

Decentralized Borrowing and Centralized Default

Yun Jung Kim* Jing Zhang[†]
University of Michigan University of Michigan[‡]

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Abstract

In the past, foreign borrowing by developing countries was comprised almost entirely of government borrowing. Recently, private firms and individuals in developing countries borrow substantially from foreign lenders. It is often asserted that this surge in private sector borrowing generates overborrowing and leads to sovereign defaults in developing countries. This paper analyzes the impact of decentralized borrowing using a quantitative model in which private agents decide how much to borrow and the government decides whether to default. Relative to a model in which the government decides both borrowing and default, decentralized borrowing drives up aggregate credit costs and sovereign default risk, and reduces aggregate welfare. Private agents do not internalize the effect of their borrowing on economy-wide credit costs and tend to overborrow. Depending on the severity of default penalties, decentralized borrowing may lead to either too much or too little debt in equilibrium.

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*Department of Economics, University of Michigan, 611 Tappan St., Ann Arbor, MI, 48109. Email: yunjungk@umich.edu

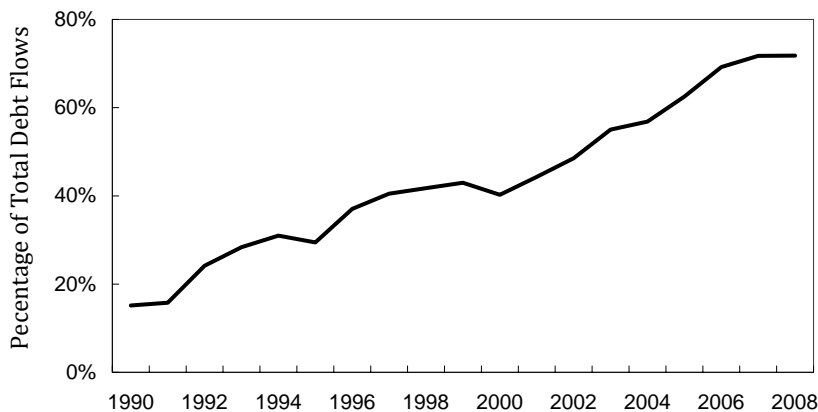
[†]Department of Economics, University of Michigan, 611 Tappan St., Ann Arbor, MI, 48109. Email: jzhang@umich.edu

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1 Introduction

In the past, foreign borrowing by developing countries was comprised almost entirely of government borrowing. Motivated by this observation, the sovereign debt literature focuses exclusively on foreign debt undertaken by sovereign governments. We refer to such models as “*centralized borrowing*” models. Recently, however, private firms and individuals in developing countries borrow substantially from abroad directly or indirectly through domestic private banks. As shown in Figure 1, private borrowing rose from less than 20 percent of total foreign borrowing in 1990 to more than 70 percent in 2008. Private borrowing is often implicitly or explicitly guaranteed by governments, which broadens the concept of sovereign debt to include also private sector debt.¹ This paper studies how *decentralized borrowing* by private agents affect aggregate debt levels, credit costs and sovereign defaults.

Figure 1: Debt Flows to Private Sectors of Developing Countries



Data source: *Global Development Finance*

This paper builds a quantitative *decentralized borrowing* model in which private agents decide how much to borrow, and the government decides whether to default. To highlight the effect of decentralized borrowing, we assume that all foreign debt is undertaken by private sectors. Empirically, governments play a crucial role in repayments of private sector debt, and sovereign credit risk is a key factor in the pricing of private sector loans.² Our assumption

¹Chile’s debt nationalization in 1982 is an example. Chilean total foreign debt reached as high as 20 billion dollars in 1982, and two thirds of it is private debt by leading domestic private banks. When the Latin American panic dried up new loans, six top private banks failed, and the Chile government assumed the responsibility for private foreign debt.

²Governments can impose capital controls to block private debt repayments or nationalize private debt to assume responsibility of repayments. For details, see section 2.1.

that governments make default decisions on private borrowing captures these features of the data. We find that decentralized borrowing generates overborrowing incentives of private agents because they fail to internalize the impacts of their individual borrowing on aggregate borrowing costs. In equilibrium, decentralized borrowing increases aggregate credit costs and sovereign default risk, and reduces aggregate welfare, relative to centralized borrowing. Depending on the severity of default penalties, decentralized borrowing may lead to either too much or too little debt in equilibrium.

In the model, a continuum of identical households borrow from foreign lenders in terms of non-contingent debt to smooth their aggregate income shocks. A benevolent government decides whether to enforce debt repayments to maximize the welfare of the representative household. If the government defaults, the country loses access to international financial markets and suffers income losses for some stochastic periods. Foreign lenders offer the households a bond price schedule that depends on the level of aggregate borrowing instead of individual borrowing, because aggregate borrowing, together with the aggregate income shock, determines the default probability. Our centralized default setup captures the lending practices to developing countries based on aggregate indicators.

We contrast model mechanisms under decentralized borrowing with those under centralized borrowing. Consider the demand and supply curves of debt over credit costs. Under centralized borrowing, the government takes into account the adverse effect of an extra unit of debt on credit costs and next-period default probabilities when choosing debt contracts. In contrast, individual households fail to internalize the adverse effect of their borrowing, and overborrow. This *overborrowing effect* shifts up the demand curve for debt and tends to increase equilibrium debt levels and credit costs. On the other hand, the government has larger default incentives under decentralized borrowing, because benefits of staying in international credit markets are reduced by anticipated future overborrowing by private agents, implying lower bond prices and higher credit costs. This *bond price schedule effect* shifts up the supply curve of debt and tends to reduce debt levels but increase credit costs. Thus, equilibrium credit costs are unambiguously higher under decentralized borrowing. However, the impact of decentralized borrowing on equilibrium debt levels depends on the relative strength of the two effects.³

³A similar type of the externality, arising from sequential borrowing from multiple *lenders*, is studied in Bizer and DeMarzo (1992). In their framework, eliminating the externality calls for either making loan terms contingent on future borrowing or disallowing future borrowing but making debt callable. Bi (2006) and Hatchondo and Martinez (2009) examine the Bizer-DeMarzo type of externality in quantitative sovereign debt models. By contrast, our paper focuses on the externality arising from a large number of *borrowers*

To examine quantitative implications of decentralized borrowing, we parameterize the model. Default penalties play a central role in quantitative results. Following Arellano (2008), we assume that income losses after default are disproportionately large under good income shocks, allowing the default model to generate an empirically reasonable default rate. We find that decentralized borrowing substantially increases credit costs and the default rate, and reduces the country’s welfare relative to centralized borrowing. All of these findings are robust to different parameter values of default penalties. Thus, some types of capital controls may reduce aggregate credit costs and the likelihood of debt crisis, and improve the welfare of developing countries.

Decentralized borrowing increases the equilibrium level of debt under lenient default penalties, but lowers it under severe default penalties, relative to centralized borrowing.⁴ To understand this result, let us consider the two counteracting effects determining equilibrium debt. The overborrowing effect arises from the failure of private agents to internalize the impact of their additional borrowing on credit costs. Thus, the more sensitive credit costs are to aggregate borrowing, the stronger is the overborrowing effect. The bond price schedule effect occurs because the overborrowing incentive lowers the repayment welfare and drives up the default likelihood of the government. Thus, the greater the decrease in the repayment welfare is, the stronger is the bond price schedule effect.

When the default penalties are lenient, credit costs are sensitive to aggregate risky borrowing because the government’s incentive to default rises rapidly with additional units of debt. In this case, decentralized borrowing, though it produces more defaults, lowers the repayment welfare only by a little because default is not that costly. Thus, the overborrowing effect is strong while the bond price schedule effect is weak, which leads to equilibrium overborrowing. When the default penalties are severe, credit costs are less sensitive to risky borrowing, but the repayment welfare is lowered substantially under decentralized borrowing. Thus, the overborrowing effect is weak while the bond price schedule effect is strong, which leads to equilibrium underborrowing.

Our work is closely related to Uribe (2006) who shows that regardless of whether a debt limit is imposed at the country level or at the individual level, the equilibrium level of debt is the same. In his analysis, debt limits and interest rates are exogenously specified, and there is no default risk. By contrast, in our model, the presence of default risk endogenizes both

in a centralized default environment. To eliminate the externality in our paper, one need either make loan terms contingent on individual borrowing or impose taxes (or subsidies) on individual borrowing.

⁴If income losses after default are a constant fraction of the income shock, the decentralized borrowing model always generates underborrowing in equilibrium. For details see section 4.2.

interest rates and debt limits. In general, the equilibrium level of debt depends on whether interest rates depend on aggregate debt or individual debt.⁵ Our work is also related to Jeske (2006) and Wright (2006), who study the effects of private borrowing under complete markets. We instead consider the effects under incomplete markets.⁶

Our model builds on the classic sovereign default framework of Eaton and Gersovitz (1981) and recent quantitative research on sovereign debt: Arellano (2008), Aguiar and Gopinath (2006), among others. Recently, different approaches have been taken to enrich and improve the sovereign debt model. One approach is to introduce production economies to the sovereign debt literature (Bai and Zhang (2009)). One approach is to introduce political economy considerations to the government’s decision (Hatchondo et al. (2009), and Cuadra and Saprizza (2008)). One approach is to introduce long-duration bonds (Arellano and Ramanarayan (2010), Chatterjee and Eyigungor (2010), and Hatchondo and Martinez (2009)). Another approach is to add renegotiations and settlements (Yue (2010), and Wright and Benjamin (2009)). All these papers examine equilibrium outcomes under centralized borrowing. Our paper instead studies outcomes under decentralized borrowing.

The remainder of the paper is organized as follows. Section 2 presents the model with decentralized borrowing. In section 3, we compare the quantitative implications of the models with decentralized and centralized borrowing. Section 4 investigates how different default penalties affect the quantitative results, in particular, the equilibrium debt level. We conclude in section 5.

2 Models

This section presents a dynamic stochastic general equilibrium model of decentralized borrowing and centralized default in which borrowing decisions are made by individual households and default decisions are made by a government. This setup is intended to capture an environment in which borrowing decisions are made by private agents and lending decisions of foreign lenders are guided by aggregate indicators rather than individual borrowers’ ability to repay.

⁵Interest rates depend on aggregate debt in the decentralized borrowing model. By contrast, one can interpret that interest rates depend on individual borrowing in the centralized borrowing model, which generates the same outcomes as a model with both decentralized borrowing and decentralized default.

⁶Bai and Zhang (2010) show that both incomplete markets and default risk are important to account for various dimensions of international data, for example, savings and investment behavior.

2.1 Model with Decentralized Borrowing

The model economy consists of three types of agents: a continuum of identical households and a sovereign government in a small open economy, and foreign lenders. The households receive stochastic aggregate income shocks y , which follow a Markov process with the transition function $f(y', y)$. In order to smooth income shocks, the households trade non-contingent bonds b with risk-neutral foreign lenders. The benevolent government, maximizing its representative household's welfare, decides whether to enforce foreign debt contracts.⁷ In each period, the country is either in the normal phase with access to international financial markets or in the penalty phase without access to financial markets.

The timing is as follows. At the beginning of each period, the income shock y is realized. If the country is in the normal phase, the government decides whether to enforce the repayment of outstanding foreign debt B .⁸ If the government enforces debt contracts, the households repay their debt b and decide on consumption c and next-period debt b' . If the government defaults, the households do not to repay their debt, and the economy goes into the penalty phase. The country in the penalty phase suffers from income loss and has probability θ of reverting to the normal phase each period.

Government

At the beginning of the normal period, the benevolent government observes current income shock y and aggregate foreign debt B . The government decides whether to enforce debt contracts to maximize the representative household's welfare. This welfare is given by $v^D(y)$ if the government chooses to default, and $v^R(B, y, \Gamma(B, y))$ if the government chooses to enforce the repayment with an anticipation that the economy will borrow $B' = \Gamma(B, y)$ this period. Thus, the government solves the following problem:

$$D(B, y) = \arg \max_{d \in \{0,1\}} \{(1-d) v^R(B, y, \Gamma(B, y)) + d v^D(y)\}, \quad (1)$$

where $d = 1$ indicates default and $d = 0$ indicates repayment. If the repayment welfare v^R is greater than the default welfare v^D , then the government enforces the repayment

⁷Broner and Ventura (2010) analyze an environment with both domestic and international trade of contingent claims among private agents. They assume that the government, when deciding whether to enforce the claims, cannot discriminate between domestic and foreign creditors. We implicitly allow discrimination. Our framework is equivalent to the one in which both domestic and international borrowing and lending are allowed, and the government always enforces domestic contracts, but not necessarily international contracts. Given that households are identical, domestic borrowing and lending are never observed in equilibrium.

⁸A positive B denotes foreign assets, and a negative B denotes foreign debt.

of individual debt contracts. Otherwise, the government decides to declare default. Our assumption that national governments make default choices highlights default risk, driven by national governments, of private debt contracts. The governments can impose exchange or capital controls to prevent private agents from repaying their debt.⁹

Foreign Lenders

Foreign lenders are risk neutral. They operate in competitive international financial markets and have the opportunity cost of funds at the risk-free interest rate r . They thus have to break even for each debt contract. Since the government's default decisions are based on aggregate debt, the bond price schedule also depends on aggregate debt. For any aggregate borrowing level B' , the lender expects to receive the repayment B' next period if and only if the government enforces repayment next period, that is $D(B', y') = 0$. Thus, the total expected repayment next period is $\int_{y'} B' (1 - D(B', y')) f(y', y) dy'$. The resource cost of this debt contract to the lender today is $q(B', y)B'$. The zero profit condition requires that the resource cost equals the present value of the expected repayment. This gives rise to the bond price schedule:

$$q(B', y) = \frac{\int_{y'} (1 - D(B', y')) f(y', y) dy'}{1 + r}. \quad (2)$$

If the government will enforce repayment under all future income shocks, the bond price is simply the inverse of the gross risk-free rate. However, if the government defaults for some future income shocks, the bond price is lower to compensate for the default risk.

The centralized default decision for decentralized borrowing implies that credit terms for private debt depends critically on aggregate debt. This implication is consistent with empirical evidence. Using firm-level observations from 30 countries for 1995–2004, Borensztein et al. (2007) show that sovereign ratings are a significant determinant of credit ratings assigned to corporations in emerging market economies. Similar findings are presented in Agca and Celasun (2009), Ferri and Liu (2003), Fernandez-Arias and Lombardo (1998), and Mendoza and Yue (2010). Major rating agencies' practices also support this implication. For example, Standard and Poor's (2001) stresses that sovereign credit risk is always a key consideration in the assessment of the credit standing of banks and corporations in emerging markets. Their main argument is that governments in financial distress or default may force private sector to default by imposing exchange controls and other restrictive measures.

⁹See discussions on national default risk in Wright (2006).

Individual Households

We now describe the individual household's problem. A measure one continuum of infinitely-lived identical households have flow utility $u(c)$ over consumption c , where $u(\cdot)$ is increasing and strictly concave. If the country is in the normal phase and the government decides to repay, then the households can trade one-period non-contingent bonds b' . The households take as given the aggregate borrowing level B' and the associated bond price $q(B', y)$. In addition, the households also take as given the default decision of the government $D(B', y')$.

Hence, a household with bond holding b and income shock y solves:

$$\begin{aligned} v^R(b, y, B') &= \max_{b'} u(y + b - q(B', y)b') & (3) \\ &+ \beta \int_{y'} [(1 - D(B', y')) v^R(b', y', B'') + D(B', y') v^D(y')] f(y', y) dy' \\ &s.t. \quad B'' = \Gamma(B', y'), \end{aligned}$$

where $0 < \beta < 1$ is the discount factor, and $B'' = \Gamma(B', y')$ is aggregate bonds that the economy will issue next period if the government continues to enforce repayment. Aggregate borrowing B' plays an important role in each household's decision; it pins down the cost of borrowing today, the government's default decision next period, and future aggregate borrowing B'' .

If the government decides to default, the households do not repay their debt but lose access to international financial markets. In each period, the economy has probability θ of regaining access to international financial markets with zero debt obligations. During the exclusion periods, the households suffer from income loss; their income drops from y to y^{def} . The default welfare is given by

$$\begin{aligned} v^D(y) &= u(y^{def}) + \beta \int_{y'} [\theta v^R(0, y', B'') + (1 - \theta) v^D(y')] f(y', y) dy', & (4) \\ &s.t. \quad B'' = \Gamma(0, y'). \end{aligned}$$

Recursive Competitive Equilibrium

The recursive competitive equilibrium of this economy is a list of (i) individual value functions and policy functions: v^R , v^D , c , and b' , (ii) a government default decision function $D(B, y)$, (iii) an actual law of motion for aggregate debt $B' = \Gamma(B, y)$, and (iv) a bond price schedule $q(B', y)$ such that

1. Given q, Γ and D , the value and policy functions solve the household's problem.

2. The household's policy function b' is consistent with Γ .
3. Given Γ , $D(B, y)$ solves the government's problem.
4. The bond price schedule $q(B', y)$ ensures foreign lenders' break-even in expected value.

2.2 Model with Centralized Borrowing

We compare our decentralized borrowing model with the standard Eaton and Gersovitz (1981) type model of centralized borrowing. All aspects of the model with centralized borrowing are identical to the model with decentralized borrowing, except one difference. In the centralized borrowing model, the government, instead of the households, makes the borrowing decision.¹⁰ We briefly describe the centralized borrowing model. The government's value function is

$$W(B, y) = \max_{d \in \{0,1\}} \{(1-d)W^R(B, y) + dW^D(y)\} \quad (5)$$

where $W^R(B, y)$ is the repayment welfare and $W^D(y)$ is the default welfare. Let $D_C(B, y)$ denote the government optimal default decision and $q_C(B', y)$ denote the bond price schedule. The repayment welfare is given by

$$W^R(B, y) = \max_{B'} u(y + B - q_C(B', y)B') + \beta \int_{y'} W(B', y') f(y', y) dy'. \quad (6)$$

Note that the government chooses aggregate debt next-period, B' , and allocates it evenly across the households. The default welfare is defined as

$$W^D(y) = u(y^{def}) + \beta \int_{y'} [\theta W(0, y') + (1-\theta)W^D(y')] f(y', y) dy'. \quad (7)$$

The bond price schedule is again given by foreign lenders' break-even condition:

$$q_C(B', y) = \frac{\int_{y'} (1 - D_C(B', y')) f(y', y) dy'}{1 + r}. \quad (8)$$

The recursive competitive equilibrium of this economy consists of a list of the government's value functions, $\{W, W^R, W^D\}$, policy functions $\{B', D_C\}$ and a bond price schedule $q_C(B', y)$ such that

1. Under q_C , the value and policy functions solve the government's problem.
2. The bond price $q_C(B', y)$ ensures foreign lenders' break-even in expected value.

¹⁰One can alternatively think of the government's problem as the representative household's problem.

2.3 Comparison of the Two Borrowing Environments

In order to facilitate exposition, we treat the value functions and the bond price functions as differentiable in this subsection.¹¹ The first order condition in the model with centralized borrowing is

$$u'(c) \left[q_C(B', y) + \frac{\partial q_C(B', y)}{\partial B'} B' \right] = \beta \int_{y'} (1 - D_C(B', y')) u'(c') f(y', y) dy'. \quad (9)$$

The $\frac{\partial q_C(B', y)}{\partial B'} B'$ term represents the change in the bond price in response to one extra unit of the bond. This term is not present in the corresponding first order condition in the model with decentralized borrowing:

$$u'(c) q(B', y) = \beta \int_{y'} (1 - D(B', y')) u'(c') f(y', y) dy', \quad (10)$$

since households take the bond price as given.

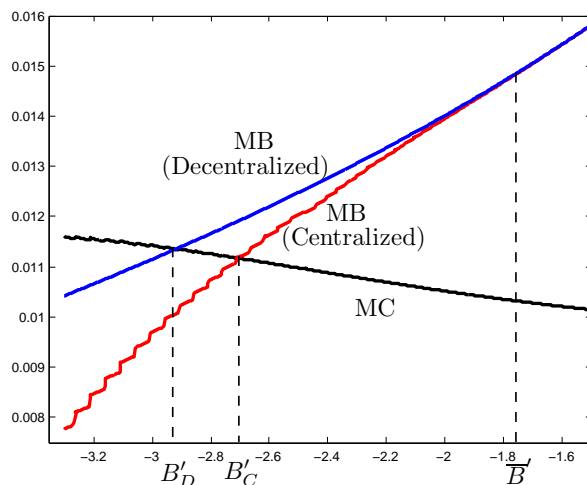
For expository purposes, let us compare the debt levels assuming that the bond price schedules and the default sets are the same in the two models, that is $q_C = q$ and $D_C = D$. Denote the optimal bond holdings in the model with centralized borrowing and in the model with decentralized borrowing by B'_C and B'_D , respectively.¹² For sufficiently low levels of debt, the government enforces repayments under all future shocks and thus the economy faces the risk-free interest rate. We denote the maximum amount of such debt by \bar{B}' and refer to it as the safe debt limit. Then it must be the case that $\frac{\partial q(B', y)}{\partial B'} = 0$ for any $B' > \bar{B}'$. This implies that $B'_C = B'_D$ if the optimal debt is below the safe debt limit in both models.

Now consider the effect of raising debt by one unit when $B' < \bar{B}'$. The marginal cost is the expected loss in future utility conditional on not defaulting next period, which is the right hand side of equation (9) and (10). The marginal benefit is the current utility gain from the resource raised by one extra unit of debt, which is the left hand side of these two equations. We plot the marginal cost and benefit for each model in Figure 2. The marginal costs are identical across the two models and rise with the debt level. The marginal benefits in both models decline with the debt level. Moreover, the marginal benefit is higher under decentralized borrowing since $\frac{\partial q(B', y)}{\partial B'} > 0$ and $\frac{B' \partial q(B', y)}{\partial B'} < 0$. At the optimal debt level, the marginal benefit equals the marginal cost. This implies that $B'_C > B'_D$, and so the households would like to borrow more under decentralized borrowing.

¹¹The solution method employed in the quantitative analysis section does not depend on the differentiability of the value functions and the bond price schedule.

¹²With decentralized borrowing, individual households choose b'_D instead of B'_D . In equilibrium, however, individual and aggregate debt coincide. Thus, we compare aggregate debt in the two models.

Figure 2: Marginal Benefits and Marginal Costs of Debt



Notes: The marginal cost and benefit of an additional unit of debt in the centralized borrowing model are plotted for a country with an endowment shock at the 60th percentile and a debt level about 34% of income using the model solution. The marginal benefit in the decentralized borrowing model is constructed using the bond price schedule in the centralized borrowing model.

When making borrowing decisions, the government internalizes the adverse effect of additional borrowing on the bond price, but individual households, acting as price takers, do not.¹³ This is an example of a pecuniary externality: one individual's actions affect another individual's welfare through prices.¹⁴ Pecuniary externalities by themselves are not a source of inefficiency since they work within the market mechanism through prices. However, they do cause efficiency losses and lower welfare if there are other market imperfections such as incomplete markets and limited enforcement in the model.¹⁵

The above discussions assume that the bond price and default schedules are the same in the two models. These assumptions automatically hold if default never occurs in equilibrium and the bond price schedule is an exogenous function of aggregate debt. In this case, decentralized borrowing unambiguously leads to overborrowing in equilibrium.¹⁶ However, in our

¹³The externality resembles the one studied in Gertler and Kiyotaki (2010). In their paper, individual banks do not internalize the effects of their own liability structure on the aggregate liability structure. This externality induces relatively more debt financing in individual banks' liability structure, and leads to an over-levered aggregate balance sheet.

¹⁴Levchenko (2005) highlights another source of externality of private borrowing. When there are heterogeneous agents and heterogeneous access to international financial markets, financial integration might break domestic risk sharing and hurt those without access to international financial markets.

¹⁵For more discussions on efficiency losses from pecuniary externalities, see Loong and Zeckhauser (1982) and Greenwald and Stiglitz (1986).

¹⁶This is one of the examples in Uribe (2006) and referred to as the debt-elastic country premium case.

model both the bond price and default schedules are endogenous. Given the overborrowing incentives of the households, borrowing costs are higher and welfare, especially the repayment welfare, is lower under decentralized borrowing. Consequently, the government has a higher incentive to default, and the bond price schedule is less favorable under decentralized borrowing, i.e., the default set changes and the bond price schedule shifts. This bond price schedule effect reduces borrowing. Hence, whether decentralized borrowing leads to equilibrium overborrowing depends on which effect dominates: the overborrowing incentive or the bond price schedule effect. We analyze quantitatively the impacts of decentralized borrowing on the equilibrium debt level in the next section.

3 Quantitative Analysis

This section investigates the quantitative implications of the decentralized borrowing model. In order to highlight the impacts of decentralized borrowing, we first compare the equilibrium dynamics of the decentralized and centralized borrowing environments. We then evaluate the ability of the decentralized borrowing model to account for observed statistical moments of the business cycle in Argentina.

3.1 Calibration and Computation

We calibrate the model at the quarterly frequency. The utility has standard CRRA form: $u(c) = \frac{c^{1-s}-1}{1-s}$, where the coefficient of relative risk aversion s is 2. The risk-free interest rate is set to 1.7%, corresponding to the average quarterly interest rate of a five-year U.S. treasury bond for the period 1983–2001. The income shock y_t follows an AR(1) process: $\ln(y_t) = \rho \ln(y_{t-1}) + \varepsilon_t$ with $|\rho| < 1$ and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. We use the time series of Argentina’s GDP to calibrate the shock process and estimate ρ to be 0.945 and σ_ε to be 0.025.

The default penalty plays a crucial role in sovereign debt models. In the benchmark calibration, we assume that the default penalty is disproportionately large for large income shocks, following Arellano (2008). Specifically, y^{def} has the following form:

$$y^{def} = \begin{cases} (1 - \lambda)\bar{y} & \text{if } y > (1 - \lambda)\bar{y} \\ y & \text{if } y \leq (1 - \lambda)\bar{y} \end{cases}, \quad (11)$$

where \bar{y} denotes the unconditional mean of income shocks, and λ characterizes the income loss after default. A larger λ makes the default penalty more severe both by lowering the threshold income shock that is subject to income loss and by raising the magnitude of

income loss. We refer to this specification as the *asymmetric default penalty*. An alternative specification is the *symmetric default penalty* where income loss is a constant fraction of the income shock.

The motivation for the asymmetric default penalty is that sovereign default is often accompanied by a drop in private credit, and so the economy would have to forgo disproportionately larger income under good shocks.¹⁷ In addition, with a symmetric default penalty, sovereign debt models rarely generate equilibrium default and fail to match the default rate observed in the data. The asymmetric default penalty makes default attractive when the country experiences bad shocks, and thus helps raise the default probability. Empirical support on either form of the default output cost is rather weak despite their common use. Given the quantitative importance of the default output cost, we conduct a sensitivity analysis on the default penalties in the next section.

The default penalty parameter λ , the discount factor β , and the re-entry probability θ are chosen such that the model with decentralized borrowing produces the 3% default probability, 14% income drop upon default, and 1.75% standard deviation of the trade balance observed in the Argentina data. The default penalty parameter λ is estimated to be 0.1 and the discount factor β is 0.97. The re-entry probability θ is estimated to be 0.1, which corresponds to 10 quarters of exclusion from international financial markets after default. This is in line with the historical evidence presented in Gelos et al. (2010).¹⁸ See the lower panel of Table 1 for the summary of these parameter values.

With the functional forms and parameters described above, we solve the models numerically using the discrete state-space technique. The decentralized borrowing model is more difficult to compute than the centralized borrowing model. The state space has three dimensions $(b, y; B')$ in the decentralized borrowing model, while it has only two dimensions (B, y) in the centralized borrowing. Moreover, in the decentralized borrowing model we need to find the aggregate borrowing function $\Gamma(B, y)$ to be consistent with individual borrowings. Such aggregate borrowing functions are not unique in general. We discuss the detailed solution algorithm and the selection method for the aggregate borrowing function in Appendix A.

After solving the model, we simulate the model for 500,000 periods and find the latest 1,000 default episodes. We extract 74 consecutive observations of the normal period before each default event and examine the mean statistics over these samples. The 74 observations

¹⁷Mendoza and Yue (2010) present a model that generates endogenously this form of income loss.

¹⁸Gelos et al. (2010) find, for all defaulting episodes during the period of 1980–2000, that the median exclusion length is 3 years after default.

prior to a default episode correspond to the number of quarters between the latest two default events in Argentina.¹⁹ In the next subsection, we compare the implications of the decentralized borrowing model with those of the centralized borrowing model.

3.2 Decentralized versus Centralized Borrowing

Table 1 presents statistics for the Argentina data and for the decentralized and centralized borrowing models. The first column shows business cycle statistics for Argentina from 1983 to 2001. The annual default probability of 3% is based on three default episodes in approximately one hundred years. The average debt over GDP ratio of 43.36% is calculated for the period from 1983 to 2001 using *Global Development Finance*. The debt statistics include total external debt of the private and public sectors. The second column of Table 1 presents the statistics in the model with decentralized borrowing. To highlight the role of decentralized borrowing, we present these statistics in the model with centralized borrowing under the same set of parameter values in the third column.

There are three striking differences between the decentralized and centralized borrowing models. First, the mean spread under decentralized borrowing is higher by a factor of more than thirty compared to the one under centralized borrowing: 11.25% versus 0.37%. Second, the model with decentralized borrowing exhibits a much higher default probability, 3.03%, far exceeding 0.11% in the model with centralized borrowing. Third, the decentralized borrowing model generates a higher debt to income ratio than the centralized borrowing model does. The mean debt to income ratio is 22.48% in the decentralized borrowing model, while it is 21.22% in the centralized borrowing model.

To understand these differences, we examine borrowing decisions in the two models. Figure 3 plots the desired borrowing conditional on not defaulting over the current bond holdings.²⁰ Desired borrowing is similar across the two models for low levels of debt. As the debt level increases, desired borrowing increases faster under decentralized borrowing. Under centralized borrowing, the government recognizes that the interest rate increases as an additional unit of debt is taken. Under decentralized borrowing, however, households do not take into account the interest rate effect of their borrowing and would like to overborrow. This negative externality becomes especially severe when current debt is large and the interest

¹⁹In the model with centralized borrowing, default is so rare that only 137 samples satisfy our criteria. We thus compute the model statistics based on these 137 samples.

²⁰All figures in this subsection are based on the income shock, which is 10% below the mean. We observe the same qualitative results for the other income shocks. Both the current and next-period bond holdings are normalized by the mean income shock.

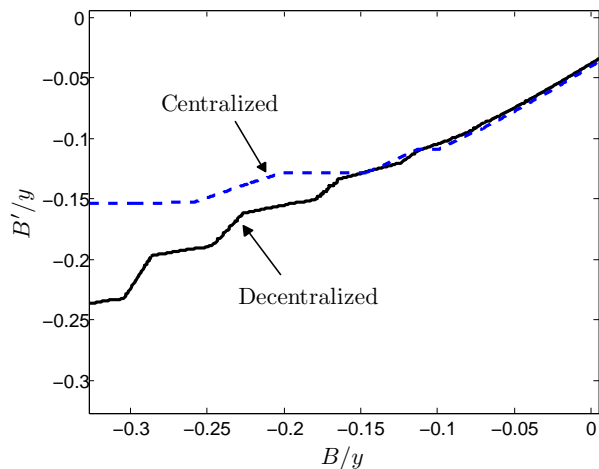
Table 1: Comparison of Decentralized and Centralized Borrowing

Statistics	Data	Model		
		Decentralized	Centralized 1	Centralized 2
mean(spread)	10.31	11.25	0.37	7.30
std(spread)	5.60	26.09	0.76	7.63
mean(B/y)	-43.36	-22.48	-21.22	-7.23
std(y)	8.12	5.52	5.63	5.71
std(C)	9.47	6.57	6.35	6.38
std(TB/y)	1.75	1.75	1.49	1.75
corr(c, y)	0.98	0.97	0.97	0.96
corr(TB, y)	-0.59	-0.47	-0.36	-0.24
corr(spread, y)	-0.89	-0.55	-0.50	-0.67
corr(spread, c)	-0.91	-0.70	-0.64	-0.73
corr(spread, TB/y)	0.70	0.85	0.83	0.47
				0.00
prob(default)	3.00	3.03	0.11	3.07
drop in y upon default	-14.21	-13.28	-15.41	-13.02
drop in c upon default	-16.01	-12.77	-15.02	-12.84
welfare		9.85	9.92	9.97
discount factor β		0.97	0.97	0.93
output loss λ		0.10	0.10	0.10
re-entry probability θ		0.10	0.10	0.65

Note: All statistics except correlations and welfare are in percentage terms. The income and consumption drops in default are based on the 2001 Argentine default episode. The interest rate spread is computed as the difference of the EMBI yield and the yield of a 5 year U.S. bond.

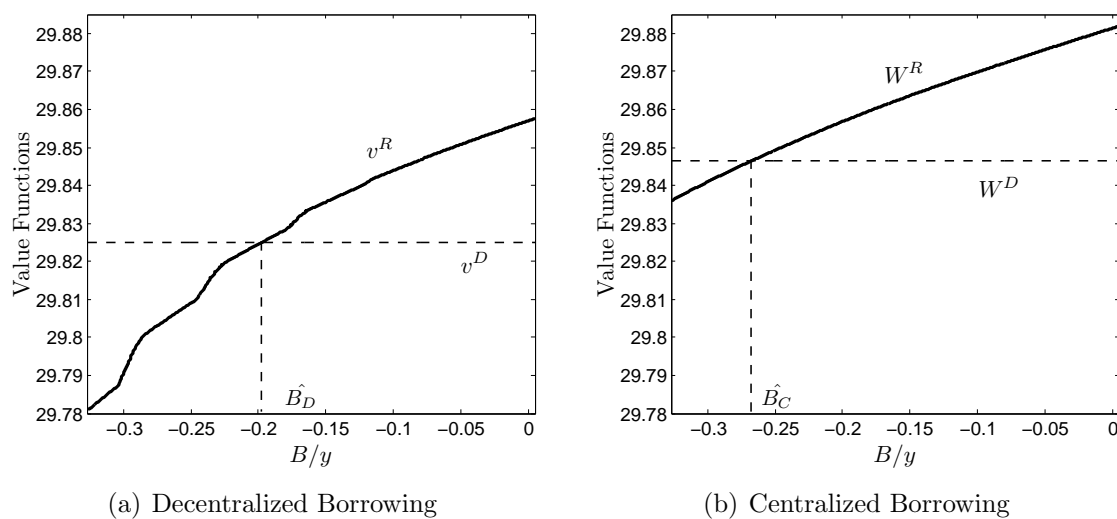
rate rises sharply with an additional unit of debt.

Figure 3: Comparison of Desired Borrowing



Borrowing and default are two instruments with which households affect their consumption path. Under centralized borrowing, the government, or equivalently the representative household, owns both instruments. Under decentralized borrowing, the households have only the first instrument and tend to take more debt since they fail to internalize the negative externality of their borrowing. Thus, welfare, especially the repayment welfare, is lower under decentralized borrowing, as shown in Figure 4. Consequently, the government finds default attractive for a wider range of debt levels under decentralized borrowing.

Figure 4: Comparison of Value Functions

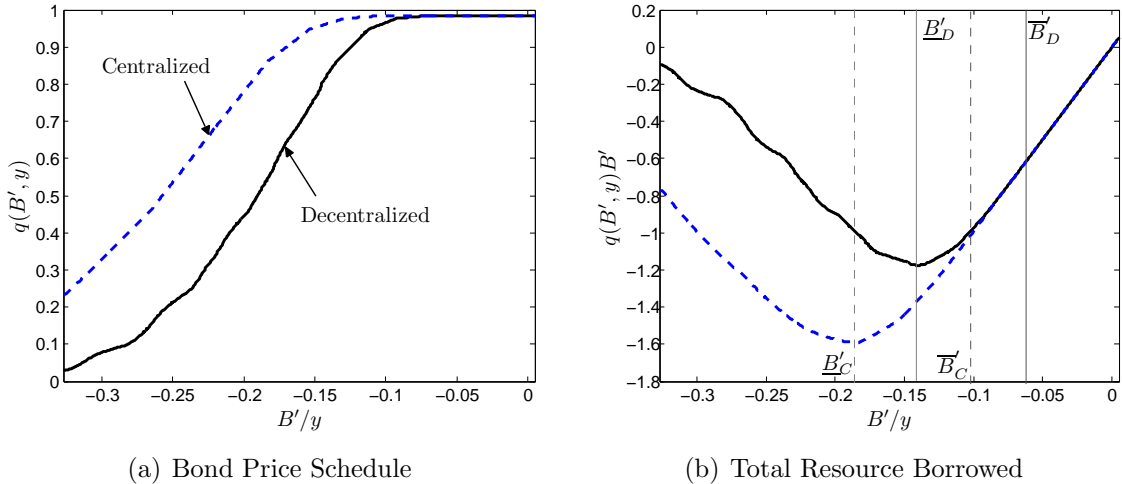


(a) Decentralized Borrowing

(b) Centralized Borrowing

The failure of individual households to internalize the effect of their borrowing on the government’s default choices lowers the bond price schedule under decentralized borrowing. As shown in the left panel of Figure 5, the prices are discounted by more for any level of bonds under decentralized borrowing. This less favorable bond price schedule generates tighter debt limits and constrains borrowing by more. The right panel displays the total resources that debt generates, $q(B', y)B'$. Any debt B' less than the safe debt limit \bar{B}' generates resources $B'/(1+r)$. Once it exceeds the safe debt limit, the debt becomes risky. Let us refer to the debt level which maximizes the resource obtained from foreign lenders, $q(B', y)B'$, as the risky debt limit, denoted by \underline{B}' . The optimal level of debt would never exceed the risky debt limit because the borrower can obtain the same amount of resources with a smaller next-period repayment. As shown in the figure, both the safe and risky debt limits are tighter under decentralized borrowing.

Figure 5: Comparison of Bond Price Schedules



The incentive to overborrow and the lower bond price schedule have opposite effects on the equilibrium level of debt. Whether decentralized borrowing leads to larger equilibrium debt depends on which force dominates. Under the benchmark calibration, desired borrowing is higher even when the bond price schedule is less favorable under decentralized borrowing. This leads to higher equilibrium debt under decentralized borrowing. Figure 6 shows the limiting distribution of bond holdings as shares of mean income for the two models. The distribution is more concentrated on high debt levels in the decentralized borrowing model. This implies that even with higher costs of borrowing, the incentive to overborrow is strong enough to induce the households to issue more debt. As a result, the interest rate spread

is substantially higher under decentralized borrowing. Also, the interest rate spread is more countercyclical under decentralized borrowing. As shown in Table 1, $\text{corr}(\text{spread}, y)$ and $\text{corr}(\text{spread}, c)$ are -0.55 and -0.70 , respectively, under decentralized borrowing, and -0.50 and -0.64 , respectively, under centralized borrowing.

Figure 6: Comparison of Bond Distributions

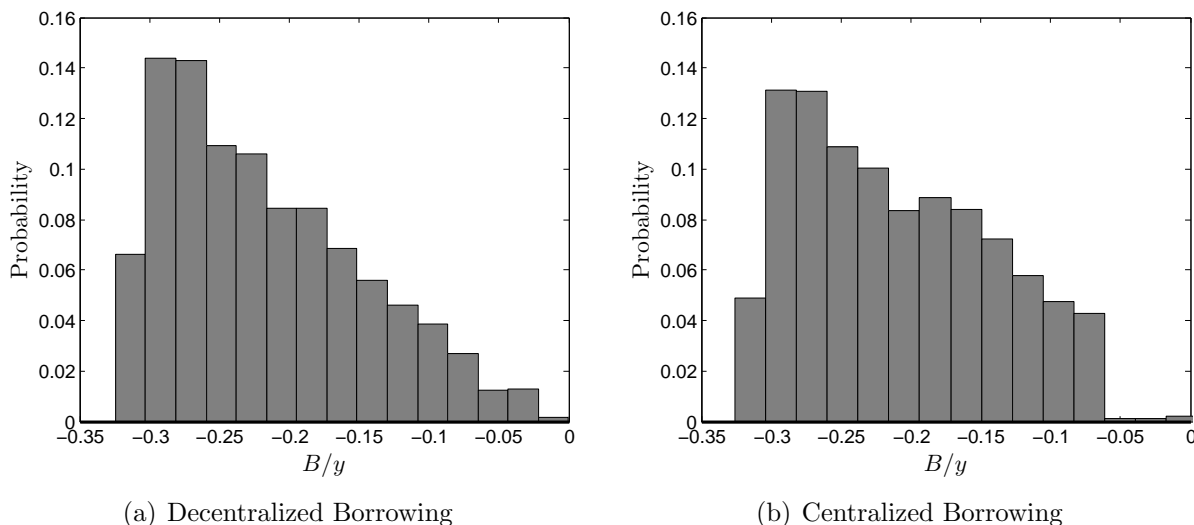


Table 1 also reports the welfare statistics, measured as permanent consumption, for each simulated model.²¹ The welfare is 1% lower under decentralized borrowing than under centralized borrowing. The 1% welfare difference is economically significant, considering that the welfare cost of business cycles estimated by Lucas (1987) is only about one-tenth of a percent of consumption.

In summary, the decentralized borrowing model generates a larger default rate, a higher mean spread, lower welfare and larger equilibrium debt than the centralized borrowing model in the benchmark calibration.²² All these findings, except the one on equilibrium debt, are robust to different default penalty parameters and specifications. We will discuss these

²¹The welfare calculation is based on the limiting distribution.

²²Models with political economy can also generate interest rate spreads higher than the standard sovereign debt model (see Cuadra and Sapriza (2008)). Political instabilities lead to short-sighted governments, who do not fully internalize the *next-period marginal cost* of their borrowing and have large incentives to default. Thus, everything else equal, interest rate spreads are higher than the one without political instabilities. By contrast, our model generates higher interest rate spreads because households do not fully internalize the *current-period marginal benefit* of their borrowing. They ignore the impact of an additional unit of debt on economy-wide borrowing costs and tend to overborrow, which increases default incentives of the government and interest rate spreads charged by international creditors.

robustness checks in section 4.

3.3 Quantitative Predictions of the Models

In this subsection, we compare the quantitative predictions of the two models with the Argentine data. Both models are calibrated to match the relevant moments in the data. In particular, the parameters β , λ , and θ are calibrated to best match the default rate, income loss, and trade-balance volatility in the data. The fourth column of Table 1 shows the statistics of the recalibrated centralized borrowing model. To generate the data moments, the centralized borrowing model needs a low discount factor of 0.93, and a large reentry probability of 0.65. That is, the average exclusion period after default is only 1.5 quarters.

The most striking difference across these two models is the equilibrium debt level. The debt to income ratio is 7.23% in the model with centralized borrowing. In contrast, it is about 22.48% in the model with decentralized borrowing—much closer to the data. We want to highlight that these quantitative results depend critically on the form and the parameter values of the default penalties, which we will analyze in details in the next section. Given the importance of the default penalties, the literature will benefit from more empirical research on the default penalties.

The model with decentralized borrowing also shows better performance in terms of replicating countercyclical trade balances. The correlation between the trade balance and income is -0.59 in the data; it is only -0.24 in the model with centralized borrowing, but -0.47 in the model with decentralized borrowing. In addition, decentralized borrowing generates a mean spread close to the data of 10.31%. The mean spread is 11.25% in the decentralized borrowing model and only 7.30% in the centralized borrowing model. On the other hand, the decentralized borrowing model overestimates the volatility of the interest spread.

Although both models match well the output drop after default in the Argentine 2001 default episode, they predict a much stronger negative relationship between output and default than is found in the historical record. As documented by Tomz and Wright (2007), the output declines from trend by only 1.6% in the first year of default in 169 default episodes over the period 1820–2004. Moreover, they document that only 62% of all the default episodes occur when the output is below the trend. However, the output declines from the trend in the first year of default by 12% under decentralized borrowing and by 11% under centralized borrowing. In both models, almost all default episodes occur when the output is below the trend.²³

²³A lower direct output cost parameter would reduce the discrepancy between the model simulation and

One caveat of the model simulation is worth mentioning. We compute model statistics based on 74 simulation periods before default to mimic the 74 quarters between the two default events of Argentina. Ideally, we should compute statistics based on the simulations where two consecutive defaults are exactly 74 periods apart. However, it is rare to find such cases in the simulation. Instead, we compute the model statistics based on the simulation episodes in which default ends and starts 70–78 periods apart, and compare these statistics with the benchmark results in Table 4 in the appendix.²⁴ Although some statistics are moderately different from the benchmark, the key patterns between the centralized and decentralized borrowing models are robust to this restricted sample of the model simulations.

4 Overborrowing or Underborrowing

This section examines the robustness of our quantitative results to different default penalty parameters and to an alternative symmetric specification of the default penalty. We find under all variations that decentralized borrowing implies a larger default rate, a higher mean spread, and lower welfare. The overborrowing result, however, is not robust: the decentralized borrowing model might generate underborrowing as we vary the default penalties.

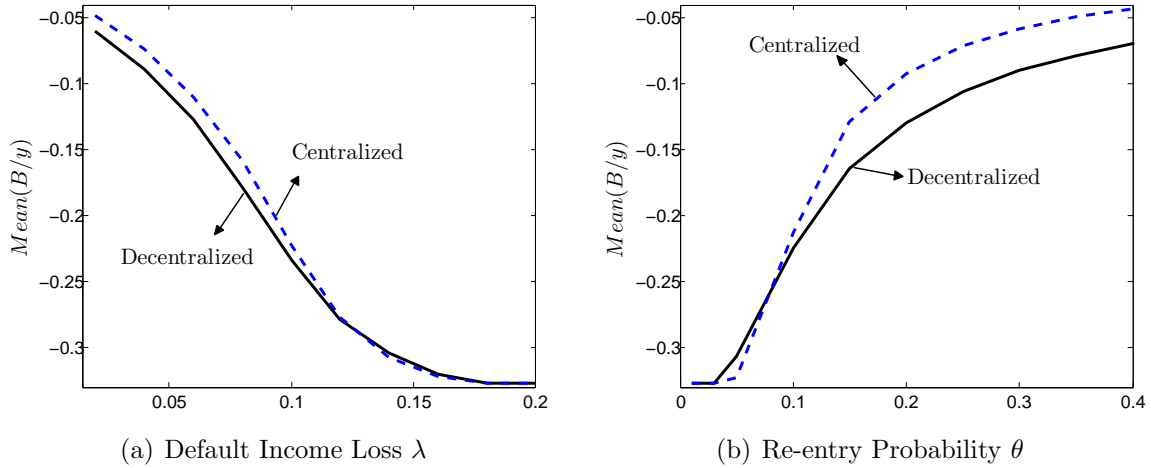
4.1 Alternative Default Penalty Parameters

Consider the two models with the benchmark parameter values. In the first set of experiments, we vary the default income loss parameter λ from 1% to 20% while fixing all the other parameters. In the second set of experiments, we vary the re-entry probability from 1% to 40% while fixing all the other parameters. We plot the equilibrium debt to income ratios of the two models for these two sets of experiments in Figure 7. First of all, equilibrium debt in both models increases with the default income loss λ and decreases with the re-entry probability θ . This is intuitive because larger values of λ or lower values of θ are associated with more severe default penalties, and this in turn implies less frequent default and more lenient bond price schedules.

Second, we find that decentralized borrowing generates overborrowing for low values of λ , but underborrowing for high values of λ , as shown in the left panel of Figure 7. The differences in equilibrium debt appear to be small in the figure, but the magnitudes of the data in these dimensions in both models. However, the models would have difficulties matching the observed frequency of default and the observed debt to output ratio.

²⁴There are about 13 such episodes in a simulation of 500,000 periods.

Figure 7: Equilibrium Debt: Varying Default Penalties



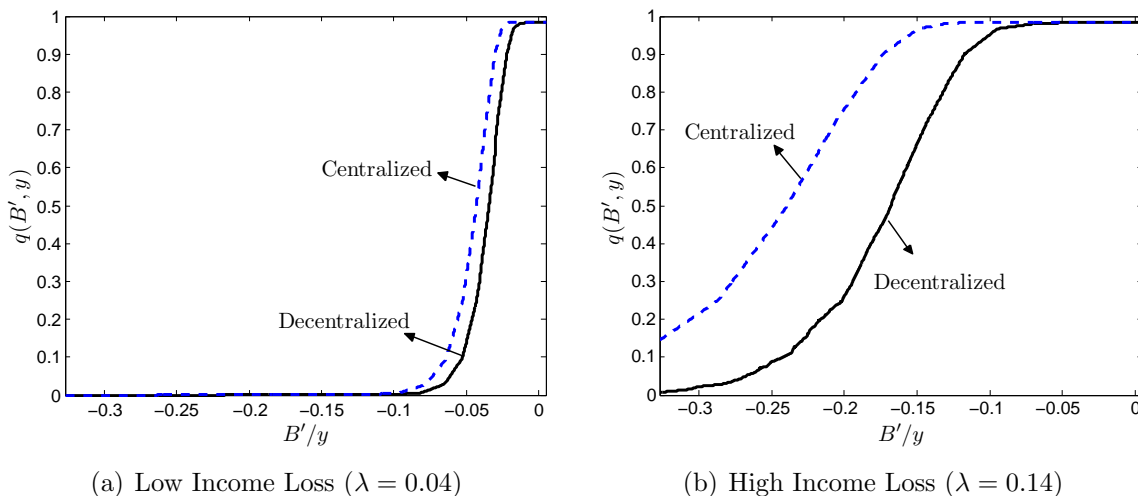
overborrowing or underborrowing are not trivial. For example, decentralized borrowing generates overborrowing by 24.3% when λ is 0.02 and underborrowing by 1.2% when λ is 0.14. Note that for λ higher than 0.18, no default happens and thus the equilibrium debt levels are identical in both models.

As we discussed earlier, decentralized borrowing has two counteracting effects on equilibrium debt: the overborrowing and bond price schedule effects. The overborrowing incentive arises from the failure of individual households to internalize the effect of their additional borrowing on the bond price. The more sensitive credit costs are to aggregate borrowing, the stronger is the overborrowing effect, which can be seen from comparing the first order conditions in equation (9) and (10). On the other hand, the bond price schedule effect occurs because the overborrowing incentive lowers the repayment welfare and drives up the default likelihood of the government. Thus, the greater the decrease in the repayment welfare is, the stronger is the bond price schedule effect.

When λ is low, the bond price schedules in both models are very sensitive to aggregate risky borrowing because the government's incentive to default rises rapidly with additional units of debt. In addition, the difference between the two bond price schedules is small, as shown in the left panel of Figure 8. This is because decentralized borrowing, though it produces more defaults, lowers the repayment welfare only by a little because default is not that costly. Thus, the overborrowing effect is strong while the bond price schedule effect is weak, which leads to equilibrium overborrowing. As λ increases, the bond price schedules in both models become flatter and the difference between them becomes larger, as shown in the

right panel of Figure 8. With increasingly severe default penalties, the government’s incentive to default does not rise quickly as aggregate debt rises. But decentralized borrowing reduces the repayment welfare substantially, which drives up the default incentives and lowers the bond price schedule greatly. Thus, the overborrowing effect is weak while the bond price schedule effect is strong, which leads to equilibrium underborrowing.

Figure 8: Bond Prices for Different Income Losses



Finally, we compare equilibrium debt of the two models for different re-entry probabilities θ . Decentralized borrowing generates overborrowing when θ is high, but underborrowing when θ is low. In particular, equilibrium debt under decentralized borrowing is 53.1% more when θ is 0.3, but 5.3% less when θ is 0.05. The equilibrium debt levels are identical across the two models for very low values of θ , which implies no default in equilibrium. The intuition for these results is similar to that for different λ . When θ is high, the bond price schedules are steep and similar in both models, which leads to equilibrium overborrowing. When θ is low, the bond price schedules become flatter and the difference between them becomes larger, implying equilibrium underborrowing.

When the default rates are zero in both models, the two models generate identical business cycle statistics including the mean spread, default rate and welfare. In all the other cases, the model with decentralized borrowing generates larger default rates, higher mean spreads, and lower welfare than the model with centralized borrowing. Even when decentralized borrowing generates equilibrium underborrowing, the substantial difference in the bond price schedule, as shown in the right panel of figure 8, is enough to make the spreads higher in equilibrium. We report the detailed statistics for these experiments in Appendix B.

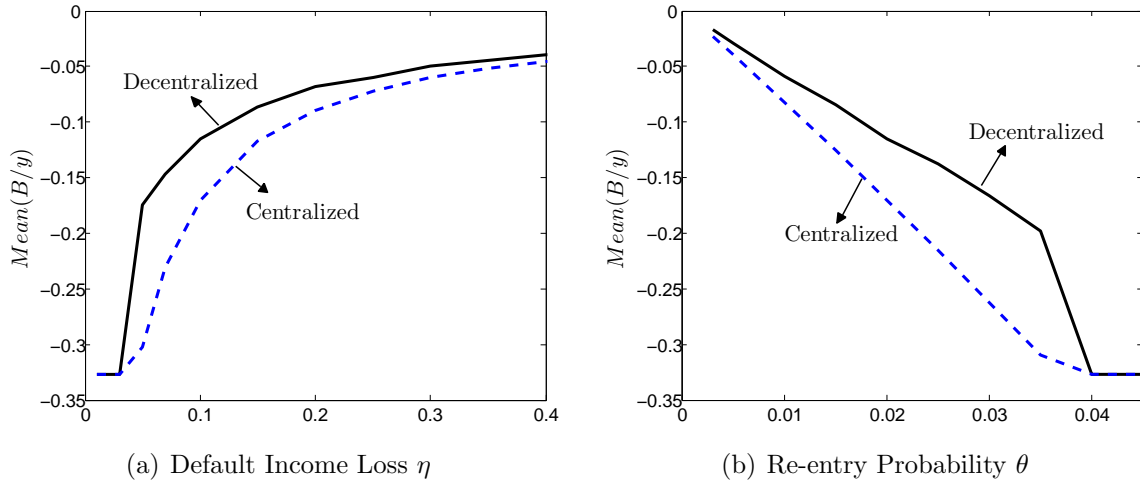
4.2 Alternative Default Penalty Specification

We now investigate the symmetric default penalty of the following form:

$$y^{def} = (1 - \eta)y, \text{ for all } y, \quad (12)$$

where η captures the constant fraction of income loss after default. Figure 9 shows equilibrium debt levels for different values of η and θ . Surprisingly, under the symmetric default penalty, decentralized borrowing consistently generates underborrowing.

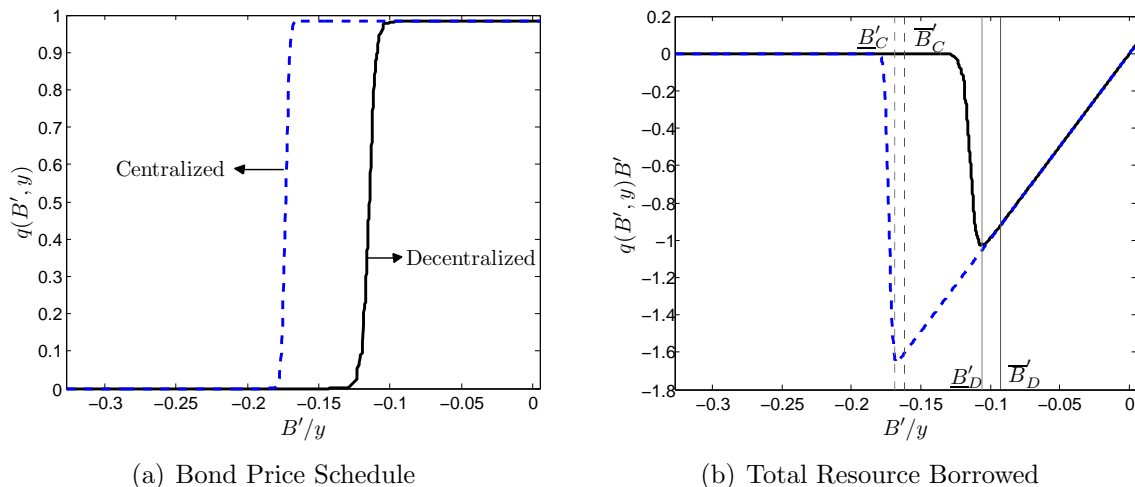
Figure 9: Equilibrium Debt under Symmetric Default Penalty



To understand this result, we plot the bond price schedules for both models under the symmetric default penalty in the left panel of Figure 10. In the right panel, we plot the resources obtained from foreign lenders, $q(B', y)B'$, as a function of next-period debt. The bond price schedules are extremely steep and the risky-debt region $[\underline{B}', \bar{B}']$ is tiny in both models. For the overborrowing effect to operate, the risky debt region needs to be large to accommodate the overborrowing incentives. Given the tiny risky debt region, equilibrium debt is mainly constrained by the safe debt limits in both models. Under decentralized borrowing, the overborrowing incentives tighten the safe debt limit greatly, and this translates into underborrowing in equilibrium.

As in the case with the asymmetric default penalty, the model with decentralized borrowing generates a higher default probability, a higher mean spread, and lower welfare for all the cases with positive default rates. We report the detailed statistics for different values of parameters in the symmetric default penalty case in Appendix B.

Figure 10: Bond Prices Under Symmetric Default Penalty



5 Conclusion

This paper presented a quantitative model of a small open economy with decentralized borrowing and centralized default. In the model, individual households make borrowing decisions and a government makes default decisions to maximize the welfare of the representative household. Accordingly, foreign lenders price debt contracts based on macroeconomic variables: aggregate income shocks and aggregate debt levels. Private agents do not internalize the negative externality of their borrowing on aggregate credit costs, and thus have incentives to overborrow. As a result, decentralized borrowing drives up the economy-wide credit costs, raises the likelihood of sovereign default, and lowers welfare.

Despite the overborrowing incentives, the model with decentralized borrowing does not necessarily lead to a higher level of equilibrium debt than the model with centralized borrowing. This is because households also face a less favorable bond price schedule under decentralized borrowing, which tends to reduce the optimal level of debt. When the income loss after default is disproportionately large under good income shocks, decentralized borrowing generates overborrowing in equilibrium for lenient default penalties, but underborrowing for severe default penalties. When the income loss is proportional to the income shock, decentralized borrowing always generates a lower equilibrium debt level for a wide range of default penalties.

Given our analysis on decentralized borrowing, regulations on private international capital flows may improve welfare. The most obvious policy would be imposing capital controls

to prohibit private borrowing. This would require that the government be able to efficiently allocate funds among private agents. Alternatively, the government can impose, on international private borrowing, either taxes if there is overborrowing in equilibrium or subsidies if there is underborrowing in equilibrium.²⁵ Future research on the optimal tax or subsidy on international private borrowing will be useful since in practice it is hard to implement capital controls.

²⁵For discussions of optimal policy under complete markets, see Jeske (2006), Kehoe and Perri (2004) and Wright (2006).

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Appendix A – Computational Algorithm

This appendix describes the computation algorithm for the decentralized borrowing model in details. We first discretize the state space (b, y, B') . We next make initial guesses for the bond price schedule and the government default decision. Specifically, we assume that $q^0(B', y) = \frac{1}{1+r}$ for all (B', y) and $d^0(B, y) = 0$ for all (B, y) . Given these guesses, we solve the individual household's optimal debt level $b'(b, y, B')$, together with the law of motion of aggregate debt $B' = \Gamma(B, y)$. Specifically, we accomplish this using the following steps.

We guess an initial law of motion of aggregate debt $\Gamma^0(B, y)$. We then solve for the optimal value functions v^R and v^D and the optimal debt policy b' using value function iteration for all combinations of $(b, y; B')$. We update the law of motion of aggregate debt $\Gamma^1(B, y)$ such that $b'(b, y, B') = B'$. We iterate these procedures until $\Gamma(B, y)$ converges. If there exist more than one fixed point, we take the B' that gives the smallest debt.

We then update the default decision $d^1(B, y)$ by solving the government's problem in equation (1). Accordingly, we update the bond price schedule. In order to minimize spurious movements in the bond price, we interpolate the bond price schedule. To do so, we first interpolate the value functions v^R and v^D over the income shock y . We next find an income level $\hat{y}(B)$ at which $v^R(B, \hat{y}(B), B') = v^D(\hat{y}(B))$ for each B . Note that $\hat{y}(B)$ is not restricted to the discrete shock levels. We then update $q^1(B', y) = (1 - \int_{-\infty}^{\hat{y}(B)} f(y'|y)dy')/(1+r)$. We iterate over the bond price schedule until it converges.

Two computational issues are worth mentioning. First, we use the discrete state-space technique with 30 endowment grid points and 1600 asset grid points. Hatchondo et al. (2010) show that the discrete state-space technique is likely to introduce spurious interest rate movements if the grid points of the state space is too coarse. Given the complexity of our model, we assume that the Markov endowment process is exogenously given by a 30-state Markov chain, obtained using a quadrature based method of Tauchen and Hussey (1991).²⁶ To check whether our asset grids are fine enough to offer robust results, we increase the number of asset grids to 2000 and find that the results remain almost unchanged.

Second, the fixed point mapping for the aggregate borrowing function has more than one fixed point. The existence of the fixed point in the aggregate borrowing function is straightforward to establish. Consider an aggregate borrowing function which specifies aggregate borrowing for all (B, y) to be so large that the government will default for sure next period.

²⁶The range of our discretized endowment shocks is larger than the range of the Argentina output process. In addition, the simulated income series from our discrete shock process captures pretty well various moments of the data.

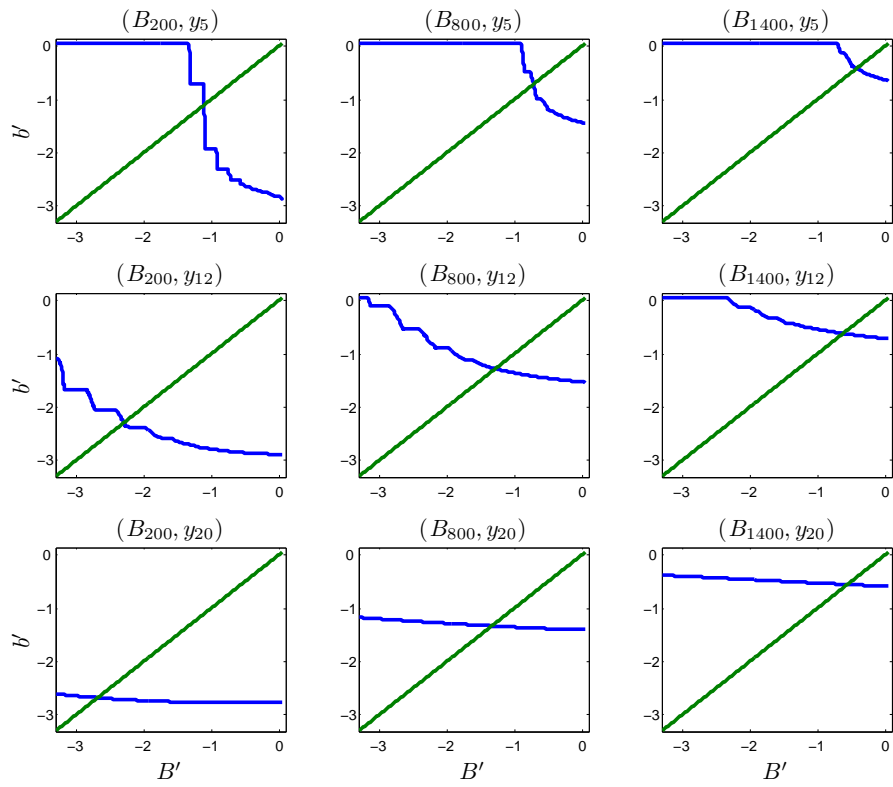
The bond price for such borrowing is zero, and the individual household is indifferent between all levels of debt. Without loss of generality, we can set individual borrowing the same as aggregate borrowing. Thus, we have established the existence of the fixed point, though this particular fixed point is not interesting because there is no borrowing and lending in equilibrium.

To find an “interesting” fixed point, we need to start with the initial guess for the bond price to be the inverse of the risk free rate instead of zero. To illustrate the fixed point mapping, we have plotted the debt choice of households $b'(B, y; B')$ over aggregate debt choices B' for different levels of aggregate debt and income (B, y) in Figure 11. The equilibrium debt choice b^* is given by the intersection of the function b' and the 45-degree line. In general, as aggregate borrowing increases, the bond price declines and individual borrowing decreases. When aggregate debt is large enough such that the bond price is zero, households are indifferent between all debt levels. Thus, the individual debt choice becomes a correspondence instead of a function for this region of B' , which produces a continuum of fixed points. In the case where $q(B', y)$ equals zero, we can set individual borrowing b' to zero without loss of generality, which implies a unique “interesting” fixed point. Or equivalently, we can select the fixed point with smallest debt.

The computation algorithm for the model with centralized borrowing is simpler. We discretize the state space (B, y) . We start with a guess for the bond price schedule, $q_C^0(B', y) = \frac{1}{1+r}$ for all (B', y) . We next solve the optimal value functions W^R , W^D and the optimal policy function B' using value function iteration. We then update the default decision based on W^R and W^D . We finally update the bond price schedule using a smoothing method analogous to that described above. We repeat the above procedures until the bond price schedule converges.

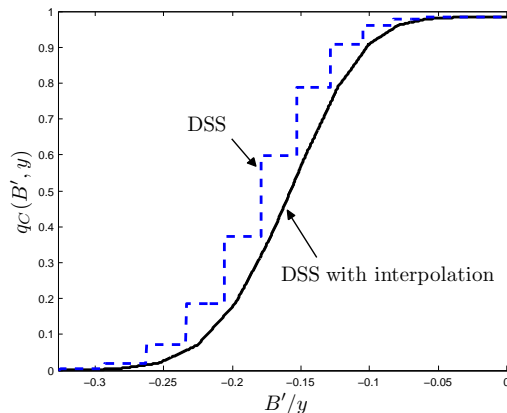
Our computation strategy is different from Hatchondo et al. (2010) in solving the optimal debt decision. Like most studies in the literature, we use the grid search method over the discretized space. By contrast, Hatchondo et al. (2010) interpolate the value functions over the asset space and use the first-order condition to solve for the optimal debt decision. Hatchondo et al. (2010) show that when the number of the grid points are large enough, the two methods give very similar solutions. On the other hand, both their and our papers smooth the bond price function by interpolating the value functions over endowment and finding the cutoff endowment level that makes the government indifferent between repaying and defaulting. To see the effect of the bond price interpolation, Figure 12 shows the bond price schedule before and after the interpolation for the model with centralized borrowing.

Figure 11: Fixed Point Mapping



The discrete state-space (DSS) technique causes discrete jumps in the bond price, while the interpolation method removes spurious movements in the bond price. Simulation results show that the interpolation greatly reduces the volatility and countercyclicality of the interest rate spread. Also, it reduces the default rate substantially, which suggests that the DSS method might overestimate the default likelihood.

Figure 12: Bond Price Schedule with and without Interpolation



Note: The displayed bond price schedules are for the median income shock.

Appendix B – Sensitivity Analysis on Default Penalties

In this appendix, we report the simulation results for different default penalties. Table 2 contrasts the relevant statistics of the two models for different parameter values of the asymmetric default penalty specification. The decentralized borrowing model generates overborrowing when default penalties are lenient (low λ and high θ), and underborrowing when default penalties are severe (high λ and low θ). Moreover, regardless of default penalties, the decentralized borrowing model generates substantially higher mean spreads, larger default rates and lower welfare than the centralized borrowing model.

Table 2: Varying Default Penalties: Asymmetric Income Loss

	λ				θ			
	0.06	0.10	0.12	0.16	0.05	0.15	0.25	0.35
	Decentralized							
mean(B/y)	-12.73	-23.41	-27.86	-32.02	-30.69	-16.45	-10.57	-7.82
mean(spread)	10.34	11.42	11.30	7.62	9.17	11.15	10.55	10.54
prob(default)	3.37	3.10	2.67	0.95	2.17	3.41	3.37	3.52
welfare	9.90	9.86	9.82	9.79	9.81	9.88	9.91	9.88
	Centralized							
mean(B/y)	-10.99	-22.25	-27.69	-32.20	-32.32	-12.88	-7.10	-4.88
mean(spread)	0.41	0.38	0.32	0.07	0.03	0.56	0.90	1.19
prob(default)	0.11	0.14	0.07	0.01	0.01	0.15	0.30	0.36
welfare	9.95	9.92	9.91	9.90	9.91	9.93	9.94	9.95

Table 3 shows the results for different parameter values of the symmetric default penalty specification. Different from the case with the asymmetric default penalty, the decentralized borrowing model consistently generates lower equilibrium debt independent of the default penalty parameter values. Similar as in the asymmetric default penalty case, decentralized borrowing generates higher mean spreads, higher default rates, and lower welfare.

Table 3: Varying Default Penalties: Symmetric Income Loss

	η				θ			
	0.005	0.01	0.02	0.03	0.05	0.1	0.15	0.35
	Decentralized							
mean(B/y)	-2.91	-5.92	-11.53	-16.59	-17.41	-11.53	-8.61	-4.45
mean(spread)	10.89	7.20	5.66	4.29	6.86	5.66	5.85	7.45
prob(default)	4.21	2.41	2.23	1.29	2.48	2.23	2.43	2.79
welfare	9.88	9.86	9.82	9.84	9.86	9.82	9.82	9.78
	Centralized							
mean(B/y)	-3.99	-8.20	-17.00	-26.22	-30.25	-17.00	-11.74	-5.17
mean(spread)	0.02	0.02	0.01	0.01	0.01	0.01	0.02	0.02
prob(default)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
welfare	9.96	9.96	9.94	9.92	9.92	9.94	9.95	10.01

Appendix C – Sensitivity on Restricted Simulation Samples

This appendix conducts sensitivity analysis on restricted simulation samples. Specifically, we compute the model statistics based on the simulation episodes in which default ends and starts 70–78 periods apart. The results are reported in Table 4 below. By contrast, the benchmark model statistics are computed on the 74 periods that are followed by default. The key difference is that the restricted simulation episodes start with zero debt, while the benchmark simulations do not necessarily start with zero debt. In the model with decentralized borrowing, the restricted sample produces a larger mean spread (12.20% versus 8.96%) and a lower debt to output ratio (19.34% versus 4.87%). Similarly, in the recalibrated model with centralized borrowing, the restricted sample produces a larger mean spread (12.20% versus 8.96%) and a lower debt to output ratio (19.34% versus 4.87%). On the other hand, the key patterns between the centralized and decentralized borrowing models are unchanged in the restricted simulation sample.

Table 4: Defaults End/Start 70–78 Periods Apart

Statistics	Data	Model	
		Decentralized	Centralized 2
mean(spread)	10.31	12.20	8.96
std(spread)	5.60	24.27	8.42
mean(B/y)	-43.36	-19.34	-4.87
std(y)	8.12	5.59	5.90
std(C)	9.47	6.42	6.38
std(TB/y)	1.75	1.90	1.51
corr(c, y)	0.98	0.95	0.97
corr(TB, y)	-0.59	-0.27	-0.19
corr(spread, y)	-0.89	-0.51	-0.56
corr(spread, c)	-0.91	-0.69	-0.62
corr(spread, TB/y)	0.70	0.79	0.46
drop in y upon default	-14.21	-12.38	-12.23
drop in c upon default	-16.01	-11.87	-12.05
discount factor β		0.97	0.93
output loss λ		0.10	0.10
re-entry probability θ		0.10	0.65
Number of episodes		118	105