Solving the Country Risk-Business Cycle Disconnect: Endogenous Output Dynamics in a Model of Sovereign Default

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Abstract

Quantitative models of business cycles in emerging economies explain key co-movements between country risk and economic activity *if* observed country risk is introduced as an exogenous interest rate on working capital. Paradoxically, models of strategic default approximate observed debt ratios and sovereign spreads *if* default is accompanied by an exogenous output collapse with particular features. We aim to resolve this country risk-business cycle disconnect by providing a joint explanation of debt ratios, country risk premia, and output dynamics. We show that the quantitative predictions of a model of sovereign default with endogenous output fluctuations are consistent with observed debt ratios, the cyclical correlations of sovereign spreads, and V-shaped output dynamics. Two features of the model are crucial for these results: First, working capital is used to pay for imported inputs, which allow firms to operate more efficiently than available domestic substitutes. Second, the payoffs of repayment and default are affected by endogenous feedback between country risk and output via the financing cost of working capital.

VERY PRELIMINARY

1 Introduction

Three key empirical regularities characterize the relationship between sovereign debt and economic activity in emerging economies:

(1) *Output displays V-shaped dynamics around default episodes.* Defaults are associated with deep recession troughs and followed by recoveries. Arellano (2006) shows that on the quarter in which default occurred, the deviations from trend in GDP were -14 percent in Argentina, -13 percent in Russia and -7 percent in Ecuador. Moreover, Levy-Yeyati and Panizza (2006) show that the recessions associated with defaults tend to begin prior to the defaults and that they generally hit bottom precisely when the defaults take place.

(2) *Interest rates on sovereign debt and domestic output are negatively correlated.* Neumeyer and Perri (2005) report that the cyclical correlations between these interest rates and GDP range from -0.38 to -0.7 in five emerging economies (Argentina, Brazil, Korea, Mexico and the Philippines), with an average correlation of -0.55. Uribe and Yue (2006) report correlations of -0.67 for Argentina, -0.51 for Brazil, -0.8 for Ecuador, -0.58 for Mexico, -0.37 for Peru, and around zero for the Philippines and South Africa.

(3) *External debt as a share of GDP is high on average, and particularly high when countries default.* Foreign debt was about a third of GDP on average over the 1998-2005 period for the entire group of emerging and developing countries as defined in IMF (2006). Within this group, the highly indebted poor countries had the highest average debt ratio at about 100 percent of GDP, followed by the Eastern European and Western Hemisphere countries, with averages of about 50 and 40 percent of GDP respectively. Regarding debt ratios when countries default, Reinhart et al. (2003) report that the external debt ratio during default episodes averaged 71 percent of GDP for all developing countries that defaulted at least once in the 1824-1999 period. The well-known default episodes of recent years are in line with this estimate: Argentina defaulted in 2001 with a 64 percent debt ratio, and Ecuador and Russia defaulted in 1998 with debt ratios of 85 and 66 percent of GDP respectively.

These empirical regularities have proven difficult to explain. On one hand, quantitative business cycle models can account for the negative correlation between country interest rates and output *if* the interest rate on sovereign debt is introduced as the exogenous interest rate faced by a small open economy in which firms require working capital to pay the wages bill.[1] On the other hand, quantitative models of sovereign default based on the classic setup of Eaton and Gersovitz (1981) can generate endogenous countercyclical sovereign spreads *if* the country faces exogenous stochastic shocks to

[1] See Neumeyer and Perri (2005), Uribe and Yue (2006) and Oviedo (2005).

GDP.[2] Moreover, these models require exogenous output costs with special features in order to support non-trivial levels of debt, and even then they produce mean debt ratios under 10 percent of GDP.[3] Thus, there is a crucial disconnect between business cycle models and sovereign default models: business cycle models lack an explanation of the default risk premia that drive their findings, while default models lack an explanation of the business cycle dynamics that are critical for their results.

The country risk-business cycle disconnect raises three important questions: Would a business cycle model in which country risk is endogenized still be able to explain the stylized facts that models with exogenous country risk have explained? Can a model of sovereign default with endogenous output dynamics produce the large output declines needed to support high ratios of defaultable debt as an equilibrium outcome? Would a model that endogenizes both country risk and output dynamics be able to mimic the V-shaped dynamics of output associated with defaults, and the countercyclical behavior of country risk?

This paper provides answers to these questions by studying the quantitative implications of a model of sovereign default with endogenous output fluctuations. The model borrows from the sovereign default literature a workhorse recursive formulation of strategic default in which a sovereign borrower makes optimal default choices by comparing the payoffs of repayment and default. In addition, the model borrows from the business cycle literature a transmission mechanism that links country risk with economic activity via the financing cost of working capital. We aim to make significant extensions to the two classes of models, because in our setup the equilibrium dynamics of output and country risk premia are determined jointly, and they influence each other via the interaction between foreign lenders, the domestic sovereign borrower, and domestic firms.

In our model, firms borrow from abroad to finance working capital, and all domestic agents (i.e. firms and government) default on their foreign obligations at the same time. Hence, the interest rates on working capital and on sovereign debt are endogenous and equal to each other at equilibrium. This treatment of the financing cost

[2] See Aguiar and Gopinath (2006), Arellano (2006), Yue (2006), and Bai and Zhang (2005).

[3] Aguiar and Gopinath (2006) obtain a *maximum* debt-GDP ratio of 0.27 assuming a cost of 2 percent of output per quarter for as long as the economy remains in financial autarky. Yue (2006) also uses the same output cost and obtains a *mean* debt ratio of 0.097 in a model with renegotiation, where at equilibrium countries remain in default only for one period. Arellano (2007) obtains a mean debt ratio of 0.06 assuming a kinked output cost such that income is the maximum of actual output or 0.97 of average output while the economy is in financial autarky.

of working capital differs from the treatment in Neumeyer and Perri (2005) and Uribe and Yue (2006), both of which treat the interest rate on working capital as an exogenous variable set to match the observed interest rate on sovereign debt. In contrast, in our setup both interest rates are endogenous and the two are driven by sovereign default risk. In addition, in the Neumeyer-Perri and Uribe-Yue models, working capital loans are used to pay the wages bill in full, while in our model firms use working capital to pay only for a fraction of imported intermediate goods. This lower working capital requirement is desirable because, using standard labor shares, working capital loans would need to be about 2/3rds of GDP to cover the wages bill, and this is difficult to reconcile with observed ratios of total credit to the private sector as a share of output in emerging economies, which are around 50 percent (including not just short-term revolving loans to firms, but all credit to households and firms at all maturities).

Imported intermediate inputs play a special role in our setup. In particular, we assume that they are "superior" to available domestic substitutes in the sense that they allow firms to operate more efficiently (i.e. they support a higher level of total factor productivity, TFP). On the other hand, the financing cost of the domestic substitutes is lower because they do not require working capital financing. As a result, firms trade off the benefit of higher TFP against the higher financing cost of imported inputs when making their production plans.

Firms choose which intermediate goods (foreign or domestic) to use in order to maximize profits. This choice depends crucially on the country interest rate because it drives the financing cost of working capital. When the country has access to world financial markets, firms choose domestic intermediate goods only if TFP is low enough and/or the country interest rate is high enough for the gain from using superior imported inputs to be outweighed by the higher financial costs. Otherwise, if TFP is high enough and/or the country interest rate is low enough, firms choose imported inputs, and in this situation equilibrium fluctuations in country risk induce fluctuations in factor demands and output, as well as in consumption.

When the economy defaults, both the government and firms are excluded from world credit markets for some time, with an exogenous probability of re-entry as is common in the recent quantitative models of default. Since the probability of default depends on whether the country's value of default is higher than that of repayment, there is feedback between the economic fluctuations induced by changes in interest rate premia, default probabilities, and endogenous country risk. In particular, rising country risk in the periods leading to a default is associated with declining economic activity as the firms' financing cost increases.

An important innovation of the model is that the output cost of default is an equilibrium outcome that follows the asymmetric pattern that Arellano (2007) found critical for supporting non-trivial levels of debt in Eaton-Gersovitz models of sovereign default. In particular, she showed that, beyond a threshold level of output below which default does not entail an output cost, the fraction of output loss at default needs to be an increasing function of the output realization. To capture this idea, she modeled the level of output at default as equal to the maximum of "actual" output or a fraction λ of average output. Hence, if actual output exceeds λ of mean output, default lowers output to its mean level, making the output cost of default larger in "better" states of nature because actual output is higher in "better" states but mean output is unchanged. On the other hand, if actual output is less or equal than λ of mean output, there is no output loss at default.

The output cost of default follows this asymmetric pattern in our model because of the optimal shift from imported inputs to inferior domestic substitutes that occurs at high interest rates and/or low TFP. Since effectively the interest rate on working capital rises to infinity at a default, firms always operate using inferior domestic inputs when the country is in financial autarky. Before default, however, firms may have already shifted to use domestic inputs if the interest rate was high enough and/or TFP was low enough. Conversely, if the interest rate was low enough and/or TFP was high enough, firms operated with imported inputs before default, and in this case they adjust abruptly to a default by shifting to inferior domestic substitutes. Thus, the fraction of output loss caused by a default is higher when the country defaults in a "better" state of nature, while below a threshold "bad" state with sufficiently high interest rates and/or low TFP, there is no output loss because firms were already operating with domestic inputs.

This asymmetric output cost of default implies that the option to default brings in more "state contingency" into the asset market. In particular, it increases the cost of default and allows the model to produce equilibria that support significantly higher mean debt ratios than those obtained in existing models of sovereign default.

The assumptions that purchases of imported inputs require working capital loans and that TFP is higher with imported inputs than with domestic substitutes play a key role in our analysis. If exclusion from world credit markets implies that firms cannot buy foreign inputs and there are no domestic substitutes available, the output collapse and the associated cost of default would be unrealistically large (infinitely large if 100 percent of the cost of imported inputs requires payment in advance). In reality, firms in emerging economies facing financial crisis substitute foreign inputs for those that can be employed at permissible financial terms, and/or look for alternative forms of credit, including inter-enterprise credit and internal financing using retained earnings or redirecting resources from capital or development expenditures. Our assumptions that firms can use domestic or imported intermediate goods, with the latter supporting higher TFP but requiring advanced payment for at least a fraction of the total cost, capture these features in a setup tractable for the recursive analysis of strategic default.

The results of our quantitative analysis show that the model can make progress in explaining the three key empirical regularities of sovereign debt mentioned earlier. In particular, the model supports high debt-GDP ratios, mimics the V-shaped pattern of output dynamics around defaults with large recessions that hit bottom during defaults, and yields countercyclical interest rates on sovereign debt. These results are obtained requiring only a small fraction of firms' factor costs to be paid with working capital (only 10 percent of the cost of imported inputs), and with a modest TFP differential between foreign and domestic inputs (TFP is 7 percent higher with imported inputs).

The rest of the paper proceeds as follows: Section 2 presents the model. Section 3 characterizes the recursive equilibrium of the model and examines the output costs of default. Section 3 conducts the quantitative analysis for a benchmark model calibration and examines the results of sensitivity analysis. Section 4 concludes.

2 A Model of Sovereign Default and Business Cycles

2.1 Overview

We study sovereign default and business cycles in a dynamic model of a small open economy. There are three agents in the economy: households, firms, and government. The economy takes the world's risk free interest rate as given. Households derive utility from consumption and disutility from providing labor services. The government cares about households and makes lump sum transfers to households. It takes as given the private sector's decision rules from a competitive equilibrium, and chooses an optimal debt policy (amounts and default) so as to maximize the households' welfare.[4]

Firms make profit-maximizing production plans. They produce a final consumption good using labor, intermediate goods and a time-invariant stock of capital $\overline{k}$. Production is subject to Markov TFP shocks ε_t, with transition probability distribution function $\mu(\varepsilon_t|\varepsilon_{t-1})$. Intermediate goods can be imported from abroad or acquired in a domestic market. A fraction θ of the purchases of imported intermediate goods requires advance payment, so firms need to obtain working capital loans κ to buy imported inputs. These are intraperiod loans that firms repay at the end of period. Domestic inputs do not require advance payment, but imported inputs are allow firms to operate at a higher TFP level, $\varepsilon^* > \varepsilon_d$, where ε^* (ε_d) is TFP with imported (domestic) inputs, and $\varepsilon_d = \lambda\varepsilon^*$, with $\lambda < 1$.[5] Alternatively, following Finn (1995), we could assume that imported inputs allow higher capacity utilization rates of capital and labor per unit of intermediate good, and introduce endogenous capacity utilization to account for the higher TFP of foreign inputs.

International creditors are risk-neutral and have perfect information about the country's output and asset position. They behave competitively and face an opportunity cost of funds equal to a constant world risk-free interest rate r^*. The government and these creditors trade one-period zero-coupon bonds, so markets of contingent claims are incomplete. The face value of a discount bond is denoted as b', specifying the amount to be repaid next period. When the country purchases bonds $b' > 0$, and when it borrows $b' < 0$. The set of bond face values is $B = [b_{\min}, b_{\max}] \subset R$, where $b_{\min} \leq 0 \leq b_{\max}$. We set the lower bound $b_{\min} < -\frac{\overline{y}}{r}$, which is the largest debt that the country could repay with full commitment. The upper bound $b_{\max}$ is the highest level of assets that the country may accumulate.[6] Let $q(b', \varepsilon)$ be the price of a bond with face value b' issued by the country agent with a productivity shock ε. This bond pricing function will be determined in equilibrium.

We assume that the sovereign country cannot commit to repay its debt. As

[4]Cuadra and Sapriza (2007) analyze optimal fiscal policy in the presence of sovereign default in an endowment economy. Our model also studies a Ramsey government's optimal default and debt policy. Yet sovereign default has feedback effect on the business cycle dynamics.

[5]We could also have ε_d as a second stochastic process, but we keep the number of state varaibles small by making the simplifying assumption that ε_d and ε have a linear relationship.

[6]$b_{\max}$ exists when the interest rates on a country's saving are sufficiently small compared to the discount factor, which is satisfied in our paper since $(1 + r^*)\beta < 1$.

is typical in the Eaton-Gersovitz class of models of sovereign debt, we assume that if the country defaults then it does not repay at date t, but the punishment is exclusion from the world credit market that same period.[7] The defaulting country can re-enter the financial markets with an exogenous probability η, and when it does it starts with a fresh record and zero debt. Also as in the Eaton-Gersovitz setup, the country cannot hold positive international assets during the exclusion period, otherwise the model cannot support equilibria with debt.

One key feature that we add to the Eaton-Gersovitz default environment is that in our model the defaulting sovereign can also divert the repayment of the firms' working capital loan to foreign lenders. Hence, effectively both firms and government default on their foreign obligations together. This assumption is made for tractability, and while we acknowledge that it is perhaps an extreme scenario, we will provide empirical evidence of a strong link between the interest rates faced by firms and the interest rates on sovereign debt. Moreover, in some actual default episodes, as was the case in Mexico's 1982-83 default, the government did take over the foreign loan obligations that the corporate sector could not meet. The following quote from a 2001 book by the IMF historian summarizes the Mexican arrangement and notes that arrangements of this type have been commonly used since then: *"A simmering concern among Mexico's commercial bank creditors was the handling of private sector debts, a substantial portion of which was in arrears...the banks and some official agencies had pressured the Mexican government to assume these debts...Known as the FICORCA scheme, this program provided for firms to pay dollar-denominated commercial debts in pesos to the central bank. The creditor was required to reschedule the debts over several years, and the central bank would then guarantee to pay the creditor in dollars. Between March and November 1983, close to $12 billion in private sector debts were rescheduled under this program... FICORCA then became the prototype for similar schemes elsewhere*" (Boughton (2001), Ch. 9, pp. 360-361)

[7] In Yue (2006), a defaulting country would make partial debt repayment based on the debt renegotiation agreement with the foreign lender. The length of financial exclusion is endogenous, depending on the amount of reduced debt. In this paper we focus on the linkage between default and output dynamics and thus abstract from debt renegotiation.

2.2 Households' Problem

Households' preferences are given by a standard time-separable utility function $E\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t - h(L_t)\right)\right]$, where $0 < \beta < 1$ is the discount factor, and c_t and l_t denote consumption and labor supply in period t respectively. $u(\cdot)$ is the period utility function, which is continuous, strictly increasing, strictly concave, and satisfies the Inada conditions. Following Greenwood, Hercowitz and Huffman (1988), we remove the wealth effect on labor supply by specifying period utility as a function of consumption net of the disutility of labor $h(L_t)$, which is increasing, continuously differentiable and convex. This formulation of preferences has been shown to play an important role in allowing international real business cycle models to approximate well observed business cycle facts, and it also simplifies the supply-side of the model by removing intertemporal considerations from the labor supply choice.

Households choose labor supply and consumption, taking as given the wage rate, profits paid by firms and government transfers. Households do not borrow directly from the rest of the world, but they will still be able to smooth consumption because the benevolent government will borrow, pay transfers, and make default decisions internalizing their utility function. This assumption is made for simplicity, so that the decentralized optimization problem of the households in the competitive equilibrium reduces to the following static problem:

$$\max_{c_t, L_t} E\left[\sum \beta^t u(c_t - h(L_t)\right] \tag{1}$$

$$s.t. \quad c_t = w_t L_t + \pi_t + T_t \tag{2}$$

Here T represents government transfers, w is the wage rate, and π are profits.

The period utility function takes the standard constant-relative-risk-aversion form $u\left(c, L\right) = \frac{\left(c - \psi \frac{L^\omega}{\omega}\right)^{1-\sigma} - 1}{1-\sigma}$ with $\omega > 1$ and $\sigma, \psi > 0$, and the labor disutility function $\psi \frac{L^\omega}{\omega}$ is defined in isoelastic form. The optimality condition for labor supply is:

$$\psi \left(L_t\right)^{\omega - 1} = w_t \tag{3}$$

The wage elasticity of labor supply is therefore given by $1/(\omega - 1)$.

2.3 Firms' Problem

Firms operate a Cobb-Douglas production technology for producing gross output, $f(m, L_f, k) = m^{\alpha_m} L_f^{\alpha_l} k^{\alpha_k}$, with factor shares α_l, α_m, and α_k for labor, intermediate goods, and capital respectively, with $0 < \alpha_l, \alpha_m, \alpha_k < 1$ and $\alpha_l + \alpha_m + \alpha_k = 1$. The firms' gross output when using imported inputs $m^*$is:

$$y^* = \varepsilon^* f(m^*, L_f, k) \tag{4}$$

Imported inputs are sold in a competitive world market at the exogenous relative price p_m^*.[8] A fraction θ of the cost of these imported inputs needs to be paid in advance, so firms require working capital financing κ_t . These working capital loans are within-period loans offered by foreign creditors at the interest rate r. This interest rate is linked to the sovereign interest rate at equilibrium, as shown in the next section. Working capital loans satisfy the standard payment-in-advance condition:

$$\frac{\kappa_t}{1 + r_t} \geq \theta p_m^* m^*$$

Profit-maximizing firms will choose working capital loans so that the above condition holds with equality. The decision rule on working capital loans is denoted by $\kappa_t = \kappa(r_t, \varepsilon_t,)$.

The firms' profits when producing with imported inputs can be written as:

$$\pi^* = \varepsilon^* m^{*\alpha_m} L_f^{\alpha_l} k^{\alpha_k} - p_m^*(1 + \theta r)m^* - wL_f$$

When firms opt for using domestic intermediate goods m the production technology and profits are given by:[9]

$$y^d = \varepsilon_d f(m, L_f, k) \tag{5}$$

$$\pi^d = \varepsilon_d m^{\alpha_m} L_f^{\alpha_l} k^{\alpha_k} - p_m m - wL_f$$

[8] We keep this price constant but it can also be modeled as an exogenous terms-of-trade shock with a given stochastic process.

[9] Note that foreign and domestic inputs are perfect substitutes except for the change in TFP, so firms only use one of them.We also explored a more general formulation with imperfect substitution between the two inputs.

where p_m is the price of domestic inputs to be determined within the economy at equilibrium. As explained earlier, domestic inputs do not require working capital financing but using them yields a lower level of TFP than using the superior foreign inputs. The assumption that domestic inputs do not require working capital is made for simplicity, the key element for the model is that at high levels of country risk (including the exclusion period) the financing cost of foreign inputs is higher than that of domestic inputs.

Consider now the optimization problem of domestic firms. They take the wage rate, the interest rate, and intermediate goods prices as given and choose whether to use domestic or imported intermediate goods *and* the optimal amount to buy in each case. This is equivalent to first evaluating the profit-maximizing plans under each alternative (domestic or imported inputs) and then choosing the one that yields higher profits:

$$\pi = \max \left[\max_{m^*, L_f} (\pi^*), \max_{m, L_f} (\pi^d) \right] \tag{6}$$

When imported intermediate goods are used, the optimality conditions are

$$\begin{aligned} \alpha_m \varepsilon^* m^{*\alpha_m - 1} L_f^{\alpha_l} k^{\alpha_k} &= p_m^* (1 + \theta r) && (7) \\ \alpha_l \varepsilon^* m^{*\alpha_m} L_f^{\alpha_l - 1} k^{\alpha_k} &= w && (8) \end{aligned}$$

Alternatively, if firms produce using the domestic intermediate good, the optimality conditions are:

$$\begin{aligned} \alpha_m \varepsilon_d m^{\alpha_m - 1} L_f^{\alpha_l} k^{\alpha_k} &= p_m && (9) \\ \alpha_l \varepsilon_d m^{\alpha_m} L_f^{\alpha_l - 1} k^{\alpha_k} &= w && (10) \end{aligned}$$

These two sets of optimality conditions are standard optimality conditions setting marginal products of labor and intermediate goods equal to the corresponding effective marginal costs (inclusive of the financing cost of working capital for imported inputs).

The economy is endowed with a fixed amount of domestic input $\overline{m}$. This endowment is owned by domestic agents who simply consume final goods they obtain in exchange from selling domestic inputs to final good producers at price p_m. Alternatively, the supply of domestic intermediate goods can be endogenized in the economy. Profit-maximizing domestic intermediate good producer produce domestic input. Production of domestic inputs is given by $m = AL_m^{\gamma}$, where L_m is labor used in production of domestic inputs. The case with fixed supply of domestic input is a special case with $\gamma = 0$.

2.4 Competitive Equilibrium of the Private Sector

A competitive equilibrium for the private sector of the economy is defined as follows:

Definition 1 *A competitive equilibrium for the private sector is given by sequences of allocations* $[c_t, T_t, L_t, L_{f,t}, m_t, \kappa_t]_{t=0}^{\infty}$ *and prices* $[w_t, \pi_t, p_t^m]_{t=0}^{\infty}$ *such that:*

1. The allocations $[c_t, L_t]_{t=0}^{\infty}$ *maximize households' utility given the price sequences* $[w_t, \pi_t, p_t^m]_{t=0}^{\infty}$ *and government transfers* $[T_t]_{t=0}^{\infty}$.

2. The allocations $[L_{f,t}, m_t, \kappa_t]_{t=0}^{\infty}$ *maximize firms profits given the price sequences* $[w_t, p_t^m]_{t=0}^{\infty}$*, the exogenous sequence of interest rates on working capital loans* $[r_t]_{t=0}^{\infty}$ *and the prices of imported inputs* p_m^**.* $[\pi_t]_{t=0}^{\infty}$ *is the maximized firm profits.*

3. The market-clearing condition of the domestic labor market holds, $L_{f,t} = L_t$*.*

One of the constraints on the problem of the sovereign borrower, who makes the default decisions, will be that the sovereign government must choose allocations that are a competitive equilibrium for the private sector. Since the sovereign government's problem and the equilibrium of the credit market will be characterized in recursive form, it is useful to also characterize the above competitive equilibrium in recursive form. To this end, we use the indicator function $\Phi(r, \varepsilon)$ to identify whether firms are using domestic or imported intermediate goods given the current state of interest rates and productivity. In particular, $\Phi(r, \varepsilon) = 0$ if $\pi = \max(\pi^*)$ and $\Phi(r, \varepsilon) = 1$ if $\pi = \max(\pi^d)$. Hence, firms use imported (domestic) inputs when $\Phi(r, \varepsilon) = 0$ ($\Phi(r, \varepsilon) = 1$).

At a competitive equilibrium, the market-clearing condition of the labor market equates the labor demand given in (8 or (10) and (**??**) with the labor supply condition (3). As a result, we can express the competitive equilibrium allocations of factor demands and working capital as functions of the current states of productivity and interest rates as follows:

$$\kappa(r,\varepsilon) = (1-\Phi(r,\varepsilon))\,\theta p_m^* m^*(r,\varepsilon^*) + \Phi(r,\varepsilon)\cdot 0 \quad (11)$$

$$m(r,\varepsilon) = (1-\Phi(r,\varepsilon))\,m^*(r,\varepsilon^*) + \Phi(r,\varepsilon)\overline{m} \quad (12)$$

$$L(r,\varepsilon) = (1-\Phi(r,\varepsilon))\,L_f^*(r,\varepsilon^*) + \Phi(r,\varepsilon)L_f^d(\varepsilon_d) \quad (13)$$

$$L_f(r,\varepsilon) = (1-\Phi(r,\varepsilon))\,L_f^*(r,\varepsilon^*) + \Phi(r,\varepsilon)L_f^d(\varepsilon_d) \quad (14)$$

where the corresponding optimal factor demands are:[10]

$$m^*(r,\varepsilon^*) = \left[\left(\frac{\alpha_l}{\psi}\right)^{\alpha_l}(\varepsilon^* k^{\alpha_k})^{\omega}\left(\frac{\alpha_m}{p_m^*(1+\theta r)}\right)^{\omega-\alpha_l}\right]^{\frac{1}{\omega(1-\alpha_m)-\alpha_l}} \quad (15)$$

$$L_f^*(r,\varepsilon^*) = \left[\left(\frac{\alpha_l}{\psi}\right)(\varepsilon^* k^{\alpha_k})^{\frac{1}{1-\alpha_m}}\left(\frac{\alpha_m}{p_m^*(1+\theta r)}\right)^{\frac{\alpha_m}{1-\alpha_m}}\right]^{\frac{1-\alpha_m}{\omega(1-\alpha_m)-\alpha_l}} \quad (16)$$

$$L_f^d(\varepsilon_d) = \left[\frac{\alpha_l \varepsilon_d k^{\alpha_k}}{\psi}\overline{m}^{\alpha_m}\right]^{\frac{1}{\omega-\alpha_l}} \quad (17)$$

Note also that given L_d, the equilibrium price of the domestic intermediate goods (when they are employed) is $p_m = \alpha_m \varepsilon_d \overline{m}^{\alpha_m-1}\left(L_f^d\right)^{\alpha_l} k^{\alpha_k}$.

It follows from the above solutions that production is not affected by interest rates when firms use domestic intermediate goods, because there is no working capital requirement in this case. In contrast, when firms use imported intermediate goods, their demand for imported inputs and labor *decreases* with country interest rates. Thus, in this situation, sovereign risk affects the firms' actions through the working capital friction. Because the interest rate on working capital loans is driven by the sovereign interest rate, firms face higher financing costs when default risk rises, and so factor demands and output fall. One special case of this situation is the state when default occurs, in which the country has no access to working capital because effectively r has gone to infinity. In this case, firms cannot import inputs and must use inferior domestic substitutes.

[10] Notice that, because of the Greenwood-Hercowitz-Huffman preferences, the marginal rate of substitution between labor and consumption in the left-hand-sides of (3) is independent of consumption, and hence labor supplies are independent of any intertemporal effects.

2.5 Country Risk and Private Interest Rates: Some Empirical Evidence

As mentioned earlier, we assume that when the sovereign defaults, the government appropriates the repayment that firms would have made to foreign lenders. We will show that at equilibrium, this will equate the interest rates on both types of debt, since risk neutral lenders will view them as having the same probability of default. This, however, raises a key empirical question: Are sovereign interest rates and the rates of interest faced by private firms closely related in emerging economies?

Empirical evidence suggests that indeed interest rates on loans to private firms and on sovereign bonds tend to move together. To study this issue, we constructed country estimates of firms' financing costs that represent aggregate measures derived from firm-level data. We measured firm-level interest rates as the ratio of total debt service divided by total debt obligations using the *Worldscope* database, which provides the main lines of balance-sheet and cash-flow statements of publicly listed corporations. We then constructed the corresponding country measure as the median or average across firms.

The comparison of this measure of interest rates faced by private firms with the standard EMBI+ measure of interest rates on sovereign debt shows two striking facts (see Table 1):[11] First, the two interest rates are highly correlated, second, the effective financing cost of firms is generally *higher* than the sovereign interest rates. In some countries the relationship between the two rates is very strong (see Figure 1). The fact that private interest rates exceed the sovereign interest rates indicates that the common conjecture that firms (particularly the large corporations covered in our data) may pay lower rates than governments with default risk is incorrect.

The studies by Arteta and Hale (2006) and (2007) provide further and more systematic evidence on the strong effects of sovereign debt on the terms of private-sector debt contracts of emerging economies. In particular, they show strong, systematic negative effects on private corporate bond issuance during and after default episodes.

Table 1: Sovereign Interest Rates and Firm Financing Cost

[11] The firm-level financing cost is measured as the ratio of total debt service payments divided by total debt obligations, using the Worldscope database for publicly listed corporations. We use country medians to measure the approximate firm financing cost in each country.

Country	Sovereign Interest Rates	Median Firm Interest Rates	Correlation
Argentina	13.32	10.66	0.87
Brazil	12.67	24.60	0.14
Chile	5.81	7.95	0.72
China	6.11	5.89	0.52
Colombia	9.48	19.27	0.86
Egypt	5.94	8.62	0.58
Malaysia	5.16	6.56	0.96
Mexico	9.40	11.84	0.74
Morocco	9.78	13.66	0.32
Pakistan	9.71	12.13	0.84
Peru	9.23	11.42	0.72
Philippines	8.78	9.27	0.34
Poland	7.10	24.27	0.62
Russia	15.69	11.86	-0.21
South Africa	5.34	15.19	0.68
Thailand	6.15	7.30	0.94
Turkey	9.80	29.26	0.88
Venezuela	14.05	19.64	0.16

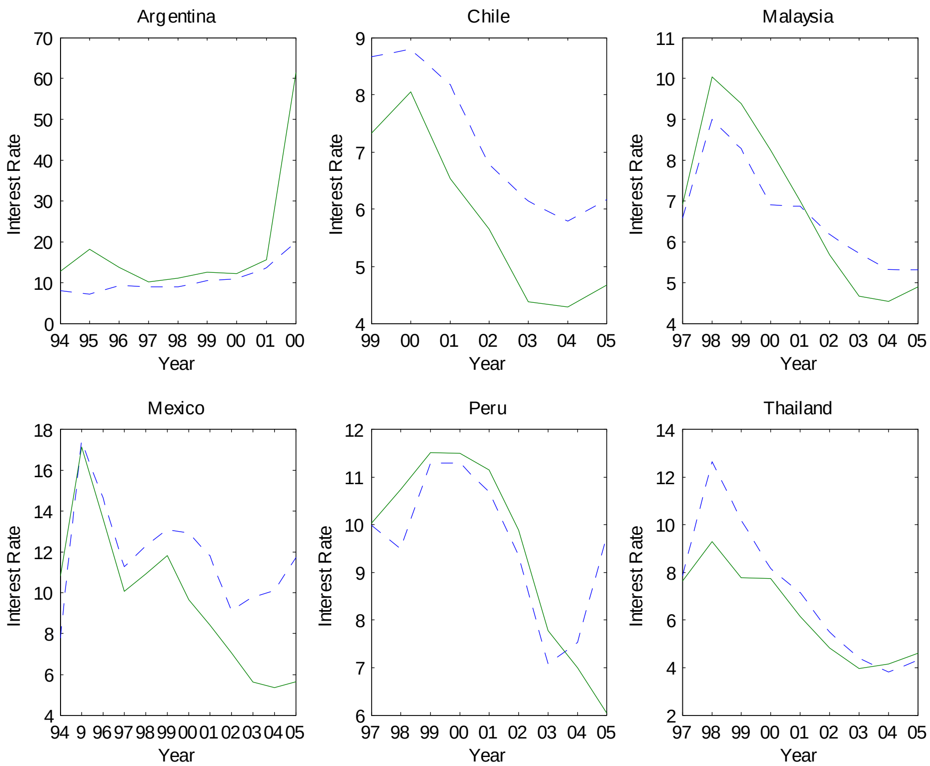


Figure 1: Sovereign Bond Interest Rates and Median Firm Financing Cost

——— Sovereign Bond Interest Rates - - - - Median Firm Financing Cost

2.6 The Sovereign Government's Problem

The sovereign government solves a Ramsey-like problem. It chooses a debt strategy that maximizes the households' welfare subject to the constraints that (a) the private sector allocations must be a competitive equilibrium; and (b) the government budget constraint must hold. The state variables are asset position, working capital last period, and productivity shock, denoted by the triplet $(b_t, \kappa_{t-1}, \varepsilon_t)$. The price of sovereign bonds is given by the bond pricing function $q_t(b_{t+1}, \varepsilon_t)$. Since at equilibrium the default risk premium in sovereign debt will be shown to be the same as in working capital loans, it follows that the interest rate on working capital can also be expressed as a function of $q_t(b_{t+1}, \varepsilon_t)$. Hence, the factor allocations and demand for working capital of the private sector's competitive equilibrium (eqs. (11)-(14)) can be rewritten as $\kappa(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, $m(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, and $L(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$.

The recursive optimization problem of the government is summarized in the value function $V(b, \kappa, \varepsilon)$, which is defined as follows:

$$V(b_t, \kappa_{t-1}, \varepsilon_t) = \begin{cases} \max\left\{v^{nd}(b_t, \varepsilon_t), v^d(\kappa_{t-1,}\varepsilon_t)\right\} & \text{for } b_t < 0 \\ v^{nd}(b_t, \varepsilon_t) & \text{for } b_t \geq 0 \end{cases} \tag{18}$$

If the country is borrowing, the value function is the maximum of the value of continuing in the credit relationship (i.e., repayment or "no default"), $v^{nd}(b_t, \varepsilon_t)$, and the value of default, $v^d(\kappa_{t-1,}\varepsilon_t)$. If the economy holds a non-negative net foreign asset position, the value function is simply the continuation value because in this case the economy is accessing the world credit market to save, receiving a return equal to the world's risk free rate.

The continuation value $v^{nd}(b_t, \varepsilon_t)$ is defined as follows:

$$v^{nd}(b_t, \varepsilon_t) = \max_{c_t, b_{t+1}} u(c_t - h(L(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t))) + \beta EV(b_{t+1}, \kappa_t, \varepsilon_{t+1}) \tag{19}$$

subject to

$$\begin{aligned} c_t + q_t(b_{t+1}, \varepsilon_t) b_{t+1} &\leq (1 - \alpha_m) \varepsilon_t f(m(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t), L(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t), k) + b_t \\ \kappa_t &= \kappa(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t) \end{aligned}$$

In this problem, the government faces a constraint that represents the resource constraint of the economy for allocations that are supported as a competitive equilibrium, just like in a Ramsey problem. This resource constraint is obtained by combining the households' budget constraint with the government budget constraint, $T_t = b_t - q_t(b_{t+1}, \varepsilon_t) b_{t+1}$. From the optimality conditions from the firms' problem, the total domestic factor payments, $w_t L_t + \pi_t$, equal the fraction $(1 - \alpha_m)$ of gross output $\varepsilon f(m, l, k)$. In this resource constraint, $m(\cdot)$ and $L(\cdot)$ are the competitive equilibrium allocations of intermediate goods and labor at the current state of productivity and interest rates, including the decision of the firms as to whether operate with imported or domestic inputs. One remark is that firms can choose the latter even when the economy has access to foreign working capital loans for imported inputs if the interest rate is "high enough" and/or the TFP differential between the two inputs is small enough.

The resource constraint captures three important features of the model: First, the government internalizes how interest rates affect output and factor demands. Second, the households cannot borrow from abroad, but the benevolent government simply transfers to them an amount equal to the negative of the balance of trade (i.e. it gives the private sector the flow of resources it needs to finance the gap between GDP and consumption). Third, the working capital loans κ_{t-1} and κ_t do not enter explicitly in the continuation value or in the resource constraint that applies to this case, because working capital payments are included in the fraction of gross output allocated to payments of intermediate goods, $\alpha_m f(m,l,k)$. Still, we need to keep track of the state variable κ_t because the amount of working capital loans taken by firms this period affects the sovereign's incentive to default next period, as explained below.

The value of default $v^d\left(\kappa_{t-1},\varepsilon_t\right)$ is:

$$v^d\left(\kappa_{t-1},\varepsilon_t\right)=\max_{c_t}\left[u\left(c_t-h(L(\varepsilon_d))\right)+\beta\left(1-\eta\right)Ev^d\left(0,\varepsilon_{t+1}\right)+\beta\eta EV\left(0,0,\varepsilon_{t+1}\right)\right] \tag{20}$$

subject to

$$c_t=(1-\alpha_m)\varepsilon_t^d f\left(m(\varepsilon_d),L_f(\varepsilon_d)\right)+\kappa_{t-1}$$

Note that $v^d\left(\kappa_{t-1},\varepsilon_t\right)$ takes into account the fact that in case of default at date $t,$ the country has no access to financial markets this period, and hence the country consumes the total income, given by the resource constraint in the default scenario. In this case, since firms cannot borrow to finance purchases of imported inputs, $m(\cdot)$, $L(\cdot)$ and $L_f\left(\cdot\right)$ are the competitive equilibrium allocations of intermediate goods and labor obtained when firms operate with domestic inputs, m^d and L_f^d given by (**??**) and (**??**) respectively. Moreover, because we assume that the defaulting government keeps the repayment of last period's working capital loan, the total income includes government transfers equal to the appropriated repayment for the amount κ_{t-1} (i.e., on the date of default, the government budget constraint is $T_t=\kappa_{t-1}$). The value of default at t also takes into account that at $t+1$ the economy may re-enter world capital markets with probability η and associated value $V\left(0,0,\varepsilon_{t+1}\right)$, or remain in financial autarky with probability $1-\eta$

and associated value $v^d(0, \varepsilon_{t+1})$.

For a debt position $b_t < 0$, given the level of working capital κ_{t-1}, default is optimal for the set of realizations of the TFP shock for which $v^d(\kappa_{t-1}, \varepsilon_t)$ is at least as high as $v^{nd}(b_t, \varepsilon_t)$:

$$D(b_t, \kappa_{t-1}) = \left\{ \varepsilon_t : v^{nd}(b_t, \varepsilon_t) \leq v^d(\kappa_{t-1}, \varepsilon_t) \right\} \tag{21}$$

This default set has a different specification than in the typical Eaton-Gersovitz model of sovereign default (as in Arellano (2007)) because we need to keep track of how the state of working capital affects the gap between the values of default and repayment. This results in a two-dimensional default set that depends on b_t and κ_{t-1}, instead of just b_t.

Despite the fact that the default set depends on κ_{t-1}, the probability of default itself remains a function of b_{t+1} and ε_t only. This is because the firms' optimality conditions imply that the next period's working capital loan κ_t depends on ε_t and the interest rate, which is a function of b_{t+1} and ε_t. Hence the default probability $p(b_{t+1}, \varepsilon_t)$ can be induced from the default set, the decision rule for working capital, and the transition probability function of productivity shocks $\mu(\varepsilon_{t+1}|\varepsilon_t)$ as follows:

$$\begin{aligned} p(b_{t+1}, \varepsilon_t) &= \int_{D(b_{t+1}, \kappa_t)} d\mu(\varepsilon_{t+1}|\varepsilon_t) \\ \text{where } \kappa_t &= \kappa(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t) \end{aligned} \tag{22}$$

The economy is considered to be in financial autarky when it has been in default for at least one period and has no access to world credit markets as of date t. As noted above, the economy exits this exclusion stage at date $t+1$ with probability η. We assume that during the exclusion stage the economy cannot build up its own stock of savings to supply working capital loans to firms, which could be used to purchase imported inputs.[12] This simplifying assumption ensures that, as long as the economy remains in financial autarky, the optimization problem of the sovereign is the same as the problem in the default period but evaluated at $\kappa_{t-1} = 0$ (i.e. $v^d(\varepsilon_t, 0)$).

We also studied an alternative setup in which we allowed for a domestic financial

[12] Alternatively, we could assume that the default punishment includes exclusion from world capital markets and from the world market of intermediate goods.

market to operate during the exclusion stage. In this case, households make saving plans to offer working capital loans to firms at a market-determined interest rate, and firms demand these loans if the endogenous domestic interest rate is low enough to make the superior TFP of the foreign inputs profitable despite their higher financing cost. In this case, we include domestic loans as a relevant state variable and endogenize the equilibrium determination of domestic interest rates. We found, however, that for parameter values around our baseline calibration this domestic financial market is not viable: The interest rate at which households would find it optimal to accumulate savings is too high for firms to optimally choose to obtain domestic working capital loans to purchase imported inputs. Hence, the equilibrium for the model with the domestic financial market operating during the exclusion stage is the same as that for the model that simply assumes that firms operate with domestic inputs whenever they are excluded from world credit markets.

Our model with endogenous output and working capital financing preserves a standard feature of the Eaton-Gersovitz class of sovereign default models: Given ε, the value of defaulting is independent of the level of debt, while the value of not defaulting increases with b', and consequently the default set grows with the country's debt and so does the equilibrium default probability.

Theorem 2 *Given a productivity shock ε and level of working capital loan κ, for $b^0 < b^1 \leq 0$, if default is optimal for b^1, then default is also optimal for b^0. That is $D\left(b^1, \kappa\right) \subseteq D\left(b^0, \kappa\right)$. The country agent's probability of default in equilibrium satisfies $p^*\left(b^0, \varepsilon\right) \geq p^*\left(b^1, \varepsilon\right)$. Hence the equilibrium bond price satisfies $q^*\left(b^0, \varepsilon\right) \leq q^*\left(b^1, \varepsilon\right)$*

Proof. See Appendix. ■

At equilibrium, bond prices depend on the risk of default. For a high level of debt, the default probability is higher. Therefore, equilibrium bond prices decrease with indebtedness. This result is consistent with the empirical evidence, for example, Edwards (1984).

2.7 The Foreign Lenders' Problem

Lenders take the bond price function as given and choose the amount of bonds they buy from the sovereign b' to maximize expected profits given by

$$U\left(b',\varepsilon\right)=\begin{cases} q\left(b',\varepsilon\right)b'-\frac{1}{1+r^{*}}b' & \text{if } b'\geq 0 \\ \frac{\left[1-p\left(b',\varepsilon\right)\right]}{1+r^{*}}\left(-b'\right)-q\left(b',\varepsilon\right)\left(-b'\right) & \text{if } b'<0 \end{cases} \tag{23}$$

where $p\left(b',\varepsilon\right)$ is the expected probability of default for a country with a productivity ε and indebtedness b'.

Because we assume that lenders behave competitively, the lenders' expected profits are zero in equilibrium. Using the zero expected profit condition, we get

$$q\left(b',\varepsilon\right)=\begin{cases} \frac{1}{1+r^{*}} & \text{if } b'\geq 0 \\ \frac{\left[1-p\left(b',\varepsilon\right)\right]}{1+r^{*}} & \text{if } b'<0 \end{cases} \tag{24}$$

When the country lends ($b'\geq 0$) the sovereign bond price is equal to the price of a risk-free bond $\frac{1}{1+r^{*}}$. When the country borrows ($b'<0$) there exists risk of default and the sovereign bond is priced to compensate the lenders for bearing default risk.

Foreign lenders also make working capital loans to firms when firms find it optimal to use imported intermediate goods. Because the sovereign government diverts the repayment of working capital loans when it defaults, the risk of default on private working capital loans is equal to the sovereign default risk. Therefore, the net interest rate on working capital loans is given by:

$$r\left(b',\varepsilon\right)=\frac{1}{q\left(b',\varepsilon\right)}-1 \tag{25}$$

3 Recursive Equilibrium and Output Costs of Default

3.1 Recursive equilibrium of the world credit market

The recursive equilibrium of the credit market is defined as follows:

Definition 3 *A recursive equilibrium for the model economy is given by (i) a decision rule $b'\left(b,\kappa,\varepsilon\right)$ for the sovereign government with the associated value function $V\left(b,\kappa,\varepsilon\right)$,* *consumption and transfers rules* $c(b,\kappa,\varepsilon)$ *and* $T(b,\kappa,\varepsilon)$, *default set* $D(b,\kappa)$ *and default probabilities* $p^*(b',\varepsilon)$*; and (ii) an equilibrium pricing function for sovereign bonds* $q^*(b',\varepsilon)$ *such that:*

1. Given $q^*(b',\varepsilon)$*, the decision rule* $b'(b,\kappa,\varepsilon)$ *solves the recursive maximization problem of the sovereign government (18).*

2. The consumption plan $c(b,\kappa,\varepsilon)$ *satisfies the resource constraint of the economy*

3. The transfers policy $T(b,\kappa,\varepsilon)$ *satisfies the government budget constraint.*

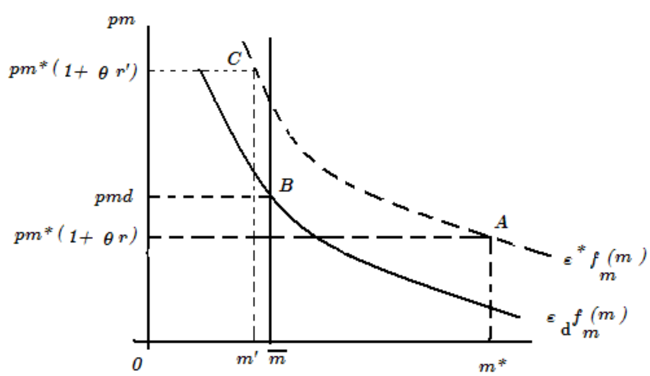
4. Given $D(b,\kappa)$ *and* $p^*(\varepsilon,b')$, *the bond price function* $q^*(b',\varepsilon)$ *satisfies the zero-expected-profits condition of foreign lenders (24).*

Condition (1) requires the sovereign government's default and saving/borrowing decisions to be optimal given the interest rates on sovereign debt. Condition (2) requires that the private consumption allocations implied by these optimal borrowing and default choices be both feasible and consistent with a competitive equilibrium (recall that the resource constraints of the sovereign's optimization problem consider only private-sector allocations that are competitive equilibria). Condition (3) requires that the decision rule for transfers shifts the appropriate amount of resources from the government to the private sector (i.e. an amount equivalent to net exports when the country has access to world credit markets, or the diverted repayment of working capital loans when a default occurs, or zero when the economy is in financial autarky beyond the date of default). Notice also that given (2) and (3), the consumption plan satisfies the households' budget constraint. Finally, condition (4) requires the equilibrium bond prices that determine country risk premia to be consistent with optimal lender behavior.

A solution for the above recursive equilibrium includes solutions for $m\left(q_t\left(b_{t+1},\varepsilon_t\right),\varepsilon_t\right)$, $L\left(q_t\left(b_{t+1},\varepsilon_t\right),\varepsilon_t\right)$, and $\kappa\left(q_t\left(b_{t+1},\varepsilon_t\right),\varepsilon_t\right)$; and a solution for equilibrium interest rates on working capital as a function of b_{t+1} and ε_t follows from (25). Expressions for equilibrium wages, profits and the price of domestic inputs as functions of r_t and ε_t follow then from the firms' optimality conditions and the definition of profits described earlier.

3.2 Endogenous Output Costs of Defaults

We now provide some intuition for how the mechanism that drives firms to shift between foreign and domestic inputs affects the dynamics of output and default. The decision to shift depends on the states of productivity and the interest rates on working capital. For simplicity, in the illustration below, we use a final good production function that depends only on m. The demand of intermediate goods is determined by the marginal product of m, which are given by the two downward sloping curves depending on which input is used. The imported intermediate goods are available at price p_m^*. The domestic intermediate input is in fixed supply at level $\overline{m}$. If the interest rate is sufficiently low, the firms' optimal plans call for using imported inputs up to the amount at which the marginal product of m equals the marginal cost $p_m^*(1+\theta r)$. This is point A in Figure 2, where for simplicity we used a production function that depends only on m.



Around point A, output fluctuates as a result of shocks to ε^* and changes in r driven by changes in the risk of default. Given that marginal products are decreasing and continuously differentiable, it follows that as the interest rate increases, the demand for imported inputs and the profits of the firms decline, until we reach a threshold value r' at which $\pi^d = \pi^*$. This is an interest rate that is high enough for firms to find it optimal to switch to the inferior domestic input, because $r \geq r'$ yields $\pi^d \geq \pi^*$. Since profits with the production function simplified to one input are just $(1-\alpha_m)\varepsilon m^{\alpha_m}$, it is easy to see that at the threshold interest rate the demand for imported inputs falls to $m' = \lambda^{1/\alpha_m}\overline{m}$, which is always lower than $\bar{m}$ because $0 < \lambda < 1$ (recall that $\varepsilon_d = \lambda\varepsilon^*$). The threshold point is shown as point C in Figure 2. Notice that, because of the higher TFP of imported inputs, when interest rates are high (but not yet at r') it can be optimal for firms to use quantities of imported inputs that are lower than $\overline{m}$. This is because they still make more profits with the foreign inputs than by switching to domestic inputs.

When the interest rate reaches r', firms switch to domestic inputs and the equilibrium price and quantity of intermediate goods are determined at point B in Figure 2. Around point B, fluctuations in output are driven by changes in ε_d,and output is no longer affected by the interest rate. This has two important implications. First, since in principle r' can be reached before the country defaults, high interest rates can trigger a switch to the inferior technology even before default occurs. Second, since r' is well defined and at default $r \to \infty$, firms will always use domestic inputs when the economy defaults.

Productivity shocks can also cause the switch from imported to domestic inputs, even if the interest rate on working capital remains constant. Consider the model's production function with three inputs. Again firms are indifferent between using imported or domestic intermediate goods when the maximal profits π^* and π^d are equal. When firms use imported inputs goods, the maximal profit is $\alpha_k \varepsilon^* k^{\alpha_k} \left(\frac{\alpha_l}{\psi}\right)^{\frac{\alpha_l}{\omega(1-\alpha_m)-\alpha_l}} \left(\varepsilon^* k^{\alpha_k}\right)^{\frac{\omega\gamma+\alpha_l}{\omega(1-\gamma)-\alpha_l}} \left(\frac{\alpha_m}{p_m^*(1+\theta r^*)}\right)^{\frac{\omega\alpha_m}{\omega(1-\alpha_m)-\alpha_l}}$. When the domestic substitutes are used, the maximal profit is $\alpha_k \varepsilon_d k^{\alpha_k} \left[\frac{\alpha_l \varepsilon_d k^{\alpha_k}}{\psi} \overline{m}^{\alpha_m}\right]^{\frac{\alpha_l}{\omega-\alpha_l}} \overline{m}^{\alpha_m}$. By equating these two expressions we can determine a threshold value of ε^* low enough for firms to find it optimal to switch to domestic inputs for given a value of r.[13] The reason is that a low ε^* lowers the marginal product of the imported inputs but firms still pay the extra marginal cost due to the working capital constraint. Hence, firms may choose to use the inferior domestic inputs and not pay the financing cost on working capital loans.

The switch from imported to domestic inputs that occurs at high interest rates has important implications for the output cost of default in our model. The findings of Aguiar and Gopinath (2006) and Arellano (2007) showed that the penalties associated with default play a crucial role in the quantitative performance of models of sovereign default in the Eaton-Gersovitz class. In contrast with the existing literature, which imposes exogenous output costs of various forms, our model generates an endogenous output loss when the country defaults. The fraction of output lost due to default is given by:

$$1 - \frac{Y^d}{\max\left(Y^*, Y^d\right)}$$

[13] Given values of ε_d and ε^*, we can also use the condition equating maximal profits to pin down the threshold interest rate at which firms switch from foreign to domestic intermediate goods in the case that production requires three inputs.

where Y^* and Y^d are given by $(1-\alpha_m)$ fraction of gross output (4) and (5) evaluated at the corresponding equilibrium allocations of labor and intermediate goods.[14]

The left panel in Figure 3 shows how Y^*, Y^d and the output loss at default change with TFP shocks for a given r. When the country has access to world financial markets, firms choose optimally whether to use imported or domestic inputs, and hence output is given by $\max\left(Y^*, Y^d\right)$. In this case, $Y^* = \max\left(Y^*, Y^d\right)$ as long as the TFP shock ε^* is above the threshold at which firms switch to domestic inputs, and $Y^d = \max\left(Y^*, Y^d\right)$ otherwise. If the country defaults, exclusion from world financial markets prevents firms from accessing working capital loans and forces them to switch to domestic inputs, so at default output is always given by Y^d. If follows therefore that the output with credit market access is higher than the output at default as long as ε^* exceeds the threshold value, and otherwise the output with credit market access is the same as the output at default and default has no effect on output. This generates the kinked output loss function also shown in Figure 3 (for a given value of r).

As Figure 3 shows, the country faces no loss in output at default if the current productivity shock is at or below the threshold value that causes firms to switch to domestic inputs, because output would be already determined by Y^d. For TFP shocks above the threshold value, however, the output loss increases with the size of the shock, because default is accompanied by a switch from Y^* to Y^d, so default is more painful at higher levels of TFP. This asymmetry in the output loss makes the default option more attractive to the country at lower states of productivity, and works as a “desirable” implicit hedging mechanism given the incompleteness of asset markets. Arellano (2007) found that an exogenous asymmetry of this form is key for explaining observed default frequencies and higher debt level in emerging economies. In our setup, this asymmetry is an endogenous feature of the model. The two key assumptions that support it are the working capital constraint on imports of intermediate goods, and the assumption that imported inputs are superior to domestic inputs in terms of TFP.

[14]Note that in our model we distinguish gross output from GDP. GDP equals gross output minus the cost of intermediate goods, and because under the assumptions of competitive factor pricing and a Cobb-Douglas production function GDP reduces to the fraction $(1-\alpha_m)$ of gross output.

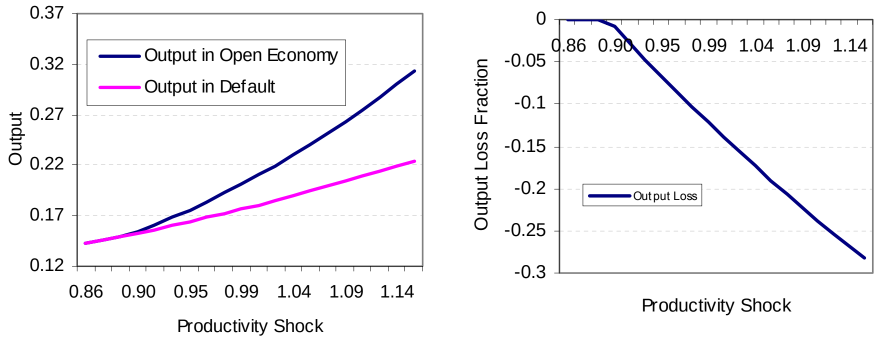


Figure 3: Output Function and Asymmetric Output Loss due to Default

4 Quantitative analysis

4.1 Calibration

This section studies the quantitative implications of the model. We conduct numerical simulations based on the following benchmark calibration. The risk aversion parameter σ is set to 2, which is standard in the quantitative business cycle literature. The world risk-free interest rate r^* is set to 1 percent, the average quarterly real interest rate on 3 month U.S. treasury bills over the 1980-2005 period. The supply for domestic intermediate good production $\overline{m}$ is chosen so that the average amount of domestic input is equal to the average amount of imported inputs that is used when $r = r^*$. With this calibration target, we ensure that the results are not driven by the relatively low supply of domestic intermediate goods. The curvature of labor in the utility function is set to 1.455, which implies a wage elasticity of labor supply of 2.2. This number is close to the value used in Mendoza (1991) and Neumeyer and Perri (2005). The probability of stochastic re-entry after default is 0.1, which implies that the country stays in exclusion for 2.5 years after default on average, consistent with the finding in Gelos et al. (2003).

The share of intermediate goods in gross output α_m is set to 0.3. This parameter is difficult to set using actual data because in the model intermediate goods are either all imported or all purchased internally, but in the data the share of total intermediate goods often hovers around 40 percent of output, and only about 1/3 to 1/2 of this share are imported inputs (see Gopinath, Itskhoki, and Rigobon (2007) and Mendoza

(2007)). Hence, setting $\alpha_m = 0.4$ would match the share of total intermediate goods but overestimate the fraction of them that are imported, while $\alpha_m = 0.15$ would match the share of imported inputs but underestimate the share of total intermediate goods. With this in mind, we chose $\alpha_m = 0.3$ as in intermediate value. Given this share for intermediate goods, the capital share in gross production α_k is set to 0.2 so as to match the typical 30 percent share of capital in value added.

Productivity shocks when firms use imported intermediate goods follow an AR(1) process:

$$\log \varepsilon_t = \rho_\varepsilon \log \varepsilon_{t-1} + v_t \tag{26}$$

with $v_t \overset{iid}{\sim} N\left(0, \sigma_v^2\right)$. Since data limitations prevent us from estimating directly this process using TFP data, we set σ_v^2 and ρ_ε (together with other parameters to be discussed below) using the simulated method of moments (SMM). The target moments used to set σ_v^2 and ρ_ε are the variability and persistence of output, which we calibrate to quarterly data for Argentina (this facilitates comparisons with the literature on quantitative models of default, which largely focuses on data for Argentina). We use seasonally-adjusted quarterly real GDP from the Ministry of Finance (MECON) for the period 1980Q1 to 2005Q4. The standard deviation and persistence of the cyclical component of H-P filtered GDP are 4.7 percent and 0.79 respectively. Given these targets, the process of productivity shocks derived using SMM features $\rho_\varepsilon = 0.90$, and $\sigma_v^2 = 1.29$ percent.

The additional parameters set using SMM are the time discount factor β, the parameter that measures the reduced efficiency of domestic intermediate goods λ, the share of imported inputs paid for with working capital θ, and the scaling parameter ψ in the disutility of labor. These parameters are targeted to match the default frequency, the average fraction of output loss due to default, the volatility of the trade balance-to-GDP ratio, and the average labor used in the economy. The target statistic for default frequency is 0.69% because Argentina has defaulted five times on its external debt since 1824 (the average default frequency is 2.78% annually or 0.69% quarterly). The average output loss due to default for Argentina is 13% based on the cyclical position of the country's GDP around the 2001 debt crisis. The standard deviation of the trade balance-to-GDP ratio for Argentina is 2%. The target for the labor allocation is 0.33.

Table 2 shows the parameters of the benchmark calibration. The SMM estimate of the subjective discount factor is 0.88, which is close to the value used in the existing studies on sovereign default. The estimate for λ implies that domestic inputs are about

7% less efficient than the imported ones. Finally, the estimate for θ implies that firms need to pay only about 1/10 of the cost of their imported intermediate goods in advance. This low estimate of θ is important because the business cycle models of Neumeyer and Perri (2005) and Uribe and Yue (2006) need to assume that 100 percent of the wages bill is paid in advance in order to produce a close approximation to actual business cycles (see also Oviedo (2005)). But with the standard labor share of about 2/3rds, working capital financing would need to be 2/3rds of GDP, and such a high share of credit exceeds estimates of the *total* ratio of credit to the private sector as a share of GDP in many emerging economies (which average about 50 percent of GDP but including all forms of credit to firms and households at all maturities, and not just short-term revolving loans to firms).

Table 2: Model Parameter Calibration

Calibrated Parameters		Value	Target Statistics	
CRRA risk aversion	σ	2		
Risk-free interest rate	r^*	1%	US interest rates	1%
Capital share	α_k	0.21	Capital income share	0.3
Intermediate share	α_m	0.3	Intermediate good share	0.3
Intermediate labor share	γ	0.3	Manufacture labor share	0.3
Labor elasticity para.	ω	1.45	Labor supply elasticity	2.2
Re-entry Probability	η	0.1	Length of exclusion	2.5 years
Productivity persistence	ρ_ε	0.90	Output persistence	0.79
Productivity std. dev.	σ_ε	1.29%	Output std. dev.	4.70%
Time discount factor	β	0.92	Default frequency	0.69%
Inefficiency	λ	0.93	Output loss in Default	13%
Working capital friction	θ	0.10	Trade balance volatility	2.03%
Labor disutility para.	ψ	0.69	Average labor	0.33

4.2 Results of the Benchmark Simulation

This subsection examines the model's ability to account for the three key empirical regularities of sovereign debt highlighted in the Introduction: V-shaped output dynamics with deep recessions that hit bottom at times of default, countercyclical country interest rates, and high debt ratios. To explore this issue, we feed the TFP process to the model and conduct 1000 simulations, each with 1000 periods and truncating the first 100 observations.

The quantitative predictions of the model approximate closely the three key stylized facts of sovereign debt, and in addition they match two key business cycle regularities (the cyclical variability of consumption and the correlation of net exports with GDP). Table 3 below compares the moments produced by the model's numerical solution with moments from Argentine data. The bond spreads data are quarterly EMBI+ spreads on Argentine foreign currency denominated bonds from 1994Q2 to 2001Q4, taken from J.P. Morgan's EMBI+ dataset.

Table 3: Model Simulation and Statistics in the Data

Statistics	Data	Model
Corr. between Bond Spreads and GDP	-0.62	-0.52
Corr. between Bond Spreads and Trade Balance	0.68	0.52
Corr. between Trade Balance and GDP	-0.58	-0.47
Consumption Std. Dev./Output Std. Dev.	1.44	1.29
Debt/GDP	35%	19.40%
Corr. between GDP and Imported Intermediate	-	0.59
Corr. between Spread and Imported Intermediate	-	-0.49
Bond Spreads Std. Dev.	0.78%	0.77%
Average Bond Spreads	1.86%	0.69%

The model mimics the positive correlation between spreads and net exports, and the negative correlations of spreads and net exports with GDP. The model replicates the negative correlation between spreads and GDP because sovereign bonds have higher default risk in bad states. Several quantitative models of sovereign debt (e.g. Arellano (2007), Aguiar and Gopinath (2005), Yue (2006)) and business cycle models of emerging economies (e.g. Neumeyer and Perri (2005), Uribe and Yue (2006)) also produce countercyclical spreads, but the former treat output as an exogenous endowment and in the latter country risk is exogenous. Hence, one of the two sides of the correlation is left unexplained. In contrast, our model nearly matches the negative correlation between GDP and spreads in a setting in which both output and country risk are endogenous, and influence each other because of the relationship between country risk and working capital financing. Moreover, our model also produces a closer approximation to the observed correlation between bond spreads and GDP.

The countercyclical net exports follow from the fact that, when the country is in a bad state, it faces higher interest rates and tends to borrow less. The country's trade balance thus increases because of the lower borrowing, leading to a negative correlation with output.

Consumption variability exceeds output variability in the Argentine data, and this is a common feature for emerging economies (see Neumeyer and Perri (2005)). The model is able to mimic this stylized fact because the ability to share risk with foreign lenders is negatively affected by the higher interest rates induced by increased default probabilities. The sovereign borrows less when the economy faces a bad productivity shock, and thus households adjust consumption by more than in the absence of default risk. On the other hand, because agents are impatient, the benevolent government borrows more to increase private consumption when the productivity shock is good. Hence, the variability of consumption rises.

The model produces a debt-to-GDP ratio of nearly 20 percent on average. This high debt ratio is largely the result of the large *and* asymmetric endogenous output drop that occurs when the country defaults. Although 20 percent it is still below the average debt-to-GDP ratio for Argentina from 1980-2004 based on the World Bank's WFD dataset, this debt ratio is several orders of magnitude larger than those typically obtained in quantitative models of sovereign default with exogenous output costs (even when these costs are targeted to improve the models' quantitative performance). Yue's (2006) model with renegotiation and an exogenous 2 percent output cost at default yields an average debt ratio of 9.7 percent. Arellano (2007) obtains a mean debt ratio of 6 percent of GDP assuming a kinked output cost such that income equals the maximum of actual output or 97 percent of average output while the economy is in default.[15]

The model also closely matches the volatility of the Argentine bond spreads observed in the data. Yet the average bond spread is lower than in the data. Because we assume a zero debt recovery rate and risk-neutral creditors, bond spreads are linked one-to-one with default probabilities. The model can generate enough time-series variability in default probabilities, but since the quarterly default frequency is 0.7% of that in the data, the model can only generate 0.7% of the average bond spread, which is about 40% of the average bond spreads observed in the data. In an extension of the model examined

[15] Aguiar and Gopinath (2006) obtained a *maximum* (not average) debt ratio of 27 percent of GDP assuming a cost of 2 percent of output per quarter for each period under in financial autarky. The maximum debt ratio in our benchmark simulation is 42.3 percent.

later, we follow Arellano (2007) in introducing risk-averse foreign lenders. This change implies that predicted bond spreads include an additional risk premium, and hence can get closer to the data.

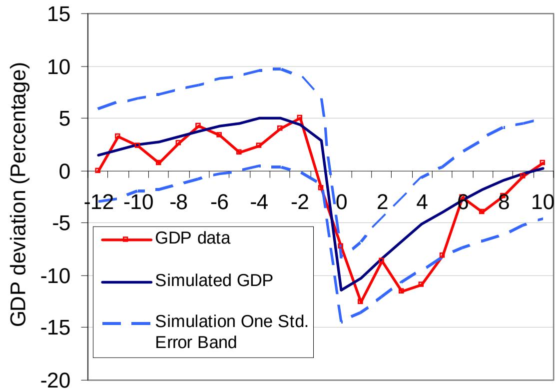
4.3 Dynamics of Output Before and After a Default Episode

We illustrate the model's ability to match V-shaped dynamics of output around a default episode by plotting in Figure 3 the model's average path of output dynamics around default episodes together with the data for Argentina around the 2001 default. The Figure covers 12 quarters before and 10 quarters after a debt crisis, with date 0 indicating the default period. The line with diamonds is the H-P detrended Argentine GDP from 1999Q1 to 2004Q2. The solid line is the average for output at each date $t = -12, ..., 10$ around defaults in the stationary distribution of the model. The dashed lines correspond to one-standard-error bands of these simulation averages.

Figure 3 shows that the model produces a substantial output drop when the country defaults, equivalent to about 13 percent of the pre-default output level. In addition, the model displays a V-shaped recovery during the financial exclusion period. In the model, the country is most likely to default when it receives a "very bad" productivity shock (the average decline in TFP is 6 percent below trend at date 0). Because the productivity shock is mean-reverting, TFP is very likely to improve in the following periods after default (for example, mean TFP rises 1 percent in date 1). Therefore, even though the country remains financially excluded on average at dates 1 to 10, the economy recovers because TFP improves.[16] Moreover, the relative magnitudes of the recession and recovery match the data quite well (the output dynamics for Argentina before and after the 2001 debt crisis are mostly within the one standard error bands of the model simulation).

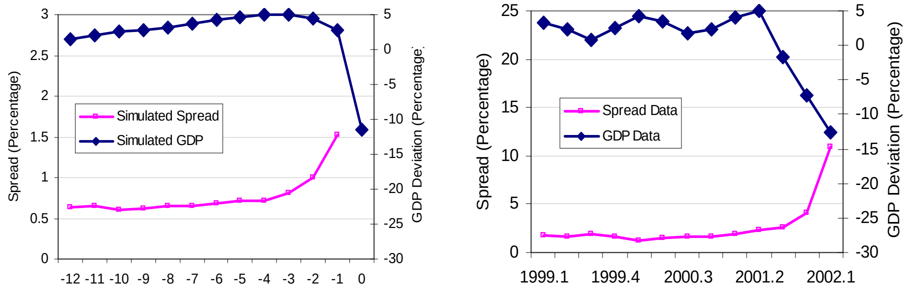
Figure 4: Dynamics of Output Before and After Default

[16] These V-shaped dynamics are qualitatively consistent with the data of many emerging markets that suffered Sudden Stops .Calvo, Izquierdo and Talvi (2006) conducted a detailed cross-country empirical analysis of the recovery of emerging economies from Sudden Stops, and found that most recoveries are not associated with improvements credit market access.



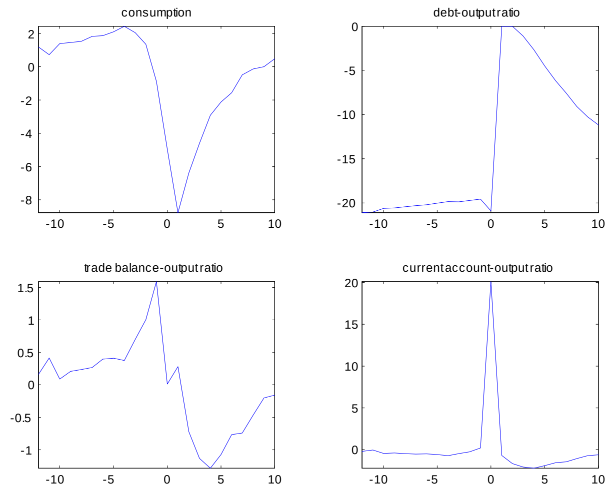
The model also matches the dynamics of sovereign bond spreads before a debt crisis. The left panel of Figure 5 presents the mean of simulated output and bond spreads up to 16 quarters before default episodes in the stationary distribution of the model. The dark line corresponds to output and the grey line plots sovereign bond spreads. Figure 5 clearly illustrates the negative correlation between output and bond spreads before a debt crisis. In particular, the spread on the sovereign bonds increases as the country approaches a debt crisis. The average quarterly spread increases from 0.7 percent to almost 1.5 percent in the 4 quarters before default in the model simulation. At the same time, the H-P detrended output starts to decline by about 2 percent, which indicates that the economy enters a recession and suffers a sharp output drop subsequently due to default. These features reflect well the Argentine experience. The right panel of Figure 5 shows the H-P detrended real GDP and EMBI+ sovereign bond spreads for Argentina from 1994Q1 to 2001 Q4. The data show a relatively stable sovereign spread before 2000 and a sharp increase in 2001, and the Argentine economy also experienced a steady output performance and then a deep recession starting in 2001. Hence, our model seems capable of generating endogenous output and sovereign spread dynamics consistent with the data.

Figure 5: Dynamics of Output and Sovereign Spreads before Debt Crisis



The model also replicates the dynamics of consumption, trade balance, and debt/output ratio around default episodes. Figure 6 below shows the average of simulations around default periods in the model. The first panel shows that consumption is highly volatile around the default episode. Consumption collapses when the country defaults and recovers gradually after default, similarly as the dynamics of output. Debt-output ratio is over 20% on average before default, and it slightly increases in the period before default. The trade balance-ouput ratio is also very volatile. The country experiences an sharp increase of trade balance close to default. And as the country regains access to the financial markets, it runs a trade deficit gradually. This model also generate a sharp reversal in current account. The country runs a small current account deficit on average before default. Default in the model causes a big reduction in the capital inflow. Accordingly, current account balance increases sharply. This mimics the "sudden stop" phenomenon in emerging economies around financial crises.

Figure 6: Dynamics of Economic Variables Before and After Default



4.4 Key Features of the Equilibrium with Default

How does the interaction between endogenous output fluctuations and endogenous default risk affect the quantitative performance of the model? This question can be answered by studying the behavior of the value function for the country with access to world financial markets, the sovereign bond price function, the saving decision rule, and output. Figure 7 shows these equilibrium functions for high and low TFP shocks as a function of the country's foreign asset position.

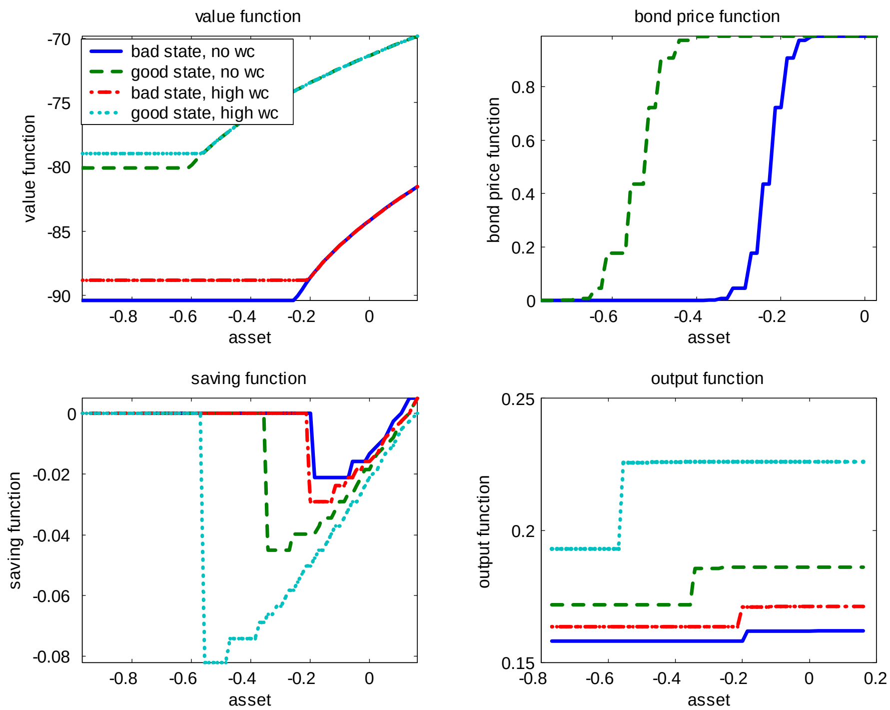
The first panel of Figure 7 shows that the value function increases with the asset position for the range of asset positions higher than the value at which default is certain (for asset positions smaller than this value, the value function becomes independent of foreign assets). As the country debt increases (i.e., its assets fall) the value of default can exceed the value of not defaulting. The country is more likely to default if the productivity shock is bad because the default option is more attractive then. This is because it is more painful to repay the debt in a bad state, while at the same time default does not lead to a high output loss compared to the case with a good productivity shock. The value function also differs as we vary the amount of working capital in place κ. Because the government transfers working capital to households when it defaults, the value of default increases with working capital.

The second panel of Figure 7 shows that sovereign bond prices increase with asset holdings (i.e., decrease with the debt position), reflecting the standard result that default risk premia are higher at higher levels of debt. Moreover, bond prices are higher when the country experiences a good TFP shock, which imply a lower default risk premium, a lower probability of default and a lower country interest rate. Working capital financing becomes less costly as a result, leading firms to increase demand for foreign inputs and produce more. This feedback from country interest rates to output dynamics also affects the country's incentives to default, reinforcing the reduction in default risk. The opposite is true when the country experiences a bad TFP shock, and this is an important result because it implies that, for any given level of debt before the country is in financial autarky, default is more likely when TFP is low than when it is high.

The lower-left panel of Figure 7 shows that the country borrows less (i.e. chooses a higher asset position) when it experiences a bad productivity shock. This property of the assets decision rule helps us to explain the countercyclical trade balance and the positive correlation between the trade balance and sovereign spreads documented earlier.

Finally, the lower-right panel of the Figure shows that the relationship between output and foreign assets follows "almost" a two-step function. The lower step corresponds to the range of high debt in which firms operate with domestic inputs, either because the country defaults or because the interest rate is sufficiently high and the state of productivity is sufficiently low. The higher step pertains to the range of debt positions when the country has access to world credit markets and firms use imported inputs. Output in this region is not independent of the asset position (so the output plots are not truly two step functions). In this region, output fluctuates with country risk because the demand for imported intermediate goods is directly affected by country spreads. When the country borrows more, sovereign default risk increases and this rises the financing cost of working capital to firms. In response, firms cut demand for imported intermediate goods and output falls. The plot also shows that the size of the output drop at default is larger with the good productivity shock because of the asymmetry of the default cost illustrated earlier.

Figure 7: Equilibrium Solutions



4.5 Sensitivity Analysis: Working Capital

The working capital constraint plays a very important role in the quantitative performance of our model. Its relevance can be illustrated further by comparing the benchmark results with the results of a simulation that abstracts from working capital (i.e., $\theta = 0$). Without working capital, the fraction of output loss at default is invariant to changes in productivity, as in the existing quantitative studies of sovereign debt that assume that income is an exogenous endowment (e.g. Aguiar and Gopinath 2006, Yue 2006). We recalibrate the output loss fraction parameter in this variant of our model so as to match the average output loss in default of 13 percent observed in the data, which we use also as a calibration target in the benchmark case. The other parameters are kept unchanged. The second column of Table 3 presents the simulation results from this no-working-capital case and the third column shows the results from the benchmark model.

The model without working capital performs much worse in terms of its ability to match all the important features of the data that the benchmark model mimicked closely.

The frequency of defaults falls from 0.7 percent to 0.04 percent. The GDP correlations of sovereign spreads and net exports increase sharply. The correlation between spreads and net exports changes from significantly positive at 0.52 to negative at -0.26. The mean debt ratio declines by nearly 13 percentage points of GDP, and the average and standard deviation of country spreads fall by about 70 basis points. These results follow from two important differences in the model without working capital relative to our setup: First, the output loss due to default becomes symmetric across states of nature, and second, bond spreads no longer have a direct impact on production. As a result, debt is not as good a hedging mechanism as in the benchmark model with working capital, making default more painful ex ante in the model without working capital, and thus reducing the amount of average debt. Moreover, the model without working capital cannot reproduce the V-shaped dynamics that the benchmark model produces (see Figure 4), because it goes back to the disconnect between country risk and output of the standard quantitative model of sovereign debt.

Table 4: Role of Working Capital Constraint

Statistics	No WC	benchmark	lager WC
Output loss	12.5%	12.5%	12.8%
GDP std. dev.	4.64%	4.85%	4.89%
Default probability	0.04%	0.69%	0.51%
Corr. between Spreads and GDP	-0.10	-0.52	-0.39
Corr. between Spreads and TB	-0.26	0.52	0.36
Corr. between TB and GDP	-0.28	-0.47	-0.42
Debt/GDP	6.91%	19.40%	7.88%
Bond spreads std. dev.	0.07%	0.77%	0.61%
Average Bond Spreads	0.04%	0.69%	0.51%

We have established that removing working capital worsens significantly the quantitative performance of the model. But how sensitive are the results to the value of the working capital requirement beyond the extreme case of $\theta = 0$? To answer this question, we solved the model for a case in which the working capital requirement is three times higher than in the baseline (i.e. firms need to pay for 30 percent of the cost of imported inputs with working capital). Thus, we set $\theta = 0.3$ instead of 0.1 as in the benchmark case. The last column of Table 4 shows the results for this simulation.

Interestingly, the higher working capital requirement increases only slightly the size of the output loss at default and the variability of GDP, but it reduces the probability of default, the mean and standard deviation of spreads, and the mean debt-GDP ratio.

These changes reflect the fact that the higher value of θ has opposing effects on default incentives and production plans. On one hand, default leads to a higher fraction of output loss on average. On the other hand, firms are more likely to switch to domestic intermediate goods, since higher θ increases the effective price of imported inputs, and changes in sovereign interest rates have a larger impact on production. These effects amplify the response of output to productivity shocks, making output more volatile. This result is also complementary to the finding in Uribe and Yue (2006) showing that the impact of output on country interest rates magnifies business cycle volatility. At the same time, this higher volatility makes the sovereign borrower more cautious ex ante, reducing the default probability, the mean debt/GDP ratio, and the volatility of bond spreads. Despite these changes, however, the overall quantitative effects of tightening the working capital constraint are small. The main reason is that the average sovereign spreads, and hence the average interest rate charged on working capital loans, do not deviate sharply from the one-percent risk free rate, and this is the case because these long-run averages include the infrequent default periods with high spreads and the more frequent "tranquil times" with small spreads. At a one-percent interest rate, even tripling the value of θ does not change the simulation results too much.

The above findings must not be viewed as suggesting that the working capital constraint is of little consequence. To the contrary, the comparison with the model without working capital showed that this financial friction plays a crucial role in the link between business cycles and sovereign default in our model. Thus, the general conclusion from the simulations in Table 4 is that the effects of the working capital constraint have significant nonlinearities in the model. The effects are substantial at low values of θ, while at higher values they tend to be small.

4.6 Sensitivity Analysis: Domestic Inputs

As we explained earlier, domestic and imported inputs differ in two key respects: First, domestic inputs are infinitely inelastic in fixed supply (at an endogenous price), while the supply of foreign inputs is infinitely elastic (at the exogenous world price p_m^*). Second, although domestic inputs do not require working capital, they reduce production efficiency, as they allow firms to operate only at a fraction λ of the TFP obtained with imported inputs. In the calibration, we calibrate the production function parameter so that the average supply of domestic input is the same as the average demand for imported input. Thus the results in the model is not driven by the relative scarcity of domestic intermediate goods. In this subsection, we conduct sensitivity analysis on λ alone to illustrate the role of the assumptions about domestic inputs in our model.

Table 5 presents simulation results comparing the benchmark case ($\lambda = 0.93$) with cases in which λ is 10 percent smaller (lower λ) and with $\lambda = 0.99$ (higher λ). All of the other parameters are the same as in the benchmark calibration. The results show that the value of λ directly affects the fraction of output loss at a default, as would be expected. The lower λ rises the loss to 29 percent, while the higher λ reduces it to about 5 percent. The lower λ yields a significantly smaller probability of default frequency and a much higher debt ratio, of 166 percent of GDP. In addition, the volatility of the spreads falls sharply from 1.76 percent with higher λ to 0.77 percent in the benchmark case and 0.12 percent with lower λ. These results have a straightforward interpretation: lower λ increases the cost of default, and the greater default penalty makes the sovereign less likely to default and able to borrow higher amounts on average. Spreads are also less volatile because the reduced frequency of defaults reduces the frequency of states with very high spreads. The opposite occurs when λ is higher, but the changes are quantitatively smaller.

Changes in λ also affect business cycle comovements but the effects are less significant than those noted above, and they are also largely non-monotonic. Lower and higher λ produce a higher correlation between GDP and spreads than in the benchmark case, but the correlation is always negative. Similarly, the correlation between net exports and GDP is always positive, but it is higher in the benchmark case than in the scenarios with lower and higher λ. The correlation between net exports and GDP is always negative. It rises to -0.11 with higher λ, but it remains unchanged at -0.47 for lower λ. The variability of GDP also remains nearly unchanged with lower λ, while it falls by nearly 1 percentage point with higher λ.

Table 5: Changes in TFP of Domestic Inputs

Statistics	Lower TFP ($\lambda = 0.82$)	benchmark ($\lambda = 0.93$)	Higher TFP ($\lambda = 0.99$)
Output loss	29.0%	12.5%	4.9%
GDP std. dev.	4.89%	4.85%	3.93%
Default probability	0.08%	0.69%	0.72%
Corr. between Spreads and GDP	-0.31	-0.52	-0.18
Corr. between Spreads and TB	0.29	0.52	0.07
Corr. between TB and GDP	-0.47	-0.47	-0.11
Debt/GDP	166.07%	19.40%	0.50%
Bond spreads std. dev.	0.12%	0.77%	1.76%
Average Bond Spreads	0.07	0.69%	0.72%

5 Conclusion

This paper examined the quantitative predictions of a model of strategic sovereign default with endogenous output dynamics. Firms have the choice of producing with imported inputs that require working capital financing from foreign lenders, or using domestic inputs that do not require external credit but reduce the firms' TFP. Lender's charge the same default risk premium on working capital loans to firms as on sovereign debt because the sovereign diverts the repayment of working capital loans when the country defaults.

The model produces endogenous output costs of default because default entails temporary exclusion from world credit markets, and firms cannot obtain foreign working capital financing when the economy defaults. Hence, the financing cost of working capital effectively grows infinitely large when default occurs, and firms optimally choose to switch from superior imported inputs to their inferior domestic substitutes. This produces endogenous output costs that are large and asymmetric (as a function of the state of productivity) in a manner consistent with the shape of exogenous output costs that the quantitative literature on sovereign debt has identified as necessary in order to obtain default incentives that trigger default in bad states of nature, at higher debt ratios and at higher spreads (see Arellano (2007)).

The model is consistent with three key stylized facts of sovereign debt: (1) the V-shaped dynamics of output around default episodes, (2) the negative correlation between interest rates on sovereign debt and output, and (3) high average debt ratios.

The model also replicates the countercyclical dynamics of net exports and the positive correlation between country spreads and GDP found in the data, as well as the observed cyclical variability of private consumption. In addition, the model's assumption that the sovereign diverts the repayment of private working capital loans implies that interest rates faced by private firms and sovereign borrowers move together, and this is a reasonable assertion because in the data the two interest rates are positively correlated.

The results of the numerical analysis suggest that the model provides a possible solution to the disconnect between sovereign debt models (which rely on exogenous output dynamics with particular properties to explain the stylized facts of sovereign debt) and models of emerging markets' business cycles (which assume an exogenous financing cost of working capital that is calibrated to match the interest rate on sovereign debt). On the other hand, we acknowledge that central to our analysis are assumptions about how sovereign default affects private sector borrowers, and about how default affects the efficiency of private production, that should be the subject of further theoretical and empirical research.

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