Firm Dynamics and Financial Development*

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Abstract

This paper studies how financial development in an economy influences firms' financing and growth. We first document empirically the debt financing and growth patterns of firms with a large and comprehensive dataset from 22 European countries. We find that in less financially developed economies, small firms grow faster and have lower debt to asset ratios than large firms. We then develop a quantitative model where financial development drives firm growth and debt financing through the availability of credit. We parameterize the model to the firm financial structure in the data and show that financial development can rationalize the difference in growth rates between firms of different sizes across countries.

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1 Introduction

Do small and large firms grow at different rates across countries? Many theoretical models of firm dynamics and financial frictions predict that small firms grow faster than large firms due to limited availability of credit for small firms.[1] This prediction implies that the relation of firm size and growth should be systematically linked to the economy's credit accessibility; but little is known about the variation of firm growth across countries. Our paper fills this gap by studying how debt financing and growth vary with firm size across countries with different financial development.

We first analyze empirically the relation of firm size with debt financing and growth using firm level data from 22 European countries. We document that small firms grow faster and finance their assets with less debt than large firms in less financially developed countries. We then develop a quantitative model where financial development drives firm growth and debt financing through the availability of credit. We assess the model's prediction regarding the cross section firm growth, when firm size and debt usage are parameterized to those in the firm level data. We find that financial development is quantitatively important in rationalizing the growth rates of firms across different sizes and countries.

Our empirical contribution consists of providing a systematic cross-country investigation of the relations of firm financing and growth with size. Our analysis is new in that we document these relations for comprehensive firm level datasets that include a large number of small private firms across 22 European countries. We focus on the *relative* behavior of firms of different sizes across countries with *varying* financial development. We first find that small firms grow faster than large firms. And this difference is stronger in countries that are less financially developed as indicated by the ratio of private credit to GDP and the availability of credit information of consumers and firms. We also find that small firms in more financially developed countries use more debt financing than large firms relative to those in less financially developed countries. Importantly, these findings are robust to controlling for country, industry or age specific characteristics.

We then develop a quantitative dynamic model of heterogeneous firms where financial development interacts with firm growth and debt financing. In the model firms borrow to finance their operations but debt is unenforceable. Lenders limit firm debt because of default risk and incur a fixed credit cost when issuing loans. We proxy differences in financial development across economies with differences in fixed credit costs. High credit costs limit debt disproportionately for small firms which make their scale inefficient. These small firms

[1] Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2006), Quadrini (2004), and DeMarzo and Fishman (2007) among others.

grow faster as they can expand their scale. Hence, in the model small firms in less financially developed economies have less debt financing and higher growth rates, just as in the data.

The framework is a dynamic stochastic model that builds on Cooley and Quadrini (2001). Firms use a decreasing returns to scale technology to transform capital into output and face uncertain productivity. They finance capital and dividends with debt and profits and have the option to default on their debt. To compensate for default risk, lenders offer each firm a limited schedule of loan contracts. The restrictions on loans impact firms' debt financing and capital choices. Increasing debt is useful for financing capital and dividends, but larger loans are also costly because of higher default risk. Hence, firms prefer to shrink their capital and become inefficiently small, to avoid excessively large loans. Firms can also be small simply because the persistent component of their productivity is low.

The loan schedule systematically varies across firms and across economies. Each firm is offered a customized schedule that depends on its default risk given the economy wide credit cost. In any economy, small unproductive firms confront more adverse loan schedules than large productive firms because they have higher default incentives and the fixed credit costs are relatively more costly for their small loans. And in economies with high credit costs, debt contracts are restricted for all firms, but disproportionately limited for the small firms.

These features in loan schedules determine firm size and growth across economies. Small unproductive firms are more likely to be inefficient in scale than large productive firms, and especially so in economies with high credit costs. Firms of inefficient scale grow faster than those of efficient scale in response to good shocks because they use the additional output to increase their scale to a more efficient level. This implies that small firms grow faster in all economies, and particularly fast in economies with high credit costs. Hence, our model matches the first empirical regularity that small firms grow faster than large firm especially in less financially developed economies.

The debt financing patterns across economies are determined not only by the firm specific loan schedules but also by the history of shocks. Unproductive small firms face the most restrictive schedules, which tend to lower the equilibrium level of debt of small firms. But inefficient small firms have larger loans due as they have built up debt after a history of bad shocks. These dynamics tend to increase the equilibrium level of debt of small inefficient firms. Hence, small firms can have higher or lower levels of debt than large firms. Nonetheless, as credit costs increase, the restrictions on loan contracts become so severe for the small unproductive firms that the level of debt of small versus large firms decreases. Thus, our model can match the second empirical regularity that the difference in debt financing of small and large firms decrease in less financially developed economies.

We quantitatively evaluate the model implications in rationalizing the cross-section financing and growth patterns jointly. We calibrate our model using the firm level data of Bulgaria and the United Kingdom, as representative countries with weak and strong financial development. Our calibration strategy consists of choosing the credit costs and the preference and technology parameters to match the financing patterns observed in the cross section of firms in each country. Specifically, the calibrated credit costs for Bulgaria equal 0.08% of output for the average firm and for U.K. these costs are zero. We then evaluate the model's predictions on growth rates for firms of different sizes. The results show that our model can deliver quantitatively the relationship between sales growth and firm size observed in the data in both countries.

For Bulgaria, we calibrate the debt to asset ratios of firms as in the data: for the mean size firm to be 0.53 and for small firms in the first asset quintile to be 0.45. The model then delivers the observed sales growth patterns of 0.77 for the small firms and 0.40 for the large firms in the fifth asset quintile. For the United Kingdom, we calibrate the debt to asset ratios of firms as in the data: for the mean size firm to be 0.84 and for small firms in the first asset quintile to be 1.18. The model generates a growth rate of 0.17 for small firms while 0.08 for large firms, which are similar to those observed in the data of 0.23 for the small firms and 0.05 for large firms. Hence, we conclude that accessibility to credit is an important determinant of the observed differential growth rate across firms.

We next evaluate our quantitative model in generating the documented cross-country variations in the debt financing and growth patterns. In particular, we decrease credit costs to zero in the Bulgarian calibration. In this experiment, the difference in growth rates between the small and large firms decreases from 0.37 to 0.18. The lower credit costs also increase the difference in leverage ratios from -0.21 to 0.09. Thus, our model is fully consistent with the evidence that in economies with better financial development the difference in growth rates of small versus large firms is lower yet the difference in leverage ratios is bigger. Lowering the credit costs also increases the output of the small firms. In this experiment, the output of the small firms increases by 19%.

The model also reveals that financial frictions that incorporate a fixed component are suitable to match the data. As an alternative experiment, we vary the probability of re-entering financial markets after default which increases the value of default proportionately to the firm's productivity. A higher re-entry probability can be linked to worse financial development because it further restricts the availability of credit ex-ante. However, we show that a higher re-entering probability reduces the difference in growth rates between small and large firms, which is at odds with the data. In addition, financial frictions with a fixed component are also needed for the model to deliver the positive size-leverage relation observed in many less financially developed countries.

Related Literature

Our empirical findings are novel as we are the first to examine the cross-section firm financing and growth patterns simultaneously across countries with a broad coverage of firms. In regards to growth, the cross-section firm-level analyses have considered only one country, as in Rossi-Hansberg and Wright (2007) for the United States.[2] In regards to firms' financing patterns, cross-country comparisons have been studied only for large public firms; Rajan and Zingales (1995) examine G7 countries, and Booth, et al. (1999) study 10 developing countries. Public firms, however, constitute a small percentage of firms in all countries, which limits the scope of these previous findings.[3]

The theoretical model is related to the literature that studies the implications of financial frictions on firm growth. Our theory is closest to Cooley and Quadrini (2001), who develop a model where financing restrictions arise from limited commitment in debt contracts. They show that these frictions can potentially deliver large differences in the growth rates between small and large firms. In our paper, we use firm level data to quantify the extent to which financial considerations impact growth rates. We further concentrate on how differences in financial development can explain the financing and growth patterns of firms across countries. Our paper is also very related to Albuquerque and Hopenhayn (2004), who analyze the effects of enforcement problems under a full set of state contingent assets. In our model, we use incomplete markets to allow firms with a history of bad shocks to decrease their value and to allow precautionary savings to play a role.[4]

Apart from financial frictions, the two leading theoretical explanations for why small firms grow faster are based on selection mechanisms and mean reversion in the accumulation of factors of production. Hopenhayn (1992) and Luttmer (2007), for example, propose theories where the growth of small firms reveals a selection effect: small firms tend to exit with bad shocks, and so they grow faster when they survive after good shocks. Rossi-Hansberg and Wright (2007) develop a model where the mean reversion in the accumulation of industry specific human capital makes small firms grow faster. We view these theories as complementary to the financial frictions theory. Nonetheless, theories of firm growth without financial frictions are silent (by construction) regarding the joint financing and growth patterns of firms across countries.

The paper is also related to the literature in corporate finance on the capital structure

[2] The cross-country analysis of growth has been restricted to industry level data as in Rajan and Zingales (1998).

[3] For example in the United Kingdom less than 4% of firms in our dataset are public firms.

[4] Clementi and Hopenhayn (2006), Quadrini (2004), and DeMarzo and Fishman (2007) also study theoretically financial constraints that arise due to informational asymmetries between lenders and entrepreneurs.

of firms.[5] Hennessy and Whited (2005) develop a dynamic model of debt financing and show that progressive taxes induce larger firms to use more debt financing. Interestingly, this theory is at odds with the data in the United Kingdom where corporate taxes are progressive, yet the relation between size and leverage is negative. Miao (2005) also studies firms' capital structure in a model with endogenous firm exit in response to productivities shocks. In his model, firms choose debt only when they enter, yet small firms have higher leverage ratios because their equity value is small. In our model, the firm's debt choice is time varying and the interest rate on debt reflects endogenous default probabilities.

The rest of the paper is organized as follows. Section 2 presents the new empirical findings on firm growth and debt financing across countries with varying financial development. Section 3 introduces and characterizes the model. Section 4 presents the quantitative analysis by calibrating our model to two countries: the United Kingdom and Bulgaria. Section 5 concludes.

2 Empirical Facts

In this section, we study the empirical relation of firm size with debt financing and growth across countries. We find that these relations vary systematically with the degree of financial development across countries. First, small firms use relatively more debt financing than large firms in more financially developed countries. Second, small firms tend to grow faster than large firms in all countries, but by more in countries with weaker financial development.

In what follows, we first describe the firm-level database, Amadeus, which we use for the analysis of firms in Europe. We then highlight our findings with two example countries: the United Kingdom and Bulgaria. We then present our main empirical findings regarding the debt financing and growth patterns of firms in 22 European countries that vary in their financial development.

2.1 Data Description

The data source is AMADEUS, which is a comprehensive, European database. Amadeus contains financial information on over 7 million private and public firms in 38 European countries covering all sectors in the economy. Nonetheless, the coverage of Amadeus is limited for some countries. Given our aim to document firms' financing and growth patterns for a comprehensive and representative sample of firms, we need to select the countries for which Amadeus contains a sufficiently large number of firms.

[5] See Harris and Raviv (1991) for a comprehensive review.

We first exclude countries that do not require private firms to report their balance sheets. We next use a simple criterion to select the countries that have a ratio of the number of firms reporting positive assets to PPP-adjusted GDP larger than 20 percent of the ratio for the United Kingdom in 2005. The dataset for the United Kingdom in Amadeus is especially attractive because it contains the largest number of firms by far relative to all the other countries. These criteria leave us with 22 countries: Belgium, Bulgaria, Croatia, Czech, Denmark, Estonia, Finland, France, Iceland, Ireland, Italy, Latvia, Lithuania, Malta, the Netherlands, Norway, Portugal, Romania, the Russian Federation, Spain, Sweden, and the United Kingdom.[6] In the appendix we show that the datasets for these 22 countries are in fact quite comparable and representative of the universe as reported by the European Commission.

We examine the firms' balance sheet data for these 22 countries in 2004 and 2005. Firm size is measured by the book value of the total assets of the firm. To measure debt financing we compute the firm's leverage ratio in 2005. Leverage is defined as the broad measure of total liabilities over total assets of the firm. We use this broad definition because it is a more consistent measure across countries and because it provides the largest sample of firms. Firm growth is measured by the net real growth rate of sales from 2004 to 2005, adjusted by CPI in each country. We exclude firms in the financial and government sectors following Rajan and Zingales (1995). We also clean the data by restricting the sample to firms that report positive assets and non-negative liabilities each year. For the growth statistics, we further restrict the sample to firms that also report positive sales in both 2004 and 2005. Finally, we remove firms with outlier observations of growth and leverage in the top 1 percentile.[7]

Financial development in these 22 countries is measured using two statistics. The first one is the average private credit to GDP ratio over 2000–2004 taken from the *World Development Indicators*. Higher ratios of private credit to GDP indicate better financial development. The second measure is the coverage of credit registries. Credit registries in countries track the loans and defaults of individuals and firms and facilitate lending by banks and financial institutions. The statistic we use is the percentage of adults that are included in the public and private credit registries in 2005 in each country.[8] Larger credit bureau coverage indicates better financial development because it implies that it is easier for financial intermediaries to make loans when credit information of borrowers is available. Credit bureau coverage is taken from the Doing Business publications of the World Bank.

Table 1 reports descriptive statistics for the firm level datasets and the two measures of

[6] The threshold of 20% is not important. If we use a threshold of 15% only Slovak is added to the sample of countries.

[7] The appendix contains more details about the data cleaning procedure.

[8] We use data for 2005 because this statistic is not available for many countries before 2005.

financial development for each country. Countries are ordered by their level of private credit to GDP. The table shows the variability of financial development is large across these 22 countries. For example, the private credit to GDP ratio is 143% in the Netherlands and only 18% in Russia; the credit bureau coverage is 100% in the Sweden and 0% in Croatia. As expected, these two financial development indices are highly correlated in our sample with a correlation equal to 0.64.

The mean and median level of assets for firms in each country are reported for 2005 in terms of current Euros in the table. Firm asset levels vary across countries, and they tend to be larger for countries with stronger financial development. Moreover, the distribution of firms in all countries is highly skewed as the mean asset levels are much larger than the median asset levels. We also report the average leverage ratio and the average growth rate across all firms in each country. Both mean leverage and mean growth vary substantially across countries. The mean leverage ratio is 0.92 in the Netherlands, but only 0.42 in Estonia; the mean net growth rate is 11% in the Netherlands, but 54% in Estonia. The table also reports the number of firms with positive assets and liabilities in the dataset of each country.

Overall, these aggregate statistics are systematically related to financial development. First, firms in countries with better financial development tend to have larger leverage ratios. The cross-country correlation of mean leverage and the private credit to GDP ratio is 0.31, and the correlation of mean leverage and the credit bureau coverage is 0.43. Second, the average firm growth rates are smaller in countries with better financial development. The cross-country correlation of mean growth and the private credit to GDP ratio is -0.58, and the correlation of mean growth and the credit bureau coverage is -0.29. Third, firms in countries with better financial development are larger. The correlation of the mean asset level and private credit to GDP equals 0.65, and the correlation of the mean asset level and credit coverage is 0.44.

2.2 Example: United Kingdom and Bulgaria

To provide a stark illustration of our main empirical findings, we analyze two example countries that differ substantially in their financial development: the United Kingdom and Bulgaria.

Let's first consider the unconditional relation of leverage and firm size in Bulgaria and in the United Kingdom. To this end, we divide firms in each country into 10 quantiles according to their assets and compute their leverage ratios. Figure 1 plots the mean leverage ratio of firms in each quantile in Bulgaria and the UK for year 2005. The figure illustrates

Table 1: European Countries: Datasets and Financial Development

	Firm Level Datasets					Financial Development	
	Mean Asset	Median Asset	Mean Leverage	Mean Growth	No. Firms	Credit Coverage (%)	Credit to GDP (%)
Denmark	5909	365	0.58	0.16	116726	7.7	147
Netherlands	13791	523	0.92	0.11	147754	68.9	143
United Kingdom	13269	86	0.84	0.11	846910	76.2	143
Portugal	2750	159	0.80	0.12	198162	63.7	138
Iceland	3295	129	0.91	0.59	16528	100	120
Ireland	7588	202	0.91	0.18	86736	100	116
Spain	5023	405	0.75	0.26	526455	42.1	109
Malta	11186	887	0.75	0.33	1749	—	108
Sweden	6496	197	0.62	0.18	192240	100	91
France	5102	215	0.74	0.09	802371	1.8	87
Norway	5020	261	0.78	0.26	144400	100	83
Italy	5247	650	0.81	0.12	528374	59.9	81
Belgium	4000	236	0.74	0.07	290332	55.3	75
Finland	4933	153	0.56	0.16	73556	14.7	60
Croatia	4729	318	0.66	0.04	18942	0.00	48
Czech Republic	3664	168	0.76	0.32	57302	24.9	37
Latvia	3068	576	0.71	0.43	4596	0.6	34
Estonia	585	34	0.42	0.54	50326	12.5	29
Bulgaria	2227	86	0.65	0.53	29731	13.6	22
Lithuania	4273	622	0.61	0.58	6006	4.4	19
Russia	4671	73	0.79	0.63	163628	0.0	18
Romania	307	16	0.98	0.46	419251	1.4	11

the remarkably distinct pattern of size and leverage across countries. In the UK the leverage-size relation is generally downward sloping: small firms have relatively higher leverage ratios than large firms. In particular, the mean leverage ratio of the smallest firms is above 1 and that of the largest firms is 0.66.[9] In Bulgaria the leverage-size relation is generally increasing, ranging from 0.35 for the smallest firms to 0.69 for the largest firms.[10]

The relation between firm size and firm growth is also different across these two countries. To analyze the unconditional relation of growth and size, we again divide firms in each country into 10 quantiles according to their assets in 2004 and compute average sales growth from 2004 to 2005 for each quantile. Figure 2 reports the mean sales growth rate for firms in

[9] When leverage is greater than one, firms have negative equity. Herranz et al. (2008) document that 21% of the small firms in the United States have negative equity in 1998.

[10] In an earlier version of this paper, we documented that in Ecuador with a similar degree of financial development as Bulgaria, small firms have lower leverage ratios than large firms, as we documente here for Bulgaria.

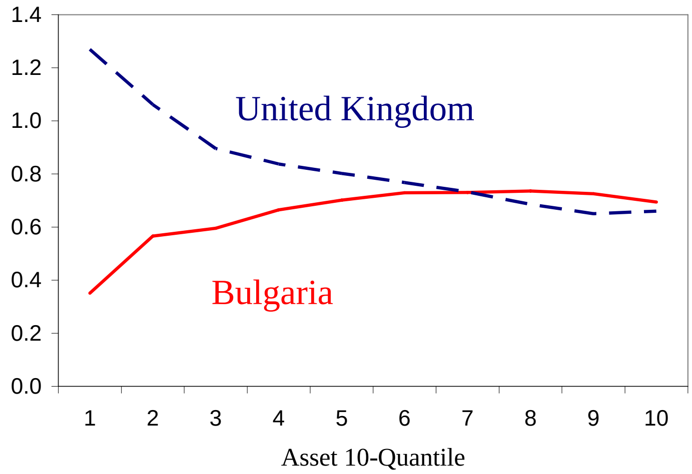

Figure 1: Firm Size and Leverage

each asset quantile in Bulgaria and in the UK. The figure illustrates that small firms grow faster than large firms in both countries. The difference in growth rates of small and large firms, however, is bigger in Bulgaria than in the UK. Small British firms in the first asset quantile grow at the rate of 54%, whereas large British firms in the tenth asset quantile grow at the rate close to zero. Small Bulgarian firms, however, grow at the rate of 157%, while large Bulgarian firms grow at about 12%.

Our findings for the UK and Bulgaria suggest that the firm growth and financing patterns might be related to the degree of financial development in each country. In the next subsection, we examine these relations with comprehensive firm-level datasets in the 22 European countries.

2.3 Cross-Country Empirical Findings

Our hypotheses are that in countries with stronger financial development small firms have higher leverage ratios and lower growth rates relative to large firms. Therefore, we pool all the countries together and estimate two regressions of the following forms:

$$Leverage_{k,c}(\text{or } Growth_{k,c}) = \beta_0 + \beta_1 \log(Asset\ Share_{k,c}) \tag{1}$$

$$+\beta_2 \log(Asset\ Share_{k,c}) \times Financial\ Development_c + Dummy\ Variables + \nu_{k,c},$$

where c denotes the country, and k the firm. The dependent variable is the firm's leverage ratio for the leverage regressions and the firm's real sales growth rate for the growth

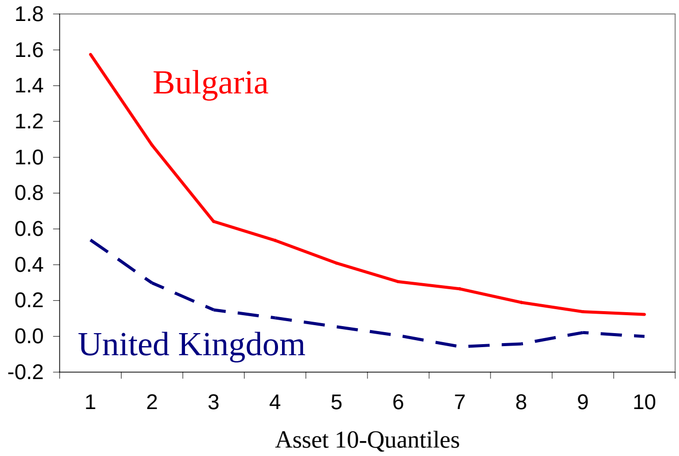

Figure 2: Firm Size and Sales Growth

regressions. *Asset Share*$_{k,c}$ is the share of the firm k's assets in the total assets of country c. Given the highly skewed firm size distribution we use the log of firms' asset shares as firm size. *Financial Development*$_c$ corresponds to the two measures of financial development in country c, namely private credit over GDP and coverage of credit registries. The term *Dummy Variables* corresponds to fixed effects at the country $\times$ industry $\times$ age level. Hence, the regression gives each country$\times$industry$\times$age group an independent intercept.

The regression specification controls for country-specific effects, 2-digit industry-specific effects, and 7 age-group-specific effects. Country effects control for any country characteristic, for instance business cycles, institutional quality, the legal system, the political system, and many others. Industry effects are at the 2-digit level constructed with NACE codes. They control for any inherent features of industries, including capital intensity, competition structure, liquidity needs, and tradability. The 7 age groups are constructed at 5 year intervals up to 30 years and a final group for firms with age greater than 30 years. Age effects control for any inherent life-cycle features of firms such as market share and technological development.

As discussed in Rajan and Zingales (1998), the use of fixed effects enables us to control for a much wider array of omitted variables. These dummy variables will capture the peculiar features of each age group within each sector of each country, such as the particular technological characteristics or specific tax treatments varying at the country$\times$industry$\times$age level. Only additional explanatory variables that vary within each of the industry-country-age groups need be included. These are firm size and the primary variable of interest, the interaction between firm size and financial development. According to our hypothesis, we must find the coefficient estimate for the interaction between size and financial development to be negative in the leverage regression and to be positive in the growth regression.

Table 2 reports the regression results using the two measures of financial development. The first two columns report the leverage regressions and the last two columns report the growth regressions. For the regressions using coverage of credit registries, we drop Malta because this statistic is not available for this country. We report the coefficient on firm size and the coefficient on the interaction term between firm size and financial development in the table. The standard errors of the regression coefficients are reported in parentheses and are robust to heteroskedasticity throughout the paper.

Table 2: Firms' Leverage, Growth and Financial Development

	Leverage		Growth	
	Private Credit to GDP	Credit Bureau Coverage	Private Credit to GDP	Credit Bureau Coverage
Size (log(firm's asset share))	-0.039***	-0.044***	-0.052***	-0.029***
	(0.0003)	(0.0003)	(0.0009)	(0.0006)
Interaction (credit to GDP × size)	-0.012***		0.030***	
	(0.0003)		(0.0007)	
Interaction (credit bureau coverage ×size)		-0.016***		0.007***
		(0.0003)		(0.0006)
Fixed Effects Country×Industry×Age	Yes	Yes	Yes	Yes
Adjusted R^2	0.06	0.06	0.05	0.05
Number Observations	4564461	4563685	2568782	2568559
Number of Groups	4773	4662	4550	4486

Let's start with the regression that analyzes the size-leverage relation. The estimated coefficient on the interaction variable is negative as expected and statistically significant at the 1-percent level under both measures of financial development. The coefficient estimate on size is also negative and statistically significant under both measures. Thus, smaller firms have on average higher leverage ratios than large firms other things equal. Moreover, when private credit to GDP or credit bureau coverage increases, the leverage ratios of small firms relative to large firms increase.

The interaction term is similar to a second derivative. To interpret its magnitude, let's look at the regression with private credit to GDP and compare a small firm with an asset share equal to 0.1% to a large firm with an asset share equal to 10% in Bulgaria and the United Kingdom. The leverage difference between these comparable small and large firms is

6.7 percentage points higher in the UK than in Bulgaria as private credit to GDP is higher in the UK by 121 percentage points. These numbers are economically significant given that the mean leverage ratio for Bulgaria equals 0.65.

Let's now look at the regressions that analyze the size-growth relation. Size continues to be a significant determinant; smaller firms overall grow faster. The estimated coefficients on the interaction term are positive as expected and statistically significant at the 1-percent level for both measures of financial development. That is, the growth difference between small and large firms decreases with both private credit to GDP and credit bureau coverage. We can interpret the coefficient on the interaction of private credit to GDP and size as follows. The difference in growth rates of a small firm with an asset share equal to 0.1% relative to a large firm with an asset share equal to 10% is 17 percentage points less in the United Kingdom than in Bulgaria.

In the appendix we present robustness checks of these results. We first experiment with employment as an alternative measure of size and find that all the results are unchanged. We then estimate the regressions with three additional interaction terms added one by one: size with industry, size with age and size with GDP per capita. The estimated coefficients on the interaction terms of firm size and private credit to GDP in the leverage and growth regressions remain with the same sign and the same significance under all of these alternative specifications. The same is true for the estimated coefficient on the interaction of firm size and credit bureau coverage in all these specifications, except in the case when that all three additional interaction terms are added.

In summary, we find that small firms use less debt financing and grow disproportionately faster than large firms in countries with worse credit coverage and lower ratios of private credit to GDP. These empirical findings are important for providing a comprehensive picture of the relation between financial development with financing and growth across firms and across countries.

In what follows, we build a model to study and quantify the mechanism by which financial development affects the growth dynamics and financing patterns of firms. In modeling differences in financial development, we are guided by the findings from the empirical analysis that small firms have less debt financing in countries where credit registries are limited. In the model, lenders incur fixed credit costs when issuing loans associated with screening any particular loan application. High fixed credit costs, analogous to limited credit registries, are more costly for small firms and can lead to lower debt financing for them.

3 Model Economy

This section presents a dynamic model of heterogeneous firms to study firms' financing choices and dynamics. The model builds on Cooley and Quadrini (2001) while incorporating differentiation across economies based on financial development. In the model, entrepreneurs decide on the level of capital and debt financing for their firms. Debt contracts are not enforceable and entrepreneurs can default on the debt they owe. Creditors pay a fixed credit cost when issuing any loan. We control the financial development with the fixed credit cost; a large cost limits the availability of loans and corresponds to weak financial development.

3.1 Firms

Entrepreneurs in the economy are infinitely lived and have access to a mass one of risky project opportunities to produce a homogeneous consumption good. Each entrepreneur owns at most one project (also referred to as firm) and decides on entry, exit, production and financing plans to maximize the present value of dividends. Every period a fraction of the firms exit due to either exogenous death shocks or endogenous exit decisions. These project opportunities are available to potential entrepreneurs, who choose to enter and operate the firm if the project drawn gives them positive expected present value.

Every period each operating firm produces output y with a stochastic decreasing returns technology with capital as input. For a given level of capital input K invested the previous period, the firm produces output y given by

$$y = zK^{\alpha} \tag{2}$$

where $0 < \alpha < 1$. The productivity of the project z follows a Markov process given by $f(z', z)$. Capital depreciates completely after production every period.

An operating firm starts the period with a loan to be paid of size B_R and the installed capital K. It produces output zK^{α} after the productivity shock z is realized. Entrepreneurs finance the new capital K' and dividends D from two options: internally with the firm's output net of debt repayment $zK^{\alpha} - B_R$ and externally by acquiring a new loan with creditors B'. The dividends are given by

$$D = zK^{\alpha} - B_R + B' - K'. \tag{3}$$

We define the leverage of this firm as the ratio of total debt due this period to capital installed B_R/K if $B_R \geq 0$. If the firm starts with assets $B_R < 0$, the firm has no liabilities due, and thus its leverage ratio is equal to zero.

The timing of decisions within the period is as follows. At the beginning of the period, δ fraction of firms exit exogenously. All surviving firms receive their shocks. An entrepreneur with debt B_R, capital K, and shock z decides whether to default or not. If the entrepreneur repays his debts, he chooses a new loan, capital for the following period and dividends. Otherwise, the firm exits. Potential entrants replace all exiting firms.

3.2 Recursive Formulation

We lay out the recursive formulation of the entrepreneur's problem. Upon observing the shock realization, the entrepreneur decides whether to default by comparing the default value V^d with the repayment value V^c:

$$V(K, B_R, z) = \max\{V^c(K, B_R, z), V^d(z)\}, \tag{4}$$

where $V(K, B_R, z)$ denotes the present value of the firm to the entrepreneur. The entrepreneur's default decision can be represented by a binary variable $d(K, B_R, z)$ that equals 1 if default is chosen and 0 if repayment is chosen. In particular, we have

$$d(K, B_R, z) = \left\{ \begin{array}{ll} 0 & \text{if } V^c(K, B_R, z) \geq V^d(z) \\ 1 & \text{otherwise} \end{array} \right\}. \tag{5}$$

If the entrepreneur chooses to default, his debts are written-off but he loses the project and the firm exits. We assume that after default the entrepreneur is excluded from financial markets and with probability θ the entrepreneur can re-enter the market and start a new project with the same productivity z. The default value is then given by:

$$V^d(z) = \theta V^e(z),$$

where $V^e(z)$ denotes the value of a potential entrant with productivity z.

If the entrepreneur repays his debt, he keeps his project in operation and decides on production and financing. Given the set of loan contracts, the entrepreneur chooses the amount to be received from the creditor this period B' and the amount to be repaid the following period B'_R conditional on not defaulting, capital K', and dividends D to maximize the repayment value:

$$V^c(K, B_R, z) = \max_{\{B', B'_R, K', D\}} D + \beta(1-\delta)EV(K', B'_R, z') \tag{6}$$

subject to a non-negative dividend condition given by

$$D = zK^\alpha - B_R + B' - K' \geq 0, \tag{7}$$

where $\beta < 1$ denotes the discount rate of the entrepreneur. $V^c(K, B_R, z)$ is increasing in K and decreasing in B_R and $V^d(z)$ is independent of these variables. Thus, default is more attractive for firms with smaller capital and larger debt due.

Optimal debt is determined by trading off costs and benefits of various loans within the set of contracts offered. Debt is beneficial for financing investment. Debt can also be used for dividends, which is attractive when loans are cheap and entrepreneurs discount the future heavily. In addition, debt can be used to relax the non-negative dividend condition when the firm's output is low and the loan due is large. On the other hand, large debt is costly because it can lead firms to default. In particular, a large loan today implies a large repayment the next period that will be costly especially when the productivity shock is low. In this case, income might be so low that the entrepreneur fails to satisfy the non-negative dividend condition, defaults and loses the project. In anticipation of a possible default, the entrepreneur might find it optimal to reduce his borrowings such that default is avoided. Hence, in our model firms reduce debt for precautionary motives.[11]

In our model with limited enforceability of debt contracts, financing decisions interact with firms' investment. In contrast, in an environment where non-contingent contracts are perfectly enforceable and the non-negative dividend condition is relaxed, firms choose capital such that the expected marginal product of capital equals the risk-free rate:

$$E(z)\alpha K_{fb}(z)^{\alpha-1} = (1 + r). \tag{8}$$

We refer to this level of capital $K_{fb}(z)$ as first-best capital for a firm with expected productivity equal to z.

With enforcement frictions, investment also depends on the set of loan contracts available. In particular, investment is distorted downward. For example, if a firm starts with large debt, it might want to borrow a big loan B' to satisfy the non-negative dividend condition and to keep the investment level at the unconstrained optimal. Nonetheless, given that the set of loans is bounded due to possible defaults, such a big loan might not be offered to the entrepreneur. Hence, the entrepreneur might have to reduce the level of investment, making the project inefficiently small.

[11] Contrary to Cooley and Quadrini (2001), our model does not impose that debt is used for capital only, which adds a lower and upper bound on debt. This feature gives more room for the precautionary savings usage and allows a better match of the data where many firms have negative equity.

The problem for a potential entrant is simple in this model. Whenever the entrepreneur receives a project opportunity of productivity z, he decides to undertake the project and enter if the expected value of the project is greater than the outside option of zero. Thus, the value for a potential entrant is given by

$$V^e(z) = \max\{0, V^c(0, 0, z)\}.$$

Note that the new entrant starts with no assets and thus the value conditional on entering is exactly equal to the value of the contract $V^c(0, 0, z)$ when K and B_R are equal to zero.

3.3 Loan Contracts

Every firm with productivity z faces a schedule of loan contracts that consists of triplets $(B', K', B'_R; z)$. B' is the transfer of funds between the firm and the creditor the current period, B'_R is the transfer the next period, and K' is the capital for the next period. If B' is positive, it represents the payment from the lender to the firm which is used for capital and dividends. B'_R is the associated payment that the firm promises the lender conditional on not defaulting. The contract depends on the capital choice K' because default probabilities the following period are influenced by the level of capital. If B' is negative, it denotes a payment from the firm to the creditor as savings, and B'_R denotes the gross saving proceeding from the lender to the firm the next period.

For every loan contract with $B' > 0$, creditors needs to pay the cost ξ. One can rationalize the expense of ξ as costs lenders pay to obtain information about the entrepreneur's total debt. Knowing this information is necessary for the lender to correctly assess the probability of default of each entrepreneur.[12] We interpret ξ as the economy's ease to acquire credit information and it controls the financial development of the model economy. The parameter ξ can be naturally linked to the coverage of credit registries across countries. When ξ is low, credit registries in the economy have wide coverage and it is very easy and cheap to access credit information. When ξ is large, the lender has to spend some resources to screen the entrepreneur and obtain his debt information.[13] As documented in the empirical section, the coverage of credit registries across countries varies widely and this variable is linked to

[12] Note that it is optimal for lenders to pay the credit cost ξ to avoid excessive default probabilities. If contracts would not condition on the total debt, entrepreneurs would have an incentive to borrow a large amount in a given period from many lenders and then default the following period. Moreover, given that creditors who are considering lending to an entrepreneur pay the credit cost, it is optimal for entrepreneurs to obtain all the debt needed only from one creditor.

[13] This specification of credit issuance costs is similar to the one used in Tertilt et al. (2008). They document that improvements in credit scoring in the U.S. is important for understanding the rise in bankruptcies and volume of debt.

the ways firms grow and finance their assets. Thus, our model focuses on variation in ξ to capture differences in financial development across economies.

Creditors in the model are assumed to be able to commit to loan contracts. They are risk-neutral, competitive, and discount time at the risk-free interest rate r. They behave passively and are willing to finance the firm's financing needs as long as they are compensated for the expected loss in case of default and for the expense of ξ. Default probabilities vary across firms with different productivity levels. Thus, for each firm with productivity z, creditors offer contracts $(B', K', B'_R; z)$ such that

$$B' + \xi = \frac{B'_R(1-\delta)}{(1+r)} \left(1 - \int d(K', B'_R, z') f(z', z) dz'\right) \text{ for } B' > 0. \tag{9}$$

The lender breaks even in expected value with every contract, as the effective interest rate required incorporates the default premium consistent with default probabilities. When the entrepreneur saves, creditors do not need to pay ξ and default probabilities are zero. Thus, savings contracts satisfy the following condition

$$B' = \frac{(1-\delta)}{(1+r)} B'_R \text{ for } B' \leq 0. \tag{10}$$

3.4 Equilibrium

Before defining the equilibrium of this economy, we make an assumption on the relation between the risk-free rate and the discount factor of entrepreneurs. The assumption imposes that the rate at which entrepreneurs discount the future is higher than the risk-free rate.

Assumption 1 The risk-free rate r is such that $1/\beta - 1 > r > 0$.

This condition can be interpreted as a general equilibrium property of economies with lack of enforcement and incomplete markets. If $\beta(1 + r) = 1$, firms strictly prefer to accumulate assets rather than distribute dividends because of the additional benefits of assets in terms of avoiding firm failure. This would generate an excessive supply of loans that would in turn drive down the risk-free rate.

The model delivers an endogenous distribution of firms, denoted by $\Upsilon(K, B_R, z)$, which depends on the decisions of firms to borrow and invest. The distribution of firms is defined as the mass of firms over the endogenous and exogenous states (K, B_R, z). Whenever existing firms in the distribution $\Upsilon(K, B_R, z)$ exit either exogenously or endogenously, their z projects are released to potential entrant entrepreneurs. New entrants start their operation with zero capital and zero loans. Thus, the measure of entrants $\mu(z)$ is given by the following:

$$\mu(z) = \int [(1-\delta)d(K, B_R, z) + \delta]\, \Upsilon(K, B_R, z)\mathbf{d}(K \times B_R).$$

Define a transition function $Q(\cdot)$ that maps current states into future states by the following

$$Q\left((K, B_R, z), (K', B_R', z')\right) = \left\{ \begin{array}{l} f(z', z) \text{ if } B_R'(K, B_R, z) = B_R', K'(K, B_R, z) = K' \\ 0 \text{ elsewhere} \end{array} \right\}$$

where $B_R'(K, B_R, z)$ and $K'(K, B_R, z)$ are the optimal decision rules for capital and debt. The evolution of the distribution of firms is given by:

$\Upsilon'(K', B_R', z') =$

$$\int Q((0,0,z),(K', B_R', z'))\mu(z)\mathbf{d}z + \qquad (11)$$
$$(1-\delta)\int (1 - d(K, B_R, z))Q((K, B_R, z), (K', B_R', z'))\Upsilon(K, B_R, z)\mathbf{d}(K \times B_R \times z)$$

The distribution of firms the following period includes the set of surviving firms that do not default and do not receive the death shock. It also includes the new firms that enter after project opportunities are released by the exiting firms.

The recursive equilibrium for this economy consists of the policy functions of firms $\{(B'(K, B_R, z),\ K'(K, B_R, z),\ B_R'(K, B_R, z)),\ D(K, B_R, z),\ d(K, B_R, z)\}$, the value functions of firms $\{V(K, B_R, z),\ V^c(K, B_R, z),\ V^d(z),\ V^e(z)\}$, the schedule of loan contracts $(B', K', B_R'; z)$ offered by creditors, and the distribution $\Upsilon(K, B_R, z)$ of firms over (K, B_R, z) such that

1. Given the schedule of loan contracts offered, the policy and value functions of firms satisfy their optimization problem.

2. Loan contracts reflect the firm's default probabilities such that with every contract creditors break even in expected value.

3. The distribution of firms follows (11) and is consistent with the policy functions of firms and shocks given the initial distribution Υ_0.

3.5 Borrowing Limits and Financial Development

Limited enforceability of debt contracts generates endogenous borrowing limits for firms because creditors do not provide loans that will be defaulted on in all future states. These borrowing constraints play a key role in determining optimal debt. Moreover, borrowing limits vary across firms and with the degree of financial development. In particular, weak financial development limits borrowing relative to assets. And this limitation is more severe for small firms than for large firms.

We provide an analytical characterization of these findings by considering the case when firms are heterogeneous with respect to z yet this productivity is constant over the firm's lifetime. In addition, for simplicity we assume that firms do not face the death shock ($\delta = 0$). We also impose the following assumption on credit costs.

Assumption 2 Credit costs are such that $\xi \leq \left(\frac{\alpha z}{1+r}\right)^{\frac{1}{1-\alpha}} \frac{1-\alpha}{\alpha}, \ \forall z.$

This assumption plays two roles. First, it guarantees that firms have an incentive to borrow to the limit every period. Second, it ensures that the borrowing limit is at least as large as the first best level of capital for all firms.

When productivity is certain and constant over time, firms will either repay or default with probability one on any loan. Thus, there is no equilibrium default, as loans that will be defaulted upon with probability one are not offered. Hence, debt contracts are offered at the risk-free rate with $B'_R = (1+r)(B' + \xi)$ for $0 < B' \leq \overline{B}(z)$, where $\overline{B}(z)$ is defined as the borrowing limit of a firm with productivity z. $\overline{B}_R(z) = (1+r)(\overline{B}(z) + \xi)$ is the associated debt repayment.

The assets of the firm are equal to the level of capital $K_{fb}(z)$, which is constant over time at the first best level as its return is equalized in equilibrium to the constant return on bonds. Given that $\beta(1+r) < 1$, the firm chooses optimally to borrow to the limit. Thus, the value of a firm with productivity z and debt repayment B_R is given by

$$V^c(K_{fb}(z), B_R, z) = [zK_{fb}(z)^\alpha - B_R + \overline{B}(z) - K_{fb}(z)] + \beta V^c(K_{fb}(z), \overline{B}_R(z), z).$$

For the case when $B_R = \overline{B}_R(z)$ the value of this firm is equal to

$$V^c(K_{fb}(z), \overline{B}_R(z), z) = \frac{1}{1-\beta}[zK_{fb}(z)^\alpha - K_{fb}(z) - r\overline{B}(z) - (1+r)\xi].$$

Given that more productive firms have larger capital, as long as debt limits are weakly increasing in productivity (which happens in equilibrium), these firms also have larger values. The borrowing limit for a firm with productivity z is the level of debt that makes the contract value equal to the default value, and is given by

$$V^c(K_{fb}(z), \overline{B}_R(z), z) = \frac{1}{1-\beta}[zK_{fb}(z)^\alpha - K_{fb}(z) - r\overline{B}(z) - (1+r)\xi] = \theta V^c(0,0,z).$$

The default value is endogenous and depends on the probability of owning a new project in the future. A new entering firm starts with zero debt and capital, borrows to the limit in the first period and invests the first best level of capital for production the following period. Its value is given by:

$$V^c(0,0,z) = [\overline{B}(z) - K_{fb}(z)] + \frac{\beta}{1-\beta}[zK_{fb}(z)^\alpha - K_{fb}(z) - r\overline{B}(z) - (1+r)\xi].$$

Combining the above two equations, we derive the debt limit as

$$\overline{B}(z) = \frac{K_{fb}(z)\left[(1+r)(1-\theta\beta) + \alpha(\theta-1)\right] - \xi\alpha(1+r)\left(1-\theta\beta\right)}{\alpha\left(r\left(1-\theta\beta\right) + \theta\left(1-\beta\right)\right)} \tag{12}$$

Large and productive firms have looser borrowing limits than small firms, independent of the degree of financial development. Also, independent of productivity, stronger financial development (lower ξ) increases the loan availability for all firms.

Furthermore, the maximum loan relative to capital for a firm with productivity z is

$$\frac{\overline{B}(z)}{K_{fb}(z)} = \frac{1+r}{\alpha\left(r\left(1-\theta\beta\right) + \theta\left(1-\beta\right)\right)}\left\{(1-\theta\beta) + \frac{\alpha(\theta-1)}{1+r} - \frac{\xi\alpha\left(1-\theta\beta\right)}{K_{fb}(z)}\right\}. \tag{13}$$

The relation between debt limits to assets and size is affected by the economy's financial development or easiness to acquire credit information which is parameterized by ξ. When credit information is free ($\xi = 0$), all firms face the same borrowing limits relative to assets. This is because the problem is homogeneous with respect to z. When credit costs are large ($\xi > 0$), small firms are constrained in their borrowing relative to large firms because the credit costs are a bigger burden for them. Moreover, the disadvantage of small firms relative to large firms becomes more pronounced as ξ increases. The following proposition summarizes this finding.

Proposition 1. *In the case without uncertainty, $\delta = 0$, and under assumptions 1 and 2, the relation between debt limits to assets and firm size is decreasing in the degree of financial development:*

$$\frac{d^2\overline{B}(z)/K(z)}{dK(z)d\xi} > 0.$$

Proof. Direct differentiation of equation (13) delivers the result.

Deriving analytical expressions for debt limits in the case with uncertain productivities is difficult due to lack of analytical solutions for the firm's decision rules of debt and investment. However, all these results regarding borrowing limits, sizes, and financial development carry through when we solve numerically the model for the more general case with uncertainty.

4 Quantitative Implications of the Model

We now assess quantitatively our model mechanism in reproducing the facts regarding the financing and growth patterns observed in the firm level data of Europe. We calibrate our model to two example countries, Bulgaria and the United Kingdom, as representative of countries with weak and strong financial development. The model can quantitatively account for the relation of firm size with growth and leverage found in each country. We also show that as in the data, our model predicts that stronger financial development alone can simultaneously generate smaller growth rates and higher leverage ratios of small firms relative to large firms.

4.1 Calibration

We calibrate the model twice to match Bulgarian and British data in 2005 respectively. The following parameters are chosen independently of the model equilibrium. The interest rate r is set to 4% per annum for Bulgaria and 2% per annum for the UK, which correspond to the real interest rates in these countries from IFS.[14] The decreasing returns parameter α is chosen to be 0.90, following Atkeson and Kehoe (2005). The probability to re-access credit markets after default θ is set to 0.10 following Chatterjee et al. (2007) so that the average number of years that defaulters are excluded from credit markets equals 10 years.

All other parameters are calibrated jointly such that our model produces relevant moments of Bulgarian and British firm datasets. We assume that firms' idiosyncratic productivity consists of a permanent component μ_z and an i.i.d. component ε such that the productivity for firm i equals $z_t^i = \mu_z^i \cdot \varepsilon_t^i$. To make the distribution of firms in our model tractable we choose a finite number of μ_z and ε_t to parameterize the distribution of productivity. We assume that μ_z can take five values $\mu_z \in \{\mu_z^1, \mu_z^2, \mu_z^3, \mu_z^4, \mu_z^5\}$ and that ε_t can take two values $\{\varepsilon_L, \varepsilon_H\}$. Each μ_z is assumed to have equal mass. Without loss of generality, we assume that transitory shocks have a mean of one, and thus the low shock ε_L and its probability p_L are sufficient to capture the transitory idiosyncratic shock process. We jointly

[14]The real interest rate is constructed as the difference between the annual nominal lending rate and the inflation rate.

calibrate $\{\mu_z^1, \mu_z^2, \mu_z^3, \mu_z^4, \mu_z^5, \varepsilon_L, p_L, \beta, \xi, \delta\}$ to match the following ten moments in the data: the median asset levels of five asset quintiles in each country, the average real sales growth rate from 2004 to 2005 of 53% in Bulgaria and 11% in the UK, the average coefficient of variation for sales across firms of 0.40 in Bulgaria and 0.3 in the UK, the mean leverage ratio of 0.65 in Bulgaria and 0.84 in the UK, the leverage ratio of firms in the first asset quintile of 0.45 in Bulgaria and 1.18 in the UK, and the mean age of firms of 10 years across countries in Europe.[15] Table 3 summarizes all the parameter values in the calibration.

Table 3: Parameter Values in Benchmark Calibration

		Values		
	Parameter	Bulgaria	United Kingdom	Target
Interest rate	r	0.04	0.02	Annual real interest rate
Re-entry prob.	θ	0.10	0.10	Chatterjee et al (2007)
Technology	α	0.90	0.90	Atkeson and Kehoe (2005)
Permanent prod.	$\mu_z^1, ..., \mu_z^5$	$1.26, 1.47, 1.61$ $1.80, 2.12$	$1.22, 1.39, 1.59,$ $1.86, 2.27$	Median quintile asset
Temporary prod.	$\varepsilon_L, \varepsilon_H$	0.21, 1.13	0.48, 1.08	Mean CV sales
	p_L	0.145	0.13	Mean sales growth rate
Death rate	δ	0.08	0.08	Mean age of firms
Credit cost	ξ	0.03	0.0	Leverage for 1st asset quintile
Discount factor	β	0.94	0.96	Mean leverage

The calibrated ξ parameter for Bulgaria equals 0.03 which corresponds to 0.08% of output for the average firm. The credit costs are higher for the smallest firms and equal 4.3% of the output of firms in the first asset quintile. The calibrated ξ parameter for the UK equals zero.[16]

4.2 Model Dynamics

Before presenting the quantitative results, we demonstrate how firms adjust debt, capital and dividend policies in response to transitory shocks. These responses drive the growth and financing dynamics over time. When experiencing sequences of bad shocks, firms reduce their scale and increase their debt financing. After good shocks, firms expand their scale and reduce their debt. These dynamics imply that firms with the same permanent productivity display different sizes that depend on the history of shocks. Across these firms, inefficiently small firms tend to have higher growth rates and higher leverage ratios.

[15] The coefficient of variation for sales is computed from the detrended time series of real sales of each firm for 2000-2005.

[16] In the calibration we restrict ξ to be non-negative, and for the UK this constraint is binding.

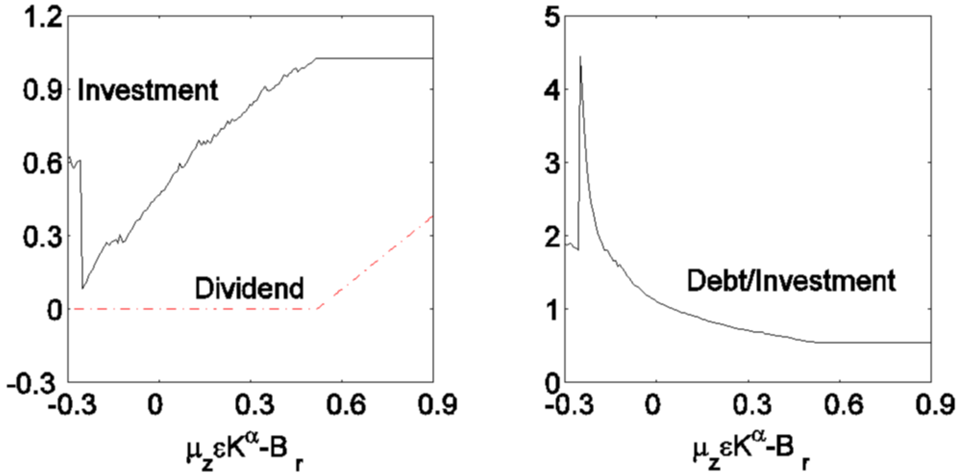

Figure 3: Policy Rules

Consider a firm with median permanent productivity. The optimal policies for this firm depend on a single endogenous state variable: the firm's 'cash on hand' which equals output minus debt repayment, $\mu_z \varepsilon K^{\alpha} - B_R$. Cash on hand encodes all the information regarding the firm's history of shocks and it is low when firms have a low productivity shock, large debt due, and small capital. In Figure 3 we plot optimal investment, dividends, and debt relative to investment as a function of cash on hand under the parameter values in the British calibration. To provide an economic interpretation of the numbers, we report investment, dividends and cash on hand relative to the mean output of this firm.

With large cash on hand, the firm invests the first best level, distributes dividends and holds a low level of debt. The low debt level is due to a precautionary savings motive as in standard precautionary savings models (Aiyagari 1994 and Huggett 1993). With uncertainty the firm may not find it optimal to exhaust its borrowing opportunities because large debt increases the likelihood of firm failure. Thus, the firm has incentives to decrease its debt level whenever possible to insure against a possible stream of bad shock realizations.

With intermediate levels of cash on hand, the firm stops paying dividends, increases new loans and decreases investment. The firm lowers investment to prevent debt from increasing too rapidly because large debt increases default risk. Avoiding future default is beneficial because the expected value of keeping the project is large. Thus, the firm is willing to be inefficiently small in its production.

With low levels of cash on hand, the firm has very large debt to repay and finds it no longer optimal to avoid default. In anticipation of default under the low shock the following period, the firm chooses high debt and adjusts investment to a more appropriate scale for the high shock only.[17] Nevertheless, investment is still smaller than that for a firm with large cash on hand because the expected marginal product of capital is lower due to the firm failure in the defaulting state.

4.3 Main Quantitative Results

We now study the quantitative implications of the calibrated model. Specifically, we compute and simulate the model twice, one under the Bulgarian calibration and one under the British calibration. In each simulation, we obtain a model economy with 15,000 firms over 500 periods. The model delivers in the long run a cross-section distribution of firms, which we use to compute the model's statistics. At every point in time, we divide the cross section of firms into five asset quintiles. In the model, firm size equals the assets of the firm: capital K plus savings B_R for firms with $B_R < 0$. We compute for every asset quintile and for the entire distribution of firms, average sales growth rates, leverage ratios, and median asset levels.

As shown in the previous section, firm size depends on its permanent productivity and also its history of transitory shocks. Specifically, a firm is small either because it has a low level of permanent productivity (*unproductive*) or because it has a sequence of bad shocks (*unlucky*). The different reasons why firms are small have different financing implications. Unproductive small firms tend to have low debt to asset ratios given the restrictive schedule of loan contracts they face. Unlucky small firms, however, tend to have high debt to asset ratios as a result of the bad shocks. When hit with good shocks, these unlucky firms also tend to have high growth rates since they can expand their size to a more efficient level. These key model mechanisms and their interaction with financial development help understand the quantitative implications of the model.

Bulgaria Calibration

Let's first consider the results from the model calibrated to Bulgarian data. The upper panel of Table 4 reports the data and the model statistics for this calibration. The results show that the target moments of Bulgarian data are successfully matched in the calibration. More importantly, the calibrated model accounts for the size-growth and size-financing patterns. The calibration targets the average leverage ratio for the whole economy and for the smallest firms, and the average sales growth of firms. The model then generates a negative size-growth relation that matches the data well. The growth rate for the smallest firms equals to 77%

[17] The jump in investment in the default region of Figure 3 is an artifact of our two discretized idiosyncratic shocks.

Table 4: Growth, Leverage and Firm Size: Bulgaria Calibration

	Data			Model			Model		
	Bulgaria			Calibration $\xi = 0.03$			Stronger Fin. Dev. $\xi = 0$		
Quintile	Assets	Growth	Lev.	Assets	Growth	Lev.	Assets	Growth	Lev.
1	1	0.73	0.45	1	0.77	0.47	1.1	0.58	0.79
2	4.6	0.51	0.63	5.1	0.56	0.62	5.2	0.55	0.76
3	12	0.51	0.72	13	0.46	0.60	13	0.50	0.71
4	38	0.49	0.73	41	0.48	0.62	40	0.41	0.68
5	198	0.39	0.71	202	0.40	0.68	207	0.40	0.70
Mean	51	0.53	0.65	52	0.53	0.60	53	0.49	0.73
	United Kingdom			Calibration $\xi = 0$			Weaker Fin. Dev $\xi = 0.03$		
Quintile	Assets	Growth	Lev.	Assets	Growth	Lev.	Assets	Growth	Lev.
1	1	0.23	1.18	1	0.17	0.95	0.7	0.25	0.42
2	5	0.13	0.87	4	0.09	0.79	3.4	0.17	0.50
3	17	0.05	0.79	14	0.10	0.82	14	0.10	0.82
4	66	0.07	0.71	71	0.10	0.82	70	0.10	0.82
5	508	0.05	0.66	516	0.08	0.80	513	0.07	0.80
Mean	120	0.11	0.84	121	0.11	0.84	120	0.14	0.67

and for the largest firms equals to 40%. The model also generates an increasing leverage pattern ranging from 0.47 for the smallest firms to 0.68 for the largest firms.

To understand the quantitative results, let's look at the composition of firms in each asset quintile. The majority of firms in asset quintile i have permanent productivity μ_z^i. For example, there are 98% of firms with permanent productivity μ_z^1 and 2% with μ_z^2 in the first asset quintile. Therefore, the overall model implications are driven by the difference in the financing and growth patterns across permanent productivity. With the presence of fixed credit cost, firms with lower permanent productivity face a more restrictive schedule of loan contracts. Thus, they tend to have lower equilibrium leverage because firms endogenize the constraints when making financing decisions. These unproductive firms also have on average more inefficient size due to the more restrictive borrowing opportunities. Thus, they tend to have higher growth as they increase capital using the extra resources coming from the good shock. Therefore, we observe a positive size-leverage relation and a negative size-growth relation in the Bulgaria calibration.

Our model is also consistent with several other empirical predictions. First, the model predicts that firms who default have larger leverage ratios than continuing firms. This implication is consistent with Campbell et al. (2006) who find that the leverage ratios of failing public firms in the United States are larger than those of continuing firms. For the Bulgaria calibration, 1.8% of firms default every period. The average leverage ratio of these firms equals 1.78, whereas the average leverage ratio of continuing firms is 0.58. Second, the model predicts that large firms are the ones who distribute dividends, which is consistent with U.S. data as documented in Fazzari et al. (1988). In this calibration of our model, 75% of firms in the first asset quintile do not pay any dividends, compared to 62% for firms in the fifth asset quintile.

Our results demonstrate that financial frictions can rationalize quantitatively the growth-size relation observed in Bulgaria. The exercise of this paper is revealing because it uses the financial variables of firms to discipline the extent to which the growth-size relation can be attributed to financial imperfections. By parameterizing the model to mirror the debt financing patterns of firms, we find that the results deliver quantitatively the growth-size relation in the data.

United Kingdom Calibration

We now analyze the results from the model calibrated to British data. The lower panel of Table 4 reports the data and model statistics for this calibration. The calibration successfully match the target moments in terms of an average leverage ratio of 0.84 and an average growth rate of 0.11. The calibration, however, produces a leverage ratio of the first asset quintile lower than that in the data, 0.95 versus 1.18. The reason is that we restrict the fixed credit cost ξ to be non-negative, which prevents the model from generating an excessively high leverage ratio. Nonetheless, the model delivers a negative relation between size and leverage and a negative relation between size and growth, though the fit is less tight than that in Bulgaria. Specifically, the leverage ratio and growth rate are 0.95 and 0.17, respectively, for small firms in the first asset quintile, while they are 0.80 and 0.08, respectively, for large firms in the fifth asset quintile.

In this calibration, most of the firms in each asset quintile have the corresponding permanent productivity level, as in the Bulgarian calibration. For example, there are 94% of firms of permanent productivity μ_z^1 and 6% of firms of permanent productivity μ_z^2 in the first asset quintile. The fraction of unlucky firms, however, is higher than that in the Bulgarian calibration because firms have better borrowing opportunities to sustain a longer sequence of bad shocks while becoming very inefficiently small. Since the model is homogeneous across permanent productivity when $\xi = 0$, the financing and growth patterns of unlucky firms drive the overall quantitative results. As shown in the previous section, unlucky firms tend to grow faster and have higher leverage ratios. Thus, we observe in the cross section that small firms grow faster and have large levels of debt, and large firms grow slower and have lower levels of debt.[18]

The British calibration also delivers that the leverage ratios of firms that default are larger than those of the continuing firms, 1.87 versus 0.83. In this calibration 1% of firms default every period. In regards to dividends, this calibration also generates that small firms are less likely to pay dividends than large firms. In particular, 61% of firms in the first asset quintile do not pay any dividends compared to 52% for firms in the fifth asset quintile. Comparing with the Bulgarian calibration, these findings also imply that firms in economies with stronger financial development are more likely to pay dividends. Paying dividends more often is intrinsically related to a lower precautionary motive for firms in economies with larger loan availability.

4.4 Varying Credit Costs

Let's now consider the comparative static of changing credit costs when all other parameters remain unchanged. Consider first this exercise in the Bulgarian calibration, where we lower ξ to zero to increase the degree of financial development. The last three columns of the upper panel of Table 4 report these results.

When credit costs equal to zero both leverage ratios and growth rates are decreasing in size. Lower credit costs reduce the difference in growth rates between the smallest and largest firms from 37 percentage points (77% relative to 40%) to 18 percentage points (58% relative to 40%). Lower credit costs also increase the difference in leverage ratios from -21 percentage points (0.47 relative to 0.68) to 9 percentage points (0.79 relative to 0.70). Thus, our model is fully consistent with the evidence documented in this paper that better financial development is associated with a smaller difference in growth rates between small and large firms but a bigger difference in leverage ratios between small and large firms.

Lowering credit costs also has a level effect for debt financing and output of all firms. When credit costs equal zero, the average leverage ratio in the Bulgarian calibration increases substantially from 0.60 to 0.73. With better financial development, firms engage in less precautionary savings because more loans are readily available in case of low shocks. The availability of loans also increases the average capital level for firms, and thus increases output. The average output for firms in the first asset quintile when $\xi = 0$ is 19% higher than that when $\xi = 0.03$ in the benchmark Bulgarian calibration. The difference in output across these two economies is smaller for bigger firms, but remains sizeable and equal to 1.5% for the firms in the fifth asset quintile.

[18] The intrinsic positive comovement of growth rates and leverage ratios present in our model with zero credit costs is similar to the one analyzed by Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), and DeMarzo and Fishman (2007).

The model delivers a similar comparative static of credit costs under British calibration. The last three columns of the lower panel of Table 4 report the model statistics under the British calibration but with weaker financial development, $\xi = 0.03$. The results show that in such an economy small firms grow faster and engage in less debt financing. Moreover, the average leverage ratio and the output of small firms are lower when $\xi = 0.03$.

These comparative static results reveal that the impact of the credit cost parameter ξ on growth and leverage are robust for alternative structural parameters. When ξ equals zero, there is no difference in the financing and growth patterns across productive and unproductive firms. The negative size-leverage and size-growth relations are driven by higher debt to asset ratios and faster growth rates of small, unlucky firms. As credit costs increase, unproductive small firms face more restrictive borrowing opportunities than productive large firms. Thus, they tend to have lower debt to asset ratios and higher growth rates due to more inefficient size. This leads to a more accentuated negative relation of firm size and growth, but a less negative relation between firm size and leverage in economies with weaker financial development. Importantly, these mechanisms are quantitatively relevant in our model economies that are calibrated to multiple and different countries.

On the other hand, the results suggest that although financial development appears as a major determinant for the differential relation of firm size with growth and leverage across countries, the overall average growth rate and size of firms are driven by other factors such as the productivity process for firms. The reason is that aggregate statistics are driven mainly by the largest firms in the economy and for them the fixed credit costs are minor.

4.5 Varying Default Value

Financial development impacts the credit accessibility of firms. In our model with limited enforceability of contracts, various parameters control the availability of loans. One is the fixed credit cost ξ, which affects small unproductive firms the most. Another one is the re-entry probability of entrepreneurs after default θ, which increases the default values of all firms proportionately. A larger θ could also proxy weaker financial development because of more limited credit available. In this subsection, we examine how the size-leverage relation and the size-growth relation vary with θ. We find that the implications of changing θ are consistent with the documented cross-country size-leverage patterns. The implications on the cross-country size-growth pattern, however, are at odds with the data.

In Table 5 we report these results for the two benchmark calibrations, Bulgaria and the UK. Let's start with the results in the right panel for the UK calibration, where the re-entry probability θ increases from 0% to 30%. We find that on average firms use less

Table 5: Sensitivity Analysis

	Bulgaria Calibration				United Kingdom Calibration			
	$\theta = 0.3$		$\theta = 0.0$		$\theta = 0.3$		$\theta = 0.0$	
Asset Tercile	Growth	Lev.	Growth	Lev.	Growth	Lev.	Growth	Lev.
1	0.70	0.54	0.76	0.63	0.19	0.90	0.17	0.95
2	0.53	0.58	0.43	0.58	0.11	0.80	0.10	0.82
3	0.37	0.66	0.34	0.56	0.08	0.72	0.06	0.73
Mean	0.53	0.59	0.51	0.60	0.13	0.81	0.11	0.83

debt financing and grow faster due to more restricted borrowing opportunities under higher default values. We also find that the size-leverage relation becomes slightly less negative and the size-growth relation remains almost the same. This is because higher default values give firms more incentive to default and less incentive to shrink their size to avoid failure after bad shocks. Thus, there are fewer unlucky firms of high permanent productivity among small asset firms, which tends to lower the negative relations of firm size with growth and leverage. The effects, however, are quantitatively small as shown in the Table.

Increasing the re-entry probability has similar qualitative effects under the Bulgaria calibration, as reported in the left panel of Table 5. The quantitative impacts, however, are bigger and more pronounced than those in the UK calibration. In particular, a higher θ leads to a positive size-leverage relation. With fixed costs $\xi > 0$, unproductive small firms have the contract values disproportionately smaller than productive large firms. Thus, the unproductive small firms increase their default incentives more when a higher θ raises the default values proportionately across firms. This implies that these small firms face a more limited loan schedule and thus reduce their debt to asset ratios more. Also, the growth rates of these firms are lowered due to their unwillingness to shrink their size under higher default incentives.

These comparative static results reveal that varying the proportional re-entry probability θ and varying the fixed credit cost ξ have different implications on the relations of size with financing and growth. When varying θ, we find that the model can deliver the comparative static we found in the data for debt financing, but not for growth. When varying ξ, we find that the model can deliver the documented cross-country patterns for both debt financing and growth. Thus, financial frictions that feature a fixed component independent of the firm size have a better chance of capturing the different patterns of debt financing and growth found in the data.

Theoretical models with limited enforcement controls the availability of credit by its modeling of pre-default conditions, such as credit costs and post-default conditions, such as bankruptcy costs, gains from diverting firms' resources, and the probability to re-access credit markets. The message of this exercise is that models of firm dynamics and financial frictions (Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2006), and DeMarzo and Fishman (2007)) have the potential to produce the quantitative patterns of leverage and growth with size across countries with a flexible specification of the default value or contract value that contains a friction with a constant component. Comparative static results on varying a constant default value in our model would deliver similar results as our comparative static on ξ. The advantage with ξ is that we can intuitively link it to the cross-country variation in credit registries.

5 Conclusion

We have studied both empirically and theoretically the growth and debt financing patterns of firms across countries. Using a broad and comprehensive firm level database from 22 European countries, we have documented that in less financially developed countries –countries with lower private credit to GDP or limited credit bureau coverage– small firms grow faster and use less debt financing than large firms. These findings are robust to controlling for age, sector and country fixed effects. Our empirical analysis provided a new picture of the relation between financial development with debt financing and growth across firms and countries.

We then developed a quantitative dynamic model of heterogeneous firms where financial development affects firm financing and growth through the accessibility to credit. By calibrating the degree of financial development to the observed debt financing of firms, we assessed the model implications on firm growth. We found that financial development is important in explaining quantitatively the difference in growth rates across firms and across countries.

A contribution of the paper is to use micro firm level data in a quantitative model to study the growth and financing patterns in the cross section of firms of multiple countries. A natural next step is to analyze a time dimension by introducing aggregate fluctuations in the model to study the cyclical features of firm dynamics. Moscarini and Postel-Vinay (2008) document that for the U.S. the variance in the firm size distribution is procyclical and the early phases of booms are mainly driven by the expansion of small firms. Our framework can prove useful in analyzing the impact of financial frictions on the cyclical cross-section firm dynamics. More generally, we view our quantitative methodology that combines firm level data with theory as a useful tool to analyze the interaction of micro decisions with macro implications.

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Empirical Appendix

In this appendix, we first examine the comparability of the country samples. We then describe in detail the procedure for cleaning the data in the regressions. We next document the unconditional statistics for leverage and growth with size across countries. Finally, we present robustness checks of the main empirical regressions reported in Table 2.

Comparability of Country Samples

This section analyzes the coverage and comparability of the Amadeus dataset across countries. The European Commission Report contains information on the distribution of the universe of firms in the business sector for most of the countries in our sample. They report the percentage of enterprises that have 1 to 9 employees, 10-49 employees, 50-250 employees and above 250 employees. Hence, we compare the fraction of firms for each employment category in our datasets with that in the universe from the report.[19]

Unfortunately, the employment information is not reported for every firm in Amadeus. The lack of employment data can be a severe problem for some countries. For example, in the Netherlands only 65% of firms in the business sector that report assets and liabilities report employment. Moreover this lack of employment information is the most severe for small firms. Hence, we impute employment measures for firms that do not report employment in Amadeus. To do this, we run regressions country by country of log(employment) on log(assets) and log(liabilities). The fit of these regressions is good with R squares above 0.6 for all countries.[20] We then impute employment for the firms that do not report it using the estimated coefficients and their assets and liabilities.

Table 6 reports the firm distribution in Amadeus and in the universe for countries that we have data for. The table shows that in our sample, the majority of firms are small with only 1 to 9 employees (micro firms) as in the data. In our sample, on average 70% of firms are micro firms, whereas in the universe of firms 89% are micro. In our sample, only about 1% of firms have more than 250 employees, which is consistent with the universe where less than 1% of firms fall into this category. Importantly the coverage in Amadeus is similar across countries, with most countries having micro firms between 60% and 80%.

[19] For this comparison we include only firms in sectors that correspond to the business sectors in the European Commission Report.

[20] Introducing additional controls such as firm age and sector dummies change the fit of the regressions only marginally.

Table 6: Coverage and Comparability of Country Datasets

	Amadeus Dataset				European Commission – Universe			
	Micro 1-9	Small 10-49	Medium 50-250	Large >250	Micro 1-9	Small 10-49	Medium 50-250	Large >250
Belgium	0.895	0.087	0.014	0.004				
Bulgaria	0.689	0.233	0.062	0.016	0.902	0.08	0.016	0.002
Croatia	0.625	0.264	0.089	0.022				
Czech Rep	0.601	0.269	0.106	0.024	0.953	0.038	0.008	0.001
Denmark	0.765	0.189	0.037	0.009	0.869	0.109	0.019	0.003
Estonia	0.788	0.179	0.029	0.004	0.815	0.151	0.03	0.004
Finland	0.797	0.160	0.033	0.010	0.924	0.061	0.012	0.003
France	0.810	0.156	0.027	0.006	0.923	0.064	0.01	0.003
Iceland	0.911	0.083	0.005	0.001				
Ireland	0.736	0.236	0.025	0.002				
Italy	0.695	0.264	0.034	0.007	0.946	0.048	0.005	0.001
Latvia	0.317	0.408	0.229	0.046	0.831	0.139	0.027	0.003
Lithuania	0.270	0.444	0.250	0.035	0.755	0.197	0.043	0.005
Malta								
Netherlands	0.750	0.198	0.040	0.012	0.89	0.091	0.016	0.003
Norway	0.795	0.182	0.020	0.003				
Portugal	0.773	0.196	0.028	0.004				
Romania	0.875	0.098	0.023	0.005	0.881	0.09	0.023	0.006
Russia	0.314	0.485	0.164	0.037				
Spain	0.727	0.238	0.030	0.006	0.923	0.068	0.008	0.001
Sweden	0.819	0.144	0.030	0.007	0.947	0.043	0.008	0.002
UK	0.755	0.190	0.040	0.015	0.864	0.114	0.018	0.004
Average	0.700	0.224	0.063	0.013	0.887	0.091	0.017	0.003

Data Cleaning Procedure

In this section, we describe the detailed procedures in assembling the cross-country datasets analyzed in the empirical section. In particular, we present step-by-step data cleaning procedures, construction methods of all the variables, and data sources for the country level statistics.

Firm Data

We download the data from the Amadeus database compiled by Bureau Van Dijk Electronic Publishing. We delete all firms in the financial and government sectors which correspond to NACE codes 65, 66, 67, and 75. We delete firms that have one or more of the following characteristics: missing total assets, non-positive total assets, missing total liabilities and negative liabilities.

For the leverage regressions, we generate the leverage variable for each firm by taking the ratio of the firm's total liabilities to total assets. We drop the outlier firms with leverage ratios in the top 1% percentile of the leverage distribution in each country. We generate the *Asset Share* variable by dividing the firm assets by the sum of total assets in its country. We generate the interaction variables by multiplying *log(Asset Share)* by private credit to GDP or by credit coverage.

For the growth regressions we follow these additional steps. We drop the firms with missing, zero or negative operating revenue (or sales) in 2004 and 2005. We generate the real growth variable as

$$\frac{\text{operating revenue}_{05} * \text{exchange rate depreciation}_{05/04}}{\text{operating revenue}_{04} * \text{cpi inflation}_{05/04}} - 1.$$

We drop outlier firms with growth rates in the top 1% percentile of the growth distribution in each country. For this new clean sample, we generate the *log(Asset Share)* variable and the interaction variables as described above for the leverage regressions.

For both regressions, we construct dummy variables for age groups. Firms are classified into 7 age groups based on the firm age in terms of years: $[0, 5)$, $[5, 10)$, $[10, 15)$, $[15, 20)$, $[20, 25)$, $[25, 30)$, $[30, \infty)$.

Country Data

The country-level statistics are obtained from various data sources. Private credit to GDP from 2000 to 2004 is from the *World Development Indicators* of the World Bank. Credit bureau coverage in 2005 is from *Doing Business 2006* published by the World Bank. Exchange rates, defined as local currency per Euro, and CPI inflation from 2004 to 2005 are from the *International Financial Statistics* of the International Monetary Fund.

Unconditional Firm Size, Leverage and Growth

In this section we report unconditional relations of firm size with growth rates and leverage ratios for the samples of firms in the main regressions. In every country we divide firms into asset quintiles according to their assets, and for every quintile we compute mean growth and mean leverage. Table 7 reports these statistics.

The table shows that on average across these 22 European countries small firms have higher leverage ratios and higher growth rates than large firms. We analyze the unconditional correlations of the difference in growth rates and leverage ratios of firms in the smallest quintile and in the largest quintile and financial development across countries. The

Table 7: Unconditional Leverage and Growth across Asset Quintiles

	Leverage					Growth				
	Asset Quintiles					Asset Quintiles				
	1	2	3	4	5	1	2	3	4	5
Belgium	0.98	0.69	0.68	0.67	0.66	0.13	0.05	0.05	0.05	0.05
Bulgaria	0.45	0.63	0.72	0.73	0.71	0.73	0.51	0.51	0.49	0.39
Croatia	0.63	0.67	0.68	0.68	0.64	0.01	0.01	0.02	0.05	0.08
Czech Rep	0.89	0.87	0.76	0.69	0.58	0.66	0.36	0.25	0.17	0.17
Denmark	0.54	0.60	0.59	0.58	0.58	0.25	0.13	0.11	0.15	0.17
Estonia	0.15	0.40	0.50	0.53	0.54	0.88	0.56	0.45	0.37	0.43
Finland	0.51	0.58	0.57	0.56	0.56	0.26	0.15	0.12	0.12	0.13
France	0.86	0.75	0.72	0.70	0.66	0.17	0.08	0.06	0.06	0.06
Iceland	1.10	1.00	0.88	0.83	0.74	0.91	0.55	0.36	0.50	0.61
Ireland	1.54	0.95	0.76	0.66	0.63	0.24	0.20	0.17	0.15	0.16
Italy	0.79	0.83	0.83	0.81	0.77	0.16	0.14	0.11	0.09	0.08
Latvia	0.76	0.75	0.71	0.69	0.66	0.78	0.42	0.27	0.33	0.34
Lithuania	0.65	0.64	0.61	0.59	0.58	1.46	0.50	0.36	0.31	0.29
Malta	1.01	0.77	0.72	0.66	0.61	0.57	0.35	0.34	0.13	0.26
Netherlands	1.38	0.95	0.82	0.75	0.69	0.20	0.12	0.07	0.07	0.10
Norway	0.86	0.80	0.78	0.76	0.70	0.44	0.23	0.21	0.20	0.21
Portugal	0.94	0.79	0.77	0.75	0.73	0.24	0.12	0.09	0.08	0.08
Romania	1.32	1.05	0.91	0.81	0.77	0.84	0.57	0.42	0.25	0.18
Russia	0.88	0.85	0.78	0.74	0.69	1.09	0.78	0.59	0.36	0.34
Spain	0.86	0.80	0.75	0.70	0.65	0.45	0.22	0.17	0.19	0.25
Sweden	0.56	0.61	0.62	0.63	0.66	0.28	0.16	0.15	0.15	0.17
UK	1.18	0.87	0.79	0.71	0.66	0.23	0.13	0.05	0.07	0.05
Average	0.89	0.76	0.72	0.69	0.66	0.51	0.30	0.23	0.20	0.21

correlations of the growth difference with private credit to GDP equals -0.63 and with the credit coverage equals -0.41. The correlations of the leverage difference with private credit to GDP equals 0.42 and with credit coverage equals also 0.42. These unconditional correlations feature similar patterns as those documented in the main regression results.

Robustness of the Main Regressions

This section analyzes the robustness of the cross-country regression results presented in Table 2. We first consider employment as an alternative firm size measure. Second, we consider adding additional interactions of size with industry, age and GDP per capita. We find that the results of the main regressions are maintained under these alternative specifications, except under one growth regression with all these interactions and financial development measured as credit bureau coverage.

Employment as Firm Size

Table 8 reports four leverage and growth regressions where firm size is defined by employment. Employment is either the actual number of employees reported by each firm or the imputed employment measure constructed in the section of the comparability of the country samples. The share of employment of a firm equals the ratio of its employment to the total employment in its country. The sample is the same as that in the main regressions with the exception of the firms that do not report employment and at the same time have zero liabilities. We exclude these firms when imputing their employment.

The results show that using employment as an alternative measure of firm size does not change our main conclusions. Small firms grow faster than large firms and use less debt financing in less financially developed countries. The interaction coefficients in the four regressions have signs as expected and are statistically significant at the 1% level.

Table 8: Robustness: Employment as Size

	Leverage		Growth	
	Private Credit to GDP	Credit Bureau Coverage	Private Credit to GDP	Credit Bureau Coverage
Size (log(employment share))	-0.015***	-0.019***	-0.056***	-0.033***
	(0.0004)	(0.0003)	(0.001)	(0.001)
Interaction (credit to GDP × size)	-0.012***		0.026***	
	(0.004)		(0.001)	
Interaction (credit bureau coverage ×size)		-0.020***		0.0013***
		(0.0003)		(0.0006)
Fixed Effects Country×Industry×Age	Yes	Yes	Yes	Yes
Adjusted R^2	0.05	0.05	0.05	0.05
Number Observations	4459799	4459799	2553490	2553490

Additional Interactions: Size with Industry, Age Group and GDP per capita

We now conduct additional robustness tests of the leverage and growth regressions by adding three additional variables: the interactions of firm size with the two-digit industry categories, with the seven age groups, and with the country's GDP per capita. By doing so, we allow the relation of size with growth and leverage to be industry and age dependent and to vary with the log of the country's GDP per capita. In particular, the industry-specific relation of growth and firm size has been studied by Rossi-Hansberg and Wright (2007). They build a model where the growth difference between small and large firms is larger in sectors that use physical capital more intensively. Their model accounts for the fact that in the US, the growth rate of firms declines faster with size in the manufacturing sector than in the service sector.

We conduct sensitivity analysis using 12 regression specifications, as reported in Table 9 and 10. We find that in 11 of these cases, the main regression results remain unchanged. For both the leverage and growth regressions with either measure of financial development, we experiment with three specifications by adding the three new interaction terms one-by-one. Table 9 shows that in the leverage regressions, the coefficient on the interaction of firm size and private credit to GDP or credit bureau coverage remains negative and significant when adding the interactions of firm size with industry, with age, and with the country's per capita GDP. Table 10 shows that in the growth regressions, the coefficient on the interaction of firm size and private credit to GDP remains positive and statistically significant with these three additional interaction variables. The table also shows that the coefficient on the interaction of firm size and credit bureau coverage remains positive and significant when additional industry and age interactions are introduced. This coefficient, however, becomes insignificant when we add the interaction of firm size and GDP per capita.

Table 9: Robustness on Leverage Regression: Industry, Age and GDP per Capita Interactions

	Private Credit to GDP			Credit Bureau Coverage		
Interaction (credit to GDP × size)	-0.012***	-0.010***	-0.020***			
	(0.0003)	(0.0003)	(0.0004)			
Interaction (credit bureau coverage ×size)				-0.014***	-0.013***	-0.018***
				(0.0003)	(0.0003)	(0.0003)
Interactions (industry ×size)	Yes	Yes	Yes	Yes	Yes	Yes
Interactions (Age Group ×size)	No	Yes	Yes	No	Yes	Yes
Interaction (log(GDP per capita)×size)	No	No	0.016***	No	No	0.010***
			(0.0004)			(0.0003)
Fixed Effects Country×Industry×Age	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	0.06	0.07	0.07	0.06	0.07	0.07
Number Observations	4564461	4564461	4564461	4563685	4563685	4563685

Table 10: Robustness on Growth Regression: Industry, Age and GDP per Capita Interactions

	Private Credit to GDP			Credit Bureau Coverage		
Interaction (credit to GDP × size)	0.033*** (0.0008)	0.041*** (0.001)	0.023*** (0.0008)			
Interaction (credit bureau coverage ×size)				0.009*** (0.0006)	0.016*** (0.0006)	-0.001 (0.0006)
Interactions (industry ×size)	Yes	Yes	Yes	Yes	Yes	Yes
Interactions (Age Group ×size)	No	Yes	Yes	No	Yes	Yes
Interaction (log(GDP per capita) ×size)	No	No	0.025*** (0.0009)	No	No	0.039*** (0.0008)
Fixed Effects Country×Industry×Age	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	0.06	0.06	0.06	0.05	0.06	0.06
Number Observations	2568782	2568782	2568782	2568559	2568559	2568559