Human Capital and the Demographic Transition: Why Schooling Became Optimal*

Edgar Vogel
MEA, University of Mannheim

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Abstract

This paper develops a model with increasing adult life expectancy as the driving force of the economic and demographic transition. We show that if parents invest their own time into children's human capital, rising adult life expectancy unambiguously increases fertility. With children educated in schools and parents paying tuition fees, the reaction of fertility to changes in longevity is ambiguous. If productivity of adult human capital is sufficiently large and parent's valuation for additional children is sufficiently low, fertility will decrease. Without a schooling system, rising life expectancy therefore initially increases fertility. As during the development process life expectancy rises, a schooling system will be endogenously adopted and the relationship between fertility and longevity reversed. We argue therefore that it is important to account for the change in the nature of the costs of child education: from time costs to monetary costs.

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1 Introduction and Motivation

The history of humanity was – until recently – characterized by dismal economic conditions: low income, low life expectancy, low investment into human capital and high fertility. Briefly summarized, "the life of man [used to be], solitary, poore, nasty, brutish, and short" (Hobbes (1651), p. 78). In modern Western economies we observe the opposite: high income, high life expectancy, highly educated individuals and low fertility. In this paper we develop a model which is able to rationalize the monotonic increase in human capital investment, the hump-shaped relationship between fertility and life expectancy and the endogenous appearance of a formal schooling system. We argue that it is important to account for the change in the nature of the costs of child education: from time costs in an underdeveloped economy to monetary costs in a developed economy.

The driving force of the model is rising adult life expectancy. We use a simple life-cycle setup in which adults differ with respect to their productivity on the labor market and decide about consumption, investment into adult and child human capital and the number of children. They also chose whether they educate their children at home or whether they endow them with human capital by sending them to a school and paying tuition fees. If parents increase children's human capital using their own time, rising adult life expectancy will unambiguously increase fertility. On the contrary, if children are educated in a formal schooling system, agents' reaction to rising life expectancy is ambiguous. We show that if adult human capital is sufficiently productive and parents' preferences for children are sufficiently concave, fertility decreases as parents' life expectancy rises. Furthermore, parents deciding to send their children to a school have – for any life expectancy – fewer children and invest more in their human capital.

The decision which educational system to choose depends on parents' life expectancy and ability level. Given tuition fees, more productive agents choose the formal schooling system whereas less productive agents decide to spend own time on children's human capital. As life expectancy – and thus lifetime income – increases, also less productive agents will opt for the formal schooling system. Thus, rising adult life expectancy induces a *composition* effect (formal vs. informal schooling) and a *behavioral* effect (effect of life expectancy can increase or decrease fertility if children are educated in schools). For initially low life expectancy, the share of agents participating in the formal schooling system is low. Higher life expectancy thereby pushes economy-wide fertility up. With rising life expectancy during the development process, formal schooling becomes efficient for more and more people, generating a drop in aggregate fertility and an increase in human capital investment.

Thus, the key contribution of this paper is to provide a novel explanation for the fertility transition and the endogenous appearance of a mass schooling system in an otherwise rather standard model. The explanation is based on a change in the nature of investment in child quality from time costs to monetary costs. We thus propose a theory why a formal schooling system emerged endogenously *without* a state intervention on a large scale. What we do *not* explain is why eventually schooling become free, i.e. why the society – via government and parliament – decided to first subsidize private schooling and then set up a public schooling system financed by taxes. This would require to develop a theory in which political decisions (tax system, educational institutions, etc.) are determined endogenously within the model. We leave this extension for further research.

This paper is not the first to provide a possible explanation for the economic and demographic change occurring in the second half of the 19th century. Possible causes for declining fertility are declining child mortality rates (Kalemli-Ozcan (2002), Kalemli-Ozcan (2003), Tamura (2006)), natural selection favoring parents with a higher preference for child quality than quantity (Galor and Moav (2002)) and the narrowing of the gender wage gap making children more expensive (Galor and Weil (1996)). Further explanations are changing marriage institutions with a rising proportion of better educated women (Gould, Moav, and Simhon (2008)), structural change and an increasing share of people investing into human capital (Doepke (2004)) or the introduction of compulsory schooling (Sugimoto and Nakagawa (2010)). In Cervellati and Sunde (2005), Cervellati and Sunde (2007) and Soares (2005) rising adult life expectancy serves as the key explanatory variable for the observed economic development.[1] Particularly, higher life expectancy induces agents to invest more into human capital and decrease fertility. The driving mechanisms are the increasing opportunity costs of fertility as adult's human capital investment rises. Empirical evidence for the differential impact of life expectancy on population growth is provided by Cervellati and Sunde (2009). They show that before the demographic transition, improvements in life expectancy primarily

[1]There is an enormous amount of papers not explicitly targeting to explain the demographic transition but provide explanations for various other aspects of the development process. Prominent papers are Galor and Weil (2000), Hansen and Prescott (2002), Jones (2001), Kremer (1993), Strulik (2004), Kögel and Prskawetz (2001), Ehrlich and Lui (1991), Ngai (2004), Lagerlöf (2003), Fernández-Villaverde (2001), Tamura (2002). An excellent overview is provided by Galor (2005).

increased fertility. Using historical time series, Clark (2005b) agues that fertility is not monotonically related to income or life expectancy. This hypothesis is supported by Lehr (2009) based on data from contemporary developing countries.

More recently, the literature started to deal with the question why schooling systems came into existence. Galor and Moav (2006) explain the rise of a general schooling system (and thus of mass education) with a regulatory intervention by a ruling capitalist class. If skills and capital are complementary in production, diminishing marginal returns to capital accumulation can be counteracted by increasing workers' human capital. They argue, that capitalists lobbied for the introduction of compulsory schooling out of a profit maximizing rationale. In Boucekkine, de la Croix, and Peeters (2007), the appearance of a public schooling system emerges from profit maximizing behavior of municipalities. As population density increased, more and more schools were constructed decreasing the distance (transportation costs) of each agent to the next school which increased school attendance rates.[2]

Section 2 provides stylized facts and section 3 contains a detailed description of the model and the solution to the individual choice problem. The dynamic behavior of the economy with a discussion of the development path and an illustrative simulation exercise can be found in section 4 whereas section 5 provides some empirical evidence. Section 6 concludes the paper. All proofs are relegated to the appendix.

2 Life Expectancy, Schooling and Fertility

After a stagnation of living standards over centuries, the 19th century was the starting point of an unprecedented change in almost all aspects of economic and social life.[3] GDP per capita and population entered steep growth paths (Fig. 1d). Simultaneously we can observe that a lengthening of life become a trend rather than an occasionally lucky event. Although increases in life expectancy at birth were initially driven by falling child mortality, survival probabilities for adults also increased substantially (Fig. 1a). These improvements in living conditions of daily life were initially reflected in higher fertility. Crude birth rates and net reproduction

[2]For instance, Acemoglu and Robinson (2000), Bertocchi and Spagat (2004) and Grossman and Kim (2003) argue that providing eduction to the masses decreases the potential for social conflict and civic disorder. According to these papers, the introduction of a free (and compulsory) education system was not necessarily an altruistic act but served rather the interests of the ruling class.

[3]In this paper we use data for England and Wales but the same pattern can be also observed in other countries around the same time with good data; one prominent and often studied example being Sweden.

rates reached their historical peaks around 1820 and started to fall soon thereafter.[4]

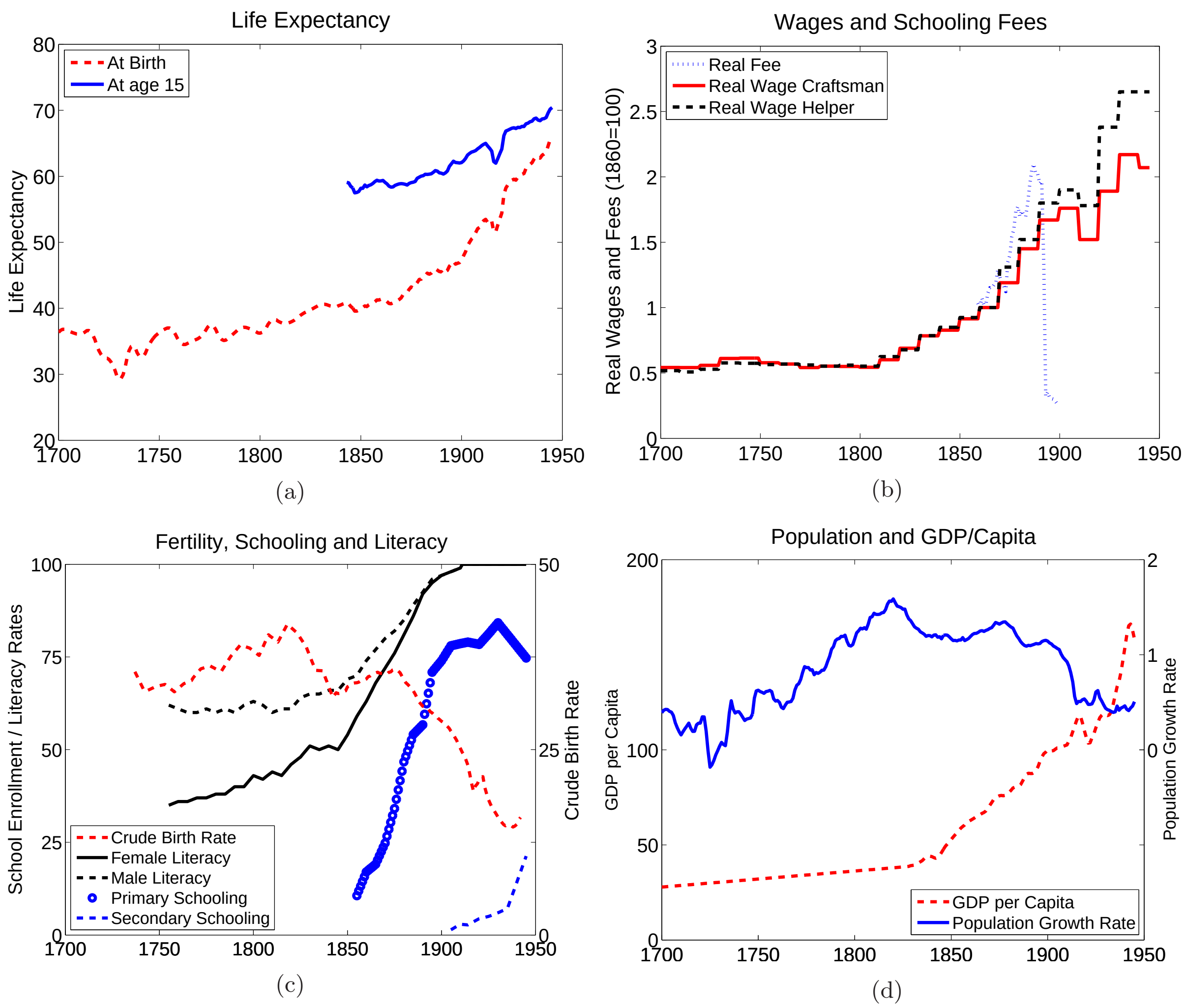
At the same time, acquisition of formal human capital started to gain momentum for the first time in history. The earliest statistics indicate that in 1850 around 10% of the children of age 5-14 attended primary school. Secondary school (10-19 years) did not enter official statistics before 1900 when the demographic transition was already well under way. Human capital measured by the ability to sign marriage contracts was, however, considerably higher. In the early 19th century, around 30% of all brides and 60% of grooms signed their marriage contracts with their names instead of using an "X" (Fig. 1c).[5] Note that the timing of the fertility reversal is closer to the introduction of primary schooling than to rising secondary school enrollment rates. It is also remarkable that primary schooling enrollment rates started to increase *before* the introduction of a compulsory schooling. The Elementary Education Act 1870 (also known as Forster's Education Act) provided only partial funding for schools in underdeveloped regions but fees were still charged. The Elementary Education Act of 1880 made schooling compulsory for children aged 5-10 (but was never aggressively enforced) and only the Free Education Act of 1891 made basic education virtually free by heavily subsidizing primary schooling.

Initially, education was thus not free but financed by parents. Data for 1834 Manchester show that up to 80% of children's education was paid for only by parents (West (1970), p. 84). On the aggregate level approximately 1% of Net National Income in 1833 was spent on day-schools (West (1970), p. 87). The development of fees relative to wages is shown in figure 1b demonstrating that the rise of tuition fees kept pace with the general wage increase and even outpaced it shortly before the Free Education Act was enacted. This suggests that education became relatively more expensive with a, *ceteris paribus*, detrimental effect on educational investment. Nevertheless, we observe that some parents decided to send their children to costly schools. These families were most likely not member of the rich bourgeois (they could afford e.g. private tutors anyway) but belonged rather to the lower or middle class indicating that they recognized the value of education, were able and willing to pay for it.

[4]There is no data available on total fertility rates before 1850. However, the few data points available show that TFR peaked around 1870 at 5 children per woman and started to decrease afterwards.

[5]Whether literacy was a useful skill before and during the industrial revolution is hotly debated in the literature. After the seminal paper by Galor and Weil (2000) human capital has been accepted as *the* key ingredient of any unified growth theory. Mokyr (2004), however, claims that literacy was restricted to a small share of the population (government officials, military personnel or members of the aristocracy) and is unlikely to serve as a good explanatory variable.

Figure 1: Stylized Facts



Data sources: Crude Birth Rate: Chesnais (1992). Tuition Fees: Mitch (1986). School Enrolment Rates: Flora, Kraus, and Pfenning (1983). GDP per Capita: Maddison (2003). Literacy: Schofield (1973), population average. Life Expectancy and Population Growth Rate: Wrigley and Schofield (1981) and Human Mortality Database (2008). Wages: Clark (2005a). The data refer to England and Wales. Enrollment data refer to the percentage of the age group (corresponding to the Net Enrollment Ratio).

3 The Model

In this section we describe the setup and solution to the model. The first subsection deals with the timing and notational conventions, followed by the description of aggregate production, production of human capital and the pricing of formal schooling. Then we move on to the households' preferences and constraints and solve the individual maximization problem. Finally, we solve for the general equilibrium and discuss the dynamic behavior of the economy.

3.1 Timing and Conventions

Consider an overlapping generations economy in which adults live for $a_B + T$ years. T is the life expectancy of an adult agent who enters adulthood at age a_B which may be regarded as the "economic" birth.[6] As a child, the agent may receive some education from her parents but is otherwise passive and does not make any own decisions. Adults can decide about consumption, number and education of children, and investment into adult human capital. Children are born right after parents enter adulthood at a_B. Parents can decide to educate children at home ("informal system") or they can decide to send their children to school and pay tuition fees ("formal system").[7] Time investment into adult human capital increases productivity on the labor market and agents differ with respect to their labor productivity.

The economy is populated by a discrete number of overlapping generations and each new generation (cohort) is indexed by τ. The new household born at time t has a life expectancy of $a_B + T_t$ where the length of childhood a_B is time invariant whereas life expectancy will change during the development process. Life expectancy is identical across agents and determined by exogenous forces outside of the households' control. Population size is the number of agents (including children) at any time t. Investment into adult human capital, child human capital and the number of children are continuous variables.[8] Reproduction is asexual, one agent in the present setting can be interpreted as a family making joint decisions. The notation in the

[6]Some papers, e.g. de la Croix and Licandro (2009) refer to this as puberty. In the context of this paper this could also be understood as marriage, see Voigtländer and Voth (2009) on the link of marriage and fertility.

[7]We use the terminology formal and informal to distinguish between education at home and education in some institution not requiring parents' time but money to "buy" the time of a teacher. In this model, formal education in a school is not free but parents have to pay tuition fees. See de la Croix and Doepke (2004) who model a framework with public (free but tax-financed) or private (tuition fees) and examine the long-run effects of the two educational system on growth and inequality.

[8]See Doepke (2005) for a model with discrete and sequential fertility decision. Although in presence of uncertainty the indivisibility assumption has an effect on the fertility behavior, he states that the "quantitative predictions of the models are remarkably similar".

paper is as follows: the subscripts τ and t denote cohort and calendar time, an individual's ability (type) is denoted by μ, a prime indicates a partial derivative, and the superscripts $j \in \{if, fo\}$ refer to the informal and formal schooling system. Variables with a bar (e.g. $\bar{x}$) denote averages and a tilde (e.g. $\tilde{x}$) indicates some threshold value for a variable. When no misunderstanding is expected, we omit indexes.

3.2 Aggregate Production

Human capital is the only productive factor in this economy and there is only one sector producing a homogeneous consumption good. We use a simple vintage model in which technological vintages are characterized by cohort specific productivity levels and each generation can operate only its cohort specific technology (each newborn generation automatically uses the new vintage).[9] Thus, agents earn over their entire working life the output (wage) of that vintage. This allows us to concentrate only on the labor market equilibrium at one point in time for one generation and avoids making assumptions about the substitutability of agents over different ages and vintages of human capital. Aggregate production for a generation τ is given by the linear technology

$$Y_\tau = A_\tau \mathcal{H}_\tau, \tag{1}$$

where A_τ denotes cohort specific productivity and $\mathcal{H}_\tau$ is the aggregate stock of effective labor supply, respectively. Effective labor supply is defined as $\mathcal{H}_\tau = P_\tau \mathcal{L}_\tau$ where P_τ is the number of workers, and $\mathcal{L}_\tau$ is average effective labor supply per worker. To exhaust total production, wages per unit of human capital and per capita income are

$$\omega_\tau = A_\tau, \tag{2}$$

$$w_\tau = \omega_\tau \mathcal{L}_\tau. \tag{3}$$

Hence, wages per unit of human capital increase in the general level of productivity and income per capita increases with higher individual effective labor supply. As will become clear later, nothing hinges on the *absolute* level of income per capita or wages. In order to focus on the

[9]See Cervellati and Sunde (2005) for similar assumptions. Chari and Hopenhayn (1991) develop a model showing that new technologies are not immediately adopted. In an empirical study Weinberg (2004) shows that older workers are more likely to operate old machines and new entrants into the labor market (young workers) will operate the most recent vintage.

main predictions of the model, we abstain therefore from including a Malthusian element by introducing a concave production function with a fixed factor.

3.3 Human Capital of Adults

Upon becoming adults, agents may decide to spend h units of time on the acquisition of adult human capital. Agents' heterogeneity translates into different productivity on the labor market where we assume that ability μ is distributed uniformly on the $[0, 1]$ interval and shifts individual productivity linearly. Then, adults' human capital is given by

$$f(h) = \mu \frac{h^\theta}{\theta}, \tag{4}$$

with $\theta < 1$.[10] The implicit assumption here is that human capital is embodied in people and therefore it has to be built up from zero by every new generation. Technological progress as a consequence of human capital accumulation therefore shows up only in the level of aggregate productivity A_τ.

3.4 The Price of Education

We do not model a detailed education sector, the main reason being the lack of consensus in the literature how a realistic modeling environment might look like. It is, for instance, conceivable that private schools are selfish profit maximizing organizations hiring teachers on the market and selling educational services but it is also equally plausible that schools are managed by non-profit organizations or run by a government attempting to recover only their costs (i.e. operating on a zero-profit basis). To begin with, we assume that each agent working in the education sector can produce one unit of educational services by using its human capital and one unit of time. This corresponds to the assumption made for education at home: each parent has to spend one unit of time and its human capital to produce one unit of "time input" into production of human capital of children. Further, we assume that the efficiency of this unit of time spent educating children increases with the average level of human capital $\bar{f}(h)$ in the economy. Using these assumptions, we have $e^{fo}(\mu) = m(\bar{f}(h))\ell^{fo}(\mu)$ where e^{fo} and ℓ^{fo} are education services produced and time spent teaching of an agent of ability μ. $m(\cdot)$ is an

[10] The uniform distribution is not crucial for the main argument of the paper. Further, the choice of this production function implies that it is never optimal for agents to choose $h = 0$. This can be relaxed at the cost of having corner solutions which would endogenously vanish once life expectancy is high enough.

increasing function mapping average human capital into a positive externality.[11] Agents are indifferent between working in the education or in the production sector and we assume that that they are randomly drawn from the population. Then, a teacher is just a representative (average) agent. Since competition on the labor market requires wages per unit of human capital ω to be identical, the price of schooling is

$$p = \omega \frac{\bar{f}(h)}{m(\bar{f}(h))}. \tag{5}$$

The price of education is increasing with the wage level and the average educational attainment of the adult population but decreasing with the externality created by a better educated population. Since this element will turn out to be crucial for the development path, we will come back to this issue in section 4.2.

3.5 Household Preferences and Constraints

The agent's utility function follows rather standard assumptions. An agent likes consumption over the life-cycle and values educated children. Conditional on the decision $j \in \{if, fo\}$ how to educate the offspring the agent from cohort τ maximizes the utility function

$$U^j = \int_0^T e^{-\rho a} \log c(a)^j da + \beta u(n^j z(e^j)) \tag{6}$$

where $\log c$ is the period utility obtained from consumption at age a, ρ is discounting future utility and $u(nz(e))$ is the intrinsic value of the quality-quantity composite weighted by β. The utility function u takes the number n of children times their quality which is captured by their human capital $z(e)$. This is a common assumption in the literature and can be understood as pure parental altruism or an implicit old-age pension system.[12] Human capital of children increases with investment e. The input into the production function is either parental time or teachers' time bought on the market for the price p per unit of time (tuition fees). Children

[11] A similar assumption concerning externalities is common in endogenous growth models with knowledge externalities, see e.g. Romer (1986). The main conclusion would not change if we required that teachers posses some minimum skill level (see also de la Croix and Doepke (2003) for a setup where teachers are average agents).

[12] The alternative formulation in which agents derive utility from the utility of their children (i.e. the dynastic approach in the spirit of Becker and Barro (1988) or Barro and Becker (1989)) requires that agents know (or form expectations) what their children will do. For models where the old-age security motive is made explicit see e.g. Boldrin and Jones (2002) and Ehrlich and Lui (1991).

survive with probability one until adulthood.[13] The timing of the agents' decisions is kept as simple as possible: agents first complete their investment into human capital and fertility and start working afterwards. Working on the labor market is an absorbing state and there is no retirement.

Since this paper does not focus on life-cycle dynamics or precise life-cycle profiles but rather on the trade-off between human capital investment (child and adult) and fertility, we assume that the discount rate and the interest rate are both zero.[14] This simplifying assumption obviously eliminates the traditional life-cycle elements for consumption, saving and labor supply. However, the "qualitative" structure of the problem is not altered: rising life expectancy has exactly the same effect as in a more realistic setup. Using the assumptions from above, the problem can now be written as

$$\{j, c^j, h^j, n^j, e^j\} = \arg\max \quad T \log c^j + \beta u(n^j z(e^j)) \tag{7}$$

subject to the constraints

$$Tc^j \leq \omega f(h^j)(T - \phi n^j - h^j) - pe^j n^j, \qquad \text{if } j = fo. \tag{8}$$

in the formal system where the educations costs are monetary costs. In the informal system the constraint is[15]

$$Tc^j \leq \omega f(h^j)(T - (e^j + \phi)n^j - h^j), \qquad \text{if } j = if. \tag{9}$$

In both constraints fixed time costs per child are identical and denoted by ϕ. The problem is in general analytically not tractable. Therefore we make the following assumptions about functional forms

[13]We ignore uncertainty about child survival. See e.g. Kalemli-Ozcan (2003) and Strulik (2004) on the theoretical and Eckstein, Mira, and Wolpin (1999) or Ram and Schultz (1979) empirical relationship between child survival and investment into human capital.

[14]Since marginal utility from consumption and the quantity-quality composite are independent, proceeding with $\rho = r \neq 0$ would not change the results (only the slope of the consumption profile would change), see the identical assumptions in e.g. de la Croix and Licandro (2009), Soares (2005) or Cervellati and Sunde (2005).

[15]In this setup we ignore the important issue of child labor. See e.g. Basu and Van (1998), Hazan and Berdugo (2002) or Baland and Robinson (2000) for models incorporating a child labor decision into growth models. However, note that we can rewrite the budget constraint by assuming that children can earn $n\varphi\omega f(h)$ where φ represents the relative wage of child labor. Then, $p = p^g - \omega f(h)$ and $\phi = \phi^g(1 - \varphi)$ where ϕ^g and p^g are *gross* and p and ϕ are *net* costs of schooling.

Assumption 1.

$$u(n,e) = \frac{(nz(e))^{1-\sigma}}{1-\sigma} \tag{10}$$

$$z(e) = \frac{e^{\gamma}}{\gamma} \tag{11}$$

with $\gamma < 1$, $\sigma > 0$.[16]

3.6 Individual Maximization Problem

The strategy is to solve the individual maximization problem – given wages and the price of schooling – conditional on the choice of the schooling system. This should highlight the conditional dynamics of fertility, investment into child, and adult human capital as life expectancy increases. Then we will analyze the effect of rising life expectancy on the choice of the parents' utility maximizing educational system. Using this two-stage procedure we can isolate the effect of rising life expectancy on the composition of the economy, i.e. formal vs. informal schooling and then the change in individual behavior conditional on this choices. Finally, we will put the individuals into a general equilibrium framework to allow for feedback effects and examine the dynamics of the aggregate economy. That is, we examine the simultaneous interaction of the behavioral and compositional change.

3.6.1 Household Solution in the Informal Education System

In this subsection we solve the household's problem assuming that agents educate their children at home. At this stage, we do not ask which schooling system is optimal for parents but examine their behavior given that they decided to stay in the informal system. This is essentially an environment without a formal schooling system where all investment into human capital is done at home by the parents. Thus, children's education consumes a share of their parents' time which is not available for productive work.

Using λ to denote the multiplier attached to the resource constraint, the first order condi-

[16] We use the log-function for sub-utility from consumption to keep fertility independent from the *level* of ω – for a given T – as the economy is growing. A non-neutral effect of wages on fertility can be brought back into the model by choosing a utility function which does not balance income and substitution effects. See Jones, Schoonbroodt, and Tertilt (2008) for an excelling literature overview how in theoretical models income is related to fertility.

tions[17] of the problem (skipping the index $j = if$) are

$$\beta u' z - \lambda \omega f(h)(e + \phi) = 0 \tag{12}$$

$$\beta u' z' - \lambda \omega f(h) = 0 \tag{13}$$

$$f'(h)(T - (e + \phi)n - h) - f(h) = 0 \tag{14}$$

where a prime denotes partial derivatives. Due to perfect capital markets human capital h maximizes lifetime income and is independent of the marginal utility of consumption λ. Furthermore, combining the FOC's for fertility and child schooling capital we obtain

$$e = \phi \frac{\gamma}{1 - \gamma} \tag{15}$$

implying that optimal investment into children's human capital is independent of adult life expectancy, parents' human capital (and ability) or wages. It is a constant determined by the relative time cost of children ϕ, and the properties of the human capital production function $z(e)$. Solving the entire household problem with informal education leads to the following proposition:

Proposition 1. *Assume that children are educated in the informal system. If adult life expectancy increases, adult schooling and fertility will increase. Investment into child human capital is constant.*

Proof. See Appendix. □

The intuition behind this result is rather simple. Adult schooling is rising since the time over which the benefits of educational investment can be reaped is increasing.[18] The result is based on the trade-off between the opportunity costs of schooling today and future benefits. Child human capital is constant since the assumption made on u implies that the agent maximizes quasi-linear utility. Under this assumption marginal utility from child schooling and marginal costs are proportional in n. However, the price of one additional unit of fertility or child schooling is linearly increasing in $f(h)$ whereas returns to child schooling are concave.

[17] To economize on notation we will not spell out the solution using the specified functional forms but rather use the general notation. For the proofs in the appendix we will, of course, switch to the specific functional forms whenever necessary.

[18] This is the Ben-Porath (1967) mechanism. See however Hazan (2009) and Kalemli-Ozcan and Weil (2002) for opposite views on the link between life expectancy, human capital investment and lifetime labor supply (including the timing of retirement).

Thus, investment into child human capital does not change and income effects are absorbed by (rising) fertility. Fertility, on the other hand, is rising because of the intratemporal optimality condition between consumption and fertility. Rising life expectancy implies rising total lifetime income and the agent will – by concavity of both utility functions - distribute some of the additional "free" income to increase consumption and fertility.[19]

3.6.2 Household Solution in the Formal Education System

Parents could also have their children educated by teachers ($j = fo$) for tuition fees p per unit of time. The crucial difference to the informal system is that the opportunity costs of child human capital are not valued by forgone adult wages but purely by monetary costs. Formally, the first order conditions associated with the problem are

$$\beta u' z - \lambda(ep + \omega f(h)\phi) = 0 \tag{16}$$

$$\beta u' z' - \lambda p = 0 \tag{17}$$

$$f'(h)(T - h - \phi n) - f(h) = 0 \tag{18}$$

In contrast to the setup with informal education, child schooling costs are no time costs any more. Thus, they do not enter the equation determining the optimal solution for adult human capital but are only monetary costs valued by the marginal utility of consumption. Combining again the first order conditions for quantity and quality of children we obtain

$$e \quad = \quad \omega f(h) \frac{\phi}{p} \frac{\gamma}{1-\gamma} \tag{19}$$

where now adult human capital increases also investment in child quality. This is an income effect stemming from the fact that the scarce factor time competing for labor supply and adult human capital accumulation is freed up. Thus, the "production function" for child human capital becomes linear instead of the convex costs caused by concave utility and concave

[19] An alternative way is to consider one equilibrium allocation of consumption c^* and time $\{n^*, h^*\}$ given a life expectancy T^*. Assume now that life expectancy rises but we hold fertility and adult schooling at $\{n^*, h^*\}$ constant. Then, per period consumption will always rise since at the given equilibrium allocation, the propensity to spend out of an additional unit of income is smaller than one but holding $\{n^*, h^*\}$ constant implies that the entire additional income is available for consumption. On the other hand, rising life expectancy makes investment into education more profitable introducing an additional "multiplier" effect increasing total lifetime income even more. Thus, marginal utility of consumption will decrease even further requiring a rise in fertility to equate marginal utilities. Consequently, the agent will sacrifice some income when young and thereby equate marginal utility from consumption and fertility. This result can also be established by assuming concavity of the production function $f(h)$ and concave utility functions $u(\cdot)$ and $z(e)$ only and does not hinge on any functional form.

production of adult human capital. However, the fact that child schooling is now purely a monetary cost implies also that the price of fertility relative to the price of schooling is rising in adult human capital (see FOC). Thus, it is not straightforward any more how fertility is changing if life expectancy (and h) is changing. Note that the introduction of a free public schooling system is still compatible with this setup. It is reasonable to assume that even without tuition fees the costs of schooling are bounded away from zero. This ensures that the household has a well defined demand for child education.

Solving the household problem now allows us to state the following proposition

Proposition 2. *Assume that children are educated in the formal system. If adult life expectancy increases, adult schooling and child schooling will increase. Fertility is always rising for $\sigma \leq 1$ but may fall for $\sigma > 1$.*

Proof. See Appendix. □

Again, investment into adult human capital rises because of the horizon effect. Child human capital rises because the price of schooling does not increase in $f(h)$ and therefore only the positive income effect is left over. Whether fertility increases or decreases depends on the coefficient of relative risk aversion with respect to the quantity-quality composite. Intuitively, if $\sigma \leq 1$, then the income effect of higher adult human capital will dominate (i.e. marginal utility from the quality-quantity composite is "less convex"). However, if $\sigma > 1$, then the effect of higher price of fertility may dominate. Given $\sigma > 1$, the reaction of fertility with respect to changes in life expectancy depends mainly on the properties of the adult human capital production function. The more adult human capital increases, the more expensive fertility becomes and therefore fertility is likely to decline.

We can draw two main conclusions from analyzing household behavior under the two schooling regimes. Firstly, if there is no formal schooling system and the only input into children's human capital is parental time we will not observe a decrease in fertility if adult life expectancy is increasing. Secondly, if there is a formal schooling system, a decrease in fertility is more likely but will not necessarily happen but depends on parameters from the utility and production function. If $\sigma > 1$, then fertility may decrease if $f(h)$ is increasing sufficiently as a consequence of more investment into adult human capital h. In either case, child schooling will (given p and ω) rise in the formal system as parental human capital increases. Aggregate fertility may decrease if the compositional effect is strong enough, i.e. sufficiently many agents decide to switch to the formal system. Being able to match the stylized facts, we proceed for the rest of the paper with

Assumption 2.

$$\sigma > 1 \tag{20}$$

While interpreting the results, we have to keep in mind that we have operated in a highly stylized environment without any frictions. Returns to education are not affected by technological progress, an assumption frequently made in the literature. We also implicitly assumed that parents and teachers are equally efficient in teaching children.[20] This may be true if we deal with educated parents (who could also be teachers) or the level of knowledge is rather low. Our assumption may be of limited use if parents are unskilled laborers or illiterate. Further, the choice of the log-utility for consumption implies that income effects – by simply raising wages (and p proportionally) – does not affect households' allocations. The purpose of these assumptions was to isolate the effect of different time allocation schemes on individual decisions. The quantitative relevance of this mechanism relative to competing explanations is ultimately an empirical question.

3.7 The Choice of Formal vs. Informal Schooling

Agents' optimal choice includes the decision in which education system their children are educated. For their decision, parents take their ability μ, wages ω, and the price of education p as given. One would expect that more able parents find it optimal to send their children to school and pay the tuition fees by spending more time on the labor market. The decision which schooling system to chose depends only on the potential income of parents. If they decide to send their children to a school, they do this because income earned on the labor market outweighs the costs of tuition fees. This is the sorting mechanism the model relies upon. The determinant is the wage-price ratio (ω/p) relative to potential earnings. Potential earnings are determined by individual ability and life expectancy. The more able agents are and the higher their life expectancy is, the cheaper and more efficient is education for their

[20]See e.g. Schultz (1964), Foster and Rosenzweig (1996) or Bartel and Sicherman (1998) on the link between technological progress and investment into human capital. The appendix contains an extension where we allow for a positive correlation between parents' ability as workers and teachers.

children in a formal system. Note that by assuming that ability is bounded from above, there may be such a vector $\{p, \omega\}$ that even the most able agents decide not to participate in the formal schooling system.[21] Then, we can state the following proposition:

Proposition 3. *If there is a vector $\{p, \omega\}$ such that agents are indifferent between the formal and informal system, agents with $\mu \geq \tilde{\mu}$ will decide to educate their children in the formal system whereas agents with $\mu < \tilde{\mu}$ will decide to educate their children at home. This threshold ability level $\tilde{\mu}$ is decreasing with rising life expectancy.*

Proof. See Appendix. □

Proposition 3 means that for a given relative price structure, also less able agents find it optimal to switch to the formal system as life expectancy is rising. The economic explanation is that lower ability is partly compensated by higher investment into human capital. In turn, rising life expectancy increases optimal investment into adult human capital thereby increasing the opportunity costs of educating children at home also for less able agents. And obviously, if for an agent with ability $\mu = \tilde{\mu}$ it was optimal to join the formal system given T, this will be optimal for higher life expectancy too: the price of schooling is constant and not increasing in h, thus the agent is always better off in the formal system. Rising life expectancy therefore implies that the indifferent agent becomes less able as T rises. However, whether this happens on the aggregate level once we allow for feedback effects of rising adult human capital investment on the price of education is not clear (see next section). Further, switching from the informal to the formal education system means that households' investment into human capital and fertility will change discontinuously.

Lemma 1. *If agents decide to educate their children in the formal system, they will increase investment in both types of human capital and decrease fertility.*

Proof. See Appendix □

This change in the optimal education system causes a change in educational attainment and fertility due to a changing composition but does not necessarily involve a behavioral change. Families choosing the formal system still could increase their fertility as their life expectancy

[21]The standardization with the upper bound of $\mu = 1$ does not matter. It is obvious that for any finite bounded ability, there is a sufficiently high price to deter even the most able agent from participating in the formal school system.

increases. Moreover, heterogeneity in ability is now also reflected in the heterogeneity of decisions.

Lemma 2. *If children are educated in the informal system, ability does not change the solution to the households' problem. If children are educated in the formal system, higher ability increases adult and child human capital investment and decreases fertility.*

Proof. See Appendix. □

The negative correlation between parental education (ability) and fertility is a well documented and widely accepted fact (Skirbekk (2008)). Note that for this pattern to emerge we need that the price of children's education is partly decoupled from parents' own human capital. Without the adoption of a formal schooling system, agents' allocations are identical despite different ability levels. Higher ability introduces an effect which proportionally raises prices of fertility and both types of human capital. Thus, more able agents enjoy only higher lifetime utility (due to higher consumption) without changing their allocation of time. Heterogenous behavior as a consequence of heterogeneity of skills requires that higher ability "buys" more time. This is, however, only the case if the price of child schooling is not perfectly linked to parents human capital.[22]

3.8 Aggregation

Aggregate human capital in goods' production for a given generation τ is given by

$$\mathcal{H}_\tau = P_\tau \left[\tilde{\mu}_\tau f(h_\tau^{if}) \ell_\tau^{if} + \int_{\tilde{\mu}_\tau}^{1} f(h_\tau^{fo}(a)) \ell_\tau^{fo}(a) da - \mathcal{E}_\tau \right] \tag{21}$$

where the first term measures effective labor supply of agents educating their children at home. The second term measures total labor supply of agents educating their children in the formal system and the last term is the labor supply of teachers not available for producing consumption goods. Total education time purchased on the market is given by

$$\mathcal{E}_\tau = \bar{f}_\tau(h) \int_{\tilde{\mu}_\tau}^{1} n_\tau^{fo}(a) e_\tau^{fo}(a) da. \tag{22}$$

[22]We could also allow for both – monetary and time costs – of child human capital with obviously qualitatively identical conclusions.

Using the assumption of uniformly distributed ability in the population, average human capital in the economy and fertility for any cohort τ are

$$\bar{f}_\tau(h) = \tilde{\mu}_\tau f(h_\tau^{if}) + \int_{\tilde{\mu}_\tau}^{1} f(h_\tau^{fo}(a))da \tag{23}$$

$$\bar{n}_\tau = \tilde{\mu}_\tau n_\tau^{if} + \int_{\tilde{\mu}_\tau}^{1} n_\tau^{fo}(a)da. \tag{24}$$

The first term is human capital and fertility of agents educating their children at home and the second term denotes the corresponding value for the families participating in the formal system.

4 The Dynamic System

The development process is shaped by the interaction of individually optimal decisions and macroeconomic externalities. Having solved the households' problem with fixed prices and for a given life expectancy, we will trace out the dynamics of simultaneous changes in prices and life expectancy. First, we study how the driving force of the model, adult life expectancy, is linked to the agents' individual decisions and how it evolves over time. Then, we will analyze the adjustment process of tuition fees when life expectancy is endogenous. Finally, we look at the dynamics of demographic variables.

4.1 Life Expectancy

Research has led to mainly two competing explanations why life expectancy has increased over the last centuries: improvements in nutrition and progress in medical knowledge. Whereas e.g. Fogel (1997) argues that the increases in the intake of calories is responsible for decreasing mortality, Cutler, Deaton, and Lleras-Muney (2006) object in their survey that it was mainly progress of medical knowledge.[23] Although is seems reasonable that life expectancy at some time will reach a biological upper bound, there are no signs that this will happen within the next generations. On the contrary, Oeppen and Vaupel (2002) show that record ("best-practice") life expectancy has risen for 150 years at a pace of 2.4 years per decade making it

[23] Cutler, Deaton, and Lleras-Muney (2006) show that there was no health-income gradient before the Age of Enlightenment and assert that ideas like germ theory, boiling water or simply washing hands are independent of the level of income. Mokyr (1993) takes a stand between the two theories writing that "...knowledge is believed to respond to market signals, social and political pressures, changes in incentives, institutions, and so on." In other words, as people escape from a state of mere subsistence, they can afford to spend resources on the advancement of knowledge.

impossible to derive a sensible prediction about maximum life expectancy. Using the insights from this literature we assume that life expectancy is a positive function of the available human capital in the economy capturing both effects. Particularly, we link the next cohort's life expectancy to the average human capital of the current cohort, implicitly assuming that parents' knowledge, health behavior, etc. determines the life expectancy of their children. We formalize this by writing

$$T_{\tau+1} = \Psi(\bar{f}(h(T_\tau))), \tag{25}$$

where Ψ is a strictly concave and non-decreasing function capturing the positive externality of average human capital on life expectancy. To escape from a trivial solution we make

Assumption 3.

$$T_{\tau+1} - T_\tau = \Delta(T_\tau) = \Psi(\bar{f}(h(T_\tau))) - T_\tau \geq 0 \quad \forall \ T_\tau > 0. \tag{26}$$

This is a simple nonlinear difference equation leading to an arbitrary high but finite life expectancy. By imposing the restriction that Ψ is non-decreasing and strictly concave we rule out possible non-monotonicities on the development path. We do this for the sake of clarity of the paper's argument: the implications of a less restrictive specification of Ψ are that we may end up with no or more than one steady-state without gaining additional insights.

4.2 Schooling Choice in General Equilibrium

It is clear that as life expectancy rises, agents invest more into human capital and thus p will increase. What matters, however, is the evolution of p relative to the evolution of potential income. Rising tuition fees do not matter as long as they are outweighed by sufficiently large increases in life expectancy. Put it differently: for any increase in potential income, there is a corresponding surge in the price of education such that the indifferent agent is characterized by exactly the same ability level. If prices increase by less, also less able agents will decide to join the formal system and $\tilde{\mu}$ will decrease. If the price of education rises faster then some less able agents will withdraw their offspring from formal schools. That is, the threshold ability level will increase undoing the positive effect of a higher potential income on children's education.

Proposition 4. *If life expectancy rises, the share of agents participating in the formal schooling system may, depending on the strength of the externality increase or decrease. Without externalities, $\tilde{\mu}$ will monotonically increase and may hit the upper bound of ability for sufficiently large life expectancy.*

Proof. See Appendix. □

For the sake of building some intuition, assume for the moment that agents are homogenous or that each agents hires a teacher from its ability group. The equilibrium price of education is then given by $p = \omega f(h(\mu))$. Plugging this price into the households' solution for child education (19) leads to a straightforward result: educational investment in the formal and informal system are identical. However, the absolute price of schooling in the formal system is higher since agents invest more into adult human capital (see lemma 1). In this case utility in the formal system is always lower than in the informal system: agents in the formal system neglect the externality on the price of schooling they generate by investing more into human capital. Hence, $U^{if} > U^{fo}$ holds for all T and optimizing agents will never chose the formal schooling system.[24] This is the very reason why we need some positive externality of higher human capital. Intuitively, if average human capital increases but there is no externality, the average will increase faster than the human capital of the agent with $\mu = \tilde{\mu}$.[25] The agent indifferent between formal and informal schooling must therefore be more skilled in order to compensate the higher price of education and make it indifferent between the two options.

4.3 Population Dynamics

The dynamics of average (aggregate) fertility is ambiguous and depends on the strengths of the different mechanisms at work. Simplifying equation (24) leads to

$$\bar{n}_\tau = \tilde{\mu}_\tau \bar{n}_\tau^{if} + (1 - \tilde{\mu}_\tau)\bar{n}_\tau^{fo}. \tag{27}$$

We know that a rising share of parents participating in the formal system decreases fertility via the composition effect. However, if the weight of these families is initially small, this effect

[24] See e.g. Bagnoud (1999) for Switzerland, Birchenough (1914) for England and Wales and Becker, Cinnirella, and Woessmann (2009) for Prussia for evidence on the reluctance of people (especially peasants and poor workers) to send their children to school despite compulsory schooling.

[25] Recall from lemma 2 that human capital investment is a positive function of μ. Then, human capital of agents participating in the formal system will raise by more than $f(h(\tilde{\mu})^{fo})$.

is likely to be dominated by the fertility of agents choosing the informal system. Secondly, even fertility of agents participating in the formal system may rise for a while as T rises. While the composition effect is then still working towards lower aggregate fertility, fertility of each subgroup will increase as life expectancy rises. Only if the share of the formal schooling is high enough and these agents also have fewer children as T goes up, will aggregate fertility of a cohort decrease unambiguously. Total population P_t and cohort size P_τ evolve according to

$$P_{t+1} = P_t + N_t(a)\,|_{a=a_B} \int_0^1 n(T_{t-a_B}, \mu) d\mu - N_t(a)\,|_{a=T_m}, \quad (28)$$

$$P_{\tau+1} = P_\tau(\bar{n}_\tau - 1), \quad (29)$$

where $T_m = T(t - T_m)$ denotes the life expectancy of the oldest agent in $t-1$ (who dies in t) born T_m years ago. $N_t(a)$ is the number of adults in t who either are of childbearing age ($a = a_B$) or die ($a = T_m$) this period.[26] The number of newborns per agent of childbearing age is determined by its life expectancy T_{t-a_B} and ability μ, and is denoted by $n(T_{t-a_B}, \mu)$. Population growth rate for any stationary life expectancy T is implicitly defined by

$$g_P = \frac{n(T)}{T} \left[\frac{(1+g_P)^T - 1}{(1+g_P)^T} \right] \quad (30)$$

The following corollary is rather obvious.

Corollary 1. *If fertility is a hump-shaped function of adult life expectancy, the population growth rate is also hump-shaped.*

Proof. See Appendix. □

Population dynamics is slightly more complicated if we start from a stationary population and let life expectancy increase. Then we have initially a positive effect on the population growth rate due to higher fertility and a (delayed) positive effect due to the fact that old agents are living longer. However, the second effect is only transitory and vanishes as life expectancy settles at a constant value. Whether in the new steady state population is growing or shrinking depends on the fertility associated with the steady-state life expectancy.

[26] Alternatively, population can be also written as the integral over all living cohorts $P_t = \int_{t-T_m}^{t+a_B} N_t(a) da$.

4.4 Technological Progress

As outlined in the introduction, technological progress occurs through the invention of better and more productive machines which can be operated only by the cohort entering the labor force at the time of the introduction of the new vintage. Following the literature, we assume that higher level of human capital facilitates invention of more productive technologies[27] and model this by assuming

$$\frac{A_\tau}{A_{\tau-1}} = g(\bar{f}_{\tau-1}(h)), \tag{31}$$

with g increasing and concave. After having defined the static solution to the households' problem and specified how aggregate variables change over time, we are ready to define the equilibrium development path of the economy.

Definition 1 (Equilibrium). *Given an initial population P_0 and initial life expectancy T_0, an equilibrium consists of a sequence of aggregate variables $\{\mathcal{H}_\tau, Y_\tau, A_\tau, T_\tau\}$, prices $\{p_\tau, \omega_\tau\}$, and individual decision rules $\{c^j_\tau, n^j_\tau, h^j_\tau, e^j_\tau, j\}$, $j \in \{if, fo\}$, such that*

1. *households optimality conditions given by equations (12) and (16) subject to the constraints (8) or (57) are satisfied,*
2. *aggregate variables are given by (1), (21), (25), and (31), prices by (3) and (5), and*
3. *life expectancy, total population, and cohort size evolve according to (25), (28), and (29).*

4.5 An illustrative simulation

The goal of this paper is to demonstrate the qualitative change in the behavior of agents as they endogenously decide to invest into the human capital of their children via a formal schooling system. In this subsection we provide therefore only an illustrative simulation without any ambition to exactly match historical time series. Since the model lacks many realistic features, it would require a lot of "twisting and tweaking" of model parameters and a very lenient attitude with respect to the choice of functional forms which is of limited use as far as further insights is concerned. Especially, the model is not able to explain the drop in tuition fees caused by the introduction of the Free Education Act from 1891 see (Fig. 1b). We therefore restrict ourselves to the choice of rather simple functional forms and have to keep in mind

[27]See e.g. Lucas (1988) or the excellent literature reviews by Jones (2005) and Klenow and Rodrígues-Clare (2005).

that at some point of the development process, a more or less exogenous drop in p took place. Our choices for Ψ and m are

$$T_{\tau+1} = \delta T_\tau + (\bar{f}_\tau(h))^\alpha, \tag{32}$$

$$m = (\bar{f}_\tau(h))^\kappa. \tag{33}$$

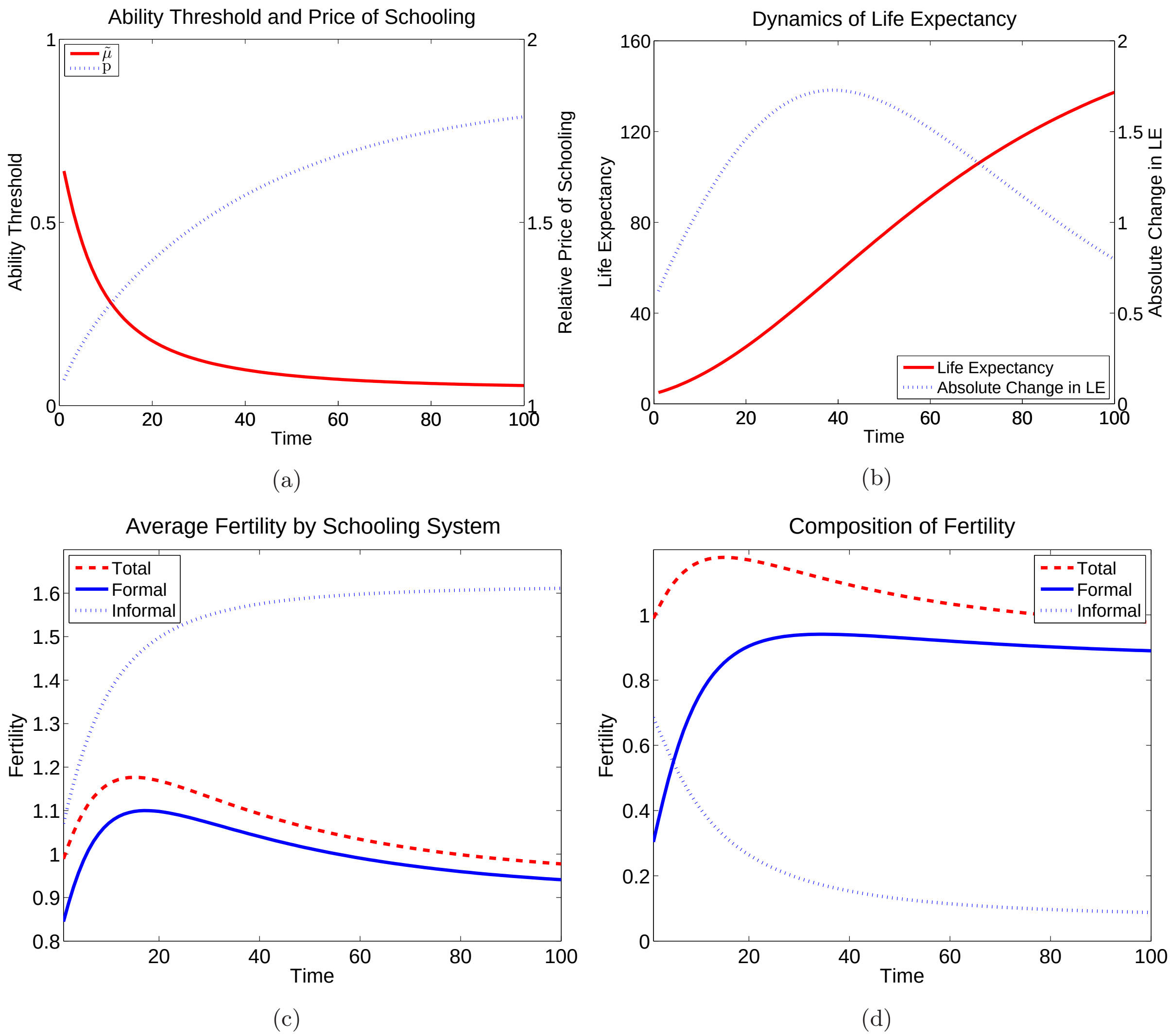
The parameters of the simulated model can be found in table 1. To simulate a situation without positive human capital externalities on the price of schooling, we set $\kappa = 0$ and change the values for σ and δ in order to keep fertility and life expectancy within reasonable bounds.

Figure 2 shows the basic patterns of the development process. Initially, life expectancy is low and only the most able agents invest into human capital of their children via the formal system. As life expectancy increases, despite a rising relative price of education the ability threshold $\tilde{\mu}$ decreases and more and more agents switch to the formal schooling system. Note that life expectancy and aggregate fertility are still rising. At this stage, average fertility in both systems is still rising and the composition effect is not sufficient to bring aggregate fertility down. However, this relationship changes during the development process. Despite the rising fertility of agents in the informal system, aggregate fertility is falling: the economy is now dominated by agents choosing the formal system and their fertility is falling as life expectancy keeps rising. On top of the compositional effect now also the behavioral effect works towards lower fertility. In figure 3 we simulate an economy without a positive human capital externality in the schooling system. The price of schooling for the indifferent agent is determined only by its human capital relative to the average. As shown in proposition 4, without externalities the ability threshold $\tilde{\mu}$ increases, and economy-wide fertility increases although life expectancy is rising.

Table 1: Model Parameters for Simulation

	β	γ	σ	ϕ	ρ	θ	κ	α	δ
With Externality	20	0.6	1.8	1.0	0.01	0.8	0.8	0.95	0.90
No Externality	20	0.6	1.2	1.0	0.01	0.8	0.0	0.95	0.85

Figure 2: A Simulation Exercise: Human Capital Externalities

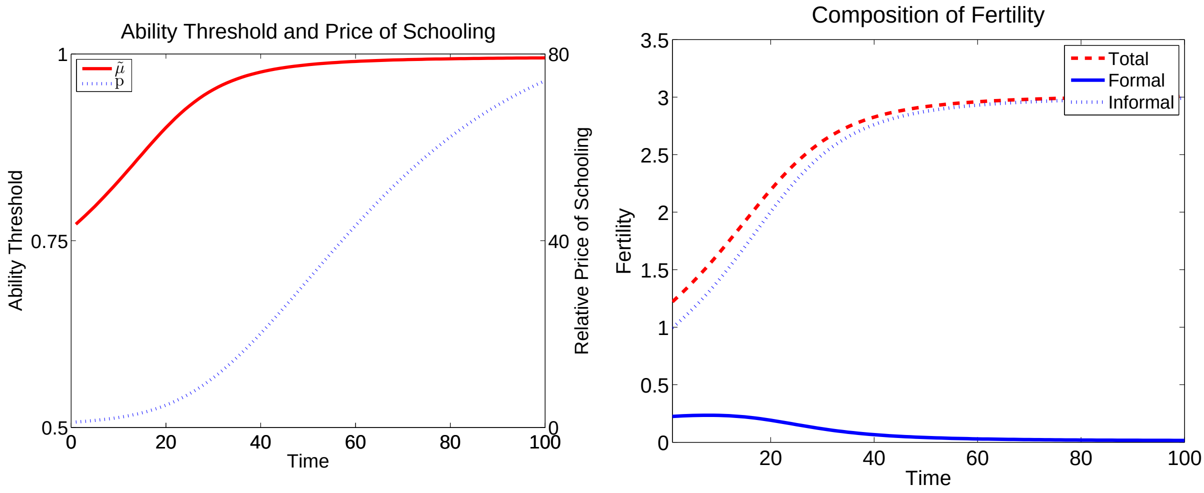


Data sources: Own simulations. See text and table for functional forms and parameter values.

5 Empirical Evidence

The theory from above provides some testable predictions. Most obviously, the reaction of fertility to changes in adult life expectancy should depend on the presence of a primary schooling system. For low levels of primary schooling, rising adult life expectancy increases fertility whereas beyond some threshold level for primary education, higher adult life expectancy should have a negative total effect on fertility. This correlation should also hold if we additionally condition on adult human capital. Although higher adult human capital investment (holding primary schooling fixed) has a direct negative effect on fertility (decreasing fertility

Figure 3: A Simulation Exercise: No Human Capital Externality



(a) (b)

Data sources: Own simulations. See text and table for functional forms and parameter values.

for those who send their children to school) this should not change the basic hump-shaped pattern. Conversely, the interaction between adult survival rates and adult human capital (even conditioning on primary schooling) should leave the negative relationship between life expectancy and fertility unchanged (or make it even stronger).

To test these two hypotheses, we will estimate a reduced form equation. The dependent variable is the crude birth rate. As a proxy for adult life expectancy we use the probability of a 15 year old agent to survive until the age of 60 in a given year. This choices ensure that the econometric setup is as close as possible to the model's spirit: agents make their fertility decisions in a given year conditional on their life expectancy at that point in time. To proxy for the availability of a primary schooling system, we use the share of 5-10 year old children with (primary) education.[28] This measure is more suitable than e.g. average years of schooling since it is naturally bounded between zero and unity and can be interpreted as the empirical counterpart to $\tilde{\mu}$. As a proxy for adult human capital we use years of secondary schooling in the population older than 15. Furthermore, we include GDP per capita and its growth rate. GDP per capita controls for income effects (which we have explicitly ruled out in the theoretical model) and the growth rate accounts for possible incentive effects of higher growth

[28] As a robustness check, we use the gross primary enrollment rate as a measure for primary education. The results leave our conclusions unchanged (results available upon request) .

rates on human capital accumulation or fertility. Our data source for adult survival rates, and the crude birth rate are the World Development Indicators (The World Bank (2010)). GDP per capita and its growth rate are from the Penn World Tables 6.3 (Heston, Summers, and Aten (2009)) and the education data is from Barro and Lee (2010).

The availability of data over long time periods is unfortunately a severe restriction. Especially data on adult life expectancy and education is only available on a ten, and respectively five year basis. The sampling period starts in 1960 and ends in 2000 with at most 5 observations for each country. Further, the estimated models suffer from strong endogeneity problems whereas the number of available instruments is limited and their applicability is highly debated in the literature. Therefore, we do not aim at establishing causality by using weak or invalid instruments but will rather show that the stylized facts presented in the historical time series are a robust finding also present in contemporaneous data for more countries. The strategy is to first estimate a reduced form equation neglecting the interaction of survival rates and primary education. Then, we re-estimate the same equations with an interaction term between schooling and adult survival rates. The estimated equations are of the form

$$\ln Cbr_{it} = \beta_0 + \beta_1 Sr_{it} + \beta_2 Sr_{it} Edu_{it} + \beta_3 Edu_{it} + \beta_4 X_{it} + \epsilon_{it} + \gamma_i, \tag{34}$$

where Sr is the adult survival rate, Edu is a measure for education, X is a set of other controls and ϵ and γ are the error terms. We expect that without the interaction term β_1 and β_3 are negative. Including $SrEdu$ we expect that the sign on the survival rate β_1 turns now positive and the sign of the interaction term β_2 negative and larger (in absolute values) than the coefficient on the survival rate. Then, for zero education the correlation between survival rates and fertility is positive ($=\beta_1 > 0$) but for a sufficiently high primary schooling participation rate the total effect of rising survival rates is negative ($=\beta_1+\beta_2 < 0$). Conversely, including an interaction term between adult survival rates and secondary education leaves – as predicted by the theory – all signs unchanged (i.e. $\beta_1 < 0$, $\beta_3 < 0$). To control for country specific unobserved heterogeneity we run a panel regression with country fixed-effects. To account for possible time trends we use either one set of year dummies for all countries or a separate set of year dummies for low- and high fertility countries. We follow the literature and define a country to be in the low fertility regime if it had a crude birth rate of less than 30 for the entire time period.

Table 2 presents the results from the models without interacting survival rates and primary education. As expected, higher survival rates and more education (primary and secondary) are negatively correlated with crude birth rates. Including different time trends and controlling for income or growth does not change this conclusion.

Table 2: Relationship Between Survival Rates and Fertility - No Interaction Effects

Survival Rate	-1.08***	-0.67***	-0.25*	-0.63***
Prim. Education	-0.19*	-0.14*	0.15*	-0.10
GDP/Capita	-0.31***	-0.13***	-0.10***	-0.15***
Growth GDP/Capita	2.13***	0.72*	0.56	0.81*
Years Sec. Schooling		-0.15***	-0.04*	-0.17***
Trend	–	–	yes	–
Trend × Low Fert.	–	–	–	yes
Constant	6.93***	5.31***	4.12***	5.44***
N	502	502	502	502

Dependent variable is the log of crude birth rate. Primary education, years of secondary schooling, survival rate and growth of GDP/capita in levels, GDP/Capita in logs. Trend is the set of year dummies. Stars indicate levels of significance (*: 10%, **: 5%, ***: 1%), robust standard errors. All regressions with country fixed effects.

Table 3 contains essentially the same regressions but includes an interaction term between survival rates and education. In columns 1-4 we interact primary, and in columns 5-8 secondary education with adult survival rates. The signs of the coefficients are in line with our expectations. Higher survival rates are correlated with higher birth rates for low values of primary education. The relationship between years of secondary education and crude birth rate is always negative. Further, the coefficient on the interaction term β_2 is larger (in absolute values) than the coefficient on the survival rates β_1 indicating that $\tilde{\mu}$ is in the support of the data: the effect of higher survival rates on fertility is initially positive but for full primary schooling negative. Repeating the same exercise but using years of secondary education instead or primary education does not change the sign of β_1: countries with higher survival rates have lower fertility rates. The interaction term is fluctuating around zero and never significantly different from zero and the effect of primary schooling varies across specifications but is in most cases insignificant. Thus, the interaction between adult survival rates and secondary schooling does not seem to have an effect on fertility.

To sum up, the results point into the direction that there is an interaction effect between adult life expectancy and primary schooling but not with secondary schooling. We interpret this as evidence if favor of the model presented in this paper but recognize that we fail to establish reliable causal evidence.

Table 3: Relationship Between Survival Rates and Fertility - With Interaction Effects

Measure for Education →	Primary Schooling				Secondary Schooling			
Survival Rate	0.43	0.60	1.52***	0.56	-0.81***	-0.66***	-0.22	-0.53***
Education	0.98**	0.85**	1.51***	0.83**	-0.20**	-0.14	-0.02	-0.08
Surv. Rate × Educ.	-1.87**	-1.59***	-2.13***	-1.50***	0.05	-0.01	-0.03	-0.10
GDP/Capita	-0.29***	-0.12***	-0.07**	-0.14***	-0.14***	-0.13***	-0.10***	-0.15***
Growth GDP/Capita	2.08***	0.71*	0.46	0.79*	0.71*	0.72*	0.56	0.85**
Education$_{-i}$		-0.15***	-0.03	-0.16***		-0.14*	0.14	-0.13
Trend	–	–	yes	–	–	–	yes	–
Trend × Low Fert.	–	–	–	yes	–	–	–	yes
Constant	5.83***	4.41***	2.71***	4.57***	5.32***	5.30***	4.09***	5.38***
N	502	502	502	502	502	502	502	502

Dependent variable is the log of crude birth rate. When Education is the share of children with primary education, Education$_{-i}$ denotes the average years of secondary schooling in the population (and vice versa). Education, Education$_{-i}$, survival rate and growth of GDP/capita in levels, GDP/Capita in logs. Trend is the set of year dummies. Stars indicate levels of significance (*: 10%, **: 5%, ***: 1%), robust standard errors. All regressions with country fixed effects.

6 Conclusion and Discussion

This paper proposed a simple model arguing that to understand the change in agents's behavior during the demographic transition, it is crucial to account for changing nature of the costs of child quality. We show that if the input in children's human capital production is only parental time, increasing life expectancy always increases fertility. This is because the price of child quality and quantity rise simultaneously with higher life expectancy.

A behavioral change can only occur if parents decide to educate their children in the formal system. This transforms time costs into monetary costs. Then, higher lifetime income "buys" also more time. In other words, if parents spend their own time to enhance children's human capital, rising life expectancy increases the price of quality and quantity. With investment into child human capital via a school, increasing labor supply and adult human capital increases only the opportunity costs of quantity but leaves the price of education unchanged. Hence, if parental human capital is sufficiently productive and the marginal valuation of an additional child is sufficiently low, the rising relative price of quantity will bias the parental decision towards more investment into quality and lower quantity.

Since at early stages of development, the share of people deciding to educate their children at home is high, gains in adult life expectancy initially increase fertility. As life expectancy rises, more agents decide to send their children to schools, thereby strengthening the composition effect but at the same time also reinforcing the negative effect of a higher life expectancy on fertility by a potential behavioral change. Once the share of parents participating the formal system is high enough, fertility will fall. Furthermore, in this paper we have proposed a theory why a formal schooling system may emerge endogenously without intervention by the state. We do not, however, make the next step and model why the society – via government and parliament – decided set up a free public schooling system financed by taxes. The extension by such a political economy element is left for future research.

7 Appendix

Since the problem has in general no closed form solution, we compute the comparative statics by implicitly differentiating the system of first order conditions. Then, for a change in variable X, the partial derivatives of n and h are given by

$$\begin{bmatrix} h_X \\ n_X \end{bmatrix} = -\begin{bmatrix} F_{hh} & F_{hn} \\ F_{nh} & F_{nn} \end{bmatrix}^{-1} \begin{bmatrix} F_{hX} \\ F_{nX} \end{bmatrix} = -|A|^{-1} \begin{bmatrix} F_{nn}F_{hX} & -F_{hn}F_{nX} \\ -F_{nh}F_{hX} & F_{hh}F_{nX} \end{bmatrix} \tag{35}$$

Proof of proposition 1. From (15) we have that e^{if} is constant. Then we can write the household problem in terms of only n and h.

$$F_h = \frac{1}{c}\left(f(h) - \mu h^{\theta-1}\ell\right) \qquad F_n = \frac{\beta}{n}\left(nz(e)\right)^{1-\sigma} - \frac{T(e+\phi)}{\ell} \tag{36}$$

with $\ell \equiv T - h - n(e+\phi)$ and $c \equiv \omega f(h)\ell T^{-1}$. Partial derivatives are given by

$$\begin{bmatrix} h_T \\ n_T \end{bmatrix} = -\begin{bmatrix} -\frac{T}{h^2}\left(\theta + \frac{h^2}{\ell^2}\right) & -T\frac{e+\phi}{\ell^2} \\ -T\frac{e+\phi}{\ell^2} & -\frac{\sigma\beta}{n^2}\left(nz(e)\right)^{1-\sigma} - \frac{T(e+\phi)^2}{\ell^2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\theta}{h} + \frac{T-\ell}{\ell^2} \\ \frac{(e+\phi)(T-\ell)}{\ell^2} \end{bmatrix} \tag{37}$$

and the determinant $|A| = F_{hh}F_{nn} - F_{hn}F_{nh} > 0$ can be shown to be positive implying that we have a maximum. Combining the elements from above establishes $n_T > 0$ and $h_T > 0$. □

Proof of proposition 2. The system of first order conditions is

$$F_h = \frac{1}{c}\left(f(h) - \mu h^{\theta-1}\ell\right) \qquad F_n = \frac{\beta}{n}(nz(e))^{1-\sigma} - \frac{1}{c}\left(pe + \phi\omega f(h)\right) \tag{38}$$

$$F_s = n\left(\beta s^{\gamma-1}(nz(e))^{-\sigma} - \frac{p}{c}\right) \tag{39}$$

with $\ell \equiv T - h - \phi n$ and $c \equiv (\omega f(h)\ell - npe)T^{-1}$. Combining F_s and F_n one obtains $e = f(h)\frac{\phi}{p}\frac{\gamma}{1-\gamma}$. Substituting this into the FOCs reduces the dimension of the system by one equation. We proceed by using F_h and F_n. For the comparative statics we have

$$\begin{bmatrix} h_T \\ n_T \end{bmatrix} = -\begin{bmatrix} -\frac{1}{c}\left(\omega\mu h^{\theta-1}\frac{1+\theta}{\theta}\right) & -\frac{1}{c}\omega\mu h^{\theta-1}\phi \\ -\left(\frac{\omega f(h)}{c}\right)^2\frac{\phi}{T(1-\gamma)} + (1-\sigma)\beta\gamma\theta\frac{(nz(e))^{1-\sigma}}{nh} & -\frac{\beta\sigma(nz(e))^{1-\sigma}}{n^2} + \left(\frac{\omega f(h)}{c}\frac{\phi}{1-\gamma}\right)^2\frac{1}{T} \end{bmatrix}^{-1}$$
$$\times \begin{bmatrix} \frac{\omega\mu h^{\theta-1}}{c} \\ \left(\frac{\omega f(h)}{T(1-\gamma)c}\right)^2\phi(h(1-\gamma)+\phi n) \end{bmatrix} \tag{40}$$

It can be again shown that $|A| = F_{hh}F_{nn} - F_{hn}F_{nh} > 0$. Further we have

$$
\begin{aligned}
h_T &= |A|^{-1}\left[\beta\sigma\frac{(nz(e))^{1-\sigma}}{n^2}\left(\frac{\omega f(h)\phi}{(1-\gamma)cT}\right)^2\omega f(h)(1+\gamma\theta)\right] > 0 && (41)\\
n_T &= |A|^{-1}\left[\left(\frac{\phi\omega f(h)}{c(1-\gamma)T}\right)^2\frac{n}{h}\omega f(h)(1+\gamma\theta) + \beta\gamma\theta^2(1-\sigma)\frac{\omega f(h)(nz(e))^{1-\sigma}}{nc}\right] \gtrless 0 && (42)
\end{aligned}
$$

where h_T is always positive. n_T is positive for $\sigma \leq 1$ but may be negative otherwise. □

Proof of Proposition 3. Substituting optimal choices for e and n into the utility function, utility of agents conditional on their education system choice is

$$
\begin{aligned}
U^{if} &= T\log\left(\omega\mu\frac{h^{1+\theta}}{T\theta^2}\right) + \frac{\beta\,(nz(e))^{1-\sigma}}{1-\sigma} & e &= \frac{\phi\gamma}{1-\gamma} && (43)\\
U^{fo} &= T\log\left(\omega\mu\frac{h^{\theta}(h+(h-T)\gamma\theta)}{(1-\gamma)T\theta^2}\right) + \frac{\beta\,(nz(e))^{1-\sigma}}{1-\sigma} & e &= \frac{\omega}{p}\frac{\mu\phi\gamma h^{\theta}}{\theta(1-\gamma)}. && (44)
\end{aligned}
$$

and for a given vector $\{p, \omega\}$ the threshold ability level $\tilde{\mu}$ is implicitly defined by setting $U^{if} = U^{fo}$. Since relative sub-utility from consumption is not affected directly by ω or μ (shift consumption proportionally), the decision which system to adopt depends only μ and $\{p, \omega\}$ to *via investment into education* (income effect). Note that a higher (lower) price p requires a proportionally higher (lower) ability μ to restore indifference (the allocation does not change for $j = if$). Write the indifference condition as

$$
T\log\left(\frac{c^{fo}}{c^{if}}\right) = \beta\left[u(n^{if}z(e^{if})) - u(n^{fo}z(e^{fo}))\right]. \qquad (45)
$$

Since the decision to join the formal system is based purely on a sufficiently large income effect, it holds that $c^{fo} \geq c^{if}$ for all solutions. For $T \to \infty$ we have

$$
\begin{aligned}
n^{if} &= \left(\frac{\beta(1-\gamma)z(e^{if})^{1-\sigma}}{(1+\theta)\phi}\right)^{\frac{1}{\sigma}}\\
h^{if} &= T\frac{\theta}{1+\theta} && (46)
\end{aligned}
$$

with e^{if} as defined above. For indifference it must always hold that $u(n^{if}z(e^{if})) - u(n^{fo}z(e^{fo})) \geq 0$. Given constant $u(\cdot)^{if}$ in the limit, we need that $u(\cdot)^{fo}$ is also constant with the difference approaching zero.[29] We also know that $\partial h^{fo}/\partial T > 0$ and hence $\partial e^{fo}/\partial T > 0$. Using $n^{if} > n^{fo}$

[29] Obviously, e^{fo} and/or n^{fo} cannot keep growing monotonically otherwise the condition $u(\cdot)^{if} - u(\cdot)^{fo} \geq 0$ would be

and $e^{fo} > e^{if}$ we know that it must hold that n^{fo} approaches n^{if} from below and e^{fo} approaches e^{if} from above. For given p and ω, this can only happen if the threshold ability level $\tilde{\mu}$ is decreasing. Since utility is non-decreasing in μ, this must hold for all μ and T. □

Proof of Lemma 1. Assume that we have found a vector $\{p, \omega, \tilde{\mu}\}$ such that $U^{if} = U^{fo}$ holds. Further, we can rewrite the FOC F_n for both schooling systems to

$$(1-\gamma)^{\sigma}\frac{T\phi\theta}{\beta} = z(e)^{1-\sigma}\frac{h(1-\gamma)}{n(h)^{\sigma}} \quad \text{if } j = if, \tag{47}$$

$$\frac{T\phi\theta}{\beta} = z(e(h))^{1-\sigma}\frac{h+(h-T)\gamma\theta}{n(h)^{\sigma}} \quad \text{if } j = fo. \tag{48}$$

The LHS if $j = if$ is always smaller than the LHS if $j = fo$ and the same ordering must hold for the RHS in equilibrium. Since RHS is increasing in h, agents switching to the formal system have lower fertility and invest more in child human capital. □

Proof of Lemma 2. Differentiating FOCs with respect to ability gives

$$F_{h\mu} = 0 \qquad F_{n\mu} = 0 \qquad \text{if } j = if \tag{49}$$

$$F_{h\mu} = 0 \qquad F_{n\mu} = \frac{\beta\gamma(1-\sigma)}{n\mu}(nz(e))^{1-\sigma} \quad \text{if } j = fo \tag{50}$$

Combining this with the Hessian from above proves that ability does not change households' allocations. For $j = fo$ and assumption 2 more able agents invest more into adult human capital and lower fertility. Higher e follows trivially from (19). □

Proof of Proposition 4. First, use the price of education from (5) to express the equilibrium investment into education by parents choosing the formal system. This gives

$$e = \phi\frac{\gamma}{1-\gamma}\frac{f(h(\tilde{\mu}))}{\bar{f}(h(T))}m(\bar{f}(h(T))) \tag{51}$$

Consider the trivial case with $m(\bar{f}(h(T))) = \bar{f}(h(T))$. Then, the price of education relative to potential earnings is always one which brings us back to the partial equilibrium situation from proposition 3: $\tilde{\mu}$ decreases in T. Consider now the polar case without any externalities ($m(\cdot) = 1$) implying that the dynamics of p is determined by the evolution of $f(h(\tilde{\mu})^{fo})$ relative

violated for some T.

to the average $\bar{f}(h)$. Substituting this into (45) gives

$$T \log\left(\frac{c^{fo}}{c^{if}}\right) = \beta \frac{z(e^{if})^{1-\sigma}}{1-\sigma}\left[(n^{if})^{1-\sigma} - (n^{fo} z(\Phi(\tilde{\mu}, T)))^{1-\sigma}\right] \tag{52}$$

$$\Phi(\tilde{\mu}, T) \equiv \frac{f(h(\tilde{\mu}, T)^{fo})}{\tilde{\mu} f(h(T)^{if}) + \int_{\tilde{\mu}}^{1} f(h(a, T)^{fo}) da} \tag{53}$$

We know that the LHS is positive for all T. Since n^{if} is converging to a constant, the same must hold for $u(n^{fo} z(e^{fo}))$. With n^{fo} approaching n^{if} from below, $\Phi(\tilde{\mu}, T)$ must converge to a constant. Further, we have $\partial\tilde{\mu}/\partial T = -(\partial\Phi/\partial T)/(\partial\Phi/\partial\tilde{\mu}) < 0$ implying that the ability level is increasing. By monotonicity of the RHS in $\tilde{\mu}$ this is holds for all T. The result hinges on the fact that p is determined by the human capital of the indifferent agent relative to the average. As T and h rise, human capital of the $j = fo$ agent rises slower than the average (note that the agent in the numerator is the first agent in the integral in the denominator). Hence, the ability level of the indifferent agent has to rise. This increases potential earnings of agent $\mu = \tilde{\mu}$ and decreases the $\bar{f}(h)$ by shifting agents from the if to fo (composition effect). Thus, $\tilde{\mu}$ will get arbitrarily close to 1 for large T (depends on fixed point of Ψ). □

Proof of Corollary 1. If fertility is a concave function of T, there are two life expectancies T_l and T_h at which fertility per family equals 2 and there must be an intermediate life expectancy T_{im} which maximizes fertility (above 2 children per couple) and population growth rate. Concavity gives us $\frac{\partial n(T_l)}{\partial T_l} > 0$ and $\frac{\partial^2 n(T_h)}{\partial T_h^2} < 0$ resulting in rising and falling population growth rate at the two stationary population levels ($g_P = 0$).

$$\left.\frac{\partial g_P}{\partial T_t}\right|_{g_P=0} = \frac{\partial n(T_t)/\partial T_t}{1 + T_t} \tag{54}$$

Maximum population growth is defined by (55) and the derivative of g_P at that point is given by (56). Thus, g_P increases at T_l, attains a maximum at T_{im} and starts to decrease thereafter and becomes even negative at T_h.[30]

$$1 - (1 + g_P)^{T_{im}} + T_{im} \log(1 + g_P)^{T_{im}} = 0 \tag{55}$$

$$\left.\frac{\partial g_P}{\partial T_{im}}\right|_{g_P>0} = -\frac{(1 + g_P)\log(1 + g_P)}{T_{im}} < 0 \tag{56}$$

[30] This can be also trivially proven by applying the mean-value theorem since aggregate fertility is continuous and differentiable in T.

□

Extensions

We have assumed that agents' labor productivity does not affect their productivity as parents (teachers). In this section we relax this simplifying assumption. To allow for a positive effect of labor market productivity on "teaching ability" we consider two polar modeling environments. In the first case, we assume that e units of time time spent are now worth of e/μ effective units of time (see e.g. Moav (2005) for a similar assumption). Thus, more productive parents need less time to teach the same amount of "effective hours". The other alternative is to introduce parental productivity directly into the human capital production function z and leave the budget constraint unchanged.

To model the first scenario, we write the budget constraint for $j = if$ as

$$Tc^j \leq \omega f(h^j) \left(T - \left(e^j/\mu + \phi\right) n^j - h^j\right) \qquad \text{if } j = if. \tag{57}$$

Then we can derive that

Lemma 3. *If children are educated in the informal system, higher ability increases adult and child human capital and decreases fertility. If adult life expectancy increases, adult schooling and fertility will increase. Investment into child human capital is constant.*

Proof. See below. □

Now we move on to the second alternative. Keeping the budget constraint unchanged, we allow for a more general relationship between investment into children and their human capital and write the production function as $z(e, \mu)$ with positive partial and cross derivatives. Equilibrium child human capital investment is implicitly defined by

$$e + \phi - \frac{z(e, \mu)}{z_e(e, \mu)} = 0, \tag{58}$$

and we show in the proof below that if the elasticity of z is increasing in parental ability, investment into children's human capital is higher for smarter parents. Then we can state that

Lemma 4. *If children are educated in the informal system, higher ability increases investment into child human capital. The effect of ability on adult human capital and fertility is ambiguous. If adult life expectancy increases, adult schooling and fertility will increase. Investment into child human capital is constant.*

Proof. See below. □

Thus, introducing a positive correlation between labor market and teaching ability does not change the comparative statics with respect to changes in adult life expectancy but has only cross-sectional implications. Rising life expectancy will still increase fertility and human capital investment.

Proof of Lemma 3. Investment into children's human capital is $\hat{e} \equiv \mu\phi\frac{\gamma}{1-\gamma} = \mu e$ implying that more able parents invest more into their offspring's human capital. The remaining FOCs are

$$F_h = \frac{1}{c}\left(f(h) - \mu h^{\theta-1}\ell\right) \qquad F_n = \frac{\beta}{n}\left(nz(\hat{e})\right)^{1-\sigma} - \frac{T(e+\phi)}{\ell} \tag{59}$$

Further, we know that the sign of the elements in the Hessian from (37) do not depend on μ (which proves the second sentence). Given assumption 2 it can be shown that $F_{n\mu} < 0$ and $F_{h\mu} = 0$. Then we have

$$\begin{bmatrix} h_\mu \\ n_\mu \end{bmatrix} = -|A|^{-1}\begin{bmatrix} -F_{hn}F_{n\mu} \\ F_{hh}F_{n\mu} \end{bmatrix} \begin{matrix} > 0 \\ < 0 \end{matrix} \tag{60}$$

□

Proof of Lemma 4. Parents' ability changes optimal investment into child human capital via

$$e_\mu = -\frac{e\frac{\partial\epsilon(\mu)}{\partial\mu}}{1 - \frac{1}{\epsilon(\mu)}} \gtrless 0 \tag{61}$$

where $\epsilon(\mu)$ is the elasticity of child human capital production function with respect to e. For the sake of clarity we assume that it depends only on μ and not on e. Given that for a solution we need $\epsilon(\mu) < 1$, e rises if the elasticity of z is increasing in μ. Differentiating the new optimality conditions w.r.t. ability gives

$$F_{h\mu} = -\frac{ne_\mu T}{\ell^2} \qquad F_{n\mu} = e_\mu \left(\beta z' u'(1-\sigma) - T\frac{T(T-h)}{\ell^2}\right). \tag{62}$$

Using the Hessian from (37) establishes that

$$\begin{bmatrix} h_\mu \\ n_\mu \end{bmatrix} = -|A|^{-1} e_\mu \begin{bmatrix} \frac{T}{\ell^2}\left(\frac{\beta\sigma(nz)^{1-\sigma}}{n} - \frac{T}{\ell}(\phi+e) + (\phi+e)\beta(1-\sigma)(nz)^{-\sigma}z'\right) \\ T\left(\frac{T}{\ell^2}\left(\frac{1}{\ell} + (T-h)\frac{\theta}{h^2}\right) - \left(\frac{\theta}{h^2} + \frac{1}{\ell^2}\right)\beta(1-\sigma)(nz)^{-\sigma}z'\right) \end{bmatrix} \begin{matrix} \gtrless 0 \\ \gtrless 0 \end{matrix} \tag{63}$$

where we only can sign $h_\mu = 0$ and $n_\mu < 0$ for $\sigma = 1$. □

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