Information Aggregation and Investment Decisions*

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Abstract

We study a role of asset prices in aggregating information and guiding real investment. First, we develop a tractable, yet flexible class of noisy REE models of a financial market with a general specification of asset's payoffs. We show that the interplay between noisy information aggregation and firm decisions leads to a systematic wedge between asset prices and expected firm value, conditional on the information contained in market prices. From an ex ante perspective, the expected wedge, i.e. the difference between the expected price and the expected dividend, is positive (negative) if the dividend function is convex (concave). The expected wedge is zero if the dividend function is linear, as is typically assumed in standard REE models. Second, we apply our framework to study the interaction between information aggregation in financial markets and firm's investment decisions. The option value inherent in firm's use of the information contained in market prices convexifies the payoff. This implies that expected share prices exceed expected dividends. Third, we show that linking managerial compensation to share prices gives managers an incentive to manipulate the firm's decisions to their own benefit at the share-holders' expense. By conditioning their decisions excessively on the information conveyed through market prices, managers can inflate the share price by taking inefficient investment decisions, which reduces expected dividends. This further amplifies the wedge between share price and expected firm value.

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1 Introduction

A key role played by asset prices is aggregation of information about the value of firms. By pooling together the dispersed knowledge of individual actors, prices provide information that helps shape investor expectations and portfolio decisions. To the extent that the information conveyed in prices is not already known within the firm, prices also affect the firm's assessment of the value of investment projects and guide real allocations. If information aggregation is perfect, asset prices fully reflect current expectations of future dividends and provide a parsimonious way of conveying information and guiding real investment. If managerial compensation is linked to share prices, the price mechanism also aligns the managers' incentives with the best interest of shareholders.

This paper reconsiders the role of asset prices in aggregating information and guiding real investment, when information aggregation is imperfect, and share prices offer only a noisy signal of the underlying fundamentals. We consider a setting in which firm's shares are traded in a financial market, in which the share price emerges as a noisy signal pooling the dispersed information investors may have about the firm's fundamental value. The firm then takes an investment decision based on the information conveyed by the share price. The investors in turn anticipate the firm's decision in their trading strategies. In equilibrium, the resulting feedback determines the firm's share price, investment decision, and its dividend value.

As our main results, we show that the interplay between noisy information aggregation and firm decisions leads to a systematic wedge between asset prices and expected firm value, conditional on the information contained in market prices. From an ex ante perspective, this wedge has a positive expected value, i.e. expected prices exceed expected firm value. Moreover, linking managerial compensation to share prices gives managers an incentive to manipulate the firm's decisions to their own benefit at the share-holders' expense. By conditioning their decisions excessively on the information conveyed through market prices, managers can inflate the share price, while reducing expected dividends. This further amplifies the wedge between share price and expected firm value.

To develop these results, we propose a tractable, yet flexible model of noisy information aggregation in financial markets. We consider a market structure in which informed traders are risk-neutral and face constraints on portfolio holdings. This environment allows us substantially more flexibility in specifying the firm's dividends, its investment decision, and the nature of informational asymmetries between investors than the canonical models of noisy information aggregation with CARA preferences and normally distributed dividends.[1]

More specifically, the paper proceeds as follows. We first develop our financial market model assuming that dividends are an exogenous function of underlying fundamentals. We show that a systematic wedge emerges between the share price and the expected dividends, whenever the share price aggregates dispersed shareholder information with noise: the expected dividends conditional on the information conveyed by the price always increase less than one-for-one with the price. In other words, prices respond more to shocks in the underlying environment than would be justified by their information content about expected future dividends. The share price then is higher (lower) than the expected dividends when expected fundamentals conditional on the price are high (low). We label this wedge between share price and expected firm value the information aggregation wedge.

The intuition for the wedge is as follows. Any shareholder's expectation of future dividends (and, hence, his trading decisions) is based both on his private information and on the information conveyed through the price. The expectation of future dividends conditional on market information, on the other hand, only reflects the information conveyed by the price. From the perspective of any individual shareholder, the noise in market information is uncorrelated with the noise in private signals. In the aggregate, however, the trading equilibrium induces a positive correlation between the noise in the private signal of those shareholders who end up holding the shares, and the noise in the market signal provided by the share price. As long as the shareholder's private information gets reflected in the share price, the price then responds more strongly to fundamental shocks than would be implied solely by its effect on expected dividends.

We show that from an ex ante perspective, the expected information aggregation wedge between the share price and the firm value may be positive or negative, depending on whether the dividend is a convex or a concave function of the underlying fundamentals. The slope of the dividend function determines shareholder's exposure to fundamental shocks, i.e. how much the dividends vary with changes in fundamentals. The shareholders' exposure determines the absolute magnitude of the information aggregation wedge, for given realizations of the share price. When dividends are a linear function of fundamentals, the shareholders' exposure is symmetric around the prior expectation, and the positive wedge on the upside exactly offsets the negative wedge on the downside. From an ex ante perspective, the expected wedge is zero. When instead the dividends are a convex function of fundamentals, the firm's upside risk is larger than its downside risk, and the positive

[1] Among many others, see Grossman and Stiglitz (1980), Hellwig (1980) and Diamond, and Verrecchia (1981), as well as recent textbook treatments by Brunnermeier 2001, Vives 2008, and Veldkamp 2011.

wedge on the upside exceeds the negative wedge on the downside. From an ex ante perspective, the expected wedge is positive. The opposite reasoning applies with concave dividend functions, where the downside risk exceeds the upside risk in dividends, and the expected wedge is negative.[2]

Second, we endogenize the dividend allowing the firm to make an investment decision after observing its own share price. We solve the model in a closed form and derive a simple expression for the average wedge across states of nature.[3] We show that the dividend function in our model of endogenous investment is convex, resulting in a positive expected wedge. We further decompose the expected wedge into two key components. First, the reaction of market prices to expected fundamentals determines the magnitude of the conditional wedge for a fixed level of investment. Second, the variability of firm's posterior expectation about fundamentals captures the value of market information for firm's investment problem. Our expression for the wedge illustrates the role of two central elements that originate the wedge in our model: the dispersed nature of information and the value of market information for firm's decision.

The option value inherent in firm's ex post use of market information convexifies its expected dividends (e.g., Dixit and Pindyck 1994). As the firm takes on more exposure to the fundamentals in good states than in bad states, the information aggregation wedge is asymmetric and larger in absolute value, when it is positive. The feedback from information aggregation to firm decisions leads to share prices that are higher than expected dividends from an ex ante perspective. This positive information aggregation wedge emerges even when firms' management acts in the best interest of its shareholders, and when shareholders are perfectly rational. Moreover, the investment decisions of the firm are efficient.

Thirdly, we consider a model in which managerial compensation is tied to market valuations. Specifically, we assume manager's objective is to maximize a weighted sum of expected dividends and the price. Because of the information aggregation wedge, investment decisions that maximize firm's share price need not maximize its expected dividend. The manager can manipulate the share price to his benefit at the shareholder's expense, by responding more aggressively to the

[2]The information aggregation wedge also emerges in a canonical CARA-normal setting. However, the restrictions imposed in the CARA normal setting only allow for the case of a linear dividend payoff and a symmetric wedge. If on average the asset is in zero net supply, the ex ante expected price then coincides with ex ante expected dividends. When the average net supply of the asset is not zero, a difference between price and expected dividends emerges due to risk premia, which are absent in our model. We further analyze this issue in the appendix.

[3]Our model with endogenous investment can be solved in a standard framework of noisy information aggregation only under a restrictive assumption that the firms have no proprietary information and firm's decisions are perfectly anticipated by the market.

information conveyed through the price. The manager overinvests and increases the exposure in high states when the wedge is positive, but under-invests and reduces exposure in low states when it is negative. In contrast to the case in which the manager acts in the shareholder's best interest, compensation tied to the price results in higher share prices and lower expected dividends. The extent of this manipulation is increasing in the degree to which compensation is linked to share prices.

Finally, we extend the model in several dimensions. First, we generalize our noise trading assumption to include uninformed traders who trade partly for exogenous motives, and partly in response to the perceived wedge between expected dividends and prices. The price elasticity of uninformed trader's demand determines the price impact of private information by the informed shareholders. The information aggregation wedge is largest in illiquid markets when informed traders have a large price impact. Thus, the better uninformed traders are able to arbitrage the discrepancy between expected dividends and price, the smaller is the information aggregation wedge and the resulting investment distortions. Second, we consider an environment in which a signal about market-specific information is also observed by the firm. This decreases firm's reliance on market prices, reducing the option value component of market information as well as the temptation to manipulate the wedge when managerial incentives are tied to prices. The resulting information aggregation wedge is smaller, and firm's investment decision becomes more efficient. Thirdly, we also explore the role of assumptions on the dividend structure and investment costs, study a variant in which firm-specific proprietary information is partially known to the market, and a variant in which firm's investment decision is perfectly anticipated by the market.

Related Literature

Our model of information aggregation shares similar features with a large literature on noisy information aggregation in rational expectations models, including the papers cited above. Our alternative formulation of the asset market draws on Hellwig, Mukherji, and Tsyvinski (2006), and may prove convenient for other applications that require a more flexible specification of dividends than the canonical setting with CARA preferences and normal distributions allows for. The price for this gain in flexibility comes in form of the assumption of risk neutrality, which imposes strong restrictions on the shareholders' preferences.

The information aggregation wedge appears to have received little attention by the literature - to our knowledge, the only explicit discussion of the excess sensitivity of prices relative to fundamentals appears in Vives (2008), and is by its nature limited to the linear, symmetric case. In contrast, the added flexibility of our model offers new results linking asset over- or under-valuation to the shape of the asset's payoff, without any reference to compensation for risk on the downside, or leverage, speculation, other frictions or behavioral trading motives on the upside.

Our model relating firm investment to market information is most closely related to the literature on REE models with the feedback effect in which real decisions depend on the information contained in the price (e.g., Goldstein and Guembel 2008, Dow and Rahi 2003, Dow, Goldstein, and Guembel 2010). These models build primarily on environments in which there is only market-specific information, and focus more on the implications for investment efficiency and firm value than on asset pricing consequences. Dow and Rahi (2003) study risk-sharing and welfare in a setting with endogenous investment. Their CARA-Normal setup imposes restrictions on the information structure, which imply that the firm's decision can be directly inferred from the share price. Goldstein and Guembel (2008) focus on the strategic aspect of the feedback effect. They show that when traders exploit the impact of their demand on prices and investment, manipulative short-selling strategies that distort the firm's investment decision can be profitable. Dow, Goldstein, and Guembel (2010) study a setup with endogenous information production. Since speculators' incentives to produce information increase with the ex-ante likelihood of an investment opportunity, the authors find that small changes in fundamentals can cause large shifts in investment and firm value. There are two important differences with those models. First, in our model of endogenous investment we derive an explicit characterization of the environment in which both the firm and market have private information. Second, we focus more on the resulting asset pricing implications of our model, as well as the link between asset prices, expected dividends and managerial incentives.

Our analysis of managerial compensation tied to share prices is most closely related to Benmelech, Kandel, and Veronesi (2010). They build a dynamic REE setting where managerial effort postpones the decline in growth opportunities of a firm. When growth rates slow down, share price compensation incentivizes managers to conceal the true state by over-investment in negative NPV projects. Price-based incentives thus imply a tradeoff: while inducing high effort in early stages, it leads to concealment and suboptimal investment in later part of the firm's life-cycle. The central difference of our model is that investment distortions in our setup do not relate to misreporting, but rather arise from the excessive weighing of market information in the signal extraction problem of managers.

Our paper is related more broadly to a large literature on REE models with investment. Leland (1992) addresses efficiency considerations in a model with insider trading, where information aggregation affects the level of available funding to the firm. Dow and Gorton (1997) study a dynamic model of feedback effects in a setup where prices accurately reflect public information, and distortions arise from differences in horizons between managers and shareholders. Subrahmanyam and Titman (1999) endogenize investment on share prices, but assume it affects a growth option independent of the dividend of shareholders: share prices thus convey information about fundamentals, but do not internalize their impact on investment. Angeletos, Lorenzoni, and Pavan (2010) model the interaction between early investment choices by entrepreneurs and the later transfer of firm property to traders. An informational advantage that originates from the dispersed nature of entrepreneur information induces a speculative motive that causes excess non-fundamental volatility in real investment and asset prices. Goldstein, Ozdendoren, and Yuan (2010) discuss efficiency considerations and trading commonality (frenzies) arising from the socially suboptimal weighting of private and exogenous public signals about fundamentals. An important difference with the latter two papers is that we allow all agents to simultaneously condition on equilibrium prices when making trading and real investment decisions. As a result, there are no strategic complementarities that deviate the weighting of public and private sources of information from the optimal signal extraction problem of a future price, which is at the heart of the mechanism highlighted by these authors.

The rest of the paper is structured as follows. Section 2 describes a simple financial market model with exogenous dividends to illustrate the basic mechanism behind the information aggregation wedge. Section 3 introduces the model with endogenous investment decisions, highlighting the central role of this mechanism for generating a positive average wedge between share prices and firm's dividends. This section also includes the analysis of the effects of tying managerial incentives to share prices. In section 4, we analyze various extensions and robustness exercises to our baseline model. Section 5 concludes.

2 A Model with Exogenous Dividend

In this section, we develop a simple model of noisy information aggregation in a financial market in which an asset's dividends are exogenously given. This model serves as a building block for later sections in which we consider endogenous investment decisions, as well as more general payoff specifications and information environments.

2.1 Agents, Information Structure, and Financial Market

We formulate the trading environment as a Bayesian game between a unit measure of risk-neutral, privately informed traders, and a 'Walrasian auctioneer'.

Initially, nature draws a stochastic "fundamental" θ, which is normally distributed according to $\theta \sim \mathcal{N}(\mu, \lambda^{-1})$, with mean μ, and unconditional variance λ^{-1},

Each informed trader is endowed with one share of a firm, which pays a final dividend $\pi(\theta)$ to its shareholders. The function $\pi(\cdot)$ is strictly increasing and twice continuously differentiable.

Each informed trader i then receives a noisy private signal x_i about firm's fundamental. This signal is normally distributed according to $x_i \sim \mathcal{N}(\theta, \beta^{-1})$, with a mean θ and a variance β^{-1}, and is i.i.d. across traders (conditional on θ). Traders then participate in an asset market in which they decide whether to hold or sell their share at the market price, P. Specifically, trader i submits a a price-contingent supply schedule $s_i(\cdot) : \mathbb{R} \to [0, 1]$, to maximize her expected wealth $w_i = (1 - s_i) \cdot \pi(\theta) + s_i \cdot P$. By restricting supply to $[0, 1]$, we assume that traders can sell at most their endowment, and cannot buy a positive amount of shares. Individual trading strategies are then a mapping $s : \mathbb{R}^2 \to [0, 1]$ from signal-price pairs (x_i, P) into the unit interval. Aggregating traders' decisions leads to the aggregate supply function $S : \mathbb{R}^2 \to [0, 1]$,

$$S(\theta, P) = \int s(x, P) d\Phi(\sqrt{\beta}(x - \theta)), \tag{1}$$

where $\Phi(\cdot)$ denotes a cumulative standard normal distribution, and $\Phi(\sqrt{\beta}(x - \theta))$ represents the cross-sectional distribution of private signals x_i conditional on the realization of θ.[4]

Nature then draws a random demand of shares by "noise traders". We assume the demand shock has the form $D(u) = \Phi(u)$, where u is normally distributed with mean zero and variance δ^{-1}, $u \sim \mathcal{N}(0, \delta^{-1})$, independently of θ. This specification is from Hellwig, Mukherji, and Tsyvinski (2006).[5] This functional form allows us to preserve the normality of posterior beliefs and retains the tractability of Bayesian updating.

Once informed traders have submitted their orders and the exogenous demand for shares is realized, the auctioneer selects a price P that clears the market. Formally, the market-clearing price function $P : \mathbb{R}^2 \to \mathbb{R}$ selects, for all realizations (θ, u) a price P from the correspondence $\hat{P}(\theta, u) = \{P \in \mathbb{R} : S(\theta, P) = D(u, P)\}$.[6]

Finally, after all trades have occurred, firm's dividends $\pi(\theta)$ are realized and disbursed to the final shareholders.

Let $H(\cdot|x, P) : \mathbb{R} \to [0, 1]$ denote the shareholders' posterior cdf of θ, conditional on observ-

[4]We assume that the Law of Large Numbers applies to the continuum of traders, so that conditional on θ the cross-sectional distribution of signal realizations ex post is the same as the ex ante distribution of traders' signals.

[5]We generalize this demand specification in Section 4.2 allowing for price-elastic demands by noise traders.

[6]If the function $\pi(\cdot)$ is bounded, we can without loss of generality restrict the range of $P(\cdot)$ to coincide with the range of $\pi(\cdot)$.

ing a private signal x, and conditional on the market price P. A *Perfect Bayesian Equilibrium* consists of supply functions $s(x,P)$ for informed traders, a price function $P(\theta,u)$, and posterior beliefs $H(\cdot|x,P)$ such that (i) $s(x,P)$ is optimal given $H(\cdot|x,P)$; (ii) the asset market clears for all (θ,u); and (iii) $H(\cdot|x,P)$ satisfies Bayes' rule whenever applicable, i.e., for all p such that $\{(\theta,u) : P(\theta,u) = p\}$ is non-empty.

2.2 Characterization

We begin by characterizing informed traders' supply of shares. With risk-neutrality, supply decisions are equal to either 0 or 1 almost everywhere – an order to hold ($s_i = 0$) or sell the share ($s_i = 1$) at P. The trader's expected value of holding the share is $\int \pi(\theta)dH(\theta|x,P)$. Since private signals are log-concave, posterior beliefs $H(\cdot|x,P)$ are first-order stochastically increasing in x, for any P that is observed in equilibrium. Since $\pi(\cdot)$ is increasing in θ, this implies that the traders' decisions are monotone in x, and characterized by a signal threshold function $\hat{x} : \mathbb{R} \to \mathbb{R} \cup \{\pm\infty\}$, such that

$$s(x_i,P) = \begin{cases} 1 & \text{if } x_i < \hat{x}(P), \\ 0 & \text{if } x_i > \hat{x}(P), \\ \in [0,1] & \text{if } x_i = \hat{x}(P), \end{cases} \tag{2}$$

so a trader sells if $x_i < \hat{x}(P)$ and holds if $x_i > \hat{x}(P)$. We call the informed trader who observes the signal equal to the threshold, $x = \hat{x}(P)$, and who is therefore indifferent, the *marginal trader*. The supply threshold is uniquely defined by

$$\begin{aligned} \hat{x}(P) &= +\infty \quad \text{if } \lim_{x\to+\infty} \int \pi(\theta)dH(\theta|x,P) \leq P, \\ \hat{x}(P) &= -\infty \quad \text{if } \lim_{x\to-\infty} \int \pi(\theta)dH(\theta|x,P) \geq P, \\ P &= \int \pi(\theta)dH(\theta|\hat{x}(P),P) \;\; \text{otherwise.} \end{aligned} \tag{3}$$

Expression (3) illustrates three cases: (*i*) if the most optimistic trader's expected dividend is lower than the price, all traders sell so the signal threshold becomes $+\infty$; (*ii*) if the most pessimistic trader's expected dividend exceeds the price, all traders keep the share and the threshold for selling is $-\infty$; (*iii*) only some traders sell, and the threshold $\hat{x}(P)$ takes an interior value at which the marginal trader's posterior expectation of the dividend must equal the price. Aggregating the individual supply decisions, the market supply is $S(\theta,P) = \int_{-\infty}^{\hat{x}(P)} 1 \cdot d\Phi(\sqrt{\beta}\,(x-\theta)) = \Phi(\sqrt{\beta}\,(\hat{x}(P)-\theta))$, which equals 1 if $\hat{x}(P) = +\infty$, and 0 if $\hat{x}(P) = -\infty$.

Next, we analyze the market-clearing condition. Since $D(u) \in (0,1)$, in equilibrium, $\hat{x}(\cdot)$ must be finite for all P on the equilibrium path, and satisfy the third condition in (3). Equating demand

and supply, we characterize the correspondence of market-clearing prices:

$$\hat{P}(\theta, u) = \left\{ P \in \mathbb{R} : \hat{x}(P) = \theta + \frac{1}{\sqrt{\beta}} u \right\}. \tag{4}$$

From now on, we focus on equilibria in which the price is conditioned on (θ, u) through the state variable $z \equiv \theta + 1/\sqrt{\beta} \cdot u$. The equilibrium beliefs are characterized in the next lemma. All proofs are provided in the appendix.

Lemma 1 (Information Aggregation) *(i) In any equilibrium with conditioning on z, the equilibrium price function $P(z)$ is invertible. (ii) Equilibrium beliefs for price realizations observed along the equilibrium path are given by*

$$H(\theta|x, P) = \Phi\left(\sqrt{\lambda + \beta + \beta\delta}\left(\theta - \frac{\lambda\mu + \beta x + \beta\delta\hat{x}(P)}{\lambda + \beta + \beta\delta}\right)\right). \tag{5}$$

Part (i) of Lemma 1 shows that in any equilibrium, the price function must be invertible with respect to z, implying that the observation of P is equivalent to observing z. If the price function is not invertible, then some price realization P would be consistent with multiple realizations of z. But (4) implies that P then cannot be consistent with market clearing in all these states simultaneously.

Part (ii) of the Lemma exploits the invertibility to arrive at a complete characterization of posterior beliefs $H(\cdot|x, P)$. With invertibility, we can summarize information conveyed by the price through z, and note that conditional on θ, z is normally distributed with mean θ and variance $(\beta\delta)^{-1}$. Thus, the price is isomorphic to a normally distributed signal of θ, with a precision that is increasing in the precision of private signals, and decreasing in the variance of demand shocks.

Using Lemma 1 we rewrite (3), the indifference condition that defines the signal threshold $\hat{x}(P)$:

$$P = \int \pi(\theta) d\Phi\left(\sqrt{\lambda + \beta + \beta\delta}\left(\theta - \frac{\lambda\mu + \beta(1+\delta)\hat{x}(P)}{\lambda + \beta + \beta\delta}\right)\right). \tag{6}$$

This condition equates the price (on the left-hand side of (6)) to the marginal trader's expectation of dividends on the right-hand side. Notice that the latter is also influenced by P, through its effect on posterior beliefs. Using the market-clearing condition, we uniquely characterize the equilibrium price, $P(z)$, and the expected dividend conditional on public information, $V(z)$, as a function of z.

Proposition 1 (Asset market equilibrium) *Define $P(z)$ as*

$$\begin{aligned} P(z) &= \mathbb{E}(\pi(\theta)|x = z, z) \\ &= \int \pi(\theta) d\Phi\left(\sqrt{\lambda + \beta + \beta\delta}\left(\theta - \frac{\lambda\mu + (\beta + \beta\delta)z}{\lambda + \beta + \beta\delta}\right)\right), \end{aligned} \tag{7}$$

The asset market equilibrium is characterized by the price function $P(z)$ and the traders' threshold function $\hat{x}(p) = z = P^{-1}(p)$. The expected dividend conditional on public information z, denoted $V(z)$, is

$$V(z) = \mathbb{E}(\pi(\theta)|z) = \int \pi(\theta) d\Phi \left(\sqrt{\lambda + \beta\delta} \left(\theta - \frac{\lambda\mu + \beta\delta z}{\lambda + \beta\delta} \right) \right). \tag{8}$$

Proposition 1 characterizes the asset market equilibrium. If (and only if) the price function is invertible, traders infer the state z from the price. In our case, this directly follows from the strict monotonicity of $\pi(\theta)$. The resulting price function $P(z)$ is uniquely defined.[7]

The price $P(z)$, and the expected dividend conditional on public information, $V(z)$, differ in how expectations of θ are formed. The price equals the dividend expectation of the marginal trader who is indifferent between keeping or selling her share. This trader conditions on the market signal z, as well as a private signal, whose realization must equal the threshold $\hat{x}(P)$ in order to be consistent with the trader's indifference condition. The trader treats these two sources of information as mutually independent signals of θ. At the same time, the market-clearing condition implies that $\hat{x}(P)$ must equal z in order to equate demand and supply of shares. The marginal trader's expectation $\mathbb{E}(\pi(\theta)|x = z, z)$ thus behaves as if she received one signal z of precision $\beta(1+\delta)$ instead of $\beta\delta$. In contrast, the expected dividends $\mathbb{E}(\pi(\theta)|z)$ conditional on P (or equivalently z) weighs z according to its true precision $\beta\delta$.

Alternatively, we can view the difference in the responsiveness of price and the expected dividend conditional on the price, as the result of a compositional change in the identity of the traders holding the shares. An increase in z raises the price and expected dividend through a direct effect on all traders' expectations. This is reflected in the weight $\beta\delta$ attributed to z in both $P(z)$ and $V(z)$. In addition, an increase in z changes the identity of the marginal trader: since the random demand for shares is larger (on average) for a higher z, market clearing requires more selling from the original shareholders. This implies that the new marginal trader must have higher expectations about the dividend, which holds in equilibrium since her private signal now corresponds to the higher realization of z.[8] The resulting extra shift in prices is captured by the additional weight β attributed to z in the price function.[9]

[7] Notice that this only implies the uniqueness of the equilibrium that conditions on the summary statistic z, not overall uniqueness of the equilibrium characterized in proposition 1.

[8] If z increases because of θ, the distribution of private signals shifts up, decreasing supply for a given P. If instead z increases because of u, the distribution of signals and hence supply remains unchanged, but the demand for shares has gone up. Of course, in equilibrium traders cannot disentangle these two possibilities from observing the price.

[9] A different way to illustrate the role of belief heterogeneity is to compare our equilibrium with a market in which all shareholders have access to a common signal z of fundamentals. In that case, all shareholders must be indifferent

2.3 The Information Aggregation Wedge

The main asset pricing implication of Proposition 1 is that at the interim stage –when the share price is observed but before dividends are realized– the equilibrium price generally differs from the expected dividend, conditional on the public information. We label this difference the *information aggregation wedge*, $W(z) \equiv P(z) - V(z)$. Our first theorem characterizes how the expectation of this wedge depends on the shape of the dividend function $\pi(\theta)$:

Theorem 1 ***(Average wedge):*** *In the equilibrium of Proposition 1, the sign of the unconditional information aggregation wedge depends on the convexity of* $\pi(\theta)$*;*

$$\begin{aligned} \pi'(\mu+\epsilon) &> \pi'(\mu-\epsilon) \text{ for all } \epsilon > 0 \Longrightarrow \mathbb{E}\left(W\left(z\right)\right) > 0 \\ \pi'(\mu+\epsilon) &= \pi'(\mu-\epsilon) \text{ for all } \epsilon > 0 \Longrightarrow \mathbb{E}\left(W\left(z\right)\right) = 0 \\ \pi'(\mu+\epsilon) &< \pi'(\mu-\epsilon) \text{ for all } \epsilon > 0 \Longrightarrow \mathbb{E}\left(W\left(z\right)\right) < 0 \end{aligned}$$

This theorem states that the unconditional expectation of the gap between prices and expected dividends is determined by the second derivative of the dividend function, as a function of the underlying state. As a starting point, suppose that $\pi(\cdot)$ is linear, $\pi(\theta) = \theta$. Panel a) of figure 1 plots the price (the thick solid line) and expected dividend (the thick dashed line) as a function of the state variable z. In this case, we can compute the price, the expected dividend and the wedge as

$$P(z) = \frac{\lambda\mu + (\beta + \beta\delta)z}{\lambda + \beta + \beta\delta}, \qquad V(z) = \frac{\lambda\mu + \beta\delta z}{\lambda + \beta\delta}, \qquad W(z) = (\gamma - 1)\,(z - \mu)\,\frac{\beta\delta}{\lambda + \beta\delta},$$

where

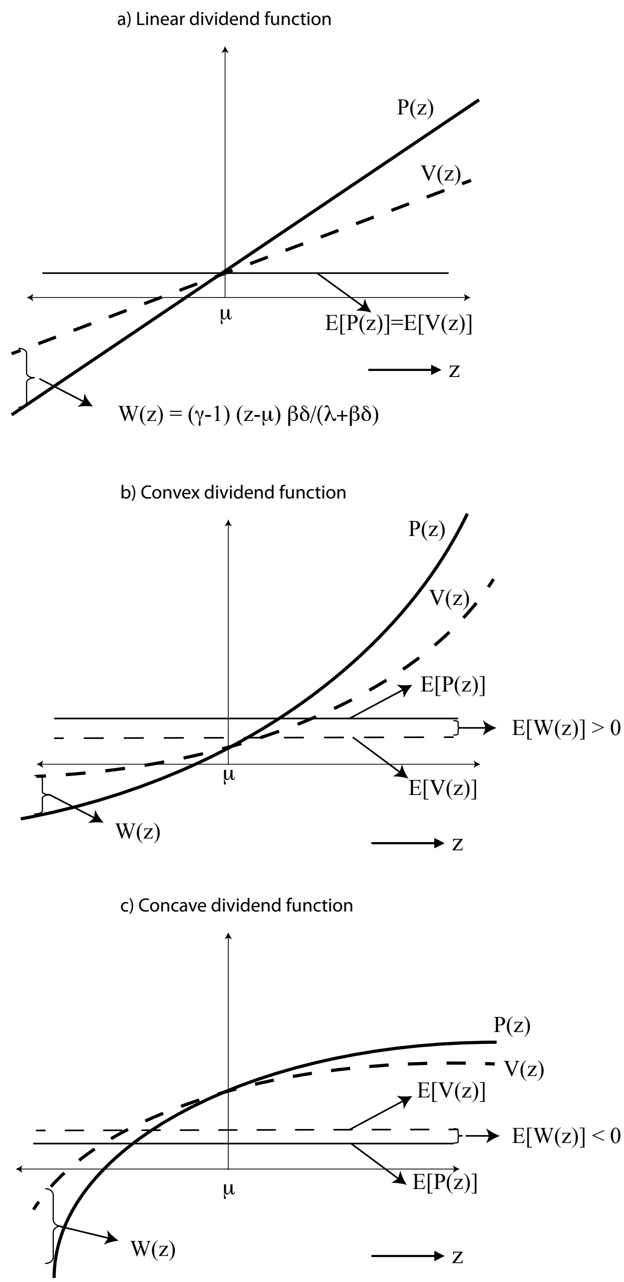
$$\gamma \equiv \frac{\beta + \beta\delta}{\lambda + \beta + \beta\delta} / \frac{\beta\delta}{\lambda + \beta\delta} > 1 \tag{9}$$

is the ratio between the Bayesian weight assigned to the market signal z by the marginal trader, and an uninformed outsider who only observes the price. The price is more sensitive to innovations in z than the expected dividend, resulting in a wedge that is negative for $z < \mu$, zero for $z = \mu$, and positive for $z > \mu$. The marginal wedge is symmetric around $z = \mu$ ($W'(\mu + \epsilon) = W'(\mu - \epsilon)$ and $W(\mu + \epsilon) = -W(\mu - \epsilon)$, for all $\epsilon > 0$), when the dividend function is linear. Since z is distributed symmetrically around μ, the unconditional expectation of the wedge is zero.

The same argument applies more generally to any dividend function whose derivative is symmetric around $\theta = \mu$ (implying $\pi(\mu + \epsilon) - \pi(\mu) = -(\pi(\mu - \epsilon) - \pi(\mu))$ for all $\varepsilon > 0$), in which case

between selling and keeping their share (otherwise the market wouldn't clear), so the price must equal the common posterior expectation of the dividend, conditional on z, i.e. $P(z) = V(z)$, and all traders are indifferent between selling or keeping the share

Figure 1: Price, Expected Dividend and Wedge with Exogenous Dividend



$W'(\cdot)$ is symmetric around $z = \mu$, and $\mathbb{E}(W(z))$ is zero. This corresponds to the second case in Theorem 1. When $\pi'(\mu + \epsilon) \geq \pi'(\mu - \epsilon)$ for all $\epsilon > 0$, the absolute value of the conditional wedge on the upside exceeds the absolute value of the conditional wedge on the downside, so the expected wedge $\mathbb{E}(W(z))$ is positive. This is the case in particular whenever the payoff function is convex in θ. When $\pi'(\mu+\epsilon) \leq \pi'(\mu-\epsilon)$ for all $\epsilon > 0$, the absolute value of the wedge in the downside exceeds the absolute value of the wedge in the upside, and the resulting expected wedge is negative. This case applies to all concave dividend functions. These two cases are plotted in panels b) and c) in figure 1.

The magnitude of the wedge at a given realization of z depends on the shareholders' exposure (the net effect in the dividend) to changes in the underlying fundamentals, which locally is captured by the derivative $\pi'(\cdot)$. With convex payoffs, this exposure is increasing in θ, and therefore higher on the upside. With concave payoffs, this exposure is decreasing in θ and hence higher on the downside.

In the online appendix, we derive the information aggregation wedge for a model with with CARA preferences and normally distributed signals and shocks.[10] This model remains tractable as long as the traders' terminal wealth is normally distributed, conditional on their available information. This restricts dividends to be a linear function of fundamentals and implies that if the average supply of the asset is zero the ex ante expected price equals the expected dividend ex ante and the expected wedge is zero. More generally any difference between unconditional expectations of prices and dividends results from risk premia when the average supply of shares differs from zero.

It is important to note that our results on differences between expected prices and dividends are not a consequence of irrational trading strategies, behavioral biases of investors, or agency conflicts. Nor are such differences accounted for by risk premia (since traders are risk neutral). Our model, and theorem 1 in particular, offers a theory in which expected prices and expected dividends can generally differ as a result of the interplay between the dividend structure and the partial aggregation of information into prices, in a context where traders hold heterogeneous beliefs in equilibrium.[11] To our knowledge, this result is new to the literature.

In the next section, we use the insights offered by our general model to study the informational feedback to endogenous investment decisions by firms. We show that the option value of responding to market information endogenously generates a convex dividend function, which results in a

[10] See, e.g., Hellwig (1980), Diamond and Verrecchia (1981), and Grossman and Stiglitz (1980).

[11] Note that our specific limits of arbitrage assumption (individual supplies bounded within $[0, 1]$) is not crucial to the result. Belief heterogeneity and hence the information aggregation wedge remains in equilibrium as long as risk-neutral traders face finite portfolio constraints.

positive wedge from an ex ante perspective.

3 A Model with Endogenous Investment

We now consider a model in which the dividend depends on a real investment decision by firm's manager. We augment the setting in section 2 to include an additional stage in which the firm's manager takes a decision after observing the market price, as well as some private information that influences dividends. The asset market stage is modeled as before, but its outcome is now influenced by the trader's anticipation of firm's manager's response to the market price.

3.1 General Formulation

Formally, we suppose that firm's dividend function takes the form $\pi : \Theta \times Y \times A \to \mathbb{R}$, where $\theta \in \Theta = \mathbb{R}$ denotes firm's fundamental, $y \in Y \subseteq \mathbb{R}$ denotes firm's private information, and $A \subseteq \mathbb{R}$ denotes a compact set from which the firm chooses an action $a \in A$ after observing y and its share price.

As before, nature initially draws the stochastic fundamental θ and the demand shock u, which are independent of each other and distributed according to $\theta \sim \mathcal{N}(\mu, \lambda^{-1})$ and $u \sim \mathcal{N}(0, \delta^{-1})$. In addition, nature draws y, which we assume is distributed according to the conditional cdf. $G : Y \times \Theta \to [0,1]$, where $G(\cdot|\theta)$ denotes the the cdf of y, conditional on θ, and $g(\cdot|\theta)$ the corresponding pdf.

At this stage, and before the market opens, the firm's manager commit to a decision rule $a : Y \times \mathbb{R} \to A$, which is selected to maximize manager's expectation of an objective function $\tilde{\pi}(\theta, y, a, P)$. This objective allows, in particular, for a manager's compensation to depend on the market price.

Each trader i then receives a noisy private signal $x_i \sim \mathcal{N}(\theta, \beta^{-1})$. Traders decide whether to hold or sell their share at the market price P. Individual trading strategies are then a mapping $s : \mathbb{R}^2 \to [0,1]$ from signal-price pairs (x_i, P) into the unit interval. The aggregate supply function $S : \mathbb{R}^2 \to [0,1]$ satisfies $S(\theta, P) = \int s(x,P) d\Phi(\sqrt{\beta}(x-\theta))$. The demand for shares takes the form $D(u) = \Phi(u)$. Once the orders to hold or sell are submitted, and the demand for shares is realized, the price P is selected to clear the asset market. The a market-clearing price function is $P : \mathbb{R}^2 \to \mathbb{R}$. This asset market stage is unchanged from the previous section.

Finally, the firm observes the price, its private information, and implements its decision $a(y, P)$.

Let $H(\cdot|x, P)$ denote the traders' posterior cdf of θ, conditional on observing a private signal

x, and a market-clearing price P. A *Perfect Bayesian Equilibrium* of the augmented game consists of a shareholder's supply function $s(x, P)$, a price function $P(\theta, u)$, a decision rule $a(y, P)$ for the firm, and posterior beliefs $H(\cdot|x, P)$, such that (i) the supply function is optimal given the shareholder's beliefs $H(\cdot|x, P)$ and the anticipated investment rule $a(y, P)$; (ii) $P(\theta, u)$ clears the market for all (θ, u); (iii) $a(y, P)$ solves manager's decision problem; and (iv) $H(\cdot|x, P)$ satisfies Bayes' Rule whenever applicable.

Discussion: The general formulation of our endogenous investment model embeds several important special cases which we will analyze to discuss the role of specific assumptions.

1. If $\pi(\theta, y, a) = \tilde{\pi}(\theta, y, a, P)$, then manager's and final shareholder's objective coincide. The manager maximizes expected dividends. Under this benchmark, manager's decisions will make ex post efficient use of the information conveyed by prices. Moreover, the assumption that the firm pre-commits to a rule is innocuous in this case, as the ex post optimal choice of manager corresponds to the rule to which the firm commits ex ante.

2. If $\pi(\theta, y, a) \neq \tilde{\pi}(\theta, y, a, P)$, then there is a conflict of interest between managers and shareholders. Such a conflict may result from agency problems, moral hazard considerations, or from the structure of manager's compensation contracts. This formulation also allows for compensation contracts explicitly tied to observable market prices. The pre-commitment assumption plays a role for our results in the case of price-based incentives. An ex post choice of the investment decision takes the price P as given, whereas the prior commitment to a rule allows the manager to internalize the effect of its investment decision on market prices.[12]

3. If $\pi(\theta, y, a) = \pi(\theta, y', a)$, for all y, y', and all (θ, a), then y is a noisy signal of the underlying fundamental θ which has no direct payoff implications for the firm.

4. If $G(y|\theta) = G(y)$, for all θ, then the firm has no additional private information about θ. However, the information contained in y is relevant to firm's decision problem. This is the case we focus on in the next section. The firm decides on an investment project whose cost is known to firm's manager. The returns are determined by the fundamental θ and are observed with noise only by shareholders.

The novelty of our investment model is that it allows for feedbacks through the price in presence of both market- and firm-specific information. Traders condition dividend expectations on the

[12] Perhaps a simple way to justify this pre commitment is that the firm's decision making is based on internal reporting, compensation rules and decision making procedures that are updated less frequently than the decisions themselves. In a dynamic environment, the design of such procedures then internalizes the impact of such decisions on future market prices.

price since it provides information about θ, and also because it affects the investment choice of the firm. This is what we call a two-way feedback effect: the price aggregates information and affects investment. The investment decision also affects traders' dividend expectations and trading decisions, which in turn determine the equilibrium price. In the CARA-normal setup, endogenous investment has been modeled assuming that either i) the marginal effect of investment in the dividend does not enter traders' payoffs (Subrahmanyam and Titman (1999); Goldstein, Ozdenoren and Yuan (2010)), or ii) the share price is a sufficient statistic for firm's investment choice (Dow and Rahi (2003)). Our model allows to characterize the two-way feedback effect under more general payoff specifications and information structures. This allows us to discuss a richer set of implications on the relation between managerial incentives, share prices, and investment decisions.

Equilibrium Characterization: The equilibrium characterization proceeds in two stages. The second stage —financial market— proceeds along the same lines as in the previous section. Suppose that firm's manager's decision is characterized by an arbitrary decision rule $a(y,P)$. If $\int \pi(\theta, y, a(y,P))\, dG(y|\theta)$ is monotone in θ, the trader's strategies are characterized as before by threshold rule $\hat{x}(P)$ such that they choose to sell whenever their private signal $x \leq \hat{x}(P)$. Lemma 1 extends immediately to this more general model. The observation of P is equivalent to observing $z = \hat{x}(P) = \theta + 1/\sqrt{\beta} \cdot u$, $z \sim \mathcal{N}(\theta, (\beta\delta)^{-1})$. Along the equilibrium path the shareholders' posterior beliefs are

$$H(\theta|x,P) = \Phi\left(\sqrt{\lambda+\beta+\beta\delta}\left(\theta - \frac{\lambda\mu + \beta x + \beta\delta\hat{x}(P)}{\lambda+\beta+\beta\delta}\right)\right).$$

We continue to focus on equilibria in which the price is conditioned on the (θ, u) through $z = \theta + 1/\sqrt{\beta} \cdot u = \hat{x}(P)$. Equilibrium price and expected dividend solve:

$$P(z) = \mathbb{E}(\pi(\theta, y, a(y, P(z)))\,|x = z, z), \tag{10}$$

$$V(z) = \mathbb{E}(\pi(\theta, y, a(y, P(z)))\,|z), \tag{11}$$

where expectations are taken both with respect to θ and y. This condition implicitly defines the price function. On the right of equation (10), $P(z)$ appears both through the public signal z that it conveys about θ and through its impact on firm's decision $a(y, P(z))$. For given $a(y,P)$, the market equilibrium exists if and only if there exists a strictly monotone solution to the condition for $P(z)$ in (10). In what follows, we disregard the monotonicity requirement at first, and then verify ex post whether it holds at the proposed equilibrium price function.

The manager's first-stage problem —before the financial market opens— is formulated as follows:

$$\max_{a(y,P),P(z)} \int \tilde{\pi}\left(\theta, y, a\left(y, P\left(z\right)\right), P\left(z\right)\right) dG\left(y|\theta\right) d\Phi\left(\sqrt{\beta\delta}\left(z-\theta\right)\right) d\Phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right)$$

s.t.

$$P\left(z\right) = \mathbb{E}\left(\pi\left(\theta, y, a\left(y, P\left(z\right)\right)\right) | x = z, z\right).$$

That is, the manager chooses a price-contingent decision rule, subject to the constraint that the price function is an equilibrium price function.

Using the fact that $P(z)$ must be invertible, we reformulate manger's decision rule as a function of y and z. After changing the order of integration between θ, y and z, the optimal decision is characterized by the solution to the pointwise optimization problem as follows,

$$\begin{aligned} & \max_{a(y,z),P(z)} \int \tilde{\pi}\left(\theta, y, a\left(y, z\right), P\left(z\right)\right) dH_F\left(\theta|y, z\right) \\ s.t.\ P\left(z\right) &= \mathbb{E}\left(\pi\left(\theta, y, a\left(y, z\right)\right) | x = z, z\right), \\ &= \int \pi\left(\theta, y, a\left(y, z\right)\right) dG\left(y|\theta\right) d\Phi\left(\sqrt{\lambda+\beta+\beta\delta}\left(\theta - \frac{\lambda\mu+\beta\left(1+\delta\right)z}{\lambda+\beta+\beta\delta}\right)\right). \end{aligned}$$

The firm's posterior conditional on firm-specific information y and market information z, denoted $H_F\left(\cdot|y, z\right)$, is

$$H_F\left(\theta|y, z\right) = \frac{\int_{-\infty}^{\theta} g\left(y|\theta\right) d\Phi\left(\sqrt{\lambda+\beta\delta}\left(\theta - \frac{\lambda\mu+\beta\delta z}{\lambda+\beta\delta}\right)\right)}{\int_{-\infty}^{\infty} g\left(y|\theta\right) d\Phi\left(\sqrt{\lambda+\beta\delta}\left(\theta - \frac{\lambda\mu+\beta\delta z}{\lambda+\beta\delta}\right)\right)}.$$

As long as the manager's payoff does not depend directly on P, i.e. when $\tilde{\pi}\left(\theta, y, a, P\right) = \tilde{\pi}\left(\theta, y, a\right)$, there is no commitment issue. The rule $a\left(y, z\right)$ that is chosen by the manager ex ante also corresponds to the rule that is ex post optimal from manager's perspective, once the price is taken as given. Moreover, when $\tilde{\pi}\left(\theta, y, a\right) = \pi\left(\theta, y, a\right)$, manager's and final shareholder's incentives are perfectly aligned, and the resulting investment decisions make efficient use of the information conveyed by z. The manager's incentive to manipulate the price to his own benefit and the shareholders' detriment by committing to an investment rule is thus directly linked to an incentive scheme that rewards managers for the share price performance. In the remainder of this section, we compare investment incentives under dividend value maximization with those induced by price-based incentives.

3.2 A Binary Action Model

In this section, we discuss the implications of the feedback effect and the role of managerial incentives in an environment where the manager makes a binary choice.

Assume that the firm's decision is binary, $a \in \{0, 1\}$, where $a = 1$ denotes the decision to invest, and $a = 0$ denotes the decision not to invest. The dividend of the firm is given by:

$$\pi(\theta, F; a) = \rho \cdot \theta + a \cdot (\theta - F), \tag{12}$$

where $\rho > 0$. The dividend has two components. The first is an exogenous effect $\rho \cdot \theta$ of the fundamental, θ, on payoffs.[13] The second component is endogenous. If the firm chooses to invest it incurs a cost F, but its revenue increases by θ. We assume that F is independent of θ, distributed with cdf $G(\cdot)$ and density $g(\cdot)$. Let $\underline{F}$ denote the lower bound of the distribution of F ($\underline{F}$ can be equal to $-\infty$). The cost F is observed by the manager before choosing investment, and is private information within the firm; traders do not observe signals about F. The *firm-specific* cost F summarizes characteristics of the project about which the firm holds precise information (for example, proprietary technical specifications). The *market-specific* fundamental θ relates to conditions about which knowledge is dispersed throughout the market (for example, demand for a new product).[14]

Suppose for now that firm's investment decision is characterized by a threshold rule $\tilde{F}(P)$:

$$a(F, P) = \begin{cases} 1 \text{ if } F \leq \tilde{F}(P); \\ 0 \quad \text{otherwise.} \end{cases} \tag{13}$$

The firm invests if and only if the investment cost is below the threshold $\tilde{F}(P)$. For now, we leave $\tilde{F}(P)$ deliberately general to characterize the market equilibrium. Below we consider two specifications of manager's objective $\tilde{\pi}(\theta, y, a, P)$ which are consistent with the threshold investment rule in (13).

Following the same steps as above, trader's supply decisions are characterized by a threshold rule $\hat{x}(P)$, which satisfies:

$$\begin{aligned} P &= \rho \int \theta dH(\theta|\hat{x}(P), P) + \int [a(F, P) \cdot (\int \theta dH(\theta|\hat{x}(P), P) - F)] dG(F) \\ &= \left(\rho + G\left(\tilde{F}(P)\right)\right) \int \theta dH(\theta|\hat{x}(P), P) - \int_{\underline{F}}^{\tilde{F}(P)} F dG(F). \end{aligned} \tag{14}$$

The first integral in the upper line of equation (14), $\rho \int \theta dH(\theta|x, P)$, is the marginal trader's expectation of the dividend if the firm does not invest. The second term in the first line is the

[13] We further discuss the role of the exogenous component in Section 4.1.

[14] See Miller and Rock (1985) and Rock (1986) for further discussions on market- and firm- specific sources of information.

expected impact of investment on the dividend. For each pair (F, P), the marginal trader considers the difference between the posterior expectation of the fundamental and the investment cost, $\int \theta dH(\theta|x, P) \ - F$. Since the trader does not observe F, the expectation is an integral over the investment range $F \leq \tilde{F}(P)$. Equation (14) compares the cost of holding the share, P (the left-hand side) to the expected dividends (the right hand side). The price enters the expected dividend through its impact on marginal trader's expectation of θ, and by its influence on firms' investment threshold $\tilde{F}(P)$.

With invertibility of the price function, we redefine the investment threshold as a function of z: $\tilde{F}(z) = \tilde{F}(P)$. Using the market-clearing condition $z \equiv \hat{x}(P)$, and the characterization of shareholder beliefs in Lemma 1, we characterize the equilibrium of the endogenous investment model:

Proposition 2 (Equilibrium with endogenous investment) *For an investment threshold $\tilde{F}(z)$, define $P(z)$ by:*

$$P(z) = \left(\rho + G\left(\tilde{F}(z)\right)\right) \frac{\lambda\mu + \beta(1+\delta)z}{\lambda + \beta + \beta\delta} - \int_{\underline{F}}^{\tilde{F}(z)} F dG(F). \tag{15}$$

If $P(z)$ is strictly increasing, the asset market equilibrium is characterized by the price function $P(z)$ and traders' threshold function $\hat{x}(p) = z = P^{-1}(p)$. The expected dividend conditional on public information z is given by

$$V(z) = \left(\rho + G\left(\tilde{F}(z)\right)\right) \frac{\lambda\mu + \beta\delta z}{\lambda + \beta\delta} - \int_{\underline{F}}^{\tilde{F}(z)} F dG(F). \tag{16}$$

The equilibrium price and expected dividend in Proposition 2 can be decomposed into three terms. First, $\rho \cdot \frac{\lambda\mu+\beta(1+\delta)z}{\lambda+\beta+\beta\delta}$ and $\rho \cdot \frac{\lambda\mu+\beta\delta z}{\lambda+\beta\delta}$ denote the expected dividend if the firm does not invest, from the marginal trader's and manager's perspective. Second, $G\left(\tilde{F}(z)\right) \cdot \frac{\lambda\mu+\beta(1+\delta)z}{\lambda+\beta+\beta\delta}$ and $G\left(\tilde{F}(z)\right) \cdot \frac{\lambda\mu+\beta\delta z}{\lambda+\beta\delta}$ are the additional expected payoff if the firm invests. The third term $\int_{\underline{F}}^{\tilde{F}(z)} F dG(F)$ is the expected investment cost.

In the next two subsections, we consider two separate cases for manager's objective and compare the resulting threshold functions $\tilde{F}(z)$, equilibrium prices, and expected dividend values.

3.3 The Benchmark Case: Dividend Maximization

We now characterize the equilibrium in which manager's objective is to maximize the expected dividend: $\tilde{\pi}(\theta, F; a) = \pi(\theta, F; a)$. The manager invests if (and only if) the realization of the cost F

is (weakly) lower than the posterior of the fundamental $\mathbb{E}(\theta|P)$. The investment threshold is given by

$$\tilde{F}(P) = \tilde{F}(z) = \int \theta dH(\theta|z) = \frac{\lambda\mu + \beta\delta z}{\lambda + \beta\delta}. \tag{17}$$

The equilibrium is given by Proposition 2 after replacing $\tilde{F}(z) = \mathbb{E}(\theta|z)$. For the discussion below, it is convenient to redefine the state z in terms of the posterior expectation $Z \equiv \mathbb{E}(\theta|z)$. We then rewrite the price, the expected dividend, and the wedge in terms of this posterior expectation Z:

$$P(Z) = (\rho + G(Z))(\mu + \gamma(Z - \mu)) - \int_{\underline{F}}^{Z} F dG(F), \tag{18}$$

$$V(Z) = (\rho + G(Z)) Z - \int_{\underline{F}}^{Z} F dG(F), \tag{19}$$

$$W(Z) \equiv P(Z) - V(Z) = (\gamma - 1)(Z - \mu)(\rho + G(Z)). \tag{20}$$

The parameter $\gamma > 1$ is given by expression (9), which corresponds to the ratio of Bayesian weights assigned to the market signal z by the marginal trader, and manager (or an uninformed outsider). The next lemma establishes a sufficient condition for the invertibility of the price function:

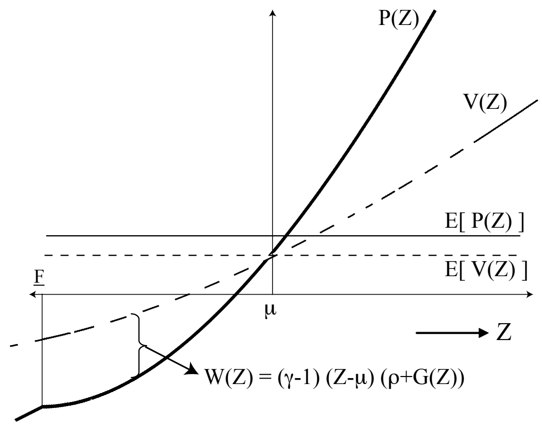
Lemma 2 (Invertibility of the Price Function) *The price function is invertible, if* $\rho + G(F) + g(F)(F - \mu) > 0$*, for all* $F \geq \underline{F}$.

Lemma 2 states a sufficient condition for price invertibility in the case of expected dividend maximization. Price non-invertibility, which is caused by price non-monotonicity, can arise if the marginal trader's valuation of investment is locally decreasing in z. If $\rho = 0$, this is inevitably the case whenever $G(F)/g(F)$ is non-decreasing and converges to 0 as $F \to -\infty$. The condition in Lemma 2 imposes a lower bound on the sensitivity of the dividend to the fundamental through the exogenous payoff component $\rho \cdot \theta$. We further expand on this issue in Section 4.1 and Appendix B.

The wedge $W(Z)$ can be decomposed as a product of two terms. The first term given by equation (Info agg wedge) corresponds to the difference between marginal trader's and manager's posterior beliefs about θ: $(\gamma - 1)(Z - \mu)$. This term determines the sign of the wedge and follows from our discussion in Section 2. The price, $P(z)$, is the expectation of dividends by the marginal trader who observes z both as private *and* public information of θ. Thus, the price reacts more strongly to the market information relative to the dividend expectation of the manager, $V(Z)$.

The second term is the marginal effect of the fundamental, θ, on the expected dividend: $(\rho + G(Z))$. This term includes the exogenous effect of the fundamental (ρ) and the endogenous effect from investment ($G(Z)$). It captures the value of market information for the investment decision: the market signal determines manager's posterior belief of θ and the probability of investment

Figure 2: Price, Expected Dividend and Wedge with Endogenous Investment



$G(Z)$, which determines the net effect of θ on the dividend and, hence, the absolute magnitude of the wedge.

Figure 2 plots the price (solid line), the expected dividend (dashed line), and the wedge as a function of the state variable Z. The expected dividend $V(Z)$ is increasing and convex, and the price function is also increasing given the condition imposed in Lemma 2. As in the case of the exogenous dividend model of Section 2, the information aggregation wedge is negative for $Z < \mu$, zero at $Z = \mu$, and positive for $Z > \mu$. The key insight is that the model with investment gives rise to an *endogenously* convex dividend function. Therefore, we can apply the results of Theorem 1 (part (ii)):

$$\mathbb{E}(W(Z)) = (\gamma - 1)\, Cov\,(G(Z), Z) > 0, \tag{21}$$

since $G(\cdot)$ is increasing. Expression (21) can be rearranged as

$$\begin{aligned} \mathbb{E}(W(Z)) &= (\gamma - 1) \int_{-\infty}^{\infty} G(Z) \frac{Z-\mu}{\sigma_Z} \phi\left(\frac{Z-\mu}{\sigma_Z}\right) dZ \\ &= (\gamma - 1)\, \sigma_Z \int_{0}^{\infty} \left(G(\mu + \sigma_Z u) - G(\mu - \sigma_Z u)\right) u \phi(u)\, du, \end{aligned} \tag{22}$$

where $\sigma_Z^2 = \beta\delta / (\lambda + \beta\delta) \cdot \lambda^{-1}$ is the ex ante variance of the firm's posterior Z. Expression (22) explicitly shows the three factors that are necessary and sufficient for obtaining a positive expected wedge, and how together they determine its magnitude. First, information heterogeneity among traders ($\gamma > 1$) is required to obtain a conditional wedge $W(Z)$. If the shareholders in the financial market had identical beliefs, then price would would still convey the shareholder's common information to the firm, and the firm would still value this information and act on it, but there would no longer be any wedge.

Second, a positive wedge requires ex ante uncertainty about the firm's posterior Z, i.e. $\sigma_Z^2 > 0$. The variance of the firm's posterior measures how strongly the information conveyed by the market affects the firm's beliefs about θ; σ_Z^2 represents the value of market information for the investment decision and is increasing in the precision of the market signal $\beta\delta$ and the prior variance of the fundamental λ^{-1}. Intuitively, precise private information (high β), or low variance of noise trading (high δ) increase the likelihood that movements in z are due to innovations in θ. This makes z a more reliable signal and increases the sensitivity of manager's posterior Z to changes in z. Also, learning about θ is more important the larger its ex-ante variance (λ^{-1}). The unconditional wedge is increasing in the variance σ_Z^2, which captures the amount of learning from market prices.

Third, the firm's investment choice must be sensitive to the market information - that is, the shape of the distribution $G(\cdot)$ matters for the wedge. This distribution measures how much the firm's investment decision responds to the market information: the higher the variance of the firm-specific cost F (i.e. the flatter $G(\cdot)$ is around the prior mean of F), or the more certain the prior expectation of the firm's investment decision (i.e. if $G(Z)$ is close to 0 or 1 around the prior mean μ), the less the firm's investment probability is going to respond to changes in market information, and hence the less the firm's exposure to θ varies with the price. If the investment decision was completely fixed before the observation of the price, the investment probability would be constant. The wedge would be symmetric around the prior mean $Z = \mu$, and its ex ante expectation equal to zero. This corresponds to the linear dividend case in Theorem 1. The expected information aggregation wedge is therefore large from an ex ante perspective when (i) the prior uncertainty about the firm's investment decision is high, and (ii) the realization of the market signal generates a significant update in the investment probability $G(Z)$. Uncertainty about the firm's investment decision is highest, when $G(F) = \mathbb{I}_{F>\mu}$, i.e. when the firm's investment cost is known to be equal to the prior mean μ (meaning that ex ante the firm would be indifferent), and any update from the price can swing the investment decision in either direction.[15]

[15] The value of market information for the firm highlights the close relation of our setup with real options (e.g., Dixit and Pindyck, 1994). Firm's payoff depends on the realization of a random variable (θ) and on an endogenous choice of investment (a). The price is a public signal of the variable θ for the firm. The firm can better match a good realization of θ by investing, while limiting the negative effects of a low realization by not investing. The value of the investment option depends on the precision of information, and the prior uncertainty about the random variable. An important novel result of our model is that the option value of information also leads to expected prices to be higher than expected dividends when traders hold heterogenous beliefs in equilibrium.

The next proposition summarizes the comparative statics of the information aggregation wedge, the expected price, and the dividend in terms of γ and σ_Z^2. We will defer a more complete discussion of the role of $G(\cdot)$ until section 4.

Proposition 3 (Comparative Statics) *(i) For a given value of γ, $\mathbb{E}(P(Z))$, $\mathbb{E}(V(Z))$, and $\mathbb{E}(W(Z))$ are increasing in σ_Z^2. (ii) For given value of σ_Z^2, $\mathbb{E}(V(Z))$ does not depend on γ; $\mathbb{E}(P(Z))$ and $\mathbb{E}(W(Z))$ are increasing in γ.*

The unconditional price, dividend, and wedge are all increasing in the prior uncertainty about the firm's posterior, σ_Z^2. Recall that the unconditional expected dividend is larger when the manager learns more information from the market (higher σ_Z^2). Moreover, since the marginal trader's posterior is more sensitive to z than manager's, the impact of σ_Z^2 on the average price is stronger than on the expected dividend. This raises the unconditional wedge. The unconditional dividend is independent of the difference between manager's and marginal trader's expectations (γ). The expected price and, hence, the wedge, scale up γ.

The primitive parameters β, δ, and λ affect expected dividends $\mathbb{E}(V(Z))$ through σ_Z^2, which is increasing in both the precision of the market signal ($\beta\delta$), and in the prior uncertainty (λ^{-1}). Both better market information and a more variable prior increase the value of the real option to invest, raising the unconditional expected dividend. The same parameters affect the unconditional information aggregation wedge $\mathbb{E}(W(z))$ through both σ_Z^2 and γ. Notice, however, that the effects go in opposite directions: γ is decreasing in λ^{-1}, decreasing in β, and decreasing in δ. The overall comparative statics on the unconditional wedge and price are therefore a priori not clear. Prior uncertainty λ^{-1} must be sufficiently high to generate option value from investment, and private information precision β needs to be large to create belief dispersion. Finally, the market information $\beta\delta$ must be retain some value for the firm, yet it cannot be not so precise that it completely crowds out the prior, eliminating the wedge. The next proposition uses expression (22) to provide tight bounds on the magnitude of the expected wedge, for a given distribution $G(\cdot)$ of the firm's investment cost. Furthermore, holding β and λ constant, we show that the expected wedge can become arbitrarily large.

Proposition 4 (Bounds on $\mathbb{E}(W(Z))$) *(i) Suppose that $G(\cdot)$ has continuous density $g(\cdot)$, and let $\|g\| = \max_{F \geq \underline{F}} g(F)$. Then*

$$\mathbb{E}(W(Z)) \leq (\gamma - 1)\,\sigma_Z^2\,\|g\| = \left(\frac{\beta+\beta\delta}{\lambda+\beta+\beta\delta} - \frac{\beta\delta}{\lambda+\beta\delta}\right)\frac{\|g\|}{\lambda} < \frac{\beta}{\lambda+\beta}\frac{\|g\|}{\lambda}.$$

(*ii*) *Holding* β *and* λ *constant,*

$$\lim_{\delta\to 0}\mathbb{E}\left(W\left(Z\right)\right)=\frac{\beta}{\lambda+\beta}\frac{g\left(\mu\right)}{\lambda}.$$

(*iii*) *For all* $K>0$*, there exist* $G\left(\cdot\right)$ *and* $\delta'>0$*, such that for any* $\delta\leq\delta'$*,* $\mathbb{E}\left(W\left(Z\right)\right)>K$*.*[16]

Part (i) shows that for a given function $G\left(\cdot\right)$, there is an upper bound on the magnitude of the expected information aggregation wedge. Part (ii) derives the limit of the information aggregation wedge when $\delta\to 0$, i.e. when the market is infinitely noisy. It also shows that the information aggregation wedge remains positive in the limit, whenever $g\left(\mu\right)>0$, i.e. whenever the distribution of F has positive density at the prior mean μ. If the distribution is such that $g\left(\mu\right)=\|g\|$, i.e. the distribution of F reaches its peak density at μ, then as an immediate corollary, $\mathbb{E}\left(W\left(Z\right)\right)$ reaches its maximum in the limit where $\delta\to 0$ (i.e. market information becomes completely uninformative). This result stems from the fact that $\left(\gamma-1\right)\sigma_Z^2$ is strictly increasing in $\beta\delta$, and reaches a finite limit as $\delta\to 0$, and in this limit, the marginal effect of the firm's posterior Z on the investment probability is highest when the posterior Z is near the prior μ, which occurs with probability 1 in the limit as $\delta\to 0$.

Part (iii) shows that, although for a given function $G\left(\cdot\right)$ the information aggregation wedge is uniformly bounded in δ, this bound becomes arbitrarily large, if the distribution of the investment cost is concentrated around the prior mean μ, and $\delta\to 0$. From (22), $\mathbb{E}\left(W\left(Z\right)\right)$ is highest, when $G\left(\cdot\right)=\mathbb{I}_{F>\mu}$, i.e. the investment cost is equal to the prior mean μ of the fundamental θ. As we discussed above, this cost scenario maximizes the informational impact of the price on the firm's investment decision. In this case, the expected wedge is $\mathbb{E}\left(W\left(Z\right)\right)=\left(\gamma-1\right)\sigma_Z/\sqrt{2\pi}$, and because we know from part (ii) that $\left(\gamma-1\right)\sigma_Z^2$ converges to a finite limit as $\delta\to 0$, it follows that $\left(\gamma-1\right)\sigma_Z$ grows unboundedly large as $\delta\to 0$ and $\sigma_Z\to 0$. A combination of a high degree of prior uncertainty about the firm's investment decision, coupled with a lot of noise in the market price (yet sufficient information such that even a small update through z can have a large effect on the firm's investment probability) thus can make the expected information aggregation wedge arbitrarily large.

[16] In part (iii) the order of limits does not matter, as long as $g\left(\mu\right)\to\infty$ along the limiting sequence. Along a sequence $\{G_n\left(\cdot\right)\}$ of normal distributions for the investment cost, with mean F_n and standard deviation σ_{F_n}, respectively, this condition requires that $\lim_{n\to\infty}\left(\mu-\mu_n\right)/\sigma_{F_n}=0$, i.e. that μ_n converges to μ at a rate faster than σ_{F_n} converges to 0.

3.4 Tying Managerial Incentives to Share Prices

In the benchmark model of endogenous investment of Section 3.2, we assumed the manager's and shareholders' objectives coincide: $\tilde{\pi}(\cdot) = \pi(\cdot)$. This assumption lead to an investment rule that maximizes $V(Z)$: the expected dividend conditional on the share price. We now discuss the effects of managerial incentives tied to stock market performance. A well known result in economies with complete markets and common information is that $P(Z) = V(Z)$. In all states, maximizing prices or expected dividends is equivalent.

This result does not hold in our model because of the wedge between the price and the expected dividend. Moreover, the wedge is directly affected by the investment decision that determines the net impact θ on the dividend. If a manager has incentives to maximize the share price as opposed to the dividend, the size of the wedge can be changed by choosing investment to increase market prices. Since investment in the benchmark model maximized expected dividends, any deviations from the investment rule defined by the threshold $\tilde{F}(Z) = Z$ reduces the expected payoffs of shareholders.

We now consider the case where manager's objective function is given by $\tilde{\pi}(\theta, F, a; P) = (1 - \alpha)\pi(\theta, F; a) + \alpha P,\ \alpha \in [0, 1]$. This objective function is a linear combination of the share price, $P(Z)$, and the dividend $\pi(\theta, F; a)$. More specifically, we assume that the manager chooses an initial decision rule $a(F, P)$, which he commits to prior to the market opening.[17] For a given value of Z, manager's problem can then be stated as follows:

$$\max_{\tilde{F}(Z)} \quad \mathbb{E}(\tilde{\pi}(\theta, F, a; P)|Z) = \alpha P(Z) + (1 - \alpha) V(Z),$$

where $P(Z)$ and $V(Z)$ are given by equations (18) and (19), and $\alpha \in [0, 1]$ measures how strongly incentives are based on the price relative to expected dividends.

The equilibrium is characterized by Proposition 2: threshold functions $\hat{x}(P)$ for traders and $\tilde{F}(P)$ for manager, and an invertible price function $P(Z)$. The investment threshold is found by maximizing $\mathbb{E}(\tilde{\pi}|Z)$ pointwise, for all Z, and then checking that the resulting price function is invertible. Formally, we have

$$\begin{aligned} \tilde{F}(Z) \quad &\in \quad \arg\max_{\tilde{F}} \{\alpha P(Z) + (1 - \alpha) V(Z)\} = \arg\max_{\tilde{F}} \{V(Z) + \alpha W(Z)\} \\ &= \quad \arg\max_{\tilde{F}} \left\{ \left(\rho + G\left(\tilde{F}\right)\right) [\mu + k(Z - \mu)] - \int_{\underline{F}}^{\tilde{F}} F dG(F) \right\}. \end{aligned} \tag{23}$$

[17] In this context, the assumption of pre-commitment is important, as the ex post optimization (taking P as given) results in the same maximization of expected dividends as before. The influence of firm's decisions on market prices (and the role of price-based incentives) results precisely from the market's anticipation of the firm's investment choices.

The parameter $k = 1 + \alpha(\gamma - 1)$ measures excess weighting of market information by the manager and captures the strength of the distortion introduced by price-based incentives. The variable $k \in [1, \gamma]$ depends on the size of the information aggregation wedge through γ and the weight given to the prices in the manager's objective function, α. At one extreme, $\alpha = 0$ and $k = 1$ correspond to our benchmark model of dividend maximization (section 3.2). At the other extreme, $\alpha = 1$ and $k = \gamma$: the manager's incentives are based only on the share price.

Taking first-order conditions to determine the investment threshold $\tilde{F}(Z)$ (and checking that price invertibility holds under the same condition as before in lemma 2), we find the following equilibrium characterization.

Proposition 5 (Equilibrium with price-based incentives) *In the PBE with price-based incentives, the investment threshold $\tilde{F}(Z)$, price $P(Z)$, and expected dividend $V(Z)$ are given by*

$$\begin{aligned} \tilde{F}(Z) &= \mu + k(Z - \mu), & (24) \\ P(Z) &= \left[\rho + G\left(\tilde{F}(Z)\right)\right](\mu + \gamma(Z - \mu)) - \int_{\underline{F}}^{\tilde{F}(Z)} F dG(F) & (25) \\ V(Z) &= \left[\rho + G\left(\tilde{F}(Z)\right)\right] Z - \int_{\underline{F}}^{\tilde{F}(Z)} F dG(F) & (26) \end{aligned}$$

Equations (25) and (26) decompose the effect that the information aggregation wedge has on the price and the expected dividend. Without price-based incentives ($\alpha = 0$; $k = 1$), the stronger reaction of the price to market signals has no impact on the expected dividend. When $\alpha > 0$ ($k > 1$), the price increases with α at an efficiency cost that reduces the expected dividend in (26). We formalize the result in the next theorem.

Theorem 2 (Tying managerial incentives to share-prices) *In the PBE with price-based incentives:*

(i) $\tilde{F}'(Z) = k > 1$: The volatility of investment is increasing in α and γ (and hence in k), for all $Z \neq \mu$.

(ii) The expected dividend $V(Z)$ is decreasing, while the share price $P(Z)$ and the wedge $W(Z)$ are increasing in α, for all $Z \neq \mu$.

(iii) The expected share price $\mathbb{E}(P(Z))$ and the unconditional wedge $\mathbb{E}(W(Z))$ are increasing in σ_Z^2 and γ.

(iv) $V(Z)$ and $\mathbb{E}(V(Z))$ are decreasing in γ, while the effect of σ_Z^2 on expected dividends, $\mathbb{E}(V(Z))$ is ambiguous.

(iv) If $\alpha > 0$, then $\lim_{\delta \to 0} \mathbb{E}(W(Z)) = \infty$, for any continuous, strictly increasing $G(\cdot)$.

Theorem 2 summarizes the comparative statics for the game with price-based incentives. Part (i) states that investment volatility is increased when incentives are tied to share prices ($\alpha > 0$). The manager reacts more strongly to the information conveyed by the price, as captured by the parameter $k > 1$. Price-based incentives induce the manager to align investment with the beliefs of the marginal shareholder. Therefore, when Z is higher than μ, firm's investment threshold is too high. The firm invests in some states in which the cost F exceeds the expected gains from investment, Z. When Z is lower than μ, the firm's investment threshold is too low. The firm foregoes investment in some states in which Z exceeds the cost F. This results in higher prices but lower expected dividends, for all $Z \neq \mu$ (part ii).

Figure 3: Effect of Price-based Incentives

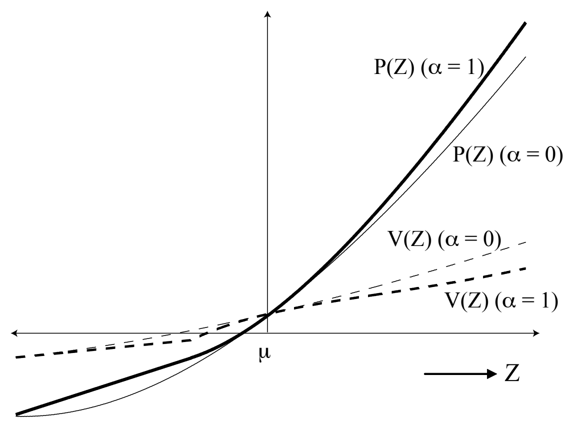


Figure 3 plots the price and expected dividend functions for the extreme cases $\alpha = 1$ (the thick lines) and $\alpha = 0$ (the thin lines). When $\alpha = 1$ ($k = \gamma$) the manager is purely concerned with price maximization. The investment rule in (24) exactly matches the marginal trader's expectation of θ. The firm behaves as if it were run by the marginal trader and achieves the largest share price for each realization of the state Z to the detriment of expected dividends. Indeed, figure 3 shows how the price in this case (the thick, solid line) is always above the one attained under dividend maximization, or $\alpha = 0$ (the thin, solid line). The expected dividend under priced-based incentives (thick, dashed line) is everywhere below its counterpart in the benchmark case (the thin, dashed line). The conditional information aggregation wedge is thus exacerbated and so is the unconditional wedge.

From an ex ante perspective, this reinforces the effects of γ and σ_Z^2 on the expected share price $\mathbb{E}(P(Z))$ and the unconditional wedge $\mathbb{E}(W(Z))$ (part iii) that were positive already in the case with dividend maximization. Things change, however, with regards to the expected dividend value

$\mathbb{E}(V(Z))$. First, an increase in γ lowers expected dividend value of the firm by increasing the manager's ability to distort the investment threshold to boost the share price. This is reflected in the fact that k is increasing in γ. Second, the overall effect of information provided through the price is ambiguous, and we can construct both cases in which better market information increases the firm's dividend value, and cases where the opposite is true. In particular, if k is not too high (so that the distortion motive is not too large), and G is highly concentrated around some value close to μ, the loss from the investment distortion is small, relative to the value of the information. On the other hand, if k is high, and $G(\cdot)$ is highly dispersed, then the price-based incentives distort the investment decision sufficiently severely so that a more informative price signal may actually reduce the firm's dividend value. This result follows the standard logic that improved information need not be socially desirable, if economic decisions do not make efficient use of this information.

Finally, part (v) shows that with price based incentives, the information aggregation wedge becomes arbitrarily large, as the market information becomes noisier. As was the case with expected dividend maximization, the comparative statics of different variables w.r.t. the underlying parameters β, δ, and λ are not unambiguous, because of the competing effects of σ_Z^2 and γ (where the latter now also induces inefficient investment decisions). However, in that case, we showed that the expected wedge was bounded, for given $G(\cdot)$. With price-based incentives, this is no longer true. In particular, we can rewrite $\mathbb{E}(W(Z))$ along the same lines as (22):

$$\begin{aligned}\mathbb{E}(W(Z)) &= (\gamma-1)\,cov\left(G\left(\tilde{F}(Z)\right), Z\right)\\ &= (\gamma-1)\,\sigma_Z\int_0^\infty \left(G(\mu+k\sigma_Z u)-G(\mu-k\sigma_Z u)\right)u\phi(u)\,du.\end{aligned}$$

Result (v) then follows immediately from the observation that $\lim_{\delta\to 0} k\sigma_Z = \infty$ for any $\alpha > 0$, and $\lim_{k\sigma_Z\to\infty}\left(G(\mu+k\sigma_Z u)-G(\mu-k\sigma_Z u)\right) = 1$ for all $u > 0$, so that $\mathbb{E}(W(Z))$ is of the same order as $(\gamma-1)\,\sigma_Z$. In contrast to the case with dividend maximization, the impact of Z on the investment probability actually increases without bound, as the market information becomes noisier. The key to this result is that with $\alpha > 0$, the weight the managers put on the market signal is bounded away from 0, even if the market information z is infinitely noisy. But this implies that the variability of the investment threshold $\tilde{F}(Z)$ becomes infinite, even though σ_Z^2 goes to zero. That is, in the limit where market information is pure noise, the manager will no longer update from the price, yet with price based incentives, the variability of the investment threshold can grow arbitrarily large, because the manager is induced to align the investment threshold $\tilde{F}(Z)$ with the marginal shareholder's expectation of θ.

As an immediate corollary, we also have the observation that the introduction of price-based incentive schemes, i.e. a shift from $\alpha = 0$ to $\alpha > 0$, can induce an arbitrarily large increase in the expected share price, a large increase in investment volatility, and a reduction in the dividend value, when market prices are sufficiently noisy.

The analysis above gives an argument against tying executive compensation too closely to market valuations. When dispersed information drives a wedge between prices and expected dividends, and when the wedge responds to firm's endogenous investment decision, price-based incentives lead to inefficient investments that drive up prices but lower firm value.

4 Discussion and Extensions

We now study the robustness of our main results by considering several extensions of our benchmark model of endogenous investment. We explore alternative specifications of dividends, exogenous asset demand, and informational environments.

4.1 The sufficient condition from lemma 2

In lemma 2, we assumed that $\rho + G(F) + g(F)(F - \mu) > 0$, for all $F \geq \underline{F}$, as a sufficient condition to guarantee that the price function was invertible. Notice that this is a joint condition on the size of the exogenous dividend component ρ and the distribution of the investment cost $G(\cdot)$. Furthermore, this condition was sufficient regardless of whether the firm maximized its expected dividend or the share price - since the potential for non-monotonicity of $P(\cdot)$ results from the disagreement between the optimal investment decision from the firms' and the marginal shareholder's point of view, the non-invertibility issue is mitigated by an increase in α, and it disappears altogether as α approaches 1. Moreover, it only occurs when the firm over invests from the marginal shareholder's point of view, i.e. when $F < \mu$.

In the online appendix, we characterize equilibria when this condition no longer holds. As long as traders continue to act on their private information, our characterization of the candidate equilibrium price function remains intact regardless of ρ or $G(\cdot)$, but the equilibrium exists only if the price function is invertible. Since the candidate price function is always uniquely determined, a violation of the non-invertibility condition necessarily entails a non-existence of equilibrium.[18]

Now, two possibilities are worth discussing. First, when $\rho \neq 0$ or when $\rho = 0$ and $\underline{F} = -\infty$, the dividend depends on θ irrespective of the investment choice, guaranteeing that traders always have an incentive to trade on their private information about θ. Hence, the price must reveal z in

[18]Non-existence here refers to non-existence of an equilibrium in which price is conditioned on z only.

equilibrium, implying that the only candidate equilibrium is the price function we characterized. Non-invertibility then arises under naturally under a thin-tail assumption on the distribution $G(\cdot)$: whenever $G(F)/g(F)$ converges to 0 as $F \to -\infty$, $G(F) + g(F)(F-\mu) < 0$ for some F and therefore, unless ρ is sufficiently large, $P(\cdot)$ is locally decreasing.[19] Non-invertibility issues are avoided for all values of ρ only if $\underline{F} \geq \mu$, i.e. only if the lowest possible investment cost realization exceeds the prior mean of θ, so that it is a priori unlikely that the investment will take place.

Non-invertibility issues of this sort also arise when the distribution $G(\cdot)$ has mass points, or, as an extreme case, when the investment cost F is deterministic. In that case, the candidate price function will have a discontinuity at any mass point of $G(\cdot)$, except when $\alpha = 1$ (under pure price maximization, $P(\cdot)$ is continuous and increasing for any distribution $G(\cdot)$). If $G(\cdot)$ has a mass point at f, the size of the jump in $P(\cdot)$ is $(G_+(f) - G_-(f))\left(\mu + \gamma\left(\tilde{F}^{-1}(f) - \mu\right) - f\right)$, where $\tilde{F}^{-1}(\cdot)$ denotes the inverse function of the firm's investment threshold. The discontinuity must be non-negative for $P(\cdot)$ to be invertible, requiring that $f \geq \mu$ for the investment threshold rules that we considered in the previous section. The distribution $G(\cdot)$ therefore cannot have any mass points below μ, nor can F be deterministic and less than μ.[20]

Second, when $\rho = 0$ and $\underline{F} > -\infty$, θ only affects dividends through the investment decision, and the shareholder's private signals only carry information if the firm invests with positive probability. the possibility of price multiplicity and indeterminacy arises from a feedback between the informativeness of the price, and the firm's investment decision. In this case, Therefore, if the firm never invests when it sees a price of 0, the shareholders will not trade on their private signals at such a price, and hte expected dividends are 0. Notice that this must occur in equilibrium, because for any $Z < \underline{F}$, the firm would be certain not to invest. For more optimistic realizations of Z it is possible to sustain trading on private information, along with a positive probability of investment in equilibrium, with the equilibrium price function characterized above. However, trade need not occur at such states: in particular suppose that $\underline{F} > \mu$ (so that the firm would never invest, based just on its prior information), and conjecture that the price is 0, for all values of Z. Then, the firm will not draw any information from the price, and hence will not invest. More generally it turns out that when $\underline{F} > \mu$, it is possible to sustain (almost) arbitrary selections from the correspondence $\{0, P(Z)\}$ as equilibrium prices, implying that the information aggregation through the price and

[19] Non-invertibility issues do not arise for any value of ρ only if $\underline{F} \geq \mu$.

[20] Checking the sufficient condition of lemma 2 for a sequence of continuous distributions $G(F)$ that approaches a mass point at $f < \mu$ also reveals that in such a case the exogenous componentρ must become infinitely large to avoid price non-monotonicity.

the firm's investment decisions are indeterminate.[21]

These multiplicity, indeterminacy and non-existence issues are interesting in their own right, but they are somewhat distracting from the main contribution of our paper. Our baseline model with an exogenous dividend component and a continuous distribution $G(\cdot)$ allows us to focus on the cases in which there is a unique equilibrium.

4.2 Price Impact of Information

In the models in section 2 and 3, the stochastic asset demand was completely inelastic. We now generalize our assumption about exogenous asset demand (noise trading) by assuming it comes from uninformed traders: they trade partly for exogenous motives, and partly in response to gaps between the price and their dividend expectation, conditional on the price. Except for the asset demand, the model is the same as in section 3. Specifically, we consider the following formulation for asset demand:

$$D(u,P) = \Phi\left(u + \eta\left(\mathbb{E}\left(\pi|P\right) - P\right)\right), \tag{elastic demand}$$

with $u \sim \mathcal{N}\left(0, \delta^{-1}\right)$. Uninformed traders' demand is increasing in the expected return conditional on the price, $\mathbb{E}\left(\pi|P\right) - P$, with an elasticity given by η. This specification of demand generalizes our previous formulation to allow for a response of uninformed traders to perceived excess returns on the asset, as well as stochastic trading motives which are unrelated to dividend expectations (for example, liquidity or hedging needs). The parameter η captures the responsiveness of uninformed traders to the expectation of dividends in excess of prices, or in other words, the extent to which they are willing or able to arbitrage away the difference between expected price and dividend value. Equivalently, η measures the price impact of private information which relates naturally to the concept of market liquidity.

We follow our previous equilibrium characterization and asset prices with minor changes to account for the endogeneity of demand to asset prices. Market-clearing implies

$$\Phi\left(\sqrt{\beta}\left(\hat{x}\left(P\right) - \theta\right)\right) = \Phi\left(u + \eta\left(\mathbb{E}\left(\pi|P\right) - P\right)\right),$$

or

$$z = \hat{x}\left(P\right) - \eta/\sqrt{\beta} \cdot \left(\mathbb{E}\left(\pi|P\right) - P\right).$$

Observing P is thus isomorphic to observing $z \sim \mathcal{N}\left(\theta, \left(\beta\delta\right)^{-1}\right)$, and Lemma 1 continues to hold without any changes. Using the fact that the expected dividend is $\mathbb{E}\left(\pi|P\right) = \mathbb{E}\left(\pi|z\right) = V\left(z\right)$, the

[21] If $\underline{F} < \mu$ a similar indeterminacy result holds, but with some additional restrictions placed on the selection between 0 and $P\left(Z\right)$.

equilibrium price function is implicitly defined by the marginal trader's indifference condition

$$P(z) = \mathbb{E}(\pi|\hat{x}(P), z) = \mathbb{E}\left(\pi|z + \eta/\sqrt{\beta} \cdot (V(z) - P(z)), z\right). \tag{27}$$

Thus, for $\eta > 0$, the increased reliance of the market signal that we discussed in sections 2 and 3 as the cause for the information aggregation wedge is partially counter-acted by the uninformed traders' response to the wedge. Using the definition of dividends from section 3 for a given investment threshold function $\tilde{F}(Z) = \tilde{F}(P(Z))$, and defining the firm's conditional expectation of θ, $Z = \frac{\lambda\mu+\beta\delta z}{\lambda+\beta\delta}$ as the state variable, the equilibrium price, expected dividend value and information aggregation wedge are characterized as

$$\begin{aligned} V(Z) &= \left(\rho + G\left(\tilde{F}(Z)\right)\right) Z - \int_{\underline{F}}^{\tilde{F}(Z)} f dG(f), && (28) \\ P(Z) &= \left(\rho + G\left(\tilde{F}(Z)\right)\right) \left(\mu + \left[1 + (\gamma - 1) \frac{\lambda + \beta + \beta\delta}{\lambda + \beta + \beta\delta + \sqrt{\beta}\eta\left(\rho + G\left(\tilde{F}(Z)\right)\right)}\right](Z - \mu)\right) \\ &\quad - \int_{\underline{F}}^{\tilde{F}(Z)} f dG(f), && (29) \\ W(Z) &= \frac{\lambda + \beta + \beta\delta}{\lambda + \beta + \beta\delta + \sqrt{\beta}\eta\left(\rho + G\left(\tilde{F}(Z)\right)\right)} (\gamma - 1) \left(\rho + G\left(\tilde{F}(Z)\right)\right)(Z - \mu). && (30) \end{aligned}$$

For a given investment threshold $\tilde{F}(Z)$, the information aggregation wedge is thus inversely related to the uninformed traders' demand elasticity η. Higher η lowers the price impact of private information, and thus the magnitude of the information aggregation wedge. At the extreme with infinite elasticity, price equals expected dividends and the wedge disappears. The other extreme ($\eta = 0$) corresponds to our baseline setup of section 3.2.

If we suppose, as before that $\tilde{F}(Z)$ is chosen to maximize $\alpha P(Z) + (1-\alpha) V(Z)$, then the corresponding first-order conditions lead to

$$\tilde{F}(Z) - \mu = \left\{1 + \alpha(\gamma - 1)\left(\frac{\lambda + \beta + \beta\delta}{\lambda + \beta + \beta\delta + \sqrt{\beta}\eta\left(\rho + G\left(\tilde{F}(Z)\right)\right)}\right)^2\right\}(Z - \mu).$$

As long as $\alpha = 0$, investment remains undistorted. When $\alpha > 0$, there is over-investment for $Z > \mu$ and underinvestment for $Z < \mu$, but the inefficiency is reduced by the demand elasticity; η. For all η, $\tilde{F}(Z) \in [Z, \mu + k(Z - \mu)]$, with $\lim_{\eta\to\infty} \tilde{F}(Z) = Z$ and $\lim_{\eta\to 0} \tilde{F}(Z) = \mu + k(Z - \mu)$. Efficient investment arises as the uninformed demand becomes infinitely elastic. In this case, it is the uninformed traders who price the shares, arbitraging away the discrepancy between price and expected dividends, conditional on market information.

Therefore, all the previous effects, regarding over-valuation and the effects of price based incentives, are inversely related to the magnitude of η. When η is higher the uninformed traders are better able to arbitrage the difference between expected value and price. This reduces the absolute value of the information aggregation wedge for all realizations of the state Z, and mitigates the distortions induced by price-based incentives on investment decisions.

4.3 Market-specific Information Observed by the Manager

We also consider an extension where the manager observes a private signal y about the fundamental: $y \sim \mathcal{N}(\theta, \kappa^{-1})$, and the investment decision is taken after the observation of the price, and conditioned on both y and z (or equivalently, z). Allowing the manager to observe a private signal does not change anything to the structure of the financial market equilibrium, i.e. the shareholders' strategies continue to be characterized by a threshold function $x^*(P)$, and on the equilibrium path, lemma 1 continues to hold, so that $x^*(P) = z$, which is normally distributed.

What changes, however is the characterization of the firm's expected dividend value and share price, as a function of z. In the appendix, we provide a full characterization of the resulting expected share prices and dividend values, for a given investment threshold $\tilde{F}(y, z)$, assuming that $\tilde{F}$ is linear in (y, z). This allows us to cover the two polar cases of dividend maximization (when $\tilde{F}(y, z)$ is set equal to $\mathbb{E}(\theta|y, z)$), and (when $\tilde{F}(y, z)$ is set equal to $\mathbb{E}(\theta|y, x = z, z)$), but not the intermediate cases in which the manager sets $\tilde{F}(y, z)$ to a weighted average of $\mathbb{E}(\theta|y, z)$ and $\mathbb{E}(\theta|y, x = z, z)$, but the weights vary with y.

The key issue here is that the market's forecast of the firm's signal y is also distorted relative to the "true" distribution of y. That is, the price is based on expectations of θ that are conditional on the market signal z, and conditional on observing a private signal x also equal to z - just as with the forecast of θ, the market assigns additional weight to z in forecasting the firm's signal y. For intermediate values of α, the manager weighs $\mathbb{E}(\theta|y, z)$ and $\mathbb{E}(\theta|y, x = z, z)$ by both α and $1 - \alpha$, and by the objective and the market densities of y, conditional on z. For the two extreme cases of price and dividend maximization ($\alpha \in \{0, 1\}$), we can however offer a clean characterization showing the following results:

Proposition 6 (Market-specific information observed by the manager) *Suppose that the manager observes a private signal $y \sim \mathcal{N}(\theta, \kappa^{-1})$. Then:*

(i) The wedge remains positive, and is bounded away from 0 for all values of κ.

(ii) The wedge is larger under price-based incentives, but the gap disappears as $\kappa \to \infty$.

Thus, the information aggregation wedge is robust to the introduction of private information observed by the manager. In fact, the wedge is reinforced by the existence of this private information, because of the additional distortion in beliefs regarding the realization of y. However, even if the wedge always remains positive, the manager's private information crowds out the use of market information in forecasting z and hence reduces the incentive to overweigh the market signal and distort investment. As a result the amplification of the wedge is not as strong, and vanishes in the limit as the private signal becomes infinitely precise.

5 Concluding Remarks

This paper examines the role of asset prices in aggregating information about fundamentals and providing guidance for real investment decisions. We first develop a model that allows to characterize a broad set of heterogenous information environments in which dividends are an (exogenous) nonlinear function of the fundamentals. We then develop a model in which a firm's payoffs depends on fundamentals and the choice of investment, thus endogenizing the dividend. Information about the fundamentals is dispersed among traders in a financial market and partially aggregated in firm's share price, upon which the firm conditions its investment choice.

We find that market-generated information enhances firm's value by encouraging investment when high prices communicate good fundamentals, but limiting the loses by discouraging investment when low prices signal poor realizations. However, the interaction between dispersed information and endogenous investment is also the source of a systematic departure between equilibrium share prices and expected dividends –*the information aggregation wedge*. The wedge originates from a higher weight assigned to market-generated signals by the marginal informed trader. This higher weight arises because of the positive correlation between the noise in the public signal revealed by the price, and the noise in private signals of those traders who end up holding (and therefore pricing) the share, and is perfectly consistent with individual rationality. Moreover, because the firm responds to the information conveyed by the price investing more in good states than in bad ones, it exacerbates the price overreaction on the upsides more than the price underreaction on the downsides. As a result, the information aggregation wedge is asymmetric, and the share price exceeds on average the expected dividend value of the firm.

We then discuss the role of price-based managerial incentives. We find that compensation tied to share prices may enhance share overvaluation and induce excess volatility in investment, as managers try to cater investment policies to those traders who have the largest impact on market prices.

Our model has two predictions that align with empirical evidence. First, the model suggests the value enhancing effects of market-specific information in guiding real investment importantly depend on the extent of informed trading activity. This fits the evidence provided by Chen, Goldstein and Jiang (2007) who study the impact of informed trading in the sensitivity of real investment to price changes. The authors find stronger sensitivity in shares with more informed trading activity, as measured by PIN (*probability of informed trading* – Easley et al. (1996)).[22] Second, Polk and Sapienza (2009) provide support to our findings regarding the impact of stock-based compensation. They test a "catering" theory using discretionary accruals as a proxy for mispricing,[23] finding a positive relation between share overvaluation and excess investment after controlling for Tobin's Q. This relation is stronger for firms with higher share turnover, which could proxy for traders' short-term horizons. Moreover, they find that firms with high excess investment subsequently have low share returns, the more so the larger is their measure of mispricing. This suggests that such investment behavior is indeed inefficient.

While our model has taken the manager's objective as given, the design of optimal incentive structures in the presence of a wedge between expected dividend and prices remains an important question for future research. Our model of financial markets with information aggregation appears to provide a promising building block for future work in this direction, as well as for other questions that require a flexible payoff structure for analyzing the interplay between managerial incentives, corporate decisions, and market prices.

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[22] Roll, Schwartz and Subrahmanayam (2009) provide related evidence arguing that developed options markets for a firm's share stimulate the entry of informed traders. They find that firms with deeper options markets have higher sensitivity of corporate investment to share prices, which translates into higher values of Tobin's Q.

[23] Discretionary accruals measure the extent to which a firm has abnormal non-cash earnings. Firms with high discretionary accruals typically have relatively low share returns in the future, suggesting that discretionary accruals artificially drive up prices temporarily.

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6 Appendix A: Proofs for sections 2-3

Proof of Lemma 1. Part (i): By market-clearing, $z = \hat{x}(P(z))$ and $\hat{x}(P(z')) = z'$, and therefore $z = z'$ if and only if $P(z) = P(z')$.

Part (ii): Since $P(z)$ is invertible, observing P is equivalent to observing $z = \hat{x}(P(z))$ in equilibrium. But $z|\theta \sim \mathcal{N}\left(\theta, (\beta\delta)^{-1}\right)$, from which the characterization of $H(\cdot|x, P)$ follows immediately from Bayes' Law. ■

Proof of Proposition 1. Substituting the market-clearing condition $\hat{x}(P) = z$, a price function $P(z)$ is part of an equilibrium if and only if it satisfies (7) and is invertible. $P'(z) > 0$ is immediate because $\pi(\cdot)$ is strictly increasing, and an increase in z represents a first-order stochastic shift in the posterior over θ. The price function $P(z)$ is monotone and continuous over its domain and spans its entire range, so all prices are observed in equilibrium (and hence out-of-equilibrium beliefs play no role). Thus, the characterization in proposition 1 defines the unique equilibrium in which prices are conditioned only on z. ■

Proof of Theorem 1. Define $\hat{\lambda}_P = \lambda + \beta + \beta\delta$, $\hat{\lambda}_z^{-1} = \lambda^{-1} + (\beta\delta)^{-1}$, and

$$\lambda_P^{-1} = \hat{\lambda}_P^{-1} + \left(\frac{\beta + \beta\delta}{\lambda + \beta + \beta\delta}\right)^2 \hat{\lambda}_z^{-1}.$$

Now, first, notice that ex ante, z is normally distributed according to $z \sim \mathcal{N}\left(\mu, \hat{\lambda}_z^{-1}\right)$. Second, some simple algebra shows

$$\int_{-\infty}^{\infty} \sqrt{\hat{\lambda}_P}\phi\left(\sqrt{\hat{\lambda}_P}\left(\theta - \frac{\lambda\mu + \left(\hat{\lambda}_P - \lambda\right)z}{\hat{\lambda}_P}\right)\right)\sqrt{\hat{\lambda}_z}\phi\left(\sqrt{\hat{\lambda}_z}\,(z-\mu)\right)dz = \sqrt{\lambda_P}\phi\left(\sqrt{\lambda_P}\,(\theta-\mu)\right).$$

Third, we have

$$\begin{aligned}\lambda_P^{-1} &= \frac{1}{\lambda + \beta + \beta\delta} + \left(\frac{\beta + \beta\delta}{\lambda + \beta + \beta\delta}\right)^2\left(\frac{1}{\lambda} + \frac{1}{\beta\delta}\right) \\ &= \frac{1}{\lambda}\left\{1 - \frac{\beta + \beta\delta}{\lambda + \beta + \beta\delta} + \left(\frac{\beta + \beta\delta}{\lambda + \beta + \beta\delta}\right)^2 \frac{\beta\delta + \lambda}{\beta\delta}\right\} > \lambda^{-1}.\end{aligned}$$

With the first two observations, we compute $\mathbb{E}\left(P\left(z\right)\right)$:

$$\begin{aligned}\mathbb{E}\left(P\left(z\right)\right) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \pi\left(\theta\right) d\Phi\left(\sqrt{\hat{\lambda}_P}\left(\theta - \frac{\lambda\mu + \left(\hat{\lambda}_P - \lambda\right)z}{\hat{\lambda}_P}\right)\right) d\Phi\left(\sqrt{\hat{\lambda}_z}\,(z-\mu)\right) \\ &= \int_{-\infty}^{\infty} \pi\left(\theta\right)\int_{-\infty}^{\infty} \sqrt{\hat{\lambda}_P}\phi\left(\sqrt{\hat{\lambda}_P}\left(\theta - \frac{\lambda\mu + \left(\hat{\lambda}_P - \lambda\right)z}{\hat{\lambda}_P}\right)\right)\sqrt{\hat{\lambda}_z}\phi\left(\sqrt{\hat{\lambda}_z}\,(z-\mu)\right)dz d\theta \\ &= \int_{-\infty}^{\infty} \pi\left(\theta\right)\sqrt{\lambda_P}\phi\left(\sqrt{\lambda_P}\,(\theta-\mu)\right)d\theta,\end{aligned}$$

Moreover, by the law of iterated expectations, $\mathbb{E}\left(V\left(z\right)\right) = \mathbb{E}\left(\pi\left(\theta\right)\right) = \int_{-\infty}^{\infty}\pi\left(\theta\right)d\Phi\left(\sqrt{\lambda}\,(\theta-\mu)\right)$. Therefore, the ex ante expectation of the wedge is

$$\begin{aligned}\mathbb{E}\left(W\left(z\right)\right) &= \int_{-\infty}^{\infty}\pi\left(\theta\right)\left(\sqrt{\lambda_P}\phi\left(\sqrt{\lambda_P}\,(\theta-\mu)\right) - \sqrt{\lambda}\phi\left(\sqrt{\lambda}\,(\theta-\mu)\right)\right)d\theta \\ &= \int_{-\infty}^{\infty}\pi'\left(\theta\right)\left(\Phi\left(\sqrt{\lambda}\,(\theta-\mu)\right) - \Phi\left(\sqrt{\lambda_P}\,(\theta-\mu)\right)\right)d\theta \\ &= \int_{0}^{\infty}\pi'\left(\mu+u\right)\left(\Phi\left(\sqrt{\lambda}u\right) - \Phi\left(\sqrt{\lambda_P}u\right)\right)du \\ &\quad + \int_{0}^{\infty}\pi'\left(\mu-u\right)\left(\Phi\left(-\sqrt{\lambda}u\right) - \Phi\left(-\sqrt{\lambda_P}u\right)\right)du \\ &= \int_{0}^{\infty}\left(\pi'\left(\mu+u\right) - \pi'\left(\mu-u\right)\right)\left(\Phi\left(\sqrt{\lambda}u\right) - \Phi\left(\sqrt{\lambda_P}u\right)\right)du,\end{aligned}$$

where the first equality proceeds by integration by parts, the second by a change in variables, and the third step uses the symmetry of the normal distribution ($\Phi\left(-\sqrt{\lambda}u\right) = 1 - \Phi\left(\sqrt{\lambda}u\right)$). Since (by the third observation above) $\lambda > \lambda_P$, $\Phi\left(\sqrt{\lambda}u\right) > \Phi\left(\sqrt{\lambda_P}u\right)$ for all $u > 0$, and the theorem then follows immediately. ■

Proof of Proposition 2. Substituting the market-clearing condition $\hat{x}(P) = z$ and the investment threshold $\tilde{F}(z) = \tilde{F}(P(z))$ into (14), a price function $P(z)$ is part of an equilibrium if and only if it satisfies (15) and is invertible. Given the investment threshold $\tilde{F}(z)$, firm's expected dividend value is $V(z) = \left(\rho + G\left(\tilde{F}(z)\right)\right) E(\theta|z) - \int_{\underline{F}}^{\tilde{F}(z)} f dG(f)$. ■

Proof of Lemma 2. Taking the derivative with respect to Z in equation (18) gives

$$P'(\cdot) \quad = \quad \gamma \cdot (\rho + G(Z)) + g(Z)(Z - \mu)(\gamma - 1).$$

Since $\gamma > 1$, we can write the following inequality for values $Z < \mu$,

$$P'(\cdot) \quad > \quad \rho + G(Z) + g(Z)(Z - \mu),$$

which is positive for all Z whenever the distribution $G(F)$ satisfies the condition stated in the Lemma. ■

Proof of Proposition 3. (*i*) Since $V'(Z) = \rho + G(Z) > 0$ and $V''(Z) = g(Z) > 0$, $V(Z)$ is increasing and convex, so an increase in σ_Z^2 strictly increases $\mathbb{E}(V(z))$. Moreover, notice that $\mathbb{E}(W(z)) = (\gamma - 1)\mathbb{E}((G(Z) - G(\mu))(Z - \mu))$. Since $(G(Z) - G(\mu))(Z - \mu)$ is strictly positive for all $Z \neq \mu$, and strictly quasi-convex in Z, an increase in σ_Z^2 strictly increases $\mathbb{E}(W(z))$.The result for $\mathbb{E}(P(z))$ then follows from the statements about $\mathbb{E}(V(z))$ and $\mathbb{E}(W(z))$. Finally, (*ii*) is immediate given that γ does not affect $V(Z)$, but linearly scales up $W(Z)$. ■

Proof of Proposition 4. (*i*) From (22), we have

$$\begin{aligned}
\mathbb{E}(W(Z)) &= (\gamma - 1)\sigma_Z^2 \int_0^\infty \frac{G(\mu + \sigma_Z u) - G(\mu - \sigma_Z u)}{2\sigma_Z u} 2u^2 \phi(u)\, du \\
&\leq (\gamma - 1)\sigma_Z^2 \|g\| \int_0^\infty 2u^2 \phi(u)\, du.
\end{aligned}$$

The result then follows from noting that $\int_0^\infty 2u^2\phi(u)\, du = 1$ and

$$(\gamma - 1)\sigma_Z^2 = \left(\frac{\beta + \beta\delta}{\lambda + \beta + \beta\delta} - \frac{\beta\delta}{\lambda + \beta\delta}\right)\frac{1}{\lambda}.$$

(*ii*) Taking the limit in the expression above as $\delta \to 0$ and $\sigma_Z^2 \to 0$, we have $\lim_{\delta\to 0}(\gamma - 1)\sigma_Z^2 = \beta/(\beta + \lambda) \cdot \lambda^{-1}$, and

$$\lim_{\sigma_Z^2 \to 0} \int_0^\infty \frac{G(\mu + \sigma_Z u) - G(\mu - \sigma_Z u)}{2\sigma_Z u} 2u^2\phi(u)\, du = g(\mu) \int_0^\infty 2u^2 \phi(u)\, du = g(\mu).$$

(iii) From (22), it is immediate that $\mathbb{E}(W(Z))$ is maximized when $G(\cdot) = \mathbb{I}_{F>\mu}$, an indicator function that assigns 1, whenever $F > \mu$, i.e.

$$\mathbb{E}(W(Z)) \leq (\gamma - 1)\sigma_Z \int_0^\infty u\phi(u)\,du = (\gamma - 1)\sigma_Z \frac{1}{\sqrt{2\pi}}.$$

This corresponds to a scenario where the investment cost distribution is highly concentrated around the prior mean μ, so that a small change in the firm's posterior can have a large effect on its investment probability. Moreover for all $\varepsilon > 0$, there exists $\varepsilon' > 0$, such that whenever $\max_F \|G(F) - \mathbb{I}_{F>\mu}\| \leq \varepsilon'$, $\mathbb{E}(W(Z)) \geq (\gamma - 1)\sigma_Z/\sqrt{2\pi} - \varepsilon$. But from the above, it follows immediately that $\lim_{\delta\to\infty}(\gamma - 1)\sigma_Z = \infty$, so that $\lim_{\delta\to\infty}\lim_{G(\cdot)\to\mathbb{I}_{F>\mu}}\mathbb{E}(W(Z)) = \infty$. Using the limit characterization from (ii), $\lim_{G(\cdot)\to\mathbb{I}_{F>\mu}}\lim_{\delta\to\infty}\mathbb{E}(W(Z)) = \infty$, as long as $\lim_{G(\cdot)\to\mathbb{I}_{F>\mu}} g(\mu) = \infty$. ■

Proof of Proposition 5. Equation (24) follows immediately from (23). Substituting this into (16) and (15) gives (26) and (25). To check that this is an equilibrium, we check that the price function is invertible:

$$P'(Z) = \gamma(\rho + G(\mu + k(Z - \mu))) + (\gamma - k)k(Z - \mu)g(\mu + k(Z - \mu)),$$

which is strictly positive under the assumption from Lemma 2. The expected dividend function (26) then follows from rearranging
$V(Z) = (\rho + G(\mu + k(Z - \mu)))Z - \int_{\underline{F}}^{\mu + k(Z-\mu)} f\,dG(f)$ along the same lines as the price function.
■

Proof of Theorem 2. Proof: Part (i) is immediate from the definition of k. For (ii) notice that

$$\begin{aligned}
\frac{\partial V}{\partial \alpha} &= \frac{\partial V}{\partial k}(\gamma - 1) = g\left(\tilde{F}(Z)\right)(Z - \mu)(\gamma - 1)\left(Z - \tilde{F}(Z)\right) \\
&= -g\left(\tilde{F}(Z)\right)(Z - \mu)^2\alpha(\gamma - 1)^2 < 0, \text{ for } Z \neq \mu, \\
\frac{\partial P}{\partial \alpha} &= \frac{\partial P}{\partial k}(\gamma - 1) = g\left(\tilde{F}(Z)\right)(Z - \mu)\left(\mu + \gamma(Z - \mu) - \tilde{F}(Z)\right)(\gamma - 1) \\
&= g\left(\tilde{F}(Z)\right)(Z - \mu)^2(1 - \alpha)(\gamma - 1)^2 > 0, \text{ for } Z \neq \mu, \\
\frac{\partial W}{\partial \alpha} &= \frac{\partial P}{\partial \alpha} - \frac{\partial V}{\partial \alpha} = g\left(\tilde{F}(Z)\right)(Z - \mu)^2(\gamma - 1)^2 > 0, \text{ for } Z \neq \mu.
\end{aligned}$$

For (iii), we write $\mathbb{E}(W(Z))$ along the same lines as (22) as

$$\mathbb{E}(W(Z)) = (\gamma - 1)\sigma_Z \int_0^\infty (G(\mu + k\sigma_Z u) - G(\mu - k\sigma_Z u))\,u\phi(u)\,du,$$

and the comparative statics for $\mathbb{E}(W(Z))$ then follow immediately. Likewise, we write $\mathbb{E}(P(Z))$ as

$$\mathbb{E}(P(Z)) = \rho\mu + \mathbb{E}\left(\int_{\underline{F}}^{\tilde{F}(Z)} G(f)\,df\right) + \frac{\gamma-k}{k} cov(G\left(\tilde{F}(Z)\right), \tilde{F}(Z)).$$

Both terms in $\mathbb{E}(P(Z))$ are increasing in the variance of $\tilde{F}(Z)$, which is equal to $k^2\sigma_Z^2$, and therefore increasing in both γ and σ_Z^2. In addition, $(\gamma-k)/k$ is increasing in γ, which completes the comparative statics arguments for $\mathbb{E}(P(Z))$ w.r.t. γ.

For (iv), notice that $\frac{\partial V}{\partial \gamma} = \frac{\partial V}{\partial k}\alpha < 0$, and therefore $\mathbb{E}(V(Z))$ is also decreasing in γ. To see that the effect of σ_Z^2 on expected dividends, $\mathbb{E}(V(Z))$ is ambiguous notice that

$$V''(Z) = \frac{2-k}{k} g\left(\tilde{F}(Z)\right) - \frac{k-1}{k} g'\left(\tilde{F}(Z)\right)\left(\tilde{F}(Z)-\mu\right).$$

Now, if $k \in (1,2)$ and $g(\cdot)$ is single-peaked with a maximum at $F=\mu$, then $g'(F)(F-\mu) < 0$ and $V''(Z) > 0$, implying that an increase in the value of market information σ_Z^2 unambiguously increases $\mathbb{E}(V(Z))$. If on the other hand $k > 2$ and $g(\cdot)$ is uniform (with $\underline{F} < \mu$), then $V''(Z) < 0$ for all Z s.t. $g\left(\tilde{F}(Z)\right) > 0$. In this case, if σ_Z^2 is sufficiently low (so that with sufficiently high probability the posterior Z is close to the prior μ), $\mathbb{E}(V(Z))$ is decreasing in the quality of market information σ_Z^2.

(v) The result follows from the expression for $\mathbb{E}(W(Z))$ derived under (iii), and from observing that $\lim_{\delta\to 0} k\sigma_Z = \infty$, so that $\lim_{\delta\to 0}(G(\mu + k\sigma_Z u) - G(\mu - k\sigma_Z u)) = 1$ for all $u > 0$. But then $\lim_{\delta\to 0}\mathbb{E}(W(Z)) = \lim_{\delta\to 0}(\gamma-1)\sigma_Z/\sqrt{2\pi} = \infty$. ■