

Optimal Monetary and Fiscal Policy for a Small Open Economy

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1 Introduction

This paper aims at analyzing the international dimension of fiscal policy. The neoclassical literature on optimal fiscal policy has focused mainly on closed economy models suggesting that, when taxes are distortionary, welfare would be maximized if taxes are smoothed over time and across states of nature (see Barro, 1979 and Lucas and Stokey, 1983). In these models, if possible, taxes would be invariant (see Lucas and Stokey, 1983 and Chari, Christiano and Kehoe, 1991) or would follow a random walk (see Barro, 1979). The small open economy, on the other hand, has an incentive to use the changes in the tax rate at its own advantage: indeed it is possible to increase welfare by diverting production to the rest of the world without a corresponding reduction in utility units of consumption. For example while in a closed economy following a positive productivity shock, the optimal policy would imply a procyclical and smooth evolution of taxes, in the small open economy taxes are initially countercyclical and vary over time.

This incentive is indeed similar to what has been emphasized in recent contributions that analyze the monetary policy stabilization problem in open economy (see Tille (2002), Corsetti and Pesenti (2001), Benigno and Benigno (2003) and De Paoli (2004)). In these papers, in which taxes are lump sum, the aforementioned incentive would imply a deviation from a price stability policy and the use of the terms of trade as a device to divert production toward foreign economies. To this end we also explore how this incentive influences the optimal policy when both fiscal and monetary stabilization issues are present.

We build a small open economy characterized by a set of very simple assumptions: specifically we assume that there are no trade frictions (i.e. the law of one price holds) and that capital markets are perfect (i.e. asset markets are complete). On the other hand, following the recent contributions by Benigno and Woodford (2003), Schmitt-Grohe and Uribe (2001) and Siu (2001) we allow for distortionary income taxation and we restrict government debt to one period nominal or real riskless bonds.

Our analysis focuses initially on the case in prices are flexible in order to highlight the open economy dimension of the fiscal policy problem. Indeed under this structure there are two policy incentives: reducing the inefficiency caused by movement in distortionary taxation and managing strategically the real exchange rate. Differently, from the closed economy framework in an open economy it is not optimal to perfectly smooth taxes so to avoid distortions on households' marginal rate of substitution between consumption and leisure. Distortionary income taxes can be used to lower domestic disutility of production without observing a corresponding decline in the utility of consumption. This is possible because higher taxes would induce a smaller depreciation of the real exchange rate, allowing domestic agents to switch consumption towards foreign produced goods. Note that, in a closed economy, this mechanism is absent because a fall in the disutility of domestic production would be accompanied by a corresponding reduction in the utility of consumption.

The denomination of government debt does not alter the key mechanism described above but it is important in order to determine the dynamic properties of our variables of interest. If public debt is indexed to consumer price inflation, and therefore yields real returns, both taxes and debt are non-stationary. On the other hand, if inflation can affect the level of real debt, taxes will follow a stationary process.

Once we allow for sticky prices a further distortion is added to the economy, namely the inefficient allocation of resource caused by positive domestic producer inflation (as in Woodford, 2003). Like income taxes, domestic producer inflation can now be used in a strategic way to reallocate production toward the rest of the world (as it is emphasized by De Paoli (2004) for the monetary stabilization case). Indeed, under sticky prices, both inflation and taxes are used to switch production. Moreover, the introduction of price rigidity reduces the variability of taxes because in this case domestic producer inflation can also be used to manipulate the consumption-leisure choice. However, from a quantitative point of view, our framework suggests that the cost of inflation will overshadow the inefficiency caused by varying distortionary taxation and, therefore, changes in domestic producer inflation are quantitatively small. Note that this result holds even in a model with real bonds and is a consequence of the conflict between price stability and the incentive to strategically affect the real exchange rate. This is different, however, from the trade-off (emphasized by the Benigno and Woodford (2003), Schmitt-Grohe and Uribe (2001) and Siu (2001)) between price stability and the use of inflation as insurance that arise only in models in which the government issues nominal bonds.

From a methodological point of view our analysis follows the technique developed by Benigno and Woodford (2003) by proposing a linear quadratic approach to the optimal policy problem. The present work encompasses as special cases the closed economy framework (Benigno and Woodford, 2003) and the small open economy case in which there are endogenous lump sum taxes (i.e. De Paoli (2004)). Under price flexibility our loss function is quadratic in the output and real exchange rate gaps. With price rigidities, the variability of inflation also affects welfare. This approach allows us to derive simple policy rules that prescribe the optimal state-contingent responses to shocks. We do so by specifying targeting rules as in Svensson (2003). In particular, the optimal plan is composed by two rules: one that specifies targeting a linear combination of domestic producer inflation, domestic output growth and changes in the real exchange rate; the other seeks to stabilize expected producer inflation to zero.

The remainder of the paper is structured as follows. Section 2 describes the structure of the model; section 3 present the model in log-linear approximation; section 4 discuss the policy problem while the analysis of the optimal policy plan is conducted in section 5. Section 6 concludes.

2 Model

2.1 Household behavior

We consider a two-country framework, a small open economy and the rest of the world. Each country is populated by agents who consume a basket of goods consisting of home and foreign produced goods. The model follows closely the one proposed by De Paoli (2003) and Gali and Monacelli (2002) for the case in which there is no fiscal policy stabilization problem. We consider a very simple small open economy model in which markets are complete and producer currency pricing holds.

There is a measure n of agents in our small open economy, that have utility function of the same form:

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\rho}}{1-\rho} - \int_0^n V(H_t(j)) dj \right] \quad (1)$$

The disutility of labour is represented by the function $V()$. The number of industries in the Home economy also has measure n and the goods in each industry j are produced using labour specific to that industry. The quantity supplied of labour of type j is denoted $H_t(j)$. Each households supply every type of labour and consume every type of good. C is a Dixit-Stiglitz aggregator of home and foreign goods as

$$C = \left[v^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

where $\theta > 0$ is the intratemporal elasticity of substitution and C_H and C_F are the two consumption sub-indexes that refer, respectively, to the consumption of home-produced and foreign-produced goods. v is a function of the relative size of the small economy with respect to the rest of the world, n , and of the degree of openness, λ : $(1-v) = (1-n)\lambda$.

Similar preferences are specified for the rest of the world:

$$C = \left[v^*{}^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-v^*)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \quad (2)$$

with $v^* = n\lambda$.

Note that the specification of v and v^* gives rise to home bias in consumption, as in Sutherland, A (2002). The size of the bias decreases with degree of openness λ . Moreover, the proportion of foreign-produced goods in Home consumption preferences is proportional to Foreign size and the proportion of home-produced goods in Foreign consumption preferences is proportional to Home size.

We have

$$C_H = \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, C_F = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}} \quad (3)$$

where $\sigma > 1$ is the elasticity of substitution for goods produced within a country. The consumption-based index for the small open economy that corresponds to the above specifications of preferences is given by:

$$P = \left[v P_H^{1-\theta} + (1-v) (P_F)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \theta > 0 \quad (4)$$

where P_H is the price sub-index for home-produced goods expressed in the domestic currency and P_F is the price sub-index for foreign produced goods expressed in the domestic currency.

$$P_H = \left[\left(\frac{1}{n} \right) \int_0^n p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, P_F = \left[\left(\frac{1}{1-n} \right) \int_n^1 p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}} \quad (5)$$

The law of one price holds: $p(h) = S p^*(h)$ and $p(f) = S p^*(f)$, where S is the nominal exchange rate (the price of foreign currency in terms of domestic currency). We define the real exchange rate as $RS = \frac{S P^*}{P}$. We assume that, as in Chari et al. (2002), markets are complete domestically and internationally. In each period t the economy faces one of the finitely many events $s^t \in \Upsilon$ (where Υ is the set of finitely many states). We denote by p^t the history of events up through and including period t . Looking ahead from period t the conditional probability of occurrence of state s^{t+1} is $\mu(s^{t+1} | p^t)$. The initial realization s_0 is given. There are complete markets in this economy both at domestic and international level. We represent the asset structure by having complete contingent one period nominal bonds denominated in the home currency (see Chari et al. 2002). We let $B^j(s^{t+1})$ denote the home

consumer's holdings of this bond, which pays one unit of the home currency if state s^{t+1} occurs and 0 otherwise, and we let $Q(s^{t+1} | p^t)$ denote the price of one unit of such a bond at date t and state s^t in units of domestic currency. The Home consumer j maximizes utility subject to the sequence of budget constraints

$$\sum_{s^t \in \Upsilon} \frac{Q(s^{t+1} | p^t) B^j(s^{t+1})}{P_t} = \frac{B^j(s^t)}{P_t} + \int_0^n \frac{W_t^j H_t^j}{P_t} dj + \frac{K_t}{P_t} + C_t^j + Tr_t$$

where W_t^j is the nominal wage for labor of type j , K_t^j are nominal profits (after taxes) for the Home consumer j .

We abstract from any monetary frictions by considering a cashless economy as in Woodford (2003, chapter 2). Given our preference specification, the total demands of the generic good h , produced in country H, and of the good f , produced in country F, are respectively:

$$\begin{aligned} y^d(h) &= \left[\frac{p(h)}{P_H} \right]^{-\sigma} \left\{ \left[\frac{P_H}{P} \right]^{-\theta} \left[vC + \frac{v^*(1-n)}{n} \left(\frac{1}{RS} \right)^{-\theta} C^* \right] + G \right\} \\ y^d(f) &= \left[\frac{p(f)}{P_F} \right]^{-\sigma} \left\{ \left[\frac{P_F}{P} \right]^{-\theta} \left[\frac{(1-v)n}{1-n} C + (1-v^*) \left(\frac{1}{RS} \right)^{-\theta} C^* \right] + G^* \right\} \end{aligned}$$

where G and G^* are country-specific government purchase shocks. To characterize our small open economy we use the definition of v and v^* and take the limit for $n \rightarrow 0$ so that

$$\begin{aligned} y^d(h) &= \left[\frac{p(h)}{P_H} \right]^{-\sigma} \left\{ \left[\frac{P_H}{P} \right]^{-\theta} \left[(1-\lambda)C + \lambda \left(\frac{1}{RS} \right)^{-\theta} C^* \right] + G \right\} \\ y^d(f) &= \left[\frac{p(f)}{P_F} \right]^{-\sigma} \{ C^* + G^* \} \end{aligned} \tag{6}$$

Because of market completeness, marginal utilities of income are equalized across countries at all times and states of nature as in Chari et al. (2002)

$$\frac{U_C(C_{t+1}^*)}{U_C(C_t^*)} \frac{P_t^*}{P_{t+1}^*} = \frac{U_C(C_{t+1})}{U_C(C_t)} \frac{S_{t+1} P_t}{S_t P_{t+1}} \tag{7}$$

2.2 Price setting mechanism

In this section we describe the price setting mechanism. We assume the technology for producing all goods is given by

$$y(z) = A_t h(z)^\delta \delta < 1$$

where $h(z)$ is labour specific to industry z . Moreover, each individual supplies a fraction $\frac{1}{n}$ of each type of labour ($H_t(z) = \frac{1}{n} h_t(z)$). Therefore, we can write the disutility of labour specified in (1) in terms of production $y(z)$ as follows:

$$V(H_t(z)) = V(y_t(z), \varepsilon_{Y,t}) = V \left(\frac{1}{n} \left(\frac{y_t(z)}{\varepsilon_{y,t}^{\delta-1}} \right)^{1/\delta} \right) \tag{8}$$

where $\varepsilon_{y,t} = A_t^{1/(\delta-1)}$. In addition, we define $\delta = \frac{1}{\eta+1}$ and rewrite (8) using the following functional form for $V(\cdot)$:

$$V(\tilde{y}_t(z), \varepsilon_{Y,t}) = \frac{1}{n} \frac{\varepsilon_{y,s}^{-\eta} y_s(j)^{\eta+1}}{\eta+1}$$

Prices follows a partial adjustment rule a la Calvo in which in each period a fraction $\alpha \in [0, 1)$ of randomly picked firms is not allowed to change the nominal price of the good it produces. The remaining fraction of firms $(1 - \alpha)$ choose prices optimally by maximizing the expected discounted value of profits¹

$$E_t \sum_{s=0}^{\infty} \alpha^s Q_{t,t+s} \left\{ (1 - \tau_{t+s}) p_{t+s}(z) y_{t+s}(z) - W_{t+s}^j h_{t+s}(z) \right\}$$

where $Q_{t,t+s}$ is the stochastic discount factor by which financial markets discount random nominal income in period $t+s$ defined as $Q_{t,t+s} \equiv \beta^s \frac{U_C(C_{t+s})}{U_C(C_t)} \frac{P_t}{P_{t+s}}$, the wage rate is given by $\frac{W_{t+s}^j}{P_t} = mc_t \frac{V_H(H_t(j))}{U_C(C_t)}$ where mc_t is an exogenous mark-up factor in the labor market. $\tilde{y}_{t,t+s}^h(z)$ is the total demand from the Home and Foreign country respectively under the condition that $\tilde{p}_t(z)$ still holds at period $t+s$:

$$\tilde{y}_{t,t+s}(z) = \left(\frac{\tilde{p}_t(z)}{P_{H,t+s}} \right)^{-\sigma} \left[\left(\frac{P_{H,t+s}}{P_{t+s}} \right)^{-\theta} \left((1 - \lambda) C_{t+s} + \lambda \left(\frac{1}{RS_{t+s}} \right) C_{t+s}^* \right) + G_{t+s} \right]$$

The associated first order condition with respect to $\tilde{p}_t(z)$ is

$$E_t \left\{ \sum (\alpha\beta)^{T-t} U_C(C_T) \left(\frac{\tilde{p}_t(z)}{P_{H,t}} \right)^{-\sigma} Y_{H,T} \left[\frac{\tilde{p}_t(z)}{P_{H,T}} \frac{P_{H,T}}{P_T} - \frac{\sigma mc_t V_y(\tilde{y}_{t,T}(z), \varepsilon_{Y,t})}{(1 - \sigma)(1 - \tau_t) U_C(C_T)} \right] \right\} = 0 \quad (9)$$

In what follows we will refer to fluctuations in this wedge as mark-up shocks. Given this price setting specification a la Calvo, the price index evolves according to the following law of motion:

$$(P_{H,t})^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1 - \alpha) (\tilde{p}_t(h))^{1-\sigma} \quad (10)$$

2.3 Government budget constraint

While we abstract from the existence of seignorage revenues, we consider different alternatives for the government in terms of the denomination of bonds that can be issued. The structure of the debt is kept exogenous. In the first case we focus on the situation in which the government issues one period nominally risk free bonds expressed in local currency units, collect taxes and faces an exogenous expenditure streams. We have that the nominal value D_t^n follows the law of motion:

$$D_t^n = D_{t-1}^n (1 + i_{t-1}) - P_{H,t} s_t$$

where s_t is the real primary budget surplus:

$$s_t \equiv \tau_t Y_t - G_t - Tr_t$$

and τ_t is income tax rate, G_t are government purchases and Tr_t are (lump-sum) government transfers exogenously given. Now define

$$\frac{D_t^n (1 + i_t)}{P_t} \equiv d_t^n$$

¹All households within a country that can modify their price at a certain time face the same discounted value of the streams of current and future marginal costs under the assumption that the new price is maintained. Thus they will set the same price.

So that we can rewrite the government budget constraint as

$$d_t^n = d_{t-1}^n \frac{(1 + i_t)}{\Pi_t} + \frac{P_{H,t}}{P_t} s_t (1 + i_t)$$

In addition to the case of nominal bonds, we consider two alternative specifications for government debt. If the government can issue a riskless real one-period bonds (D_t^r) - more specifically, a bond indexed to consumer price inflation - its budget constrain can be written as:

$$d_t^r = d_{t-1}^r (1 + i_t) + \frac{P_{H,t}}{P_t} s_t (1 + i_t^r)$$

where

$$d_t^r = D_t^r (1 + i_t^r)$$

Alternatively we assume that government debt is issued in the form of foreign currency denominated bonds (D_t^f), we can rewrite the government budget constraint as:

$$d_t^f = d_{t-1}^f \frac{(1 + i_t^f) e_t}{\Pi_t} - \frac{P_{H,t}}{P_t} s_t (1 + i_t^f)$$

where

$$d_t^f = \frac{S_t D_t^f (1 + i_t^f)}{P_t}$$

and CPI (consumer price index) inflation is denoted by $\Pi_t = \frac{P_t}{P_{t-1}}$ and the devaluation rate is defined as $e_t = \frac{S_t}{S_{t-1}}$. The implication for fiscal and monetary policy of the different debt characterizations are explored later in the text.

2.4 Rest of the world

As shown in the previous section 2.1, the structure and the preference specification for the rest of the world variables are identical to those of the small open economy. However, given the preference specification v^* , the rest of the world's demand will not be affected by the small open economy's dynamics (see equation (6)).

We can also write equations analogous to (9) and (10) for the foreign country, representing its pricing equation and price index evolution.

Moreover, the dynamic of the flow budget constraint for the foreign government is given by:

$$D_{t+1}^* = D_t^* (1 + i_{t-1}^*) - P_t^* s_t^*$$

where

$$s_t^* \equiv \tau_t^* C_t^* - G_t^* - Tr_t^*$$

3 A log-linear representation of the model

We approximate the model around a steady state in which the exogenous variables $(\varepsilon_{y,t}, G_t, mc_t)$, all take constant values such that $\bar{\varepsilon}_y, \bar{G} > 0$ and $mc \geq 1$. We further focus on a steady-state in which $\Pi_{H,t} \equiv P_{H,t}/P_{H,t-1} = 1$. In this steady-state $\bar{T} = \bar{RS} = 1$, $\bar{C} = \bar{C}^*$, $\bar{Y} = \bar{Y}^*$ and $\bar{U}_C(\bar{C}, 0) = \bar{\mu}\bar{V}_y(\bar{Y}, 0)^2$. Log deviations from the steady state are denoted with a hat.

The small open economy system of equilibrium conditions derived from log linearizing equations (4), (6), (7) (9) and the government solvency condition is:

$$(1 - \lambda)\hat{p}_H + \lambda\widehat{RS} = 0 \quad (11)$$

$$\hat{Y}_t = -\theta\hat{p}_H + (1 - \lambda)\hat{C} + \lambda\hat{C}^* + \theta\lambda\widehat{RS}_t + \hat{g}_t \quad (12)$$

$$\hat{C}_t = \hat{C}_t^* + \frac{1}{\rho}\widehat{RS}_t \quad (13)$$

$$\hat{\pi}_t^H = k \left(\rho\hat{C}_t + \eta\hat{Y}_t - \hat{p}_H + \widehat{mc}_t + \omega\hat{\tau}_t - \eta\widehat{\varepsilon}_{Y,t} \right) + \beta E_t \hat{\pi}_{t+1}^H \quad (14)$$

$$\begin{aligned} -\rho d_{ss}\hat{C}_t + \hat{d}_{t-1} - d_{ss}(a\Delta RS_t + b\hat{\pi}_t^H) &= d_{ss}(1 - \beta)\left(-\rho\hat{C}_t - \frac{\lambda}{1 - \lambda}RS_t\right) + \bar{\tau}(\hat{\tau}_t + \hat{Y}_t) - \hat{g}_t \\ &+ \beta E_t \left[-\rho d_{ss}\hat{C}_{t+1} + \hat{d}_t - d_{ss}(a\Delta RS_{t+1} + b\hat{\pi}_{t+1}^H) \right] \end{aligned} \quad (15)$$

Equation (11) describes the relationship between domestic relative prices ($p_{H,t} = P_{H,t}/P_t$) and the real exchange rate. Equation (12) characterizes the demand for domestic goods, with \hat{g}_t defined as $\frac{G_t - \bar{G}}{\bar{Y}}$. The risk sharing condition is described in equation (13). Finally, the last equation represents the small open economy Phillips Curve. We define $k = \frac{(1 - \alpha\beta)(1 - \alpha)}{\alpha(1 + \sigma\eta)}$, $\omega = \frac{\bar{\tau}}{1 - \bar{\tau}}$ and π_t^H denotes the producer price inflation, i.e. $\pi_{H,t} \equiv \ln(P_{H,t}/P_{H,t-1})$. Moreover, it is clear from equation (14) that a policy of pure domestic price stabilization, that sets $\hat{\pi}_t^H = 0$ in every state, leads to the same equilibrium allocation than the case in which prices are perfectly flexible, i.e. when $\alpha = 0$ and therefore $k \rightarrow \infty$.

Equation (15) represents the government budget constraint in a compact form in order to allow for different type of bonds. In order to obtain this expression, we used agents optimal intertemporal choice (i.e. the Euler equation). In the case of nominal bonds, $\hat{d}_t = \hat{d}_t^n = \frac{d_t^n - \bar{d}}{\bar{Y}}$, $a = \lambda/(1 - \lambda)$ and $b = 1$. Alternatively, $\hat{d}_t = \hat{d}_t^r = \frac{d_t^r - \bar{d}}{\bar{Y}}$ and $a = b = 0$ in the case of real bonds. Finally, $\hat{d}_t = \hat{d}_t^f = \frac{d_t^f - \bar{d}}{\bar{Y}}$, $a = -1$ and $b = 0$ in the case of foreign currency denominated bonds. Moreover, the steady state debt to GDP ratio is denoted by d_{ss} , more specifically $d_{ss} = \frac{\bar{d}}{\bar{Y}}$. Note that in the case of zero steady state government debt, the denomination of government debt is irrelevant for the dynamics of the small open economy. In this case, equation (15) becomes:

²This specification implies a specific level of initial distribution of wealth across countries. Appendix A contains the full characterization of the steady state.

³We denote $\hat{d}_t^r = \frac{d_t^r - \bar{d}}{\bar{Y}}$ in order to allow for zero steady state debt. In the appendix we also present a rescaled version of (15), where $\hat{d}_t = \frac{d_t - \bar{d}}{\bar{d}}$, and therefore $\hat{d}_t = d_{ss}\hat{d}_t^r$.

$$\widehat{d}_{t-1} = \bar{\tau}(\widehat{\tau}_t + \widehat{Y}_t) - \widehat{g}_t + \beta \widehat{d}_t \quad (16)$$

The system of structural equilibrium conditions is closed by specifying the monetary and fiscal policy rules. Given the domestic exogenous variables $\widehat{\varepsilon}_{y,t}, \widehat{g}_t, \widehat{m}c_t$ and the external shock \widehat{C}_t^* , we can determine the dynamics of $\widehat{Y}_t, \widehat{RS}_t, \widehat{C}_t, \widehat{\pi}_t^H, \widehat{d}_t$ and $\widehat{p}_{H,t}^4$.

Foreign dynamics are governed by the foreign Phillips curve, demand condition and government budget constraint:

$$\widehat{\pi}_t^* = k \left(\rho \widehat{C}_t^* + \eta \widehat{Y}_t^* + \widehat{\mu}_t^* - \eta \widehat{\varepsilon}_{Y,t}^* \right) + \beta E_t \widehat{\pi}_{t+1}^*$$

$$\widehat{Y}_t^* = \widehat{C}_t^* + \widehat{g}_t^*$$

$$\begin{aligned} -\rho d_{ss}^* \widehat{C}_t^* + \widehat{d}_{t-1}^* - d_{ss}^* \widehat{\pi}_t^* &= -\rho d_{ss}^* \widehat{C}_t^* + \bar{\tau}^* (\widehat{\tau}_t^* + \widehat{Y}_t^*) - \widehat{g}_t^* \\ &+ \beta E_t \left[-\rho d_{ss}^* \widehat{C}_t^* + \widehat{d}_t^* - d_{ss}^* \widehat{\pi}_{t+1}^* \right] \end{aligned}$$

The specification of the foreign policy rules complete the system of equilibrium conditions which determine the evolution of $\widehat{Y}_t^*, \widehat{C}_t^*$ and $\widehat{\pi}_t^*$. We should note that the dynamics of the rest of the world is not affected by *Home* variables. Therefore, the small open economy can treat \widehat{C}_t^* as exogenous shock. Moreover, the policy choice of the rest of the world modifies the way foreign structural shocks affect \widehat{C}_t^* but does not influence how \widehat{C}_t^* affects the small open economy.

4 Welfare measure

The policy objective for the small open economy is given by the expected utility of the agents belonging to the economy:

$$W = E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[U(C_t) - \int_0^n V(y_t(h), \varepsilon_{Yt}) dh \right] \right\}$$

We assume that policy makers can commit to maximize its objective function.

In the appendix we derive a second order approximation of the policy objective. In order to eliminate the discounted linear terms that appear in the Taylor approximation, we follow the method of Benigno and Woodford (2003) and Sutherland (2002) by using a second order approximation to some of the structural equilibrium conditions in order to obtain a complete second order solution for the evolution of the endogenous variables of interest. We also assume that policy makers are committed to past promises following a timeless perspective commitment (as in Woodford, ch 7). Using this form of commitment we obtain that the loss function for our small open economy can be expressed as a quadratic function of $\widehat{Y}_t, \widehat{RS}_t$, and $\widehat{\pi}_t^H$:

$$L_{t_0}^i = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} \Phi_Y \widehat{y}_t^2 + \frac{1}{2} \Phi_T \widehat{r} s_t^2 + \frac{1}{2} \Phi_\pi (\widehat{\pi}_t^H)^2 \right] + t.i.p \quad (17)$$

⁴In order to retrieve the value of the nominal exchange rate and interest rate we can use households' intertemporal choice (i.e. the Euler equation) and the definition of the real exchange rate.

where $\widehat{r\hat{s}}_t = (\widehat{RS}_t - \widehat{RS}_t^T)$, $\widehat{y}_t = (\widehat{Y}_t - \widehat{Y}_t^T)$, and \widehat{Y}_t^T and \widehat{RS}_t^T are the target variables which are functions of the shocks, and the weights Φ_Y, Φ_{RS} and Φ_π depend on the structural parameters of the model. These are defined in appendix B.

Not surprisingly, there is no tax smoothing objectives for the policy makers as in the closed economy work by Benigno and Woodford (2003). However, the open economy dimension of the model gives rise to the real exchange rate term as an important policy objective (this term would arise in general in a small open economy setting without the fiscal stabilization problem, see De Paoli, 2003 on this). Policy should seek to minimize the discounted value of a weighted sum of squared deviations of inflation from zero and squared fluctuations in output gap and real exchange rate gap where the target levels depend on various exogenous disturbances (see the Appendix).

By inspection of the weights Φ_π, Φ_Y , and Φ_{RS} in the appendix, we can see which are the factors that affect the determinants of the loss function in our small open economy. Our economy is characterized by two distortions that are in common with ones commonly assumed in the literature on monetary policy stabilization, namely an inefficient output level due to monopolistic distortions and the staggered price setting mechanism that create dispersion of demand across differentiated goods. In open economy, another incentive arises, namely the incentive to decrease the expected disutility of producing goods by diverting production towards the rest of the world by strategically manipulating the real exchange rate (see on this Corsetti and Pesenti, 2001). The other distortions that we have introduced in this economy, i.e. the tax distortions, are relevant only to the extent to which they modify the target variables and the weights in the loss function (see the Appendix on this). The only case in which there is no concern for the real exchange rate gap in the policy objective function is the special parametric case when $\rho = \theta = 1$ and the monopolistic distortions are set at a value equal to the openness parameter, i.e. $\lambda = \phi$ (as in Benigno and Benigno, 2003 and Gali and Monacelli, 2003 for the application to the small open economy case).

5 Optimal Policy

5.1 The case of Flexible Prices

In this section we start by considering the case where prices are perfectly flexible (that is, $\alpha = 0$). Our objective is to understand the open economy dimension of the optimal (fiscal) problem for our small open economy case. The assumption that $\alpha = 0$ implies that $\Phi_\pi = 0$ and $k^{-1} = 0$. We then have that the loss function for our small open economy becomes:

$$\min U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} \Phi_Y \widehat{y}_t^2 + \frac{1}{2} \Phi_T \widehat{r\hat{s}}_t^2 \right] + t.i.p$$

The system of equilibrium conditions for the small open economy defined in section (3) determines the set of constraints faced by the policymaker. The optimal policy can be represented by the following Lagrangian:

$$\mathcal{L} = E_{t_0} \sum \beta^{t-t_0} \left[\begin{aligned} & \frac{1}{2} \Phi_Y^2 \hat{y}_t^2 + \frac{1}{2} \Phi_T \hat{r}_t^2 + \varphi_{1,t} \left(\eta \hat{Y}_t + (1-\lambda)^{-1} \widehat{RS}_t - \omega \hat{\tau}_t \right) + \\ & + \varphi_{2,t} \left(\begin{aligned} & -d_{ss} \widehat{RS}_t + \hat{d}_{t-1} - d_{ss} (a \Delta RS_t + b \hat{\pi}_t^H) + d_{ss} (1-\beta) \left(\frac{1}{1-\lambda} \widehat{RS}_t \right) \\ & - \bar{\tau} (\hat{\tau}_t + \hat{Y}_t) + \beta E_t \left[d_{ss} \widehat{RS}_{t+1} - \hat{d}_t + d_{ss} (a \Delta RS_{t+1} + b \hat{\pi}_{t+1}^H) \right] \end{aligned} \right) \\ & + \varphi_{3,t} \left(\hat{Y}_t - \frac{(1+l)}{\rho(1-\lambda)} \widehat{RS}_t \right) \end{aligned} \right] + t.i.p$$

where $\varphi_{1,t}$, $\varphi_{2,t}$ and $\varphi_{3,t}$ are the Lagrange multiplier associated with the policy constraints. The first order conditions of this minimization problem with respect to $\hat{\pi}_t^H$, \hat{Y}_t , \widehat{RS}_t , $\hat{\tau}_t$ and \hat{d}_t are given by

$$-b d_{ss} (\varphi_{2,t} - \varphi_{2,t-1}) = 0 \quad (18)$$

$$\Phi_Y \hat{y}_t + \eta \varphi_{1,t} - \bar{\tau} \varphi_{2,t} + \varphi_{3,t} = 0 \quad (19)$$

$$\begin{aligned} & \Phi_T \hat{r}_t + \frac{1}{(1-\lambda)} \varphi_{1,t} - d_{ss} (a+1) (\varphi_{2,t} - \varphi_{2,t-1}) \\ & + d_{ss} \frac{(1-\beta)}{(1-\lambda)} \varphi_{2,t} + \beta a d_{ss} (E_t \varphi_{2,t+1} - \varphi_{2,t}) - \frac{(1+l)}{\rho(1-\lambda)} \varphi_{3,t} = 0 \end{aligned} \quad (20)$$

$$\omega \varphi_{1,t} - \bar{\tau} \varphi_{2,t} = 0 \quad (21)$$

$$-\varphi_{2,t} + E_t \varphi_{2,t+1} = 0 \quad (22)$$

Moreover, the first order condition with respect to $\hat{\pi}_t^H$ at time t_0 implies $d_{ss} b \varphi_{2,t_0} = 0$. We can combine the first order conditions to obtain the following equilibrium relationship that holds under the optimal plan:

$$\Phi_T \hat{r}_t + \frac{(1+l)}{\rho(1-\lambda)} \Phi_Y \hat{y}_t = -m_0 \varphi_{2,t} + (a+1) d_{ss} (\varphi_{2,t} - \varphi_{2,t-1}), \quad (23)$$

where m is defined in the appendix.

From the aggregate supply equation and the intertemporal budget constraint, we can determine the evolution of the tax rate and the evolution of government debt⁵:

$$\hat{\tau}_t = \hat{\tau}_t^{T'} - \omega^{-1} \left(\frac{\eta(1+l) + \rho}{(1+l)} \right) \hat{y}_t \quad (24)$$

where $\hat{\tau}_t^{T'}$ is defined in the appendix

$$\hat{d}_t = -\varphi_{2,t} \frac{n_2}{1+l} + \frac{1}{1+l} E_t f'_{t+1}$$

where $\hat{\tau}_t^{T'}$ and f'_t are function of the different disturbances and the coefficients n_1 and n_2 are defined in the appendix. The composite disturbance f'_t has been labelled fiscal stress in Benigno and Woodford (2003) as it summarizes the shocks affecting the government intertemporal budget constraint.

⁵See the appendix for a full derivation.

The expressions above illustrate the link between the dynamic of our endogenous variables and the marginal value, measured in utility terms, of one unit of the government revenue in period t , i.e. $\varphi_{2,t}$. However the evolution of $\varphi_{2,t}$ depends on the features of the government budget constraint. In particular, if (i) $d_{ss} = 0$ and/or $b = 0$ then $\varphi_{2,t}$ follows a non-stationary process given by

$$\varphi_{2,t} = \frac{f_t - E_{t-1}f_t}{(n_1 + n_2)} + \varphi_{2,t-1} \quad (25)$$

On the other hand when (ii) $d_{ss} \neq 0$ and $b \neq 0$ (this is the case in which inflation influences the burden of government debt) then

$$\varphi_{2,t} = 0 \quad (26)$$

Therefore the dynamic properties of the output gap, the real exchange rate gap and the tax gap rate depend as well on the features of the government solvency conditions. In particular under (i) these variables will be non-stationary, while under (ii) they will be stationary.

Case (i) $d_{ss} = 0$

To gain some intuition on the open economy dimension of the fiscal policy we now assume $d_{ss} = 0$, $\theta = \rho = 1$ and $\mu = 1/(1 - \lambda)$. The first assumption implies that the government budget constraint is affected only by the evolution of the primary surplus and not by the interest payment on existing debt (see equation (16)). In this case the denomination of the government debt is irrelevant and only real variables are determined. The second restriction implies that there are no trade imbalances while the third restriction implies that the monopolistic distortions are at the efficient level from the small open economy perspective. Under this specification, using a model with lump sum taxes, Gali and Monacelli (2004) and De Paoli (2003) found that the small open economy is isomorphic to a closed economy and that the flexible price allocation is equivalent to the constraint efficient allocation. As mentioned in the previous section, differently from the monetary policy stabilization problem (see De Paoli (2004)), under the assumption that $\theta = \rho = 1$, fluctuations in the real exchange rate will still affect the loss function. A further restriction in terms of the degree of monopolistic competition is instead required here.

In general, under the special case that we are considering, our restrictions will be such that the small open economy is isomorphic to the closed economy (i.e. the case in which $\lambda = 0$). First it is easy to prove that, under our restrictions, the desired tax rate is zero as long as the economy is subject to domestic productivity shocks. Moreover under the assumption that $d_{ss} = 0$ ⁶ we have that government debt follows a unit root process:

$$\hat{d}_t = \hat{d}_{t-1} + \frac{1}{1+l} E_t \Delta f'_{t+1}$$

and we can rewrite (23) so that under the optimal plan, the expected growth of the output gap is zero from which it follows that the expected growth of the tax gap is zero as well:

$$E_t \Delta \hat{y}_{t+1} = E_t \Delta (\hat{\tau}_{t+1} - \hat{\tau}_{t+1}^T) = 0. \quad (27)$$

⁶When $d_{ss} = 0$ then $n_1 = 0$ (see the appendix).

Table 1: Parameter Values used in the Quantitative Analysis

Parameter	Value	Notes:
β	0.99	Specifying a quarterly model
η	0.47	Following Rotemberg and Woodford (1997)
θ	3	Following Obstfeld and Rogoff (1998) (unless specified otherwise)
λ	0.4	This implies a 40% import share of the GDP
ρ	1	Specifying a Log utility function
α	0.66	Characterizing an average length of price contract of 3 quarters
σ	10	Following Benigno and Woodford (2003)
d_{ss}	2.6	Steady state debt to GDP of 60% (unless specified otherwise)
τ	0.2	Steady state taxes of 20% of GDP

Therefore in this special case in which there is no open economy dimension and steady state output is efficient from a small open economy perspective, taxes are constant after productivity shocks. Indeed, taxes are smoothed in order to avoid a further distortion that would affect the marginal rate of substitution between leisure and consumption. This result is consistent with the findings of Barro (1979) and Aiyagari et al. (2002). As shown in De Paoli (2004) with lump sum taxes and under the restrictions that we have imposed, the flexible price allocation is efficient and the output gap is closed. Here lump-sum taxes are not available and when there is a positive productivity shock the best that the policy authority can do is to lower taxes at the new level forever. Figures (1) and (2) illustrate this case⁷.

We now move to the case in which $\theta \neq 1$, $\rho \neq 1$, with $d_{ss} = 0$ and $\mu = 1/(1 - \lambda)$ so that we analyse the open economy dimension of our stabilization problem by allowing for trade imbalances. By combining the first order conditions we obtain the following expression:

$$\Phi_T E_t \Delta \widehat{r}_{t+1} + \frac{(1+l)}{\rho(1-\lambda)} \Phi_Y E_t \Delta \widehat{y}_{t+1} = 0, \quad (28)$$

so we can see that under the optimal plan for $d_{ss} = 0$ we need to stabilize not only the expected growth rate of the output gap but also the expected change in the real exchange rate gap. Equivalently, we can rewrite the above equation in terms of the tax and exchange rate gap:

$$\Phi_T E_t \Delta \widehat{r}_{t+1} + \frac{\omega(1+l)^2}{\rho(1-\lambda)(\eta(1+l) + \rho)} \Phi_Y E_t \Delta (\widehat{\tau}_{t+1} - \widehat{\tau}_{t+1}^{T'}) = 0,$$

In this case is no longer optimal to keep the expected rate of change of taxes constant because under the optimal plan it is also important to manage the expected rate of change of the real exchange rate. Figure (3) illustrates this case. Differently from the case in which the small open economy is isomorphic to the closed economy, now the tax target is no longer zero even under productivity shocks. Following a positive productivity shock we observe an increase in the level of taxes and the real exchange rate depreciates. In particular we note that the real exchange rate depreciates by less than what it would if lump sum taxes were available. The increase in taxes induces a lower level of production than the one that would arise if only lump sum taxes were available. In this case the welfare of the small open economy can be increased by reducing the disutility of producing home goods. This is achieved through

⁷The specification of the parameter values used in our quantitative exercise is shown in table (1)

a combined increase in the distortionary taxations (that reduces directly the home production) and a relatively less depreciated real exchange rate that divert domestic consumption toward foreign produced goods.

In this case, (the open economy case) the reduction in production is not accompanied by a corresponding reduction in consumption. This is because the movement in the real exchange rate helps in redirecting demand toward foreign produced goods. Note that when $\theta = \rho = 1$ the income and the substitution effects coming from real exchange rate changes will offset each other and a marginal reduction in the utility value of output will be accompanied by a corresponding reduction in utility value of consumption so that distortionary taxes will remain constant over time and will move across states just to satisfy the intertemporal solvency condition. In the open economy case the increase in taxes (that reduce output) implies a less depreciated real exchange rate (that allows to consume more from the rest of the world).

Case (ii) $d_{ss} \neq 0$ with nominal bonds

We now turn our attention to the case where the path of inflation affects the intertemporal solvency condition and layout an economy in which government debt is denominated in domestic currency. More specifically $d_{ss} \neq 0$, $a = \frac{\lambda}{1-\lambda}$ and $b = -1$. As mentioned above in this case $\varphi_{2,t} = 0 \forall t$. Moreover given that (18) implies (22) we can see that the system of equilibrium conditions is underidentified. So that by inspection of (15) we have that expected producer inflation and government debt are indeterminate. In this case, by combining the first order conditions, it is possible to derive a simple targeting rule for our optimal policy problem:

$$\Phi_T \widehat{r}_t s_t + \frac{(1+l)}{\rho(1-\lambda)} \Phi_Y \widehat{y}_t = 0. \quad (29)$$

Note that under the assumption that bonds are issued in domestic currency, the endogenous variables do not exhibit the non stationary property.

To understand, as before, the open economy dimension of our stabilization problem we first focus on the special case in which our small open economy is isomorphic to the closed economy (i.e. $\rho = \theta = 1$ and $\mu = 1/(1-\lambda)$). Here we have that our targeting rule simplifies to that the output gap is fully stabilized as in Benigno and Woodford (2003):

$$\widehat{y}_t = 0.$$

and domestic producer inflation is going to be determined by the following equation

$$\widehat{d}'_{t-1} - d_{ss} b \widehat{\pi}_t^H = f'_t,$$

In this case the efficient allocation is achieved. Differently from case (i) described above, taxes do not need to adjust because domestic inflation is used to satisfy the intertemporal solvency condition. In an environment in which prices are flexible inflation is not costly, while taxation is distortionary. Following a productivity shock the optimal plan prescribes a change in the domestic price level. The resulting allocation is the same as the one that would be achieved if lump sum taxes were available and this result is consistent with the findings of Bohn (1990) and Chari, Christiano and Kehoe (1991). (see figure (4))

As in case (i), once we assume that $\theta \neq 1$, $\rho \neq 1$, following a domestic productivity shock, the small open economy benefits from an increase in distortionary taxation that will induce a lower disutility of production. As prescribed by (29), there exists an incentive to manage the real exchange rate gap in this case. The amount of real exchange rate depreciation is reduced so that domestic households can switch their demand toward foreign produced goods (see figure (5)).

Summary

As it becomes clear from the above results, the stabilization problem in an open economy is more complex than the closed economy one even under flexible prices. Apart from the incentive to avoid the distortions implied by taxation, the small open economy also wants to strategically affect the real exchange rate. That is production taxation (as opposed to a situation in which there are lump sum taxes) introduces a distortion in the economy but also brings in an instrument which is able to influence the level of domestic production in a welfare-improving manner. While in a closed economy following a positive productivity shock, the optimal policy would imply a procyclical and smooth evolution of taxes, in the small open economy taxes are initially countercyclical and vary over time. Indeed, the small open economy has an incentive to increase taxation in order to reduce domestic production. Moreover, the depreciation of the real exchange rate is limited (if compared with the case of endogenous lump sum taxes) in order to divert expenditure towards the rest of the world.

5.2 Fiscal and Monetary interactions: the case of sticky prices

We now turn to the optimal policy problem in the case of sticky prices so that $\alpha > 0$. We characterize the general optimal fiscal and monetary plan, in which there are two policy instruments, inflation and taxes and three policy objectives: that is minimizing the distortions created by sticky prices and taxation, and managing the real exchange rate. As before we allow for different types of bond denominations. Therefore, the policy problem consists of choosing the path of $\{\widehat{\pi}_t^H, \widehat{Y}_t, \widehat{RS}_t, \widehat{d}_t, \widehat{\tau}_t\}$ as to minimize (17), subject to the equilibrium conditions shown in section 3 and the initial condition for \widehat{d}_{t_0-1} and the constraints that $\widehat{\pi}_{t_0}$ and \widehat{Y}_{t_0} are equalized to the precommitted values⁸. The Lagrangian for this problem can be written as

$$\mathcal{L} = E_{t_0} \sum \beta^{t-t_0} \left[\begin{aligned} & \frac{1}{2} \Phi_Y \widehat{y}_t^2 + \frac{1}{2} \Phi_T^2 \widehat{r}_t^2 + \frac{1}{2} \Phi_\pi \left(\widehat{\pi}_t^H \right)^2 + \\ & + \varphi_{1,t} \left(-k^{-1} \pi_t^H + \eta \widehat{Y}_t + (1-\lambda)^{-1} \widehat{RS}_t - \omega \widehat{\tau}_t + \beta E_t \pi_t^H \right) + \\ & + \varphi_{2,t} \left(\begin{aligned} & -d_{ss} \widehat{RS}_t + \widehat{d}_{t-1} - d_{ss} (a \Delta R S_t + b \widehat{\pi}_t^H) + d_{ss} (1-\beta) \left(\frac{1}{1-\lambda} \widehat{RS}_t \right) \\ & - \bar{\tau} (\widehat{\tau}_t + \widehat{Y}_t) + \beta E_t \left[d_{ss} \widehat{RS}_{t+1} - \widehat{d}_t + d_{ss} (a \Delta R S_{t+1} + b \widehat{\pi}_{t+1}^H) \right] \end{aligned} \right) \\ & + \varphi_{3,t} \left(\widehat{Y}_t - \frac{(1+l)}{\rho(1-\lambda)} \widehat{RS}_t \right) \end{aligned} \right] + t.i.p$$

The first order conditions with respect to $\widehat{\pi}_t^H, \widehat{Y}_t, \widehat{RS}_t, \widehat{\tau}_t$ and \widehat{d}_t are given by:

$$\Phi_\pi \widehat{\pi}_t^H - (\varphi_{1,t} - \varphi_{1,t-1}) - b d_{ss} (\varphi_{2,t} - \varphi_{2,t-1}) = 0 \quad (30)$$

$$\Phi_Y \widehat{y}_t + k \eta \varphi_{1,t} - \bar{\tau} \varphi_{2,t} - \varphi_{3,t} = 0 \quad (31)$$

⁸This should be consistent with the 'timeless perspective' equilibrium. For a discussion on the timeless perspective of optimal rule see Woodford, 2003.

$$\begin{aligned} & \Phi_T \widehat{r}_t + \frac{k}{(1-\lambda)} \varphi_{1,t} - d_{ss}(a+1)(\varphi_{2,t} - \varphi_{2,t-1}) \\ & + d_{ss} \frac{(1-\beta)}{(1-\lambda)} \varphi_{2,t} + \beta a d_{ss} (E_t \varphi_{2,t+1} - \varphi_{2,t}) + \frac{(1+l)}{\rho(1-\lambda)} \varphi_{3,t} = 0 \end{aligned} \quad (32)$$

$$k\omega\varphi_{1,t} - \bar{\tau}\varphi_{2,t} = 0 \quad (33)$$

$$-\varphi_{2,t} + E_t \varphi_{2,t+1} = 0 \quad (34)$$

These conditions along with the policy constraints and the initial conditions will determine the paths of $\{\widehat{\pi}_t^H, \widehat{Y}_t, \widehat{R}S_t, \widehat{d}_t, \widehat{\tau}_t, \varphi_{1,t}, \varphi_{2,t}, \varphi_{3,t}\}$.

As before, we express the optimal state-contingent response to shocks as described above in terms of policy rules that the monetary and fiscal authorities of our small open economy should follow. In doing so, we specify the policy behavior in terms of targeting rules following Giannoni and Woodford (2003) and Svensson (2003). These rules can be obtained by eliminating the Lagrange multiplier from the above system of first order conditions of the optimal policy problem.

$$\left[\frac{(1+l)\Phi_y}{(1-\lambda)\rho} \right] \Delta \widehat{y}_t + \Phi_T \Delta \widehat{r}_t + \left[\frac{k\Phi_\pi}{(1-\bar{\tau}) + b d_{ss} k} \right] (\gamma \widehat{\pi}_t^H + d_{ss}(a+1)\widehat{\pi}_{t-1}^H) = 0 \quad (35)$$

where $\gamma = \left[d_{ss} \left(\frac{(1-\beta)}{(1-\lambda)} - (1+a) \right) + \left(\frac{(1+l)(\eta(1-\bar{\tau})-\bar{\tau})}{\rho(1-\lambda)} + \frac{1-\bar{\tau}}{(1-\lambda)} \right) \right]$. We first note that the variables of interest in this targeting rule are current and past domestic producer inflation, the rate of change in the real exchange rate gap and the rate of change of the output gap. We can first compare our targeting rule with the one in De Paoli (2004) in which there is no fiscal stabilization problem. In general the relative weights on the target variables are different since now Φ_y , Φ_T , and Φ_π are affected by the degree of distortionary taxation in steady state. Moreover here the past producer inflation term enters in the targeting rule except in the case when government debt is issued in foreign currency and/or the steady state level of government debt is zero.

The last first order condition, (34) implies the following target criterion:

$$E_t \widehat{\pi}_{t+1}^H = 0, \quad (36)$$

so that, in our small open economy case, expected producer inflation is set to zero.

In the appendix we show that the dynamic of the Lagrange multiplier and the government debt are given by:

$$\varphi_{2,t} = \frac{f'_t - E_{t-1} f'_t}{(n'_1 + n'_2)} + \varphi_{2,t-1} \quad (37)$$

$$\widehat{d}_t = \frac{E f'_{t+1}}{1+l} - \frac{n'_2 \varphi_{2,t}}{1+l} + \frac{n_3 d_{ss}}{(1+l)} (\varphi_{2,t} - \varphi_{2,t-1}) \quad (38)$$

so that the Lagrange multiplier follows a unit root process as in the flexible price case and the government debt is non-stationary. Similarly the output gap and the exchange rate gap evolve according

to

$$\Phi_T \widehat{r}_t + \frac{(1+l)}{\rho(1-\lambda)} \Phi_Y \widehat{y}_t = -m_0 \varphi_{2,t} + (a+1) d_{ss} (\varphi_{2,t} - \varphi_{2,t-1}), \quad (39)$$

where n'_1, n'_2 and n_3 are defined in the appendix.

Case (i) $d_{ss} = 0$

In order to understand how the open economy dimension changes the stabilization problem under sticky prices we first focus on the special case in which $d_{ss} = 0$, $\theta = \rho = 1$ and $\mu = 1/(1-\lambda)$. Under the assumption that $d_{ss} = 0$, as discussed above, the denomination of the government debt is irrelevant for determining the optimal response to shocks. We can then rewrite our targeting rules as

$$\left[\frac{\Phi_y}{(1-\lambda)} \right] \Delta \widehat{y}_t + \left[\frac{k\Phi_\pi}{(1-\bar{\tau})} \right] \left(\frac{(\eta(1-\bar{\tau}) - \bar{\tau}) + 1 - \bar{\tau}}{(1-\lambda)} \right) \widehat{\pi}_t^H = 0$$

$$E_t \widehat{\pi}_{t+1}^H = 0,$$

Combining the previous targeting rules and the Phillips curve to substitute in for taxes we obtain the following equilibrium relationship that holds under the optimal plan:

$$\omega E_t \Delta (\widehat{\tau}_{t+1} - \widehat{\tau}_{t+1}^{T'}) + k^{-1} \pi_t^H = 0. \quad (40)$$

Differently from the stabilization problem under flexible prices, here taxes cannot be smoothed over time. This can be seen by comparing equation (27) and (40) for the flexible and the sticky prices cases respectively. Figure (6) illustrate this comparison for both closed and open economy. When lump sum taxes are not available, domestic producer inflation is not fully stabilized (see figure (7)). In this case taxes and inflation will vary in order to satisfy the intertemporal solvency condition following a domestic productivity shock. A permanent change in taxes is not an option (as in the flexible price case) because, given the link between inflation and taxes coming from the aggregate supply relationship, that would imply non stationarity in the domestic producer inflation (and an explosive path for the domestic price level). Moreover since there is a cost of inflation under sticky prices, the optimal plan would require both taxes and inflation to be varied.

When $\theta \neq 1$, $\rho \neq 1$, with $d_{ss} = 0$ and $\mu = 1/(1-\lambda)$, as we have emphasized before, in our small open economy there is an incentive to divert production toward the rest of the world. This is the reason for which, as shown in figure (9), both taxes and inflation vary more compared to the closed economy case (i.e. $\lambda = 0$). Note that equation (39) implies

$$\Phi_T E_t \Delta \widehat{r}_{t+1} + \frac{(1+l)}{\rho(1-\lambda)} \Phi_Y E_t \Delta \widehat{y}_{t+1} = 0, \quad (41)$$

This is identical to equation (28) obtained under flexible prices for $d_{ss} = 0$. Therefore when there is no nominal burden from existing debt (i.e. $d_{ss} = 0$), the optimal policy under both flexible and sticky prices prescribes the stabilization of expected growth rate of the output gap and expected change in the real exchange rate gap. Indeed, figure (8) illustrates that quantitatively there is no significant difference between the evolution of output, the output gap and the real exchange rate under sticky and flexible prices. The difference among these two cases arises in the use of the two stabilization tools, i.e. inflation and taxes. Under sticky price, both inflation and taxes are used to switch production. Under flexible prices only taxes can be used to improve welfare and therefore the required change in taxes is larger.

What is important to note here is that in a closed economy taxes are procyclical, here the incentive to divert production makes taxes moving in a countercyclical way (see figures (6) and (8)). Moreover, taxes are more volatile under flexible prices compared to the sticky price case, because in the former case domestic producer inflation does not affect the supply of home produced goods and so it cannot be used as an instrument to redirect production toward the rest of the world.

Case (ii) $d_{ss} \neq 0$ with nominal bonds

In terms of our parametrization this case requires $a = \frac{\lambda}{1-\lambda}$ and $b = -1$ so that domestic producer inflation can influence the intertemporal solvency condition. While in the nominal bonds case under flexible prices, equation (29) imply a stationary process for the real exchange rate gap and the output gap, now under sticky prices and nominal bonds equation (39) shows that those gaps follow a non stationary process. This is because under flexible prices, domestic producer inflation varies to affect the level of real debt so that the taxes, the output gap and the real exchange rate gap are stationary. Under sticky prices domestic producer inflation is costly and therefore it varies less under the optimal plan (see figure (11)). This result is compatible with the findings of Schmitt-Grohe and Uribe (2001), Siu (2001) and Benigno and Woodford (2003). However in an open economy framework with price stickiness, under the optimal plan, both the variability of inflation and taxes are reduced compared to the case in which prices are flexible. As mentioned earlier, taxes will move less because domestic producer inflation can be used to manipulate the consumption-leisure choice.

6 Conclusion

(to be written)

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A Appendix A: Steady state equations

In this appendix we derive the steady state conditions and define some parameters that depend on these conditions. All variables in steady state are denoted with a bar.

From the demand equation at Home, we have:

$$\begin{aligned} y^d(h) &= \left[\frac{p(h)}{P_H} \right]^{-\sigma} \left[\frac{P_H}{P} \right]^{-\theta} \left[vC + \frac{v^*(1-n)}{n} \left(\frac{1}{RS} \right)^{-\theta} C^* \right] \\ y^d(f) &= \left[\frac{p(f)}{P_F} \right]^{-\sigma} \left[\frac{P_F}{P} \right]^{-\theta} \left[\frac{(1-v)n}{1-n} C + (1-v^*) \left(\frac{1}{RS} \right)^{-\theta} C^* \right] \end{aligned} \quad (\text{A.1})$$

Normalizing $\bar{P}_H = \bar{P}_F$, we have:

$$\bar{Y} = v\bar{C} + \frac{v^*(1-n)}{n}\bar{C}^* + \bar{G} \quad (\text{A.2})$$

$$\bar{Y}^* = \frac{(1-v)n}{1-n}\bar{C} + (1-v^*)\bar{C}^* \quad (\text{A.3})$$

If we specify the proportion of foreign-produced goods in home consumption as $1-v = (1-n)\lambda$ and the proportion of home-produced goods in foreign consumption is $v^* = n\lambda$, and take the limiting case where $n = 0$, we have.

$$\bar{Y} = (1-\lambda)\bar{C} + \lambda\bar{C}^* + \bar{G} \quad (\text{A.4})$$

And from the Foreign demand we have

$$\bar{Y}^* = \bar{C}^* \quad (\text{A.5})$$

For further reference, let's define some steady state dependent constants:

$$\begin{aligned} d_g &= \frac{\bar{G}}{\bar{Y}} \\ d_b &= (1-\lambda)\frac{\bar{C}}{\bar{Y}} \end{aligned}$$

Moreover, using equation (A.4), we can notice that:

$$\frac{\lambda\bar{C}^*}{\bar{Y}} = 1 - d_b - d_g$$

Moreover, from the government budget constraint we have:

$$\delta\bar{D} = \bar{P}\bar{s} \quad (\text{A.6})$$

with $\delta = \beta^{-1}(1-\beta)$ and with the steady state fiscal surplus defined as follows:

$$\bar{s} = \bar{\tau}\bar{Y} - \bar{G} - \bar{T}r \quad (\text{A.7})$$

And the steady state constants defined in the text are:

$$s = \bar{s}^{-1}\bar{Y}$$

$$s_\tau = \bar{s}^{-1} \bar{\tau} \bar{Y}$$

The Symmetric Steady State:

From the complete asset market assumption we have:

$$RS_t = \kappa_0 \left(\frac{C_t}{C_t^*} \right)^\rho \quad (\text{A.8})$$

where

$$\kappa_0 = RS_0 \left(\frac{C_0}{C_0^*} \right)^\rho \quad (\text{A.9})$$

So if we assume an initial level of wealth such that $\kappa_0 = 1$, the steady state version of (A.8) imply $\bar{C} = \bar{C}^*$.

If, moreover we assume $\bar{G} = 0$ we have:

$$d_g = 0$$

$$d_b = (1 - \lambda)$$

If we also assume $\bar{T}r = 0$, we have:

$$s = 1/\bar{\tau}$$

$$s_\tau = 1$$

Applying our normalization to the price setting equations we have:

$$U_C(\bar{C}) = \mu V_y (\lambda \bar{C}^* + (1 - \lambda) \bar{C}) \quad (\text{A.10})$$

$$U_C(\bar{C}^*) = \mu^* V_y (\bar{C}^*). \quad (\text{A.11})$$

Where

$$\mu = \frac{\sigma \bar{m} \bar{c}}{(1 - \bar{\tau})(\sigma - 1)}$$

$$(1 - \phi) = \frac{1}{\mu}$$

$$\phi = 1 - \frac{(1 - \bar{\tau})(\sigma - 1)}{\sigma \bar{m} \bar{c}}$$

$$0 \leq \phi < 1; \mu > 1$$

B Appendix B

In this appendix, we derive the 1st and 2nd order approximation of the equilibrium conditions of the model. Moreover, we show the second order approximation of the utility function in order to address welfare analysis. To simplify and clarify the algebra, we use the following isoelastic functional forms:

$$U(C_t) = \frac{C_t^{1-\rho}}{1-\rho}$$

$$V(y_t(h), \varepsilon_{Y,T}) = \frac{\varepsilon_{y,t}^{-\eta} y_t(h)^{\eta+1}}{\eta+1}$$

B.1 Demand

As shown in the text, home demand equation is:

$$Y_{H,t} = \left[\frac{P_{H,t}}{P_t} \right]^{-\theta} \left[(1-\lambda)C_t + \lambda \left(\frac{1}{RS_t} \right)^{-\theta} C_t^* \right] + g_t \quad (\text{B.12})$$

Using the "steady state depend" parameters defined in Appendix A, and with home relative price defined as $P_{H,t}/P_t = p_{H,t}$, we obtain the following first order approximation to demand in the small open economy:

$$\hat{Y}_H = -\theta(1-d_g)\hat{p}_H + d_b\hat{C} + (1-d_b-d_g)\hat{C}^* + \theta(1-d_b-d_g)\widehat{RS} + \hat{g} \quad (\text{B.13})$$

Note that fiscal shock \hat{g}_t is defined as $\frac{G_t - \bar{G}}{\bar{Y}}$, allowing for the analysis of this shock even when the zero steady state government consumption is zero. In the symmetric steady state, where $d_b = 1 - \lambda$ and $d_g = 0$, the equation (B.13) becomes:

$$\hat{Y}_H = -\theta\hat{p}_H + (1-\lambda)\hat{C} + \lambda\hat{C}^* + \theta\lambda\widehat{RS} + \hat{g} \quad (\text{B.14})$$

And the second order approximation to the demand function is:

$$\begin{aligned} \hat{Y}_H = & -\theta(1-d_g)\hat{p}_H + d_b\hat{C} + (1-d_b-d_g)\hat{C}_t^* + \theta(1-d_b-d_g)\widehat{RS} + \hat{g} \\ & + \theta^2(1-d_g)d_g\frac{\hat{p}_H^2}{2} + \theta^2(1-d_b-d_g)(d_b+d_g)\frac{\widehat{RS}^2}{2} + (1-d_b)d_b\frac{\hat{C}^2}{2} \\ & - \theta d_b d_g \hat{p}_H \hat{C} + \theta(1-d_b-d_g)(d_b+d_g)\widehat{RS}\hat{C}_t^* - \theta^2 d_g(1-d_b-d_g)\hat{p}_H\widehat{RS} \\ & - \theta d_g(1-d_b-d_g)\hat{p}_H\hat{C}_t^* - \theta(1-d_b-d_g)d_b\widehat{RS}\hat{C} - (1-d_b-d_g)d_b\hat{C}_t^*\hat{C} \\ & + \theta(1-d_g)\hat{p}_H\hat{g} - d_b\hat{C}\hat{g} - (1-d_b-d_g)\hat{C}_t^*\hat{g} - \theta(1-d_b-d_g)\widehat{RS}\hat{g} + s.o.t.i.p. \end{aligned} \quad (\text{B.15})$$

In matrix notation, we have:

$$\sum \beta^t \left[d'_y y_t + \frac{1}{2} y'_t D_y y_t + y'_t D_e e_t \right] + s.o.t.i.p = 0$$

where

$$y_t = \begin{bmatrix} \hat{Y}_t & \hat{C}_t & \hat{p}_{Ht} & \hat{\tau}_t & \widehat{RS}_t \end{bmatrix}$$

$$e_t = \begin{bmatrix} \hat{\varepsilon}_{yt} & \widehat{m}c_t & \hat{g}_t & \hat{C}_t^* \end{bmatrix}$$

$$d'_y = \begin{bmatrix} -1 & d_b & -\theta(1-d_g) & 0 & \theta(1-d_b-d_g) \end{bmatrix}$$

$$D'_y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & (1-d_b)d_b & -\theta d_b d_g & 0 & -\theta(1-d_b-d_g)d_b \\ 0 & -\theta d_b d_g & \theta^2(1-d_g)d_g & 0 & -\theta^2 d_g(1-d_b-d_g) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\theta(1-d_b-d_g)d_b & -\theta^2 d_g(1-d_b-d_g) & 0 & \theta^2(1-d_b-d_g)(d_b+d_g) \end{bmatrix}$$

$$D'_e = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -d_b & -(1-d_b-d_g)d_b \\ 0 & 0 & 0 & \theta(1-d_g) & -\theta d_g(1-d_b-d_g) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\theta(1-d_b-d_g) & \theta(1-d_b-d_g)(d_b+d_g) \end{bmatrix}$$

Moreover, in the symmetric equilibrium:

$$d'_y = [-1 \quad 1 - \lambda \quad -\theta \quad \theta\lambda(1 - \lambda)^2]$$

$$D'_y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda(1 - \lambda) & 0 & 0 & -\theta\lambda(1 - \lambda) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\theta\lambda(1 - \lambda) & 0 & 0 & \theta^2\lambda(1 - \lambda) \end{bmatrix}$$

$$D'_e = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(1 - \lambda) & -\lambda(1 - \lambda) \\ 0 & 0 & 0 & \theta & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\theta\lambda & \theta\lambda(1 - \lambda) \end{bmatrix}$$

B.2 Risk Sharing Equation

In a perfectly integrated capital market, the value of the intertemporal marginal rate of substitution is equated across borders:

$$\frac{U_C(C_{t+1}^*)}{U_C(C_t^*)} \frac{P_t^*}{P_{t+1}^*} = \frac{U_C(C_{t+1})}{U_C(C_t)} \frac{S_{t+1}P_t}{S_tP_{t+1}} \quad (\text{B.16})$$

Assuming the symmetric steady state equilibrium, the log linear approximation to the above condition is:

$$\hat{C}_t^* = \hat{C}_t + \frac{1}{\rho} \widehat{RS}_t \quad (\text{B.17})$$

Given our utility function specification, equation (B.16) gives rise to a exact log liner expression and therefore the first and second order approximation are identical.

In matrix notation, we have:

$$\sum E_t \beta^t \left[c'_y y_t + \frac{1}{2} y'_t C_y y_t + y'_t C_e e_t \right] + s.o.t.i.p. = 0$$

$$c'_y = [0 \quad -1 \quad 0 \quad 0 \quad \frac{1}{\rho}]$$

$$c'_e = [0 \quad 0 \quad 0 \quad 1]$$

$$C'_y = 0$$

$$C'_e = 0$$

B.3 The Real Exchange Rate

Given our preference specification in the small open economy, and knowing that in the rest of the world $P_F = SP^*$, we can write the price level in the following form:

$$\left(\frac{P}{P_H} \right)^{1-\theta} = (1 - \lambda) + \lambda \left(RS \frac{P}{P_H} \right)^{1-\theta} \quad (\text{B.18})$$

Therefore, the first order approximation to the above expression is:

$$\tilde{p}_H = -\frac{\lambda \widehat{RS}}{1 - \lambda} \quad (\text{B.19})$$

Moreover, the second order approximation to equation (B.18) is:

$$(1 - \lambda)\hat{p}_H = -\lambda\widehat{RS} + \lambda(\theta - 1) \left[(1 - \lambda(1 - \lambda))\frac{\widehat{RS}^2}{2} - \widehat{RS}\widehat{p}_H \right] \quad (\text{B.20})$$

In matrix representation, we have:

$$\begin{aligned} \sum E_t \beta^t \left[f'_y y_t + \frac{1}{2} y'_t F_y y_t + y'_t F_e e_t \right] + s.o.t.i.p. &= 0 \\ f'_y &= [0 \quad 0 \quad -(1 - \lambda) \quad 0 \quad -\lambda] \\ F_y &= \lambda(\theta - 1) \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & (1 - \lambda(1 - \lambda)) \end{bmatrix} \end{aligned}$$

B.4 Exogenous Foreign Dynamics

$$\widehat{\pi}_t^* = k \left(\rho \widehat{C}_t^* + \eta \widehat{C}_t^* + \widehat{m}c_t^* + \omega \widehat{\tau}_t^* - \eta \widehat{\varepsilon}_{Y,t}^* \right) + \beta E_t \widehat{\pi}_{t+1}^* \quad (\text{B.21})$$

And the dynamic of the flow budget constraint for the foreign government is given by:

$$D_{t+1}^* = D_t^* (1 + i_{t-1}^*) - P_t^* s_t^*$$

where

$$s_t^* \equiv \tau_t^* C_t^* - G_t^* - Tr_t^*$$

And the system is closed by specifying a monetary and a fiscal rule that will determine the variables of interest, i.e. C_t^* and $\widehat{\pi}_t^*$. Note that in general a structural exogenous disturbance might affect both C_t^* and $\widehat{\pi}_t^*$.

B.5 Price Setting

The first and second-order approximation to the price setting equation follow Benigno and Benigno (2003)⁹. The introduction of the tax component is done in the same manner as in Benigno and Woodford (2003). They are derived from the following first order condition of sellers that can reset their prices:

$$E_t \left\{ \sum (\alpha\beta)^{T-t} U_c(C_T) \left(\frac{\tilde{p}_t(h)}{P_{H,t}} \right)^{-\sigma} Y_{H,T} \left[\frac{\tilde{p}_t(h) P_{H,T}}{P_{H,T} P_T} - \frac{\sigma m_t V_y(\tilde{y}_{t,T}(h), \varepsilon_{Y,t})}{(1 - \sigma)(1 - \tau_t) U_c(C_T)} \right] \right\} = 0 \quad (\text{B.22})$$

where

$$\tilde{y}_t(h) = \left(\frac{\tilde{p}_t(h)}{P_{H,t}} \right)^{-\sigma} Y_{H,t} \quad (\text{B.23})$$

and

$$(P_{H,t})^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1 - \alpha) (\tilde{p}_t(h))^{1-\sigma} \quad (\text{B.24})$$

and mc_t is a markup shock, and income taxes are represented by τ_t .

We can write the second order approximation for equation (B.22) as follows:

⁹For a detail derivation of the first-order approximation to the price setting see technical appendix in Benigno and Benigno (2001). Benigno and Benigno (2003) have the details on the second-order approximation.

$$V_o = E_0 \left\{ \sum_t \beta^t z_t + \frac{1}{2} z_t X_t + \frac{1}{2} \frac{\sigma(1+\eta)}{k} \pi_t^{H2} \right\} + s.o.t.i.p. \quad (\text{B.25})$$

where:

$$z_t = \eta \widehat{Y}_t + \rho \widehat{C}_t - \widehat{p}_H + \widehat{m}_t - \widehat{q}_t - \eta \widehat{\varepsilon}_{Y,t}$$

$$X_t = (2 + \eta) \widehat{Y}_t - \rho \widehat{C}_t + \widehat{p}_H + \widehat{m}_t + \widehat{q}_t - \eta \widehat{\varepsilon}_{Y,t}$$

we define $q_t = 1 - \tau_t$ and therefore we have:

$$\widehat{q}_t = -\omega \widehat{\tau}_t - \frac{1}{2} \frac{\omega}{1 - \bar{\tau}} \widehat{\tau}_t^2$$

where $\omega = \frac{\bar{\tau}}{1 - \bar{\tau}}$

The first order approximation to the price setting equation can be written in the following way:

$$\widehat{\pi}_t^H = k \left(\rho \widehat{C}_t + \eta \widehat{Y}_t - \widehat{p}_H + \widehat{m}_t + \omega \widehat{\tau}_t - \eta \widehat{\varepsilon}_{Y,t} \right) + \beta E_t \widehat{\pi}_{t+1}^H \quad (\text{B.26})$$

where $k = (1 - \alpha\beta)(1 - \alpha)/\alpha(1 + \sigma\eta)$.

And the second order approximation to the price setting can be written as follows:

$$Q_{to} = \phi \sum E_t \beta^t \left[a'_y y_t + \frac{1}{2} y'_t A_y y_t + y'_t A_e e_t \right] + s.o.t.i.p. \quad (\text{B.27})$$

with

$$a'_y = \begin{bmatrix} \eta & \rho & -1 & \omega & 0 \end{bmatrix}$$

$$A'_y = \begin{bmatrix} \eta(2 + \eta) & \rho & -1 & \omega & 0 \\ \rho & -\rho^2 & \rho & -\rho\omega & 0 \\ -1 & \rho & -1 & \omega & 0 \\ \omega & -\rho\omega & \omega & \omega & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A'_e = \begin{bmatrix} -\eta(1 + \eta) & 1 + \eta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

B.6 Government Budget Constraint

We assume at first that all public debt consists of riskless nominal one-period bonds. We have that the nominal value D_t follows the law of motion:

$$D_t = D_{t-1}(1 + i_{t-1}) - P_{H,t} s_t$$

where

$$s_t \equiv \tau_t Y_t - G_t - Tr_t$$

Now define

$$\frac{D_t(1 + i_t)}{P_t} \equiv d_t$$

So that we can rewrite the government budget constraint as

$$d_t = d_{t-1} \frac{(1 + i_t)}{\Pi_t} + \frac{P_{H,t}}{P_t} s_t (1 + i_t)$$

Let's define $p_{H,t} \equiv \frac{P_{H,t}}{P_t}$. In log-linear terms the government budget constraint is given by

$$\beta \hat{d}_t = \hat{d}_{t-1} + \frac{\beta s^{-1}}{1-\beta} i_t - s^{-1} \hat{p}_{H,t} - s_\tau s^{-1} (\hat{Y}_t + \hat{\tau}_t) + \hat{g}_t + \hat{T}r - \frac{s^{-1}}{1-\beta} (\hat{\pi}_t^H + \lambda/(1-\lambda)\Delta R S_t)$$

In order to derive a second order approximation to the intertemporal government solvency condition we define:

$$W_t = E_t \sum_{T=t}^{\infty} U_C(C_T, \xi_{C,T}) s_t p_{H,t}$$

with

$$W_t = \frac{d_{t-1}}{\Pi_t} U_C(C_T, \xi_{C,T})$$

where

$$s_t \equiv \tau_t Y_t - G_t - T r_t \quad (\text{B.28})$$

$$U_C(C_T, \xi_{C,T}) s_t p_{H,t} = U_c \bar{s} (1 - \rho \hat{C} + \hat{p}_H + \bar{s}^{-1} \tilde{s} + \frac{1}{2} \rho^2 \hat{C}^2 + \frac{1}{2} \hat{p}_H^2 - \rho \bar{s}^{-1} \tilde{s} \hat{C} + \bar{s}^{-1} \tilde{s} \hat{p}_H) \quad (\text{B.29})$$

$$\tilde{s} = \tilde{\tau} \bar{Y} + \tilde{Y} \bar{\tau} - \tilde{G} - \tilde{T}r + \tilde{\tau} \tilde{Y}$$

$$\bar{s}^{-1} \tilde{s} = \bar{s}^{-1} \bar{\tau} \bar{Y} (\hat{\tau} + \hat{Y}) + \bar{s}^{-1} \bar{\tau} \bar{Y} (\hat{\tau} + \hat{Y})^2 - \bar{s}^{-1} \bar{Y} \hat{g} + s.o.t.i.p$$

Where \hat{g} is defined as $\frac{G_t - \bar{G}}{\bar{Y}}$ to allow for zero steady state values of government consumption.

Defining $\bar{s}^{-1} \bar{Y} = s$ and $\bar{s}^{-1} \bar{\tau} \bar{Y} = s_\tau$, we have:

$$\bar{s}^{-1} \tilde{s} = s_\tau (\hat{\tau} + \hat{Y}) + s_\tau (\hat{\tau} + \hat{Y})^2 - s \hat{g} + s.o.t.i.p \quad (\text{B.30})$$

Note that, if $\bar{G} = 0$, $\bar{s} = \bar{\tau} \bar{Y}$

Substituting (B.30) into (B.29), we have:

$$U_C(C_T, \xi_{C,T}) s_t p_{H,t} = U_c \bar{s} \left\{ \begin{array}{l} 1 - \rho \hat{C} + \hat{p}_H + [s_\tau (\hat{\tau} + \hat{Y}) + s_\tau (\hat{\tau} + \hat{Y})^2 - s \hat{g}] \\ + \frac{1}{2} \rho^2 \hat{C}^2 + [s_\tau (\hat{\tau} + \hat{Y}) + s_\tau (\hat{\tau} + \hat{Y})^2 - s \hat{g}] (-\rho \hat{C} + \hat{p}_H) \end{array} \right\} \\ U_c \bar{s} \left\{ \begin{array}{l} 1 + s_\tau \hat{Y} - \rho \hat{C} + \hat{p}_H + s_\tau \hat{\tau} + \frac{1}{2} s_\tau \hat{Y}^2 - \rho s_\tau \hat{Y} \hat{C} + s_\tau \hat{Y} \hat{p}_H \\ + s_\tau \hat{\tau} \hat{Y} + \frac{1}{2} \rho^2 \hat{C}^2 + \frac{1}{2} \hat{p}_H^2 + s_\tau \hat{\tau}^2 \\ - \rho s_\tau \hat{\tau} \hat{C} + s_\tau \hat{\tau} \hat{p}_H + \rho s \hat{g} \hat{C} - s \hat{p}_H \hat{g} \end{array} \right\}$$

Therefore, defining $\widetilde{W}_t = \frac{W_t - \bar{W}}{\bar{W}}$, we have:

$$\widetilde{W}_t = (1 - \beta) \left[b'_y y_t + \frac{1}{2} y'_t B_y y_t + y'_t B_e e_t \right] + \beta E_t \widetilde{W}_{t+1} + s.o.t.i.p.$$

$$b'_y = [s_\tau \quad -\rho \quad 1 \quad s_\tau \quad 0]$$

$$B'_y = \begin{bmatrix} s_\tau & -\rho s_\tau & s_\tau & s_\tau & 0 \\ -\rho s_\tau & \rho^2 & 0 & -\rho s_\tau & 0 \\ s_\tau & 0 & 1 & s_\tau & 0 \\ s_\tau & -\rho s_\tau & s_\tau & s_\tau & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B'_e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \rho s & 0 \\ 0 & 0 & -s & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Moreover, we have that

$$\widetilde{W}_t = -\rho\widehat{C}_t + \widehat{d}_{t-1} - \widehat{\pi}_t = -\rho\widehat{C}_t + \widehat{d}_{t-1} - \left(\frac{\lambda}{1-\lambda}\Delta RS_t + \widehat{\pi}_t^H\right)$$

Note that

$$\Delta RS_t = \Delta S_t + \pi_t^* - \pi_t$$

and

$$\pi_t = (1-\lambda)\widehat{\pi}_t^H + \lambda(\Delta S_t + \pi_t^*)$$

Substituting we get:

$$\pi_t = \widehat{\pi}_t^H + \frac{\lambda}{1-\lambda}\Delta RS_t$$

The first order approximation of the intertemporal budget constraint is:

$$\begin{aligned} -\rho\widehat{C}_t + \widehat{d}_{t-1} - \left(\frac{\lambda}{1-\lambda}\Delta RS_t + \widehat{\pi}_t^H\right) &= (1-\beta)(-\rho\widehat{C}_t + \widehat{p}_{H,t} + s_\tau(\widehat{\tau}_t + \widehat{Y}_t) - \widehat{g}_t) \\ &\quad + \beta E_t \left[-\rho\widehat{C}_{t+1} + \widehat{d}_t - \left(\frac{\lambda}{1-\lambda}\Delta RS_{t+1}^* + \widehat{\pi}_{t+1}^H\right) \right] \end{aligned}$$

Throughout the text we also use a different representation of the budget constraint where $\widehat{d}'_t = d_{ss}\widehat{d}_t$:

$$\begin{aligned} -\rho d_{ss}\widehat{C}_t + \widehat{d}'_{t-1} - \left(\frac{\lambda}{1-\lambda}d_{ss}\Delta RS_t + d_{ss}\widehat{\pi}_t^H\right) &= (1-\beta)d_{ss}(-\rho\widehat{C}_t + \widehat{p}_{H,t}) + \bar{\tau}(\widehat{\tau}_t + \widehat{Y}_t) - \widehat{g}_t \\ &\quad + \beta E_t d_{ss} \left[-\rho\widehat{C}_{t+1} + \widehat{d}'_t - \left(\frac{\lambda}{1-\lambda}\Delta RS_{t+1}^* + \widehat{\pi}_{t+1}^H\right) \right] \end{aligned}$$

Note that $s_\tau = \frac{\bar{\tau}}{d_{ss}(1-\beta)}$. The two equations are identical, but the second one is rescaled in order to allow for a zero steady state government debt.

We now turn to the characterization of the **Government Budget Constraint in the case of Real Bonds**:

In this case we assume at first that all public debt consists of riskless real one-period bonds (RD_t)

$$D_t^r = D_{t-1}^r(1+r_{t-1}) - p_{H,t}s_t \quad (\text{B.31})$$

where

$$s_t \equiv \tau_t Y_t - G_t - T r_t \quad (\text{B.32})$$

Now define

$$D_{t-1}^r(1+r_{t-1}) \equiv d_{t-1}^r \quad (\text{B.33})$$

So that we can rewrite the government budget constraint as

$$d_t^r = d_{t-1}^r(1+r_t) - p_{H,t}s_t(1+r_t) \quad (\text{B.34})$$

given that

$$U_C(C_t) = (1+r_t)\beta E_t[U_C(C_{t+1})] \quad (\text{B.35})$$

$$d_t^r \beta E_t U_C(C_{t+1}) = d_{t-1}^r U_C(C_t) - p_{H,t}s_t U_C(C_t) \quad (\text{B.36})$$

In recursive formulation:

$$RW_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} U_C(C_T) s_t p_{H,t} \quad (\text{B.37})$$

with

$$RW_t = d_{t-1}^r U_C(C_t) \quad (\text{B.38})$$

The approximation of $U_C(C_T) s_t p_{H,t}$ follows the same steps as in the previous section. Defining $\widetilde{RW}_t = \frac{RW_t - \overline{RW}}{\overline{RW}}$, we have:

$$\widetilde{RW}_t = (1 - \beta) \left[r b'_y y_t + \frac{1}{2} y'_t R B_y y_t + y'_t R B_e e_t \right] + \beta E_t \widetilde{RW}_{t+1} + s.o.t.i.p. \quad (\text{B.39})$$

$$r b'_y = \begin{bmatrix} s_\tau & -\rho & 1 & s_\tau & 0 \end{bmatrix}$$

$$R B'_y = \begin{bmatrix} s_\tau & -\rho s_\tau & s_\tau & s_\tau & 0 \\ -\rho s_\tau & \rho^2 & 0 & -\rho s_\tau & 0 \\ s_\tau & 0 & 1 & s_\tau & 0 \\ s_\tau & -\rho s_\tau & s_\tau & s_\tau & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R B'_e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \rho s & 0 \\ 0 & 0 & -s & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

However, we have that

$$RW_t = d_{t-1}^r U_C(C_t) \quad (\text{B.40})$$

Therefore the evolution of \widetilde{RW}_t do not depend on inflation:

$$\widetilde{RW}_t = -\rho \widehat{C}_t + \widehat{d}_{t-1}^r = -\rho \widehat{C}_t + \widehat{d}_{t-1}^r \quad (\text{B.41})$$

The first order approximation of the intertemporal budget constraint is:

$$\begin{aligned} -\rho \widehat{C}_t + \widehat{d}_{t-1}^r &= (1 - \beta)(-\rho \widehat{C}_t + \widehat{p}_{H,t} + s_\tau(\widehat{\tau}_t + \widehat{Y}_t) - s \widehat{g}_t) \\ &+ \beta E_t \left[-\rho \widehat{C}_{t+1} + \widehat{d}_t^r \right] \end{aligned} \quad (\text{B.42})$$

Therefore, we conclude that the first order approximation to the budget constraint changes and so does the constraint of the policy problem. However, the welfare function just depend on $r b'_y$, $R B'_y$ and $R B'_e$ which are equal to b'_y , B'_y and B'_e .

We now turn to the characterization of the **Government Budget Constraint in the case of Foreign Currency denominated Bonds**

Total government debt is issued in the form of foreign currency denominated D_t^* bonds.

$$S_t D_t^f = S_t D_{t-1}^f (1 + i_{t-1}^f) - P_{H,t} s_t \quad (\text{B.43})$$

From the household's optimal choices:

$$U_C(C_t) = (1 + i_t^f) \beta E_t \left[\frac{U_C(C_{t+1}) e_t}{\Pi_t} \right] \quad (\text{B.44})$$

where

$$\Pi_t = \frac{P_t}{P_{t-1}} \quad (\text{B.45})$$

$$e_t = \frac{S_t}{S_{t-1}} \quad (\text{B.46})$$

Now define

$$\frac{S_{t-1}D_{t-1}^f(1+i_{t-1}^f)}{P_{t-1}} \equiv d_{t-1}^f \quad (\text{B.47})$$

So that we can rewrite the government budget constraint as

$$d_t^f = d_{t-1}^f \frac{(1+i_t^f)e_t}{\Pi_t} - p_{H,t}s_t(1+i_t^*) \quad (\text{B.48})$$

$$\beta E_t \left[\frac{U_C(C_{t+1})e_t}{\Pi_t} \right] d_t^f = d_{t-1}^f \frac{U_C(C_t)e_t}{\Pi_t} - p_{H,t}s_t U_C(C_t) \quad (\text{B.49})$$

In order to derive a second order approximation to the intertemporal government solvency condition we define:

$$FW_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} U_C(C_T) s_t p_{H,t} \quad (\text{B.50})$$

with

$$FW_t = \frac{d_{t-1}^f e_t}{\Pi_t} U_C(C_T) \quad (\text{B.51})$$

Therefore the evolution of \widetilde{FW}_t depend real devaluation rate:

$$\widetilde{FW}_t = -\rho \widehat{C}_t + \widehat{d}_{t-1}^f + \Delta R S_t - \pi_t^* \quad (\text{B.52})$$

The first order approximation of the intertemporal budget constraint is:

$$\begin{aligned} -\rho \widehat{C}_t + \widehat{d}_{t-1}^f + \Delta R S_t - \pi_t^* &= (1-\beta)(-\rho \widehat{C}_t + \widehat{p}_{H,t} + s_\tau(\widehat{\tau}_t + \widehat{Y}_t) - s\widehat{g}_t) \\ &+ \beta E_t \left[-\rho \widehat{C}_{t+1} + \widehat{d}_t^f + \Delta R S_{t+1} - \pi_{t+1}^* \right] \end{aligned} \quad (\text{B.53})$$

We should note that in this case the second order moments of the budget constraint are also identical to the case of nominal and real bonds. Therefore, only the linear terms in the budget constraint depend on the characteristics of the bonds.

Morover, we can write the budget constraint as follows:

$$\begin{aligned} -\rho \widehat{C}_t + \widehat{d}_{t-1} - (a\Delta R S_t + b\widehat{\pi}_t^H) &= (1-\beta)\left(-\rho \widehat{C}_t - \frac{\lambda}{1-\lambda} R S_t + s_\tau(\widehat{\tau}_t + \widehat{Y}_t) - s\widehat{g}_t\right) \\ &+ \beta E_t \left[-\rho \widehat{C}_{t+1} + \widehat{d}_t - (a\Delta R S_{t+1} + b\widehat{\pi}_{t+1}^H) \right] \end{aligned} \quad (\text{B.54})$$

where: $a = \lambda/(1-\lambda)$ and $b = 1$ in the case of nominal bonds; $a = b = 0$ in the case of real bonds and; $a = -1$ and $b = 0$ in the case of foreign currency denominated bonds.

Alternatively, using $\widehat{d}_t^f = d_{ss}\widehat{d}_t$ we obtain equation (15) of the text.

B.7 Welfare

Following Benigno and Benigno (2003), the second order approximation to the utility function can be written as:

$$U_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} [U(C_s^j) - V(y_s^j, \xi_{Y,s})] \quad (\text{B.55})$$

$$W_{t0} = U_c \bar{C} E_{t0} \sum \beta^t \left[w'_y y_t - \frac{1}{2} y'_t W_y y_t - y'_t W_e e_t - \frac{1}{2} w'_\pi \pi_t^2 \right] + s.o.t.i.p \quad (\text{B.56})$$

$$w'_\pi = \frac{\sigma}{\mu k}$$

$$w'_y = [-1/\mu \quad 1 \quad 0 \quad 0 \quad 0]$$

$$W'_y = \begin{bmatrix} \mu(1+\eta) & 0 & 0 & 0 & 0 \\ 0 & -(1-\rho) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$W'_e = \begin{bmatrix} \frac{\eta}{\mu} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Using the second order approximation of the equilibrium condition, we can eliminate the term $w'_y y_t$. Do so, we will derive the vector Lx , such that:

$$[a_x \quad d_x \quad f_x \quad c_x \quad b_x] Lx = w_y$$

Giving the values of a_y, b_y, c_y, d_y , defined in this appendix, we have:

Note: these parameters were derived under the special case where there is a symmetric steady state ($G = 0$).

Then we will have

$$Lx_1 = \frac{((-1+\phi)(1-\theta\rho)\lambda^2 + (-(-1+\phi)2\theta\rho - 1 + 2\phi)\lambda - \phi)(-1+\tau)}{\Omega}$$

$$Lx_2 = \frac{\Sigma\rho(-1+\phi) + (\Psi(-1+\lambda))}{\Omega}$$

$$Lx_3 = \frac{(-1+\phi)\Sigma(1-\theta\rho)\lambda - \theta(-1+\phi)\Sigma\rho + \Psi\theta + \phi\Sigma}{\Omega}$$

$$Lx_5 = \frac{(-1+\phi)(1-\theta\rho)\lambda^2 + (-(-1+\phi)2\theta\rho - 1 + 2\phi)\lambda - \phi}{\Omega}$$

where:

$\Psi = ((\eta+1)\tau - \eta)$, $\Sigma = (-1 + \tau - d_{ss} + d_{ss}\beta)$, $\Omega = -\Psi l - \Sigma\rho - \Psi$, $l = (\rho\theta - 1)\lambda(2 - \lambda)$ and $1 - \phi = 1/\mu$.

And the loss function L_{t0} will have the following form:

$$L_{t0} = U_c \bar{C} E_{t0} \sum \beta^t \left[\frac{1}{2} y'_t L_y y_t + y'_t L_e e_t + \frac{1}{2} l'_\pi \pi_t^2 \right] + s.o.t.i.p \quad (\text{B.57})$$

where:

$$L_y = W_y + Lx_1A_y + Lx_2D_y + Lx_3F_y + Lx_5B_y$$

$$L_e = W_e + Lx_1A_e + Lx_2D_e$$

$$L_\pi = w_\pi + Lx_1a_\pi$$

Note that Lx_4 is irrelevant since $C_y = 0$

To write the model just in terms of the output, real exchange rate, taxes and inflation, we define the matrixes N and N_e mapping all endogenous variables into $[Y_t, T_t]$ and the errors in the following way:

$$y'_t = N [Y_t, RS_t, \tau_t] + N_e e_t \quad (\text{B.58})$$

$$N = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -\frac{l+\lambda}{\rho(1-\lambda)} & 0 \\ 0 & -\frac{\lambda}{(1-\lambda)} & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$N_e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Equation (B.57) can therefore be expressed as:

$$L_{t_0} = U_c \tilde{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} [\hat{Y}_t, \widehat{RS}_t, \hat{\tau}_t]' L'_y [\hat{Y}_t, \widehat{RS}_t, \hat{\tau}_t] + [\hat{Y}_t, \widehat{RS}_t, \hat{\tau}_t]' L'_e e_t + \frac{1}{2} l_\pi \pi_t^2 \right] + t.i.p \quad (\text{B.59})$$

where:

$$L'_y = N' L_y N$$

$$L'_e = N' L_y N_e + N' L_e$$

The last step is to eliminate the cross variables terms $\hat{Y}_t \widehat{RS}_t$. For that we use the following identity (derived from combining the demand function with the risk sharing condition):

$$2\hat{Y}_t \widehat{RS}_t = \frac{\rho(1-\lambda)}{(1+l)} \hat{Y}_t^2 + \frac{(1+l)}{\rho(1-\lambda)} \widehat{RS}_t^2 + s.o.t.i.p. \quad (\text{B.60})$$

Therefore

$$\begin{aligned} [\hat{Y}_t, \widehat{RS}_t, \hat{\tau}_t]' L'_y [\hat{Y}_t, \widehat{RS}_t, \hat{\tau}_t] &= [\hat{Y}_t, \widehat{RS}_t, \hat{\tau}_t]' \begin{bmatrix} l_{yy} & l_{yt} & 0 \\ l_{yr} & l_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} [\hat{Y}_t, \widehat{RS}_t, \hat{\tau}_t] \\ &= (l_{yy} + \frac{\rho(1-\lambda)}{(1+l)} l_{yt}) \hat{Y}_t^2 + (l_{tt} + \frac{(1+l)}{\rho(1-\lambda)} l_{yr}) \widehat{RS}_t^2 \end{aligned} \quad (\text{B.61})$$

Substituting (B.61) into (B.59), we have:

$$L_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \left[\begin{array}{c} \frac{1}{2}(l_{yy} + \frac{\rho(1-\lambda)}{(1+l)}l_{yt})\widehat{Y}_t^2 + (l_{tt} + \frac{(1+l)(1-\lambda)}{\rho}l_{yr})\widehat{RS}_t^2 \\ + [\widehat{Y}_t, \widehat{RS}_t]' L'_e e_t + \frac{1}{2}l_\pi \pi_t^2 \end{array} \right] + s.o.t.i.p \quad (B.62)$$

Finally, we rewrite the previous equation as deviations from the target variables:

$$L_{to}^i = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2}\Phi_Y(\widehat{Y}_t - \widehat{Y}_t^T)^2 + \frac{1}{2}\Phi_T(\widehat{RS}_t - \widehat{RS}_t^T)^2 + \frac{1}{2}\Phi_\pi(\widehat{\pi}_t^H)^2 \right] + s.o.t.i.p \quad (B.63)$$

where:

$$\begin{aligned} \Phi_Y &= Lx5 \left\{ -2\rho\tau + \rho^2 d_{ss} (1 - \beta) + \tau + \frac{[\rho\tau - \rho^2 d_{ss} (1 - \beta)] (l + \lambda) - \tau\lambda\rho}{1 + l} \right\} \\ &+ Lx2 \left\{ (1 - \lambda)\lambda + \frac{(-\lambda)(l + \lambda)(1 - \lambda) - \theta\lambda(1 - \lambda)^2 \rho}{1 + l} \right\} \\ &+ Lx1 \left\{ 2\rho - \rho^2 + (2 + \eta)\eta + \frac{-(\rho - \rho^2)(l + \lambda) - (-1 + \rho)\lambda\rho}{1 + l} \right\} \\ &+ (\eta + 1)(1 - \phi) - 1 - \frac{(-1 + \rho)(l + \lambda)}{1 + l} + \rho^2 \end{aligned}$$

$$\begin{aligned} \Phi_{RS} &= Lx5 \left\{ \frac{d_{ss}(1 - \beta) [(l + \lambda)^2 + \lambda^2 - (1 + l)(l + \lambda)] + \frac{1}{\rho}(1 + l)l\tau}{(1 - \lambda)^2} \right\} \\ &+ Lx3 \left\{ \frac{2\lambda^2(\theta - 1)}{1 - \lambda} + \lambda(\theta - 1)(1 - (1 - \lambda)\lambda) \right\} \\ &+ Lx2 \left\{ \frac{(l + \lambda)\lambda\left(\theta - \frac{1}{\rho}\right) - \lambda\theta(1 - \lambda)}{\rho} + \theta^2\lambda(1 - \lambda) \right\} \\ &+ Lx1 \left\{ \frac{l(l + 2\lambda) + \frac{1+l}{\rho}l(\rho - 1)}{(1 - \lambda)^2} \right\} \\ &- \frac{(l + \lambda)(-1 + \rho)}{(1 - \lambda)\rho^2} \end{aligned}$$

$$\Phi_\pi = \frac{\sigma(1 - \phi)}{k} + (1 + \eta)\frac{\sigma}{k}Lx1$$

and

$$\widehat{Y}_t^T = q_y^e e_t$$

$$\widehat{RS}_t^T = q_{rs}^e e_t$$

with

$$q_y^e = \frac{1}{\Phi_T} \left[\begin{array}{cccc} \frac{\eta}{\mu} + Lx1(1 + \eta)\eta & -Lx1(1 + \eta) & -q_{yg}^e & Lx2(1 - \lambda)\lambda \end{array} \right]$$

$$\begin{aligned} q_{yg}^e &= Lx5 \{ \rho(1 + \tau) - \rho^2 d_{ss}(1 - \beta) \} \\ &+ Lx1 \{ \rho(\rho - 1) \} \\ &+ Lx2 \{ \lambda^2 - 1 \} \\ &+ 1 - \rho \end{aligned}$$

$$q_t^e = \frac{1}{\Phi_T} \left[\begin{array}{cccc} 0 & 0 & -q_{rg}^e & -Lx_2 \left\{ \frac{(l+\lambda)\lambda}{\rho} + \theta\lambda(1-\lambda) \right\} \end{array} \right]$$

$$q_{rg}^e = Lx_5 \left\{ \frac{(l+\lambda)\rho dss(1-\beta) - l}{1-\lambda} \right\}$$

$$+ Lx_2 \left\{ \frac{(l+\lambda)(1+\lambda)}{\rho} - \frac{\lambda\theta}{1-\lambda} - \theta\lambda^2 \right\}$$

$$+ Lx_1 \left\{ \frac{-l\rho}{1-\lambda} \right\}$$

$$+ \frac{(l+\lambda)(-1+\rho)}{\rho(1-\lambda)}$$

B.7.1 Special Cases

Special Case 1: $\rho = \theta = 1$

$$Lx_1 = \frac{-(\lambda - \phi)}{(\eta + 1)} \quad (\text{B.64})$$

$$Lx_2 = (1 - \lambda) + \frac{\rho(\lambda - \phi)}{(1 - \tau)(\eta + 1)} \quad (\text{B.65})$$

$$Lx_3 = -\theta \quad (\text{B.66})$$

$$Lx_5 = \frac{\omega(\lambda - \phi)}{(\eta + 1)}$$

$$\Phi_Y = (\eta + 1)(1 - \lambda) + \frac{\lambda\omega(\lambda - \phi)}{(\eta + 1)}$$

$$\Phi_{RS} = \frac{\lambda\omega(2\lambda - 1)(\lambda - \phi)}{(\eta + 1)(1 - \lambda)^2\rho}$$

$$\Phi_\pi = (1 - \lambda)\frac{\sigma}{k}$$

Note that we can always use the following identity:

$$\widehat{Y}_t = \frac{(1+l)}{\rho(1-\lambda)} \widehat{RS}_t + \widehat{g}_t + \widehat{C}_t^*$$

Moreover we can see that, if there are no monopolistic distortions $\lambda = \phi$, or the economy is closed $\lambda = 0$, or no steady state taxes $\omega = 0$:

Special case 1.2: $\rho = \theta = 1$ and $\lambda\omega(\lambda - \phi) = 0$

$$\frac{\Phi_Y}{1-\lambda} = (\eta + 1)$$

$$\frac{\Phi_{RS}}{1-\lambda} = 0$$

$$\frac{\Phi_\pi}{1-\lambda} = \frac{\sigma}{k}$$

Moreover, in the case $\lambda = \phi$, we have:

$$\widehat{Y}_t^T = q_y^e e_t$$

with

$$q_y^e = \frac{1}{1+\eta} \begin{bmatrix} \eta & 0 & -1 & 0 \end{bmatrix}$$

$$\widehat{RS}_t^T = 0$$

Special Case 2: $\rho\theta = 1$

$$Lx_1 = (1-\tau) \frac{-(\lambda-\phi)}{(\rho+\eta)(1-\tau) + \tau(\rho-1)} \quad (\text{B.67})$$

$$Lx_2 = (1-\lambda) + \frac{\rho(\lambda-\phi)}{(\rho+\eta)(1-\tau) + \tau(\rho-1)} \quad (\text{B.68})$$

$$Lx_3 = -\theta \quad (\text{B.69})$$

$$Lx_5 = \tau \frac{(\lambda-\phi)}{(\rho+\eta)(1-\tau) + \tau(\rho-1)}$$

$$\Phi_Y = \frac{(1-\lambda)(\rho+\eta) + (\lambda-\phi)(\rho-1)}{(\rho+\eta)(1-\tau) + \tau(\rho-1)} + \frac{\tau(\lambda-\phi) [\lambda\rho^2 + (\rho-1)(1+\eta)]}{(\rho+\eta)(1-\tau) + \tau(\rho-1)}$$

$$\Phi_{RS} = \frac{\lambda^2(2-\lambda)\lambda(\rho-1)}{(1-\lambda)\rho^2} + \frac{\tau(\lambda-\phi)\lambda(2\lambda-1)}{(1-\lambda)^2 [(\rho+\eta)(1-\tau) + \tau(\rho-1)]}$$

$$\Phi_\pi = \frac{\sigma(1-\tau)(\eta+1)(1-\lambda) + (\rho-1)(1-\phi)}{k((\rho+\eta)(1-\tau) + \tau(\rho-1))}$$

Special Case 2.2: $\rho\theta = 1$ and $\phi = \lambda$

$$\frac{\Phi_Y}{(1-\lambda)} = \eta + \rho$$

$$\frac{\Phi_{RS}}{(1-\lambda)} = \frac{(\rho-1)\lambda(2-\lambda)}{\rho^2} \left[\frac{\lambda^2}{(1-\lambda)} \right]^2$$

$$\frac{\Phi_\pi}{(1-\lambda)} = \frac{\sigma}{k}$$

Special Case 2.3: $\rho\theta = 1$ and $\tau = 0$

$$\Phi_Y = (\eta + \rho)(1-\lambda) + (\rho-1)(\lambda-\phi)$$

$$\Phi_{RS} = \frac{\lambda^2(2-\lambda)\lambda(\rho-1)}{(1-\lambda)\rho^2}$$

$$\Phi_\pi = \frac{\sigma(\eta + \rho)(1-\lambda) + (\rho-1)(\lambda-\phi)}{k(\rho + \eta)}$$

C Appendix: Optimal Fiscal Policy under Flexible Prices

The optimal policy can be represented by the following Lagrangian:

$$\mathcal{L} = E_{t_0} \sum \beta^{t-t_0} \left[\begin{aligned} & \frac{1}{2} \Phi_Y (\hat{Y}_t - \hat{Y}_t^T)^2 + \frac{1}{2} \Phi_T (\widehat{RS}_t - \widehat{RS}_t^T)^2 + \varphi_{1,t} \left(\eta \hat{Y}_t + (1-\lambda)^{-1} \widehat{RS}_t - \omega \hat{\tau}_t \right) + \\ & + \varphi_{2,t} \left(\begin{aligned} & -d_{ss} \widehat{RS}_t + \hat{d}_{t-1} - d_{ss} (a \Delta R S_t + b \hat{\pi}_t^H) + d_{ss} (1-\beta) \left(\frac{1}{1-\lambda} \widehat{RS}_t \right) \\ & - \bar{\tau} (\hat{\tau}_t + \hat{Y}_t) + \beta E_t \left[d_{ss} \widehat{RS}_{t+1} - \hat{d}_t + d_{ss} (a \Delta R S_{t+1} + b \hat{\pi}_{t+1}^H) \right] \end{aligned} \right) \\ & + \varphi_{3,t} \left(\hat{Y}_t - \frac{(1+l)}{\rho(1-\lambda)} \widehat{RS}_t \right) \end{aligned} \right] + t.i.p$$

And these are the first order conditions:

$$-b d_{ss} (\varphi_{2,t} - \varphi_{2,t-1}) = 0 \quad (C.70)$$

$$\Phi_y (\hat{Y}_t - \hat{Y}_t^T) + \eta \varphi_{1,t} - \bar{\tau} \varphi_{2,t} + \varphi_{3,t} = 0 \quad (C.71)$$

$$\begin{aligned} & \Phi_y (\widehat{RS}_t - \widehat{RS}_t^T) + \frac{1}{(1-\lambda)} \varphi_{1,t} - d_{ss} (a+1) (\varphi_{2,t} - \varphi_{2,t-1}) \\ & + d_{ss} \frac{(1-\beta)}{(1-\lambda)} \varphi_{2,t} + \beta a d_{ss} (E_t \varphi_{2,t+1} - \varphi_{2,t}) - \frac{(1+l)}{\rho(1-\lambda)} \varphi_{3,t} = 0 \end{aligned} \quad (C.72)$$

$$\omega \varphi_{1,t} - \bar{\tau} \varphi_{2,t} = 0 \quad (C.73)$$

$$-\varphi_{2,t} + E_t \varphi_{2,t+1} = 0 \quad (C.74)$$

Moreover, the first order condition at time t_0 implies $b \varphi_{2,t_0} = 0$. Moreover, substituting we $\hat{C}_t = \hat{C}_t^* + \frac{1}{\rho} \widehat{RS}_t$ into (15), the government budget constraint in his general form can be written as

$$\begin{aligned} & -\rho d_{ss} \hat{C}_t^* - d_{ss} \widehat{RS}_t + \hat{d}_{t-1} - d_{ss} (a \Delta \widehat{RS}_t + b \hat{\pi}_t^H) = d_{ss} (1-\beta) \left(-\rho \hat{C}_t^* - \frac{1}{1-\lambda} \widehat{RS}_t \right) + \\ & + \tau (\hat{\tau}_t + \hat{Y}_t) - \hat{g}_t + \beta E_t \left[-\rho d_{ss} \hat{C}_{t+1}^* - d_{ss} \widehat{RS}_{t+1} + \hat{d}_t - d_{ss} (a \Delta \widehat{RS}_{t+1} + b \hat{\pi}_{t+1}^H) \right] \end{aligned} \quad (C.75)$$

Furthermore, under the assumption that $\alpha = 0$ we can substitute in the Phillips curve

$$-\omega^{-1} \left(\frac{\eta(1+l) + \rho}{(1+l)} \right) (\hat{Y}_t - \hat{Y}_t^T) = (\hat{\tau}_t - \hat{\tau}_t^T) \quad (C.76)$$

By integrating equation (C.75) forward we can rewrite the intertemporal budget constraint of the government as

$$\begin{aligned} & \hat{d}_{t-1} - d_{ss} b \hat{\pi}_t^H = \hat{f}_t + d_{ss} a \Delta \frac{\rho(1-\lambda)}{(1+l)} (\hat{Y}_t - \hat{Y}_t^T) + \\ & \frac{d_{ss} \rho (1-\lambda)}{1+l} (\hat{Y}_t - \hat{Y}_t^T) + \frac{(1-\beta)}{(1+l)} d_{ss} E_t \sum_{s=0}^{\infty} \beta^{t+s} \left[m (\hat{Y}_t - \hat{Y}_t^T) \right], \end{aligned} \quad (C.77)$$

where

$$m \equiv \left(\frac{\bar{\tau} (1 - \omega^{-1} \eta) (1+l)}{(1-\beta)} - d_{ss} \rho (1 + \omega^{-1}) \right)$$

and

$$\begin{aligned} \widehat{f}_t \equiv & d_{ss} a \Delta \frac{\rho(1-\lambda)}{(1+l)} \left[-\widehat{C}_t^* - \widehat{g}_t + \widehat{Y}_t^T \right] + \frac{\rho(l+\lambda)}{1+l} d_{ss} \widehat{C}_t^* - \frac{\rho(1-\lambda)}{1+l} d_{ss} g_t + \frac{\rho(1-\lambda)}{1+l} d_{ss} Y_t^T + \\ & + (1-\beta) d_{ss} E_t \sum_{s=0}^{\infty} \beta^{t+s} \left[\left(\frac{\bar{\tau}}{(1-\beta) d_{ss}} - \frac{\rho}{1+l} \right) \widehat{Y}_t^T + \frac{\bar{\tau}}{(1-\beta) d_{ss}} \widehat{\tau}_t^{T'} + \frac{\rho - \bar{s}(1+l)}{1+l} g_t - \frac{\rho l}{1+l} C_t^* \right] \end{aligned}$$

The combination of the first order implies:

$$\begin{aligned} & \frac{\Phi_T \rho(1-\lambda)}{(1+l)} (\widehat{Y}_t - \widehat{Y}_t^T - \delta_t) + \frac{(1+l)}{\rho(1-\lambda)} \Phi_Y (\widehat{Y}_t - \widehat{Y}_t^T) \\ & + m_0 \varphi_{2,t} - d_{ss} (1+a) (\varphi_{2,t} - \varphi_{2,t-1}) = 0, \end{aligned} \quad (\text{C.78})$$

where $\delta_t \equiv \widehat{C}_t^* + \widehat{g}_t - \widehat{Y}_t^T + \frac{(1+l)}{\rho(1-\lambda)} \widehat{R} S_t^T$ and $m_0 = \frac{1}{1-\lambda} \left(\bar{s} + \frac{\eta(1+l)\bar{\tau}}{\omega\rho} + \frac{\bar{\tau}}{\omega} - \frac{\bar{\tau}(1-l)}{\rho} \right)$. Alternatively we can write:

$$(\widehat{Y}_t - \widehat{Y}_t^T) = m_1 \delta_t - m_2 \varphi_{2,t} + (1+a) d_{ss} m_3 (\varphi_{2,t} - \varphi_{2,t-1}) \quad (\text{C.79})$$

where

$$\begin{aligned} m_1 &= \left(\frac{(1+l)^2 \Phi_Y + \Phi_T \rho^2 (1-\lambda)^2}{\Phi_T \rho^2 (1-\lambda)^2} \right)^{-1} \\ m_2 &= \left(\frac{(1+l)^2 \Phi_Y + \Phi_T \rho^2 (1-\lambda)^2}{\rho(1+l)} \right)^{-1} m_0 \\ m_3 &= \left(\frac{(1+l)^2 \Phi_Y + \Phi_T \rho^2 (1-\lambda)^2}{\rho(1-\lambda)(1+l)} \right)^{-1} \end{aligned}$$

Moreover, substitution equation (C.79) into equation (C.77) we have:

$$\varphi_{2,t} = \frac{-(1+l)}{(n_1+n_2)} \widehat{d}_{t-1} + \frac{1}{(n_1+n_2)} f'_t + \frac{n_1}{(n_1+n_2)} \varphi_{2,t-1} \quad (\text{C.80})$$

and using (C.74) we have

$$\varphi_{2,t} = \frac{-(1+l)}{n_2} \widehat{d}_t + \frac{1}{n_2} E_t f'_{t+1} \quad (\text{C.81})$$

where:

$$\begin{aligned} n_1 &= -((1-\beta)m - d_{ss}\rho(1-\lambda))m_3 d_{ss} \\ n_2 &= (d_{ss}\rho(1-\lambda) + m)m_2 \end{aligned}$$

Moreover, we have that:

$$\varphi_{2,t} = \frac{1}{(n_1+n_2)} (f'_t - E_{t-1} f'_t) + \varphi_{2,t-1} \quad (\text{C.82})$$

$$\widehat{d}_t = \widehat{d}_{t-1} - \frac{n_2}{(1+l)} \frac{1}{(n_1+n_2)} (f'_t - E_{t-1} f'_t) + \frac{1}{(1+l)} E_t f'_{t+1} \quad (\text{C.83})$$

Case (i) : $d_{ss} = 0$

In this case, equation (C.80) implies:

$$\varphi_{2,t} = \frac{-(1+l)}{n_2} \widehat{d}_{t-1} + \frac{1}{n_2} f'_t \quad (\text{C.84})$$

and

$$\widehat{d}_t = \widehat{d}_{t-1} + \frac{1}{(1+l)} E_t \Delta f'_{t+1} \quad (\text{C.85})$$

Case (ii) : $d_{ss} \neq 0$ and of Nominal Bonds

In this case $a = \frac{\lambda}{(1-\lambda)}$ and $b = -1$, therefore the first order condition at time t_0 implies $b\varphi_{2,t_0} = 0$ combined with (C.70) implies that $\varphi_{2,t} = 0$. In this case the first order conditions (C.70) to (C.74) can be expressed as the following targeting rule:

$$\Phi_T(\widehat{RS}_t - \widehat{RS}_t^T) + \frac{(1+l)}{\rho(1-\lambda)} \Phi_Y(\widehat{Y}_t - \widehat{Y}_t^T) = 0.$$

Moreover, in the special case where $\rho = \theta = 1$ and $\mu = 1/(1-\lambda)$

$$(\widehat{Y}_t - \widehat{Y}_t^T) = 0. \quad (\text{C.86})$$

And equation (C.77) implies

$$\widehat{d}_{t-1} - d_{ss} b \widehat{\pi}_t^H = \widehat{f}_t$$

D Appendix: Optimal Fiscal and Monetary Policy when Prices are Sticky:

In this case the Lagrangian is

$$\mathcal{L} = E_{t_0} \sum \beta^{t-t_0} \left[\begin{aligned} & \frac{1}{2} \Phi_Y (\widehat{Y}_t - \widehat{Y}_t^T)^2 + \frac{1}{2} \Phi_T (\widehat{RS}_t - \widehat{RS}_t^T)^2 + \frac{1}{2} \Phi_\pi (\widehat{\pi}_t^H)^2 \\ & + \varphi_{1,t} \left(-k^{-1} \pi_t^H + \eta \widehat{Y}_t + (1-\lambda)^{-1} \widehat{RS}_t - \omega \widehat{\tau}_t + \beta E_t \pi_t^H \right) + \\ & + \varphi_{2,t} \left(\begin{aligned} & -d_{ss} \widehat{RS}_t + \widehat{d}_{t-1} - d_{ss} (a \Delta R S_t + b \widehat{\pi}_t^H) + d_{ss} (1-\beta) \left(\frac{1}{1-\lambda} \widehat{RS}_t \right) \\ & - \bar{\tau} (\widehat{\tau}_t + \widehat{Y}_t) + \beta E_t \left[d_{ss} \widehat{RS}_{t+1} - \widehat{d}_t + d_{ss} (a \Delta R S_{t+1} + b \widehat{\pi}_{t+1}^H) \right] \end{aligned} \right) \\ & + \varphi_{3,t} \left(\widehat{Y}_t - \frac{(1+l)}{\rho(1-\lambda)} \widehat{RS}_t \right) \end{aligned} \right] + t.i.p$$

And the first order conditions are:

$$\Phi_\pi \widehat{\pi}_t^H + (\varphi_{1,t} - \varphi_{1,t-1}) k^{-1} \omega^{-1} - b d_{ss} (\varphi_{2,t} - \varphi_{2,t-1}) = 0 \quad (\text{D.87})$$

$$-\Phi_\pi \widehat{\pi}_t^H \left(\frac{k\omega}{\bar{\tau} - k\omega b d_{ss}} \right) = (\varphi_{2,t} - \varphi_{2,t-1}) \quad (\text{D.88})$$

$$\Phi_Y (\widehat{Y}_t - \widehat{Y}_t^T) + \eta \varphi_{1,t} - \bar{\tau} \varphi_{2,t} + \varphi_{3,t} = 0 \quad (\text{D.89})$$

$$\begin{aligned} & \Phi_T (\widehat{RS}_t - \widehat{RS}_t^T) + \frac{1}{(1-\lambda)} \varphi_{1,t} - d_{ss} (a+1) (\varphi_{2,t} - \varphi_{2,t-1}) \\ & + d_{ss} \frac{(1-\beta)}{(1-\lambda)} \varphi_{2,t} + \beta a d_{ss} (E_t \varphi_{2,t+1} - \varphi_{2,t}) - \frac{(1+l)}{\rho(1-\lambda)} \varphi_{3,t} = 0 \end{aligned} \quad (\text{D.90})$$

$$-\omega \varphi_{1,t} = \bar{\tau} \varphi_{2,t} \quad (\text{D.91})$$

$$-\varphi_{2,t} + E_t \varphi_{2,t+1} = 0 \quad (\text{D.92})$$

These equations imply that:

$$E_t \widehat{\pi}_{t+1}^H = 0 \quad (\text{D.93})$$

And the first order conditions can be combined as:

$$\left[\frac{(1+l)\Phi_y}{(1-\lambda)\rho} \right] \Delta \widehat{y}_t + \Phi_T \Delta \widehat{r}_{s_t} + \left[\frac{k\Phi_\pi}{(1-\bar{\tau}) + bd_{ss}k} \right] (\gamma \widehat{\pi}_t^H + d_{ss}(a+1)\widehat{\pi}_{t-1}^H) = 0 \quad (\text{D.94})$$

where $\widehat{r}_{s_t} = (\widehat{RS}_t - \widehat{RS}_t^T)$, $\widehat{y}_t = (\widehat{Y}_t - \widehat{Y}_t^T)$, and $\gamma = \left[d_{ss} \left(\frac{(1-\beta)}{(1-\lambda)} - (1+a) \right) + \left(\frac{(1+l)(\eta(1-\bar{\tau})-\bar{\tau})}{\rho(1-\lambda)} + \frac{1-\bar{\tau}}{(1-\lambda)} \right) \right]$

Moreover, combining the first order condition with the government budget constraint and the Phillips Curve leads to the following expressions:

$$\varphi_{2,t} = \frac{f'_t - E_{t-1}f'_t}{(n'_1 + n'_2)} + \varphi_{2,t-1} \quad (\text{D.95})$$

$$\widehat{d}_t = \frac{E f'_{t+1}}{1+l} - \frac{n'_2 \varphi_{2,t}}{1+l} + \frac{n_3 d_{ss}}{(1+l)} (\varphi_{2,t} - \varphi_{2,t-1}) \quad (\text{D.96})$$

$$\Phi_T \widehat{r}_{s_t} + \frac{(1+l)}{\rho(1-\lambda)} \Phi_Y \widehat{y}_t = -m_0 \varphi_{2,t} + (a+1) d_{ss} (\varphi_{2,t} - \varphi_{2,t-1}), \quad (\text{D.97})$$

where:

$$n'_1 = -((1-\beta)m + d_{ss}\rho(1-\lambda))(a+1)m_3 d_{ss} + d_{ss}m_4 + m_5$$

$$n'_2 = (d_{ss}\rho(1-\lambda) + m)m_2$$

$$n_3 = -a\rho(1-\lambda)(a+1)d_{ss}m_3$$

$$m_4 = \left[a\rho(1-\lambda)(-m_2 + (a+1)d_{ss}m_3) + b \frac{(1+l)}{\Phi_\pi} \left(\frac{\bar{\tau}}{k\omega} + bd_{ss} \right) \right]$$

$$m_5 \equiv \frac{\bar{\tau}(1+l)}{(1-\beta)} \frac{\omega^{-1}}{k} \frac{1}{\Phi_\pi} \left(\frac{\bar{\tau}}{k\omega} + bd_{ss} \right)$$

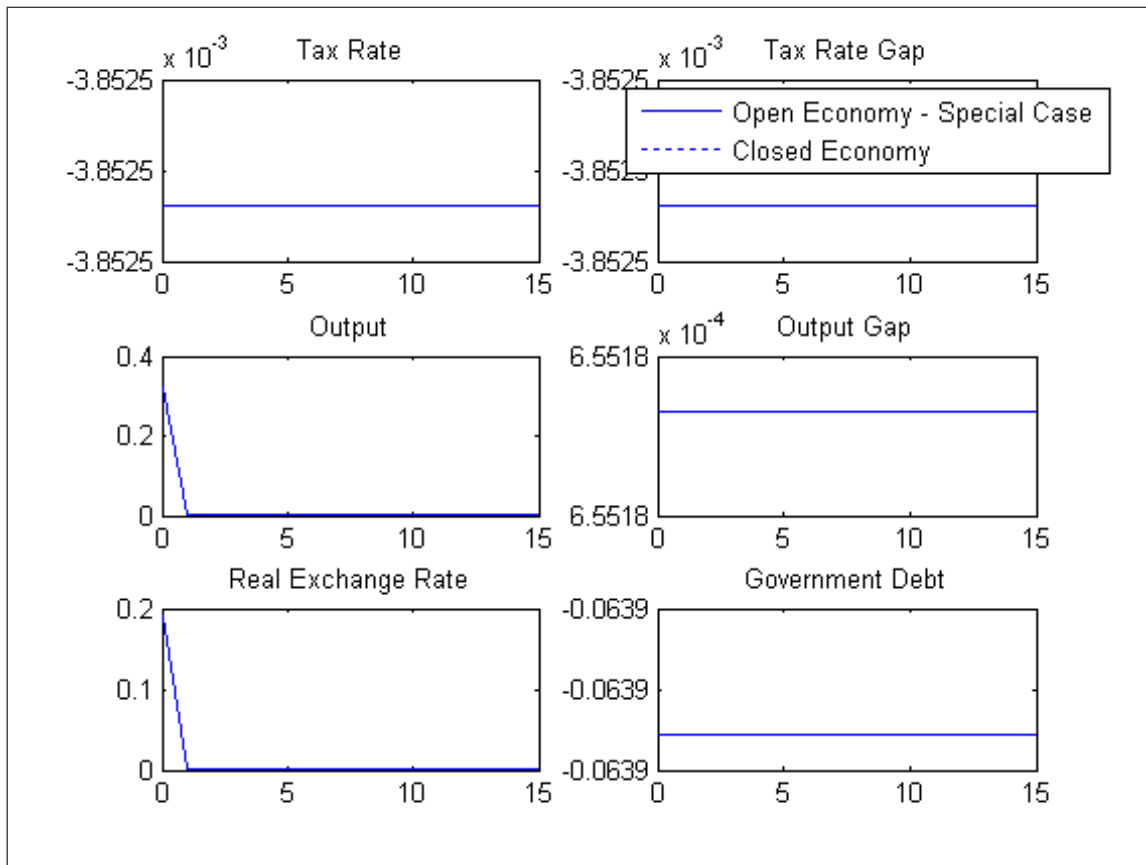


Figure 1: Impulse responses following a productivity shock- the case of flexible prices, $\rho = \theta = 1$ and $d_{ss} = 0$

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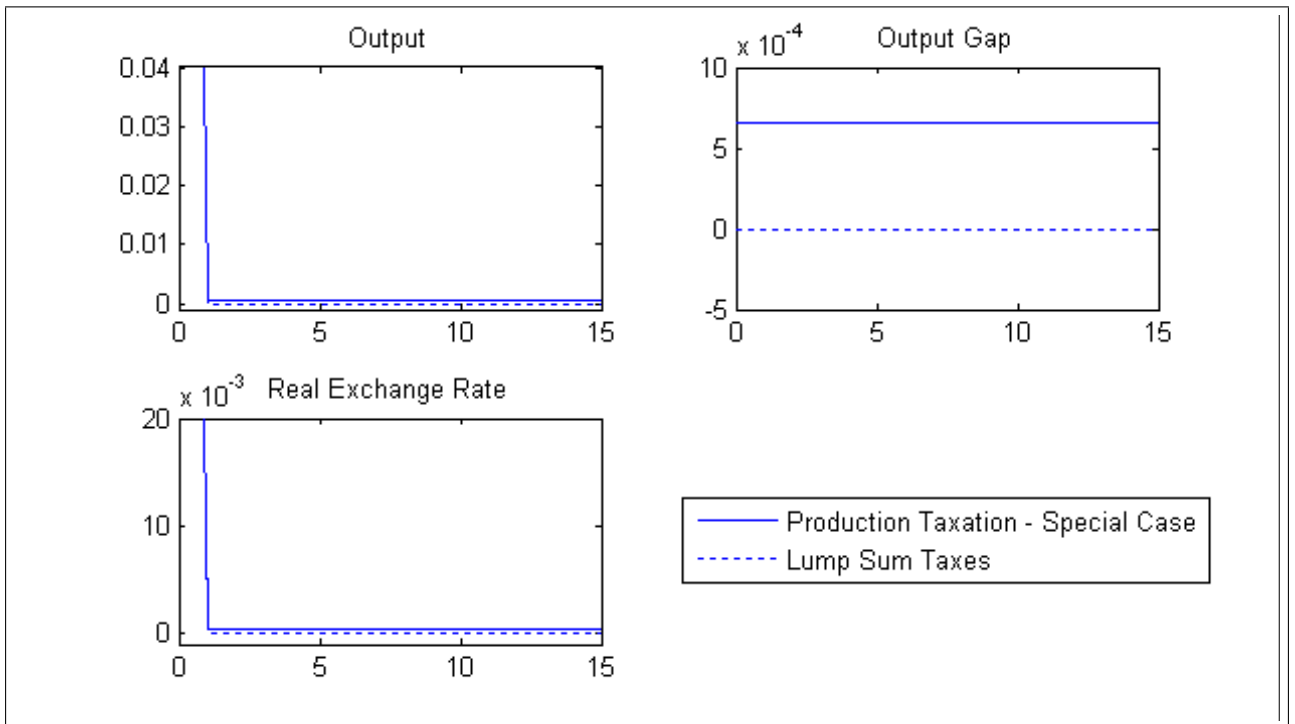


Figure 2: Impulse responses following a productivity shock- the case of flexible prices, $\rho = \theta = 1$ and $d_{ss} = 0$

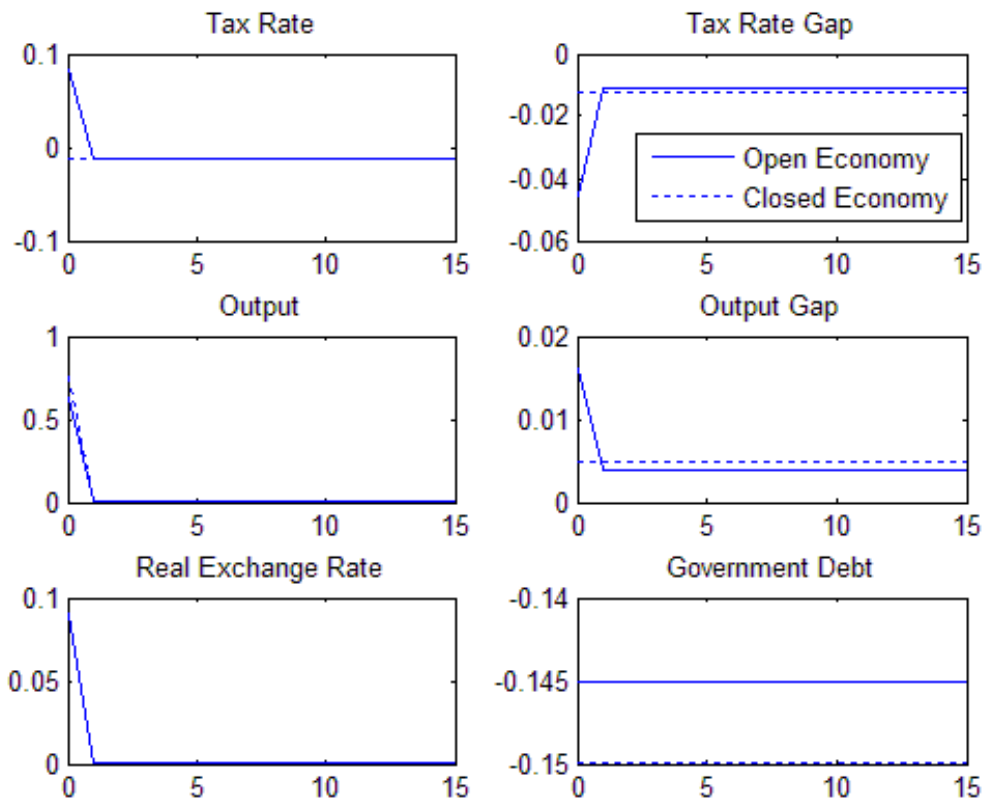


Figure 3: Impulse responses following a productivity shock- the case of flexible prices and $d_{ss} = 0$

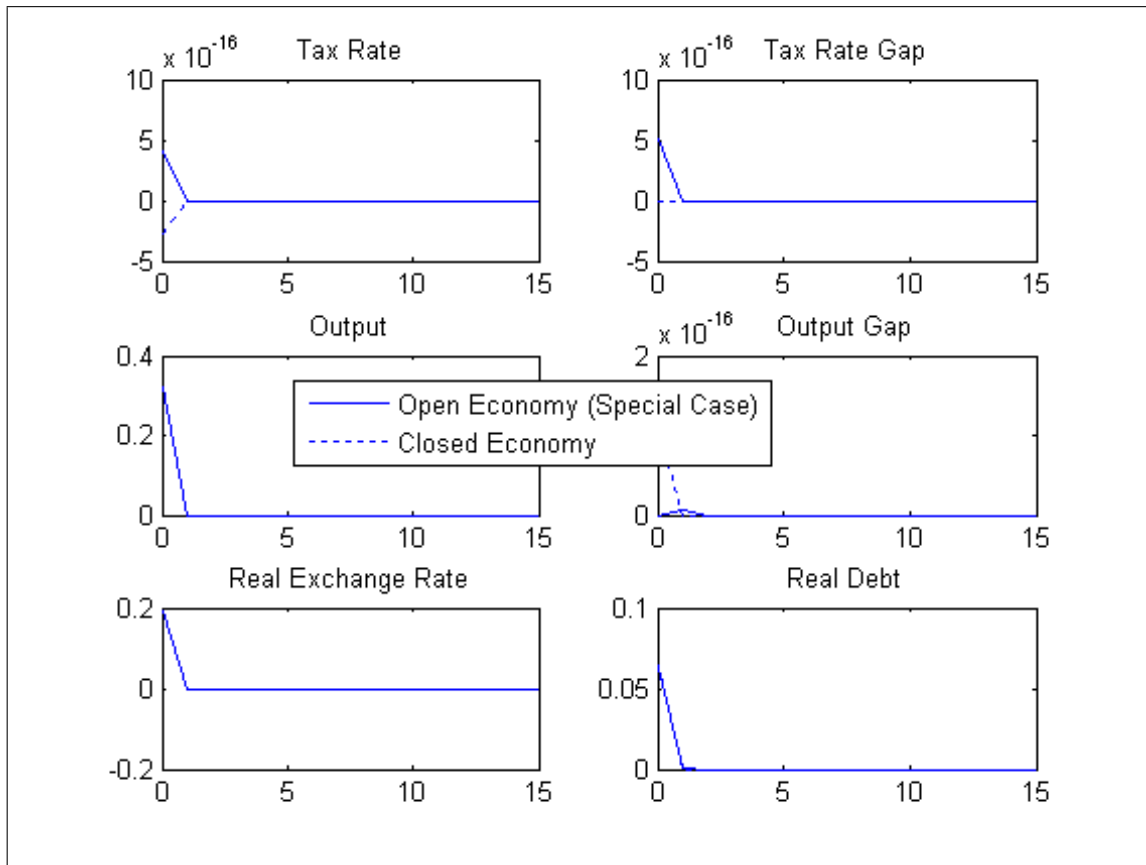


Figure 4: Impulse responses following a productivity shock- the case of flexible prices, nominal bonds and $\rho = \theta = 1$

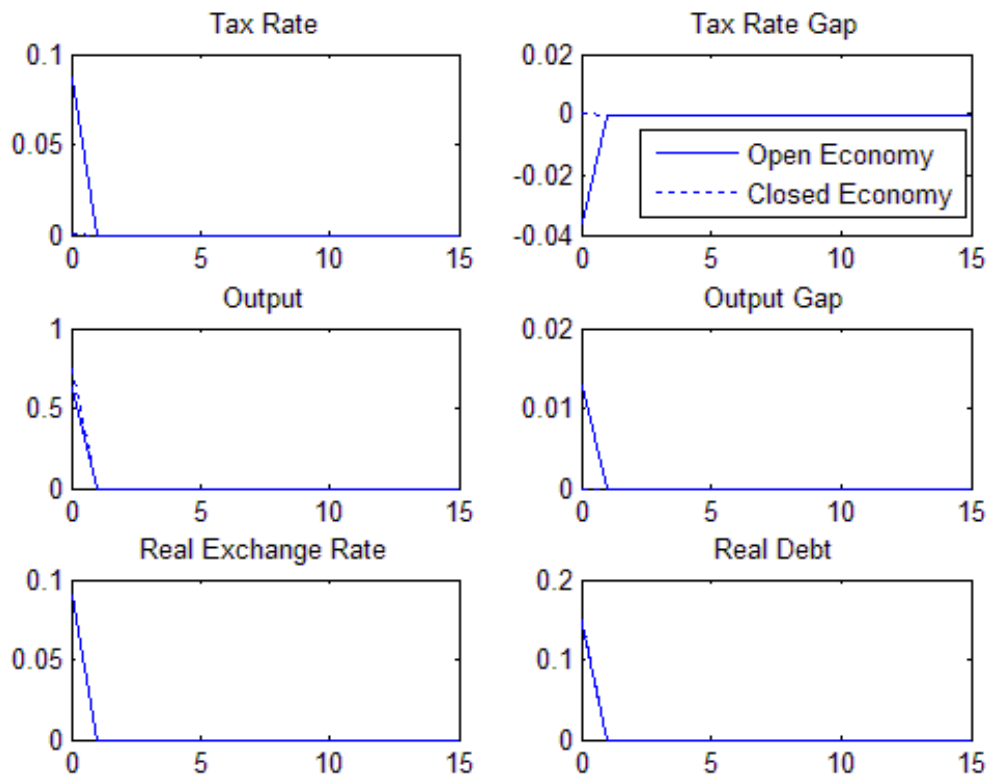


Figure 5: Impulse responses following a productivity shock- the case of flexible prices and nominal bonds

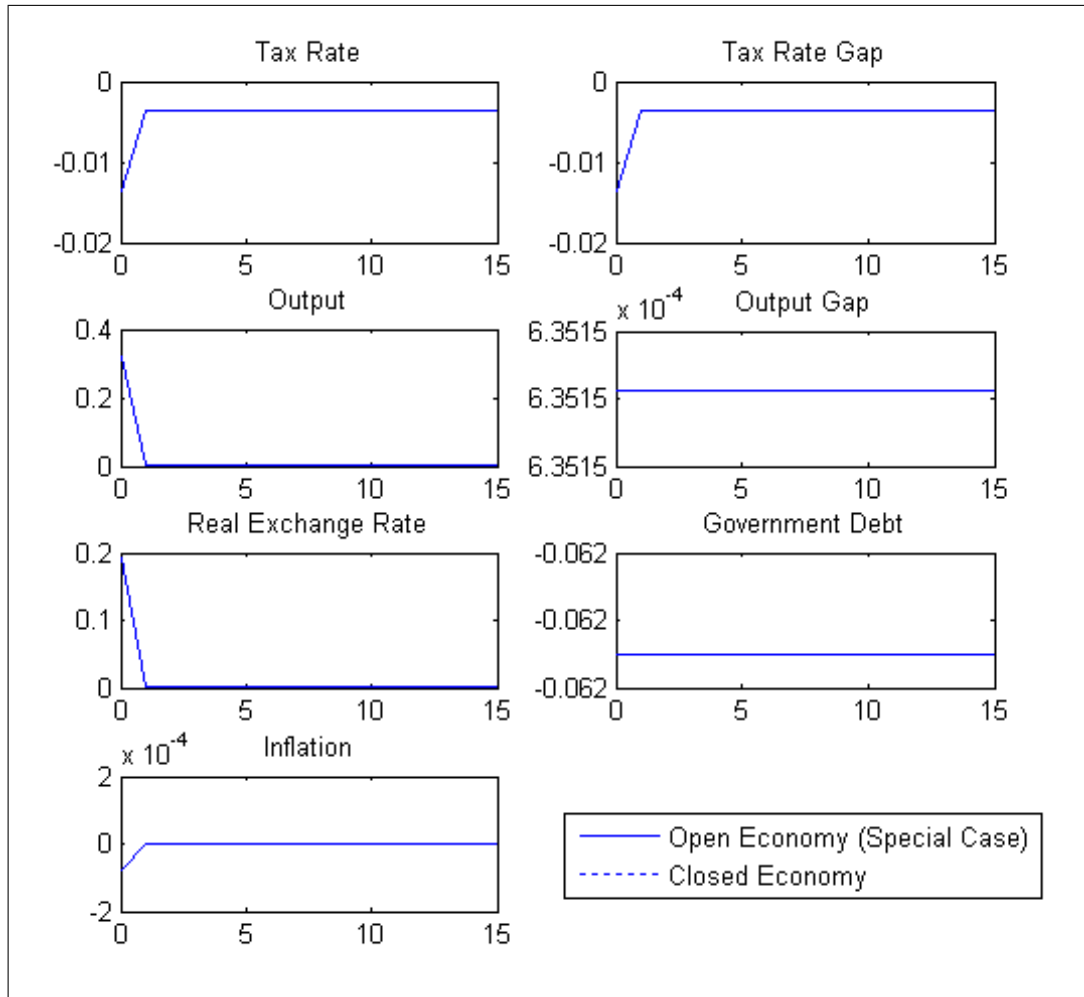


Figure 6: Impulse responses following a productivity shock- the case of sticky prices, $\rho = \theta = 1$ and $d_{ss} = 0$

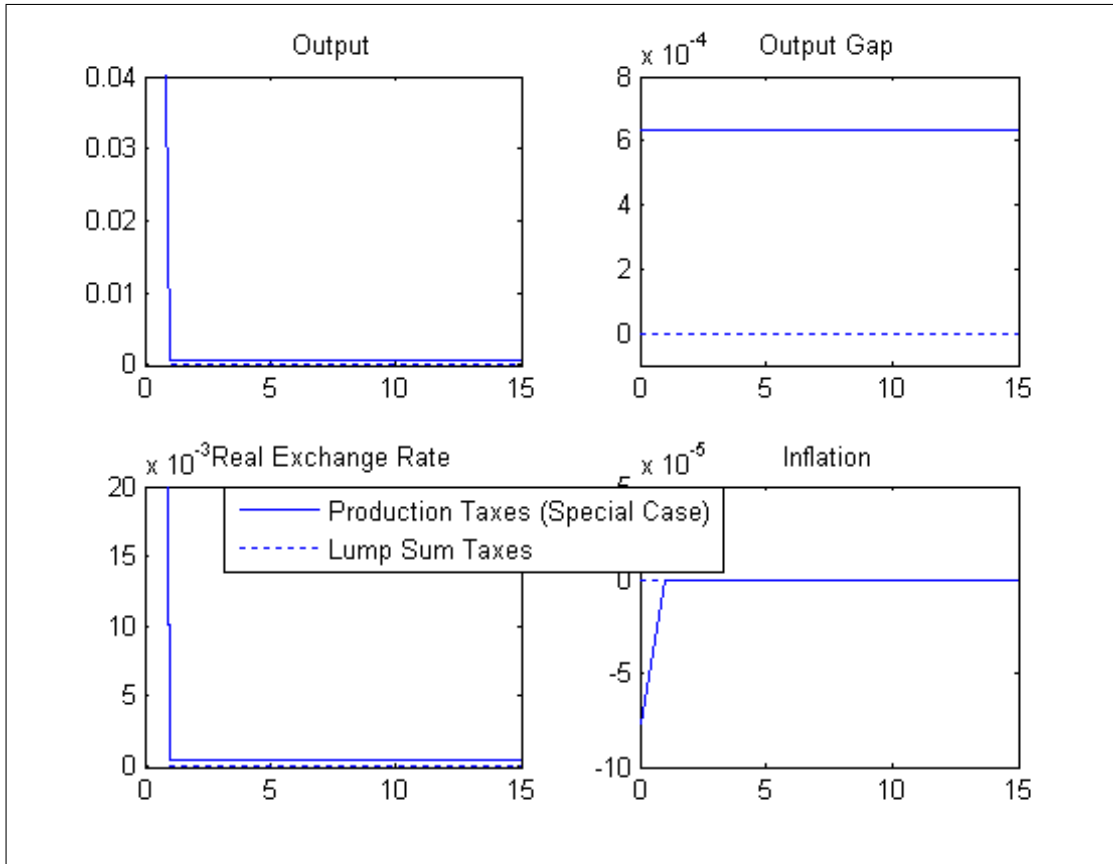


Figure 7: Impulse responses following a productivity shock- the case of sticky prices, $\rho = \theta = 1$ and $d_{ss} = 0$

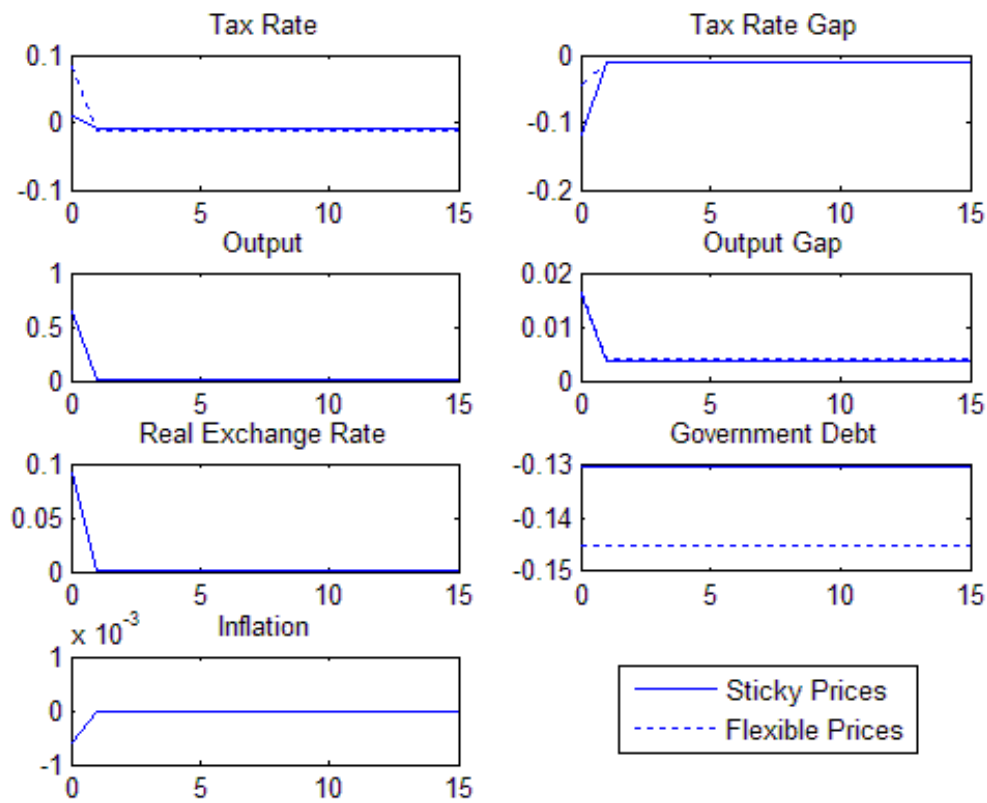


Figure 8: Impulse responses following a productivity shock- the case of $d_{ss} = 0$

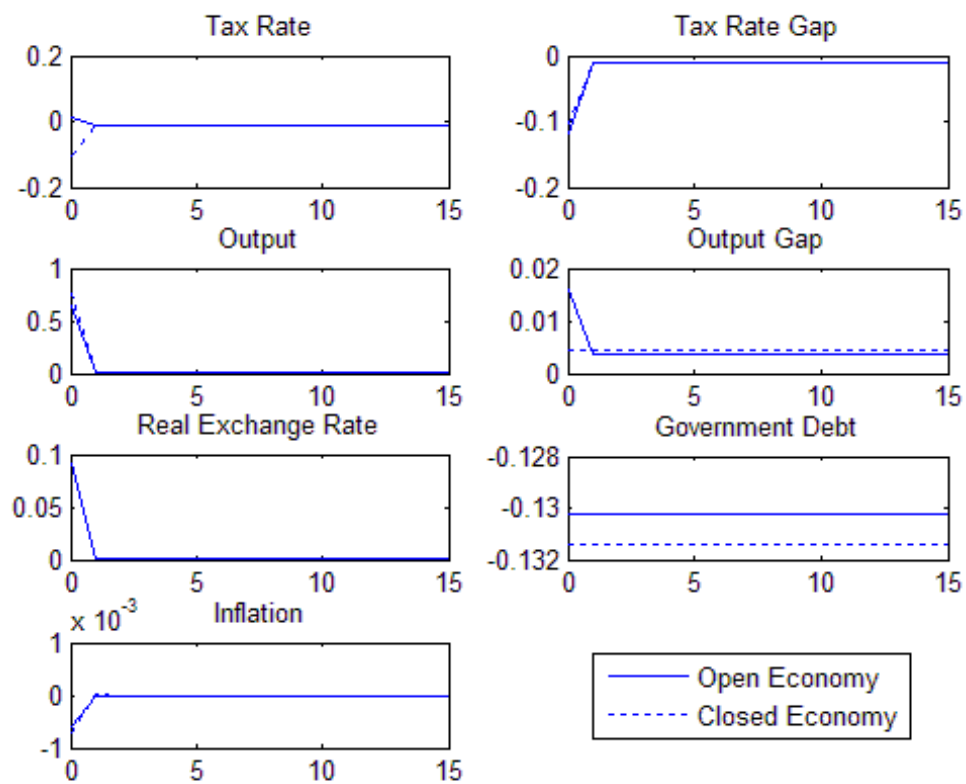


Figure 9: Impulse responses following a productivity shock- the case of sticky prices and $d_{ss} = 0$

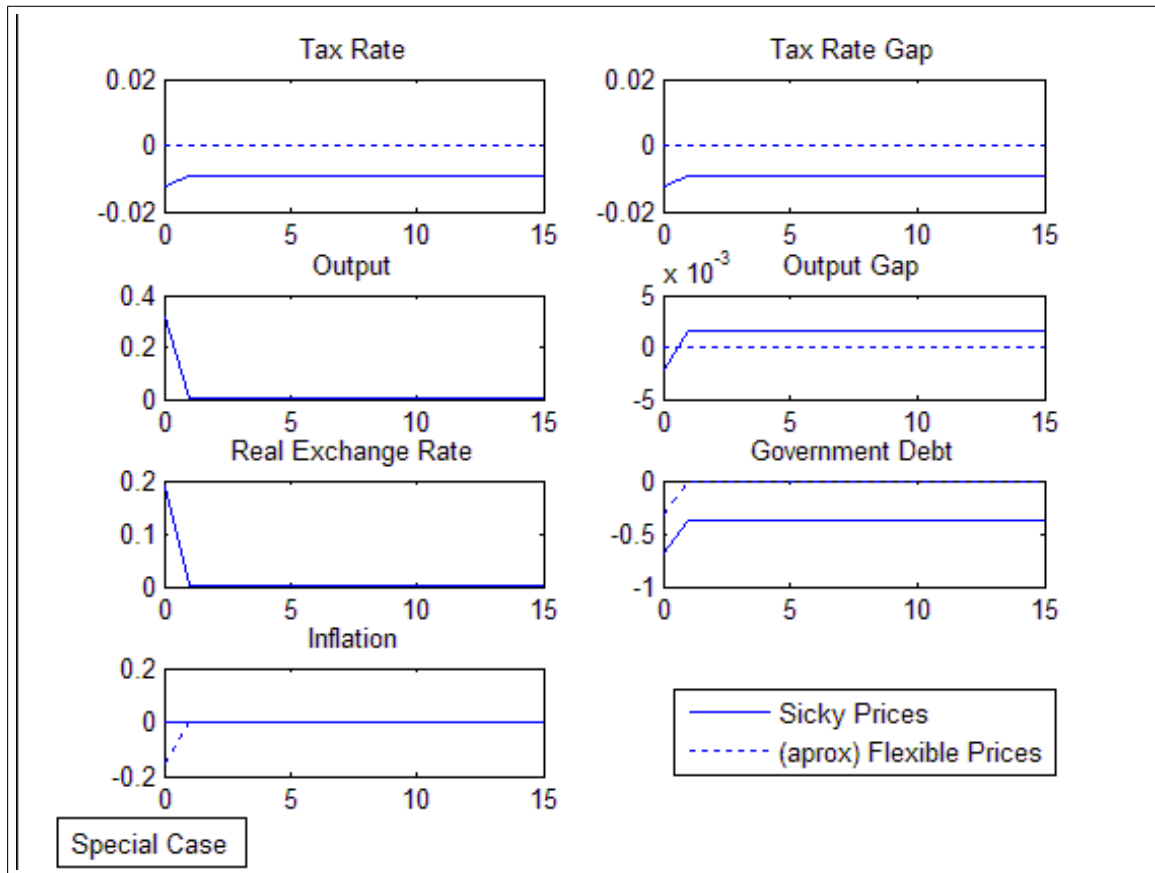


Figure 10: Impulse responses following a productivity shock- the case of nominal bonds and $\rho = \theta = 1$

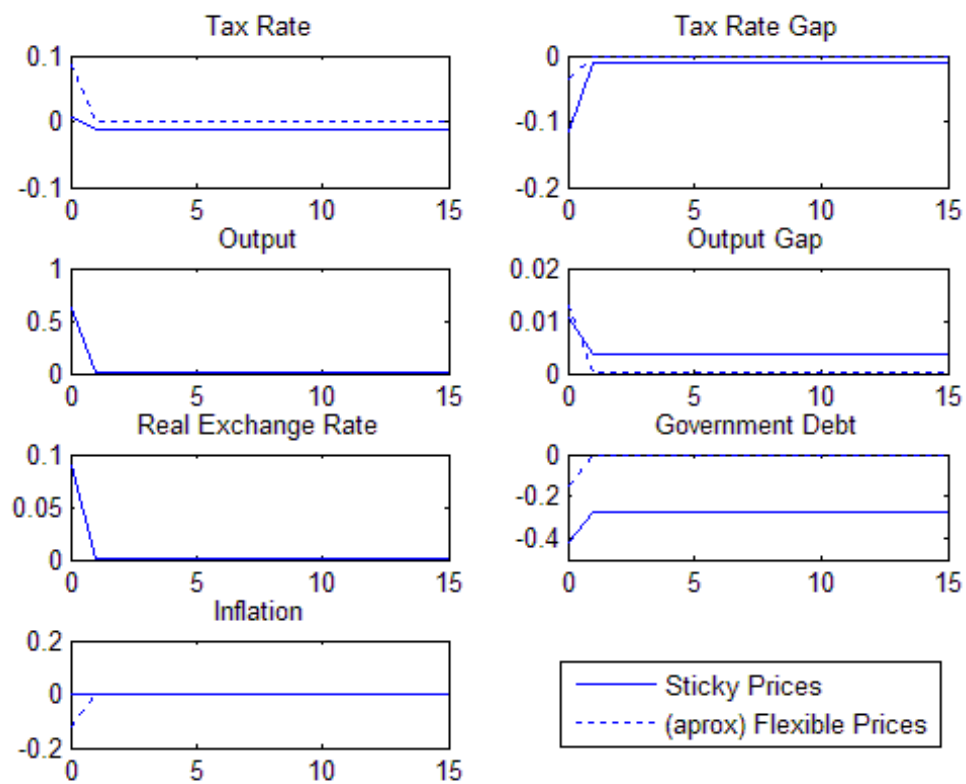


Figure 11: Impulse responses following a productivity shock- the case of nominal bonds