Dynamic Macro-Prudential Policy

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December 7, 2011

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PRELIMINARY AND INCOMPLETE[1]

Abstract

I propose a new dynamic general equilibrium framework, in which government guarantees induce financial institution to take on too much risk through excessive leverage. In response, the regulator sets capital requirements to trade-off growth with financial stability (tax-payer exposure to a banking sector collapse). This trade-off depends on the state of the economy and optimal capital requirements are therefore time varying. I solve the model numerically to characterize the optimal requirements and show that they should crucially react to *aggregate bank capital* and *credit expansion*. For most parametrization, optimal requirements are higher in good times, which resonates with the notion of "counter-cyclical capital buffers".

I also compare the optimal policy to the (best possible) constant capital requirement and show that the latter not only generates excess volatility but also episodes of extremely excessive credit expansion. In such cases, there is no longer a trade-off: an increase in capital requirement would both be good for growth and improve financial stability.

[1]Please do not circulate. I am grateful to Nuno Coimbra for outstanding research assistance, and to Helene Rey, Enrique Mendoza, Romain Ranciere, and Martin Eichenbaum for extremely fruitful discussions during the early phases of the project, and to seminar participants at Warwick, BU, and the ECB. Correspondence: fmalherbe@london.edu.

1 Introduction

Motivation

The recent financial crises has reinforced the shift of banking supervision focus toward macro issues. A consensus seems indeed to have emerged, among academics and regulators, that:

- The financial stability of the system as a whole is a major concern. In other words, prudential regulation should take systemic risk into account (Brunnermeier et al., 2009, French et al., 2010, Hanson et al. (2011), Vickers (2011), BIS (2011).
- Current regulation has pro-cyclical effects, which should be corrected, or at least mitigated (See Borio, 2003, and Kashyap and Stein, 2004, among many others).

However, no consensus has emerged yet on the means by which the regulator can achieve these goals. Furthermore, there is no widely accepted analytical framework upon which basis concrete policies can be assessed (Goodhart, 2010; Woodford, 2010; Galati and Moessner, 2011). It is therefore hard to identify the relevant operational objectives and to discuss the relative efficacy of the various possible tools.

Still, the recent crisis has opened a window of opportunity for regulatory reforms and it is very likely that "counter-cyclical capital buffers" will be introduced, as it is an explicit recommendation of the Basel Committee on Banking Supervision[2]. However, no explicit reference to an analytical framework is provided in the supporting document. The reason is perhaps that such concept is not well defined in economic theory, and can be quite confusing indeed[3]. Here is my tentative interpretation.

If "high" capital requirements are contractionary[4], such a cost has to be balanced with the benefits in terms of financial stability, or of tax-payer exposure to systemic financial crisis (Kashyap and Stein (2004)). If these costs and benefits are dependent on the state of the economy, an optimal capital requirement rule may be time-varying. Exploring what such a dynamic macro-prudential regulation should consist of is the main purpose of this paper.

To do so, I propose a new dynamic general equilibrium framework, in which demand deposit insurance (which can also be interpreted as an implicit promise of a bailout due to a too-big-to-fail concern) induces financial institution to take on too much risk through excessive leverage. In response, the regulator sets capital requirements to trade-off growth with financial stability (tax-payer exposure to a banking sector collapse).

[2] See appendix A for an excerpt.

[3] See Rochet (2011)

[4] Which is actually disputed by some prominent economists (see Admati et al. (2010) and Hellwig (2010)).

Description of the model's main features

I study an overlapping generation economy in which there are risk-neutral bankers and risk averse depositors, which can either store their savings (they work and receive a wage when young) at a normalized zero rate of return, or deposit them at a bank. Building on the representation hypothesis of Dewatripont and Tirole (1994), my starting point is that demand deposits are insured by the government, which then monitors the bankers on behalf of the depositors.

Bankers enjoy limited liabilities and their capital is usually scare. They collects deposit and lend to a representative firm which operates a neoclassical production function, with stochastic productivity shocks. Bank lending is the only source of funding in the economy. Firms always make zero profits, and bankers are in effect the residual claimants of the production. If banks default, the government bails them out and levies lump-sum taxes to break even. Note that a decreasing marginal return to capital is a crucial ingredient of the model (see below). It is a standard assumption in macroeconomics, but less so in the banking literature, which often takes a partial equilibrium approach.

The government is represented by a regulator, which seeks to maximizes social welfare by restricting bank leverage. The regulator faces thus the following trade-off: allowing higher leverage increases lending and increases thus the expected size of the economy, but it also increases the probability of bank default, and the severity there of (see Malherbe, 2011).

Results and insights

Before turning to the results, let me first expose the important basic intuitions.

If bankers' skin-in-the-game is sufficient to cover all potential losses, bankers' and social optima coincide and correspond to the level of lending that maximizes the expected size of the economy. That is, banks lend until the expected marginal return to capital equates the return to storage. However, when banking capital is scarce, bankers fund the marginal loan with deposits. Since deposit are insured, their required return is insensitive to bank risk-taking and equates the return to storage. Hence, since bankers repay deposits in full only when they are not broke, their expected private marginal cost of lending is lower than the return to storage, and they have all incentives to lend more than when they internalize all the loss.

What makes the situation even worse is that when banking capital is not sufficient to absorb all the potential losses, the social marginal cost of deposit is actually higher than the return to storage. Indeed, when future payoffs are discounted with the right pricing kernel, lending commends a risk-premium. This premium is decreasing in banking capital (when the buffer increases, tax-payer exposure to systemic crisis decreases) and in depositor wealth (their marginal utility of consumption decreases with it), it depends thus on the state of the economy.

Numerical solution of the model provides a characterization of the optimal time-varying capital requirements. For most parametrization, they are higher in good times than in bad times. The main driver of this result is that aggregate banking capital accumulates in good times. If leverage is kept constant, then lending increases proportionally. Credit expansion in good times is generally good because of the two effects mentioned above, and because expected productivity is high, but the decreasing marginal productivity of capital sets a limit to that. Actually, during a boom, credit should expand, but less than proportionally to banking capital. Therefore, to optimally control credit expansion, capital requirements should go up; that is, they should increase with *aggregate bank capital*.

These optimal time-varying capital requirement can be compared to a constant, or "through the cycle" capital requirement. What results with the latter is not only excess volatility but also episodes of extremely excessive credit expansion. Indeed, after a few good shocks, banking capital becomes relatively abundant and banks start funding negative expected net present value project (discounted at the return to storage). In such a case, there is no longer a trade-off: a temporary increase in capital requirement would both be good for growth and improve financial stability. Note that such an episode is consistent with the idea that excessive risks are being piled up by the banking sector during good times (see Borio and Drehmann, 2009 for instance).

Discussion of the modeling strategy

To construct a tractable dynamic framework that features many of the relevant dimension to macro-prudential regulation, one has to make some concessions. The main advantages of my approach are the following:

- The trade-off between the size of the economy and the volatility of tax-payers consumption is micro-funded;
- Systemic risk is endogenous (the probability of a banking sector failure is endogenous);
- It accounts for risk-shifting incentive and excessive leverage.

And the main costs and limitations are:

- While banking capital is endogenous (its law of motion depends capital adequacy ratios, investment behavior, and on the shock history), the dividend rule is exogenous;
- Deposit insurance and lump-sum tax funded bailouts are taken as given institutional features;

- The rate of return to storage is exogenous.

Review of the literature (to complete)

The powerful effect of deleveraging spirals and fire sale externalities have been learned the hard way during the recent crisis. Most of the micro-funded models of macro-prudential regulation that are currently burgeoning are developed around such mechanisms (See Korinek (2011) and Gersbach and Rochet, 2011 for good examples, and Hanson et al. (2011) for the state of the literature). When there are fire sale externalities, capital requirements are likely to magnify the business cycle by the following mechanism: In a recession, more borrower default. Loan losses thus erode bank capital. Banks have therefore to raise more capital, or sell assets, or cut on lending. Since the first option seems limited during a recession (or is not attractive enough to existing shareholders because bank market valuation is low), let me focus on the two other options. If bank sell assets, this puts downwards pressure to asset prices, and mark-to-market accounting will translate in other banks capital erosion. If it cuts on lending, this will depress economic activity further and increase the probability of default of existing loans, which in turn implies that more capital is required to hold these assets. One of the main message of such papers is that the equilibrium level of lending in the economy may not be efficient, which opens the door to policy intervention.

2 The Static Model

I expose in this section the basic driving forces in a stripped down version of the model. This serves two purposes: first it enables me to provide important intuitions on basis of analytical solutions (the dynamic model has to be solved numerically); and second, it makes the dynamic model more transparent, in the sense that it sheds light on some of its main driving forces.

2.1 The environment

This is a one-period model with a unique consumption good, which builds on Malherbe (2011).

2.1.1 The consumers

There is a measure 1 of risk-averse consumers. They are endowed with one unit of the consumption good at the beginning of the period and derive utility from end-of-period consumption. They have access to a storage technology, whose safe rate of return is normalized to 0, and they also deposit at the bank. Nevertheless, they either do not have the skills to properly monitor bankers, or there is a free-rider problem that prevents them to do so. For this reason, the government guarantees bank deposits and, following the representation hypothesis (Dewatripont and Tirole, 1994), monitors the bankers on behalf of the depositors. Provided that deposits are insured by the government, and assuming excess potential supply, deposits return matches that of storage and depositors supply their funds perfectly elastically. Their utility function is:

$$E[\ln c],$$

where c is their end-of-period consumption.

2.1.2 The banker

There is a representative banker which is endowed with e units[5] of the consumption good at the beginning of the period. They collect deposits and issue loans. Bankers have the specific skills required to (costlessly) screen and select loan applicants. They are risk-neutral, enjoy limited liabilities, and maximize the expected end-of-period wealth, which I denote v^+:

$$v^+ = \max\{0, Rl - d\}$$

Loans yield a stochastic return $R(\theta, l)$ per unit invested, where θ is a random variable, and l is the total amount of lending. For simplicity, I assume that the banker issues a single loan, of size l and that $\tilde{\theta}$ is log-normally distributed : $\ln\tilde{\theta} \sim N(\mu, \sigma)$. This random variable captures thus aggregate uncertainty and the probability distribution of the loan return might thus be interpreted as that of the whole portfolio of the representative banker, that is the whole banking sector. Similarly, e represents the whole banking sector capitalization. To capture the idea that the expected return of the marginal loan in the economy is decreasing, I set:

$$R(\tilde{\theta}, l) = \tilde{\theta} l^{\alpha - 1}$$

2.1.3 The regulator

Knowing that he will have to bail out the depositors when the banker fails, the regulator sets (risk-weighted) leverage restrictions in order to maximize a social welfare function which is the sum of the banker's and a representative consumer's utility:

$$v^+ + E[\ln c]$$

[5] e is assumed small compared to consumer aggregate endowment (which is 1).

2.2 Solving the banker's problem

To simplify the intuition, I consider here the case in which the representative banker realizes that the more credit it extends, the lower the quality of the marginal borrower. In the case the banker takes the return as given, he doesn't internalize this effect and his incentive to over-invest is even higher[6]. Since one of the main point is here to explain the mechanism that gives bankers incentive to leverage too much, this assumption is thus inconsequential.

2.2.1 The banker's problem

Ignoring capital requirements, the bankers problem is:

$$\max_{l} E\left[v^{+}\right]$$

$$st: \begin{cases} v^{+} & = \max\left(0, \tilde{\theta} l^{\alpha} - d\right) \\ l & = e + d \end{cases}$$

The second equality ensures that the banker's initial wealth is at risk, that is, that he cannot divert its initial wealth (by consuming it at the beginning of the period for instance).

2.2.2 The privately optimal lending policy

Defining θ^{0} as the threshold for the realization of $\tilde{\theta}$ below which the bank cannot repay the deposits in full:

$$\theta^{0} = \begin{cases} \frac{l-e}{l^{\alpha}}; & d > 0 \\ 0; & d = 0 \end{cases}$$

the first order condition is:

$$\frac{\partial}{\partial l}\left[\int_{\theta^{0}}^{\infty} (\theta l^{\alpha} - l + e) f(\theta) d\theta\right] = 0$$

which gives:

$$\int_{\theta^{0}}^{\infty} \left(\alpha \theta l^{\alpha-1} - 1\right) f(\theta) d\theta = 0 \tag{1}$$

which implicitly defines $l^{*}(e)$, the privately optimal level of lending for the banker.

[6]In the dynamic model bankers take lending return as given.

2.2.3 Analysis

First define $\hat{l}$, the lending threshold from which the marginal loan has a negative expected net present value (from a social and risk neutral perspective, that is when it is discounted at the return to storage):

$$\hat{l} = \left(\frac{1}{\alpha\bar{\theta}}\right)^{\frac{1}{\alpha-1}}$$

where $\bar{\theta} \equiv E[\theta]$. This level of lending is thus the one that maximizes the expected size of the economy.

Lemma 1

A non-fully capitalized bank ($l^*(e) > e$) will optimally decide to fund negative present value loans ($l^*(e) \geq \hat{l}$).

Proof:

The banker's first order condition (1) can be rewritten:

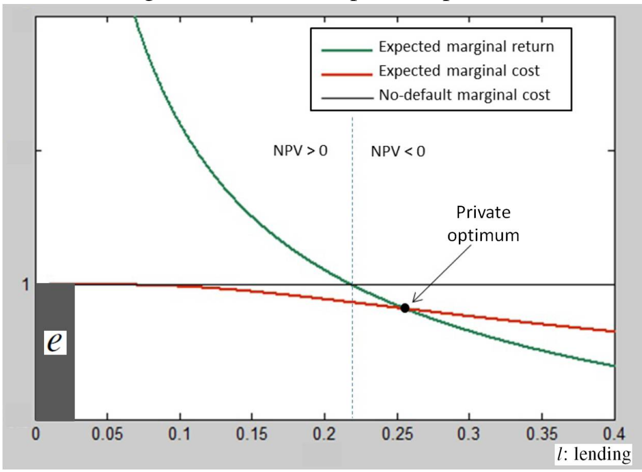
$$\alpha\bar{\theta}l^{\alpha-1} = \frac{1-\Phi\left[\frac{\ln\theta^0-\mu}{\sigma}\right]}{1-\Phi\left[\frac{\ln\theta^0-\mu}{\sigma}-\sigma\right]} \equiv 1-m(\theta^0), \tag{2}$$

where $\Phi[.]$ is the cumulative normal distribution. The right-hand side is bounded above by 1 (it tends to 1 when θ^0 tends to 0, that is when e tends to l), and is strictly decreasing with θ^0. It is thus smaller than 1 when $l > e$, which proves proposition 1.

Interpretation:

When $e < l$, the banker funds the marginal loan with deposits. Since the borrowing cost is, by assumption, insensitive to the risk, and since the banker repays deposits in full only when he is not bankrupt, his expected marginal cost of borrowing is smaller than 1. There is thus a wedge, at the banker's optimum, between the expected return of the marginal loan and the risk-neutral social opportunity cost of deposits (that is even without taking consumer risk-aversion into account). Figure 1 illustrates this.

Figure 1: The bank's private optimum



2.2.4 Comparative statics

- Since $m(\theta^0)$ is increasing in θ^0, it is
  - increasing in l
  - decreasing in e

which reflects the well known result that increasing leverage increases the option value of equity.

2.3 Solving the regulator's problem

Let $v^- \equiv \min\{0; \theta l^\alpha - d\}$ be the value of the bank when it is negative. Hence, $-v^-$ is the amount corresponding to the depositor bailout (it is positive when the bank defaults, that is when $\theta l^\alpha < d$). Given this bailout, the depositors are always paid in full and the return they require to deposit at the bank is insensitive to the bankers' risk exposure. Hence, $-v^-$ is also a measure of the bankers' subsidy implicitly implied by deposit insurance.

2.3.1 The regulator problem

When the regulator bails out the depositor, he has to raise taxes to break even. Since only the depositors have positive wealth, they end up bearing the whole burden of the bailout. Their consumption is thus $1+v^-$.

The regulator objective is to maximize the following welfare function:

$$E\left[v^+\right] + E\left[\ln\left(1+v^-\right)\right]$$

which is a sum of the banker's and the representative consumer's expected utility.

2.3.2 The socially optimal level of lending

The regulator's first order condition with respect to l is:

$$\alpha\bar{\theta}l^{\alpha-1} - 1 - E[v^{-'}(l)] + E\left[\frac{v^{-'}(l)}{1+v^{-}(l)}\right] = 0$$

which can be rewritten:

$$\alpha\bar{\theta}l^{\alpha-1} = 1 + q(l)$$

with $1+q(l) \equiv 1+E\left[v^{-'}(l)\left(1-\frac{1}{1+v^{-}(l)}\right)\right] \geq 1$.

This condition says that the optimal level of lending should reflect tax-payers' exposure to the failure of the banking sector. There is indeed a wedge $q(l) \geq 0$ between the actual social marginal cost of lending and its physical opportunity cost (the return to storage). One can therefore interpret $q(l)$ as a "systemic risk premium".

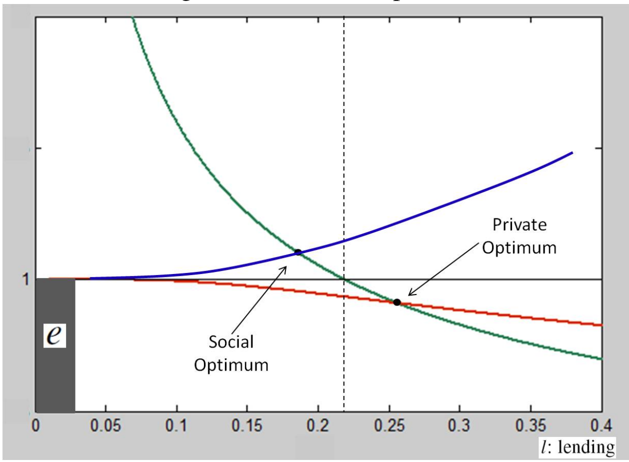
Proposition 1 **(policy trade-off)**

When banks are not fully funded, tax-payers are exposed to bank failures, and the socially optimal level of lending is inferior to the level that maximizes the size of the economy: $l^{soc} < \hat{l} < l^*$.

Proof: straightforward.

Interpretation: if consumers were risk neutral, the regulator would just make sure to maximize the size of the economy and would set x such that $l^* = \hat{l}$. However, they are risk averse. When the bank is not fully funded, there is always a draw for $\tilde{\theta}$ such that the bank will fail, which puts their consumption at risk. Therefore, the regulator trades off this risk with the size of the economy and restricts lending so that $l^* < \hat{l}$. This is illustrated in figure 2.

Figure 2: The social optimum



In blue is the social marginal cost. The social optimum is at the crossing with the marginal return to lending (in green). This optimal level of lending trades off the size of the economy (getting as close as possible to the vertical dotted line) with tax-payers' exposure to banking sector collapse (which increases with leverage).

2.4 Implementation

Knowing e and the probability distribution of $\tilde{\theta}$, the regulator can implement l^{soc} with a capital requirement ratio x, which puts an upper bound to bank leverage. The constraints takes thus the form:

$$e \geq xl$$

Remarks:

- The macro-prudential dimension lies in the fact that e represents aggregate banking capital in the economy.
- Since l^{soc} depends on the probability distribution of $\tilde{\theta}$, this can be interpreted as risk-weighted capital requirement;
- The regulator could adopt a Pigovian approach and correct for the wedge in the first order conditions with a tax:
  - $t(l) = \frac{1+q(e,l)}{1-m(e,l)}$
  - However, a single tool would work for a singled-bank economy only. In the case of several banks, one would need:
  - $t(e_i, l_i) = \frac{1+q(e,l)}{1-m(e_i,l_i,l)}$ because the depositors valuation wedge depends on aggregate capital (e) and lending (l), the bank's wedge depends on his own level of capital and lending.

2.5 Comparative statics

Assuming $e < \hat{l}$, it is then easy to check that:

1. $\frac{\partial q(e,l)}{\partial e} < 0$: *more banking capital decreases tax-payers' exposure and therefore the social marginal cost of lending.*

2. $\frac{\partial m(e,l)}{\partial e} < 0$: *more banking capital decreases excessive risk-taking incentives.*

3. $0 < \frac{\partial l^{soc}}{\partial e} < \frac{1}{x}$: *the optimal marginal increase in lending following is positive but smaller than the initial optimal leverage*.

Proof: see appendix.

2.5.1 Policy implication

Proposition 2 **(controlled credit expansion)**

The first element above implies that the optimal aggregate lending in the economy increases with aggregate bank capital. However, the third element tells us that the stringency of the optimal capital requirement should increase with aggregate bank capital:

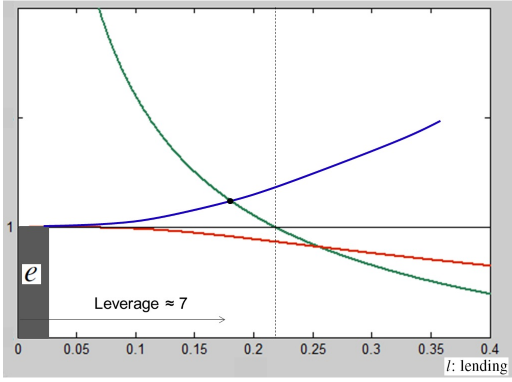
$$\frac{\partial x^*}{\partial e} > 0$$

Intuition:

This is driven by the assumption that the expect return of the marginal loan in the economy is decreasing. Therefore, starting from the optimal capital requirement for a given level of bank capital, an increasing in bank capital should optimally trigger an increase in lending, but a decrease in leverage.

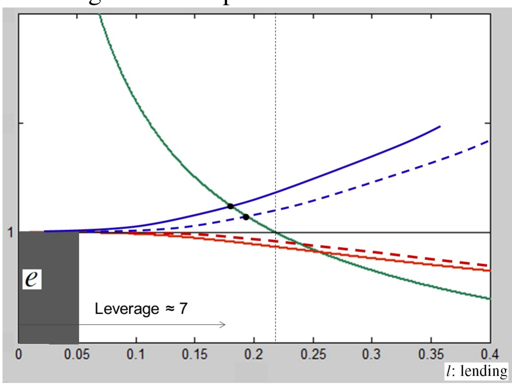
Since bank capital is likely to increase during a boom, this is a strong rationale for setting a counter-cyclical rule: banks should be allowed to growth, but only to the extent that they "build-up" capital at the same time, that is that they decrease leverage.

Figure 3: Comparative statics 1 of 3



In this example, a leverage of around 7 implements the initial social optimum.

Figure 4: Comparative statics 2 of 3



A doubling of e, increases the loss absorbing power of banking capital, which decreases the social marginal cost of deposits. Therefore, the social optimum is now closer to the vertical dotted line. The regulator hence, should allow banks to expand lending.

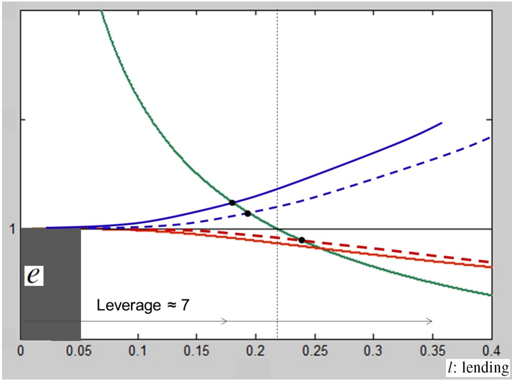
3 The dynamic model

In this section, I describe the dynamic model and expose the solution method.

3.1 The environment

This is an overlapping generation economy, with individuals living two periods. There are thus always two generations cohabiting the economy.

Figure 5: Comparative statics 3 of 3



However, keeping the same capital requirements (keeping the leverage ratio constant) would allow banks to expand way too much (up to $l = 0.35$ in this example). Actually, the constraint would not even be binding, and the banker would choose its private optimum. Clearly, implementing the new social optimum implies a decrease in leverage, and thus an increase in capital requirements.

3.1.1 The young generation

There is a measure 1 of agents born at time t. They are endowed with one unit of labor, which they supply inelastically during the first period of their life for a wage w_t. These agents may face taxes. If b_t denotes the total amount levied by the government and $(1-\gamma)$ the share of the young generation, then their after tax income is $w_t - (1-\gamma)b_t$. At the end of the period, these agents incur a preference shock, which determines whether they become bankers (which occurs with probability η) or consumers (with probability $1-\eta$) in the later part of their life.

The ones that become consumers retire and can choose between depositing their net wage at the bank or store it. As before, the rate of return to storage is normalized to 0, and deposit are insured by the government and pay the same rate.

Those that become bankers use their net wage to take over the bank.

3.1.2 The firms

Each period, there is a continuum of firms that combine young's labor and capital goods (which can costlessly be obtained 1 to 1 from consumption goods, that is any lending l from bankers translates into firm capital k) to produce output (a quantity of consumption good). The production function (identical for each firm) is a standard neo-classical constant return to scale production function:

$$F(\tilde{\theta}, k, n) = \tilde{\theta} k^{\alpha} n^{1-\alpha},$$

where n is labor, and $\tilde{\theta}$ is a stochastic variable that can be interpreted as a random aggregate productivity shock. Capital fully depreciates in the production process.

Firms pay a competitive wage:

$$w = (1-\alpha)\tilde{\theta}k^{\alpha}, \tag{3}$$

and they borrow capital from the bankers to maximize expected profit. Output is verifiable and firms enjoy limited liabilities. Firms have no initial wealth, so that the equilibrium repayment to the bankers satisfies:

$$\begin{cases} rk \leq F(\tilde{\theta},k,n) - w \\ E[F(\tilde{\theta},k,n) - rk - w] = 0 \end{cases}$$

where r is the repayment per unit of capital. The first condition reflects limited liabilities and is a feasibility constraint: firms can never repay more than the production minus the cost of labor, and the second that firms make no profit on expectation. Together with 3, it yields the standard result that since bankers are the owner of capital, they get a share α of the production:

$$rk = \alpha\theta k^{\alpha}$$

3.1.3 The old generation: consumers

During the second period of their life, consumers rest and either store the proceed of their wage or deposit it at the bank. At the end of period 2, they receive their deposit back, pay taxes if any, and consume. Their aggregate consumption is denoted c^D (D stands for depositors) and their individual utility is: $\ln \frac{c^D}{1-\eta}$.

3.1.4 The old generation: bankers

There is a measure $\eta \ll 1$ of new bankers (the young that did not become consumers), which bring their net wage as fresh capital in the bank (to make it simple, they supply it inelastically and receive full ownership of the bank). They are risk-neutral, enjoy limited liabilities, and maximize their expected end-of-period consumption: $E[c^b]$, where the superscript b stands for bankers.

These bankers raise deposits and compete to lend to firms. They take the (stochastic) return to lending $r_t = \alpha\theta_t k_t^{\alpha-1}$ as given.

Let v^+ denote the end-of-period private value of the bank:

$$v_t^+ = \max\left\{0, \alpha\theta_t k_t^{\alpha-1} l_t - l_t + e_{t-1}\right\},$$

where e_{t-1} is banking capital inherited from period $t-1$, and assume an exogenous dividend[7] rule: the old bankers receive a fraction δ of the end-of-period value. Therefore, $c_t^B = \delta v_t^+$ and the law of motion for banking capital is:

$$e_t = (1-\delta)v_t^+ + \eta s_t,$$

where $s_t = w_t - \gamma b_t$ is the net wage (s_t stands for savings at the end of period t). Note that under such a linear dividend rule, it is optimal for the bankers to maximize $E[v^+]$.

3.2 Solving the model

3.2.1 The bankers' problem

The bankers' problem is:

$$\max_{l_t} E_{t-1}\left[v_t^+\right]$$

subject to:

$$\begin{cases} v_t^+ = \max\left\{0, r_t l_t - l_t + e_{t-1}\right\} \\ E_{t-1}\left[v_t^+\right] \geq s_{t-1} - E[t_t] \end{cases}$$

The first condition gives the private value of the bank, it takes thus limited liabilities into account. The second is a participation constraint: the expected end-of-life wealth when being a banker should at least be as large that if being a depositor (t_t is the bailout tax the agent would face if he opts out). Note that l_t must be chosen before the realization of θ_t, this is why expectations are specified to be formed according to the information available a the end of the previous period: $E_{t-1}[.]$.

The first order condition is:

$$\int_{\theta_t^0}^{\infty} (r_t(\theta_t) - 1) f(\theta_t) d\theta_t \geq 0, \tag{4}$$

where

$$\theta_t^0 = \begin{cases} \frac{l_t - e_{t-1}}{k_t^\alpha}; & d > 0 \\ 0; & d = 0 \end{cases}$$

is the threshold for the realization of the shock below which the banks go bankrupt and there is a bailout.

Remarks:

[7] This dividend can be broadly interpreted as the remuneration of shareholders and managers, including "bonuses".

- If the bankers had unlimited liabilities, it would correspond to $\theta_t^0 = 0$, in which case the bankers private optimum would correspond to $\alpha\bar{\theta}_t l_t^{\alpha-1} = 1$.
- Evaluated at $E[r_t(\theta_t)] = 1$, *condition 4* is never binding[8]. Therefore, bankers always have an incentive to fund negative net present value loans.
- If $\ln\tilde{\theta}_t \sim N(\mu, \sigma)$, *condition 4* can be rewritten:

$$E_{t-1}[r_t] \geq \frac{1-\Phi\left[\frac{\ln\theta_t^0-\mu_t}{\sigma_t}\right]}{1-\Phi\left[\frac{\ln\theta_t^0-\mu_t}{\sigma_t}-\sigma_t\right]},$$

in which case bankers wants to be infinitely leveraged, and hence no banking equilibrium can exist without government intervention.

3.2.2 The regulator's problem

When banks go bankrupt, the government bails out the depositors with a total cost of:

$$v_t^- = \min\left\{0; \alpha\theta_t k_t^\alpha - (k_t - e_{t-1})\right\},$$

and levy taxes to break even: $b_t = v_t^-$. Each member of the old generation that has some resources (this excludes thus bankers, which are broke) pays a tax $\frac{\gamma v_t^-}{1-\eta}$ and each member of the young generation pays a tax $(1-\gamma)v_t^-$. γ is thus the share of the burden borne by the old generation. I take this parameter as exogenous here as I want to focus on optimal capital requirements, but endogenizing it would obviously be an interesting extension.

The regulator's objective is to maximize the following social welfare function, which is a discounted sum of each generation representative agents utility:

$$\max_{\{l_t\}_{t=1}^\infty} \sum_{t=1}^{\infty} \beta^t E_0\left[c_t^b + (1-\eta)\ln\left(\frac{c_t^d}{1-\eta}\right)\right]$$

where $c_t^d = s_{t-1} + \gamma v_t^-$ and $c_t^B = \delta v_t^+$, and subject to the following laws of motion:

$$st: \begin{cases} e_t &= \eta\left[(1-\alpha)\theta_t l_t^\alpha - (1-\gamma)v_t^-\right] + (1-\delta)v_t^+ \\ s_t &= (1-\eta)\left[(1-\alpha)\theta_t l_t^\alpha - (1-\gamma)v_t^-\right] \end{cases}$$

3.2.3 The solution algorithm

This program has to be solved numerically. To make it simpler, I assume that θ_t can take only a finite number of values: $\theta_t \in \Omega = \{\theta_1, \theta_2 ... \theta_n\}$; and follows a markov process with transition matrix Γ.

[8]Except in the degenerate case where θ can only take a single value.

I construct the following value function:

$$V_t(e_t, s_t, \theta_t) = \max_l E_t \left[c_t^b + (1-\eta)\log\left(\frac{c_t^d}{1-\eta}\right) + \beta V_{t+1}(e_{t+1}, s_{t+1}, \theta_{t+1}) \right]$$

and I solve numerically for a fixed point:

$$V_t(e_t, s_t, \theta_t) = V(e, s, \theta), \forall t$$

and the solution to the regulator's problem is

$$l^{soc}(e, s, \theta) \equiv \arg\max_l V(e, s, \theta).$$

The algorithm is:

- Initialization
  - Set parameters $\{\alpha, \beta, \gamma, \delta, \eta\}$
  - Choose Γ and Ω;
  - Set a grid for e, s, and l;
  - Set an initial value function $V^0(e, s, \theta) = V_0$;
  - $n = 0$;
- Repeat until convergence ($V_{n+1} \simeq V_n$)
  - $V^{n+1}(e_t, s_t, \theta_t) = \max_{l_t(e_t, s_t, \theta_t)} E_t \left[c_{t+1}^b + (1-\eta)\log\left(\frac{c_{t+1}^d}{1-\eta}\right) + \beta V^n(e_{t+1}, s_{t+1}, \theta_{t+1}) \right]$;
  - $n = n + 1$;

The optimal capital requirement policy Then, assuming that *condition 4* is never binding, the regulator can implement $l^{soc}(e, s, \theta)$ through the following capital requirement:

$$l_t \leq x_t e_t,$$

where: $x_t = \frac{l^{soc}(e_t, s_t, \theta_t)}{e_t}$.

When bankers are price takers, *condition 4* indeed never binds at the social optimum. In *subsection 4.3*, I explore the case where bankers have some monopoly power, and discuss the conditions under which this condition binds, and the implications in terms of resource allocation.

4 Results

4.1 A representative example of optimal capital requirements

I present here a representative case (in the sense that the qualitative results are not very sensitive to parameter changes) in which the optimal capital requirements are higher in good states than in bad states (which corresponds to the idea that banks should "accumulate" bigger capital buffers during good times).

Parametrization:

- $\{\alpha = 0.33, \beta = 0.99, \gamma = 0.5, \delta = 0.9, \eta = 0.025\}$

The value for α reflects the traditional share of capital in the macro literature. The regulator discount factor β should be smaller than 1 to ensure convergence. $\gamma = 0.5$ says that a bailout burden is shared equally by the two generations present at the time of the crisis. $\delta = 0.9$ corresponds to a case where most of the bank value is captured by shareholders and managers.

- $\Omega = \{0.8; 1; 1.2\}$
- $\Gamma = \begin{matrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{matrix} \begin{pmatrix} 0.75 & 0.15 & 0.10 \\ 0.33 & 0.34 & 0.33 \\ 0.10 & 0.15 & 0.75 \end{pmatrix}$

Note that I haven chosen the transition matrix so that the bad and the good state are relatively persistent: the probability to remain in the given state is 0.75.

Optimal policy function: Under this parametrization the optimal requirements $x(e, s, \theta)$ are:

1. Increasing in e : this is the same effect as in the comparative statics exercise exposed in *section 2*. Due to the decreasing marginal return to lending, socially optimal lending increases in e but less than proportionally to leverage.

2. Decreasing in θ: this is a "persistence effect". Being in the bad state today tells us that it is most likely to be in the bad state tomorrow. The conditional production function is thus at the left of the unconditional one, which means that optimal lending moves to the left.

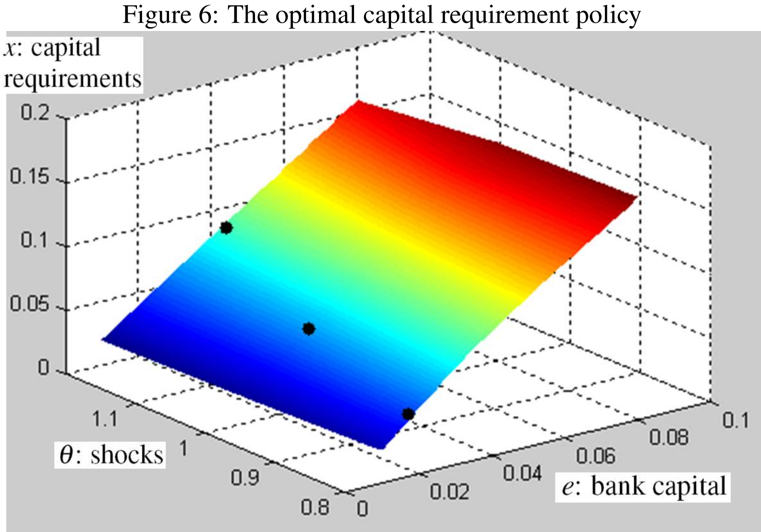
3. Decreasing in s : when depositors are richer, their marginal utility of consumption is lower, they can therefore tolerate more risk.

Optimal capital requirements over the cycle Here is a description of what happens after a good draw ($\theta_3 = 1.2$):

- e increases because of both "retained profits" ($(1-\delta)v^+$) and large input of fresh capital (ηw_t) due to high wages;
- The most likely state in the next period is the good state, therefore the production function is to the right of the unconditional one;
- s increases because of high wages and no bailout tax.

We thus have different effects going in different directions. While effect 1 (described above) pleads for more stringent capital requirements, effects 2 and 3 suggest them to be relaxed. For most possible parametrization, and as illustrated in *figure 6,* the first effect dominates. This means that capital requirements should increase in good times (that is they should be "pro-cyclical" in the sense that they should increase when productivity is high) so that bankers hold proportionally more capital in good times than in bad times. This seems to correspond to what is usually called "counter-cyclical capital buffers", in the sense that, compared to constant capital requirements, they tend to mitigate the cycle rather than magnifying it.

Figure 6: The optimal capital requirement policy

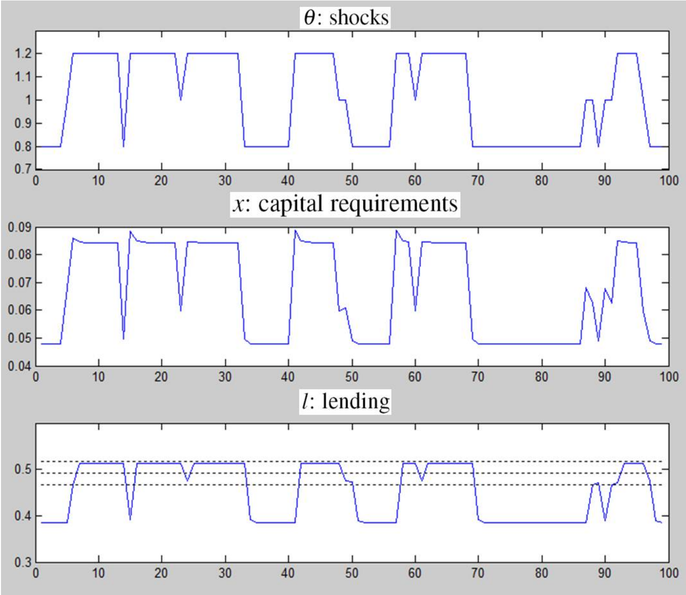


The colored surface gives the the optimal capital requirement (x) as a function of aggregate bank capital (e) and the last realization of the shock (θ). The optimal capital requirement (x) is increasing in the former, and decreasing in the latter.

The black points represent the average optimal capital requirement for the three possible values and the average value of e in the respective states. This illustrates that the first effect dominates.

To fix ideas, *figure 7* displays a simulation of 100 periods with the same parametrization.

Figure 7: The persistence case



The first panel shows the realizations of the shocks (the possible states are $\Omega = \{0.8; 1; 1.2\}$, and the transition matrix is Γ. (Recall that the probability to stay in the bad or the good state is 0.75.).
The second panel shows the optimal capital requirements.
In the third panel, the solid line corresponds to the resulting aggregate level of lending, and the dashed line correspond to the level that would maximize the size of the economy in the three states. In line with the intuition of the static model, the regulator chooses to be closer to that level when there is more banking capital available to cover the potential losses.

4.2 A comparison with "through the cycle" capital requirements

I present here a comparison with a constant (or through-the-cycle) capital requirement.

4.2.1 Exercise

The exercise is the following: I restrict the regulator to the choice of a single number x^{fixed}, which can therefore not be contingent on e, s, and θ. To find the "optimal" x^{fixed}, I simulate the economy for different values of this number and let the regulator pick the one that maximizes the average utility of 10000 generations.

Remark:

When lending is very high, each banker has still an incentive to increase leverage, but the sector becomes unprofitable at some point. In such a case, I consider that some bankers simply do not enter and chose to store their savings. The bankers that enter the industry still find it optimal to leverage to the max. Then measure of bankers that enter at equilibrium is pined down by the condition that if they all leverage up to x^{fixed}, they get their reservation utility. Bankers are thus indifferent whether to enter or not.

4.2.2 Results

First, note the first two following results:

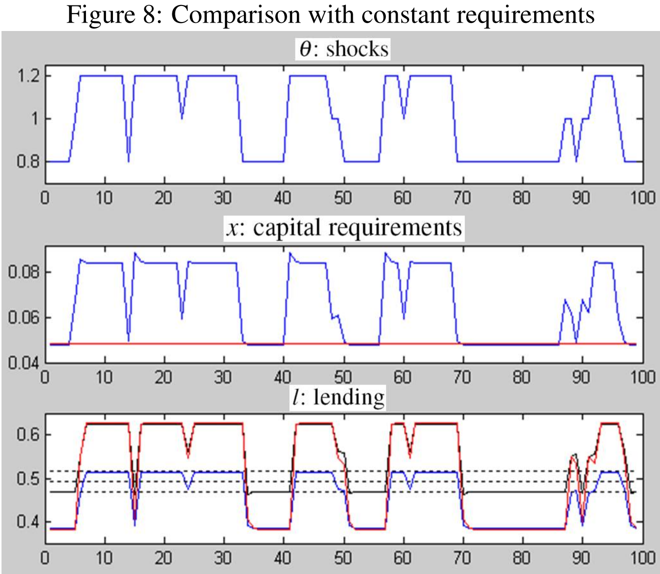
- Lending is much more volatile under constant capital requirement, which illustrates that such regulation can have "pro-cyclical" effects (in the sense that it exacerbates the cycle).
- Negative present value (when discounted at the rare of return to storage) loans are issued during booms. This is what I call "extremely excessive credit expansion". "Excessive credit expansion" would correspond to too much lending with respect to the social optimum, but the loans still being positive net present value when discounted at the rate of return to storage.

Second, there are redistribution issues:

- When credit expands, there is more capital in the economy and therefore wages go up. This implies that despite a decrease in total output, depositors' mean consumption is higher than under dynamic optimal requirement.
- It may not be a desirable feature, but it explains why the optimal constant capital requirement is relatively low. It is indeed close to the minimum optimal dynamic capital requirement. The intuition is that the regulator favors "v shaped" recoveries because an increase in wages not only directly increases depositors consumption, but also next period banking capital.

Figure 8 illustrates this comparison with a simulation similar to that above (same realization of the shocks).

Figure 8: Comparison with constant requirements



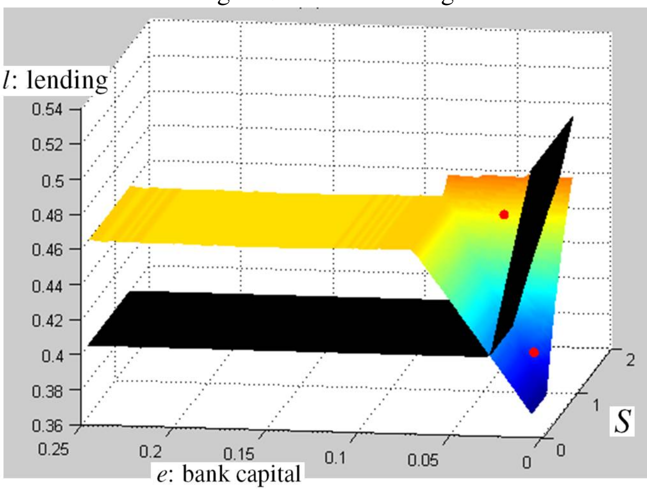
4.3 Risk-taking incentives, excessive lending and... insufficient lending (to complete)

I explore here the case where bankers have some monopoly power.

To capture this, I assume that when they issue a loan, they understand that it decreases the marginal return, but still, they do not internalize the fact that more capital increases the equilibrium wage.

In such a case, it happens that the bankers' first order condition is binding at levels below the social optimum, as is illustrated in the following figure.

Figure 9: Under-lending



The multicolor locus is the socially optimal level of lending. The black locus is the bankers' private optimum. The two red balls represent the two possible cases: depending on parameter values, the constraint may be binding or not.

5 Conclusion

To do.

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A Basel III (A. 4. 18)

4. Reducing procyclicality and promoting countercyclical buffers
18. One of the most destabilising elements of the crisis has been the procyclical amplification of financial shocks throughout the banking system, financial markets and the broader economy. The tendency of market participants to behave in a procyclical manner has been amplified through a variety of channels, including through accounting standards for both mark-to-market assets and held-to-maturity loans, margining practices, and through the build up and release of leverage among financial institutions, firms, and consumers. The Basel Committee is introducing a number of measures to make banks more resilient to such procyclical dynamics. These measures will help ensure that the banking sector serves as a shock absorber, instead of a transmitter of risk to the financial system and broader economy.