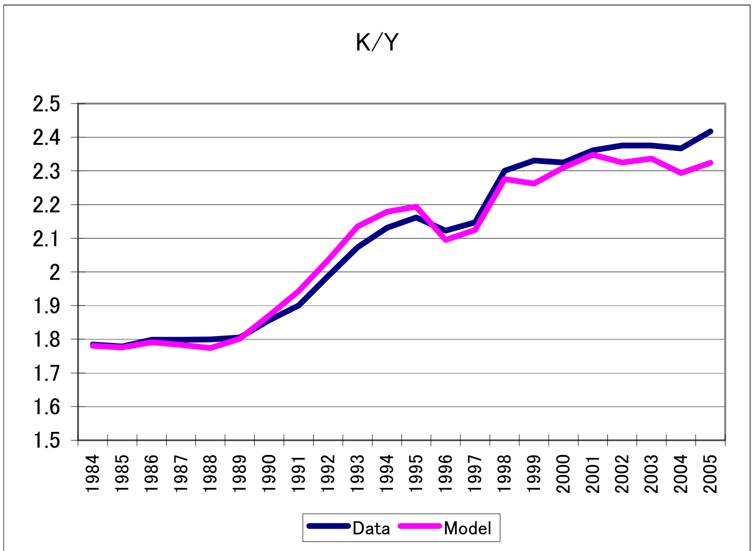
[9] Hayashi, Fumio and Edward C. Prescott (2002) "The 1990s in Japan: A Lost Decade." *Review of Economic Dynamics*, vol. 5 (1), pages 206-235.

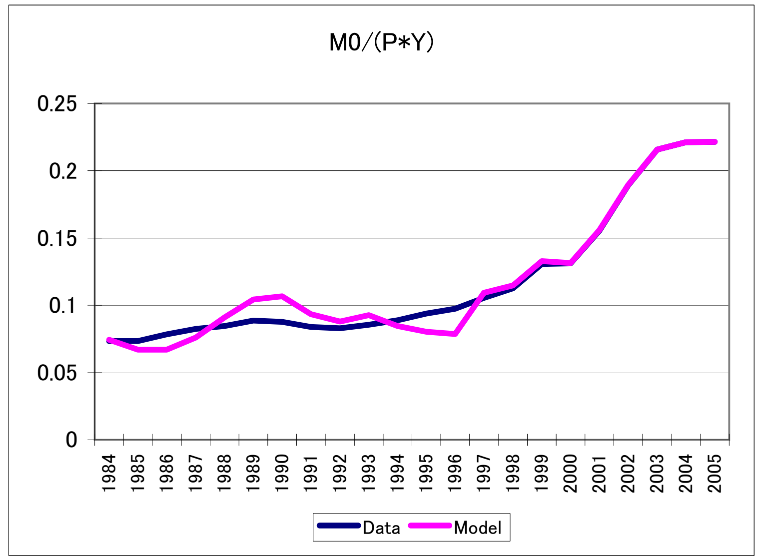
[10] Ireland, Peter N. (2005) "The Liquidity Trap, the Real Balance Effect, and the Friedman Rule" *International Economic Review*, vol. 46, pages 1271-1301, November.

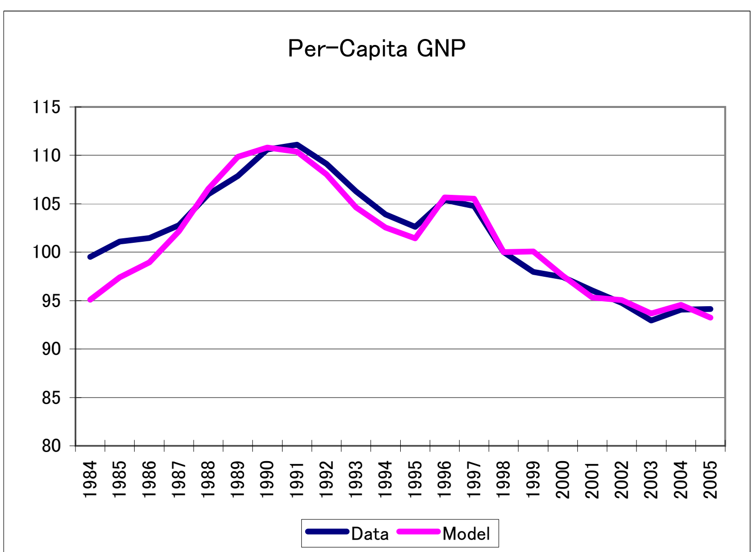
[11] Krugman, Paul R. (1998) “It’s Baaack: Japan’s Slump and the Return of the Liquidity Trap.” *Brookings Papers on Economic Activity*, pages 137-187.

[12] Lucas, Robert E. and Nancy Stokey (1987) ”Money and Interest in a Cash-in-Advance Economy.” *Econometrica*, vol. 55, pages 491-513, May.

Figure 1
Model and Japanese Data







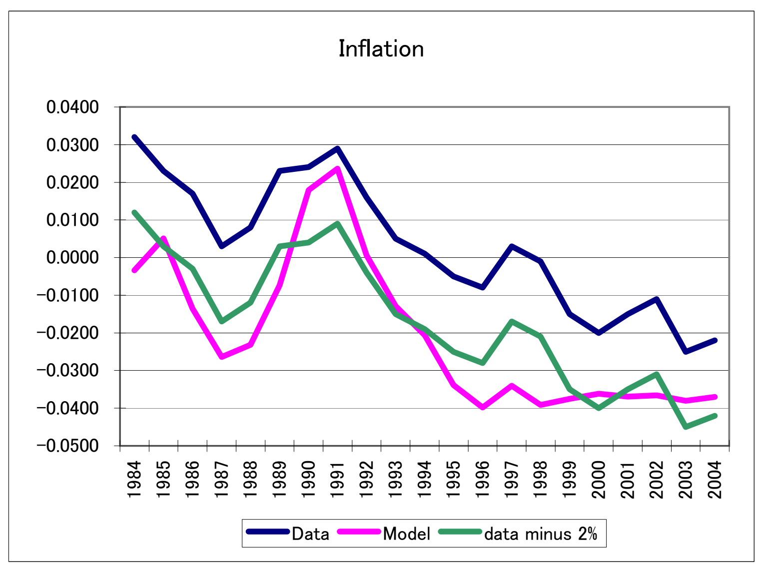


Table 1
Model Steadystates for Alternative Growth Rates of Money

Specification with Age Specific Efficiency Units

Growth rate of money (Percentage)	Cash Good Consumption*	Credit Good Consumption*	Money Output ratio	Capital Output Ratio	Output*	Real Interest Rate (Percentage)	Inflation Rate (Percentage)	Nominal Interest rate (Percentage)	Welfare
7.00	91.7	99.7	0.06	2.12	98.2	4.6	3.9	8.8	-68.86
3.00	95.4	99.8	0.07	2.14	99.0	4.5	0.1	4.6	-68.75
0.00	98.5	100.0	0.07	2.16	99.7	4.5	-2.9	1.5	-68.67
-1.43	100.0	100.0	0.08	2.17	100.0	4.4	-4.2	0.0	-68.63
-1.60	99.6	99.6	0.25	2.13	98.8	4.6	-4.4	0.0	-68.83
-1.80	99.2	99.2	0.48	2.08	97.5	4.8	-4.6	0.0	-69.10
-2.00	98.9	98.9	0.73	2.04	96.4	5.0	-4.8	0.0	-69.42
-2.20	98.6	98.6	1.01	1.99	95.3	5.3	-5.0	0.0	-69.80

* Cash good consumption, credit good consumption and output are expressed as a percentage of the respective variable under the Friedman rule.

Table 2

Simulation Results: Prices

Period	Inflation				Nominal Interest rate				Real Interest rate				Wage rate*			
	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing
1991-1995	-1.30	0.17	2.56	-4.27	2.24	3.73	6.20	0.07	4.77	4.78	4.78	4.56	100.1	100.0	100.0	101.4
1996-2000	-4.01	-3.69	-2.33	-3.79	0.00	0.19	1.28	0.00	4.18	4.28	4.42	3.94	100.6	103.1	99.1	102.2
2001-2005	-3.64	-3.69	-3.83	-2.25	0.00	0.00	0.00	1.81	3.77	3.83	3.98	3.50	100.4	106.1	99.0	102.3
2006-2010	-3.12	-3.12	-3.66	0.50	0.61	0.65	0.00	4.53	3.61	3.64	3.80	3.38	100.2	107.5	98.9	101.8
2011-2015	-1.04	-1.03	-3.73	2.20	3.43	3.46	0.00	5.67	3.58	3.59	3.88	3.63	100.1	107.8	98.1	99.8

* Wage rates areexpressed relative to baseline wage rate in the same period

Table 3

Simulation results: Allocations

Period	Consumption*				labor input*				Output*				Money Growth			
	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing
1991-1995	99.7	100.0	100.6	100.0	100.7	100.0	99.1	102.3	100.8	100.0	99.0	103.7	-2.06	-2.06	-2.06	0.66
1996-2000	100.5	95.6	99.7	101.7	99.9	99.4	99.2	100.6	100.6	96.9	98.3	102.8	9.50	12.33	12.33	4.17
2001-2005	100.4	91.6	99.4	102.0	99.8	97.8	99.4	99.9	100.3	90.3	98.5	102.2	1.00	4.17	7.88	-15.05
2006-2010	100.2	87.5	99.5	102.4	99.9	98.0	98.9	98.0	100.1	86.1	97.8	99.8	-12.73	-15.05	2.59	-7.42
2011-2015	100.1	84.7	97.9	100.4	100.0	96.5	101.2	98.4	100.1	81.5	99.3	98.1	-7.42	-7.42	-16.73	7.40

* Consumption, Output and labor are expressed as percentages relative to the baseline scenario.

Table 4

Real Balances, Capital and Total Debt as a Fraction of GNP

Period	Ratio of real balances to output				Ratio of capital to output			
	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing
1991-1995	0.10	0.09	0.08	0.14	2.10	2.10	2.10	2.15
1996-2000	0.13	0.11	0.09	0.20	2.24	2.21	2.18	2.30
2001-2005	0.16	0.20	0.16	0.13	2.34	2.33	2.29	2.42
2006-2010	0.12	0.12	0.21	0.07	2.39	2.38	2.34	2.46
2011-2015	0.07	0.07	0.20	0.05	2.40	2.39	2.31	2.38

Table 5

Taxes, interest rate and borrowing constraints

	Lump-sum taxes*				Real interest rate				Number of borrowing constrained cohorts			
Period	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing
1991-1995	0.06	0.06	0.06	0.06	4.77	4.78	4.78	4.56	1.0	1.0	1.0	1
1996-2000	0.02	0.02	0.02	0.03	4.18	4.28	4.42	3.94	1.0	1.0	1.0	1.2
2001-2005	0.10	0.09	0.08	0.15	3.77	3.83	3.98	3.50	1.0	1.0	1.0	1.2
2006-2010	0.24	0.25	0.19	0.22	3.61	3.64	3.80	3.38	3.8	4.6	1.8	1.6
2011-2015	0.18	0.18	0.22	0.16	3.58	3.59	3.88	3.63	5.8	6.0	3.4	10

* Lump-sum taxes are expressed as a fraction of total average consumption