Non-Balanced Endogenous Growth

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Abstract

This paper constructs a model of non-balanced endogenous growth. The economy features two sectors with different skill and capital intensities. Technological progress in the two sectors, population growth and capital accumulation drive economic growth. In the long run equilibrium, both output and technology in the two sectors grow at constant rates, but the rate of growth of the more capital-intensive sector is higher. Despite this non-balanced nature of growth, the model is consistent with the Kaldor facts as the growth rate of aggregate output, the real interest rate and the share of capital in GDP are constant in the long run. For the range of parameter values consistent with capital-skill complementarity, the model also features skill-biased technical change in the long run. Finally, technical change in this economy is capital augmenting away from the asymptotic equilibrium, but gradually becomes purely labor augmenting (Harrod neutral) as the economy converges to the asymptotic equilibrium path.

1 Introduction

Two sets of well-known facts about long-run U.S. growth are the Kaldor facts and the Kuznets facts. The first set of facts, which have been central to macroeconomics ever since the 1960s, refer to the relative constancy of the growth rate, the capital-output ratio, share of capital income in GDP and the real interest rate (Kaldor, 1963, see also Denison, 1974, Barro and Sala-i-Martin, 2004). The Kuznets facts, a term we borrow from Kongsamut, Rebelo and Xie (2001), on the other hand, refer to the structural transformation, that is to the systematic change in the relative importance of agriculture, manufacturing and services (see Kuznets, 1957, 1973, Chenery, 1960, Kongsamut, Rebelo and Xie, 2001). While the Kaldor facts emphasize the balanced nature of economic growth, the Kuznets facts highlight the non-balanced nature of growth.

The past two decades have witnessed a renewed interest in theories of economic growth following the onset of the endogenous growth literature, investigating the role of profit motives in determining equilibrium technology and the rate of aggregate economic growth.[1] This literature has almost exclusively focused on balanced growth models, mostly because of their relative tractability, but also because, like the previous exogenous growth literature, it was natural to focus on the Kaldor facts. It is important to understand, however, whether models in which technology in the aggregate and in different sectors is endogenous can generate a realistic growth path where growth is non-balanced consistent with the Kuznets facts, while also maintaining constant growth rate, real interest rate and factor shares in the long run, consistent with the Kaldor facts. This paper presents such a model.

Before discussing the model, it is useful to briefly look at the evidence motivating this investigation. Consistent with the theoretical structure of the model that will follow, Figures 1, 2 and 3 divide the two-digit sectors from the National Income and Product Accounts into two groups: high-skill services and the rest (which includes, agriculture, manufacturing and low-skill services). The high-skill services include health, education, legal services, movies, and insurance.[2] Figure 1 shows that nominal value added in

[1] See, among others, Romer (1986, 1990), Lucas (1988), Rebelo (1991), Segerstrom, Anant and Dinopoulos (1990), Grossman and Helpman (1991a,b), Aghion and Howitt (1992), Jones (1995), Young (1993, 1998), Howitt (2000). Aghion and Howitt (1998) and Barro and Sala-i-Martin (2004) provide excellent introductions.

[2] The average share of college graduates in the total employment of the high-skill services sector over

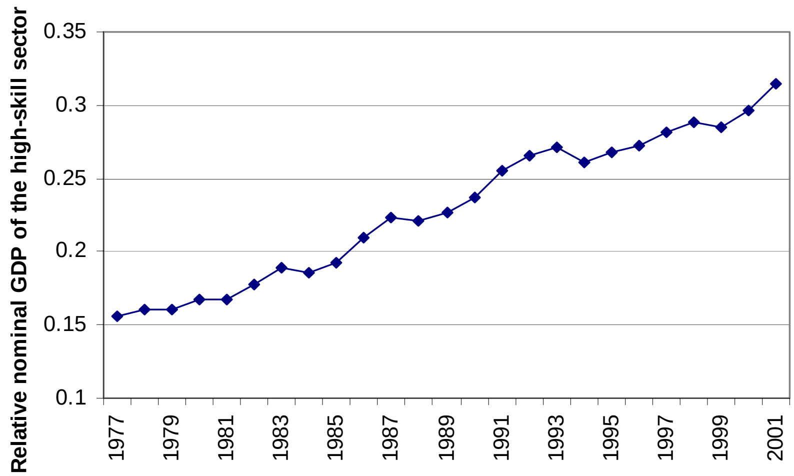

Figure 1: Nominal value added of the high-skill services divided by notional value added in the rest of the economy, 1977-2001.

high-skill services have risen considerably faster than in the rest of the economy.

Figure 2 shows that the relative price of these high-skill services have risen rapidly. Figure 3 combines the information from the previous two figures and shows that the high-skill services group has grown more slowly than the rest of the economy in real terms. It is well-known that measuring real output in the services sector is difficult (e.g., Griliches, 1994), so the trends shown in Figure 3 should be interpreted with caution. Nevertheless, these figures are consistent with a pattern of non-balanced growth where the high-skill services sector is growing faster than the rest of the economy in nominal terms, but more slowly in real terms. Another relevant fact for our model below is that the high-skill services sector is considerably less capital intensive than the rest of the economy; while the average share of capital income in value added between 1977 and 2001 is .21 in this sector, it is .41 in the rest of the economy (see the Appendix). Figure 4 illustrates one aspect of the Kaldor facts by depicting the shares of GDP accruing to labor and capital

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the 1977-2001 period is .36, while the same number for the rest of the economy is .17.

The categories of business services and other services are excluded from both groups. These two categories cannot be separated before 1987, and the other services category appears to include a different composition of subsectors before this date (and it is labeled "miscellaneous professional services").

The Appendix shows that we obtain similar patterns if we divide sectors according to their skill intensity (share of college graduates or college equivalent workers in total employment).

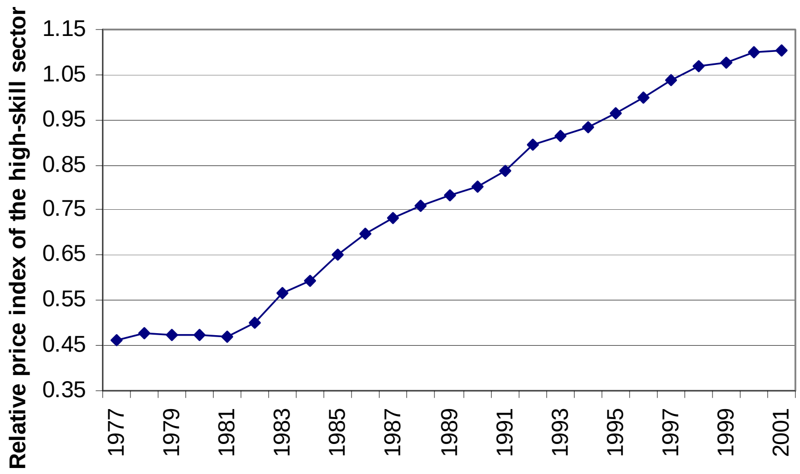

Figure 2: Relative price index of the high-skill services, 1977-2001.

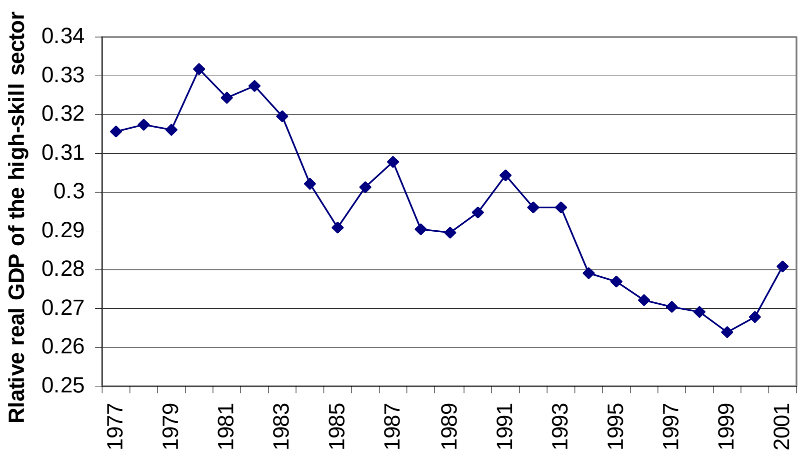

Figure 3: Real value added in the high-skill services divided by real value added in the rest of the economy, 1977-2001.

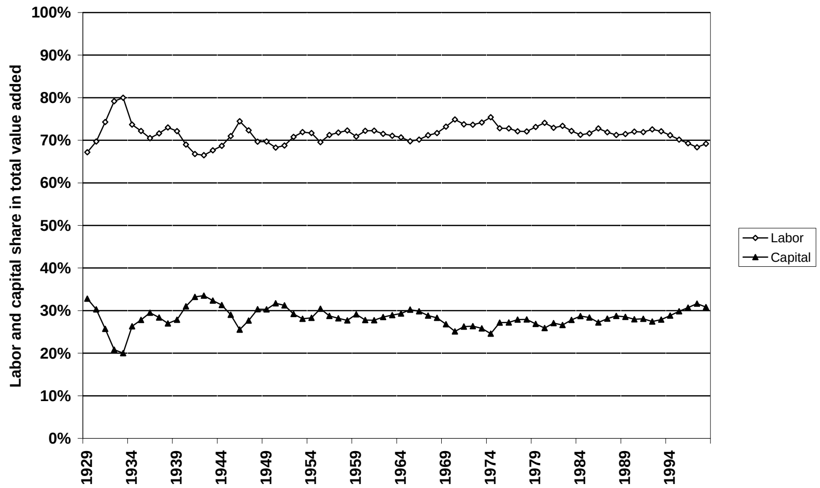

Figure 4: Capital and Labor Shares in the U.S., 1929-1999.

in the United States over the past 80 years.[3] Despite substantial capital deepening over this period and significant fluctuations, the shares show no long-run trend, implying that either exogenously or endogenously the long-run elasticity of substitution between capital and labor is equal to 1.

Finally, Figure 5 shows another important dimension of the postwar U.S. technology, the increasing skill bias of technology.[4] This figure shows a substantial increase in the premium that more skilled (college-educated) workers earn relative to less skilled workers during the past 60 years, despite a very large increase in the relative supply of skills. This pattern requires the marginal product of more skilled workers to have increased substantially relative to those of less skilled workers (e.g., Katz and Murphy, 1992, Autor, Katz and Krueger, 1998, Acemoglu, 2002a).

[3]From Piketty and Saez (2001), who in turn use the National Income and Product Accounts data.

[4]From Acemoglu (2002a). Data from 1959 come from 1940, 1950 and 1960 censuses, and 1964-1997 March CPSs. The college premium is the coefficient on workers with a college degree or more relative to high school graduates in a log weekly wage regression. The relative supply of skills is calculated from a sample that includes all workers between the ages of 18 and 65. It is defined as the ratio of college equivalents to non-college equivalents, calculated as in Autor, Katz and Krueger (1998); college equivalents=college graduates+0.5×workers with some college, and noncollege equivalents=high school dropouts+high school graduates+0.5×workers with some college.

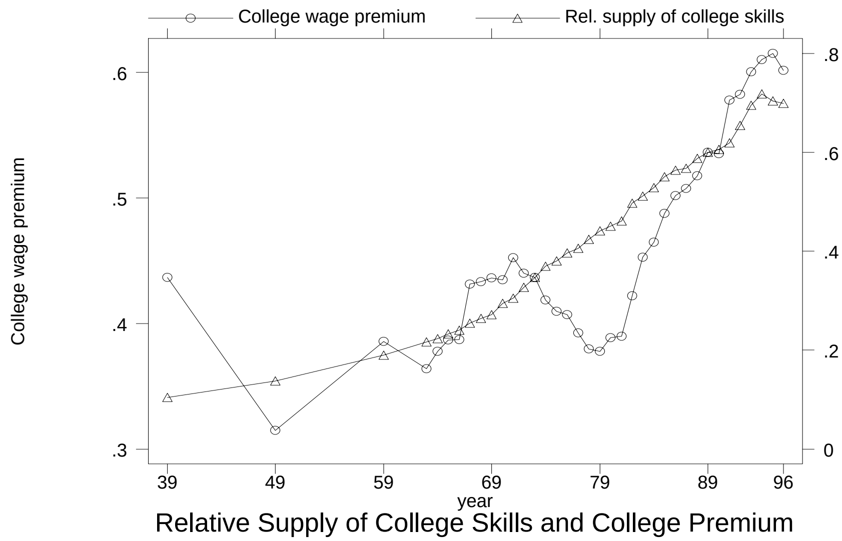

Figure 5: Relative Supply of College Equivalent Workers and the College Premium in the U.S., 1949-1995.

This paper develops a model of endogenous technology that features non-balanced and skill-biased growth, while simultaneously accounting for the Kaldor facts.[5] The model also generates endogenously skill-biased technical change in the long run, and features capital-augmenting technical change away from the asymptotic equilibrium, but purely labor-augmenting (Harrod neutral) technical change and a unitary elasticity of substitution between capital and labor along the asymptotic equilibrium path. Despite these non-standard features, the asymptotic equilibrium of the model is relatively tractable.

The model features a two-sector (two-activity) structure, with population growth, capital accumulation, and profits-driven endogenous technology in both sectors. The two sectors differ with respect to their capital intensity and skill intensity. The basic mechanism responsible for the main results in the model is that, with capital deepening in the economy, the sector with greater capital intensity tends to grow faster (because a proportional increase in the capital stock of both sectors increases output more capital-

[5] Kongsamut, Rebelo and Xie (2001) refer to this as a "generalized balanced growth path".

intensive sector more). Which factor benefits from this non-balanced growth depends on the elasticity of substitution between the two sectors.

Consistent with a large body of evidence, we take the short-run elasticity of substitution between capital and labor (unskilled plus skilled combined) to be less than 1.[6] This implies that the capital-intensive sector will grow faster in real terms, but slower in nominal terms. In addition, if, as our classification above suggests, the capital-intensive sector is less skill intensive than the rest of the economy, the model also implies capital-skill complementarity.[7]

Given this pattern of elasticities and factor intensities, the model delivers a unique asymptotic equilibrium with a constant growth rate of output, capital, and consumption, but non-balanced growth. In particular, the capital-intensive sector experiences less technological progress, but still grows faster than the less capital-intensive sector, and experiences a declining relative price. This declining equilibrium price path is just sufficient to encourage the amount of capital deepening and technical change consistent with faster growth in this sector than in the rest of the economy. Moreover, because the elasticity of substitution between the two sectors is less than 1, spending on the less capital-intensive sector grows faster. In addition, technical change in the long run is skill biased, and relative demand for skilled workers increases steadily. Finally, along this asymptotic growth path, the share of capital in GDP is constant (as if the elasticity of substitution between capital and labor were equal to 1) and technical change is Harrod neutral (purely labor augmenting), but away from the asymptotic growth path, the share of capital varies with the capital-labor ratio and there is capital-augmenting as well as labor-augmenting technical change. Only in the long run equilibrium, technical

[6] For example, Nadiri (1970), Nerlove (1967) and Hamermesh (1993) survey a range of early estimates of the elasticity of substitution, which are generally between 0.3 and 0.7. David and Van de Klundert (1965), similarly estimate this elasticity to be in the neighborhood of 0.3. Using the translog production function, Griffin and Gregory (1976) estimate elasticities of substitution for nine OECD economies between 0.06 and 0.52. See also Eisner and Nadiri (1968) and Lucas (1969). Berndt (1976), on the other hand, estimates an elasticity of substitution equal to 1, but does not control for a time trend, creating a strong bias towards 1. Using more recent data, and various different specifications, Krusell, Ohanian, Rios-Rull, and Violante (2000) and Antras (2001) also find estimates of the elasticity significantly less than 1. Estimates implied by the response of investment to the user cost of capital also typically yield an elasticity of substitution between capital and labor significantly less than 1 (see, e.g., Chirinko, 1993, Chirinko, Fazzari and Mayer, 1999, and 2001, or Mairesse, Hall and Mulkay, 1999).

[7] That capital and skills are complements during the postwar era is well-established. See, for example, Griliches (1969) or Krusell, Ohanian, Rios-Rull, and Violante (2000).

change ceases to be capital augmenting and becomes purely labor augmenting.

The rest of the paper is organized as follows. The next section discusses the relationship of this paper to the literature. Section 3 lays out the basic model and derives the equilibrium conditions for the allocation of capital between the two sectors, and equilibrium factor and good prices for a given vector of state variables, capital stock, population and technology levels. Section 4 characterizes the dynamic equilibrium and establishes the existence of the unique constant growth path equilibrium. Section 5 studies the stability of the equilibrium path. Section 6 concludes, while the Appendix contains some of the derivations and proofs not contained in the text and some further data details.

2 Related Literature

This paper is related to a number of other contributions in the literature, but to the best of our knowledge, there are no other papers investigating this class of models, nor any other models of non-balanced endogenous growth.

Most closely related is the small literature on non-balanced growth, including Echevarria (1997), Laitner (2000), and especially Kongsamut, Rebelo and Xie (2001) and Baumol (1967). Kongsamut, Rebelo and Xie (2001) construct a model of growth with changes in the sectoral composition of output because of non-homothetic preferences. Technology is exogenous, and an equilibrium with non-balanced growth consistent with the Kaldor facts emerges only under special assumptions. Although the assumption of non-homothetic preferences is clearly reasonable and most likely linked to the non-balanced nature of economic growth, it seems unlikely to account for the structural transformation by itself. Our model, in contrast, goes to the other extreme and uses homothetic preferences, and exploits the non-unity elasticity of substitution between sectors. In this respect, it is closely related to the older contribution by Baumol (1967), which also noted the effect of non-unitary elasticity of substitution on the sectoral composition of output.

Because profit incentives determine not only the aggregate rate of technology, but also the types of technologies that are developed, our paper is also related to the directed technical change literature. Acemoglu (1998, 2002b) and Kiley (1999) construct models with endogenous skill bias of technology. These models show how changes in relative supplies and international trade affect the degree of skill bias of technology, but do not generate non-balanced growth. Moreover, in these models, technical change becomes more skill biased only in response to changes relative supplies. With fixed relative supplies, there is no long-run tendency for technology to become more skill biased. In contrast, the current model generates steady skill bias even with fixed relative supplies.

Also closely related to the current study are Krusell, Ohanian, Rios-Rull and Violante (2000) and Beaudry and Green (2001, 2003). Krusell et al. construct a model in which skilled and unskilled workers are imperfect substitutes and there is capital accumulation. Capital-skill complementarity implies that capital deepening is associated with greater skill premia over time. This model does not feature endogenous technical change, however, and the equilibrium is not asymptotically balanced, so in the limit, the share of capital income in output goes to 1. Beaudry and Green analyze a model where firms choose between two activities (technologies), where, as in the current paper, the capital-intensive one is also intensive in unskilled labor. They show how the skill premium and the activity mix of the economy are related to factor proportions, and how the skill premium may increase in the relative supply of skills. Their framework is essentially static, and does not allow capital accumulation nor generate non-balanced growth or technical change.[8]

The literature on why technical change may be labor augmenting includes the older contributions by Kennedy (1964), Samuelson (1965), and Drandakis and Phelps (1965), and more recently Acemoglu (2003), Hornstein and Krusell (2003) and Jones (2004). These papers also do not generate non-balanced growth, and typically have to rely on more special assumptions on the form of technology than in the current paper.

[8]In addition there is a large literature, including among others Acemoglu (1999), Aghion, Howitt and Violante (2002), Caselli (1999), Galor and Tsiddon (1997), Galor and Maov (2000), Greenwood and Yorkuoglu (1997), Heckman, Lochner, and Taber (1998), analyzes the recent changes in the returns to skills.

3 The Model

3.1 Preferences and Technology

The economy consists of $H(t)$ skilled and $L(t)$ unskilled (for example, college and high-school educated) workers at time t, supplying their labor inelastically. The numbers of both types of workers grow exponentially at the rate n, thus:

$$H(t) = \exp(nt) H(0) \text{ and } L(t) = \exp(nt) L(0). \tag{1}$$

We assume that all households have constant relative risk aversion (CRRA) preferences over total household consumption (rather than per capita consumption), and all population growth takes place within existing households (thus there is no growth in the number of households).[9] This implies that the economy admits a representative agent with CRRA preferences (see, for example, Caselli and Ventura, 2000):

$$\int_0^\infty \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt,$$

where $C(t)$ is aggregate consumption at time t, ρ the rate of time preferences and $\theta \geq 0$ is the inverse of the intertemporal elasticity of substitution (or the coefficient of relative risk aversion). Throughout, we drop the time arguments to simplify the notation whenever this causes no confusion.

To simplify the exposition, we assume that there is no depreciation of capital, so the flow budget constraint for the representative consumer is:

$$\dot{K} = rK + W + \Pi - C - X_H - X_L, \tag{2}$$

where K denotes the total capital stock in the economy, $W = w_H H + w_L L$ is total labor income, Π is total net corporate profits received by the consumers, $X_H \geq 0$ and $X_L \geq 0$ denote the R&D expenditures, and r is the equilibrium interest rate.

The unique final good is produced by combining a skill-intensive and an unskilled-labor-intensive good with elasticity of substitution $\varepsilon \in [0, \infty)$:

$$Y = \left[\gamma Y_H^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) Y_L^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

[9] The alternative would be to specify population growth taking place at the extensive margin, in which case the discount rate of the representative agent would be $\rho - n$ rather than ρ.

where γ is a distribution parameter which determines the relative importance of the two goods in the aggregate production. In terms of the classification used in the Introduction, we think of Y_H as corresponding to the high-skill services sector and Y_L to the rest of the economy.

The resource constraint of the economy, in turn, requires that consumption, investment and R&D expenditures are less than total output, $Y = rK + W + \Pi$, thus

$$\dot{K} + C + X_H + X_L \leq Y. \tag{3}$$

The two goods Y_H and Y_L are produced competitively using constant elasticity of substitution (CES) production functions with elasticity of substitution between intermediates equal to $\nu > 1$:

$$Y_H = \left(\int_0^{M_H} y_H(i)^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}} \text{ and } Y_L = \left(\int_0^{M_L} y_L(i)^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}}, \tag{4}$$

where $y_H(i)$'s and $y_L(i)$'s denote the intermediates in the sectors that are intensive in skilled and unskilled labor, and M_H and M_L represent the technology terms. In particular M_H denotes the number of intermediates produced in the skill-intensive sector and M_L is the number of goods produced in the unskilled-labor-intensive sector.

Intermediate goods are supplied by monopolists that hold the relevant patent and are produced with the following Cobb Douglas technologies

$$y_H(i) = h(i)^{\eta} k_H(i)^{1-\eta} \text{ and } y_L(i) = l(i)^{\alpha} k_L(i)^{1-\alpha}, \tag{5}$$

where $h(i)$ and $k_H(i)$ are skilled labor and capital used in the production of good i of type H and $l(i)$ and $k_L(i)$ are unskilled labor and capital used in the production of good i of type L.

All factor markets are competitive, and market clearing for the three factors imply

$$\int_0^{M_H} h(i)\, di = H \text{ and } \int_0^{M_L} l(i)\, di = L \tag{6}$$

and

$$\int_0^{M_H} k_H(i) di + \int_0^{M_L} k_L(i)\, di \equiv K_H + K_L = K, \tag{7}$$

where the last the quality of (7) defines K_H and K_L as the capital stocks used in the two sectors.

To close the model we need to specify the innovation possibilities frontier, i.e., the technology to transform resources into blueprints for new varieties in the two sectors. We assume that:

$$\dot{M}_H = b_H M_H^{-\varphi} X_H \text{ and } \dot{M}_L = b_L M_L^{-\varphi} X_L, \tag{8}$$

where $X_H \geq 0$ and $X_L \geq 0$ are research expenditures in terms of the final good, b_H and b_L are strictly positive constants measuring the technical difficulty of creating new blueprints in the two sectors, and $\varphi \in (-1, \infty)$ measures the degree of spillovers in technology creation. When $\varphi = 0$, there are no spillovers from the current stock of knowledge to future innovations. With $\varphi < 0$, there are positive spillovers and the stock of knowledge in a particular sector makes further innovation in that sector easier. With $\varphi > 0$, there are negative spillovers ("fishing out") and further innovations are more difficult in sectors that are more advanced (see, for example, Jones, 1995, Kortum, 1997).[10] Similar to the results in Jones (1995), Young (1999) and Howitt (1999), there will be endogenous growth for a range of values of φ because of population growth. In the remainder, we will typically think of $\varphi > 0$, so that there are negative spillovers, though this is not important for any of the asymptotic results.

Finally, we assume that there is free entry into research, and a firm that invents a new skill-intensive or unskill-intensive intermediate becomes the monopolist producer with a perpetually enforced patent.

3.2 Equilibrium

Let w_H and w_L denote the wage rates for skilled and unskilled workers, $[q_H(i)]_{i=1}^{M_H}$ and $[q_L(i)]_{i=1}^{M_L}$ the prices for skill-intensive and unskill-intensive intermediates, and p_H and p_L the prices of the Y_H and Y_L goods. Let us also normalize the price of the final good at each instant to 1 as the numeraire.

An equilibrium in this economy is given by paths for factor, intermediates and final

[10] This innovation possibilities frontier assumes that only the final good is used to generate new technologies. The alternative is to have a scarce factor, such as labor or scientists, in which case some amount of positive spillovers are necessary. Acemoglu (2003), for example, considers a model with no population growth, and the results there rely on a specific form of the innovation possibilities frontier whereby:

$$\dot{M}_H = b_H M_H S_H \text{ and } \dot{M}_L = b_L M_L S_L,$$

where S_H and S_L denote scientists (or labor) allocated to generating new blueprints in the two sectors.

goods prices r, w_L, w_H, $[q_H(i)]_{i=1}^{M_H}$, $[q_L(i)]_{i=1}^{M_L}$, p_H and p_L, employment, capital allocation and research expenditure decisions $[h(i)]_{i=1}^{M_H}$, $[l(i)]_{i=1}^{M_L}$, $[k_H(i)]_{i=1}^{M_H}$, $[k_L(i)]_{i=1}^{M_L}$, X_H and X_L such that firms maximize profits and markets clear, and consumption and savings decisions, C and $\dot{K}$, that maximize consumer utility.

It is useful to break the characterization of equilibrium into two pieces: static and dynamic. The static part takes the state variables of the economy, which are the capital stock, the labor supplies and the technology, K, H, L, M_H and M_L, as given, and determines the allocation of capital and labor across sectors and factor and good prices. The dynamic part of the equilibrium determines the evolution of the three endogenous state variables, K, M_H and M_L (the dynamic behavior of H and L is given by (1)).

First, our choice of numeraire implies that the price of the final good, P, satisfies:

$$1 \equiv P = \left[\gamma^{\varepsilon} p_H^{1-\varepsilon} + (1-\gamma)^{\varepsilon} p_L^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}.$$

Next, since Y_H and Y_L are supplied competitively, their prices are equal to the value of their marginal product, thus

$$p_H = \gamma \left(\frac{Y_H}{Y}\right)^{-\frac{1}{\varepsilon}} \quad \text{and} \quad p_L = (1-\gamma)\left(\frac{Y_L}{Y}\right)^{-\frac{1}{\varepsilon}}, \tag{9}$$

and the demands for intermediates, $y_H(i)$ and $y_L(i)$, are given by the familiar isoelastic demand curves:

$$\frac{q_H(i)}{p_H} = \left(\frac{y_H(i)}{Y_H}\right)^{-\frac{1}{\nu}} \quad \text{and} \quad \frac{q_L(i)}{p_L} = \left(\frac{y_L(i)}{Y_L}\right)^{-\frac{1}{\nu}}. \tag{10}$$

The value of the monopolist for intermediate i in the s-intensive sector is given by

$$V_s(i,t) = \int_t^{\infty} \exp\left[-\int_t^{v} r(z)dz\right] \pi_s(i,v)dv, \tag{11}$$

for $s = H, L$, where $\pi_s(i,t) = (q_s(i,t) - mc_s(i,t))\, y_s(i,t)$ is the flow profits for firm i at time t, with q_s given by the demand curves in (10), and mc_s is the marginal cost of production in this sector. Given the production functions in (5), the cost functions take the familiar Cobb-Douglas form $mc_H(i) = \eta^{-\eta}(1-\eta)^{\eta-1} r^{1-\eta} w_H^{\eta}$, and $mc_L(i) = \alpha^{-\alpha}(1-\alpha)^{\alpha-1} r^{1-\alpha} w_L^{\alpha}$. In equilibrium, all firms in the same sector will make the same profits, so we $V_s(i,t) = V_s(t)$, and we use $V_H(t)$ and $V_L(t)$ to denote the value firms in the two sectors at time t.

Each monopolist will choose its price to maximize (11). Since prices at time t only influence revenues and costs at that point, profit-maximizing prices will be given by a constant mark-up over marginal cost:

$$q_H(i) = \left(\frac{\nu}{\nu-1}\right) \eta^{-\eta} (1-\eta)^{\eta-1} r^{1-\eta} w_H^{\eta} \tag{12}$$

$$q_L(i) = \left(\frac{\nu}{\nu-1}\right) \alpha^{-\alpha} (1-\alpha)^{\alpha-1} r^{1-\alpha} w_L^{\alpha} \tag{13}$$

Equations (12) and (13) imply that all intermediates in each sector sell at the same price $q_H = q_H(i)$ and $q_L = q_L(i)$. This combined with (10) implies that the demand for and the production of the same type of intermediate will be the same. Thus:

$$\begin{aligned} y_H(i) &= h(i)^{\eta} k(i)^{1-\eta} = h^{\eta} k^{1-\eta} \qquad \forall i \leq M_H \\ y_L(i) &= l(i)^{\alpha} k(i)^{1-\alpha} = l^{\alpha} k^{1-\alpha} \qquad \forall i \leq M_L. \end{aligned}$$

Market clearing conditions, (6) and (7), then yield:

$$y_H = \frac{H^{\eta} K_H^{1-\eta}}{M_H} \quad \text{and} \quad y_L = \frac{L^{\alpha} K_L^{1-\alpha}}{M_L}. \tag{14}$$

Substituting (14) into (4), we obtain the total supply of skill- and unskill-intensive goods as

$$Y_H = M_H^{\frac{1}{\nu-1}} H^{\eta} K_H^{1-\eta} \text{ and } Y_L = M_L^{\frac{1}{\nu-1}} L^{\alpha} K_L^{1-\alpha}, \tag{15}$$

and the aggregate output of the economy is:[11]

$$Y = \left[\gamma \left(M_H^{\frac{1}{\nu-1}} H^{\eta} K_H^{1-\eta}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) \left(M_L^{\frac{1}{\nu-1}} L^{\alpha} K_L^{1-\alpha}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{16}$$

The parameters η and α determine which sector is more capital intensive.[12] When $\eta > \alpha$, the skill-intensive sector is less capital intensive, and the converse applies when $\eta < \alpha$.

[11] Equation (16) also shows that this model generalizes those in Acemoglu (2002b, 2003) in an important respect. The model of Acemoglu (2002b) can be thought of as the special case where $\eta = \alpha$, while Acemoglu (2003) is the special case where $\eta = 1$ and $\alpha = 0$ or $\eta = 0$ and $\alpha = 1$.

[12] We use the term "capital intensive" as corresponding to a greater share of capital in value added. While this share is constant because of the Cobb-Douglas technologies, the equilibrium ratios capital to labor in the two sectors depend on prices.

Using (14) and (15) we can rewrite the prices for the skill- and unskill-intensive intermediates as $q_H = \gamma M_H^{\frac{1}{\nu-1}} \left(\frac{Y_H}{Y}\right)^{-\frac{1}{\varepsilon}}$ and $q_L = (1-\gamma) M_L^{\frac{1}{\nu-1}} \left(\frac{Y_L}{Y}\right)^{-\frac{1}{\varepsilon}}$, and the flow profits from the sale of the skill- and unskill-intensive intermediates are:

$$\pi_H = \frac{\gamma}{\nu} \left(\frac{Y_H}{Y}\right)^{-\frac{1}{\varepsilon}} \frac{Y_H}{M_H} \text{ and } \pi_L = \frac{1-\gamma}{\nu} \left(\frac{Y_L}{Y}\right)^{-\frac{1}{\varepsilon}} \frac{Y_L}{M_L}. \tag{17}$$

Finally, factor prices and the allocation of capital between the two sectors are determined by:[13]

$$w_H = \left(\frac{\nu-1}{\nu}\right) \eta\gamma \left(\frac{Y}{Y_H}\right)^{\frac{1}{\varepsilon}} \frac{Y_H}{H} \tag{18}$$

$$w_L = \left(\frac{\nu-1}{\nu}\right) (1-\gamma)\,\alpha \left(\frac{Y}{Y_L}\right)^{\frac{1}{\varepsilon}} \frac{Y_L}{L} \tag{19}$$

$$r = \left(\frac{\nu-1}{\nu}\right) \gamma\,(1-\eta) \left(\frac{Y}{Y_H}\right)^{\frac{1}{\varepsilon}} \frac{Y_H}{K_H} \tag{20}$$

$$r = \left(\frac{\nu-1}{\nu}\right) (1-\gamma)\,(1-\alpha) \left(\frac{Y}{Y_L}\right)^{\frac{1}{\varepsilon}} \frac{Y_L}{K_L}. \tag{21}$$

These factor prices take the familiar form, equal to the marginal product of a factor from (16) with a discounted due to the markup $(\nu-1)/\nu$.

3.3 Comparative Statics

Let us now analyze how changes in the state variables, H, L, K, M_H and M_L impact on equilibrium factor prices and factor shares.

Equations (20) and (21) determine the allocation of the capital stock between two sectors. Let us denote the fraction of capital employed in the skill-intensive sector by $\kappa \equiv K_H/K$ (clearly $1-\kappa \equiv K_L/K$). Then (20) and (21) imply:

$$\kappa = \left(\frac{1-\eta}{1-\alpha}\right) \left(\frac{\gamma}{1-\gamma}\right) \left(\frac{Y_H}{Y_L}\right)^{\frac{\varepsilon-1}{\varepsilon}} \left[1 + \left(\frac{1-\eta}{1-\alpha}\right) \left(\frac{\gamma}{1-\gamma}\right) \left(\frac{Y_H}{Y_L}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{-1} \tag{22}$$

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[13] To obtain these equations, start with the cost functions above, and derive the demand for factors by using Shepherd's Lemma. For example, for the H sector, these are $h = \left(\frac{\eta}{1-\eta}\frac{r}{w_H}\right)^{1-\eta} y_H$ and $k_H = \left(\frac{\eta}{1-\eta}\frac{r}{w_H}\right)^{-\eta} y_H$. Combine these two equations to derive the equilibrium relationship between r and w_H. Then using equation (12), eliminate r to obtain a relationship between w_H and q_H. Now combining with the demand curves in (10), the market clearing conditions, (6) and (7), and using (15) yields (18). The other equations are obtained similarly.

Equation (22) then implies:

$$\frac{d\ln\kappa}{d\ln K}=\frac{(1-\kappa)\left(\frac{1-\varepsilon}{\varepsilon}\right)(\eta-\alpha)}{1+\left(\frac{1-\varepsilon}{\varepsilon}\right)\left[(1-\eta)(1-\kappa)+(1-\alpha)\kappa\right]}>0\Leftrightarrow(\eta-\alpha)(1-\varepsilon)>0. \tag{23}$$

This result states that when the elasticity of substitution between sectors, ε, is less than 1, the fraction of capital allocated to the capital-intensive sector declines in the stock of capital. For example, when $\eta>\alpha$, the unskilled sector is more capital intensive, and a greater capital stock causes an increase in the share of capital allocated to the skilled sector. To obtain the intuition for this result, which is useful for understanding many of the results that will follow, note that if K increases and κ remains constant, then the capital-intensive sector grows by more than the other sector. Equilibrium prices given in (9) imply that when $\varepsilon<1$, the relative price of the capital-intensive sector falls by more than proportionately, inducing a greater fraction of capital to be allocated to the sector that is less intensive on capital. The converse result obtains when $\varepsilon>1$.

Similarly, simple differentiation also establishes:

$$\frac{d\ln\kappa}{d\ln M_L}=-\frac{d\ln\kappa}{d\ln M_H}=\frac{(1-\kappa)\left(\frac{1-\varepsilon}{\varepsilon}\right)\left(\frac{1}{\nu-1}\right)}{1+\left(\frac{1-\varepsilon}{\varepsilon}\right)\left[(1-\eta)(1-\kappa)+(1-\alpha)\kappa\right]}>0\Leftrightarrow\varepsilon<1. \tag{24}$$

When the elasticity of substitution, ε, is less than 1, an improvement in the productivity of a sector causes the share of capital going to that sector to fall. The intuition is again the same: increased production in a sector causes a more than proportional decline in its relative price, inducing a reallocation of capital away from its towards the other sector (again the converse results and intuition apply when $\varepsilon>1$).

Next, combining (18) and (19) gives the skill premium as:

$$\omega\equiv\frac{w_H}{w_L}=\left(\frac{\eta}{\alpha}\right)\left(\frac{\gamma}{1-\gamma}\right)\left(\frac{Y_L}{Y_H}\right)^{\frac{1-\varepsilon}{\varepsilon}}\left(\frac{L}{H}\right).$$

Straightforward differentiation implies that:

$$\frac{d\ln\omega}{d\ln K}=\frac{\left(\frac{1-\varepsilon}{\varepsilon}\right)(\eta-\alpha)}{1+\left(\frac{1-\varepsilon}{\varepsilon}\right)\left[(1-\eta)(1-\kappa)+(1-\alpha)\kappa\right]}>0\Leftrightarrow(\eta-\alpha)(1-\varepsilon)>0.$$

This result is important for what will follow later. It implies that capital-skill complementarity, which is a well-documented feature of the data (see footnote 7), is consistent with two configurations of parameters: either the elasticity of substitution, ε, is greater than 1 and the skilled sector is more capital intensive, $\eta < \alpha$; or $\varepsilon < 1$ and the skill sector is less capital intensive, i.e., $\eta > \alpha$. We saw in the Introduction that the high-skill services sector is considerably less capital intensive, so in terms of our classification, the case with $\eta > \alpha$, and by implication with $\varepsilon < 1$, is the relevant one. It is also useful to discuss the intuition for this case. An increase in the capital stock of the economy causes the output of the more capital-intensive sector to increase relative to the output in the less capital-intensive sector (despite the fact that the share of capital allocated to the less-capital intensive sector increases as shown in equation (23)). This then increases the production of the more capital-intensive sector, and since $\varepsilon < 1$, it reduces the relative reward to the factor used more intensively in the more capital-and intensive sector. When $\eta > \alpha$, the skill-intensive sector is less capital intensive, and an increase in the capital stock raises its relative reward.

Moreover, it can be established that

$$\frac{d\ln\omega}{d\ln L} = 1 + \frac{\alpha\left(\frac{1-\varepsilon}{\varepsilon}\right)}{1+\left(\frac{1-\varepsilon}{\varepsilon}\right)\left[(1-\eta)(1-\kappa)+(1-\alpha)\kappa\right]} > 0,$$

and

$$\frac{d\ln\omega}{d\ln H} = -1 - \frac{\eta\left(\frac{1-\varepsilon}{\varepsilon}\right)}{1+\left(\frac{1-\varepsilon}{\varepsilon}\right)\left[(1-\eta)(1-\kappa)+(1-\alpha)\kappa\right]} < 0.$$

so for given capital stock and technology, the relative demand curve facing the economy is downward sloping, and

$$\frac{d\ln\omega}{d\ln M_L} = -\frac{d\ln\omega}{d\ln M_H} = \frac{\left(\frac{1-\varepsilon}{\varepsilon}\right)\left(\frac{1}{\nu-1}\right)}{1+\left(\frac{1-\varepsilon}{\varepsilon}\right)\left[(1-\eta)(1-\kappa)+(1-\alpha)\kappa\right]} > 0 \Leftrightarrow \varepsilon < 1,$$

so that an increase in M_L is skill biased and an increase in M_H is biased towards unskilled workers. The intuition for why an increase in the productivity of the sector that is intensive in skills is biased toward unskilled workers (and vice versa) is again related to the fact that the elasticity of substitution between the two sectors, ε, is less than 1.

To relate our results to the patterns shown in Figure 1, it is important to look at the behavior of the capital share,

$$s_K \equiv \frac{rK}{Y} = \gamma(1-\eta)\left(\frac{Y_H}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}\frac{1}{\kappa}.$$

In the Appendix we show that

$$\frac{d \ln s_K}{d \ln K} < 0 \Leftrightarrow \varepsilon < 1.$$

This result implies that a short-run elasticity of substitution between capital and labor is only consistent with $\varepsilon < 1$ in this model. Therefore, the evidence on the short-run elasticity of substitution between capital and labor and on capital-skill complementarity (see footnotes 6 and 7), as well as the evidence of capital intensities, implies that we should focus on the parameter configuration with $\eta > \alpha$ and $\varepsilon < 1$. Hence, in the rest of the analysis, we assume:

$$\varepsilon < 1 \tag{A1}$$

and

$$\eta > \alpha, \tag{A2}$$

where we have separated these two assumptions because A2 is necessary only for some of the results.

Finally, we can also establish that

$$\frac{d \ln s_K}{d \ln M_H} = -\frac{d \ln s_K}{d \ln M_L} > 0 \Leftrightarrow (\eta - \alpha)(1 - \varepsilon) > 0.$$

This result implies that under Assumptions A1 and A2, M_H corresponds to capital-biased technology, while M_L is labor-biased (but capital-augmenting).

4 Dynamic Equilibrium

4.1 Equilibrium Conditions

We now turn to the characterization of the dynamic equilibrium path of this economy. We start with the Euler equation for consumers

$$\frac{\dot{C}}{C} = \frac{1}{\theta}(r - \rho) \tag{25}$$

To write the transversality condition, note that the financial wealth of the representative consumer comes from future payments to capital and profits. Therefore, the wealth of the representative consumer at time t is $A(t) = K(t) + M_H(t) V_H(t) + M_L(t) V_L(t)$, where recall that $V_H(t)$ is the present discounted value of the profits of a firm in the

H sector at time t and there are $M_H(t)$ such firms, and similarly for $V_L(t)$ and $M_L(t)$. The transversality condition is then:

$$\lim_{t\to\infty} A(t)\exp\left(-\int_0^t r(\tau)\,d\tau\right) = 0, \tag{26}$$

which together with the resource constraint given in (3) determines the dynamic behavior of consumption and capital stock, C and K. In addition, equations (1) give the behavior of H and L. Finally, given the value for being the monopolist for intermediate in (11), we have two free entry conditions:

$$V_H \leq \frac{M_H^{\varphi}}{b_H} \text{ and } V_L \leq \frac{M_L^{\varphi}}{b_L}, \tag{27}$$

with each condition holding as equality when there is positive R&D expenditure for that sector, i.e., when $X_H > 0$ or $X_L > 0$.

Therefore, we can summarize a dynamic equilibrium as paths of interest rates and capital allocation decisions, r and κ, satisfying (20) and (21), and of consumption, capital stock, technology, values of innovation and research expenditures satisfying (3), (8), (11), (25), (26), and (27). It is also useful to define for later a path that satisfies all of these equations except the hospitality condition, (26), as a quasi-equilibrium.

We first characterize the asymptotic equilibrium paths, which are defined as equilibrium paths that the economy tends to as $t \to \infty$.

4.2 Asymptotic Equilibrium Paths

Let us also introduce the following notation for growth rates of the key objects in this economy:

$$\frac{\dot{K}_H}{K_H} \equiv z_H,\ \frac{\dot{K}_L}{K_L} \equiv z_L,\ \frac{\dot{K}}{K} \equiv z$$

$$\frac{\dot{Y}_H}{Y_H} \equiv g_H,\ \frac{\dot{Y}_L}{Y_L} \equiv g_L,\ \frac{\dot{Y}}{Y} \equiv g,\ \frac{\dot{M}_H}{M_H} \equiv m_H \text{ and } \frac{\dot{M}_L}{M_L} \equiv m_L,$$

so that z_s denotes the growth rate of capital stock, m_s denotes the growth rate of technology, and g_s denotes the growth rate of output in sector s. Moreover, whenever they exist, we denote the corresponding asymptotic growth rates by asterisks, i.e.,

$$z_s^* = \lim_{t\to\infty} z_s,\ m_s^* = \lim_{t\to\infty} m_s \text{ and } g_s^* = \lim_{t\to\infty} g_s.$$

We now state and prove three lemmas that will be useful both in this section, and again later on.

Lemma 1 Suppose A1 holds, then $z_H \gtreqless z_L \Leftrightarrow g_H \lesseqgtr g_L$.

Proof. Differentiating (20) and(21) (with respect to time) yields

$$\frac{\dot{r}}{r} + z_H = \frac{1}{\varepsilon} g + \frac{\varepsilon - 1}{\varepsilon} g_H \text{ and } \frac{\dot{r}}{r} + z_L = \frac{1}{\varepsilon} g + \frac{\varepsilon - 1}{\varepsilon} g_L. \tag{28}$$

Subtracting the second from the first and using Assumption A1 (i.e., $\varepsilon < 1$) gives the desired result. ■

This lemma establishes the straightforward, but at first counter-intuitive, result that when the elasticity of substitution between the two sectors is less than 1, capital will be allocated so that the growth rate of the capital stock in the sector that is growing faster will be less than in the other sector.

Lemma 2 Suppose A1 holds and the asymptotic growth rates g_H^* and g_L^* exist, then $g^* = \min\{g_H^*, g_L^*\}$.

Proof. Differentiating the production function for the final good (16) we obtain:

$$g = \frac{\left[\gamma Y_H^{\frac{\varepsilon-1}{\varepsilon}} g_H + (1-\gamma) Y_L^{\frac{\varepsilon-1}{\varepsilon}} g_L\right]}{\left[\gamma Y_H^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) Y_L^{\frac{\varepsilon-1}{\varepsilon}}\right]} \tag{29}$$

which, combined with Assumption A1 (i.e., $\varepsilon < 1$) implies that as $t \to \infty$, $g^* = \min\{g_H^*, g_L^*\}$. ■

This lemma states that, because the elasticity of substitution is less than 1, the asymptotic growth rate of aggregate output will be determined by the sector that is growing more slowly.

The next lemma shows that provided that (i) Assumption A1 holds, (ii) there exists a constant asymptotic interest rate r^* (i.e., $\lim_{t\to\infty} \dot{r} = 0$), and (iii) there is positive population growth, the free entry conditions in (27) will both hold as equality:

Lemma 3 Suppose that A1 holds, that $n > 0$ and that $\lim_{t\to\infty} \dot{r} = 0$, then $\lim_{t\to\infty} (V_H - M_H^{\varphi}/b_H) = 0$ and $\lim_{t\to\infty} (V_L - M_L^{\varphi}/b_L) = 0$.

Proof. We will prove this lemma in two steps.

Step 1: $\limsup_{t\to\infty}(V_H - M_H^{\varphi}/b_H) \geq 0$ and $\limsup_{t\to\infty}(V_L - M_L^{\varphi}/b_L) \geq 0$, and $\limsup_{t\to\infty} m_H > 0$ and $\limsup_{t\to\infty} m_L > 0$.

Step 2 : $\lim_{t\to\infty}(V_H - M_H^{\varphi}/b_H) = 0$ and $\lim_{t\to\infty}(V_L - M_L^{\varphi}/b_L) = 0$.

Proof of Step 1: First note that $\limsup_{t\to\infty}(V_H - M_H^{\varphi}/b_H) < 0$ and $\limsup_{t\to\infty}(V_L - M_L^{\varphi}/b_L) < 0$ imply that the free entry conditions, (27), are asymptotically slack, so $\lim_{t\to\infty} m_H = m_H^* = 0$ and $\lim_{t\to\infty} m_L = m_L^* = 0$ (since they cannot be negative).

Next observe that $m_H^* = m_L^* = 0$ imply $g_L^* = g_H^* = n$. To see this, combine equations (20) and (21) to obtain

$$\left(\frac{Y_L}{Y_H}\right)^{\frac{1-\varepsilon}{\varepsilon}} = \left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1-\alpha}{1-\eta}\right)\frac{K_H}{K_L}.$$

Differentiating gives

$$\frac{1-\varepsilon}{\varepsilon}(\alpha-\eta)\,n + \frac{1-\varepsilon}{\varepsilon}(1-\alpha)\,z_L - \frac{1-\varepsilon}{\varepsilon}(1-\eta)\,z_H = z_H - z_L. \tag{30}$$

To derive a contradiction, suppose that $g_L^* > g_H^*$. Then, from Lemma 1 $z_L^* < z_H^*$ and from Lemma 2, $g^* = g_H^*$. Next, differentiating (15), we obtain

$$g_H = \eta n + (1-\eta)\,z_H + \frac{1}{\nu-1}m_H \text{ and } g_L = \alpha n + (1-\alpha)\,z_L + \frac{1}{\nu-1}m_L. \tag{31}$$

Moreover, since $\lim_{t\to\infty}\dot{r} = 0$, equation (28) and the fact that $g^* = g_H^*$ imply that $g_H^* = z_H^*$. This combined with equation (31) and $m_H^* = 0$ implies that $z_H^* = n$, which together with (30) implies

$$z_H^* - z_L^* = \frac{1-\varepsilon}{\varepsilon}(1-\alpha)\left(z_L^* - z_H^*\right),$$

yielding a contradiction. The argument for the case in which $g_L^* < g_H^*$ is analogous.

Now to derive a contradiction, suppose that $m_L^* = m_H^* = 0$, which, as shown above, implies $g_H^* = g_L^* = g^* = n > 0$. Then, differentiating the second equation in (17), we obtain:

$$\lim_{t\to\infty}\frac{\dot{\pi}_L}{\pi_L} = g^* > 0. \tag{32}$$

Combining this with $\lim_{t\to\infty}\dot{r} = 0$ and the value function in (11) yields: $\lim_{t\to\infty} V_L = \infty$. Since $m_L^* = 0$ by hypothesis, $M_L^{-\varphi}$ is constant, and we have $\lim_{t\to\infty} V_L = \infty >$

$\lim_{t\to\infty} M_L^{\varphi}/b_L$, violating the free entry condition (27). This proves that m_L^* and m_L^* cannot both equal to 0, and thus $\limsup_{t\to\infty}(V_L - M_L^{\varphi}/b_L) < 0$ and $\limsup_{t\to\infty}(V_H - M_H^{\varphi}/b_H) < 0$ is not possible.

We next prove that $\limsup_{t\to\infty}(V_H - M_H^{\varphi}/b_H) \geq 0$, $\limsup_{t\to\infty}(V_L - M_L^{\varphi}/b_L) \geq 0$, $\limsup_{t\to\infty} m_H > 0$ and $\limsup_{t\to\infty} m_L > 0$.

Suppose, to derive a contradiction, $\limsup_{t\to\infty} m_L = m_L^* = 0$ and $\limsup_{t\to\infty}(V_L - M_L^{\varphi}/b_L) < 0$ (the other case is proved analogously). Since, as shown above, $m_L^* = m_H^* = 0$ is not possible, we must have $\limsup_{t\to\infty}(V_H - M_H^{\varphi}/b_H) \geq 0$ and $\limsup_{t\to\infty} m_H > 0$.

We start by noting that $\limsup_{t\to\infty} m_H > m_L^* = 0$ implies that $\limsup_{t\to\infty} g_H(t) > g_L(t) = g_L^* = n$. To see this, take an interval (t_0, t_1) such that $m_H(t) > 0 = m_L^*$ for all $t \in (t_0, t_1)$. Suppose, to derive a contradiction that $g_H(t) \leq g_L(t)$ during this interval. Then from Lemma 1, we also have $z_H(t) \geq z_L(t)$, and from (31), we have

$$g_L(t) = \alpha n + (1-\alpha) z_L(t) < g_H(t) = \eta n + (1-\eta) z_H(t) + \frac{1}{\nu - 1} m_H(t),$$

which gives a contradiction, showing that $g_H(t) > g_L(t)$ for all $t \in (t_0, t_1)$, and equations (28) and (31) imply that $g_L(t) = n > 0$. Moreover, when $m_H(t) = m_L(t) = 0$, by the same argument as above, $g_H(t) = g_L(t) = n$. Thus $g_H(t) \geq g_L(t)$ and $g_H(t) > g_L(t) = n$ whenever $m_H(t) > 0$. Since $\limsup_{t\to\infty} m_H > m_L^* = 0$ by hypothesis, we also have $Y_L/Y_H \to 0$. Then equation (29) implies that $g \to g_L^* = n$, and hence $g_L^* = g^* = n$, completing the proof of the claim.

Now, to obtain a contradiction suppose that $\limsup_{t\to\infty} m_H > m_L^* = 0$. Then, as shown above, $g_H \geq g_L^* = g^* = n > 0$. Differentiating the first equation in (17), we again obtain (32), and since $\lim_{t\to\infty} \dot{r} = 0$ and $m_L^* = 0$, by the same argument as in Step 1, we have $\lim_{t\to\infty} V_L = \infty > \lim_{t\to\infty} M_L^{\varphi}/b_L$, violating the free entry condition (27). A similar argument for the H sector completes the proof that $\limsup_{t\to\infty}(V_H - M_H^{\varphi}/b_H) \geq 0$ and $\limsup_{t\to\infty}(V_L - M_L^{\varphi}/b_L) \geq 0$.

Proof of Step 2. From the free entry conditions in (27), we have that $V_H - M_H^{\varphi}/b_H \leq 0$ and $V_L - M_L^{\varphi}/b_L \leq 0$, thus $\limsup_{t\to\infty}(V_H - M_H^{\varphi}/b_H) \leq 0$ and $\limsup_{t\to\infty}(V_L - M_L^{\varphi}/b_L) \leq 0$. Combined with Step 1, this implies $\limsup_{t\to\infty}(V_H - M_H^{\varphi}/b_H) = 0$ and $\limsup_{t\to\infty}(V_L - M_L^{\varphi}/b_L) = 0$. Hence, we just have to prove that $\liminf_{t\to\infty}(V_H - M_H^{\varphi}/b_H) \geq 0$ and $\liminf_{t\to\infty}(V_L - M_L^{\varphi}/b_L) \geq 0$. We prove the first inequality (the proof of the second is similar).

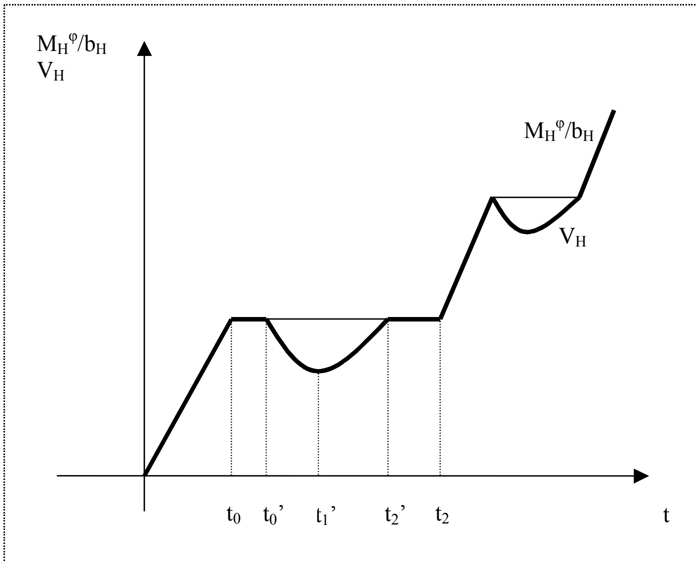

Figure 6:

Suppose, to derive a contradiction, that $\liminf_{t\to\infty} (V_H - M_H^{\varphi}/b_H) < 0$. This implies that there exists an interval (t_0', t_2') with $t_0' \to \infty$ such that $V_H(t) - M_H^{\varphi}(t)/b_H < 0$ for all $t \in (t_0', t_2')$. Suppose that (t_0', t_2') is unbounded; this would imply that $\limsup_{t\to\infty} m_H = m_H^* = 0$, yielding a contradiction with Step 1. Thus (t_0', t_2') must be bounded, so there exists $(t_0, t_2) \supset (t_0', t_2')$ such that for $t \in (t_0, t_2) \setminus (t_0', t_2')$, we have $V_H(t) - M_H^{\varphi}(t)/b_H = 0$. Moreover, since $\limsup_{t\to\infty} m_H > 0$, there also exists an interval $(t_0'', t_2'') \supset (t_0, t_2)$ such that for all $t \in (t_0, t_2) \setminus (t_0'', t_2'')$, $m_H > 0$.

Next, since $m_H = 0$ for all $t \in (t_0', t_2')$, we also have $M_H(t_0') = M_H(t_2')$. This implies

$$V_H(t_2') = \frac{M_H^{\varphi}(t_0')}{b_H} = V_H(t_0'). \tag{33}$$

Figure 6 shows this diagrammatically.

Let us rewrite (11) in the Bellman equation form

$$\frac{\dot{V}_H(t)}{r} = V_H(t) - \frac{\pi_H(t)}{r} \qquad \forall t. \tag{34}$$

Equation (34) also shows that $\dot{V}_H(t)$ is well-defined, so $V_H(t)$ is continuously differentiable in t. Equation (33) and the fact that $V_H(t) - M_H^{\varphi}(t_0')/b_H < 0$ for all $t \in (t_0', t_2')$ imply that $V_H(t)$ reaches a minimum over (t_0', t_2') with $\dot{V}_H(t) = 0$. Let $t_1' < t_2'$ be such that $V_H(t_1')$ is the first local minimum after t_0', which naturally satisfies $V_H(t_1') < V_H(t_0')$. Moreover, using (17) and (21), we have that for all t

$$\frac{\pi_H(t)}{r^*} = \frac{1}{(\nu-1)(1-\eta)} \frac{K_H(t)}{M_H(t)},$$

where $r^* = \lim_{t\to\infty} r(t)$ is the asymptotic equilibrium interest rate, which exists by the hypothesis that $\lim_{t\to\infty} \dot{r} = 0$. Also, using the fact that $\lim_{t\to\infty} \dot{r} = 0$ and the interest rate condition, (20), we obtain that since $m_H = 0$ and $n > 0$, $K_H(t_1') > K_H(t_0')$. In addition, since $M_H(t_1') = M_H(t_0')$, we can use (34) to write

$$\frac{\dot{V}_H(t_1')}{r^*} = V_H(t_1') - \frac{1}{(\nu-1)(1-\eta)} \frac{K_H(t_1')}{M_H(t_1')} < V_H(t_0') - \frac{1}{(\nu-1)(1-\eta)} \frac{K_H(t_0')}{M_H(t_0')} = \frac{\dot{V}_H(t_0')}{r^*} < 0,$$

which contradicts the fact that $V_H(t_1')$ is a local minimum, completing the proof of the lemma. ■

This lemma is an important result for our analysis. It establishes that asymptotically both free entry conditions must hold as equality. The economic intuition for the lemma comes from population growth; with population growth, it is always optimal to allocate more capital to each sector, which increases the profitability of intermediate producers in that sector. Consequently, the value of a new blueprint increases asymptotically. This rules out asymptotic equilibrium paths with slack free-entry conditions, because along such paths, the cost of creating a new blueprint would remain constant, ultimately violating the free-entry condition.

4.3 Constant Growth Paths

Lemma 3 in the previous subsection established that all asymptotic equilibria satisfy the free entry conditions, (27), as equality. We now show that under some additional parameter restrictions there are no "explosive paths," i.e., quasi-equilibria where consumption and output grow more than exponentially. We then establish the existence of a unique asymptotic equilibrium where consumption grows at a constant rate.

A constant growth path (CGP) is defined as an equilibrium path where the asymptotic growth rate of consumption exists and is constant, i.e.,[14]

$$\lim_{t\to\infty}\frac{\dot{C}}{C}=g_C^*.$$

From the Euler equation (25), this also implies that the asymptotic interest rate must be constant, thus the condition $\lim_{t\to\infty}\dot{r}=0$ in Lemmas 2 and 3 is satisfied.

To establish the existence of a CGP, we impose the following parameter restriction:

$$\zeta>\max\left\{\frac{1}{\eta},\frac{1}{\eta}\left[1-\frac{(1-\theta)}{\rho}n\right]^{-1}\right\} \tag{A3}$$

where $\zeta\equiv(\nu-1)(1+\varphi)$. This assumption ensures that the transversality condition (26) holds. A fortiori, it also rules out quasi-equilibrium paths where output and consumption grow more than exponentially.

Lemma 4 Suppose A1, A2 and A3 hold, then there exists no quasi-equilibria with $\lim_{t\to\infty}\dot{C}/C=\infty$.

This lemma is proved in the Appendix, where, for completeness, we also show that Assumption A3 is "tight" in the sense that, if first inequality in this assumption, $\zeta>1/\eta$, did not hold, there always exist quasi-equilibria with more than exponential growth.

Combined Lemmas 3 and 4 imply that the asymptotic equilibrium has to converge either to a limit with constant growth of consumption, or to an asymptotic cycle. The next proposition, which is the main result of the paper, establishes the existence of a unique constant growth path, where growth is non-balanced, with the unskilled-intensive sector growing faster than the skill-intensive sector.

Proposition 1 Suppose A1, A2 and A3 hold and $n>0$, then there exists a unique CGP where

$$g^*=g_C^*=g_H^*=z_H^*=\frac{\eta\zeta}{\eta\zeta-1}n \tag{35}$$

$$z_L^*=\frac{\zeta\left[\alpha(\eta\zeta-1)(\varepsilon-1)+\eta\zeta\right]}{(\eta\zeta-1)\left[(\varepsilon-1)(\alpha\zeta-1)+\zeta\right]}n<g^* \tag{36}$$

[14] Since, given the Euler equation (25), a constant growth rate of consumption is only consistent with a constant interest rate, our constant growth path is equivalent to the "generalized balanced growth path" of Kongsamut, Rebelo and Xie (2001).

$$g_L^* = \frac{\varepsilon z_L - z_H}{(\varepsilon - 1)} > g^* \tag{37}$$

$$m_H^* = \frac{1}{1+\varphi} z_H \quad \textit{and} \quad m_L^* = \frac{1}{1+\varphi} z_L \tag{38}$$

Moreover, the CGP features $\dot{\omega}/\omega > 0$.

Proof. We prove this proposition in three steps.

Step 1: Provided that $g_L^* \geq g_H^* > 0$, then there exists a unique CGP defined by equations (35), (36), (37) and (38) satisfying $g_L^* > g_H^* > 0$.

Step 2: All CGP must satisfy $g_L^* \geq g_H^* > 0$.

Step 3: Equilibrium prices satisfy $\dot{\omega}/\omega > 0$.

Proof of Step 1. Lemma 3 establishes that as $t \to \infty$ the free-entry conditions (27) must asymptotically hold as equality. Combining (27) with (34) (and the equivalent for the L sector), we obtain the following conditions that must hold as $t \to \infty$:

$$\frac{\frac{\gamma}{\nu}\left(\frac{Y_H}{Y}\right)^{-\frac{1}{\varepsilon}} Y_H}{r - \varphi m_H} = \frac{M_H^{1+\varphi}}{b_H} \text{ and } \frac{\frac{\gamma}{\nu}\left(\frac{Y_L}{Y}\right)^{-\frac{1}{\varepsilon}} Y_L}{r - \varphi m_L} = \frac{M_L^{1+\varphi}}{b_L} \tag{39}$$

Differentiating (39) yields:

$$g_H - \frac{1}{\varepsilon}(g_H - g) - (1+\varphi) m_H = 0 \text{ and } g_L - \frac{1}{\varepsilon}(g_L - g) - (1+\varphi) m_L = 0. \tag{40}$$

Then $g_L^* > g_H^* > 0$ and Lemma 2 imply that we must also have $g^* = g_H^*$. This condition together our system of equations, (28), (31), (40) solves uniquely for z_H^*, z_L^*, m_H^*, m_L^*, g_H^* and g_L^* as given in equations (35), (36), (37) and (38). Note that this solution is consistent with $g_L^* > g_H^* > 0$, since Assumptions A1 and A2 immediately imply that $g_L^* > g_H^*$ and $g_H^* > 0$ (which is also consistent with Lemma 3). Finally, $C \leq Y$, (2) and (26) imply that the consumption growth rate, g_C^*, is equal to the growth rate of output, g^* (suppose not, then since $C/Y \to 0$ as $t \to \infty$, the budget constraint (2) implies that asymptotically $\dot{K}(t) = Y(t)$, and integrating the budget constraint gives $K(t) \to \int_0^t Y(s)\,ds$, implying that the capital stock grows more than exponentially, since Y is growing exponentially; violating the transversality condition (26)).

Finally, we can verify that an equilibrium with z_H^*, z_L^*, m_H^*, m_L^*, g_H^* and g_L^* satisfies the transversality condition (26). Note that the transversality condition (26) will be satisfied if

$$\lim_{t\to\infty} \frac{\dot{A}(t)}{A(t)} < r^*, \tag{41}$$

where r^* is the constant asymptotic interest rate. Since from the Euler equation (25) $r^* = \theta g^* + \rho$, (41) will be satisfied when $g^* (1 - \theta) n < \rho$. Assumption A3 ensures that this is the case with $g^* = \frac{\eta\zeta}{\eta\zeta - 1} n$.

Proof of Step 2. We now prove that along all CGPs $g_L^* > g_H^* > 0$ must be true. To derive a contradiction suppose that either $g_H^* \geq g_L^*$, or $g_L^* \geq g_H^*$ but $g_H^* \leq 0$.

1. Suppose $g_H^* \geq g_L^*$, then following the same reasoning as in Step 1 the unique solution to the equilibrium conditions (28), (31), (40) is:

$$g^* = g_L^* = z_L^* = \frac{\alpha\zeta}{\alpha\zeta - 1} n \tag{42}$$

$$z_H^* = \frac{\zeta \left[\eta \left(\alpha\zeta - 1\right)\left(\varepsilon - 1\right) + \alpha\zeta\right]}{\left(\alpha\zeta - 1\right)\left[\left(\varepsilon - 1\right)\left(\eta\zeta - 1\right) + \zeta\right]} n \tag{43}$$

$$m_H^* = \frac{1}{1+\varphi} z_H^* \quad \text{and} \quad m_L^* = \frac{1}{1+\varphi} z_L^* \tag{44}$$

$$g_H^* = \frac{\varepsilon z_H^* - z_L^*}{\varepsilon - 1}. \tag{45}$$

   But combining these equations with Assumption A1 implies that $g_H^* < g_L^*$, which contradicts the hypothesis $g_H^* \geq g_L^* > 0$.

2. Suppose $g_L^* \geq g_H^*$ and $g_H^* \leq 0$, then the same steps as above give the following unique solution to equilibrium conditions (28), (31), (40): (35), (36), (37) and (38). But now, $m_H^* \leq 0$, which contradicts Lemma 3.

Step 3: Finally, using the equilibrium wages given in (18) and (19) immediately implies $\dot{\omega}/\omega > 0$, which completes the proof. ■

This proposition is the major result of the paper. It shows the existence of an asymptotic equilibrium where all key objects grow at constant rates. Growth is driven by technology, in the sense that m_H^*, $m_L^* > 0$ (and if $m_H^* = m_L^* = 0$, there would be no growth in per capita income).

Although consumption and output grow at a constant rate, growth rates are not equal across sectors. In particular, the more capital-intensive sector grows faster ($g_L^* > g_H^*$). This is despite the fact that both the capital stock and the technology in the less capital-intensive H sector grow faster (it can be verified that $m_H^* > m_L^*$ and $z_H^* > z_L^*$). However, because increases in the capital stock have a greater effect in the more capital-intensive sector (i.e., because $\eta > \alpha$), the capital-intensive sector achieves a higher growth rate. Because substitution between the two sectors is imperfect, the relative price of the less capital-intensive sector grows asymptotically; in fact, p_L defined in (9) goes to 0 as $t \to \infty$.[15] Using (9), it is also easy to verify that

$$\frac{\dot{p}_H}{p_L} - \frac{\dot{p}_L}{p_L} \to \left(\frac{\zeta (\eta - \alpha)}{(\eta\zeta - 1)\left[(\varepsilon - 1)(\alpha\zeta - 1) + \zeta\right]} \right) n > 0,$$

so the relative price of the skill-intensive sector increases asymptotically. Moreover, because the elasticity of substitution between the two sectors, ε, is less than 1, the nominal output of the skill-intensive sector grows faster than that of the unskill-intensive sector. In particular, defining nominal outputs (spending levels) as $N_H = p_H Y_H$ and $N_L = p_L Y_L$, it is straightforward to verify that $\dot{N}_H / N_H \to g_H^*$ and $\dot{N}_L / N_L \to z_L^* < g_H^*$ (recall (36)). As a result, the constant growth path equilibrium is consistent with the Kaldor and Kuznets facts discussed in the Introduction; output growth and the interest rate are constant, while the nominal output of the high-skill sector grows faster than that of the low-skill sector, but the real output growth is faster in the low-skill sector.

A consequence of the relative price movements of the two sectors is that the wages of workers employed in the less capital-intensive sector grow steadily relative to those in the more capital-intensive sector, generating the result that $\dot{\omega}/\omega > 0$. This is the source of steady skill-biased technical change in the model.

In addition, asymptotic technology satisfies $m_H^* > m_L^* > 0$, which makes technical change capital-augmenting as well as labor-augmenting. However, it can be seen that as $t \to \infty$, $\kappa \to 1$, i.e., essentially all of the capital stock of the economy is allocated to the less capital-intensive sector. As a result, at this limit point (and importantly, *only* at this limit point), all technical change becomes purely labor-augmenting (Harrod neutral), the share of capital in GDP convergence to a constant, and the elasticity of substitution between capital and labor has converge to 1. This feature that limiting technical progress is purely labor-augmenting ensures the existence of an asymptotic equilibrium with constant growth rate and constant interest rate. Away from this limit,

[15] An important feature of the equilibrium is that despite the steady decline in the price of the more capital-intensive sector, p_L, there is growth both in the capital stock allocated to this sector and technical progress directed at this sector. This is because, even though $p_L \to 0$, at any point in time, it is positive, and its level is determined in equilibrium precisely to encourage sufficient capital and technology for this sector to keep its growth rate above that of the rest of the economy.

there is capital-augmenting technical change, the share of capital changes with factor proportions and technology, and the elasticity of substitution between capital and labor is less than 1.

The source of non-balanced growth in this economy is the differential capital intensities in the two sectors. This feature, combined with capital deepening in the economy, ensures faster growth in the more capital-intensive sector. Intuitively, if capital were allocated proportionately to the two sectors, the more capital-intensive sector would grow faster. Because of the changes in prices, the equilibrium allocation favors the less capital-intensive sector, but not enough to fully offset the faster growth in the more capital-intensive sector. With a similar intuition, there is also technological progress in the capital-intensive sector, though again slower than in the less capital-intensive sector, and similarly, this differential is not sufficient to offset the faster growth of the more capital-intensive sector.

Proposition 1 establishes that there exists a unique constant growth path. In addition, from Lemma 3, we know that there cannot be any other equilibria with constant interest rates, and from Lemma 4, that there exist no quasi-equilibria where consumption grows more than exponentially. This still leaves the possibility of equilibrium paths that asymptotically do not converge to a limit with constant growth rates, and instead converge to asymptotic cycles. Unfortunately, we are generally unable to rule out the existence of asymptotic cycles.

5 Dynamics (Incomplete)

The previous section characterized the asymptotic equilibrium, and established the existence of a unique constant growth path. This growth path has a number of attractive features, including the presence of skill-biased technical change, purely labor-augmenting technical change and non-balanced growth between the two sectors. We also showed that away from this constant growth path, the economy experiences capital-augmenting as well as labor-augmenting technical change. We now further study the equilibrium behavior of the economy away from this asymptotic equilibrium.

Equilibrium behavior away from the asymptotic equilibrium path is generally difficult to analyze because there are a large number of endogenous variables. We now show that we can represent the dynamic equilibrium of this economy by 8 differential equations, for 5 state variables and 3 control variables. The 5 state variables are H, L, K, M_H, and M_L, and the 3 control variables are C, V_H, and V_L.

The behavior of the first two state variables, H and L, is given by (1). The dynamics of aggregate consumption, C, is given by the Euler equation (25), and the behavior of V_H and V_L are given by standard Bellman equations obtained from (11):

$$rV_H - \dot{V}_H = \pi_H \tag{46}$$

$$rV_L - \dot{V}_L = \pi_L \tag{47}$$

where π_H and π_L denote the flow monopoly profits given by (17). The free entry conditions in (27) regulate the dynamics of M_H, and M_L. In particular,

$$\text{if } V_s < M_s^{\varphi}/b_s, \text{ then } \dot{M}_s = 0,$$

and

$$\text{if } V_s = M_s^{\varphi}/b_s, \text{ then } \frac{\dot{M}_s}{M_s} = \frac{1}{\varphi}\frac{\dot{V}_s}{V_s} \tag{48}$$

Finally, from (3) and (8), the behavior of K is given as

$$\dot{K} = Y - C - b_H^{-1} M_H^{\varphi} \dot{M}_H - b_L^{-1} M_L^{\varphi} \dot{M}_L \tag{49}$$

where clearly $\dot{M}_s = 0$ when $V_s < M_s^{\varphi}/b_s$.

The characterization of the dynamic equilibrium path is equivalent to solving the following problem: start with a vector of initial values for the state variables, $H(0)$, $L(0)$, $K(0)$, $M_H(0)$, and $M_L(0)$, and choose initial values for the control variables, $V_H(0)$, $V_L(0)$ and $C(0)$, such that given the initial values the differential equations (25), (46), (47), (48) and (49) define a path (quasi-equilibrium) that does not violate the feasibility or the transversality conditions. We refer to the dynamic equilibrium as stable if this path converges to the unique constant growth path equilibrium.

The analysis of this system of differential equation is relatively difficult not only because of the relatively large number of equations, but also because many of these variables grow at different rates in the asymptotic equilibrium. Nevertheless, it is possible to make some progress by dividing the space of H, L, K, M_H, M_L, C, V_H, and V_L into four regions, and characterizing equilibrium behavior in each of these regions separately.

First, we can start with the region where (48) holds for both $s = L$ and H (i.e., the free entry conditions, (27), hold as equality). In this region, which we refer to as region I, we can reduce the system of differential equations to an autonomous system of four variables,

$$\mu_H \equiv \frac{M_H}{K_H},\ \mu_L \equiv \frac{M_L}{K_L},\ k \equiv \frac{K}{H^{g^*/n}},\text{ and } c \equiv \frac{C}{H^{g^*/n}},$$

where K_H and K_L are given by (20) and (21) as functions of the state variables, H, L, K, M_H, and M_L. It can be verified that the constant growth path equilibrium characterized above corresponds to a steady state in terms of these four variables, μ_H, μ_L, k, and c (i.e., μ_H, μ_L, k, and c will be constant— $\dot{\mu}_H = 0$, etc—in the constant growth path equilibrium). The behavior of this four equation system can be analyzed numerically to determine the stable manifold around the steady-state values of μ_H, μ_L, k, and c (i.e., around the constant growth path). We find that the set of differential equations linearized around the constant growth path have three positive and one negative eigenvalues, so in the neighborhood of the constant growth path the stable manifold, $\mathbf{M}$, is one-dimensional around the cost of growth path.

Similarly, the behavior of the set of differential equations can also be characterized in the regions where the free entry condition is slack for one sector and binds for the other, i.e., (48) holds for one sector and fails to hold for the other. Let us take region II where this condition is slack for $s = L$ and holds for $s = H$ (region III is defined as the alternate case). In this region, $\dot{M}_L = 0$, we have a system of four differential equations in K, M_H, C, and V_L given by

$$\dot{K} = Y - C - b_H^{-1} M_H^{\varphi} \dot{M}_H, \tag{50}$$

$\dot{M}_H / M_H = \dot{V}_H / \varphi V_H$ combined with (46), equation (25), and equation (47).

Finally, in region IV where both free entry conditions are slack, we have a system of four differential equations in K, C, V_H and V_L given by

$$\dot{K} = Y - C, \tag{51}$$

and equations (25), (46), and (47).

The numerical procedure is implemented as follows: we start with initial conditions $H(0)$, $L(0)$, $K(0)$, $M_H(0)$, and $M_L(0)$, and characterize equilibrium dynamics backwards, starting in region I. Suppose at time T, the system reaches region I at some point along the stable manifold $\mathbf{M}$. It is clear that if it reached region I at some point not belonging to $\mathbf{M}$, the constant growth path would never be reached, so the quasi-equilibrium would either violate the transversality condition (because V_H or V_L grow faster than exponentially) or the resource constraint (because C grows faster than exponentially). Let us denote the corresponding point along $\mathbf{M}$ reached at time T by $K(T)$, $M_H(T)$, $M_L(T)$ and $C(T)$, and moreover we have $V_H(T) = M_H(T)^{\varphi}/b_H$ and $V_L(T) = M_L(T)^{\varphi}/b_L$ by the definition of region I where both free entry conditions hold as equality. Starting in region I at $K(T)$, $M_H(T)$, $M_L(T)$ and $C(T)$, the set of differential equations converge to the unique constant growth path equilibrium as $t \to \infty$ by definition of the stable manifold $\mathbf{M}$.

Then the problem in region II is to find an initial point $\tilde{T} \leq T$, $K(\tilde{T})$, $C(\tilde{T})$ and $V_L(\tilde{T})$ such that the differential equations (25), (46), (47) and (50), starting at $K(\tilde{T})$, $M_H(0)$, $M_L(0)$, $C(\tilde{T})$, $V_H(\tilde{T}) = M_H(0)^{\varphi}/b_H$, $V_L(\tilde{T}) \leq M_L(0)^{\varphi}/b_L$ at time $\tilde{T}$ take us exactly to $K(T)$, $M_H(T)$, $M_L(0)$ and $C(T)$ at time T. Note that we require the point reached along $\mathbf{M}$ at time T to have $M_L(T) = M_L(0)$ because we are traveling to region I via region II where $\dot{M}_L = 0$. Since $\mathbf{M}$ is one-dimensional as noted above, $K(T)$, $M_H(T)$, and $C(T)$ are uniquely defined given $M_L(0)$. Moreover, we impose $M_H(\tilde{T}) = M_H(0)$ since before $\tilde{T}$ the system had not reached regions I or II, so $\dot{M}_H(t) = 0$ for all $t \leq \tilde{T}$. This also implies that $V_H(\tilde{T}) = M_H(0)^{\varphi}/b_H$ since at $\tilde{T}$ the system reaches region II, where the free entry condition for sector H holds as equality. Therefore the problem here is to find $T - \tilde{T}$, $K(\tilde{T})$, $C(\tilde{T})$, and $V_L(\tilde{T})$. The problem in region III is identical except that sectors L and H are switched.

Finally in region IV, we find the initial values of the control variables, $V_H(0)$, $V_L(0)$ and $C(0)$ and the value $\tilde{T}$ so that the system reaches $K(\tilde{T})$, $C(\tilde{T})$, $V_L(\tilde{T})$, and $V_H(\tilde{T}) = M_H(0)^{\varphi}/b_H$ traveling along the differential equations (25), (46), (47) and (51) exactly at $\tilde{T}$. With a similar argument, any other values of $V_H(0)$, $V_L(0)$ and $C(0)$ would either violate the transversality condition or the resource constraint.

Although it is not possible to analytically prove that $V_H(0)$, $V_L(0)$ and $C(0)$, and thus the resulting dynamic equilibrium path, are unique, we check uniqueness in our numerical calculations.

We now report a number of numerical examples. We take the following parameterization of the model.

To Be Completed

6 Conclusion

This paper constructs a model of non-balanced endogenous growth. The economy consists of two sectors, with different skill and capital intensities. Profit-maximizing incentives determine the rate at which the technologies of the two sectors progress. Growth is driven by population growth, technical progress and capital accumulation. In the asymptotic equilibrium, consumption and output grow at a constant rate, but output and technology of the two sectors grow at different rates. This last feature is a consequence of the different degrees of capital intensities combined with capital deepening. In particular, capital deepening implies that the more capital-intensive sector tends to grow faster.

Despite non-balanced growth and the fact that the price of one of the sectors tends to zero asymptotically, the equilibrium generates technical progress directed towards both sectors. This is because the rate at which the price of the more capital-intensive sector goes to zero is determined endogenously to ensure equilibrium technical progress.

We show that in this framework technical progress is skill biased and generally capital augmenting, but in the asymptotic equilibrium, it becomes purely labor augmenting.

The main patterns of growth implied by the model are consistent with the broad trends in the U.S. economy, where a group of high-skill services have been growing faster in nominal terms, but slower in real terms than the rest of the economy, while the aggregate growth rate, real interest rate and capital share in the economy have remained stable.

We view this model as a potential alternative to existing models of growth that impose balanced growth, and in addition either assume technical progress to be exogenous or constrain the form and direction of technical change. Its attractive feature is that it offers a potential unified framework for the analysis of both the Kaldor and the Kuznets facts, while also generating endogenous technical change in both sectors. Future work should investigate whether this framework can be useful in thinking about other features of technical progress and economic growth, such as cross-country technology transfers, patterns of convergence and the effects of various policies and institutions on economic growth and technical progress.

7 Appendix

7.1 Derivation of Comparative Statics

Here we show that

$$\frac{d\ln s_K}{d\ln K} < 0 \Leftrightarrow \varepsilon < 1$$

and

$$\frac{d\ln s_K}{d\ln M_H} = -\frac{d\ln s_K}{d\ln M_L} > 0 \Leftrightarrow (\eta - \alpha)(1-\varepsilon) > 0.$$

From the interest rate expression (20), we have that

$$s_K \equiv \frac{rK}{Y} = \gamma(1-\eta)\left(\frac{Y_H}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}\frac{1}{k_H}$$

where

$$\left(\frac{Y_H}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \frac{1}{\gamma + (1-\gamma)\left(\frac{Y_H}{Y_L}\right)^{\frac{1-\varepsilon}{\varepsilon}}}$$

We then have

$$\frac{d\ln s_K}{d\ln K} = \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{d\ln\frac{Y_H}{Y}}{d\ln K} - \frac{d\ln\kappa}{d\ln K} \tag{52}$$

and

$$\frac{d\ln s_K}{d\ln M_s} = \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{d\ln\frac{Y_H}{Y}}{d\ln M_s} - \frac{d\ln\kappa}{d\ln M_s} \tag{53}$$

for $s = H, L$.

Recall that

$$\kappa = \left(\frac{1-\eta}{1-\alpha}\right)\left(\frac{\gamma}{1-\gamma}\right)\left(\frac{Y_H}{Y_L}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left[1+\left(\frac{1-\eta}{1-\alpha}\right)\left(\frac{\gamma}{1-\gamma}\right)\left(\frac{Y_H}{Y_L}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{-1}$$

and

$$\ln\frac{Y_H}{Y_L} = \frac{1}{\nu-1}\ln\left(\frac{M_H}{M_L}\right) + \ln\left(\frac{H^\eta}{L^\alpha}\right) + (1-\eta)\ln\kappa - (1-\alpha)\ln(1-\kappa) + (\alpha-\eta)\ln K$$

Differentiating these equations, we obtain

$$\frac{d\ln\frac{Y_H}{Y_L}}{d\ln K} = \left(\frac{(1-\eta)(1-\kappa)+(1-\alpha)\kappa}{1-\kappa}\right)\frac{d\ln\kappa}{d\ln K} + (\alpha-\eta) \tag{54}$$

$$\frac{d\ln\frac{Y_H}{Y_L}}{d\ln M_H} = \left(\frac{(1-\eta)(1-\kappa)+(1-\alpha)\kappa}{1-\kappa}\right)\frac{d\ln\kappa}{d\ln M_H} + \frac{1}{\nu-1} \tag{55}$$

and

$$\frac{d\ln\frac{Y_H}{Y_L}}{d\ln M_L}=\left(\frac{(1-\eta)(1-\kappa)+(1-\alpha)\kappa}{1-\kappa}\right)\frac{d\ln\kappa}{d\ln M_L}-\frac{1}{\nu-1}. \tag{56}$$

Combining equations (52) and (54) together with (23) yields

$$\frac{d\ln s_K}{d\ln K}=\frac{\left(\frac{1-\varepsilon}{\varepsilon}\right)(\eta-\alpha)\left((1-\gamma)\left(\frac{Y_L}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}-(1-\kappa)\right)}{\left[1-\left(\frac{\varepsilon-1}{\varepsilon}\right)((1-\eta)(1-\kappa)+(1-\alpha)\kappa)\right]}$$

which can be rewritten as

$$\frac{d\ln s_K}{d\ln K}=\frac{\left(\frac{1-\varepsilon}{\varepsilon}\right)(\eta-\alpha)\left(\left[1+\left(\frac{\gamma}{1-\gamma}\right)\left(\frac{Y_H}{Y_L}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{-1}-\left[1+\left(\frac{1-\eta}{1-\alpha}\right)\left(\frac{\gamma}{1-\gamma}\right)\left(\frac{Y_H}{Y_L}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{-1}\right)}{\left[1-\left(\frac{\varepsilon-1}{\varepsilon}\right)((1-\eta)(1-\kappa)+(1-\alpha)\kappa)\right]}$$

which establishes that

$$\frac{d\ln s_K}{d\ln K}<0\Leftrightarrow\varepsilon<1$$

Similarly, by combining equations (53), (55) and (56) together with (24) we can derive

$$\frac{d\ln s_K}{d\ln M_L}=-\frac{d\ln s_K}{d\ln M_H}=\frac{\left(\frac{1-\varepsilon}{\varepsilon}\right)\left(\frac{1}{\nu-1}\right)\left((1-\gamma)\left(\frac{Y_L}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}-(1-\kappa)\right)}{\left[1-\left(\frac{\varepsilon-1}{\varepsilon}\right)((1-\eta)(1-\kappa)+(1-\alpha)\kappa)\right]}$$

which can be rewritten as

$$\frac{d\ln s_K}{d\ln M_L}=-\frac{d\ln s_K}{d\ln M_H}=\frac{\left(\frac{1-\varepsilon}{\varepsilon}\right)\left(\frac{1}{\nu-1}\right)\left(\left[1+\left(\frac{\gamma}{1-\gamma}\right)\left(\frac{Y_H}{Y_L}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{-1}-\left[1+\left(\frac{1-\eta}{1-\alpha}\right)\left(\frac{\gamma}{1-\gamma}\right)\left(\frac{Y_H}{Y_L}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{-1}\right)}{\left[1-\left(\frac{\varepsilon-1}{\varepsilon}\right)((1-\eta)(1-\kappa)+(1-\alpha)\kappa)\right]},$$

which establishes the desired result that

$$\frac{d\ln s_K}{d\ln M_H}=-\frac{d\ln s_K}{d\ln M_L}>0\Leftrightarrow(\eta-\alpha)(1-\varepsilon)>0.$$

7.2 Proof of Lemma 4 and The Converse Result

Proof of Lemma 4: First, recall that $C\leq Y$, (2). Hence it is enough to prove that $\lim_{t\to\infty}g=\infty$ will violate the resource constraint. We will prove this separately in two cases, when $g_L^*\geq g_H^*$ and when $g_L^*<g_H^*$

Suppose $g_L^*\geq g_H^*$ and $g^*=\infty$. Then, Lemma 2 implies $g_H^*=g^*=\infty$. and equation (28) together with (31) yields

$$g=n-\left(\frac{1-\eta}{\eta}\right)\frac{\dot{r}}{r}+\frac{1}{\eta(\nu-1)}m_H$$

Given $n < \infty$ and $\lim_{t\to\infty} \dot{r}/r > 0$, it must be that asymptotically

$$g^* = \frac{1}{\eta(\nu - 1)} m_H^*. \tag{57}$$

Combining the technology possibility frontier (8) and (57) we have

$$\lim_{t\to\infty} \frac{\dot{X}_H}{X_H} = \lim_{t\to\infty} \frac{\dot{m}_H}{m_H} + \eta(1-\varphi)(\nu-1) g^*$$

Then, the first inequality in Assumption A3, $\zeta > 1/\eta$, implies

$$\lim_{t\to\infty} \frac{\dot{X}_H}{X_H} > g^*$$

which violates the resource constraint (3).

Next suppose that $g_H^* > g_L^*$and $g^* = \infty$. Then, following the steps of above, Lemma 2 implies $g_L^* = g^* = \infty$, and equation (28) together with (31) yields

$$\left(\frac{1-\eta(1-\varepsilon)}{\varepsilon}\right) g_H = \eta n + \left(\frac{1-\eta}{\varepsilon}\right) g - (1-\eta)\frac{\dot{r}}{r} + \left(\frac{1}{\nu - 1}\right) m_H$$

Since $g^* = \infty$, a fortiori $g_H^* = \infty$, and, given $n < \infty$ and $\lim_{t\to\infty} \dot{r}/r > 0$, we have that asymptotically

$$m_H^* = (\nu - 1)\left[\left(\frac{1-\eta(1-\varepsilon)}{\varepsilon}\right) g_H^* - \left(\frac{1-\eta}{\varepsilon}\right) g^*\right]. \tag{58}$$

Once again the innovation possibilities frontier (8) implies

$$\lim_{t\to\infty} \frac{\dot{X}_H}{X_H} = \lim_{t\to\infty} \frac{\dot{m}_H}{m_H} + \eta(1-\varphi) m_H^*$$

Then equation (57) together with the first inequality in Assumption A3, $\zeta > 1/\eta$, implies

$$\lim_{t\to\infty} \frac{\dot{X}_H}{X_H} > \eta(1-\varphi)(\nu-1) g^* > g^*$$

which violates the resource constraint (3), completing the proof that when Assumption A3 holds any quasi-equilibrium with more than exponential growth violates the resource constraints.

For completeness, we also prove the converse of Lemma 4, which shows that the use of the first inequality in Assumption A3, $\zeta > 1/\eta$, in this lemma is “tight” in the sense that, if it were relaxed, the converse result would obtain.

Lemma 4': Suppose A1 and A2 hold, but $\zeta \equiv (\nu - 1)(1 - \varphi) \leq \frac{1}{\eta}$, then there exists quasi-equilibria with $\lim_{t\to\infty} g = \infty$.

Proof. This lemma will be proved by showing that in this case

$$g_L^* = g_H^* = g^* = \infty \quad \text{and} \quad z_L^* = z_H^* = z^* \tag{59}$$

$$z^* = g^* - \frac{\dot{r}}{r} \tag{60}$$

$$\frac{\dot{m}_H}{m_H} = \frac{\dot{m}_L}{m_L} = [1 - \eta\zeta] g \tag{61}$$

is a quasi-equilibrium.

From the interest rate conditions in the two sectors (20) and (21), and (59) we obtain

$$z^* = g^* - \lim_{t\to\infty} \frac{\dot{r}}{r}$$

which is exactly condition (60). By substituting into (31), we obtain

$$g^* = n - \left(\frac{1-\eta}{\eta}\right) \lim_{t\to\infty} \frac{\dot{r}}{r} + \frac{1}{\eta(\nu - 1)} m_H^*$$

$$g^* = n - \left(\frac{1-\alpha}{\alpha}\right) \lim_{t\to\infty} \frac{\dot{r}}{r} + \frac{1}{\alpha(\nu - 1)} m_L^*$$

which gives $g^* = \frac{1}{\eta(\nu-1)} m_H^*$ and $g^* = \frac{1}{\alpha(\nu-1)} m_L^*$, and hence

$$m_H^* = \frac{\eta}{\alpha} m_L^*.$$

Differentiating this condition gives equation (61).

Finally, we need to check feasibility, i.e., that the R&D expenditures do not grow faster than output. From the technology possibilities frontiers, (8), this requires

$$\lim_{t\to\infty} \frac{\dot{X}_H}{X_H} = \frac{\dot{m}_H^*}{m_H^*} + \eta\zeta g^* \leq g^*$$

$$\lim_{t\to\infty} \frac{\dot{X}_L}{X_L} = \frac{\dot{m}_L^*}{m_L^*} + \alpha\zeta g^* \leq g^*$$

and both these conditions are satisfied given (61) and $\eta > \alpha$. ■

7.3 Data Sources and Additional Results

7.3.1 Data Sources

Our main source of data is the Gross Domestic Product by Industry Data from the National Income and Products Accounts (NIPA). We use these data to calculate nominal and real value added, price indices and the capital shares between 1977 and 2001 (the period for which chain-type quantity and price indexes are provided). All GDP figures refer to value added,[16] and take 1996 as baseline year, to calculate the growth rate of the industrial price indexes.

Capital share is calculated from compensation of employees and property income (excluding indirect business tax and nontax liabilities, see McCahill and Moyer, 2002). Property income in the NIPA data also includes the self-employed income in proprietors' income. We allocate two-thirds of this amount to labor income and the rest of capital income.[17]

Throughout, we exclude the government and private household sectors. Our baseline analysis also excludes the categories business services and miscellaneous professional services, which cannot be separated before 1987 (miscellaneous professional services are replaced by other services after 1987).

To measure skill intensity in the various sectors, we use the concordance developed by Autor, Katz and Krueger (1998), which gives us 47 consistent industries (excluding government and private households sectors). We also use Autor, Katz and Krueger's (1998) calculations of education shares. They compute industry shares of college graduates and college equivalents (college graduates plus 0.5 workers with some college) in total employment using the sample of employed workers aged 18-64, from the Merged Outgoing Rotation Group (MORG) files of the Current Population Survey (CPS) for the years 1980, 1990 and 1998 and the Census PUMS for 1960, 1970, 1980 and 1990.

7.3.2 Sectoral Classifications and Additional Results

Our baseline classification used for Figures 1-3 separates the economy into high-skill services and the rest. High-skill services include legal services, health services, banking

[16] The variable is "Current Dollar GDP by Industry", which is equal to an industry's gross output (sales or receipts and other operating income, commodity taxes and inventory change) minus its intermediate inputs (consumption of goods and services purchased from other industries or imported).

[17] The results are not affected by this allocation.

and finance, insurance, motion picture and theatres, and a broad category which covers educational, business, engineering and social services. The first column of Table A1 gives average growth rates of nominal and real value added, growth rate of price indices, average capital share, average share of college graduates in employment and average share of college equivalent workers in employment for this high-skill services sector and the rest of the economy. GDP growth rates rates are calculated as the geometric average of the total nominal or real value added of the entire sector. Price indices and capital share numbers use nominal GDP as weights.

The numbers in column 1 of Table A1 confirm the patterns shown in Figures 1-3. The high skill-services sector, which is more skill intensive and less capital intensive, grows more rapidly in nominal terms and more slowly in real terms than the rest of the economy.

The second column shows the same pattern once we exclude the other sectors aggregated together with business services (education services, engineering services and social services) in the Autor, Katz and Krueger (1998) concordance.

The third column separates industries into high and low skill groups according to the fraction of college graduates in total employment, using the cutoff of 25 percent college employment over the whole sample. With this classification, the high-skill sector is about 20 percent of total value added (as opposed to just above 18 percent in our baseline classification). The overall pattern is the same as in the baseline: the high-skill sector, which is also less capital intensive, has a faster nominal growth rate and a slower real growth rate. Finally, the fourth column shows the same pattern using a classification based the fraction of college equivalents in total employment (the cutoff is 40 percent, which makes the high-skill sector about 70 percent of total output).

	Baseline classification	Baseline classification w/o ind. 51	College grads. (threshold .25)	College eq. (thresh .4)
Growth of nom. GDP (L)	0.063	0.063	0.063	0.063
Growth of price ind. (L)	0.026	0.026	0.026	0.026
Growth of real GDP (L)	0.031	0.031	0.031	0.031
Capital share (L)	0.406	0.406	0.404	0.405
Rel. size of sec. L	0.814	0.836	0.790	0.813
Coll. grad. share (L)	0.165	0.165	0.162	0.165
Growth rate of nom. GDP (H)	0.094	0.097	0.090	0.094
Growth rate of price ind. (H)	0.064	0.066	0.059	0.064
Growth rate of real GDP (H)	0.026	0.026	0.026	0.026
Capital share (H)	0.210	0.236	0.241	0.216
Rel. size of sec. H	0.182	0.161	0.201	0.171
Coll. grad. share (H)	0.361	0.352	0.351	0.359

Table A1: Average growth rates of nominal and real value added, average growth rate of price indices, capital share, share of total output, share of college graduates in total employment. H refers to the high-skill service sector or the higher education sector, and L refers to the rest of the economy. All numbers, except education college graduate shares, calculated from National Income and Product Accounts, and college graduates shares from Autor, Katz and Krueger (1998).

References

[1] Acemoglu, Daron, "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality" *Quarterly Journal of Economics*, CXIII (1998), 1055-1090.

[2] Acemoglu, Daron, "Changes in Unemployement and Wages Inequality: An Alternative Theory and Some Evidence" *American Economic Review*, LXXXIX (1999), 1259-1278.

[3] Acemoglu, Daron, "Technical Change, Inequality and the Labor Market", *Journal of Economic Literature*, XL (2002a), 7-72.

[4] Acemoglu, Daron, "Directed Technical Change" *Review of Economic Studies*, LXIX (2002b), 781-810.

[5] Acemoglu, Daron, "Labor- and Capital-Augmenting Technical Change" *Journal of European Economic Association*, I (2003), 1-37.

[6] Acemoglu, Daron and Joshua Angrist, "How Large Argument Capital Externalities? Evidence From Compulsory Schooling Laws" *NBER MacroAnnual, 2000*, edited by Ben Bernanke and Kenneth Rogoff, MIT Press, Cambridge, 9-58.

[7] Aghion, Philippe and Peter Howitt, "A Model of Growth Through Creative Destruction" *Econometrica*, LX (1992), 323-351.

[8] Aghion, Philippe and Peter Howitt, *Endogenous Growth Theory*, Cambridge, MA, MIT Press, 1998.

[9] Aghion, Philippe, Peter Howitt and Gianluca Violante, "General Purpose Technology and Wage Inequality" *Journal of Economic Growth*, VII (2002), 315-345.

[10] Antras, Pol, "Is the U.S. Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution" MIT mimeo (2001).

[11] Autor, David, Lawrence Katz and Alan Krueger, "Computing Inequality: Have Computers Changed the Labor Market?" *Quarterly Journal of Economics*, CXIII (1998), 1169-1214.

[12] Barro, Robert and Xavier Sala-i-Martin. *Economic Growth*, McGraw Hill, New York (2004, 2nd edition).

[13] Baumol, William J., "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis", *American Economic Review*, LVII (1967), 415-426

[14] Berndt, Ernst, "Reconciling Alternative Estimates of the Elasticity of Substitution," *Review of Economics and Statistics*, LVIII (1976), pp. 59-68.

[15] Beaudry, Paul and David Green, "Cohort Patterns in Canadian Earnings: Assessing the Role of Skill Premia in Inequality Trends", *The Canadian Journal of Economics*, XXXIII (2000), 907-936.

[16] Beaudry, Paul and David Green, "The Changing Structure of Wages in the US and Germany: What explains the differences?", *American Economic Review*, XCIII (2003), 573-603.

[17] Caselli, Francesco, "Technological Revolutions", *American Economic Review*, LXXXIX (1999), 78-102.

[18] Chenery, Hollis, "Patterns of Industrial Growth", *American Economic Review*, V (1960), 624-654.

[19] Chirinko, Robert S. "Business Fixed Investment: a Critical Survey of Modeling Strategies, Empirical Results and Policy Implications" *Journal of Economic Literature*, XXXI, (1993), 1875-1911.

[20] Chirinko, Robert S., Steven M. Fazzari and Andrew P. Mayer, "How Responsive Is Business Capital Formation to Its User Cost?" *Journal of Public Economics*, LXXV, (1999), 53-80.

[21] Chirinko, Robert S., Steven M. Fazzari and Andrew P. Mayer, "That Elusive Elasticity: A Long-Panel Approach to Estimating the Price Sensitivity of Business Capital" Federal Reserve Bank of St. Louis, working paper (2001).

[22] David, Paul, and Th. Van de Klundert, "Biased Efficiency Growth and Capital-Labor Substitution in the U.S., 1899-1960," *American Economic Review,* LV, (1965), 357-393.

[23] Denison, Edward F, "Accounting for United States Economic Growth, 1929-1969" Washington, DC: Brookings Institution (1974).

[24] Drandakis, E. and Edmund Phelps, "A Model of Induced Invention, Growth and Distribution" *Economic Journal*, CXXVI (1965), 823-840.

[25] Echevarria, Cristina, "Changes in Sectoral Composition Associated with Economic Growth", *International Economic Review*, XXXVIII (1997), 431-452.

[26] Eisner, Robert, and M. I. Nadiri, "Investment Behavior and the Neoclassical Theory," *Review of Economics and Statistics*, L (1968), 369-382.

[27] Galor, Oded and Daniel Tsiddon, "Technological Progress, Mobility and Economic Growth", *American Economic Review*, LXXXVII (1997), 363-382.

[28] Galor, Oded and Omer Maov "Ability Biased Technological Transition, Wage Inequality and Economic Growth" *Quarterly Journal of Economics*, CXV (2000), 469-497.

[29] Greenwood, Jeremy and Mehmet Yorukoglu, "1974", *Carnagie-Rochester Conference Series on Public Policy*, XLVI (1997), 49-95.

[30] Griffin, James M. and Paul R. Gregory, "An Intercountry Translog Model of Energy Substitution Responses", *American Economic Review*, LXVI (1976), 845-857.

[31] Griliches, Zvi, "Capital-Skill Complementarity", *Review of Economics and Statistics*, LI (1969), 465-468.

[32] Griliches, Zvi, "Productivity, R&D, and the Data Constraint", *American Economic Review*, LXXXIV (1994), 1-22.

[33] Grossman, Gene and Elhanan Helpman, "Quality Ladders in the Theory of Growth" *Review of Economic Studies*, LIIX (1991a), 43-61.

[34] Grossman, Gene and Elhanan Helpman, *Innovation and Growth in the Global Economy,* Cambridge, MA, MIT Press, 1991b.

[35] Hamermesh, David S., *Labor Demand*, Princeton University Press, Princeton 1993.

[36] Heckman, James. Lance Lochner and Christopher Taber "Tax policy and human capital formation," *American Economic Review Papers and Proceedings* 88, (1998) 293-297.

[37] Hornstein, Andreas And Per Krusell "Implications of the Capital-Embodiment Revolution For Directed R&D and Wage Inequality" *Federal Reserve Bank of Richmond Economics Quarterly*, XIC, Fall (2003).

[38] Howitt, Peter, "Steady Endogenous Growth with Population and R&D Inputs Growing" *Journal of Political Economics*, CVII (1999), 715-730.

[39] Howitt, Peter, "Endogenous Growth and Cross-Country Income Differences", *American Economic Review*, XC (2000), 829-846.

[40] Jones, Charles I., "R&D-Based Models of Economic Growth", *Journal of Political Economics*, CIII (1995), 759-784.

[41] Jones, Charles I., "The Shape of Production Functions and the Direction of Technical Change", U.C. Berkeley mimeo (2004).

[42] Kaldor, Nicholas , "Capital Accumulation and Economic Growth", in Friedrich A. Lutz and Douglas C. Hague, eds., Proceedings of a Conference Held by the International Economics Association, London, Macmillan.(1963)

[43] Katz, Lawrence and Kevin Murphy, "Changes in Relative Wages, 1963-1987: Supply and Demand Factors", *Quarterly Journal of Economics*, CVII (1992), 35-78.

[44] Kennedy, Charles, "Induced Bias in Innovation and the Theory of Distribution" *Economic Journal,* LXXIV (1964), 541-547.

[45] Kiley, Michael, "The Supply of Skilled Labor and Skill-Biased Technological Progress" 1999, *Economc Journal.*

[46] Kongsamut, Piyabha , Sergio Rebelo and Danyang Xie, "Beyond Balanced Growth", *Review of Economic Studies*, LXVIII (2001), 869-882.

[47] Kortum, Samuel, "Research, Patenting and Technological Change" *Econometrica*, LXV (1997), 1389-1431.

[48] Krusell, Per; Lee Ohanian and Victor Rios-Rull and Giovanni Violante, "Capital Skill Complementary and Inequality", *Econometrica*, LXIIX (2000), 1029-1053.

[49] Kuznets, Simon, "Quantitative Aspects of the Economic Growth of Nations: II, Industrial Distribution of National Product and Labour Forcce", *Economic Development and Cultural Change*, V Supplement (1957).

[50] Kuznets, Simon, "Modern Economic Growth: Findings and Reflections", *American Economic Review*, LXIII (1973), 829-846.

[51] Laitner, John, "Structural Change and Economic Growth", *Review of Economic Studies*, LXVII (2000), 545-561.

[52] Lucas, Robert E., "Labor-Capital Substitution in U.S. Manufacturing," in *The Taxation of Income from Capital*, edited by Arnold Harberger and Martin J. Bailey, Washington, DC: The Brookings Institution, (1969), 223-274.

[53] Lucas, Robert E., "On the mechanics of economic development." *Journal of Monetary Economics*, XXII (1988), 3-42.

[54] Mairesse, Jacques, Bronwyn H. Hall and Benoit Mulkay, "Firm-Level Investment in France and the United States: An Exploration over What We Have Returned in Twenty Years" *Annales d'Economie et Statistiques*, LV (1999), 27-67.

[55] McCahill, Robert J. and Brian C. Moyer, "Gross Domestic Product by Industry for 1999-2001", BEA Survey of Current Business, November 2002

[56] Nadiri, M. I. "Some Approaches to Theory and Measurement of Total Factor Productivity: a Survey" *Journal of Economic Literature*, VIII, (1970), 1117-77.

[57] Nerlove, Mark, "Recent Empirical Studies of the CES and Related Production Functions" in M. Brown (editor) *The Theory and Empirical Analysis of Production*New York (1967).

[58] Piketty, Thomas and Emannuel Saez, "Income Inequality in the United States, 1913 - 1998." CEPREMAP and Harvard mimeo, 2001.

[59] Rebelo, Sergio, "Long-Run Policy Analysis and Long-Run Growth" *Journal of Political Economy* XCIX (1991), 500-521.

[60] Romer, Paul M., "Increasing Returns and Long-Run Growth" *Journal of Political Economy*, XCIV (1986), 1002-1037.

[61] Romer, Paul M., "Endogenous Technological Change" *Journal of Political Economy*, IIC (1990), S71-S102.

[62] Samuelson, Paul, "A Theory of Induced Innovations Along Kennedy-Weisacker Lines" *Review of Economics and Statistics*, XLVII (1965), 444-464.

[63] Segerstrom, Paul S., T. C. A. Anant, and Elias Dinopoulos, "A Schumpeterian Model of the Product Life Cycle" *American Economic Review* LXXX (1990), 1077-1092.

[64] Young, Alwyn "Invention and Bounded Learning by Doing" *Journal of Political Economy* CI (1993), 443-472.

[65] Young, Alwyn "Growth without Scale Effect" *Journal of Political Economy*, CVI (1998), 41-63.