Worker Matching and Firm Value

Very Preliminary and Incomplete

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Abstract

This paper studies the value of firms and their hiring and firing decisions in an environment where the productivity of the workers depends on how well they match with their co-workers and the firm acts as a coordinating device. Match quality derives from a production technology whereby workers are randomly located on the Salop circle, and depends negatively on the distance between the workers. It is shown that a worker's contribution in a given firm changes over time in a nontrivial way as co-workers are replaced with new workers. The paper derives optimal hiring and replacement policies, including an optimal stopping rule, and characterizes the resulting equilibrium in terms of employment, wages and distribution of firm values. The paper stresses the role of horizontal differences in worker productivity, as opposed to vertical, assortative matching issues.

Key words: firm value, complementarity, worker value, Salop circle, hiring, firing, match quality, optimal stopping

JEL codes: E23, J62, J63

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1 Introduction

How does the value of the firm depend on the value of its workers? When one considers firms that have little physical capital – such as IT firms, software development firms, investment banks and the like – the neo-classical model does not seem to provide a reasonable answer. The firm has some value that is not manifest in capital in the usual sense. Rather, Prescott and Visscher's (1980) 'organization capital' may be a more relevant concept in this context. One aspect of the latter form of capital, discussed in that paper, is the formation of teams and this is the issue taken up here. We ask how workers affect each other in production and how does this interaction affect firm value. The paper studies the value of firms and their hiring and firing decisions in an environment where the productivity of the workers depends on how well they match with their co-workers and the firm acts as a coordinating device. This role of the firm is what generates value.

In the model, match quality derives from a production technology whereby workers are randomly located on the Salop (1979) circle, and depends negatively on the distance between them. It is shown that a worker's contribution in a given firm changes over time in a nontrivial way as co-workers are replaced with new workers. The paper derives optimal hiring and replacement policies, including an optimal stopping rule, and characterizes the resulting equilibrium in terms of employment, wages and distribution of firm values.

The paper stresses the role of horizontal differences in worker productivity, as opposed to vertical, assortative matching issues. The literature on the latter – see for example the survey by Li (2008) – deals with the matching of workers of different types, with key importance given to the vertical or hierarchical ranking of these types. These models are defined by assumptions on the information available to agents about types, the transfer of utility among workers (or other mating agents), and the particular specification of complementarity in production (such as supermodularity of the joint production function). In the current paper, workers are ex-ante homogenous, there is no prior knowledge about their complementarity with other workers before joining the firm, and there are no direct transfers between them.

The paper proceeds as follows: in Section 2 we outline the model. We describe the set up, we delineate the interaction between workers, derive the

[1] *We thank conference participants at the 2009 annual SED meeting in Istanbul for helpful comments and the UCL Department of Economics for its hospitality. All errors are our own.*

optimal hiring and firing policy and study the implications for firm value. Section 3 describes the plan of work for future versions of this paper, including simulation studies. Section 4 concludes.

2 The Model

In this section we first describe the set-up of the firm and the production process (2.1). We then define worker interaction and the emerging state variables (2.2). Next, we discuss the optimal hiring and firing policy and derive the value of the firm (2.3). We then close the model (2.4).

2.1 The Set-Up

The firm starts off with three workers with given productivity. Workers are located on the Salop (1979) circle, with their placement randomly drawn from a uniform distribution. Any new worker will be located with the same distribution. The worker's contribution to the firm's output depends negatively on the distance between her and the other two workers. Each period the firm faces an exogenous death probability.

In each period the firm can replace at most one worker. It does so by first firing one of the existing workers without recall, and then sampling – from outside the firm – one worker. Thus, we do not allow the firm to compare the existing and the sampled worker and hire the more productive one. We rationalize this by assuming that it takes a period to learn a worker's productivity. Replacing a worker is costly.

2.2 Workers' Productivity and Interactions

The three workers are located on the unit circle. The one in the middle (out of the three) is the j worker who satisfies

$$\min_{j} \sum_{i=1}^{3} d_{ij} \tag{1}$$

where d_{ij} is the distance between worker i and j. We shall define two state variables δ_1, δ_2 as follows:

$$\delta_1 = \min_{i,j} d_{ij} \tag{2}$$

$$\delta_2 = \min_{i,j} d_{ij} \quad , i,j \neq i^*, j^* \quad i^*, j^* = \arg\min_{i,j} d_{ij} \tag{3}$$

The first state variable δ_1 expresses the shortest distance between any pair of workers. The second state variable δ_2 expresses the distance between the worker who is not one of the latter pair and the worker (out of the pair) who is closest.

The following figure illustrates:

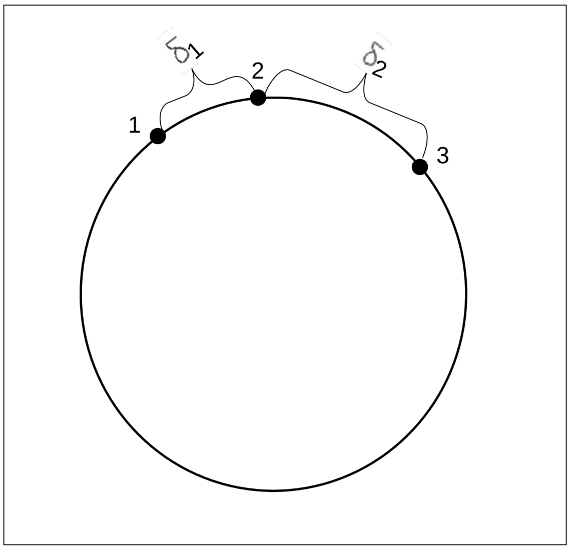

Figure 1: The State Variables

The firm's task is to find what we refer to as a common ground for the three workers; in what follows we assume that the firm chooses the middle worker as the focal point and all distances are measured going via the middle worker.

Every period, each worker works together with both co-workers to produce output. Production y_{ij} is negatively related to the distance d_{ij} :

$$y_{ij} = \frac{\widetilde{y}}{3} - d_{ij} \tag{4}$$

The firm's total output is then given by the linear additive function:

$$\begin{aligned} Y &= y_{12} + y_{13} + y_{23} \\ &= \widetilde{y} - \sum_{i=1}^{3} d_{ij} \\ &= \widetilde{y} - 2(\delta_1 + \delta_2) \end{aligned} \tag{5}$$

The profits (π) of the firm are given by:

$$\begin{aligned} \pi &= Y - W \\ &= \widetilde{y} - \sum_{i=1}^{3} d_{ij} - W \\ &= y - \sum_{i=1}^{3} d_{ij} \end{aligned} \tag{6}$$

where W is total wage bill and y is production net of wages ($\widetilde{y} - W$).

2.3 Optimal Hiring and Firing

The firm decides every period whether to replace a worker, at a cost c, and, if replacing, who should be replaced. We ask whether there is an optimal replacement rule.

Let $X = \sum_{i=1}^{3} d_{ij}$ denote total distance between the workers. Optimal replacement does not imply a unique cut-off for X, as X is not a "sufficient statistic." To see why, note that the lowest possible distance after replacement is $2\delta_1$. Hence, the value of replacement depends negatively on δ_1 – the lower is δ_1, the higher is the expected gain from one more round of replacement. So the optimal cut-off depends on the distance between the two retained workers δ_1 rather than on X.

We aim to derive an optimal stopping rule of the form: "search until $\delta_2 < \overline{\delta}_2(\delta_1)$." To derive this rule consider the value function of the firm (with the discount factor $\beta = \frac{1}{1+r}$ including a firm death probability):

$$\begin{aligned} V &= \pi(\delta_1, \delta_2) + \beta \max[V(\delta_1, \delta_2), EV(\delta_1^{'}, \delta_2^{'}) - c] \\ &= y - 2(\delta_1 + \delta_2) + \max[\frac{y - 2(\delta_1 + \delta_2)}{r}, \frac{EV - c}{1 + r}] \end{aligned} \tag{7}$$

where $\overline{\delta}_2$ is the threshold to be derived.

The expected value of replacement is given by:

$$\begin{aligned} EV &= y - (\frac{1}{2} + \delta_1 + \frac{\delta_1^2}{2}) \\ &\quad + \frac{(\delta_1 + 2\overline{\delta}_2)y - 2\overline{\delta}_2(2\delta_1 + \overline{\delta}_2) - 2\delta_1^2}{r} \\ &\quad + (1 - \delta_1 - 2\overline{\delta}_2)\frac{EV - c}{1 + r} \end{aligned} \tag{8}$$

The first term is derived as follows:

(i) The expected distance between the new worker and the existing workers is $\frac{1+\delta_1^2}{2}$ as there is an expected distance of $\frac{1}{2}(\frac{1}{2}+\frac{\delta_1}{2})$ from the new worker to one existing worker going via the middle worker with probability $\frac{1}{2}+\frac{\delta_1}{2}$ and there is an expected distance of $\frac{1}{2}(\frac{1}{2}-\frac{\delta_1}{2})$ from the new worker to the other existing worker going via the middle worker with probability $\frac{1}{2}-\frac{\delta_1}{2}$.[2] This can happen on both sides of the circle so:

$$2\left[\left(\frac{1}{2}(\frac{1}{2}+\frac{\delta_1}{2})*(\frac{1}{2}+\frac{\delta_1}{2})\right)+\left(\frac{1}{2}(\frac{1}{2}-\frac{\delta_1}{2})*(\frac{1}{2}-\frac{\delta_1}{2})\right)\right]=\frac{1+\delta_1^2}{2}$$

(ii) The two existing workers have a distance of δ_1 between them.

Hence the first term is the sum of (i) and (ii), i.e., $y-(\delta_1+\frac{1+\delta_1^2}{2}) = y-(\frac{1}{2}+\delta_1+\frac{\delta_1^2}{2})$.

The second term is derived as follows:

(i) With probability $(\delta_1+2\overline{\delta}_2)$ the new worker is below the $\overline{\delta}_2$ threshold. With probability δ_1 the new worker is between the two existing workers. The firm has a distance of δ_1 between existing workers and on average $\frac{\delta_1}{2}$ between the new worker and each of the existing workers. Hence, in total, $y-2\delta_1$.

(ii) With probability $2\overline{\delta}_2$ the new worker is below the threshold but not between the two existing workers. The firm has a distance of δ_1 between existing workers and expects a distance (on average) of $\delta_1+\frac{\overline{\delta}_2}{2}$ between the new worker and one of the existing workers and $\frac{\overline{\delta}_2}{2}$between the new worker and the other existing worker. Hence in total $y-2\delta_1-\overline{\delta}_2$.

Summing up (i) and (ii) we get $y-2\delta_1$ and $y-2\delta_1-\overline{\delta}_2$ and the firm gets $(\delta_1+2\overline{\delta}_2)y-2\overline{\delta}_2(2\delta_1+\overline{\delta}_2)-2\delta_1^2$.

The third term comes from the fact that with probability $(1-\delta_1-2\overline{\delta}_2)$ the new worker is above the $\overline{\delta}_2$ threshold. The firm will keep replacing and pay the cost c again.

Let us write:

$$\begin{aligned} & (\delta_1+2\overline{\delta}_2)y-2\overline{\delta}_2(2\delta_1+\overline{\delta}_2)-2\delta_1^2 \\ = \; & (\delta_1+2\overline{\delta}_2)(y-2(\delta_1+\overline{\delta}_2))+2\overline{\delta}_2^2+2\delta_1\overline{\delta}_2 \end{aligned}$$

Hence we can re-write (8) as follows:

[2] Note that the middle worker may change following replacement.

$$EV = y - (\frac{1}{2} + \delta_1 + \frac{\delta_1^2}{2}) + \frac{(\delta_1 + 2\overline{\delta}_2)(y - 2(\delta_1 + \delta_2)) + 2\overline{\delta}_2^2 + 2\delta_1\overline{\delta}_2}{r} + (1 - \delta_1 - 2\overline{\delta}_2)\frac{EV - c}{1+r} \quad (9)$$

We can now derive the optimal stopping rule from equation (7) using equation (9):

$$\frac{y - 2(\delta_1 + \overline{\delta}_2(\delta_1))}{r} = \frac{EV - c}{1+r} \quad (10)$$

which gives the rule:

$$c + \frac{1}{2} + \frac{\delta_1^2}{2} - \delta_1 - 2\overline{\delta}_2 = \frac{2\delta_1\overline{\delta}_2 + 2\overline{\delta}_2^2}{r} \quad (11)$$

The LHS represents net costs, evaluated at the threshold ($\overline{\delta}_2$).

If not replacing the worker, the total distance is given by $2(\delta_1 + \overline{\delta}_2)$.When replacing the worker, the firm expects to have a distance of $\frac{1}{2} + \delta_1 + \frac{\delta_1^2}{2}$,because the expected distance of the new worker is $\frac{1}{2} + \frac{\delta_1^2}{2}$ and between the existing workers it is δ_1. The firm pays c when replacing the worker. So its costs are $c+$ the net cost of replacing. The latter is the total distance after replacement less the total distance without replacement $\frac{1}{2} + \delta_1 - 2(\delta_1 + \overline{\delta}_2)$. Thus:

$$c + \frac{1}{2} + \frac{\delta_1^2}{2} + \delta_1 - 2(\delta_1 + \overline{\delta}_2) = c + \frac{1}{2} + \frac{\delta_1^2}{2} - \delta_1 - 2\overline{\delta}_2$$

which is the LHS of (11).

The RHS of (11) represents the gains from replacement, i.e. expected savings in total distance in present value terms.

With probability δ_1 the new worker will be between the two existing workers who have a distance of δ_1 between them. The expected savings in total distance is $2\overline{\delta}_2$ because $2\delta_1 - 2(\delta_1 + \overline{\delta}_2)$.Why? Existing total distance is $2(\delta_1 + \overline{\delta}_2)$.The new worker will be $\frac{\delta_1}{2}$ away from each on average so in sum δ_1 and the two existing workers are δ_1 away from each other. So the new distance is $2\delta_1$. Probability δ_1 multiplied by savings $2\overline{\delta}_2$ is the first term of the RHS of (11).

With probability $2\overline{\delta}_2$ the worker is within $\overline{\delta}_2$ away from one existing worker. The expected savings in total distance is $\overline{\delta}_2/2$ on each side because

$\delta_1 + \frac{\overline{\delta}_2}{2} + \delta_1 + \frac{\overline{\delta}_2}{2} - 2(\delta_1 + \overline{\delta}_2)$. Existing total distance is $2(\delta_1 + \overline{\delta}_2)$.The new worker will be $\frac{\overline{\delta}_2}{2}$ away from one worker on average and $\delta_1 + \frac{\overline{\delta}_2}{2}$ from the other worker and the two existing workers are δ_1 away from each other. So the new distance is $\delta_1 + \frac{\overline{\delta}_2}{2} + \delta_1 + \frac{\overline{\delta}_2}{2}$.Probability $2\overline{\delta}_2$ multiplied by savings $\overline{\delta}_2$ is the second term of the RHS of (11).

Equation (11) implies that when δ_1 is higher then the net cost is lower and that the gain (savings in distance) is higher. This means that the higher is δ_1 the worse is the team and the more the firm is willing to replace. Thus $\overline{\delta}_2$ declines.

In terms of the value function we get:

$$\frac{\partial EV}{\partial \delta_1} = -\frac{1+r}{r}\left[1 + \frac{\delta_1(1+r)}{\delta_1 + 2\overline{\delta}_2 + r}\right] < 0 \tag{12}$$

Expected firm value declines with δ_1.

The following figure illustrates this optimal behavior:

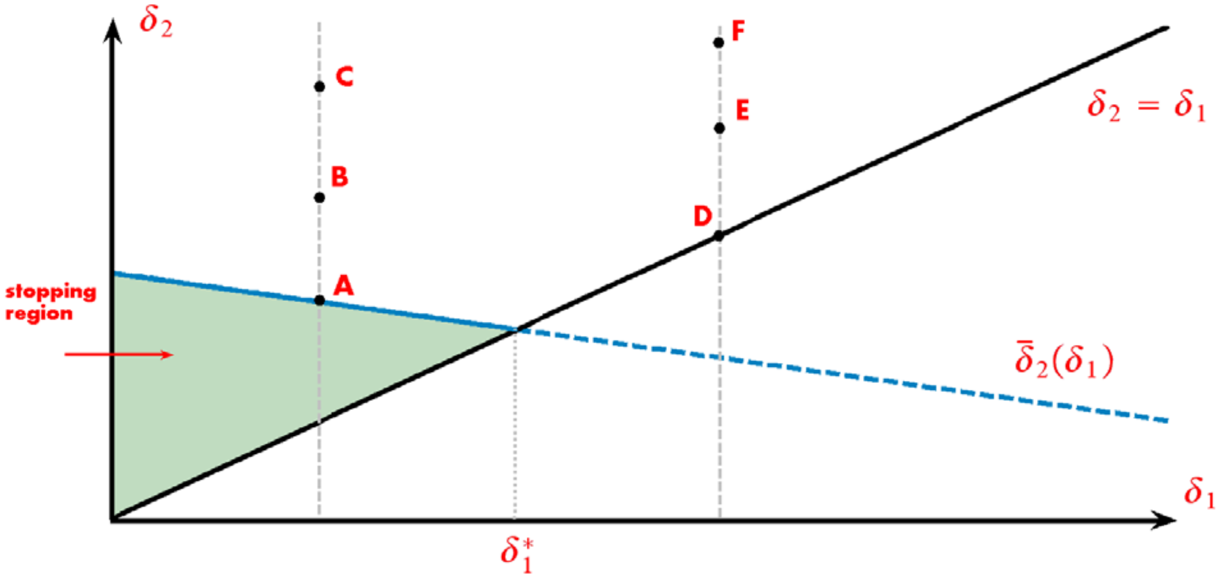

Figure 2: Optimal Policy

The space of the figure is that of the two state variables, δ_1 and δ_2. The feasible region is above the 45 degree as $\delta_2 \geq \delta_1$ by definition. The downward sloping line shows the optimal replacement threshold $\overline{\delta}_2$ as a function of δ_1.

Beyond the $\delta_1^* = \overline{\delta}_2(\delta_1^*)$ point, the firm replaces according to the 45 degree line.

With the replacement of a worker, the firm may move up and down a vertical line for any given value of δ_1 (such as movement between A, B and C or between D, E and F). This is what happens till the firm gets into the absorbing state of no further replacement in the triangle formed by the $\delta_1^* = \overline{\delta}_2(\delta_1^*)$ point, the intersection of $\overline{\delta}_2(\delta_1)$ line with the vertical axis, and the origin ($\delta_1 = \delta_2 = 0$).

This absorbing state is achieved as follows: first, note that δ_1 may decrease after worker replacement, i.e., $\delta_1' < \delta_1$, when the new worker is closer to one existing worker than the old distance δ_1. In this case the latter distance becomes the new δ_2, i.e., $\delta_2' = \delta_1$. But the new δ_2 is below the threshold, i.e., $\delta_2' < \overline{\delta}_2(\delta_1)$, as $\delta_2' = \delta_1 < \overline{\delta}_2(\delta_1)$. Hence, in the region below the $\overline{\delta}_2(\delta_1)$ line and above the 45 degree line, to the left of the $\delta_1^* = \overline{\delta}_2(\delta_1^*)$ point, the replacement process stops.[3] Optimal replacement policy therefore behaves according to the following:

Proposition 1 *The cut-off level $\overline{\delta}_2$ is decreasing in δ_1, and $\overline{\delta}_2 > \delta_1$. Furthermore, the replacement process has the following threshold property: If the firm is indifferent between replacing and not replacing a worker at $\delta_2 = \overline{\delta}_2(\delta_1)$, then it will always strictly want to stop if $\delta_2 < \overline{\delta}_2(\delta_1)$.*

Note, too, that in the replacement dynamics just described, the degree of complementarity between existing workers may change, unlike the contributions to the match of the workers in the assortative matching literature, where they are of fixed types.

2.4 Closing The Model

There are costs $K \geq 3c$ to open a firm. A zero profit condition pins down the wage ($w = \frac{W}{3}$):

$$E^{\delta_1 \delta_2} V(\delta_1, \delta_2; w; \widetilde{y}, c) = K \tag{13}$$

The equilibrium is efficient.

3 Plan of Work

At first stage we plan to fully characterize the value function and the resulting equilibrium. We shall then undertake a simulation of the distribution

[3] Since $\overline{\delta}_2(\delta_1)$ is decreasing, $\overline{\delta}_2(\delta_1') > \overline{\delta}_2(\delta_1)$, δ_2 is now below the threshold and the process stops.

of firms' values in the steady state. This will be done by calibrating the model's parameters $(c, r, \widetilde{y}, K)$, repeated random sampling from a uniform distribution of worker distances, and computation of δ_1 and δ_2.

As a second step we would like to extend the current set-up. The current model focuses on the organization of the firm given constant external conditions, It would be of interest to model the effects of changing these external conditions, such as states of the business cycle. This could be done in a number of ways:

- Endogenize search frictions, which are currently exogenously given by c. This could take the form of a different levels of c to reflect changing conditions w.r.t. the difficulty of recruiting workers.
- Allow for different productivity levels, i.e. levels of $\widetilde{y}$.

  This set up could also be extended to cater for more realistic features of the economic environment:
- Allow for exogenous worker turnover.
- Allow for more than three workers and derive stopping rules.
- Model the wage solution as a result of bargaining between the workers and the firm.
- Allow for taxes, such as a firing tax or severance payments.
- Generalize the production function, $Y = f(y_{12}, y_{13}, y_{23})$

In future work we would like to enlarge the set-up by modelling competitive search and vertical matching.

4 Conclusions

The paper has characterized the firm in its role as a coordinating device. Thus output depends on the interactions between workers. The paper has derived optimal policy, using a threshold on a state variable and allowing endogenous hiring and firing. Firm value comes from optimal coordination done in this manner and it fluctuates as the quality of the interaction between the workers changes.

References

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