Foreign Borrowing, Portfolio Allocation and Bailouts

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VERY PRELIMINARY

Abstract

This paper investigates sovereign debt sustainability in open economies where the domestic government borrows both from domestic and foreign investors. Domestic debt repayment hinges on the government's ability to prevent domestic losses either by defaulting selectively or by bailing out perfectly domestic agents. When the government imperfectly obsrves portfolios and when these portfolios are heterogenous, costless default precluded and bailouts provide only partial insurance against the default. At the same time, this partial insurance makes domestic debt less risky for domestic investors and, in turn, equilibrium domestic holdings incentivize debt repayment and allow for external borrowing. Furthermore, capital controls make the government more accountable by retaining home domestic savings, but at the cost of a lack of country diversification, even if the sovereign can hedge against domestic shocks in the international market.

Keywords: sovereign debt, liquidity management, bailouts, capital controls.

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Research field: International economics, finance.

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1 Introduction

The role of reputational concerns in explaining country repayment in a world of limited sanctions has been a major focus of scholarly work on sovereign debt[1]. This paper analyses a different motive for honoring one's sovereign liabilities: the collateral damage inflicted by a default on the country's private sector. Sovereign default does not only expropriate foreign investors, but also affects domestic residents and companies holding government securities to meet their liquidity needs.

The collateral-damage theory of sovereign credibility rests on the premise that the country cannot operate a perfectly selective default in which only non-residents are impacted by the default decision, but also that the country cannot default wholesale and bail out domestic residents and companies by an amount equal to their loss from default. Indeed, defaulting selectively or implementing perfect bailouts both require tracking all domestic exposures to domestic debt, both direct and indirect (CDS and other derivative instruments, private sector exposures to foreign institutions or foreign subsidiaries that might be jeopardized by the default, etc).

Hence, the foundations of an internal cost of default rely, at least, on two necessary conditions: 1) government's information on domestic exposures is imperfect; 2) domestic exposures are heterogenous. Indeed, even when portfolios are not observable but are homogenous, the government is able to bail out perfectly domestic residents, as it is able to acquire sufficiently accurate information[2].

This paper shows that these conditions are both necessary and sufficient to make sovereign debt sustainable. More precisely, it builds a model in which the government has only limited information on domestic portfolios and where these portfolios are endogenously heterogenous and shows that sovereign debt repayment is an equilibrium government's decision.

Empirical evidence negates the costless-default hypothesis that foreigners could be perfectly targeted through selective default or finely tailored bailouts. The key reason is that financial institutions adapt immediately to bailouts. He et al. (2010) provide evidence that in 2008's financial crisis hedge funds and brokers-dealers sold assets to commercial banks in anticipation of a bailout of the latter[3]. Besides, the use of bailouts by banks is hard to track as money is fungible (cf. COP (2011)) and public authorities can only rely on broad proxies

[1] E.g., Eaton and Gersovitz (1981), Bulow and Rogoff (1989) and Hellwig and Lorenzoni (2009).

[2] When portfolios are observable, the government will indifferently default selectively or default wholesale and bail out.

[3] Such as monetary bailouts: Federal Reserve's lending facilities, or government's guarantees such as the Trouble Asset Relief Program I and II.

such as the overall lending activities. Internal costs of default through the banking sector are documented by Rajan and Zingales (1998). They show that the probability of banking crises increases significatively after a domestic default. Brutti (2011) provides further evidence on the association of sovereign debt crises and domestic private sector's liquidity crises, and Gennaioli et al. (2011) document the effect of a default on the domestic private credit. Cf. also Arteta and Hale (2008) or Borensztein and Panizza (2009) and Panizza et al. (2009) for recent surveys.

Section 2 builds a model of collateral damage, in which domestic firms manage their risk. At date 0, the government needs to borrow money and issues bonds to this purpose; the price fetched by these bonds is endogenous and depends on repayment expectations. The country's private sector, described as a set of firms needing cash at date 1 to finance a decreasing-returns-to-scale investment, and foreign investors decide whether to buy these bonds or to go for a safe alternative abroad. At date 1, the government takes two decisions: whether to engage in (non-selective) default; and in case of default whether to bail out the domestic sector. The efficiency of a bailout is hindered by the government's lack of knowledge about individual portfolio positions[4].

Portfolio choices hinge on the expectation of repayment and, for domestic residents, on the prospect of a bailout in case of default (section 3); conversely, the collateral-damage cost, and therefore the country's incentive to default and bail out depend on past foreign and domestic portfolio allocations (section 4). This mutual feedback between portfolio allocations and policy leads to ùy first insight, the existence of a coordination problem among domestic and foreign investors and the complementarity between the domestic public debt held domestically and the debt held abroad (section 5).

The second insight is that bailouts and ability to borrow can exhibit some complementarity. The option to bail out the private sector alleviates the cost of default; this collateral-damage-reduction effect makes the country less trustworthy on the international capital market. Yet, bailouts also provide some liquidity insurance to domestic residents, who may then be less averse to hold risky government securities. Notably, bailouts make possible a whole range of equilibria with probabilistic default. This risk-management effect may actually boost the country's access to international lending.

The third insight concerns the role of the dispersion of domestic private holdings on bailouts and, thus, on defaults. The dispersion of domestic agents' holdings narrows the

[4]I show that this can also be rationalized through a captured government and lack of knowledge on taxpayers' side.

government's ability to extract information on individual portfolios from aggregate data. As an effect, bailouts are not finely tuned as they take too large values for investors with small exposure to domestic debt and too small values for investors with large exposure.

The fourth insight is on the determinants on the sovereign debt creditworthiness. This paper argues that the allocation of sovereign debt is the cornerstone of credibility, not only in aggregate terms (foreignly/domesticaly held debt) but at agents' level. This challenges rating agencies' methodology, as they focus only on aggregate criteria[5]. Besides, this paper also argues that the domestic investors' willingness to hold domestic public debt, and hence, the domestic country's creditworthiness are also affected by the relative risk aversion between foreign and domestic investors.

The fifth insight concerns capital controls (section 6). A response to the coordination problem emphasized in this paper would be to implement capital controls to force domestic agents to invest in domestic assets. I argue that at least two forces can circumscribe the effects of such controls: a need for diversification for domestic agents and the possibility to skirt the controls. Agents try to diversify their portfolios when domestic bonds are subject to an additional macroeconomic shock. The ability to skirt depends essentially on the degree of openness of the financial account[6]. One additional contribution of this paper is to show that, because of sovereign risk, the domestic country usually cannot insure itself against countrywide shocks.

Section 7 broadens these insights to other policy responses: *ex ante* the government can implement capital controls to force domestic agents to hold domestic bonds or it may adjust the amount of bonds it issues. *Ex post* the government is able to default partially. Under realistic assumptions, these policies do not prevent mutliple equilibria. In a nutshell, to be efficient, capital controls requires as much information as selective defaults. A reduction of bonds' issuance *ex ante* restricts the set of equilibria, but without eliminating no-borrowing outcomes. When the government is allowed to default partially, we also obtain multiple equilibria, which now affect the haircut on the debt.

The remaining of this paper is organized as follows. Section 2 presents the environment. Section 3 derives the optimal responses of private agents and section 4 details the optimal bailout and default strategy of the government. Using these results, section 5 studies the

[5]Standard and Poor's (2011) considers five broad criteria: institutional effectiveness and political risks, economic structure and growth prospects, external liquidity and international investment position, flexibility and fiscal performance combined with debt burden and monetary flexibility.

[6]This degree of openness can be measured by indexes *de jure* by Chinn and Ito (2006) or *de facto* by Milesi-Ferretti and Lane (2005).

equilibrium outcome and focuses on the no-commitment problem, which is the heart of the paper. Section 6 introduces capital controls and macroeconomic shocks in the baseline setting. Finally, section 7 studies partial defaults, the government's capture and *ex ante* bonds issuance.

Related literature This paper is related to several bodies of the literature.

Foreign borrowing has been studied extensively emphasizing international portfolio diversification. This literature suggested several explanations as agency problems as in Tirole (2003) or lack of commitment as in the strand of literature initiated by Kehoe and Levine (1993) or Hellwig and Lorenzoni (2009) for a recent contribution.

Macroeconomic needs of liquidity may have several causes. Here these needs result from a mistiming between endowment and investment opportunities as in Woodford (1990). Other causes of liquidity demand are able to replicate the main features of this paper as in Holmstrom and Tirole (1998), where firms hoard reserves to face reinvestment shocks. More generally, the argument is robust to approaches of liquidity demand where the domestic private sector has a net demand of stores of value and where its welfare is a function of the satiation of this demand[7].

The idea that foreign borrowing is linked to internal factors goes back at least to Cole and Kehoe (1998), where a sovereign default represents a negative signal to domestic agents, who adapt in turn their behaviour, making the option of default less worthy. Recent contributions include Guembel and Sussman (2009) who consider the internal redistributive effects of sovereign default and the corresponding political economy problem, or Brutti (2011) who introduce an internal cost of default also based on liquidity needs as in Holmstrom and Tirole (1998)[8]. My approach differs from theirs in, at least, two dimensions: the possibility of bailouts and the competition between stores of value. Broner et al. (2010) argue that the internal cost is endogenous and react to the expected policy: when the government is expected to default, foreign investors resell their bonds to domestic agents, making the default worthless. I discuss later in the paper the interaction between bailouts and their key ingredient: secondary markets.

The problem of coordination among creditors for sovereign debt was highlighted by Wright (2005). From his point of view, lenders have to coordinate on the punishments they will impose in case of default. I consider here a different problem of coordination: the one between

[7]For empirical evidence on the use of public debt as private liquidity, cf. Kumhof and Tanner (2005) or Krishnamurthy and Vissing-Jorgensen (2010) among others.

[8]For similar approaches, cf. also Basu (2009), Bolton and Jeanne (2011) or Gennaioli et al. (2011).

domestic and foreign investors for purchasing domestic bonds. Here punishments are endogenously chosen as they are function of the final asset allocation.

A by-product of this coordination problem is multiple equilibria as in Kareken and Wallace (1981), Calvo (1988) or Chang and Velasco (2000). None of them considers the possibility of both default and transfers.

Many studies have recently been led on bailouts as Philippon and Skreta (2012), Tirole (2012) and Farhi and Tirole (2012). The latter consider monetary bailouts, i.e. decreases of interest rates to make borrowing less costly. They argue that such bailouts are distortionary, in contrast to fiscal bailouts, i.e. direct transfers to private agents. This paper focuses on these direct transfers and shows that they have, nevertheless, distortionay effects on private decisions. My approach shares with Philippon and Skreta (2012) and Tirole (2012) the idea that bailouts are costly because of asymmetric information: their focus is, however, more on asset quality which resembles to an adverse selection problem *à la* Akerlof, while my focus is on entrepreneurs' needs for funds.

My paper links with the recent empirical literature on public debt such as Reinhart and Rogoff (2011). My model produces as outcome one of the main stylized fact they emphasize: a high level of external debt leads to a banking crisis (understood here as losses suffered by the domestic private sector) and banking crises accompany sovereign debt crisis. However, banking crises do not precede sovereign debt crisis, contrary to what they highlight.

The introduction of capital controls to deal with foreign borrowing has been studied extensively, especially to put under the spotlight their prudential role against potential overborrowing (cf. Qureshi et al. (2011) or Korinek (2011) for a recent survey). However, the interaction with sovereign default has received less attention except by Wright (2006), who only considers private capital flows. Capital controls may interfere with public finances, as mentionned by Dooley (1996), who recalls that capital controls could be implemented to decrease the cost of public debt. I emphasize here another interaction between public debt and capital controls: countries try to limit capital outflows in order to force the coordination of domestic entrepreneurs towards equilibria where they purchase more domestic bonds, making possible foreign borrowing. This is reminiscent of Kremer and Mehta (2000)'s result on transaction costs, in a more general framework, however, where bailouts are allowed to mitigate domestic risk aversion. The effect of capital controls here is similar to Gennaioli et al. (2011) or Broner and Ventura (2011). As in the fomer, capital controls help to increase the borrowing capacity of the domestic government. My main contribution here is to show that capital controls lead to a lack of diversification of portfolios that cannot be reduced by country's hedging.

2 The environment

Consider a three-period model of a small economy. This economy is populated by a government, a continuum of domestic entrepreneurs and the rest of the world consists of foreign investors. There is a single tradable good. I denote by $t = 0, 1, 2$ the three dates.

2.1 The agents

Domestic entrepreneurs There is a continuum of entrepreneurs with mass normalized to 1. Entrepreneurs are risk-neutral and make decisions so as to maximize utility $u(c_0, c_1, c_2) = c_0 + c_1 + c_2$ where c_t is their consumption at date t.

Each of them receives an endowment of 1 unit of good at period 0 but can only invest in period 1. He produces in period 2 with a concave technology: $F(I) = \rho_1 \min(1, I) + \rho_2 \max(0, I - 1)$ where I is what they invested in period 1. I assume that $\rho_1 > \rho_2 > 1$, i.e. that F is concave. This concavity of the production function introduces entrepreneurial risk aversion. For simplicity, I assume that date-2 income is not pledgeable and, thus, he cannot borrow in period 1.

Because entrepreneurs do not invest before period 1, they need to transfer their endowments from date 0 to date 1. To do so, they may purchase either domestic public bonds or foreign risk-free bonds. I denote by $x^i \in [0, 1]$ what an entrepreneur i invests in foreign bonds and then $1 - x^i$ denotes what he invests in domestic bonds. The price of public bonds is p. Investing $1 - x^i$ in domestic public debt allows to hold $b^i = (1 - x^i)/p$ public bonds which promise z^i as nominal repayments in period 1.

Finally, domestic entrepreneurs can be bailed out in period 1. A bailout consists of a transfer B^i from the government in that period.

Aggregation I define, respectively, aggregate investment I, aggregate portfolio decision x, aggregate repayments Z^E to entrepreneurs and aggregate bailout B as:

$$I = \int_0^1 I^i di \quad x = \int_0^1 x^i di \quad Z^E = \int_0^1 z^i di \quad B = \int_0^1 B^i di \tag{1}$$

I denote by H the equilibrium cumulative distribution function of the x^is and by h its density.

Finally, I assume that the weight accorded by the government to each entrepreneur is identical across entrepreneurs, so we only need consider the aggregate welfare of entrepreneurs $W^e = \int_0^1 F(I^i) di$.

Foreign investors and foreign bonds Foreign investors are risk-neutral agents endowed in period 0 with an infinite wealth. Their utility function is: $u(c_1^*) = c_1^*$ where c_1^* is their consumption at date 1. Throughout the paper, * refers to foreign investors.

As they consume only in period 1, they also need to transfer resources, and, thus, they can purchase either domestic government bonds or foreign bonds.

I denote by Z^* the external debt, namely the nominal repayment foreign investors are entitled to because of their holdings of domestic public bonds.

Foreign bonds are risk-free bonds in unlimited supply. Each yields one unit of good in period 1 for one unit of good invested in period 0.

Remark. The difference in assets' returns may illustrate differences in commitment abilities. Interpretating foreign assets as US treasuries implies that US government can perfectly commit not to default, as in Gourinchas et al. (2010).

Market for domestic bonds Domestic entrepreneurs and foreign investors compete with each other to purchase both domestic public bonds and foreign bonds. The value each of them puts on domestic bonds depends on the beliefs they have in period 0 on the decisions taken by the government in period 1. As each agent has access to the same information, their beliefs are identical among both domestic entrepreneurs and foreign investors.

Agents form beliefs in period 0 by observing both market conditions in that period (p) and the portfolio distribution (H) and by anticipating the best response of the government.

I denote by π the ex ante probability of no default.

Government The government has access to a deterministic production technology: by investing $G \leq \overline{G}$ [9] in period 0, it produces RG units of goods in period 1, with $R > 1$. The government has no resources in period 0 and must therefore borrow from domestic entrepreneurs and/or foreign investors. However the government cannot commit to repay its debt and contracts are not enforceable.

In period 0, the government issues Z bonds.

In period 1, the government chooses its repayment and bailout policies so as to maximize *ex post* domestic welfare W. Domestic welfare includes both the welfare of entrepreneurs and the production of the governement, net of repayments. This can be summarized by:

$$W = RG - P + \beta W^e \tag{2}$$

[9] This constraint will not be binding in equilibrium.

where P are the total payments to foreign investors and to domestic entrepreneurs and $\beta > 0$ is government's weight on entrepreneurs.

The government has only partial information, as it cannot observe the repayments to an individual firm (formally the $1 - x^i$'s) but only the aggregate distribution of domestic repayment to domestic investors (formally the H)[10].

This assumption also implies that the government is not able to discriminate between domestic and foreign lenders. As a consequence, it cannot default on the fraction of its debt held by foreign investors. This is a key ingredient for an internal cost of default, as a default always implies losses for at least some domestic agents.

Default and bailouts Repayments by the government P depend on the possibility to default and on the bailouts to domestic firms.

First government chooses either to default or to repay. I consider only complete default for the moment. Second, in case of default, the government may choose to bail out domestic firms. Given that the government cannot observe choices of investment made by firms, the bailout consists of a transfer B that is not contingent on the x^i. Every domestic firm receives then the same transfer B.

Bailouts imply an additional cost proportional to the bailout, cB. This cost may arise for example from parasite entrepreneurs who benefit opportunistically from the bailout[11]. The cost c is required to have equilibrium lending abroad as it will become clear later in the paper.

Remark. The cost c represents a measure of opacity. A major source of opacity can be

[10]Because of secondary markets as in Broner et al. (2010), because of anonymity as argued by Borensztein and Panizza (2009) and Brutti (2011) or because of costly information acquisition. The assumption of observing only the aggregate distribution can be rationalized in several ways. First disaggregated data are not always available and even aggregated data on net foreign positions are built *ex post* as in Gourinchas et al. (2010). Besides, this assumption implies no loss of generality: policies can also be an equilibrium outcome, where the government chooses these policies using beliefs. In equilibrium, the set of equilibria of this extended game is exactly the set of equilibria of the game considered here.

[11]This cost could be rationalized in several ways: either by assuming that there exists a young generation of entrepreneurs receiving endowments in period 1 or through parasite entrepreneurs. The latter are endowed also with a production technology. They produce $\rho_1' I'$ out of an investment I where $0 \leq \rho_1' < \rho_1$. For $\rho_1' = 0$, parasite entrepreneurs do not produce anything and just consume. They received initially no endowment. However, in case of bailout, as government cannot distinguish them from entrepreneurs, they are bailed out too. More precisely, government may identify the type of entrepreneurs using their liquidity hoardings, as parasite entrepreneurs hold nothing, but I assume that the government is not able to measure individual liquidity hoarding, and thus, it cannot identify the type of entrepreneurs. As a consequence, if the government decides to bail out domestic firms, it has no other choice to bail out indifferently good and parasite entrepreneurs. Finally $c = -\beta\rho_1' + 1$

complexity. Indeed, domestic agents may be exposed indirectly to their public debt through exposures to foreign banks, derivatives or any other financial instruments that can be affected by a domestic default.

Finally, I make the following assumption throughout this paper:

Assumption 1. $\beta\rho_2 < 1 + c$

This assumption makes it suboptimal to finance (under symmetric information) domestic firms which have enough cash to fund their most profitable activities (with return ρ_1).

2.2 Description of the game and equilibrium definition

Timing The timing of the game is summarized by the following graph:

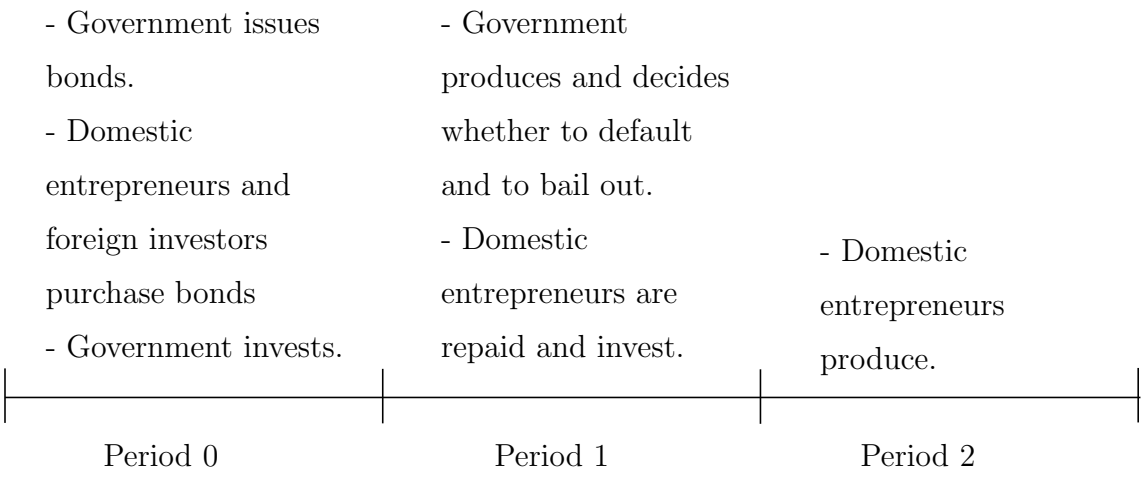

Figure 1: Timing

Strategies and equilibrium At date 0, domestic entrepreneurs and foreign investors compare their beliefs on governement's future moves (π, B) and the price of domestic bonds on the market (p) to choose portfolios (the x^i's). To do so, domestic entrepreneurs and foreign investors infer from equilibrium distribution H and equilibrium price p what the government will do in period 1.

The environment yields two equilibrium conditions: first, the equilibrium among investors in period 0, which determines p and the portfolio allocation, and second, the equilibrium of the no-commitment game between investors and the government. This equilibrium determines π and B.

3 Portfolio allocations

This section characterizes domestic and foreign investors' optimal portfolio allocation.

Domestic Entrepreneurs Each entrepreneur i observes the aggregate distribution h and infers beliefs about future policies $\{\pi, B\}$. He maximizes expected utility. His program is:

$$\begin{aligned} \max E_0 u(c_0^i, c_1^i, c_2^i) = c_0^i + E_0 c_1^i + E_0 c_2^i \\ \text{s.t. } c_0^i + x^i + pb^i = 1 \text{ and } c_2^i = F(I^i) \\ \text{if no default : } c_1^i + I^i = x^i + z^i \\ \text{if default with bailout } B : c_1^i + I^i = x^i + B \end{aligned}$$

The maximization shows that entrepreneurs take into account two factors: a speculative factor as they purchase domestic bonds depending on their expectations of default (π) and a liquidity factor as they try to guarantee a minimum of resources in every situation. This motive shows up when government's promised repayment exceeds the bailout B. In this case, a default corresponds to less liquidity for them. To determine their holdings above B, they compare the expected marginal profit of holding x^i (or conversely the cost to hold $1 - x^i$) without default: $\pi\rho_2(1 - 1/p)$, with the expected marginal profit in case of default: $(1 - \pi)\rho_1$.

Proposition 1. *Entrepreneurs invest everything both in period* 0 *and in period* 1 *($c_0^i = c_1^i = 0$). The demand function for domestic bonds* $1 - x^i$ *is:*

$1 - x^i = 0$ *if* $p > \pi$; $= [0, B]$ *if* $p = \pi$; $= B$ *if* $p \in [\overline{p}, \pi]$; $= [B, 1]$ *if* $p = \overline{p}$; $= 1$ *if* $p < \overline{p}$

with

$$\overline{p} = \frac{\pi\rho_2}{\pi\rho_2 + (1 - \pi)\rho_1} \leq \pi \tag{3}$$

Proof. See appendix. □

Figure 2 plots this demand function for arbitrary values of π and B. It gives some hints for the role of bailouts. When entrepreneurs expect that the government is not going to bail out, they prefer to invest their endowments in foreign assets. In the opposite polar case in which the value of the bailout equals 1, domestic entrepreneurs accept to purchase domestic bonds. Indeed, they prefer domestic bonds ($x^i = 0$) for a wider range of price ($p \leq \pi$ compared with $p \leq \overline{p} < \pi$).

Thus, bailouts limit flight to quality towards foreign bonds, which are here safer assets compared with domestic bonds. These results are robust: as long as domestic agents are

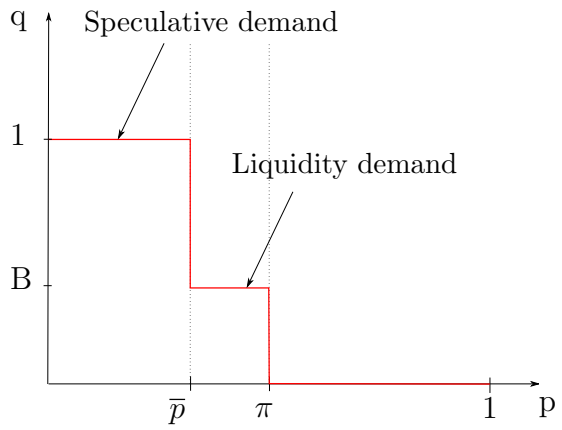

Figure 2: Entrepreneurs' demand for domestic bonds

de facto risk averse, either because they are risk-neutral or risk-averse entrepreneurs with a concave production function or because they are households with a concave utility function.

Remark. Additionally, for some values of the price p, entrepreneurs are indifferent between a large range of portfolios. This allows for *ex post* heterogeneity, even though entrepreneurs are homogeneous *ex ante*. This feature holds as long as domestic public bonds may be riskless, and hence, be perfect substitutes for riskless foreign assets.

Example (No-diversification). If entrepreneurs can only hold either domestic or foreign bonds, but not both, the bailout can take two values depending on the allocation of investment: $B = 1$ or $B = 0$. The demand function at the level of an entrepreneur is:

- if $B = 0$**:** $1 - x^i = 0$ if $p > \overline{p}$; $= \{0, 1\}$ if $p = \overline{p}$; $= 1$ if $p < \overline{p}$,

- if $B = 1$**:** $1 - x^i = 0$ if $p > \pi$; $= \{0, 1\}$ if $p = \pi$; $= 1$ if $p < \pi$.

If $p = \overline{p}$ in the first case or if $p = \pi$ in the second case, entrepreneurs are indifferent between investing in domestic bonds and in foreign bonds, and thus some hold domestic bonds while the others hold foreign bonds. The distribution of repayments to domestic agents is degenerate and has two peaks at 0 and at $1/p$.

Foreign investors Foreign investors may also invest either in foreign assets or in domestic debt. 1 unit invested in foreign assets yields 1 and 1 unit invested in domestic debt yields π in expectation. Then, if $p < \pi$ their demand for domestic bonds is infinite and if $p > \pi$, their demand is zero.

Aggregation As $\overline{p} < \pi$ for $\pi < 1$, foreign investors are always the marginal buyers in the market for domestic bonds. For any beliefs on the probability of default π and on the bailout

B, foreign investors accept to pay a higher price than domestic entrepreneurs. This is a direct consequence of the concavity of the production function. More precisely, if the government never bails out and if $\pi < 1$, foreign investors purchase all the public bonds at a price for which domestic entrepreneurs prefer foreign bonds. To anticipate on the next sections, this will imply that the government defaults for sure ($\pi = 0$). As this mechanism is at work for every value of π such that $\overline{p} < \pi$, no equilibrium without bailouts exists where $0 < \pi < 1$. This result plays a crucial role for the existence and the properties of equilibria.

4 Government's optimal response

In period 1, the government faces nominal repayments Z to its lenders, including Z^e to domestic lenders. Crucially it is not able to discriminate among lenders. If the government were able to discriminate, its choice would be trivial: it would default only on the debt held by foreign investors and bail out every firm with exactly what each of them has lost because of the default. Indeed, because of full observability, the government is able to identify both parasite entrepreneurs (the cost c vanishes) and to implement entrepreneur-specific bailouts (B^i depends on i). The following proposition gathers these insights and gives further details:

Proposition 2. *If the government could perfectly observe individual portfolios as well as engage in targeted bailouts: $B^i = 1 - x^i$, the government would default for any strictly positive level of foreign debt: $Z^* > 0$.*

Proof. See appendix □

No-discrimination introduces an internal cost which allows external debt. Indeed, assuming no-discrimination, the two value functions for the government are then: in case of no-default:

$$W_1 = RG - Z + \beta \int_0^1 F(z^i + x^i) di \tag{4}$$

In case of default:

$$W_0 = RG - (1 + c)B + \beta \int_0^1 F(B + x^i) di \tag{5}$$

These two expressions make clear the cost and benefit of a default. The benefit consists of the absence of repayment Z. The cost is the detrimental effect of the default on domestic entrepreneurs: $\beta \int_0^1 [F(z^i + x^i) - F(B + x^i)] \, di$. This internal cost of default depends crucially on the difference for each agent between nominal repayments of domestic public bonds

z^i and the transfer received in case of bailout B. The sign of this difference varies across entrepreneurs. However, I will later show that on aggregate, this cost is positive. Bailouts allow the government to decrease this cost, which is, moreover, an increasing function of β, the weight of entrepreneurs in the welfare function[12].

Government's problem The government chooses π and B so as to maximize:

$$\pi W_1 + (1-\pi)W_0(B)$$

I characterize first the optimal bailout in case of default and then the decision to default itself.

4.1 Optimal bailouts

In case of default, the government has the option to bail out domestic firms. The transfer chosen by the government maximizes (5) with respect to B. The following proposition describes the outcome of this maximization:

Proposition 3. *The optimal bailout* B *is* $\max\{b|H(1-b) \geq 1-\hat{x}\}$ *with*
$\hat{x} = \frac{\beta\rho_1 - 1 - c}{\beta(\rho_1 - \rho_2)} \in (0,1)$.
As a consequence, $B \leq 1$.

Proof. See appendix. □

This proposition states that the government chooses a bailout $B > 0$ only if there are enough domestic entrepreneurs who invested enough domestically. The two conditions matter : the government chooses not to implement large bailouts ($B << 1$) either if a lot of entrepreneurs have invested only a little or if only a few have invested a lot in domestic bonds.

Because of the concavity of the production function and due to assumption 1, the government does not want to transfer wealth to entrepreneurs who are already able to invest at least 1 in their projects. Recall that the marginal welfare for an entrepreneur who invested x^i in foreign assets is $\beta\rho_1 - 1 - c > 0$ if $x^i + B \leq 1$ but $\beta\rho_2 - 1 - c < 0$ if $x^i + B > 1$. As a consequence, when the government increases the transfer in case of bailout B, welfare only raises if there is more entrepreneurs in the former case than in the latter. $H(1-b)$ measures the number of entrepreneurs with holdings x^i less than $1-b$. Then, as long as, $H(1-b)$

[12] In Section 7, I consider another feature that limits bailouts: a possible capture of the government by the domestic private sector.

is large enough, the governement can increase b. Formally, this condition can be written as follows:

$$H(1-b)\left[\beta\rho_1 - 1 - c\right] + \left(1 - H(1-b)\right)\left[\beta\rho_2 - 1 - c\right] \geq 0 \tag{6}$$

Finally, the bailout B is the greatest value of b such that $H(1-b)$ is still large enough to obtain welfare gains.

In terms of economic interpretation, Proposition 3 shows that the ability to bail out hinges on the government's disposable information, measured here by the cost c and the distribution H. More accurately, the more the domestic economy is transparent, the more the government can implement efficient bailouts.

Example (Non-diversification). I assume here that domestic entrepreneurs cannot diversify their portfolios. They hold either domestic bonds or foreign bonds. As a consequence, the average holding of foreign bonds x corresponds to the fraction of entrepreneurs who hold foreign bonds only. Then $H(1-b) = 1-x$ for $b \in [0,1]$. Therefore, $B = 1$ if $x \leq \hat{x}$ and $B = 0$ otherwise.

Example (Symmetric holdings). I assume here that domestic entrepreneurs have symmetric holdings. Now, at the level of each entrepreneur, x corresponds to the fraction of the portfolio invested in foreign bonds. Then $H(1-b) = 1$ for $b < 1-x$ and 0 otherwise. And so, $B = 1-x$.

Symmetry implies here that the government knows exactly what each domestic firm holds. It can then bail out without giving too much to some entrepreneurs, and no one invests more than 1.

Incentive compatible bailouts So far, I have restricted bailouts to untargeted direct transfers, only in period 1.

In this paragraph, I relax the benchmark model assumptions to allow for a more general class of bailouts: the government offers $\{T_1(\hat{x}^i), T_2(\hat{x}^i)\}$, where T_t is the date-t transfer and $\hat{x}^i$ is the portfolio announced by an entrepreneur.

The delayed component T_2 could help achieve incentive compatibility. Furthermore, I allow T_1 to be random[13]. By contrast, without loss of generality, T_2 can be assumed to be deterministic, as agents are risk-neutral in period 2.

A key ingredient here is the pledgeability constraint of domestic entrepreneurs: intuitively, incentive compatible bailouts would have been implemented by loans with interest rates high enough such that only entrepreneurs with sufficiently high marginal returns (ρ_1 here) accept

[13] See Maskin and Riley (1984); Stiglitz (1987) on stochastic mechanisms.

to participate. However, these entrepreneurs would not reimburse them, ruling out such loans. Consequently T_2 has to be positive.

The following proposition summarizes the results of the mechanism design problem:

Proposition 4 (Incentive-compatible bailouts)**.** *Incentive-compatible bailouts are such that:*

(i) *Either:*

$$\frac{\beta\rho_1 - (1+c)}{\beta(\rho_1 - \rho_2) - (1+c)(1-\rho_2)} \geq \int_0^1 x^i di \tag{7}$$

T_1 is deterministic and is such that $T_1 = 1 - x^i$ and $T_2 = \rho_2 x^i$.

(ii) *Or 7 is violated and then: $T_1 = T_2 = 0$.*

(iii) *As long as*

$$(1 + c - \beta)\rho_2 \geq 1 + c - \beta\rho_2 \tag{8}$$

there is no loss of generality to consider only uniform bailouts.

Proof. See appendix. □

An incentive-compatible mechanism implies an additional transfer in period 2 (T_2) to domestic entrepreneurs. This transfer is a strictly increasing function of entrepreneurs' announcement $\hat{x}^i$ in order to give an incentive to entrepreneurs with large net positions in foreign assets to reveal their type. A lower bound on this subsidy is obtained by compensating the production that an entrepreneur would have obtained if he had announced holding only domestic bonds. Then, to choose the form of its bailout, the government compares the costs associated with the two transfers T_1 and T_2 with the loss implied a uniform bailout:

$$\begin{aligned} \max_B \left\{ \beta \int_0^1 \left[f(x^i + B) \right] di - B(1+c) \right\} \\ - \int_0^1 \left[\beta\rho_1 - (1+c)(1-x^i) + (\beta - (1+c))Rx^i \right] di \geq 0 \end{aligned} \tag{9}$$

A sufficient condition for this inequality to hold is (8), which compares the cost of an uniform bailout $B = 1$: $1 + c - \beta\rho_2$ with the cost of the targeted bailout: $(1 + c - \beta)\rho_2$. The cost of a uniform bailout consists of the loss implied by bailing out too much some entrepreneurs, while the cost of untargeted bailout corresponds to the cost of the additional bailout T_2.

Note that (8) holds if and only if $\rho_2 \geq 1$: when the production function is not too concave, having some entrepreneurs producing at lower marginal productivity is less costly than implementing two transfers. We do not need any additional assumption to consider only uniform bailouts.

4.2 Optimal default

Given an optimal value for the bailout B, the government chooses to default by comparing W_0 and W_1. More precisely, the government chooses a strategy π so as to maximize $\pi W_1 + (1-\pi)W_0$.

The comparison of W_0 and W_1 is equivalent to the comparison of losses on the one hand and repayments on the other hand:

$$W_0 \geq W_1 \Leftrightarrow (1+c)B + \beta \int_0^1 \left[F(z^i + x^i) - F(B + x^i)\right] di \geq Z \tag{10}$$

The government prefers to default if the cost of repayment Z is less than the sum of the losses incurred by domestic entrepreneurs $\beta \int_0^1 [F(z^i + x^i) - F(B + x^i)]$ and the direct cost of the bailout $(1+c)B$.

The higher the value of default W_0, the more likely the government defaults. As the option to bail out gives the ability to raise this value of default, the effect of information propagates from bailouts to the default decision: the more opaque the domestic economy, the less the government defaults.

Example (Non-diversification). Depending on the value of xB, inequality (10) becomes:

- $(1+c) + \beta\rho_2(Z^e - 1 - x) \geq Z$ if $x \leq \overline{x}$.
- $\beta\left[\rho_2(Z^e - 1) + \rho_1(1-x)\right] \geq Z$ if $x \geq \overline{x}$.

When $x \leq \overline{x}$, the internal cost to default is lower due to the bailout.

Example (Symmetric holdings). In this case, as $B = 1 - x$,

$$W_0 \geq W_1 \Leftrightarrow (1+c)(1-x) + (Z^e - (1-x))\beta\rho_2 \geq Z$$

If repayments to domestic entrepreneurs Z^e equal $1-x$, which holds if agents anticipate that the government will repay for sure, this expression boils down to $(1+c)(1-x) \geq Z$. In that case, W_0 can be lower than W_1 only when the cost c is strictly positive.

5 Equilibrium

5.1 Commitment

To begin with, as a benchmark, consider the case, where the government can commit. Optimally, the probability of repayment is $\pi = 1$ and there is no bailout ($B = 0$). Thus the price of public bonds is equal to 1 in period 0. This leads to the following situation:

Proposition 5. *In case of commitment,*

(i) *Domestic public bonds are risk-free assets.*

(ii) *Domestic entrepreneurs invest their entire endowments indifferently between the two assets, invest* 1 *in period 1 and produce* ρ_1 *in period 2. Foreign investors are indifferent between domestic and foreign assets.*

(iii) *Government borrows* $\overline{G}$ *from foreign investors and produce* $R\overline{G}$ *in period 1.*

This proposition provides us with a benchmark for the results of the remaining of the section. Under commitment, the government is unconstrained and can freely borrow. More importantly, the allocation of investment of domestic entrepreneurs has no effect.

5.2 No commitment

From now on, the government cannot commit at date 0 to reimburse its debt at date 1. Investors form expectations $\{\pi^e, B^e\}$ on these decisions and select their investment as described in section 3. Let $\Gamma(\pi, B)$ denote the set of probabilities of repayment and bailouts that are consistent with $\{\pi, B\}$: the expectation of $\{\pi, B\}$ leads to some optimal reactions in the form of a distribution of assets h, which in turn generates a correspondance of equilibrium probabilities of repayment and bailouts $\Gamma(\pi, B)$. An equilibrium is then a fixed point of this correpondance. As a consequence, equilibria of the no-commitment problem consist of belief π and expected bailout B such that:

$$\{\pi, B\} \in \Gamma(\{\pi, B\}) \tag{11}$$

I denote by S the set of these equilibria. I index equilibria by their repayment decision (π). S_π denotes the subset of equilibria with probability of repayment π[14].

In the following, an equilibrium $\{\pi, B\}$ is stable if, when investors change by arbitrarily small amount their beliefs, the equilibrium portfolio distribution remains sustainable and thus, the government would still choose $\{\pi, B\}$ as policies. Otherwise, an equilibrium is unstable.

Three kinds are equilibria may exist depending on their repayment probability: no-default equilibria where $\pi = 1$, the default equilibrium where $\pi = 0$ and, in between, a continuum of random default equilibria where $\pi \in (0, 1)$. The following proposition describes the set of equilibria using this classification:

[14]Notice that S_π is a partition of the set of all equilibria S: $S = \bigcup_\pi S_\pi$.

Proposition 6. *No-default equilibria exist ($S_{\pi=1}$ is non-empty) if and only if repayment is such that $Z < \overline{Z}$, where $\overline{Z}$ is an increasing function of β and c. An upper bound of $\overline{Z}$ is $1+c$.*

$\{\pi = 0, B = 0\}$ is always an equilibrium ($S_{\pi=0}$ is never empty).

$\{\pi, B\}$ with $\pi \in (0,1)$ is an equilibrium if and only if $B > 0$.

Besides, only no-default equilibria are stable.

Proof. Cf. Appendix. □

The three classes of equilibria

No-default equilibria When the government is expected to repay for sure ($\pi = 1$), domestic bonds are safe assets like foreign bonds and their price equals 1 ($p = 1$).

External debt can be sustained without reputational arguments as long as this external debt is sufficiently small ($Z \leq \overline{Z}$). This is in line with Brutti (2011) or Gennaioli et al. (2011). Similarly, the higher the government gives weight to domestic entrepreneurs' losses (β), the more likely it will default[15].

Default equilibrium Whatever the value of parameters, there is always at least one equilibrium in which the government defaults for sure ($\pi = 0$) and where the domestic public debt is worthless ($p = 0$).

In this equilibrium, $p = 0$ and h is degenerate and peaks at $x = 1$. Domestic entrepreneurs hold only foreign bonds. In period 1, they receive 1 from their investment and produce ρ_1 in period 2. Foreign investors may hold all the domestic debt, clearing the domestic bond market. As this debt is worthless, it has no real effect[16]. The government is not able to borrow anything since $p = 0$.

This equilibrium is not stable as every small deviations $\{\pi, B\}$ with $\pi > 0$ and $B > 0$ may sustained as an equilibrium.

Random default equilibria Between default and repayment, there are a continuum of equilibria where repayment is random, i.e. $\pi \in (0,1)$. In this case, the price of domestic bonds equals $p = \pi$. These equilibria resemble mixed strategies in standard non-cooperative

[15] Indeed the derivative of $W_1 - W_0$ with respect to β is equal to $\int_0^1 F(z^i + x^i) - F(B + x^i)di$. The sign of this expression is the same as $W_1 - W_0$, which is positive for equilibria where $\pi = 1$.

[16] Worthless domestic debt can also be held by the domestic private sector. However, because $\beta\rho_2 < 1 + c$, the government does not want to repay.

games as the beliefs of some participants (the investors) are taken into account by the other participants (the government).

The comparative statics for random default equilibria are limited to asymptotic properties of S_π: when repayment goes to infinity, random equilibria tend not to exist when repayment (Z) is too large or when the political weight of domestic entrepreneurs (β) is too small [17].

Notice that the unstability of random equilibria has a specific shape: when modifying investors' beliefs by arbitrarily small changes, the equilibrium repayment probability and bailout move continuously. Although riskless countries have a stable creditworthiness, as soon as this creditworthiness has begun to be questionable, country's debt repayment is unstable and perfectly self-fullfilling.

Notice that there is not an unique equilibrium per repayment probability, as the value of the bailout may differ, and, hence, the physical outcome of the equilibrium[18].

The role of bailouts The possibility to bail out is crucial for debt repayment, even though, actually, the government does not implement transfers. First it acts as an off-equilibrium condition for no-default equilibria. This is measured through the dependence to c: when it is costly to bail out, the value of default cannot be increased easily, and thus, it is often better to repay. c represents the relative cost of bailouts to debt repayment. This cost may be interpreted as domestic financial opacity or complexity, as it may derive from various sources such as opportunistic parasite entrepreneurs, political reputation or costs to plan and implement the bailout. Empirically, this suggests that measures of financial opacity or complexity are negatively correlated with the frequency of default. Second it allows no-default equilibria to be stable. Indeed, bailouts make domestic debt holdings less risky and, thus, still attractive for domestic residents even with anticipated default risks.

This mechanism plays also a crucial role for random default equilibria. Indeed, in that equilibria, the government should bail out domestic entrepreneurs. Indeed, suppose that $B = 0$. Then domestic entrepreneurs' willingness to pay is $\overline{p} = \pi\rho_2 / (\pi\rho_2 + (1-\pi)\rho_1)$. This

[17] Cf. appendix for a discussion of this point

[18] For example, if one distribution features no-diversification and the other symmetric portfolios, B differs for these two distributions. In the symmetric case, the government will bail out entrepreneurs so that they will produce ρ_1 in aggregate in case of default and $\rho_1 + \rho_2(1/\pi - 1)$ when there is no default. In the no-diversification case, entrepreneurs produce $\rho_1 + \rho_2(1/\pi - 1)$ when there is no default and, when there is a default, $x\rho_1 + (1-x)(\rho_1 + \rho_2)$ if $B = 1$ and $(1-x)\rho_1$ if $B = 0$. When two portfolio distributions lead to the same repayment probability and to the same bailout, the physical outcome would be the same on aggregate, but not at entrepreneurs' level. For example, with no-diversifiation, the outcome for an entrepreneur when a default occurs depends on whether he invested in foreign or in domestic bonds.

expression equals π, foreign investors' willingness to pay, if and only if $\pi = 1$ or $\pi = 0$. This corresponds exactly to the argument raised in section 3. Without bailouts, domestic entrepreneurs prefer to invest abroad when the repayment probability is strictly less than 1 as the price of domestic bonds is too high for them when considering the default risk. The anticipation of a bail out reduces the anticipated consequences of a default and thus convinces domestic investors to buy risky domestic public debt. In turn, these domestic holdings of domestic bonds motivate foreign investors to lend to the government.

Risk aversion Risk aversion is a key factor shaping the role of bailouts. In the case where the production function is linear and not concave (i.e. $\rho_2 = \rho_1$), that is when there is no risk aversion domestically, all probabilities $\pi \in [0, 1]$ are equilibria as domestic and foreign agents value the default risk in the same way. Then bailouts play no role.

More generally, when domestic agents are less risk averse than foreign investors, they are more willing to hold domestic debt than foreigners and, hence, bailouts are not required. Conversely, the bailouts' role of keeping domestic savings home is at work when domestic agents are more risk averse than foreign agents.

Remark (Domestic savings capacity). As the domestic savings capacity is normalized to 1, all the results are in relative terms with this savings capacity. In particular, note that no-default equilibira always exist when the repayment is lower than this capacity ($Z \leq 1$).

Repayment rate as a function of portfolios It is a well-known result that the properties of equilibria are not continuous functions of parameters (cf. Kreps and Wilson (1982) among others). However, this gives no indication where discontinuity may appear.

In this economy, a large class of equilibria featuring such discontinuities includes the equilibria where portfolios are very heterogenous: for example, when some domestic entrepreneurs invest mainly in foreign assets while the other purchase mostly domestic debt. The intuition of this result is the following: when portfolios are sufficiently dispersed, a very little change may shift the bailout from $B > 0$ to 0, making domestic debt too risky for domestic entrepreneurs and, thus, probabilistic equilibria are not sustainable anymore: this corresponds to a discontinuous jump in the repayment rate. I let the formal description of this class of equilibria for the appendix.

In terms of assessment of sovereign debt risk, this means that a sensitive factor of sovereign debt repayment is portfolio allocations, not only in terms of allocation between domestic and foreign agents, but also how this debt is dispersed among domestic agents' portfolios. Moreover, dispersion is not only crucial for restraining the ability to bail out, but it can also

be destabilizing as small changes in domestic agents' choices (i.e. portfolios) lead to major aggregate outcomes (i.e. a default).

5.3 Two examples

This subsection illustrates the general results of the previous section with two subclasses of equilbria: when domestic entrepreneurs cannot diversify portfolios and when they have symmetric portfolios. These examples are the two polar cases in terms of dispersion of portfolios and thus, they illustrate some of the main features of the set of equilibria S.

I leave for the appendix the derivations of these two subclasses of equilibria.

Undiversified portfolios In this case, domestic entrepreneurs' portfolios are either invested in domestic bonds ($x^i = 0$) or in foreign assets ($x^i = 1$). Then, the relevant variable is x, the fraction of domestic entrepreneurs who invested in foreign assets.

Figure 3 plots then equilibria as a function of domestic holdings (x) and repayments (Z):

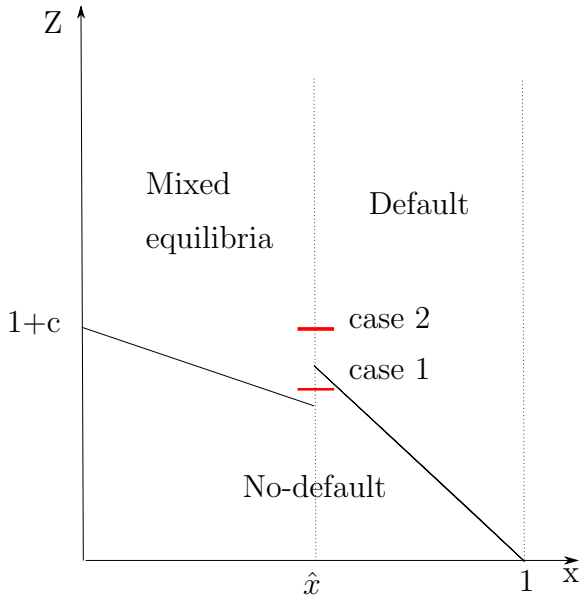

Figure 3: Equilibria as functions of domestic holdings (x) and repayments (Z) for undiversified portfolios

We obtain here a discontinuity in the ability to borrow with respect to x as in proposition 15. In the neighborhood of $\hat{x}$, small variations of x may trigger two major changes depending on the value of repayments Z. When considering changes in portfolios around $\hat{x}$, two cases arise as plotted in Figure 3 by the two barres:

Case 1: a change in portfolios along the barre may shift repayment from $\pi = 1$ (repayment for sure) to a lower but strictly positive probability.

Case 2: a change in portfolios may shift government's decision from a probability strictly posivitve to a complete default $\pi = 0$ (case 2).

Symmetric portfolios In this case, domestic entrepreneurs hold the same portfolio: $x^i = x$. Here the relevant variable is also x, which denotes now the fraction of foreign assets in every domestic entrepreneurs' portfolio. Because of symmetry, the government knows exactly how much to transfer to each domestic resident. The cost of bailing out is reduced to c, the relative cost of bailout compared with repayment.

Then, we can obtain a clear picture of this subclass of equilibra as illustrated by Figure 4.

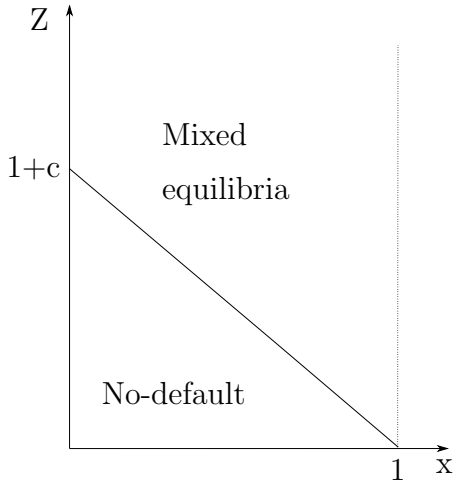

Figure 4: Equilibria as functions of domestic holdings (x) and repayments (Z)

Here the effect of Z, or equivalently Z^* the external debt, on default is straightforward: The higher Z is, the more it is possible to encounter mixed equilibria with a low probability not to default. This is related to Reinhart and Rogoff (2011) who show that banking crisis follow surges in external debt. I interpret losses suffered by the domestic private sector as a banking crisis in my model.

5.4 Discussion

Bailouts Bailouts in the baseline model can be implemented in several ways. It can represent direct transfers, loans at discounted interest rates to banks or to any other financial institutions, or all the forms that have been used to redistribute resources in order to sustain economic activity.

Nevertheless, it is worth to notice that some of these tools have been really used after defaults. In 2001 default of Argentina, banks' deposits convertibility has been suspended

(the so-called "corralito" decided the 3rd december 2001). Even though the efficiency of such suspension has been critized thereafter, the other suggested solutions would have turned to be also bailouts, as central banks interventions as lender of last resort as suggested by Bolton (2003). Similar suspensions on banks' liabilities were decided by Russian authorities in August 1998 after having defaulted on their debt.

Besides, monetary policy, devaluation and inflation can also implement the redistribution of resources after the default, as long as they allow for transfers towards residents who actually suffered from the default.

Secondary markets As highlighted by Broner et al. (2010), secondary markets for sovereign debt play a key role as foreign investors can resell their domestic debt holdings to domestic residents, making default undesirable. Similarly, introducing such secondary markets in this paper's framework reinforces the unability to discriminate among bondholders, consistently with the assumption I have made on the impossibility of selective default.

Turning to bailouts, notice that using bailouts are an easier way to preempt secondary markets than through selective default, as a selective default requires more information than implementing transfers (especially uniform transfers as in my framework). For sure, as for selective default, implementing perfect bailouts would require to track all positions and cannot cope with possible anonymity in bond markets. When government's information is scarce, as, for example, when the government has only indications on the aggregate distribution of portfolios, selective default cannot be implemented anymore, unlike bailouts that can be made independent on current holdings as in this paper's framework.

A additional possible effect of secondary markets would be to allow domestic entrepreneurs to adjust their exposures to domestic debt. For example, high productivity domestic entrepreneurs may want to resell their bonds to low productivity entrepreneurs. However, when anticipating a default, domestic bonds are worthless as transfers do not depend on bondholdings and, thus, no trades take place[19]

Complexity Conversely, in the absence of secondary markets, restrictions to bailouts may derive from the financial system's complexity. Indeed, domestic agents may be exposed to domestic debt in various ways that exceed direct holdings: when domestic public bonds are used as collateral in transactions or when domestic agents are exposed to domestic debt through foreign institutions or foreign subsidiaries. As soon as information acquisition is

[19]Notice that loans between domestic entrepreneurs would allow to reallocate resources towards high productivity entrepreneurs. However such loans cannot be granted because of no pledgeability.

costly[20], keeping track of all the domestic exposures becomes inefficient. Additionally, an agency cost to this information acquisition may arise as private agents have no incentive to reveal their exposure to domestic debt as in Proposition 4.

Exchange rate risk and costs of transactions The benchmark model considers that domestic bonds and public bonds are perfect substitutes besides sovereign risk. For sure, in a more general setting which would include different currencies, public bonds have the comparative advantage to be issued in domestic currency. This reduced exchange rate risk would compensate, at least partially, domestic sovereign risk. Similarly, paying higher costs for purchasing foreign bonds compared with domestic bonds makes domestic bonds more attractive.

6 The pros and cons of capital controls

In the benchmark model, domestic agents may underinvest in domestic bonds and, thus, reduce inefficiently the government's borrowing capacity. In response, it can use capital controls to mitigate or to prevent them.

However capital controls can be problematic: their implementation have negative collateral effects such as a lack of diversification of domestic portfolios. In most cases, I show that this lack of diversification cannot be solved using insurance contrast with the rest of the world. Indeed, such a contract implies payment that adds to the country promised repayment, on which the country may default.

As a consequence, capital controls may be desirable only if the gains from borrowing abroad overweight the losses due to lower diversification. However, I show that standard forms of capital controls inefficiently regulate capital flows[21] and do not achieve for sure the optimal outcome.

More precisely, I argue that price instruments are not efficient as they do not prevent no-borrowing equilibria and because, their effects can be turned off easily as long as the financial

[20] Cf. Caballero and Simsek (2009) for a theoretical understanding of information acquisition and complexity.

[21] Capital controls' efficiency is highly dependent on their implementation: a complete and longlasting closing of the financial account and strong limitations on currency exchanges are much more efficient than temporary and targeted measures. Capital controls can be focused on either capital inflows or outflows and enforced either through prices or quotas (see Neely (1999) for an overview). In our context, two forms are particularly relevant: first, the government observe directly domestic entrepreneurs' portfolios and force them to hold domestic assets. Second, the government controls flows of funds that go in and out of the country. For the former and the latter, information is crucial for the efficiency of capital controls.

account is sufficiently open. Quantity instruments are better to boost domestic borrowing but they are restricted by the lack of information on domestic portfolios. Attempts to increase such information have ambiguous effects as they increase at the same time the ability to bail out.

6.1 A macroeconomic shock

In the baseline model, domestic entrepreneurs have no diversification motive. This subsection adds such a diversification motive and derives its effect when capital controls are introduced.

Suppose now that the return on government's technology is stochastic and is equal to R with probability γ and 0 otherwise. In the latter case, the government is not able to repay nor to bail out. This constitutes a macroeconomic shock against which domestic entrepreneurs are willing to insure. In appendix, I give a general characterization of these macroeconomic shocks.

Without capital controls, domestic agents prefer foreign debt:

Proposition 7. *As long as $\gamma < 1$ and without capital controls, the only equilibrium is such that domestic entrepreneurs do not hold domestic debt.*

Consequently, the government never repays ($\pi = 0$).

Proof. See appendix. □

The macroeconomic shock reinforces the flight-to-quality towards the safest asset, the foreign asset, and bailouts are unable to prevent this preference. Consequently, in a world without capital controls but with macroeconomic shocks, the domestic private sector uses only foreign liquidity as stores of value.

Remark. Interpreting foreign liquidity as US treasuries, this sheds light on the "savings glut" experienced by the US in the 2000s: domestic sovereign risk and country-specific macroeconomic risks prevent domestic uses of public debt. This constitutes an alternative view of liquidity explanations of global imbalances compared with Caballero et al. (2008), who emphasize the ability to generate financial assets from real investment. This view however requires the United States to be less risk-averse than the rest of the world and U.S. treasuries to be perfectly sage. The relative development of the US financial sector allows to make both assumptions: a better financial sector is able to better diversify and it corresponds to a greater cost of default as emphasized by Gennaioli et al. (2011) (cf. Gourinchas et al. (2010) for similar assumptions).

Could the government insure against this shock? In response to the macroeconomic shock, the government can try to insure itself by contracting with foreigners: foreigners transfer resources to the government when the shock is bad (i.e. the return is 0) against a payment when the shock is good (i.e. the return is R). As in the baseline model, the participation of foreigners to this contract depends on the observability of the promised payments:

(i) When the repayment associated with the insurance contract is clearly identified by the government, the government never repays: as it has no incentives to repay foreigners. Consequently, such a contract cannot exist.

(ii) When the repayment of the insurance contract cannot be disentangled from other repayment, this fungibility leads to a similar commitment problem as in section 5.

Let me consider this new commitment problem.

To cover repayments in the bad state, the transfer should be greater than or equal to Z. Foreigners' break even condition pins down payment by the government in the good state:

$$\gamma\pi P^* = (1-\gamma)Z$$

with P^* this payment and π in the repayment probability in the good state. As a result, nominal repayment to foreigners in the good state equals: $(1-\gamma)/(\pi\gamma)Z + Z^*$. Plugging this new value of repayment into the baseline model, I obtain:

Proposition 8. *When Z is high enough or γ low enough, the government defaults with probability* 1 *in the good state.*

Proof. See appendix. □

In other words, when total repayments in the good state, i.e. the repayment associated with the insurance contract and the repayment of public bonds Z, are too high, foreigners refuse to insure the government against the macroeconomic shock. Indeed, they anticipate that the government will default for sure in the good state, and especially on the repayment to the insurance contract. Total repayments are too high either when the public debt is already too large (Z too large), or when the macroeconomic risk is very likely (γ low), which implies big repayment in the good state.

Remark. This result is reminiscent of the debate around contingent sovereign debt as discussed by Grossman and Van Huyck (1988), Borensztein and Mauro (2004) or Chamon et al. (2005). According to this literature, using such instruments rather than non-contingent debt allows to achieve better risk-sharing and to avoid pro-cyclical policies in bad times. However,

proposition 8 puts restrictions on these gains, as the possibility of default limits the promised repayment in good times.

Optimal portfolios Without macroeconomic shocks, optimal portfolios are completely invested in domestic bonds, so as to maximize the government's ability to borrow. Macroeconomic shocks introduce a trade-off for portfolios because of the additional need of diversification. The gain of domestically-invested portfolios is the greater government's ability to borrow $\gamma RG = \gamma RpZ$. The cost is what domestic entrepreneurs cannot invest in bad times: $\beta(1-\gamma)\int_0^1 (F(z^i + x^i) - F(x^i))$.

As long as R or γ are large enough, the government strictly prefer to increase its borrowing capacity, even at the cost of lack of diversification. Thus, the decentralization, as described by Proposition 7, is not desirable. In the following, I consider the effects of price and quantity instruments to achieve the optimal portfolio allocation.

6.2 Introducing a tax on flows of funds

As in the benchmark model, portfolios are not observable, the only dimension on which the governement may intervene is the flows of funds received by investors.

Let $\kappa \in (0,1)$ be the tax rate on flows of funds.

When the government is able to observe who initiated a transfer of funds, it can tax fully domestic entrepreneurs investing abroad and let foreign investors purchase domestic bonds.

When the government has only an imperfect ability to do so, it is forced to tax the flows of funds initiated by both domestic and foreign agents. The government can tax outflows or inflows only, or both. As domestic investment abroad and foreign investment domestically imply both outflows and inflows, without loss of generality, I consider only a tax on inflows in period 0 and 1[22].

As a result of this tax, capital controls contract purchases of foreign bonds by domestic agents along with investment by foreigners in domestic bonds.

6.2.1 General outcome

Let me first consider the interaction between capital controls and macroeconomic shocks.

[22]Remark that, because of the timing of the model, taxing outflows in period 0, or inflows in period 1, would be sufficient to stop domestic entrepreneurs to invest abroad.

Demand for domestic bonds Foreign investors accept to purchase domestic bonds only if $p \leq \gamma\pi\,(1-\kappa)$. Domestic entrepreneurs' payoff as a function of foreign bonds' holding is:

$$P(x^i) = \gamma\left(\pi F\left(x^i(1-\kappa) + \frac{1-x^i}{p}\right) + (1-\pi)F(x^i(1-\kappa)+B)\right) + (1-\gamma)F(x^i(1-\kappa)) \tag{12}$$

Equilibria The following proposition describes the set of equilibria:

Proposition 9 (Tax on transactions)**.** *With capital controls $\kappa > 0$,*

(i) *no-default equilibria exist. In that case, the price of domestic bonds is: $\gamma(1-\kappa)$.*

(ii) *Default $\{\pi = 0, B = 0\}$ is always an equilibrium.*

The introduction of a tax on financial flows prevents domestic agents to purchase foreign assets rather than domestic debt and, in turn, this gives incentive to the government to repay.

6.2.2 An example: when there is no macroeconomic shock.

This case corresponds to $\gamma = 1$, there is no macroeconomic shock anymore.

Each entrepreneur invests his endowment only in domestic bonds ($x^i = 0$). This corresponds to a symmetric equilibrium where $x = 0$. The price of bonds is $1-\kappa$.

The tax on external flows delivers $c\kappa$ to the government. This implies a trade-off between government financing and the risk of multiple equilibria. Overall the government obtains for sure $(1-\kappa)(1+c)+\kappa c = 1+c-\kappa$ from the issuance of bonds and capital controls. Without capital controls, the government faces multiple equilibria but it can obtain as much as $1+c$ in some equilibria.

Remark (Ex ante problem of bonds issuance). To anticipate the next section, when the government issues only a mass 1 of bonds rather than $1+c$, they obtain 1 of resource, which has to be compared with $(1-\kappa)(1+c)+\kappa c$. As long as $c \geq \kappa$, the government gains to issue more bonds than a mass of 1.

6.2.3 The openness of the financial account

Another caveat for capital controls is their width: when capital controls affect only some markets, evading structures can be created, dwindling their efficiency. A recent example is Brazil, which introduced capital controls on stocks and bonds: in response to the increase of the tax rate on bonds, intra-companies loans skyrocketed[23]. When the financial account is

[23] Numbers?

completely closed, evading is not easily implementable: the tools used to avoid the capital controls also face these controls. In this subsection, I add to the model a banking sector so as to give to domestic entrepreneurs the ability to avoid the controls.

This banking sector consists of a continuum of small banks which can collect savings and invest in foreign risky assets. This asset yields 1 with probability ζ, with $\zeta \in [0,1]$ and 0 otherwise: then the expected return equals 1. Because of competition, the return on savings put into banks is what the foreign risky asset yields.

Capital controls κ can be implemented only for sovereign debt markets or also for risky foreign asset markets. The results of introducing these costs are:

Proposition 10. *In absence of capital controls, domestic entrepreneurs purchase directly only foreign bonds: they do not save through banks (and banks do not purchase risky foreign debt) and do not purchase domestic bonds (same as in proposition 7).*

With capital controls restricted to sovereign debt markets, there exists a threshold $\overline{\zeta} \in [\gamma, 1]$ *such that, for any* $\zeta > \overline{\zeta}$*, domestic entrepreneurs save their whole endowment through banks in risky foreign assets.*

With capital controls on both markets, domestic entrepreneurs do not save through banks (banks do not purchase risky foreign debt): the outcome follows the previous subsection.

As a consequence, when capital controls are incomplete and when evading structures are sufficiently interesting (ζ is not too low), capital controls are unefficient to keep savings at home.

6.3 Verification of net positions

The verification of domestic entrepreneurs' net positions is another way to enforce capital controls in my setting. As an example, Italy and South Korea implemented such controls on portfolio choices in the past (cf. Neely (1999) and the references therein).

When the government is able to observe these net positions, the form of capital controls are easy to choose and to implement: the government forces domestic entrepreneurs to invest only in domestic assets. However, domestic agents may use strategies to avoid capital controls[24], making this ability only imperfect. In parallel to the coordination problem in the baseline model, choices of evading strategies has to be coordinated as there is complementarity between evading strategies of entrepreneurs. This coordination problem delivers similar equilibria, where bailouts and the repayment probability of the government depend endogenously on portfolios. Such a form of capital controls turns out to be ineffective.

[24]These strategies include, for example, the issuance of liabilites abroad. See Forbes (2007) for a survey.

In between, the government may identify a fraction $\mu \in [0,1]$ of investors and force them to hold domestic bonds. In case of default, it bails out completely these identified investors ($B = 1$). For the other investors, it proceeds as in the baseline case.

Then the government compares the value of its two options:

$$W_1(\mu) = -Z + \beta\mu\rho_1 + \beta(1-\mu)\int_0^1 F(z^i + x^i)di >$$
$$W_0(\mu) = -\mu - (1-\mu)(1+c)B + \beta\mu\rho_1 + \beta(1-\mu)\int_0^1 F(B + x^i)di$$

This comparison should be independent from the distribution of holdings among the other domestic entrepreneurs:

$$(1-\mu)\left(W_1(0) - W_0(0)\right) + \mu(1-Z) > -(1-\mu)Z + \mu(1-Z) > 0$$

If μ is sufficiently high, this is sufficient to tie the hands of the government and force the coordination towards domestic bonds:

Proposition 11. *For $Z < 1$, the government does not default for sure as long as the fraction μ of investors identified by the government satisfies:*

$$\mu > \mu^* = \frac{1}{1 + \frac{(1-Z)}{Z}} \tag{13}$$

whatever the distribution of holdings among the unidentified domestic entrepreneurs.

For $Z > 1$, $\mu = \mu^* = 1$.

An easier access to information on some portfolios may be achieved in several ways: 1) with legal disclosure of information for prudential policies, 2) through the structure of the bonds market (identified buyers, development of secondary markets). In terms of institution design, this suggests that the agency in charge of the allocation of bailouts should be also in charge of banking regulation, triggering potentially more risks of collusion with banks (domestic entrepreneurs in this model). In appendix, I show that a potential capture of the government cause these gains from additional information to vanish.

Remark. In this case, Z is compared to 1 and not to $1 + c$ as bailouts of identified investors are costless. The identification of investors represents both gains and costs: it facilitates debt sustainability but in a limited amount.

7 Further extensions

7.1 Partial defaults

This subsection allows the government to partially default on its debt. Indeed creditors' losses in case of default are usually less than 100% of their investment. For example Sturzenegger and Zettelmeyer (2007) document that debt restructuring in the 1998-2005 period implied losses between 13% and 75% for creditors. Partial defaults replicate a complete default followed by a bailout as they leave domestic entrepreneurs with some wealth, but they have the comparative advantage not to bear the costs linked to bailouts.

When defaulting partially, a government can either repay only a fraction of each bond it has issued or it can repay fully only some of the bonds, as in a lottery. I assume in this section that when agents prefer to perfectly diversify their portfolios and when domestic bonds are sufficiently divisible. Hence, using the law of large numbers, these two implementations are equivalent and in the following I consider only default on fractions of bonds.

Notice that, in equilibrium, entrepreneurs are not more risk averse that foreign investors against partial defaults, making partial defaults *a priori* more desirable. I show that even in this case, equilibria with complete default and bailouts still exist.

7.1.1 Partial default equilibria

I denote by λ the fraction of the debt that the government repays. From lenders' perspective, as the government is not able to observe portfolios, it has no choice but to randomize across the lenders who is going to be repaid. Thus λ represents also the probability for a lender to be repaid.

Foreign investors compare p with λ and so do domestic entrepreneurs. The government's problem is to maximize the following value function:

$$W = RG - \lambda Z - (1+c)B + \beta \int_0^1 F(\lambda z^i + B + x^i)di \tag{14}$$

The maximization of W with respect to the fraction λ requires to know the optimal bailout and its dependence with respect to the fraction λ. After having established this dependence, we obtain:

Proposition 12 (Optimal partial default)**.** *The government chooses the fraction of default and bailout such that $\lambda = \lambda^e$ and $B = 0$.*

Proof. See appendix. □

Equilibria Every $\lambda \in [0,1]$ is an equilibrium. Partial default has the comparative advantage not to require bailouts. Notice, however, that the production function introduces a bias towards partial default, as domestic entrepreneurs are risk-neutral against such defaults.

7.1.2 Welfare comparison

I now turn to the comparison of partial default with complete default and bailouts: the government can choose *ex post* between partial default and complete default with a bailout. The choice results from the following comparison, for every distribution H compatible with λ^e:

$$-\lambda^e Z + \beta\rho_1 \leq \max_B \beta \int_0^1 F(x^i + B)di - (1+c)B$$

The right hand term corresponds to the welfare outcome of partial default and the left hand term to the outcome of a complete default with a bailout. A sufficient condition for this inequality to hold is that:

$$\lambda^e \geq \frac{1+c}{Z} \tag{15}$$

In other words, when $Z < 1+c$, there exist some distribution H for which complete default are striclty preferred to partial default. For sure, this result can be magnified by having more continuous risk aversion among domestic agents.

7.2 Government's issuance of bonds

Throughout the paper, repayment Z is taken as given. This subsection endogenizes it and thus considers the *ex ante* problem of the government, when it wants to issue bonds.

Its objective is to maximize its borrowing capacity, i.e. $G = pZ$. Intuitively this borrowing capacity G is a concave function of Z as what the government gains on the one hand by increasing the number of circulating bonds Z, it may lose it on the other hand as the price p declines.

To deal with the multiplicity of equilibria, which implies that for each Z correspond several prices p, I assume that investors select one equilibrium, by maximizing their ex ante utility. This yields a function $p(Z)$: for every repayment Z, there exists one equilibrium price $p(Z)$.

The problem of the government is $\max_Z G = pZ$ s.t. $p = p(Z)$. This maximization yields the following proposition:

Proposition 13. *The optimal amount of bonds issued by the government is:* $Z^{max} = 1 + c$.

Proof. See appendix. □

The previous sections emphasized the centrality of the relative cost of bailouts c in the ability of the government to borrow. This result confirms that by continuity this cost is also critical for the optimal bonds' issuance.

7.3 Taxpayers and a captured government

In the baseline model, bailouts are limited because of asymmetric information between the government and entrepreneurs. Here I consider another source of limitation: the asymmetry of information between taxpayers and the government.

The government has now perfect information on entrepreneurs' wealth allocation (x^i), while taxpayers only have information on the aggregate distribution h. Importantly, they are able to observe the transfers accorded to domestic entrepreneurs.

Without this friction, as in proposition 2, optimal bailouts without capture would be $1-x^i$ and production $F(1)$.

However, each domestic entrepreneur can try to bribe the government by offering a share $\xi_i > 0$ of additional benefits linked to excessive bailout: $F(B^i + x^i) - F(1)$. Symmetry and the production function monotonicity imply that $x^i = 1$ and $B^i = B$, which diverges to ∞. In response, taxpayers limit the transfer by imposing a upper bound $\overline{B}$ using their information on h.

Proposition 14. *With a captured government,* $\overline{B}$ *is:* $\max\{b|H(1-b) \geq 1-\hat{x}\}$ *with* $\hat{x} = \frac{\beta\rho_1 - 1 - c}{\beta(\rho_1 - \rho_2)} \in (0,1)$ *and* $B = \overline{B}$.

$\overline{B}$ is obtained in a similar way as in proposition 3. Bailouts are restricted by the least informed agents: the taxpayers, although the government is better informed than in the baseline case.

8 Conclusion

This paper studies sovereign debt repayment when the government has no reputation concerns and when it can compensate domestic losses due to default. I argue that bailouts' restrictions hinge on the government's ability to acquire information, and, hence, on the heterogeneity of domestic portfolios, but conversely, that some ability to bail out make domestic debt less risky, boosting domestic holdings and, thus, country's ability to borrow abroad.

Turning to future research, the results I obtain are derived in a stylized model, with, for example, risk neutral agents or undifferentiable production functions. Further research should relax these assumptions and consider other forms of domestic risk aversion, but this would not change the qualitative results of the paper.

Furthermore, assets and possible stores of value are limited to domestic public bonds and foreign bonds. I could also add a richer set of assets, with different levels of risk or, even, with different maturities. By the way, the main conclusions on multiplicity derived with two assets would still hold.

Finally, the model in this paper is in real terms and does not allow for monetary policy. The framework could be enriched to include such policies in order to study the interactions between default and monetary policy standard objective such as inflation targeting. Indeed, buybacks of public bonds by central banks give rise to another endogenous cost of default through inflation.

I leave these questions for future research.

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Appendix

Comparative statics with random default equilibria

Comparative statics Contrary to no-default equilibria, comparative statics cannot be performed easily with random equilibria. On the one hand, the set of equilibria S_π is not upper- nor lower- hemi-continuous with respect to changes in parameters (e.g. β, Z or c). Indeed, after a change of parameter, S_π includes only equilibria with distributions that do not correspond to equilibria before the change. Each distribution that corresponds to an equilibrium before a change in parameters does not correspond to an equilibrium thereafter. On the other hand, there always exists some distribution h such that $\{\pi, B, h\}$ is an equilibrium with the new set of parameters (These two properties are formalized by lemma 18 in appendix.). However, the existence of these new equilibria is based on a non-constructive proof. This limits the comparative statics to asymptotic properties of S_π: when repayment goes to infinity, random equilibria tend not to exist when repayment (Z) is too large or when the political weight of domestic entrepreneurs (β) is too small. As a general picture, for a distribution of portfolios h, the higher is repayment Z, the lower is the probability of repayment π:

the probability of repayment equals one for a range a repayment values and then decreases continuously to converge to 0 as repayment goes up.

Class of discontinuous equilibria

The following proposition formalizes the class of equilibria featuring discontinuity:

Proposition 15 (Discontinuity)**.** *Let H be a distribution associated to a bailout B (as derived as in proposition 3) such that: H is flat on $[B, 1)$ and H equals $1 - \hat{x}$ on this segment. Any change of portfolio leading to a distribution H' such that $H'(x) < H(x)$ for any $x \in [0, 1)$ implies that the bailout associated to the distribution H' is $B' = 0$.*

As a consequence, if H sustains a mixed equilibrium with a repayment probability π, $H + dH$ does not sustain any equilibrium.

Proof. Consider H such that $H(1) = 1$. Given $\epsilon > 0$, let dH a small deviation such that $(H + dH)(1) = 1$, such that $\|dH\|_\infty < \epsilon$.

$$H(1^-) + dH(1) < 1 - \hat{x} \text{ and } H(1 - B) > 1 - \hat{x}$$
$$\text{implies that } 0 < H(1^-) - H(1 - B) < -dH(1)$$

H is flat for $1 - B$ and 1. When there is a discontinuity of B with respect to dH, there is H, for Z high enough, mixed equilibria exist but not for $H + dH$. □

This is only an example of a class of distributions which feature potentially some discontinuities. However it is easy to see that the two ignredients defining this class of equilibria are almost necessary conditions to obtain these discontinuities.

Derivations of the two subclasses

Equilibria with symmetric portfolios

Strategies of agents Strategies of agents both in period 0 and in period 1 are special cases of sections 3 and 5.

Domestic agents and the demand for bonds First, recall that without diversification, the demand function for any entrepreneur is either:

(if $B = 1$): $1 - x^i = 0$ if $p > \pi$; $= \{0, 1\}$ if $p = \pi$; $= 1$ if $p < \pi$

(if $B = 0$): $1 - x^i = 0$ if $p > \overline{p}$; $= \{0, 1\}$ if $p = \overline{p}$; $= 1$ if $p < \overline{p}$

As a consequence $z^i = 1/p$ or $z^i = 0$ are two only possible repayments. The distribution of repayment has two peaks at 0 and 1/p. As in the general case, for every beliefs $\{\pi, B\}$ foreign investors accept to pay a higher price (π) than domestic investors for domestic bonds.

Government policies For this subclass of equilibria, optimal bailouts are such that: if $x > \hat{x}$, $B = 0$ and if $x < \hat{x}$, $B = 1$. The government does not default as long as:

$$(1 + c) + \beta\rho_2(Z - Z^f - 1) \geq Z \text{ if } x < \hat{x} \text{ and } \beta\left[\rho_2(Z - Z^f - 1) + \rho_1(1 - x)\right] \geq Z \text{ if } x > \hat{x}.$$

Equilibria Like previously, equilibria split into three subspecies: when the government either defaults or does not default for sure, and probabilistic default (or mixed strategies). However, another view is better here to look over the set of equilibria: the S_x's.

The condition for mixed equilibria to exist can be rewritten here:

Corollary 1. *Mixed equilibria exist as long as* $x \leq \hat{x}$

Indeed, with no diversification of portfolios, $x \leq \hat{x}$ is equivalent to $B > 0$. Using this result, I can describe the whole set of equilibria.

Equilibria where the government defaults for sure $\{\pi = 0, B = 0\}$ is still an equilibrium and the outcome properties are the same as before ($p = 0$). These equilibria correspond to $S_{x=1}$.

Equilibria where $x \geq \hat{x}$ In this case, only no-default equilibria may exist. The condition for them to exist is stated by the following proposition:

Proposition 16. *The condition for no-default equilibria to exist is* $Z \leq \beta\rho_1 (1 - x)$.

This completes the results found in the general case. For every $x \geq \hat{x}$, the set $S_x \cap S_{\pi=1}$ is expanding with β and with ρ_1. Besides, it is expanding with $1 - x$.

Remark. Recall that $x \geq \hat{x}$ then, an upper bound here for Z is $Z \leq \rho_1/(\rho_1 - \rho_2)(1 - \beta\rho_2)$.

Equilibria where $x \leq \hat{x}$ In this case both no-default and mixed equilibria may exist:

Proposition 17. *The function* $Z_x(\pi) = 1 + c - \beta\rho_2 \frac{x-(1-\pi)}{\pi}$ *is such that:*

- *For every* $Z \leq Z_x(1)$, $\{\pi = 1, B = 1\}$ *is an equilibrium.*
- *For every* Z *such that there exists* $\pi \in (0, 1)$ *which satisfies* $Z_x(\pi) = Z$, $\{\pi, B = 1\}$ *is an equilibrium.*

Remark. The set of functions $\{Z_x(\pi), x \in [0, 1]\}$ satisfies two properties:

1. The function $Z_x(\pi)$ is decreasing and takes values between $Z_x(1) = 1 + c - \beta\rho_2 x$ and $\lim_{\pi \to 0} Z_x(\pi) = \infty$. Morevover, this is a continuous function.
2. For every $x \in [0, 1]$, $Z_x(1) = 1 + c - \beta\rho_2 x \geq 1 + c - \beta\rho_2 > 0$ using assumption 1.

The set of equilibria - Properties of equilibria as a function of Z I describe here how equilibria of the game between domestic entrepreneurs and foreign investors depend on the amount of bonds issued by the government. The upper bounds for Z such that no default occurs are essential here and more precisely the way they are ordered.

Upper bounds are ordered as follows:

$$Z_{\hat{x}}(1) \leq \beta\rho_1 (1 - \hat{x}) \leq Z_1(1) = 1 + c \tag{16}$$

Indeed, the first inequality is equivalent to:

$$1 + c - \beta\rho_2 \frac{\beta\rho_1 - 1}{\beta(\rho_1 - \rho_2)} \Leftrightarrow c \geq 0$$

which is true by definition. The second inequality is equivalent to:

$$(1 + c)(1 - \rho_2/\rho_1) \geq (1 - \beta\rho_2) \tag{17}$$

Symmetric portfolios

Given the information structure, symmetric portfolios is a degenerate case: in this case the government knows exactly what each domestic entrepreneurs holds. This feature simplifies a lot this subclass: bailouts are designed to match exactly the needs of domestic entrepreneurs and the government always bails out when defaulting, except when entrepreneurs hold no domestic bonds.

Strategies of agents The demand function of each agent i is:

$$x^i = 1 \text{ if } p > \pi \ ; \ = [x, 1] \text{ if } p = \pi \ ; \ = x \text{ if } p \in [\overline{p}, \pi] \ ; \ = [0, x] \text{ if } p = \overline{p} \ ; \ = 0 \text{ if } p < \overline{p}$$

Aggregation implies that $x^i = x$.

Government policies Given that the government knows perfectly how many public bonds each domestic entrepreneur holds, optimal bailout is $B = 1 - x$. The governement default condition becomes:

$$W_0 \geq W_1 \Leftrightarrow (1+c)(1-x) + \left(\frac{1-x}{p} - (1-x)\right)\beta\rho_2 \geq Z$$

Equilibria First, recall that $\{\pi = 0\}$ is an equilibrium which belongs to this subclass of equilibria. Besides, the following corollary gives a complete description of the subclass in the form of lemma 18:

Corollary 2. *For each distribution $x \in [0, 1)$, there exists a function $Z_x(\pi)$ which is such that:*

1. *Its expression is: $Z_x(\pi) = (1+c)(1-x) + (\frac{1-x}{\pi} - (1-x))\beta\rho_2$.*
2. *For $Z \leq Z_x(1) = (1+c)(1-x)$, $\{\pi = 1, B = 1-x\}$ is an equilibrium.*
3. *For Z such that there exists $\pi \in (0, 1)$ such that : $Z = Z_x(\pi)$, $\{\pi, B = 1-x\}$ is an equilibrium.*

Proof. These results come from lemma 18 and the results on government policies in case of no-diversification. □

Remark. $Z_x(\pi)$ is a continuous and strictly decreasing function of π which takes values from $(1+c)(1-x)$ to ∞. As a consequence, for every $Z \geq (1+c)(1-x)$, there exists an unique π such that $Z = Z_x(\pi)$.

Lemma 18 - General characterization

The following lemma gives an intermediary result for the description of the set of equilibria S:

Lemma 18. *Let h be a distribution of portfolios, which is not degenerate at $x = 1$ and B the corresponding optimal bailout as derived in proposition 3. For every $\pi \in [0, 1]$, there exists a function $Z_h(\pi)$, such that:*

- *If $B > 0$, $Z_h(\pi)$ is a continuous and decreasing function of π. For $Z \leq Z_h(1)$, $\{\pi = 1, B, h\}$ is an equilibrium and for $Z = Z_h(\pi)$, $\{\pi, B, h\}$ is an equilibrium.*
- *If $B = 0$, $Z_h(\pi)$ is constant and for $Z \leq Z_h(1)$, $\{\pi = 1, B, h\}$ is an equilibrium.*

Proof of proposition 1

Consider an entrepreneur who invests x^i in foreign bonds and has beliefs $\{\pi, B\}$ about the policies of the government. The expected payoff of his portfolio is:

$$P(x^i) = \pi F\left(x^i + \frac{(1-x^i)}{p}\right) + (1-\pi)\left(F\left(x^i + B\right)\right) \tag{18}$$

$P(x^i)$ is a linear function of x^i and its derivative with respect to x^i is:

$$\frac{\partial P(x^i)}{\partial x^i} = \pi\rho_2(1-1/p) + (1-\pi)\rho_2 \text{ for } x^i \geq 1-B \text{ and } \frac{\partial P(x^i)}{\partial x^i} = \pi\rho_2(1-1/p) + (1-\pi)\rho_1 \text{ for } x^i \leq 1-B$$

This leads to the following demand function at the level of an entrepreneur:

$$x^i = 1 \text{ if } p > \pi \; ; \; = [1-B, 1] \text{ if } p = \pi \; ; \; = 1-B \text{ if } p \in [\overline{p}, \pi] \; ; \; = [0, 1-B] \text{ if } p = \overline{p} \; ; \; = 0 \text{ if } p < \overline{p}$$

with $\overline{p} = \pi\rho_2/((1-\pi)\rho_1 + \pi\rho_2)$.

Proof of proposition 2

With full observability, the value functions for the government would be: in case of no-default:

$$W_1 = RG - Z + \beta \int_0^1 F(z^i + x^i)di \tag{19}$$

In case of default:

$$W_0 = RG - \int_0^1 B^i di + \beta \int_0^1 F(B^i + x^i)di \tag{20}$$

Because of full observability, the government is able to identify both parasite entrepreneurs (the cost c vanishes) and to implement bailouts contingently on entrepreneurs (B^i depends on i).

The maximization of W_0 with respect to B^i yields $B^i = 1 - x^i$, as the government does not want to increase entrepreneurs' resources above 1 because of assumption 1. W_0 shrinks to:

$$W_0 = RG - \int_0^1 (1-x^i)di + \beta \int_0^1 \rho_1 di \tag{21}$$

A necessary and sufficient condition to have equilibria where the government does not default for sure is $W_0 \leq W_1 \Leftrightarrow Z \leq \int_0^1 (1-x^i)di + \beta \int_0^1 \left(F(z^i + x^i) - \rho_1\right) di$. As the integral term is less than 0, we can focus on: $Z \leq \int_0^1 (1-x^i)di$.

On the other hand, repayment to domestic agents are: $Z^e = \int_0^1 (1-x^i)/p di \geq \int_0^1 (1-x^i)di$. Using the two inequalities, we have: $Z^e = Z$ and there is no external debt: $Z^* = Z - Z^e = 0$.

Proof of proposition 3

Government chooses B in order to maximize its objective function. Then the derivative of this objective function with respect to B is:

$$\begin{aligned}
\frac{\partial W_1}{\partial B} &= -1 - c + \beta \frac{\partial}{\partial B} \int_0^1 F(B + x^i)di = -1 - c + \beta \frac{\partial}{\partial B} \int_0^1 F(B+x)h(x)dx \\
&= -1 - c + \beta \frac{\partial}{\partial B} \left[\int_0^{1-B} \rho_1(B+x)h(x)dx + \rho_1(1 - H(1-B)) + \int_{1-B}^1 \rho_2(B+x-1)h(x)dx \right] \\
&= -1 - c + \beta \left[\int_0^{1-B} \rho_1 h(x)dx + \int_{1-B}^1 \rho_2 h(x)dx \right] = -1 - c + \beta \left[(\rho_1 - \rho_2)\left(H(1-B)\right) + \rho_2 \right]
\end{aligned}$$

Even though the production function is not differentiable everywhere, W_1 is differentiable. This derivative is positive as long as:

$$-1-c+\beta\left[(\rho_1-\rho_2)\left(H(1-B)\right)+\rho_2\right]\geq 0 \Leftrightarrow H(1-B)\geq \frac{1+c-\beta\rho_2}{\beta(\rho_1-\rho_2)} \tag{22}$$

Using $\hat{x}=\frac{\beta\rho_1-1-c}{\beta(\rho_1-\rho_2)}$, I have:

$$H(1-B)\geq 1-\hat{x}$$

Proof of proposition 4

Each entrepreneur i has a private type x^i which is its own wealth. The government implements transfers $T_1(x^i)$ and $T_2(x^i)$.

Using the revelation principle, we have:

$$\begin{aligned}\max \int_0^1 \beta\left[f\left(x^i+T_1(x^i)\right)+T_2(x^i)\right]-(1+c)\left[ET_1(x^i)+T_2(x^i)\right]di\\ \forall x^i,\ \forall \tilde{x}^i,\ f\left(x^i+T_1(x^i)\right)+T_2(x^i)\geq f\left((x^i+T_1(\tilde{x}^i)\right)+T_2(\tilde{x}^i)\\ f\left((x^i+T_1(x^i)\right)+T_2(x^i)\geq 0\end{aligned}$$

Derivates are such that:

$$\frac{\partial T_1}{\partial \tilde{x}^i}\rho_1\left(x^i+T_1\right)+\frac{\partial T_2}{\partial \tilde{x}^i}\leq 0 \text{ when } x^i+T_1\leq 1 \text{ and } \frac{\partial T_1}{\partial \tilde{x}^i}\rho_2\left(x^i+T_1\right)+\frac{\partial T_2}{\partial \tilde{x}^i}\geq 0 \text{ when } x^i+T_1\geq 1$$

One can check that: $T_1(\tilde{x}^i)=1-\tilde{x}^i$ and hence:

$$\frac{\partial T_2}{\partial \tilde{x}^i}\geq \rho_2\ \forall \tilde{x}^i\leq x^i\ ;\ \frac{\partial T_2}{\partial \tilde{x}^i}\leq \rho_1\ \forall \tilde{x}^i\geq x^i$$

In particular: $T_2(\tilde{x}^i)=-R(1-\tilde{x}^i)+T_2(0)$ satisfies the IC constraints when $R\in(\rho_2,\rho_1)$, without taking into account the pledgeability constraint. This pledgeability constraint requires that: $T_2\geq 0$. Maximization yields $R=\rho_2$. Then we have to compare as in equation 9

$$\begin{aligned}\max_B\left\{\beta\int_0^1\left[f(x^i+B)\right]di-B(1+c)\right\}-\int_0^1\left[\beta\rho_1-(1+c)(1-x^i)-(1+c-\beta)\rho_2x^i\right]di\\ \max_B\left\{\beta\int_0^1\left[f(x^i+B)-\rho_1\right]di-(1+c)\left(B-\int_0^1\left[1-x^i+\frac{(1+c-\beta)}{1+c}\rho_2x^i\right]di\right)\right\}\end{aligned}$$

As, for $B=1$, we have:

$$\left(\beta\rho_2+(\frac{1+c-\beta}{1+c}R-1)(1+c)\right)\int_0^1 x^i di\geq 0 \tag{23}$$

$$\Leftrightarrow (1+c-\beta)\rho_2\geq 1+c-\beta\rho_2 \tag{24}$$

For $B=0$, the condition is as follows:

$$\int_0^1\left[x^i\right]di\geq\frac{\beta\rho_1-1-c}{\beta(\rho_1-\rho_2)+(1+c)(\rho_2-1)}$$

Randomization with T_1 and T_2 The government can try to decrease the cost of bailouts by randomizing T_1. First remark that the support of T_1 is a subset of $[0, 1-\tilde{x}^i]$. Indeed, as $\tilde{x}^i \leq x^i$, over $1-\tilde{x}^i$ the marginal return for the government is negative almost surely.

Without loss of generality, I can consider only distributions where the government $T_1 = 1 - x^i$ with some probability and 0 otherwise, using the piecewise linearity of the production function. Consequently, $T_2(x^i) = p\rho_2 x^i$.

$$\int_0^1 \beta \left[p\rho_1 + (1-p)\rho_1 x^i + p\rho_2 x^i\right] - (1+c)\left[p(1-x^i) + p\rho_2 x^i\right] di$$

The derivative with respect to p is positive as long as:

$$\int_0^1 \beta \left[\rho_1(1-x^i) + \rho_2 x^i\right] - (1+c)\left[(1-x^i) + \rho_2 x^i\right] di \geq 0$$

$$\frac{\beta\rho_1 - (1+c)}{\beta(\rho_1-\rho_2) - (1+c)(1-\rho_2)} \geq \int_0^1 x^i di$$

As a consequence, for $\frac{\beta\rho_1-(1+c)}{\beta(\rho_1-\rho_2)-(1+c)(1-\rho_2)} \geq \int_0^1 x^i di$, the probability is such that $p = 1$. In that case, the results of the last paragraph hold. Otherwise, $p = 0$: there are no transfers at all.

Proof of proposition 6

- β: $\frac{\partial(W_1-W_0)}{\partial\beta} = \int_0^1 F(z^i + x^i) - F(B + x^i)di$: the sign is ambiguous and depends on the distribution.
- c: $\frac{\partial(W_1-W_0)}{\partial c} = B \geq 0$.

Thresholds Consider an equilibrium i where $\pi = 1$. There exists then a distribution of repayments to domestic entrepreneurs, but these repayments do not exceed 1 as $p = 1$. Then for $Z > 1$, W_1 is a decreasing function of Z on the one hand and W_0 does not depend on Z. There exists then a value Z_i such that $W_0 > W_1$ for $Z > Z_i$ but $W_0 \leq W_1$ for $ZleqZ_i$.

Defining $overlineZ = sup_{\{i\}} Z_i$, we have that for $Z \leq overlineZ$, there exists an equilibrium where $\pi = 1$.

Proof of proposition 9

The maximization of domestic entrepreneurs payoff's leads to Figure 5 which plots domestic entrepreneurs' demand function (see appendix for the derivation and the details of the computation. p_1, p_2, p_3 and p_4 are thresholds derived in the appendix.), for arbitrary values of π and B.

Red dotted lines indicate that entrepreneurs have to compare their payoffs with the two remaining portfolios: either $P((1-B)/(1-\kappa))$ and $P(1)$ or $P(0)$ and $P(1)$. Arrows denote the sign of the derivate with respect to x^i in the region delimited by small and black dashed lines.

As in the baseline model, $\pi = 0$ is an equilibrium. When investors anticipate that $\pi = 0$, they will systematically pay the tax to invest abroad, whatever its value ($\kappa \in (0, 1]$): a tax does not allow to avoid default equilibria.

$\pi = 1$ can be sustained as an equilibrium. A necessary and sufficient condition is that $p_1^S(\pi = 1) \geq \gamma(1-\kappa)$. Such a condition can be fulfilled as long as κ is such that:

$$p_1^S(\pi = 1) \geq \gamma(1-\kappa) \Leftrightarrow (1-\kappa)\left((1-\gamma)\frac{\rho_1}{\rho_2}(1-\kappa) + \gamma\left(1 - \kappa\frac{\rho_1}{\rho_2}\right)\right) \geq 1 \tag{25}$$

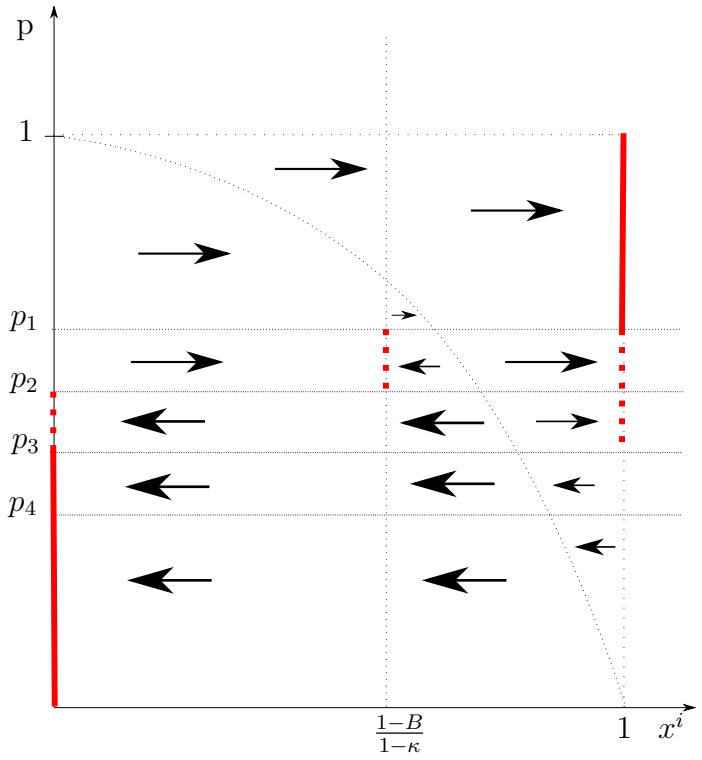

Figure 5: Domestic entrepreneurs' demand for foreign bonds

This last inequality has a solution for $\kappa < (1-\gamma) + \gamma\frac{\rho_2}{\rho_1} < 1$ (there is also a solution for $\kappa > 1$).

Thus, there exists capital controls such that $\pi = 1$ can be sustained as an equilibrium. Now, the price of bonds is $\gamma(1-\kappa)$.

Additionally, mixed equilibria exist as well.

Proof of proposition 8

For any distribution of portfolios h, π is a fixed point:

$$\pi^* = \arg\max_{\pi}\max_{B} \pi W_1\left(Z\left[1+\frac{1-\gamma}{\pi^*\gamma}\right]\right) + (1-\pi)W_0(B)$$

Let $Z_h(\pi)$ be a function as in lemma 18. If $Z\left[1+\frac{1-\gamma}{\gamma}\right] \leq Z_h(1)$, then $\pi = 1$ is still an equilibrium. Otherwise, there exists π such that $Z\left[1+\frac{1-\gamma}{\gamma}\right] = Z_h(\pi)$. We have to change the value of repayment and then π is the solution of the following equation:

$$Z\left[1+\frac{1-\gamma}{\pi\gamma}\right] = Z_h(\pi)$$

As $Z_h(\pi)$ is of the form: $Z_h(\pi) = \frac{\zeta_1}{\pi} + \zeta_2$ and $\zeta_2 > 0$, the only solution of the equation is $\pi = 0$.

Proof of proposition 12

I begin with the following lemma:

Lemma 19 (Optimal bailout)**.** *Bailouts in case of partial default are such that:*

$$B = \max\{b, H(1-\frac{b}{1-\frac{\lambda}{\lambda^e}}) \geq 1-\hat{x}\} \text{ for } \lambda < \lambda^e \text{ and } B = 0 \text{ for } \lambda \geq \lambda^e$$

Proof.

$$\frac{\partial W(\lambda)}{\partial B} = -1 - c + \beta \frac{\partial}{\partial B} \int_0^1 F\left(\lambda \frac{1-x^i}{\lambda^e} + B + x^i\right) di$$

Remark that:

$$\lambda \frac{1-x^i}{\lambda^e} + B + x^i \leq 1 \Leftrightarrow \left(\frac{\lambda}{\lambda^e} - 1\right)(1-x^i) \leq -B \Leftrightarrow$$

$$\text{either for } \lambda > \lambda^e,\ (1-x^i) \leq -\frac{B}{\frac{\lambda}{\lambda^e} - 1} \text{ or for } \lambda < \lambda^e,\ (1-x^i) \geq -\frac{B}{\frac{\lambda}{\lambda^e} - 1}$$

The first inequality is violated and thus $B = 0$ in that case. In the other case, using the same computation as in proposition 3. □

Notice that B is a continuous function of λ.

The derivative of welfare with respect to the fraction of default is:

$$\frac{\partial W}{\partial \lambda} = -Z + \beta \frac{\partial}{\partial \lambda} \int_0^1 F(\lambda \frac{1-x}{\lambda^e} + B + x) h(x) dx$$

We can notice that:

$$\lambda \frac{1-x}{\lambda^e} + B + x > 1 \Leftrightarrow B > \left(1 - \frac{\lambda}{\lambda^e}\right)(1-x)$$

For $\lambda^e \leq \lambda$, this is always satisfied. Then:

$$\frac{\partial W}{\partial \lambda} = -Z + \beta \frac{\partial}{\partial \lambda} \int_0^1 \left[\rho_2 \left(\lambda \frac{1-x}{\lambda^e} + x - 1\right) + \rho_1\right] h(x) dx = -Z + \beta \int_0^1 \left[\rho_2 \frac{1-x}{\lambda^e}\right] h(x) dx$$

For $\lambda^e \geq \lambda$,

$$\frac{\partial W}{\partial \lambda} = -Z + \beta \frac{\partial B}{\partial \lambda} + \beta \int_0^{\min(1 - \frac{B}{1-\frac{\lambda}{\lambda^e}}, 1)} \rho_1 \frac{1-x}{\lambda^e} h(x) dx + \beta \int_{\min(1 - \frac{B}{1-\frac{\lambda}{\lambda^e}}, 1)}^1 \rho_2 \frac{1-x}{\lambda^e} h(x) dx$$

As B is such that:

$$B = \max\{b, H\left(1 - \frac{B}{1 - \frac{\lambda}{\pi^e}}\right) \geq 1 - \hat{x}\}$$

we have easily that:

$$\frac{\partial B}{\partial \lambda} \geq 0 \text{ and } \frac{\partial^2 B}{\partial \lambda^2} \leq 0$$

Proof of proposition 13

The following lemma describes which equilibria are selected.

Lemma 20.

- *For $Z < Z^{max}$, $p = 1$.*
- *Similarly, for some values of the parameter, $p = \pi = 1$ for $Z_{max} < Z < Z^M$.*
- *For $Z > Z^M$, $p = (x^B)^{-1}(x)$ $p = \beta\rho_2 \frac{x-1}{1+c-\beta\rho_2 - Z}$*

This defines a function: $pZ = \beta\rho_2 Z \frac{x-1}{1+c-\beta\rho_2 - Z}$ for $Z > Z^M$ and $pZ = Z$ for $Z \leq Z^M$.

The derivate for $Z \leq Z^M$ is strictly positive and the derivate for $Z > Z^M$:

$$\frac{\partial pZ}{\partial Z} = \beta\rho_2 \frac{x-1}{1+c-\beta\rho_2 - Z} + \beta\rho_2 Z \frac{x-1}{(1+c-\beta\rho_2 - Z)^2} = 0$$

$$\beta\rho_2 \frac{x-1}{1+c-\beta\rho_2 - Z}(1 + \frac{Z}{(1+c-\beta\rho_2 - Z)} = 0 \Leftrightarrow -1 - c + \beta\rho_2 < 0$$

Then $Z = Z^M = 1 + c$.