De-regulating Markets for Financial Information

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Abstract

The 2010 Dodd-Frank Act gave the SEC a mandate to remove ratings requirements for credit products. One reason for doing this is the argument that if the information is valuable, issuers or investors will choose to pay for it anyway. But information leakage and free-rider problems can prevent the market from operating efficiently. Asset issuers might not pay for ratings if they believe that some investors will buy them anyway and many investors might not buy ratings because they can partially infer what others have learned through asset prices. This paper studies what effect removing ratings requirements has on market provision of financial information and on welfare. It finds that when an asset has a large investor base, most investors will purchase ratings themselves. When the investor base is small, the asset issuer will want to provide a rating. In between these extremes, the market does not provide full information and asset prices will fall when the ratings requirement is removed. Similarly, assets with moderate accuracy of ratings or of prior information experience the largest price declines. However, even when the private market for information collapses, neither asset issuers nor investors prefer the ratings requirement.

In July 2010, congress passed the Dodd-Frank act. One of the features of the act was its mandate for the SEC to remove ratings requirements for many credit products. Currently, some investors (pension funds for example) can only purchase credit products that achieve a minimum level of credit-worthiness, as determined by a nationally-recognized ratings agency. Eliminating such requirements would remove a major incentive for issuers of credit products to obtain ratings, allowing them to decide for themselves whether or not to pay a ratings agency to rate their asset. If such a rating was not provided by the asset issuer, investors themselves might want to purchase a rating, or some equivalent summary statistic about the quality of a credit product that they consider buying. This paper examines the effect of such a policy change on credit asset prices, on the allocation of productive capital, and on macroeconomic efficiency.

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We abstract from the potential biases or conflicts of interest in the ratings process and simply model ratings as noisy signals about the future value of a risky asset. These signals are produced, at a cost, by competitive ratings agencies. Agencies can either sell the rating service to the issuer, and disclose the rating free of charge to all investors, or can sell the rating to each investor individually. In this latter case, they must take into account that investors can free-ride by using equilibrium asset prices to partially infer what others have learned. We derive conditions under which an unregulated market for information will follow one business model or the other, or not exist at all.

Signals affect asset prices in two ways: First, a positive signal will push the price of the asset up, while a lower-than-expected signal will reduce the price investors are willing to pay for an asset. In expectation, signals are neutral. The second effect is that the rating makes the asset's payoff less uncertain. In doing so, it makes the asset less risky. Lowering risk lowers the equilibrium return and systematically raises the asset's price.

Given that information raises asset prices by lowering risk, the question of which assets would be most affected by the repeal of ratings requirements would appear to be straightforward: the assets for which the ratings convey the most information or those which investors have the least prior information about. That intuition turns out to be incorrect. Our analysis shows that when prior beliefs are very uncertain or signals are very informative, either asset issuers or investors will opt to purchase information, even without regulatory incentives. Conversely, when signals are uninformative or prior beliefs are very precise, ratings will indeed disappear but their disappearance has little price impact. Thus, the assets mostly likely to be affected by the policy change are neither the highest- nor the lowest-information securities, but the ones in between.

We also investigate whether the repeal of ratings requirements would have a greater effect on assets with a smaller or larger investor base. Again, we find that assets in the middle are most affected, but the reason is different. When the investor base is large, each investor needs to bear a small amount of risk, so prices are close to expected values regardless of whether ratings are provided. Conversely, when the investor base is small, each investor must hold many shares of the asset, in order for the market to clear. Since they are bearing lots of risk from this asset, the price they are willing to pay for the asset is very sensitive to changes in risk. Since providing information reduces the risk the asset poses to an informed investor, the incentive for an asset issuer to acquire a rating and provide it to all investors is high because providing that rating will greatly increase the price the asset sells for. Thus, for both large- and small-investor-base assets the market will produce information on its own, without a ratings requirement. For assets with medium-sized investor bases, this may not be the case. Following the ratings repeal, such assets may see their prices fall, on average, as information becomes more scarce.

Besides its effects on asset prices and risk-sharing, the information in credit ratings could also have an effect on real investment decisions. We model the interaction of financial markets and the real economy in the following way. At time 1, an entrepreneur can choose how much to invest. His payoff depends on the price the asset sells for in the time-2 financial market. If financial asset prices are very sensitive to changes in the value of the capital stock (they are informationally efficient), then the entrepreneur has incentives to invest the optimal amount. This force points towards a social benefit of providing information.

The efficiency benefits of information provision do not imply that requiring ratings is always good policy. Regulation might result in information over-provision in situations where the social benefit is less than the cost. Conversely, since externalities invalidate the welfare theorems, the information market also might not get the tradeoff right. To understand the relationship between information and welfare, we compared investors' and issuers' expected utilities with and without the mandate.

Our results reveal that typically, neither asset issuers nor investors prefer ratings requirements. Asset issuers can always choose to purchase and disclose ratings. Whenever they choose not to purchase a rating, they are better off without the rating, and otherwise they are indifferent. Surprisingly, even though mandatory ratings produce more efficient capital investment and a higher expected asset payoff, and even though the asset issuer pays for the ratings, investors typically prefer not to have ratings. Of course, the risk-averse investors like the fact that ratings make assets less risky. But less risk also implies a lower expected equilibrium asset return. The net effect is to make investors worse off. They prefer not to have a low-risk, low-return security, which in the limit, becomes redundant with the risk-free asset they already have access to. Each investor individually prefers more information to less. But all investors are better off when everyone is less informed. Thus, when the market for ratings collapses, investors benefit. The only situation in which investors like ratings mandates is when they prevent the possibility of severe asymmetric information. Our results characterize this region of the parameter space where, in the absence of ratings mandates, a large fraction of investors would become informed and others would remain uninformed. Because of the resulting asymmetry, investors might benefit from ratings mandates.

In many endowment economies, information acquisition or disclosure is bad for welfare because is it costly to produce, does not increase the total amount of resources available to agents and distorts the incentives to share risk evenly (e.g. Diamond (1985)). Yet, in reality, policy makers tend to consider more information a good thing because informative asset prices facilitate the efficient allocation of real capital. Therefore, we evaluate welfare in a production economy where informative asset prices facilitate efficient real investment, which maximizes output and consumption.[1] Thus, one contribution of this paper is to construct a noisy rational expectations market where information can have a positive welfare effect. But despite the fact that information can increase total surplus, both investors and producers (asset issuers) oppose the mandate. The reason is that, in our model, the surplus goes entirely to the producer. So, the producer typically prefers more information to less. But the producer can choose to pay for information without the presence of the mandate.

Of course, there are other reasons why one might want to provide information to market participants. For example, asset managers might face a moral hazard problem that leads them to take on too much risk. This problem might be alleviated if the riskiness of credit products is more transparent. But the logic of our results carries over to this setting. If the problem is severe, investors will purchase the information or managers will provide it to induce investors to invest with them. If information is not very informative or prior beliefs are already precise, then information might not be provided, but the omission will have little effect on asset prices or investor welfare.

The model is an equilibrium model of portfolio choice and costly information acquisition. Our analysis is related to work on costly information acquisition, such as Grossman and Stiglitz (1980), Verrecchia (1982), Peress (2004), Peress (2010) and Fishman and Parker (2011). But it extends this work by considering the trade-offs between issuer- and investor-purchased information. If the issuer does not provide the signal, investors themselves can choose to purchase the information from an information market. We model the market for information in a richer way than most of the previous literature by solving for the endogenous market price of a piece of information. This allows us to consider whether in the absence of ratings regulation, either issuer-provided or investor-purchased information markets will fill in the void. Finally, the model connects financial information choices to real investment choices, output and welfare.

Previous models of ratings provision (e.g. Sangiorgi, Sokobin, and Spatt (2008), Bolton, Freixas, and Shapiro (2007), Bolton, Freixas, and Shapiro (2008), Skreta and Veldkamp (2009), Damiano, Li, and Suen (2008), and Becker and Milbourn (2008)) consider ratings inflation and conflicts of interest in the ratings system. Similarly, Manso (2011) examines how ratings affect firm performance and

[1] Work by Goldstein, Ozdenoren, and Yuan (2011), Albagli, Hellwig, and Tsyvinski (2009) and Angeletos, Lorenzoni, and Pavan (2010) also models an interaction between financial markets and the real economy. But these are papers where financial investors can affect real investment through their asset purchasing decisions. This feedback creates complementarities in demand among investors and the potential for multiple equilibria. Our model shuts down these strategic interactions by having real investment take place first. Furthermore, these models have binary actions and outcomes. By keeping outcomes normally distributed in our model, we can solve for information choices analytically.

vice-versa. This paper abstracts from these incentive and performance issues and instead focuses on how many investors will get any information, in the absence of ratings mandates. Finally, this work is related to a microeconomics literature on welfare and information disclosure (e.g. Shavell (1994), Diamond (1985) and Jovanovic (1982)). Our model differs in a few respects. First, it features a continuum of investors who can participate in an information market that has an equilibrium price. Second, those investors can learn some of what others observe because it is reflected in the market price of the risky asset. Third, informed trade in asset markets results in a more efficient allocation of productive capital and therefore more output. All three of these features make the efficiency properties of the problem substantially different from preceding work.

Although there is an existing literature on third-party certification (e.g. Lizzeri (1999)), that literature considers only one possible certifier. Our model contributes to this literature by predicting whether a private market for certification will arise when public mandates are not present. Many of our results would also apply to ratings of consumer goods with uncertain utility payoffs. Thus, they inform the debate about how and when to require information provision for a wide range of both financial and non-financial products.

1 Model

The entrepreneur and real investment An risk-neutral entrepreneur chooses $k \geq 0$, how much real capital to invest in period 0, and whether to have his asset rated ($D = 1$) or not ($D = 0$) at a price C. If the entrepreneur has his asset rated, that rating is disclosed to all investors. The level of investment k is the entrepreneur's private information.

In period 1, the entrepreneur auctions off his investment. The equilibrium price is p per share. His expected utility is

$$(\mathbb{E}\,(p|k, D) - k)\bar{x} - CD. \tag{1}$$

In period 2, the investment will produce output

$$y = f(k) + u$$

where $f(k)$ is a concave production function, $f(0) = 0$ and $u \sim N\left(0, \frac{1}{h_u}\right)$. The units of output and investment are per-share of the project. If there are $\bar{x}$ shares of the project, total output is $\bar{x}y$ and capital investment is $\bar{x}k$. Ratings are noisy signals about output per-share: $\theta = y + \eta$ where $\eta \sim N(0, \frac{1}{h_\theta})$.

When making his rating decisions, the entrepreneur knows the function f and the distribution of u, but does not know what the realization of u will be or what the rating θ will be. Likewise, when making his investment choice, the entrepreneur knows his rating decision D, but does not know u or θ.

Investors and financial markets There is a continuum of ex-ante identical investors with measure Q. They have CARA expected utility with coefficient of risk aversion ρ:

$$EU = E\left[-e^{-\rho W}\right], \tag{2}$$

where W is their realized wealth. They have an initial endowment of wealth w_0.

Investors can purchase shares in the entrepreneur's project. They can also store their initial endowment with zero net return.[2] Each share in the entrepreneur's project pays off y, the output per share.

The price of the risky asset p is determined in an auction. Each investor submits a bidding function $b_i(q)$ that specifies the maximum amount that he is willing to pay for q units of the risky asset as a function of his information. These bid functions determine the aggregate demand. The auctioneer specifies a market-clearing price p that equates aggregate demand and supply, and each trader pays this price for each unit purchased (a Walrasian auction).[3]

Each investor i also chooses whether to purchase a rating ($d_i = 1$) or not ($d_i = 0$) at a price c. If p is the market clearing price and the number of shares investor i demands at price p is $q_i = b_i^{-1}(p)$, the budget constraint is

$$W = w_0 + q_i(y - p) - d_i c. \tag{3}$$

When making their ratings decisions, investors know the entrepreneur's rating decision D and they have rational expectations about k and therefore can infer the equilibrium $f(k^*)$. But they do not know the output shock u or what the realized rating will be. When making their bids, investors know the rating θ if the issuer pays for the asset to be rated ($D = 1$) or if they themselves have purchased the rating ($d_i = 1$). Since investors have rational expectations, when they determine the quantity of the risky asset they demand at each price, they consider what information would be conveyed if that were the realized price. It is as if the realized market price is in the information

[2]Results for the model with a gross riskless return $r > 1$ are available on request. None of the result change qualitatively.

[3]As shown by Reny and Perry (2006), this formulation of the financial market is equivalent to proposing a Walrasian rational-expectations equilibrium.

set of every investor when they form their asset demand. Let this information set at the time when investor i invests be denoted $\mathcal{I}_i$, where $\mathcal{I}_i = \{p, f(k^*)\}$ if i has not observed a rating and $\mathcal{I}_i = \{p, f(k^*), \theta\}$ if the issuer has disclosed the rating or i has chosen to purchase it.

Asset supply noise There is a set of agents who are subject to random shocks that force them to buy or sell the asset, at any current price. The demand of this group of agents is normally distributed with mean zero: $\xi \sim N(0, \frac{1}{h_x})$. Let x denote the net supply of the asset, after accounting for the noise trader demand: $x \equiv \bar{x} - \xi$. Thus, $x \sim N(\bar{x}, \frac{1}{h_x})$. This noise ensures that the price investors condition on is not perfectly informative about information that others may know.

Rating agencies and information markets Credit-rating agencies produce noisy, unbiased signals about the risky asset payoff y: $\theta = y + \eta$ where $\eta \sim N(0, \frac{1}{h_\theta})$. We call these signals "ratings." θ can be discovered at a fixed cost χ. This can be interpreted as the cost of hiring staff to interview the firm managers, analyze financial information, etc. The information, once discovered, can be distributed at zero marginal cost.

Rating agencies may sell the rating service to the entrepreneur for a fee C, in which case we assume both parties commit to publishing the result for free to all investors. Alternatively, they can sell it to individual investors, at a price c. For the latter case, we assume that the information is protected by intellectual property law and reselling it is forbidden.[4]

In either setup, we assume that the market is perfectly contestable, so that ratings agencies make zero profits.[5] This implies that, if the entrepreneur buys the rating, $C = \chi$, whereas if individual investors are the ones paying for it, and a measure λ of them choose to purchase it $c = \frac{\chi}{\lambda}$.

That information markets are competitive is crucial. The exact market structure is not. Veldkamp (2006) analyzes a Cournot and a monopolistic competition market as well. All three markets produce information prices that decrease in demand.

Order of Events

1. The ratings agency chooses a price C to charge the entrepreneur

2. The entrepreneur decides whether or not he will pay for the rating

[4]This prohibition may be difficult to enforce. We analyze the consequences of this difficulty in section 4.4.

[5]One way to ensure that the market is contestable is to force agents to choose prices in a first stage and choose entry in a second stage.

3. The entrepreneur chooses capital investment k.

4. (a) If the entrepreneur pays for the rating, the agency finds out θ and publishes it

   (b) If the entrepreneur does not pay for the rating, the ratings agency decides whether to find out θ and, if it does, chooses its price c. Investors then simultaneously decide whether or not to buy the signal. Those who do observe θ.

5. Investors submit menus of prices and quantities of assets they are willing to purchase at each price $b_i(q)$.

6. Asset auction takes place. The auctioneer sets a market-clearing price.

7. y is realized and all payoffs are received.

Equilibrium An equilibrium is a rating decision D by the entrepreneur, a capital choice $k(D)$, investor's beliefs about that capital choice $k^*(D)$ given the entrepreneur's rating decision, a rating demand d_i by each investor, ratings prices for the entrepreneur and investors C and c, bidding functions $b(q|\mathcal{I}_i)$ for each possible information set and an asset price $p(\theta, D, \{d_i\}, \xi)$ such that: entrepreneurs choose a rating demand D to maximize (1); taking D as given, the entrepreneur chooses $k^*(D)$ to maximize (1); investors choose d_i and bidding functions to maximize (2) subject to (3); ratings agencies make zero profits, the asset market clears: $\int_0^Q q_i di = x$ and investors' belief about investment is correct: $k = k^*(D)$.

2 Solving the model

To solve the model, we start with the second-period financial market equilibrium for given real investment and information choices. Then we determine the outcome of information markets and finally we solve for real investment.

2.1 Equilibrium asset prices

We begin by deriving the investors' optimal bid function for risky assets and verifying that it constitutes an equilibrium. Since the asset payoff y is normally distributed, expected utility (2) takes the form $EU = -e^{-\rho(w_0 + q(E(y|\mathcal{I}_i) - p) - d_i c) + (\rho^2/2)Var(y|\mathcal{I}_i)}$, where $E(y|\mathcal{I}_i)$ and $Var(y|\mathcal{I}_i)$ are the mean and variance of the risky asset's payoff, conditional on the investor's information. This investor maximizes EU subject to the budget constraint (3). This objective function of this constrained maximization problem is concave in q, so that the first-order condition describes the optimal portfolio:

$$q_i = \frac{1}{\rho} Var[y|\mathcal{I}_i]^{-1}(E[y|\mathcal{I}_i] - p). \tag{4}$$

To implement this optimal portfolio, the investor submits a bidding function. Each bidder is infinitesimal, which implies that he takes the market-clearing price as given. Thus, the bidding function (5) is the inverse demand function of a trader who seeks to maximize (2) subject to (3), taking p as given. Note that bids depend on each investor's information set $\mathcal{I}_i$, which includes information inferred from b being the price paid per unit:

$$b_i(q) = E(y|\mathcal{I}_i) - q\rho Var(y|\mathcal{I}_i). \tag{5}$$

Because (5) is an inverse of (4), it is a best response given everyone else's bid function.

The expectation and variance in (4) are conditional on an information set that includes beliefs about the entrepreneur's capital investment $k^*(D)$ and knowledge of the distribution of u. Thus, prior to observing any signals, $E[y] = f(k^*(D))$ and $Var[y] = h_u$. The information set of investors who have observed the rating (either because it was provided by the issuer or because they bought it) also includes θ. For these informed investors, Bayes' law says that

$$E\left[y|\theta\right] = \frac{f(k^*(D))h_u + \theta h_\theta}{h_u + h_\theta} \tag{6}$$

$$Var\left[y|\theta\right] = \frac{1}{h_u + h_\theta}. \tag{7}$$

Thus, informed traders' inverse bid function (demand) is

$$q_I = \frac{1}{\rho}\left(f(k^*(D))h_u + \theta h_\theta - p\left(h_u + h_\theta\right)\right) \tag{8}$$

For investors who have not observed the rating, the market-clearing auction price of the risky asset partially reveals the rating that others (if any) have observed. Since the price depends on asset demand and demand depends on information in the price, there is a fixed point problem. We solve by guessing a linear price rule

$$p = \alpha + \beta\xi + \gamma(\theta - f(k^*(D))), \tag{9}$$

and solving for the coefficients α, β and γ. A linear transformation of the price $f(k^*(D)) + \frac{1}{\gamma}(p - \alpha)$

is an unbiased signal about the project output y, with variance $h_p^{-1} = \frac{1}{h_u} + \left(\frac{\beta}{\gamma}\right)^2 \frac{1}{h_x}$. Thus, h_p is a measure of the informativeness of prices.

The posteriors of the uninformed investors will be:

$$E[y|p] = \frac{f(k^*(D))h_u + \left[f(k^*(D)) + \frac{1}{\gamma}(p-\alpha)\right] h_p}{h_u + h_p} \quad (10)$$

$$Var[y|p] = \frac{1}{h_u + h_p} \quad (11)$$

Therefore, the menu of prices and quantities bid by each uninformed trader will be:

$$q_U = \frac{1}{\rho}\left(f(k^*(D))h_u + \left[f(k^*(D)) + \frac{1}{\gamma}(p-\alpha)\right] h_p - p\,(h_u + h_p)\right)$$

If a measure λ of traders choose to become informed, total demand will be $\lambda q^I + (Q-\lambda)\, q^U$. Equating this total demand to asset supply x yields coefficients for the linear price rule and confirms the conjecture of a linear price. The following price coefficients are derived in appendix A.1:

$$\alpha = f(k^*(D)) - \frac{\rho\bar{x}}{\lambda(h_u + h_\theta) + (Q-\lambda)(h_u + h_p)} \quad (12)$$

$$\beta = \frac{\rho}{\lambda h_\theta}\gamma \quad (13)$$

$$\gamma = \frac{\lambda h_\theta + (Q-\lambda)\, h_p}{\lambda(h_u + h_\theta) + (Q-\lambda)(h_u + h_p)} \quad (14)$$

Substituting in for β and γ in the previous formula for h_p tells us that price informativeness is

$$h_p = \frac{\lambda^2 h_\theta^2 h_x}{\lambda^2 h_\theta h_x + \rho^2}. \quad (15)$$

The average price is α, and it consists of the expected payoff $f(k^*(D))$ less a term that accounts for investors' risk aversion ρ, the size of the average investment they must make to clear the market $\bar{x}$ and the amount of information they have, which depends on the precision of the rating, the informativeness of prices and how many investors buy the rating. The sensitivity of the price to the rating is given by γ. γ takes values between 0 and 1, and is greater when ratings are very precise relative to the prior and a large fraction of investors buy them. The sensitivity of the price to noise in demand is given by β. Prices will tend to be relatively sensitive to demand noise when investors are risk averse, when few have bought the rating or when the ratings are not very informative.

For the case where the entrepreneur provides the rating, formulas (12) - (15) still apply, setting

$\lambda = Q$, while for the case where no one buys the rating, the formulas apply taking the limit as $\lambda \to 0$.

2.2 Investor-based information market

Suppose that the entrepreneur decides not to provide a rating for the market. Investors must individually choose whether or not to acquire the rating at the price c. Since investors are ex-ante identical, they will only make different choices when those choices yield identical expected utility. Appendix A.2 computes the expected utility of an informed investor and the expected utility of an uninformed investor when a measure λ of the population of investors is informed. If there exists a $\lambda \in [0, Q]$ that equates the two expected utilities, then this is an equilibrium. Appendix A.2 shows that the equilibrium measure of informed investors is

$$\lambda = \frac{\rho}{\sqrt{h_x h_\theta}} \sqrt{\frac{h_\theta}{(h_u + h_\theta)(1 - \exp(-2\rho c))} - 1} \tag{16}$$

If this λ is not between 0 and Q, then there is a corner solution. If expected utility for uninformed is higher, the corner solution is $\lambda = 0$, otherwise, the solution is $\lambda = Q$. If the right side produces an imaginary number, it signifies that there is no positive measure of informed investors that equates expected utility for the informed and uninformed. In these instances, the only solution is for all investors to remain uninformed.

Equation (16) implies that demand for the rating is decreasing in the price c, decreasing in the precision of the prior h_u and increasing in the variability of noise trader demand $\frac{1}{h_x}$, which makes prices less informative. The effect of rating precision h_θ is ambiguous. On the one hand, more precise information is more valuable; on the other, it induces informed traders to take larger positions in the asset, which makes equilibrium prices more informative as well.

Equilibrium implies that, if the issuer does not provide the rating, (16) and the zero-profit condition $c = \frac{\chi}{\lambda}$ must hold.

Proposition 1 ***(Investors do not buy low-precision ratings)*** *If*

$$\frac{h_\theta}{h_u} < \exp\left(\frac{2\rho\chi}{Q}\right) - 1 \tag{17}$$

investors will not buy a rating

Proof in appendix A.3. Proposition 1 implies that an investor-based information market will not exist if:

- the information content of the rating h_θ is small relative to the precision of the prior h_u, since this makes information less valuable
- either the fixed cost of information discovery χ is high or the investor base Q is small (which makes the price c that the ratings agency needs to charge high, or
- investors are very risk averse, which makes the utility loss from paying the ratings cost c higher.

Proposition 2 ***(Investors do not buy high-precision ratings)*** *Investors will not buy a rating if h_θ is sufficiently high.*

Proposition 2 reveals a subtlety about the investor-driven information market. If the ratings contain very precise information, informed investors will take large positions, which makes prices highly informative. With a fixed price c for the rating, this would imply that as precision increases, only a vanishing measure of investors choose to become informed, as is the case in the model of Grossman and Stiglitz (1980). However, because the ratings agency must cover the fixed cost χ, low demand means it must raise prices. For sufficiently high precision, there is simply no price at which this market is viable.

2.3 Real investment decision

Replacing the equilibrium price into the entrepreneur's objective function in (2) and noting that $\theta = f(k) + u + \eta$, the entrepreneur solves

$$\max_k \left(E\left[\alpha + \beta\xi + \gamma\left(f(k) + u + \eta - f(k^*(D))\right)\right] - k\right)\bar{x}$$

Note that, because investment is unobserved, the entrepreneur cannot affect beliefs about $k^*(D)$ through the investment decision. The reason for the entrepreneur to undertake investment is to affect the rating and therefore to indirectly affect the selling price.

The first order condition for investment is

$$f'(k) = \frac{1}{\gamma}$$

The value of γ depends on whether the entrepreneur has provided a rating and, if he has not, on how many investors have purchased it. Since by equation (14) $\gamma < 1$, investment always falls below its first-best level, which is defined by $f'(k) = 1$. Furthermore, since γ is increasing in λ, investment will be higher when more investors are informed. Therefore whenever the equilibrium value of λ in an investor-driven market is less than Q, investment will be higher under issuer-provided ratings. Note further that if the rating is not produced at all then $\gamma = 0$ and therefore $k = 0$.

Information is socially valuable in this model because it allows entrepreneurs to appropriate part of the marginal product of additional investment even though the investment itself is unobserved. Thus it promotes a level of investment that is closer to the efficient level.

2.4 Entrepreneur's rating decision

The entrepreneur will provide a rating iff expected payoffs net of the information cost χ exceed expected payoffs without information. He takes into account that his decision to rate the asset will affect his decision of how much to invest and will affect the price the asset sells for in the financial market. Let p_h be the price of an asset when investment $k^*(1)$ is undertaken and all investors observe the asset's rating. Let p_l be the price of the asset when investment $k^*(0)$ is undertaken and no ratings are observed by any investor. Then, the entrepreneur will rate the asset when $E[p_h] - k^*(1) - \chi > E[p_l] - k^*(0)$.

Proposition 3 ***(Ratings provision by entrepreneur)***

1. If
$$\left[f(k^*(1)) - f(0) - k^*(1) + \frac{\rho}{Q}\frac{h_\theta}{h_u\left(h_\theta + h_u\right)}\right]\bar{x} > \chi, \tag{18}$$
then either the issuer will provide a rating or at least *some* investors will buy it

2. If condition (18) does not hold, the entrepreneur will not provide a rating.

Proof in appendix A.5. When the entrepreneur considers whether or not to provide a rating, the entrepreneur takes into account both the equilibrium measure of investors that will buy the rating if he doesn't provide it (λ) and how his own incentives to invest will change with the information structure. In case providing the rating results in more information (which will be the case unless $\lambda = Q$) this brings about two sources of gains. First, better information will result in closer-to-efficient investment. By equation (12), the average price moves one for one with expected output, so the entrepreneur appropriates the entire efficiency gain $f(k^*(1)) - k^*(1) - [f(k^*(0)) - k^*(0)]$. Second, by providing investors with information, the entrepreneur reduces the risk they have to bear, which increases average prices. The entrepreneur trades off these two sources of gains against the cost χ of the rating.

Condition (18) says that the gains from providing information outweigh the cost, assuming that is the entrepreneur does not provide information, the investors will not buy it either. If the condition holds, then either the entrepreneur expects a sufficient number of investors to buy the rating on their own, or will buy the rating himself. If the condition doesn't hold, then the entrepreneur prefers not to buy the rating even if he expects investors to remain uninformed

Proposition 3 implies that entrepreneurs will not provide ratings is the precision h_θ is too low, because the value they add is to little, if their cost χ is too high. Also, if the efficiency gains alone are not large enough, they will not provide ratings if the

Taking partial derivatives of (18) reveals the following conditions under which an entrepreneur will not provide a rating.

Corollary 1 *The entrepreneur will not provide a rating if h_θ, ρ or $\bar{x}$ is sufficiently low or if χ or h_u is sufficiently high.*

In summary, combining the results from propositions (1), (2) and (3) reveals when no ratings will be produced. Ratings will not be produced at all if signal precision h_θ is sufficiently low, the information fixed cost χ is sufficiently high, or if prior belief precision h_u is sufficiently high. These are the instances where de-regulation will have the most dramatic effects on information provision. Instead of all investors being informed with credit rating mandates, the market will not provide information for anyone. These are situations that we refer to as "information market collapse."

3 Effects of De-regulation on Asset Prices

Another question that financial market participants will want to answer is what will happen to the prices of assets after de-regulation. The following proposition shows that on average, the price of credit assets will fall as information becomes less abundant.

Proposition 4 *For an issuer that does not provide a rating, the average asset price is increasing in λ.*

Proof in appendix A.7. To see why this is true, note that ratings affect asset prices in two ways: First, a positive signal will push the price of the asset up, while a lower-than-expected signal will reduce the price investors are willing to pay for an asset. In expectation, signals are neutral. Thus on average, the positive and negative effects of the signal cancel out. The second effect is that the rating makes the asset's payoff less uncertain. In doing so, it makes the asset less risky. Lowering risk lowers the equilibrium return and systematically raises the asset's price.

The next two results describe which assets are likely to be most affected by de-regulation. These assets are neither the largest or smallest investor base securities, but the ones in between. Likewise, for assets with both high and low precision prior beliefs, the market will produce information on its own, without a ratings requirement. For assets with medium-sized investor bases and medium-precision prior beliefs, this may not be the case. Following the ratings repeal, such assets may see their prices fall, on average, as information becomes more scarce.

To formalize these ideas, we consider the difference between the price of an asset with mandatory ratings and the price of an asset without mandatory ratings, but in an environment where either the entrepreneur or the investor can choose to purchase a rating.

Proposition 5 ***(De-regulation reduces the price of a medium- or large-investor-base asset)*** *Let $\bar{p}^M$ be the average price of the asset if ratings are mandatory and $\bar{p}^E$ be the average price of the asset if the ratings decision is an equilibrium outcome. Then*

1. *If Q is sufficiently low, $\bar{p}^M = \bar{p}^E$*

2. *If Q is sufficiently high, $\exists\, h_x$ sufficiently small such that $\lambda > 0$, but $\bar{p}^M \geq \bar{p}^E$*

3. *For all $\chi > \chi^*$, there is an interval (a, b) such that $\lambda = 0$ and $\bar{p}^M > \bar{p}^E$ for all $Q \in (a, b)$,*

Proof in appendix A.8. The first part of this result says that when the size of the investor base Q is sufficiently low, the entrepreneur will pay to have his own asset rated. The reason is as follows: When the measure of investors is small, each investor must hold more of the asset for the market to clear. If the investor is bearing lots of risk by holding lots of the asset, then reducing that risk by giving the investor information has a large effect on the price the investor is willing to pay for the asset. The fact that the auction price for the asset is sensitive to the amount of information investors have means that entrepreneurs get much higher profits from selling a rated asset versus an unrated asset. This profit differential gives the entrepreneur a strong incentive to pay for his asset to be rated.

The second part of the result says that when the investor base is sufficiently large, and when the noise in prices is big enough to prevent excessive information leakage, then some investors will buy the ratings. The price of a rating in a competitive information market decreases as more investors buy it. Thus when there is a large measure of investors, ratings can be produced at a low per-copy price. The low price of information ensures that there is an active market for information.

In between the two extremes, repealing ratings mandates could collapse the market for information. If the cost of information is not too low, then there is a measure of investors big enough so that entrepreneurs do not want to buy the rating, but small enough that it is too expensive for the investors to pay for ratings. For such assets, de-regulation will severely reduce the amount of information investors have about the asset, increase the riskiness of the asset and reduce the equilibrium asset price.

Proposition 6 ***(De-regulation reduces the price of a medium-precision asset)*** *Let $\bar{p}^M$ be the average price of the asset if ratings are mandatory and $\bar{p}^E$ be the average price of the asset if the ratings decision is an equilibrium outcome. Suppose condition (37) holds. Then*

1. *If h_u is sufficiently low, $\bar{p}^M = \bar{p}^E$*

2. $\lim_{h_u \to \infty} \left(\bar{p}^M - \bar{p}^E\right) = 0$

3. *There is an interval (a, b) such that $\bar{p}^M > \bar{p}^E$ for all $h_u \in (a, b)$,*

Proof in appendix A.9. This result considers what happens as prior beliefs become more or less precise. When priors are very imprecise, signals are valuable and will be acquired by issuers or investors. When priors are very precise, no information will be acquired. But any information acquired would have a tiny effect of already precise prior beliefs. Since ratings affect beliefs (mean and variance) very little, they affect asset prices very little. In the limit as the prior precision tends to infinity, the difference between the asset's price with mandatory ratings and without disappears. In between these extremes, there exists a region where not all investors are informed and where the asset price is strictly less than it would be under the mandatory ratings regime. The same result can be proven for the precision of the public signal as well.

4 Ratings Regulation and Welfare

The primary friction in the model is that investors' imperfect information about capital investment decisions of the firm reduces the entrepreneur's return to investing in capital. In other words, if investors don't know that the entrepreneur invested more, he won't be compensated for that investment when he sells his firm. Efficiency requires that the marginal return to investment be equal to its unit marginal cost: $f'(k) = 1$. Therefore if we somehow manage to ensure that that the private return to a marginal unit of investment is equal to its social return, $\frac{\partial \mathbb{E}(p|k)}{k} = f'(k)$, then investment will be efficient. With imperfect information, the left side is typically smaller than the right because prices can only respond to changes in k to the extent that investors know k. Providing information to financial markets helps to remedy this friction because it makes p more responsive to k. This section looks at whether government mandated information disclosure helps or hurts economic welfare.

There are a few different ways we might think about a policy maker's objective in this model. We examine each in turn.

4.1 Maximizing output

One possible objective a government might have is to simply maximize the production of real goods. This is obviously a simplification, but it makes for a good starting point. The relevant question becomes: Which ratings policies maximize output $f(k)$?

Since the production function is concave, a higher $f(k)$ corresponds to a lower marginal product of capital $f'(k)$. The entrepreneur's first-order condition tells him to set $f'(k) = 1/\gamma$. The pricing coefficient γ (equation 14) is increasing in the measure of informed investors λ, as long as $h_\theta \geq h_p$.. Inspecting equation (15) reveals that $h_\theta \geq h_p$. This makes sense because prices cannot reveal more information that what is contained in the signals they are revealing.

If ratings are mandated by the government, $\lambda = 1$, This maximizes γ, minimizes $f'(k)$ and thus maximizes $f(k)$ over all feasible values ($\lambda \epsilon [0, 1]$). Thus, mandating ratings provides the maximum possible information, which maximizes output of real economic goods. Since information facilitates the efficient allocation of capital, mandatory information disclosure maximizes output.

4.2 Maximizing output net of costs

One obvious objection to the policy objective in the previous subsection is that it does not take into account the cost of investment or information production. In particular, it treats information as if it were free. More information might always be better. But if information is costly, it must be sufficiently valuable to justify its cost. Thus, another possible objective is to maximize $f(k) - k - \delta\chi$, where $\delta = 1$ if any agent (entrepreneur or investor) discovers information and $\delta = 0$ otherwise.

Since prices are uninformative when no agents observe a rating ($h_p = 0$ when $\lambda = 0$), the required $f'(k)$ is infinite, meaning that no investment takes place when the project is not rated:

$k^*(0) = 0$.

Next, note that since $f(k) - k$ is maximized when $\lambda = 1$, this means that if anyone incurs the cost χ to discover information, the output-maximizing outcome is for all investors to observe that information. Any $\lambda \neq \{0, 1\}$ does not maximize output net of costs. That leaves the question: In what circumstances is the higher output associated with $\lambda = 1$ large enough to compensate for the cost of information? In other words, what are the parameters of the problem for which $f(k^*(1)) - k^*(1) - \chi > 0$? Substituting in k from the first-order condition in this inequality yields

$$f\left((f')^{-1}\left(1 + \frac{h_u}{h_\theta}\right)\right) - (f')^{-1}\left(1 + \frac{h_u}{h_\theta}\right) > \chi.$$

For example, if production is $f(k) = k^\alpha$, then the high-information level of capital is $k^*(1) = ((1 + h_u/h_p)/\alpha)^{1/(\alpha-1)}$. This level of investment produces more output, net of investment and information costs when $k^*(1)((k^*(1))^{1/(\alpha-1)} - 1) > \chi$.

For a general, concave production function f, we know that $f'(k^*(1)) > 1$, so that anything that increases $k^*(1)$ also increases $f'(k^*(1)) - k^*(1)$ and therefore makes the inequality more likely to hold. A higher ratio of the precision of prices as signals to prior precision (h_p/h_θ) makes $k^*(1)$ higher. When all investors are informed ($\lambda = 1$), then $h_p/h_\theta = \lambda^2 h_\theta h_x/(\lambda^2 h_\theta h_x + \rho^2)$. This is increasing in signal precision h_θ and the inverse variance of noise trader demand shocks h_x, and decreasing in risk aversion ρ. Therefore these conditions make it more likely that the high-information level of capital is the one that maximizes output net of investment and information costs.

4.3 Maximizing a weighted sum of utilities

This is the most commonly used social welfare criterion. In this setting, the objective this produces depends on how one weights the issuer (a single entity) versus the investors (a continuum of agents). The question of how one models the noise traders then also comes into play. Since we have no guidance on how to weight these various constituencies, we sill simply examine their utilities separately in order to answer the question of who gains and who loses from reform.

A simple revealed preference argument establishes that the asset issuer is always weakly better off without the ratings mandate. Without the mandate, the asset issuer can always choose to pay for and disclose the rating. But with the mandate, he cannot choose to forgo a rating.

Thus, the question becomes: How does the ratings mandate affect investors? On the one hand, information produces more efficient investment decisions that increase the total production and therefore the total payoffs to all risky assets. This makes investors better off. On the other hand, proposition 4 tells us that information increases the price investors must pay issuers for the asset, which makes them worse off.

Proposition 7 ***(Investors prefer information market collapse)*** *Investors have higher ex-ante expected utility when no information is provided ($\lambda = 0$) than when ratings are mandatory ($\lambda = 1$).*

Proof in appendix A.11. Investors benefit from access to a high-risk, high-return asset. They are indifferent between holding the last, marginal share of a risky asset, but earn a utility benefit from holding all the inframarginal shares. When ratings are issued, it is as if the asset is replaced by a lower-risk, lower return asset. Investors earn less of a utility benefit from holding this asset at the new, higher equilibrium price.

The one situation in which investors might prefer mandatory ratings is when the alternative involves asymmetric information. If issuers will not provide the rating and only some investors are willing to buy the rating at the equilibrium information price, then there will be asymmetric information, with some investors knowing θ and others not. The informed and uninformed investors will hold different quantities of risky and riskless assets. But since all investors are identical ex-ante, holding different portfolios entails sharing risk inefficiently. Inefficient risk sharing reduces investor welfare. If this welfare effect is strong enough, investors prefer that a mandatory ratings statute restore information symmetry. The next result characterizes this information asymmetry region where mandatory ratings are preferable using threshold values of the information fixed cost χ.

Proposition 8 ***(Investors prefer mandatory ratings when information is cheap.)*** *There exists a cutoff χ^* such that for $\chi < \chi^*$, investor welfare with mandatory ratings is higher than with investor-purchased ratings.*

Proof in appendix A.11. This result is surprising because one might think that it is when information is very expensive that investors would prefer for asset issuers to pay for it and provide it to them for free. Instead, when information is expensive, investors know that few among them will buy ratings, so there will be few informed investors to drive up asset prices and excess returns will be available. Instead, when information is cheap, most investors will buy it. This leaves the individual investor with the alternative of either paying for the rating or trading with a large pool of better-informed investors. In this scenario, they will prefer that ratings be provided for free.

4.4 The Photocopier Problem and Information Resale

The model rules out the possibility that investors could purchase information, copy it, and resell it to others. In reality, intellectual property laws also prohibit this behavior. But because information is easy to replicate, such laws are difficult to enforce. In fact, prior to the 1970's, investors purchased books with credit ratings. But with the advent of the photocopier, such a business model because untenable (White, 2010). Rating agencies started charging asset issuers to rate their assets. With more efficient technologies for rapidly disseminating information today, an investor-pay model may still not be sustainable. If information resale cannot be prevented, rating agencies might not be able to sell enough copies of the information at a high enough price to make ratings discovery for investors profitable. If in the wake of the Dodd-Frank act, only asset issuers could pay for ratings, that would strengthen our welfare conclusions. Asset issuers would still prefer no ratings regulation because then they can choose to provide the rating or not. Investors would also always prefer not to have the regulation. In the previous analysis, the one case when investors preferred regulation was when the alternative was asymmetric information. But if an investor-pay market for ratings is not sustainable, then whenever information is provided, it is provided to all investors. If there is no possibility of asymmetric information, investors strictly prefer no information provision, which is only possible with the repeal of the ratings requirement.

5 Conclusions

The paper investigated the likely consequences of repealing ratings mandates. It characterizes the types of assets for which a free market for information will provide ratings to investors. Information could be purchased by an entrepreneur who wants to provide the information to investors to make his project less risky and therefore more valuable to them so that it can fetch a higher price at auction. Alternatively, it could be purchased by investors who want to know how much of the risky asset to buy.

When the private market provides information to most investors, repealing the ratings mandate will have little effect on asset prices or welfare. But in some instances, that private market does not provide information. In these cases, entrepreneurs are always better off without the ratings mandate. Surprisingly, investors are often better off without the mandate as well. Investors' welfare is maximized when no information about the asset payoff is available to anyone.

There are obvious limitations to interpreting these welfare results. This model included only a couple of potential benefits of ratings: facilitating the allocation of productive capital and preventing the inefficient risk-sharing that comes with asymmetrically informed investors. These benefits must be weighed against the cost of information discovery and the loss of investors surplus when an asset becomes less risky. There are other possible benefits of ratings, such as the ability to limit risk-taking by banks or portfolio managers or the ability to effectively summarize the average credit quality of large pools of assets. There are also other possible problems with credit ratings such as ratings inflation, the possibility that ratings crowd out some richer more nuanced sources of information, or outright investor deception. None of these in incorporated in the model. Yet, the ability of ratings to ameliorate asymmetric information problems and to improve the efficiency of asset prices are certainly two of the most widely-acknowledged benefits of ratings. And some of the weaknesses of the ratings system might be addressed by reforms that are less drastic than eliminating the ratings requirement system altogether. Thus, the conclusions provide some insight by weighing some of the most important advantages and disadvantages of credit ratings.

The results could also be re-interpreted more broadly in the context of a consumer goods market. We typically assume that when a seller provides customers with more complete information, customers benefit. But when more information makes the good less risky and thereby increases the price that the seller can charge for the good, the seller may extract all the rents from the reduction in risk and leave the buyers worse off. Services that rate products, like Consumer Reports, benefit the buyers that obtain their information, but harm the other buyers who are left with a market for lemons. The resulting inefficiency in the allocation of goods could be severe enough that buyers prefer sellers to disclose information. But the argument for mandatory provision of information is more nuanced in an equilibrium model than it is in a partial equilibrium model where the good's price is taken as a given.

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A Mathematical Appendix

A.1 Solving for the financial market equilibrium

This appendix solves for the equilibrium price in the risky asset market. It verifies the conjecture of the existence of a price that is linear in signals and asset supply and it derives the formulas for the linear weights.

Beginning with the market clearing condition $\lambda q^I + (Q-\lambda)\, q^U = x$ we use the formulas for q^I and q^U and to solve for p:

$$\begin{aligned} Qf(k^*(D))h_u + \lambda\left[\theta h_\theta - p\left(h_u + h_\theta\right)\right] + (Q-\lambda)\left[\frac{p-\alpha}{\gamma}h_p - p\left(h_u + h_p\right)\right] &= \rho x \\ Qf(k^*(D))h_u + \lambda\theta h_\theta - (Q-\lambda)\frac{\alpha}{\gamma}h_p + p\left[-\lambda\left(h_u+h_\theta\right) - (Q-\lambda)\left(h_u+h_p\right) + (Q-\lambda)\frac{h_p}{\gamma}\right] &= \rho x \end{aligned}$$

$$p = \frac{Qf(k^*(D))h_u + \lambda\theta h_\theta - (Q-\lambda)\frac{\alpha}{\gamma}h_p - \rho x}{\lambda\left(h_u+h_\theta\right) + (Q-\lambda)\left(h_u+h_p\right) - (Q-\lambda)\frac{h_p}{\gamma}} \tag{19}$$

which has a linear form as conjectured. Equating coefficients:

$$\begin{aligned} \alpha &= \frac{Qf(k^*(D))h_u - (Q-\lambda)\frac{\alpha}{\gamma}h_p - \rho\bar{x}}{\lambda\left(h_u+h_\theta\right) + (Q-\lambda)\left(h_u+h_p\right) - (Q-\lambda)\frac{h_p}{\gamma}} \qquad (20) \\ \beta &= \frac{-\rho}{\lambda\left(h_u+h_\theta\right) + (Q-\lambda)\left(h_u+h_p\right) - (Q-\lambda)\frac{h_p}{\gamma}} \\ \gamma &= \frac{\lambda h_\theta}{\lambda\left(h_u+h_\theta\right) + (Q-\lambda)\left(h_u+h_p\right) - (Q-\lambda)\frac{h_p}{\gamma}} \end{aligned}$$

Computing price informativeness yields

$$h_p = \frac{1}{\frac{1}{h_\theta} + \left(\frac{\beta}{\gamma}\right)^2 \frac{1}{h_x}}. \tag{21}$$

Substituting in expressions for β and γ yields (15).

A.2 Solving for the equilibrium fraction of informed investors

Recall the utility function:

$$V = -E\left[\exp\{-W\}\right]$$

where

$$W_i = (w_0 - cd) + q_i\left[u - p\right]$$

where c is the price of the rating and $d = 1$ if the investor bought it and zero otherwise.

Because of the CARA-Normal structure, given an information set for investor i, utility is

$$V_i = -\exp\left\{-\rho\left[E_i\left(W_i^I\right) - \frac{\rho}{2}Var_i\left(W_i\right)\right]\right\} \tag{22}$$

Use that $q_i = \frac{E_i[y|I_i]-p}{\rho Var[y|I_i]}$ so that

$$W_i^I = w_0 - cd + \frac{E\left[y|I_i\right] - p}{\rho Var\left[y|I_i\right]}\left[y-p\right]$$

to conclude that

$$E_i\left(W_i^I\right) = (w_0 - cd) + \frac{\left[E_i\left(y\right) - p\right]^2}{\rho Var_i\left(y\right)} \tag{23}$$

and

$$Var_i\left(W_i^I\right) = \frac{\left[E_i\left(y\right) - p\right]^2}{\rho^2 Var_i\left(y\right)} \tag{24}$$

Replacing (23) and (24) in (22):

$$V_i = -\exp\left(-\rho\left(w_0 - cd\right)\right)\exp\left\{-\frac{1}{2}\frac{\left[E_i\left(y\right) - p\right]^2}{Var_i\left(y\right)}\right\} \tag{25}$$

Utility of the informed investor The information set of an informed investor includes θ and p. Let

$$\Sigma_I \equiv Var\left[E_I(y) - p\right] \tag{26}$$

$$Z_I \equiv \frac{E_I(y) - p}{\sqrt{\Sigma_I}} \tag{27}$$

Replacing (26) and (27) into (25):

$$V_I = -\exp\left(-\rho\left(w_0 - c\right)\right)\exp\left\{-\frac{\Sigma_I}{2Var_I(y)}Z_I^2\right\} \tag{28}$$

Conditional on p, Z_I follows a Normal distribution with mean $A_I = \frac{E(y|p)-p}{\sqrt{\Sigma_I}}$ and standard deviation 1. Using that, by the law of total variance

$$Var\left(y|p\right) = \Sigma_I + Var_I(y)$$

and the MGF of a noncentral χ^2 distribution to take conditional expectations of (28), we conclude that

$$E\left[V_I|p\right] = -\exp\left(-\rho r\left(w_0 - c\right)\right)\sqrt{\frac{Var_I(y)}{Var(y|p)}}\exp\left(-\frac{\left(E(y|p) - p\right)^2}{2Var(y|p)}.\right) \tag{29}$$

Utility of the uninformed investor Equation (25) directly implies

$$E\left[V_U|p\right] = -\exp\left(-\rho w_0\right)\exp\left(-\frac{\left(E(y|p) - p\right)^2}{2Var(y|p)}\right) \tag{30}$$

Utility comparison From (29) and (30):

$$E\left[V_I|p\right] - E\left[V_U|p\right] = \left[\exp\left(\rho c\right)\sqrt{\frac{Var(y|\theta)}{Var(y|p)}} - 1\right]V_U(p)$$

Taking expectations over p, ex-ante indifference requires:

$$\exp\left(\rho c\right)\sqrt{\frac{Var(y|\theta)}{Var(y|p)}} = 1 \tag{31}$$

Using

$$Var(y|\theta) = \frac{1}{h_u + h_\theta} \tag{32}$$

$$Var(y|p) = \frac{1}{h_u + h_p} \tag{33}$$

and equation (15) to solve for λ yields equation (16).

A.3 Proof of proposition 1

From the expression for λ, a positive solution for λ requires

$$\frac{1}{h_\theta\exp\left(-2\rho c\right) - h_u\left(1 - \exp\left(-2\rho c\right)\right)} - \frac{1}{h_\theta} > 0 \tag{34}$$

which reduces to

$$h_\theta\exp\left(-2\rho c\right) - h_u\left(1 - \exp\left(-2\rho c\right)\right) > 0 \tag{35}$$

Since the ratings agency must make nonnegative profits and at most a measure 1 of investors purchase the rating, this means that $c \geq \chi$. Therefore (35) cannot hold unless (17) holds.

A.4 Proof of proposition 2

Rewrite (16) as

$$\lambda = \frac{\rho}{\sqrt{h_\theta h_x}} \sqrt{\frac{\frac{h_\theta + h_u}{h_\theta} \exp\left(-2\rho c\right) - \frac{h_u}{h_\theta}}{\frac{h_\theta + h_u}{h_\theta} \left(1 - \exp\left(-2\rho c\right)\right)}} \tag{36}$$

Fixing c, (36) implies $\lim_{h_\theta \to \infty} \lambda = 0$. Letting $c = \frac{\chi}{\lambda}$ does not alter this conclusion because λ is decreasing in c. However, it could still be that for any large h_θ, $\lambda > 0$. The following shows that this is not the case.

Assume the contrary. This means that even for large h_θ, the expression for λ has a solution with $c = \frac{\chi}{\lambda}$. Rearrange (36) and use $c = \frac{\chi}{\lambda}$:

$$\sqrt{h_\theta} = \frac{1}{\lambda} \frac{\rho}{\sqrt{h_x}} \sqrt{\frac{\frac{h_\theta + h_u}{h_\theta} \exp\left(-2\rho \frac{\chi}{\lambda}\right) - \frac{h_u}{h_\theta}}{\frac{h_\theta + h_u}{h_\theta} \left(1 - \exp\left(-2\rho \frac{\chi}{\lambda}\right)\right)}}$$

Take limits on both sides:

$$\lim_{h_\theta \to \infty} \sqrt{h_\theta} = \lim_{\lambda \to 0} \frac{1}{\lambda} \frac{\rho}{\sqrt{h_x}} \sqrt{\frac{\exp\left(-2\rho \frac{\chi}{\lambda}\right)}{\left(1 - \exp\left(-2\rho \frac{\chi}{\lambda}\right)\right)}} = 0$$

A contradiction.

A.5 Proof of proposition 3

1. Suppose to the contrary that the issuer does not provide information, and investors do not buy it either. Expected profits for the issuer will be:

$$\Pi^0 = f(k^*(0)) - \frac{\rho}{Q h_u} \bar{x} - k^*(0)$$

   If instead the issuer paid for a rating, expected profits would be:

$$\Pi^I = f(k^*(1)) \frac{h_u}{h_u + h_\theta} - \frac{\rho}{Q(h_\theta + h_u)} \bar{x} + f(k^*(1)) \frac{h_\theta}{h_\theta + h_u} - k^*(1) - \chi$$

   Rearranging $\Pi^I - \Pi^0 > 0$ this and using the result that $k^*(0) = 0$ yields condition (18). If the condition holds, it contradicts the assumption that the issuer does not provide information.

2. If condition (18) does not hold, then $\Pi^I \le \Pi^0$, so an issuer will not provide a rating even if he expects investors not to buy it either. By Proposition 4, this implies that the issuer will not provide a rating regardless of what he expects investors to do.

A.6 Additional comparative statics for λ

Proposition 9 *If condition (17) holds and h_x is sufficiently low $\lambda = Q$*

This is immediate from the formula for λ

Corollary 2 *If condition (17) holds and h_x is sufficiently low, the issuer will not provide a rating, but prices will be the same as if a rating were mandatory*

Proposition 10 *λ is decreasing in h_u*

Fixing c, this is immediate from the formula for λ. But the zero profit condition for the ratings agency $c = \frac{\chi}{\lambda}$ implies that a decrease in λ increases c, which implies a further decrease in λ

Consider condition

$$\frac{\rho}{\sqrt{h_x h_\theta \frac{1 - \exp(-2\rho\chi/Q)}{\exp(-2\rho\chi/Q)}}} > Q \tag{37}$$

Proposition 11 *If condition (37) holds and h_u is sufficiently low, then $\lambda = Q$*

From the formula for λ,

$$\lim_{h_u \to 0} \lambda = \frac{\rho}{\sqrt{h_x h_\theta \frac{1-\exp(-2\rho c)}{\exp(-2\rho c)}}}$$

If condition (37) holds, then setting $c = \chi/Q$ we have $\lim_{h_u \to 0} \lambda = Q$, which makes $c = \chi/Q$ the equilibrium price for the rating.

Corollary 3 *If condition (37) holds and h_u is sufficiently low, the issuer will not provide a rating, but all investors will buy it*

A.7 Proof of proposition 4

Let $\bar{p}$ be the price that arises when $x = \bar{x}$ and $\theta = f(k^*(D))$. Due to linearity, it suffices to show that this is increasing in λ. Using the market clearing condition and the definition of z:

$$\lambda \frac{f(k^*(D))h_u + f(k^*(D))h_\theta - \bar{p}\,(h_u + h_\theta)}{\rho} + (Q - \lambda)\,\frac{f(k^*(D))h_u + f(k^*(D))h_p - \bar{p}\,(h_u + h_p)}{\rho} = \bar{x}$$

Solving for $\bar{p}$:

$$\bar{p} = f(k^*(D)) - \frac{\rho\bar{x}}{Qh_u + \lambda h_\theta + (Q-\lambda)h_p}$$

so

$$\frac{\partial \bar{p}}{\partial \lambda} = \frac{h_\theta - h_p + (Q-\lambda)\frac{\partial h_p}{\partial \lambda}}{\left(Qh_u + \lambda h_\theta + (Q-\lambda)h_p\right)^2}\rho\bar{x} > 0$$

because $\frac{\partial h_p}{\partial \lambda} > 0$

A.8 Proof of proposition 5

Case 1: Q sufficiently low. For any given set of other parameters, there is a Q sufficiently low such that (18) holds. Proposition 3 tells us that if (18) holds, then either the issuer will provide a rating or some investors will buy it. But proposition 1 tells us that for Q sufficiently low, no investors will buy a rating. Therefore, $\exists \tilde{Q}$ such that for all $Q < \tilde{Q}$, (18) and (17) both hold. For any such Q, the issuer will provide the rating. If the issuer chooses to provides the rating, the information sets of all agents and therefore the asset prices are the same as if the issuer were required to buy the rating. Therefore $\bar{p}^M = \bar{p}^E$.

Case 2: Q sufficiently high. Substituting the zero-profit condition for rating agencies $c = \chi/\lambda$ into the indifference condition for investors (16) yields

$$\lambda = \frac{\rho}{\sqrt{h_x h_\theta}}\sqrt{\frac{h_\theta}{(h_u + h_\theta)(1-\exp(-2\rho\chi/\lambda)} - 1} \tag{38}$$

An equilibrium $\lambda > 0$ exists if there is a real $\lambda \in (0, Q]$ that solves this equation. For there to be a real positive solution, the term inside the square root must be positive. This implies that

$$\lambda \geq \frac{2\rho\chi}{\ln(h_u + h_\theta) - \ln(h_u)}. \tag{39}$$

Since $\lambda \leq Q$, this inequality holds for some λ in the feasible set if and only if

$$Q \geq \frac{2\rho\chi}{\ln(h_u + h_\theta) - \ln(h_u)} \equiv \tilde{Q}. \tag{40}$$

Suppose $Q > \tilde{Q}$ and consider a $\lambda\,\tilde{Q}$. Then the square root term some positive value. Call this $\xi(\lambda)$. Then λ is an equilibrium if $\lambda = \rho/\sqrt{h_x h_\theta}\xi(\lambda)$. For any ξ, ρ and h_θ, if $\sqrt{h_x} = \rho/(\lambda\sqrt{h_\theta})\xi(\lambda)$, then λ will solve (16) and will be an equilibrium. Since ξ, ρ, h_θ and λ are all real, positive numbers, such an $h_x > 0$ exists.

From proposition 4, we know that $\bar{P}$ is increasing in λ. With mandatory ratings, $\lambda = Q$. With equilibrium ratings and parameter values such that $\lambda > 0$, it must also be that $\lambda \leq Q$, for it to be in the feasible set. Therefore, $\bar{P}^E \leq \bar{P}^M$.

Case 3: intermediate-valued Q. **Step 1**: Show that no investors buy information for $Q \leq Q^*$. From the previous case, we know that $EV^I < EV^U$ if and only if

$$\exp\left(\frac{\rho\chi}{\lambda}\right)\sqrt{1 - \frac{\rho^2 h_\theta}{(h_u + h_\theta)(\lambda^2 h_\theta h_x + \rho^2)}} > 1. \tag{41}$$

If this inequality holds $\forall \lambda \in [0, Q]$, then the only equilibrium measure of informed investors is $\lambda^* = 0$.

Consider a small level of Q. As $Q \to 0$, any λ in the feasible set $[0, Q]$ also converges to 0. As $\lambda \to 0$, information costs become infinite $\exp(\rho\chi/\lambda) \to \infty$ and the square root term converges to a finite number:

$$\lim_{\lambda \to 0} \sqrt{1 - \frac{\rho^2 h_\theta}{(h_u + h_\theta)(\lambda^2 h_\theta h_x + \rho^2)}} = \sqrt{\frac{h_u}{h_u + h_\theta}} \tag{42}$$

Note that this square root term is monotonically increasing in λ. Therefore, at $\lambda = 0$, this term takes on its minimum value for all real λ. The minimum value for $\exp(\rho\chi/\lambda)$ for all λ in the feasible set $[0, Q]$ is $\exp(\rho\chi/Q)$. Thus, a sufficient condition for (41) to hold is

$$\exp\left(\frac{\rho\chi}{Q}\right)\sqrt{\frac{h_u}{h_u + h_\theta}} > 1 \tag{43}$$

$$\frac{\rho\chi}{Q} + \frac{1}{2}\log\left(\frac{h_u}{h_u + h_\theta}\right) > 0 \tag{44}$$

$$Q < \frac{2\rho\chi}{\log(h_u + h_\theta) - \log(h_u)}. \tag{45}$$

Let Q^* be the Q that makes this inequality an equality. Then, there $\exists$ a set of $Q < Q^*$ such that $EV^I < EV^U$ $\forall \lambda \in [0, Q]$.

Step 2: Show that if $Q > \underline{Q}$ and χ sufficiently large, entrepreneurs do not buy information. Proposition 3 tells us that when (18) does not hold, entrepreneurs do not purchase the rating. Note that the left side of (18) is strictly decreasing in χ and Q. Let Let $\underline{Q}$ be the cost that makes (18) hold with equality:

$$\underline{Q} = \frac{\rho\bar{x}}{\chi - f(k^*(1)) + f(0) + k^*(1)}\frac{h_\theta}{h_u\,(h_\theta + h_u)}. \tag{46}$$

Then, for all $Q > \underline{Q}$), (18) is violated, meaning that entrepreneurs do not buy information.

Step 3: The last step is to show that there exists a χ^* such that when $\chi \geq \chi^*$, $\underline{Q} < Q^*$. If this is the case, then for all $Q \in (\underline{Q}, Q^*)$, neither investors nor entrepreneurs pay for ratings.

Note that when ( ) holds with equality, Q^* is monotonically increasing in χ. For a sufficiently high χ, Q^* can be arbitrarily high. From (46), we see that $\underline{Q}$ is monotonically decreasing in χ. Furthermore, there exists a χ that makes $\underline{Q}$ arbitrarily small. Therefore, there exists a χ^* such that when $\chi \geq \chi^*$, $Q^* > \underline{Q}$. Letting $a = \underline{Q}$ and $b = Q^*$, then when $\chi \geq \chi^*$, there exist a and b: $\forall Q \in (a, b)$, neither investors nor entrepreneurs pay for ratings.

From proposition 4, we know that $\bar{P}$ is increasing in λ. With mandatory ratings, $\lambda = Q$. With equilibrium ratings and Q, χ such that $\lambda = 0$, it must be that $\bar{P}^M > \bar{P}^E$.

A.9 Proof of proposition 6

1. This follows from Corollary (3)
2. This follows because as $h_u \to \infty$, $p \to f(k^*(D))$ no matter whether there is a rating or not
3. Let h_u^* be the maximum value of h_u such that $\lambda = 1$. This value must exist due to Proposition 11. Let Π^I be the issuer's expected profits if he provides a rating and Π^B be his profits if he lets investors buy the ratings themselves. Proposition (11) implies $\Pi^I - \Pi^B = -\chi$ for $h_u < h_u^*$. Furthermore, because both Π^I and Π^B are continuous in h_u and λ is continuous in h_u at h_u^*, $\Pi^I - \Pi^B$ is continuous in h_u at h_u^*. This implies there is a positive ε such that if $h_u \in (h_u^*, h_u^* + \varepsilon)$, the issuer prefers not to provide a rating even though $\lambda < Q$. Since a mandatory rating has the same effect on asset prices as $\lambda = Q$, Proposition (4) implies that for $h_u \in (h_u^*, h_u^* + \varepsilon)$, $\bar{p}^M > \bar{p}^E$

A.10 Welfare of investors

Expected utility conditional on an information set is given by (25). Let

$$
\begin{aligned}
A_i &\equiv E\left[E_i\left(y\right)-p\right] \\
\Sigma_i &\equiv Var\left[E_i\left(y\right)-p\right] \\
Z_i &\equiv \frac{E_i\left(y\right)-p}{\sqrt{\Sigma_i}}
\end{aligned}
$$

Ex-ante, $Z_i \sim N\left(\frac{A_i}{\sqrt{\Sigma_i}}, 1\right)$.

Rewrite (25) as

$$
V_i = -\exp\left(-\rho w_0\right)\exp\left\{-\frac{1}{2}\frac{1}{Var_i\left(y\right)}\Sigma_i Z_i^2\right\}
$$

Using the formula for the moment-generating function of a chi-square distribution, the ex-ante expected utility is

$$
E(V_i) = -\exp\left(-\rho w_0\right)\frac{\exp\left\{\frac{-\frac{1}{2}{A_i}^2\frac{1}{Var_i(y)}}{1+\frac{1}{Var_i(y)}\Sigma_i}\right\}}{\sqrt{1+\frac{1}{Var_i(y)}\Sigma_i}}
$$

or, re-normalizing:

$$
W_i \equiv -2\log\left[\frac{-E(V_i)}{\exp\left(-\rho w_0\right)}\right] = \frac{{A_i}^2}{Var_i\left(y\right)+\Sigma_i} + \log\left(Var_i\left(y\right)+\Sigma_i\right) - \log\left(Var_i\left(y\right)\right) \tag{47}
$$

1. In case the issuer supplies the rating, then, using (12) - (15):

$$
\begin{aligned}
E_I\left(y\right)-p &= \frac{\rho x}{Q\left(h_u+h_\theta\right)} \\
Var_I\left(y\right) &= \frac{1}{h_u+h_\theta}
\end{aligned}
$$

   Therefore

$$
\Sigma_I = \left[\frac{\rho}{Q\left(h_u+h_\theta\right)}\right]^2\frac{1}{h_x} \tag{48}
$$

$$
A_I = \frac{\bar{x}}{Q}\frac{\rho}{h_u+h_\theta} \tag{49}
$$

2. In case the issuer does not supply the rating, there are two expected utilities to consider, that of the informed agent and that of the uninformed. But in an interior equilibrium, the two must be equal. So, it suffices to look only at the expected utility of the uninformed agent. Using (12) - (15):

$$
\begin{aligned}
E_U\left(y\right)-p &= A_U + B_U\left(x-\bar{x}\right) + C_U\left(\theta - f\left(k^*\left(D\right)\right)\right) \\
Var_U\left(y\right) &= \frac{1}{h_u+h_p}
\end{aligned}
$$

   where

$$
A_U = \beta\bar{x} = \frac{\rho}{\lambda h_\theta}\frac{\lambda h_\theta + \left(Q-\lambda\right)h_z}{\lambda\left(h_u+h_\theta\right)+\left(Q-\lambda\right)\left(h_u+h_z\right)}\bar{x} \tag{50}
$$

$$
\begin{aligned}
B_U &= \left[\frac{h_p}{h_u+h_p}-\gamma\right]\frac{\beta}{\gamma} = -\left[\frac{h_p}{h_u+h_p} - \frac{\lambda h_\theta + \left(Q-\lambda\right)h_p}{\lambda\left(h_u+h_\theta\right)+\left(Q-\lambda\right)\left(h_u+h_p\right)}\right]\frac{\rho}{\lambda h_\theta} \\
C_U &= \left[\frac{h_p}{h_u+h_p}-\gamma\right] = \left[\frac{h_p}{h_u+h_p} - \frac{\lambda h_\theta + \left(Q-\lambda\right)h_p}{\lambda\left(h_u+h_\theta\right)+\left(Q-\lambda\right)\left(h_u+h_p\right)}\right]
\end{aligned}
$$

   so

$$
\Sigma_U \equiv {B_U}^2\frac{1}{h_x} + {C_U}^2\left(\frac{1}{h_\theta}+\frac{1}{h_u}\right) = \left[\left(\frac{\rho}{\lambda h_\theta}\right)^2\frac{1}{h_x} + \left(\frac{1}{h_u}+\frac{1}{h_\theta}\right)\right]\left[\frac{h_z}{h_u+h_z} - \frac{\lambda h_\theta + \left(Q-\lambda\right)h_z}{\lambda\left(h_u+h_\theta\right)+\left(Q-\lambda\right)\left(h_u+h_z\right)}\right]^2 \tag{51}
$$

3. In case the issuer does not supply the rating but in equilibrium $\lambda = 0$, utility can be found either by setting $h_\theta = 0$ in (48) and (49):

$$\Sigma_0 = \left[\frac{\rho}{Qh_u}\right]^2 \frac{1}{h_x} \tag{52}$$

$$A_0 = \frac{\bar{x}}{Q}\frac{\rho}{h_u} \tag{53}$$

4. Finally, for the case where the issuer does not provide a rating but in equilibrium all investors strictly prefer to buy the rating, utility for each is as in the issuer-provided rating, subtracting the fixed cost $c = \frac{\chi}{Q}$, so that

$$W_Q = W_I - 2\rho\frac{\chi}{Q}$$

A.11 Proof of proposition 8

From, (50) and (49), it follows that $\lim_{\lambda\to Q} A_U = A_I$. From (51), (48) and (15), it follows that $\lim_{\lambda\to Q} \frac{1}{h_u+h_p} + \Sigma_U = \frac{1}{h_u+h_\theta} + \Sigma_I$. This implies that

$$W_I - \lim_{\lambda\to Q} W_U = \log\left(\frac{h_u + h_\theta}{h_u + h_p}\right) > 0$$

so for λ sufficiently close to Q, $W_I > W_U$.

Furthermore, replacing (52) , (53), (48) and (49) respectively into (47)

$$W_0 - W_I = \log\left(\frac{1 + \frac{1}{h_\theta}\left(\frac{\rho}{Q}\right)^2 \frac{1}{h_x}}{1 + \frac{1}{h_u+h_\theta}\left(\frac{\rho}{Q}\right)^2 \frac{1}{h_x}}\right) > 0$$

so by continuity it follows that for λ sufficiently close to 0, $W_I < W_U$.

The result then follows from the fact that for a sufficiently large χ, the equilibrium value of λ will be 0 and for sufficiently small χ, the equilibrium value of λ will be Q.