Prospect Theory Applications in Finance

Nicholas Barberis
Yale University

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Overview

- the goal of behavioral finance is to further our understanding of financial markets by improving the psychological realism of our models
  - e.g. by allowing for less than fully rational thinking

Preferences

- prospect theory
- ambiguity aversion

Beliefs

- representativeness, law of small numbers
- non-belief in the law of large numbers
- conservatism, belief perseverance, confirmation bias
- overconfidence

- here, we look at *prospect theory* applications in finance

Overview, ctd.

- almost all models of financial markets assume that investors evaluate risk according to Expected Utility
  - but this framework has had trouble matching many empirical facts
- can we make progress by replacing EU with a psychologically more realistic preference specification?
  - e.g. with prospect theory

(Cumulative) Prospect Theory

Consider the gamble:

$$(x_{-m}, p_{-m}; \ldots; x_{-1}, p_{-1}; x_0, p_0; x_1, p_1; \ldots; x_n, p_n),$$

where $x_i < x_j$ for $i < j$ and $x_0 = 0$

- under EU, it is assigned the value

$$\sum_{i=-m}^{n} p_i U(W + x_i)$$

- under prospect theory, it is assigned the value

$$\sum_{i=-m}^{n} \pi_i v(x_i)$$

Prospect Theory, ctd.

Four key features:

- the carriers of value are *gains* and *losses*, not final wealth levels
- $v(\cdot)$ has a kink at the origin, capturing "loss aversion"
  - a greater sensitivity to losses (even small losses) than to gains of the same magnitude
  - inferred from aversion to $(110, \frac{1}{2}; -100, \frac{1}{2})$
- $v(\cdot)$ is concave over gains, convex over losses
  - inferred from $(500, 1) \succ (1000, \frac{1}{2})$ and $(-500, 1) \prec (-1000, \frac{1}{2})$
- "probability weighting," i.e. weight outcomes with decision weights π_i obtained with the help of a weighting function $w(\cdot)$
  - in CPT, this means that the agent overweights the *tails* of distributions
  - inferred, in part, from our simultaneous liking of lotteries and insurance, e.g. $(5, 1) \prec (5000, 0.001)$ and $(-5, 1) \succ (-5000, 0.001)$

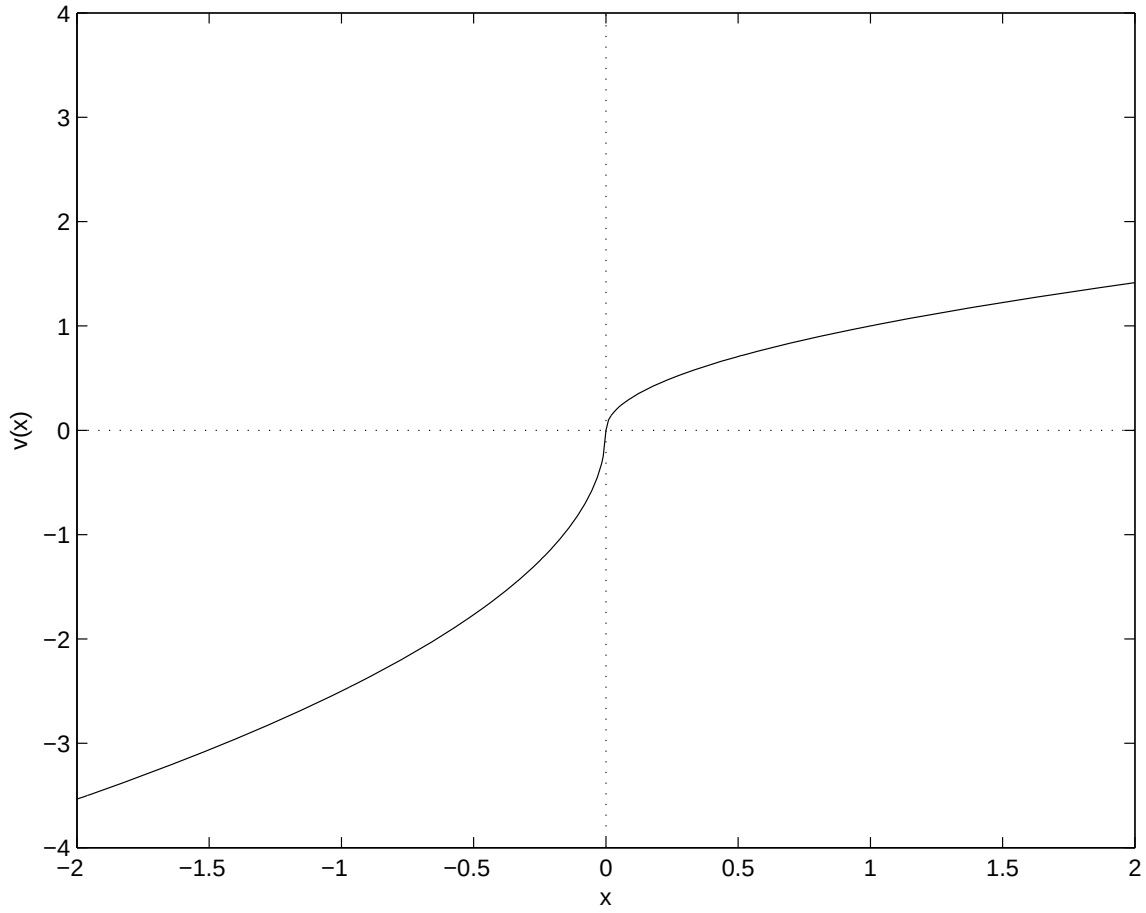

Figure 1. The graph plots the value function proposed by Tversky and Kahneman (1992) as part of their cumulative prospect theory, namely $v(x) = x^{\alpha}$ for $x \geq 0$ and $v(x) = -\lambda(-x)^{\alpha}$ for $x < 0$. The authors estimate $\alpha = 0.88$ and $\lambda = 2.25$ from experimental data. The plot uses $\alpha = 0.5$ and $\lambda = 2.5$ so as to make the loss aversion and diminishing sensitivity easier to see.

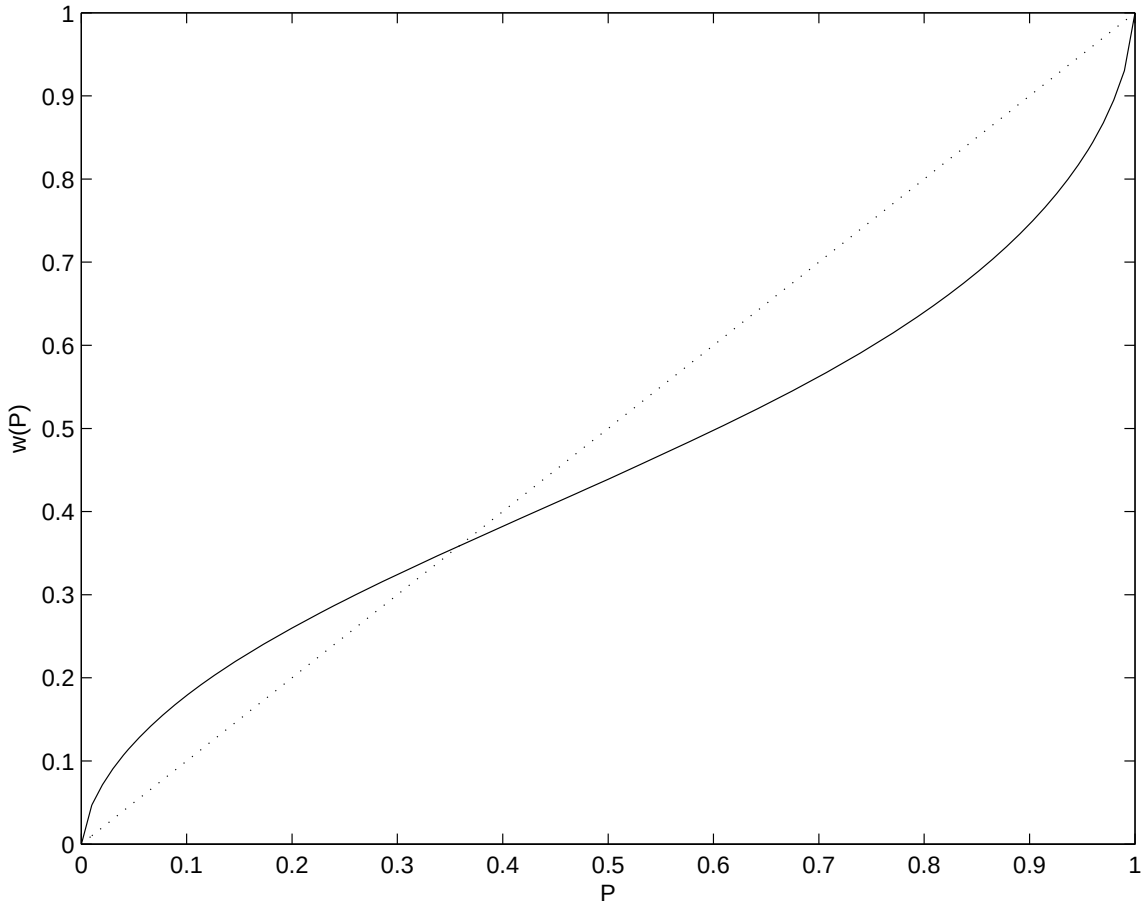

Figure 2. The graph plots the probability weighting function they propose, namely $w(P) = P^{\delta}/(P^{\delta} + (1-P)^{\delta})^{1/\delta}$, for two values of δ. The solid line corresponds to $\delta = 0.65,$ the value estimate by the authors from experimental data; and the dotted line to $\delta = 1$.

Prospect Theory, ctd.

Note:

- decision weights do not reflect erroneous beliefs
- there is growing interest in the psychological foundations of probability weighting
  - diminishing sensitivity (Tversky and Kahneman, 1992)
  - affect (Rottenstreich and Hsee, 2001)
  - salience (Bordalo, Gennaioli, Shleifer, 2012)

Prospect Theory, ctd.

- Tversky and Kahneman (1992) also suggest functional forms for $v(\cdot)$ and $w(\cdot)$ and calibrate them to experimental evidence:

$$v(x) = \begin{cases} x^{\alpha} & \text{for } x \geq 0 \\ -\lambda(-x)^{\alpha} & \text{for } x < 0 \end{cases}$$

$$w(P) = \frac{P^{\delta}}{(P^{\delta} + (1-P)^{\delta})^{1/\delta}}$$

with

$$\alpha = 0.88, \lambda = 2.25, \delta = 0.65$$

Prospect theory applications

[1]

- the *cross-section of stock returns*
  - one-period models
  - new prediction: the pricing of skewness
  - probability weighting plays the most critical role

[2]

- the *aggregate stock market*
  - intertemporal representative agent models
  - try to address the equity premium, volatility, predictability, and non-participation puzzles
  - loss aversion plays a key role; but probability weighting also matters

[3]

- *trading behavior*
  - multi-period models
  - try to address the disposition effect and other trading phenomena
  - all aspects of prospect theory play a role

Prospect theory applications, ctd.

- a critical issue in applying prospect theory is defining what a "gain" or a "loss" represents
  - a gain or loss in total wealth, value of stock market holdings, value of an individual stock?
  - and does a gain mean that an asset's return exceeded zero, the risk-free rate, the expected return?
- these are questions about framing (broad vs. narrow) and about the reference point
- by trying various plausible assumptions and taking the predictions to the data, we are *slowly* figuring out answers to these questions

The cross-section

Barberis and Huang (2008)

- single period model; a risk-free asset and J risky assets with multivariate Normal payoffs
- agents have identical expectations about security payoffs
- agents have identical CPT preferences
  - defined over gains/losses in *wealth* (i.e. no narrow framing)
  - reference point is initial wealth scaled up by the risk-free rate, so utility defined over $\hat{W} = \tilde{W}_1 - W_0 R_f$
  - full specification is:

    $V(\hat{W}) = \int_{-\infty}^{0} v(W)\, d\pi(P(W)) - \int_{0}^{\infty} v(W)\, d\pi(1-P(W))$

    (continuous distribution version of Tversky and Kahneman, 1992)

- it turns out that, in this economy, asset prices are still described by the CAPM!
  - see also De Giorgi, Hens, Levy (2011)

The cross-section, ctd.

- to make progress, introduce a small, independent, positively skewed security into the economy
- in a representative agent economy with concave EU preferences, the security would earn an average excess return of zero
- we find that, in an economy with CPT investors, the security can earn a *negative* average excess return
  - skewness itself is priced, in contrast to EU models, where only coskewness matters

- equilibrium involves *heterogeneous holdings*

  (for now, assume short-sale constraints)
  - some investors hold the old market portfolio and a large, undiversified position in the new security
  - others hold the old market portfolio and no position at all in the new security
  - heterogeneous holdings arise from non-unique global optima, not from heterogeneous preferences

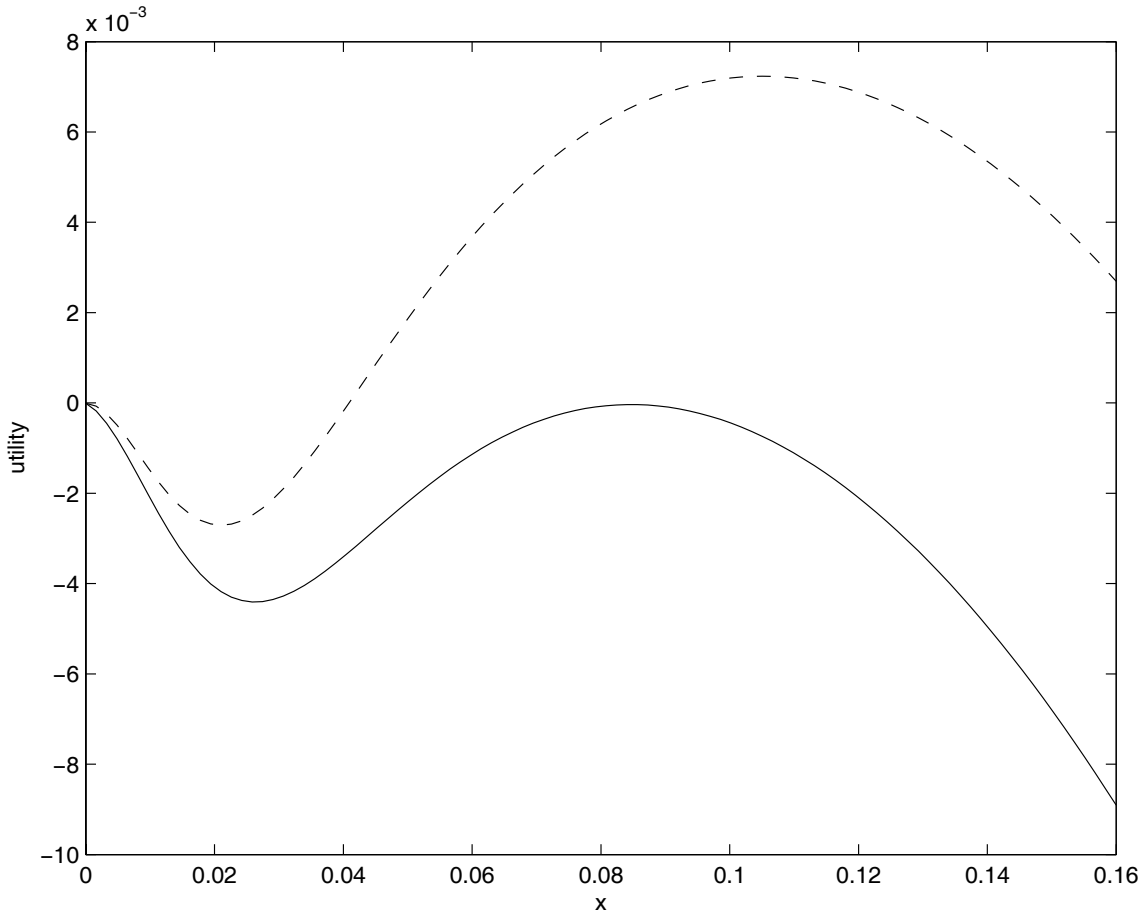

FIGURE 3. A HETEROGENEOUS HOLDINGS EQUILIBRIUM. *Notes:* The figure shows the utility that an investor with cumulative prospect theory preferences derives from adding a position in a positively skewed security to his current holdings of a Normally distributed market portfolio. The skewed security is highly skewed. The variable x is the fraction of wealth allocated to the skewed security relative to the fraction of wealth allocated to the market portfolio. The two lines correspond to different mean returns on the skewed security.

The cross-section, ctd.

Intuition:

- since it contributes skewness to the portfolios of some investors, the new security is valuable, and so earns a low average return
- not surprising that a CPT investor likes a skewed *portfolio*
  - more surprising that he likes a skewed *security*, even if it is small

Note:

- the prediction that skewness is priced appears to be robust to alternative framing and reference point assumptions

The cross-section, ctd.

Empirical evidence: Basic tests

- several papers test the model's basic prediction that skewness is priced in the cross-section
  - Zhang (2006)
  - Boyer, Mitton, Vorkink (2010)
  - Bali, Cakici, Whitelaw (2011)
  - Conrad, Dittmar, Ghysels (2012)
- all of these studies find supportive evidence

The cross-section, ctd.

Empirical evidence: Applications

- low average return on IPOs
  - Green and Hwang (2012) show that IPOs predicted to be more positively skewed have lower long-term returns
- low average returns on distressed stocks, bankrupt stocks, stocks traded on OTC markets
  - Eraker and Ready (2011), Conrad, Kapadia, Xing (2012)
- "overpricing" of out-of-the-money options
  - Boyer and Vorkink (2011) find that stock options predicted to be more positively skewed have lower returns
- low average return on stocks with high idiosyncratic volatility (Ang et al., 2006; Boyer, Mitton, Vorkink, 2010)
- diversification discount (Mitton and Vorkink, 2008)
- under-diversification
  - Mitton and Vorkink (2010) find that undiversified individuals hold stocks that are more positively skewed than the average stock

The cross-section, ctd.

Remarks:

- an example of how psychology can lead us to useful new predictions
- new theories of the foundations of probability weighting will likely lead to more predictions
  - e.g. affect (Rottenstreich and Hsee, 2001)
  - e.g. salience (Bordalo, Gennaioli, Shleifer, 2012)
- as may the system I/system II framework
  - e.g. Kumar et al.'s work on religion

Prospect theory applications

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  - one-period models
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  - probability weighting plays the most critical role

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- the *aggregate stock market*
  - intertemporal representative agent models
  - try to address the equity premium, volatility, predictability, and non-participation puzzles
  - loss aversion plays a key role; but probability weighting also matters

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- *trading behavior*
  - multi-period models
  - try to address the disposition effect and other trading phenomena
  - all aspects of prospect theory play a role

The aggregate stock market

- can prospect theory help us understand the properties of, and attitudes to, the aggregate stock market?
  - e.g. equity premium, volatility, predictability, and non-participation puzzles
- Benartzi and Thaler (1995) note that a model in which investors are loss averse over annual changes in their financial wealth predicts a large equity premium
- three elements:
  - loss aversion
  - annual evaluation
  - narrow framing
- Benartzi and Thaler (1995) emphasize the first two elements
  - "myopic loss aversion"

The aggregate stock market, ctd.

- there are few empirical tests of this hypothesis
  - but it is nonetheless gaining acceptance
  - and see Dimmock and Kouwenberg (2010)

Subsequent developments:

- formalizing the argument in more traditional asset pricing frameworks
  - Barberis, Huang, Santos (2001), Andries (2011), Pagel (2011)
- emphasizing the role of narrow framing (Barberis, Huang, Thaler, 2006)
- showing that probability weighting further increases the equity premium (De Giorgi and Legg, 2012)
- trying to also address the volatility puzzle through the dynamics of loss aversion (Barberis, Huang, Santos, 2001)

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Trading behavior

- can prospect theory help us understand how people trade stocks over time?
- a particular target of interest is the "disposition effect"
  - individual investors' greater propensity to sell stocks trading at a gain relative to purchase price, rather than at a loss

- at first sight, prospect theory, in combination with stock-level narrow framing, appears to be a promising approach
- but it turns out to be harder than expected to formalize this idea
  - prospect theory defined over annual stock-level trading profits does *not* generate a disposition effect very reliably (Barberis and Xiong, 2009)
  - but need to look at other reference point assumptions (Meng, 2012)

Trading behavior, ctd.

Other approaches?

- one idea is that people derive utility from *realized* gains and losses
  - "realization utility"
  - Shefrin and Statman (1985), Barberis and Xiong (2009, 2012)
- e.g. if you buy a stock at \$40 and sell it at \$60
  - you get a burst of positive utility *at the moment of sale,* based on the size of the realized gain
- what is the source of realization utility?
  - people often think about their investing history as a series of investing episodes
  - and they think of selling a stock at a gain (loss) as a "good" ("bad") episode

    $\Rightarrow$ when an investor sells an asset at a gain, he feels a burst of pleasure because he is creating a positive new investing episode
- realization utility with a prospect theory form predicts a disposition effect more reliably (Barberis and Xiong, 2009)

Trading behavior, ctd.

- Barberis and Xiong (2012), "Realization Utility," study *linear* realization utility, coupled with a positive time discount factor
  - the investor derives utility from the sale price of an asset minus the purchase price
- this generates a disposition effect

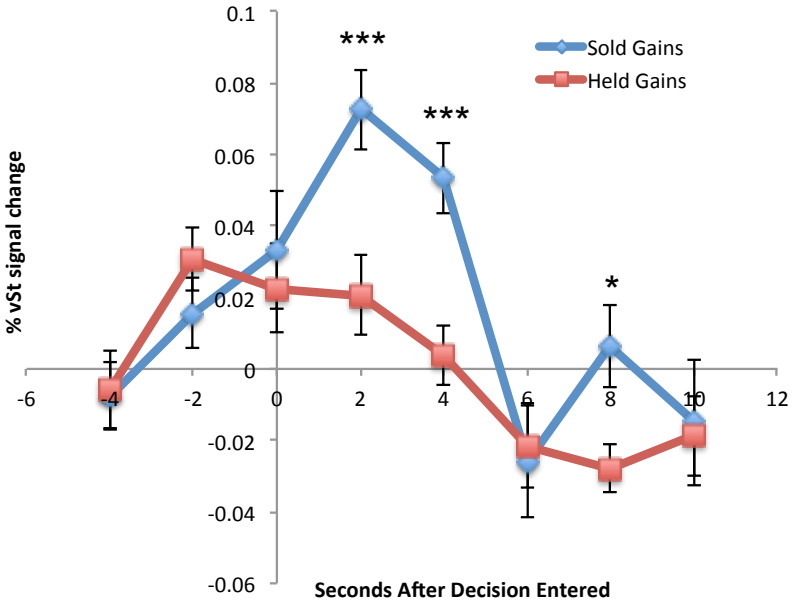
  but also:
  - "excessive trading"
  - the underperformance of individual investors even before transaction costs
  - the greater turnover in bull markets
  - the greater selling propensity above historical highs
  - the individual investor preference for volatile stocks
  - the negative premium to volatility in the cross-section
  - the fact that overpriced assets are also heavily traded
  - momentum

Trading behavior, ctd.

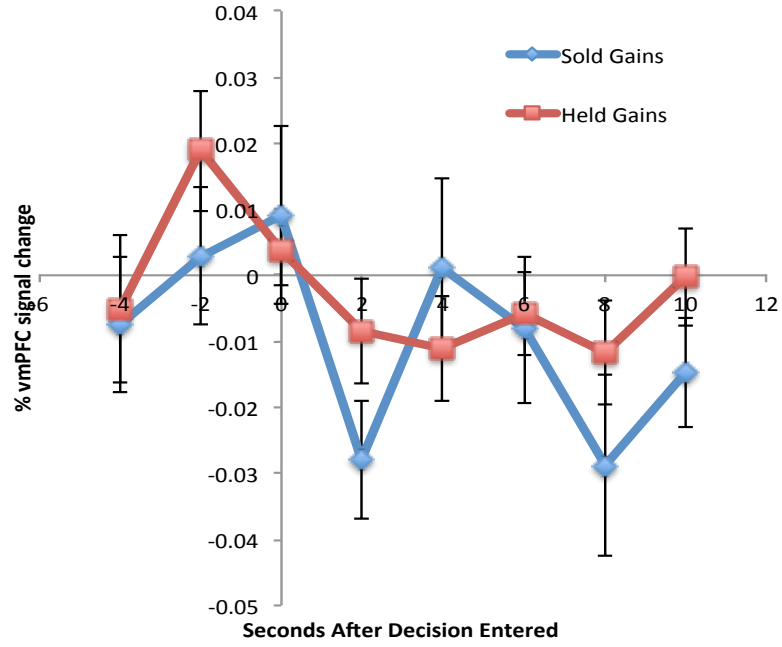
- *neural* data offer some support for realization utility
  - Frydman, Barberis, Camerer, Bossaerts, Rangel (2011)
- 28 participants trade stocks in an experimental market while their brain activity is recorded with fMRI
  - stock returns are positively auto-correlated
  - but participants nonetheless exhibit a disposition effect
- the ventral striatum (vSt) is believed to encode subjective feelings of pleasure
  - examine activity in the vSt when a participant sells a stock at a gain vs. holds a stock with a similar embedded gain

Figure 7. **Direct tests of the realization utility hypothesis.** Average activity in the vSt (Panel A) and vmPFC (Panel B) during trials when subjects were offered the opportunity to sell capital gains. The blue time series plots the average activity in trials where subjects realized capital gains, while the red time series plots the average activity in trials where subjects decided to hold capital gains. *** denotes p<0.001, ** denotes p<0.01, * denotes p<0.05 (paired t-test). *t=0* corresponds to the instant at which the subject enters his trading decision on a hand-held device.

A)



B)



Summary

- the *cross-section of stock returns*
  - one-period models
  - new prediction: the pricing of skewness
  - probability weighting plays the most critical role
  - no narrow framing needed
- the *aggregate stock market*
  - intertemporal representative agent models
  - try to address the equity premium, volatility puzzles
  - loss aversion plays a key role; but probability weighting also matters
  - typically assume stock market-level narrow framing
- *trading behavior*
  - multi-period models
  - try to address the disposition effect and other trading phenomena
  - all aspects of prospect theory play a role
  - typically assume stock-level narrow framing

Themes

- prospect theory seems to be helpful for thinking about financial phenomena
  - particularly a model that applies prospect theory to gains and losses in financial wealth
- for finance applications, probability weighting may be the most useful element of prospect theory
  - new theories of probability weighting are likely to lead to useful new predictions
  - Rottenstreich and Hsee (2001), Bordalo, Gennaioli, Shleifer (2012)
- we need to keep thinking about reference points and narrow framing
  - e.g. Koszegi and Rabin, 2006
- and we need a better understanding of dynamics
  - how past gains and losses affect risk attitudes