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Equilibrium Government

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A model of equilibrium institutions*

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Abstract

In an environment populated by ex-ante identical agents, institutions (the "rules of the game") are set to maximize payoffs of those individuals in power. They are constrained by the threat of rebellion, where the rebels would be similarly constrained by further threats. Equilibrium institutions are the fixed point of this constrained maximization problem. With the possibility of private investment and the threat of government expropriation, power is shared among a larger group of people. Endogenously, this enables the group in power to act as government committed to protection of property rights, which would otherwise be time inconsistent. But since sharing power implies sharing rents, capital taxation is inefficiently high.

JEL classifications: D7; H1; H2; O1; P4.

Keywords: government; political economy; public goods; capital taxation; time inconsistency.

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And Samuel told all the words of the LORD unto the people that asked of him a king.

And he said, This will be the manner of the king that shall reign over you: He will take your sons, and appoint *them* for himself, for his chariots, and *to be* his horsemen; and *some* shall run before his chariots. ...

And he will take your daughters *to be* confectionaries, and *to be* cooks, and *to be* bakers. ...

He will take the tenth of your sheep: and ye shall be his servants.

And ye shall cry out in that day because of your king which ye shall have chosen you; and the LORD will not hear you in that day.

Nevertheless the people refused to obey the voice of Samuel; and they said, Nay; but we will have a king over us;

That we also may be like all the nations; and that our king may judge us, and go out before us, and fight our battles.

1 SAMUEL 8:10–20

1 Introduction

Institutions are defined by North (1990) as the "rules of the game", or the "humanly devised constraints that shape human interaction". Differences in institutions go a long way in explaining the huge disparities in income across the globe as they affect incentives for agents to invest, produce and exchange.[1] This paper studies the institutions that arise in a world where those in power can establish rules for their own benefit, but are constrained by the threat of conflict. The model is then put to work to understand the institutions that emerge when private investment is possible but requires institutions that prevent expropriation of its fruits.

Institutions determine who is in power (the "incumbent army") and how output is distributed among individuals both inside the army and those outside (the "producers"). The institutions are determined to maximize the payoffs of individuals in the group that holds power. Institutions can be destroyed by a "rebellion", which makes way for the building of new ones. But rebellions are costly, reflecting the fact that conflict is costly. Importantly, the threat of conflict comes not just from outside the group in power: members of the incumbent army can defect from the government and take part in a rebellion. The cost of conflict for rebels is proportional to the number of individuals in the incumbent army who do not defect.

All individuals are ex-ante identical and self-interested. Thus, the notion of equilibrium institutions is independent of the competence, benevolence, or factional affiliation of the individuals comprising the ruling group. After any group takes power, there are always further opportunities for rebellions. Hence, once a rebellion has succeeded in replacing the government, the rebels will have the same objectives and face the same constraints as the army formerly in

[1] For example, see (North and Weingast, 1989), Engerman and Sokoloff (1997), Hall and Jones (1999) and Acemoglu, Johnson and Robinson (2005).

power. As in George Orwell's *Animal Farm*, there is no intrinsic difference between the "men" and the "pigs", but in equilibrium, some individuals will be "more equal" than others.

The model assumes that the institutions established by the incumbent army determine the allocation of resources once production has taken place. But how would those institutions manage to affect the allocation of goods ex post? As pointed out by Basu (2000) and Mailath, Morris and Postlewaite (2001), laws and institutions do not change the physical nature of the game, all they can do is affect how agents coordinate on some pattern of behaviour. But in reality, laws and institutions are seen to have a strong impact on behaviour, and this feature must be present in any model of institutions.

The view of this paper is similar to the application of Schelling's (1960) notion of focal points in the organization of society, as put forward by Myerson (2009). The "rules of the game" are self-enforcing as long as society coordinates on punishing whomever deviates from the rules — and whomever deviates from punishing the deviators. For example, if the laws specify how much an individual must pay or receive from another, and if both expect to be harshly punished if they fail to comply (along with any "higher-order" deviators) then the laws will be self enforcing.

Following this, theorizing about institutions is theorizing about (i) how rules (or focal points) are chosen, and (ii) how rules can change. For example, Myerson (2004) explores the idea of justice as a focal point influencing the allocation of resources in society. This paper takes a more cynical view towards our fellow human beings. Here, the individuals in power choose the laws and institutions to maximize their own payoffs, and those institutions can only be destroyed by a rebellion — wiping out the old institutions, and making way for new ones. There is no modelling of the post-production game. The model is thus about the incentives for creating and destroying institutions, and has nothing to say on the "technology of building institutions". Although the latter might be important, we believe that the former is the main factor in explaining the key differences in institutions across the world.

In the model, it is not conflict per se, but rather the threat of conflict that shapes institutions by constraining the actions of those in the incumbent army. No conflict occurs in equilibrium owing to the absence of uncertainty in the conflict technology and the actions of individuals.

The incentives to rebel depend on what members of a new incumbent army can extract once in power, which is also true of rebellions against the rebels, and so on. Given that all individuals are ex-ante identical, there is no fundamental reason why different institutions would be chosen following a rebellion, so the paper focuses on Markovian equilibria. The equilibrium institutions are thus the fixed point of the constrained maximization problem of the group in power subject to the threat of rebellion, where the rebels would be similarly constrained by further threats of rebellion.

There is no restriction on the composition of a group launching a rebellion (a "rebel army"). Since the most dissatisfied individuals have the most to gain from a rebellion, discouraging the "most profitable" rebellion a fortiori discourages all possible rebellions. It follows that the equilibrium institutions assign payoffs to producers according to a maximin rule and hence, producers' payoffs are equalized. Furthermore, as defections from the incumbent army reduce the overall cost of rebellion, payoffs of those inside the incumbent army must also be equalized. In other words, sharing power implies sharing rents.

There is a basic trade-off that characterizes the equilibrium institutions. The larger the army, the greater the amount of taxes it can levy, but the proceeds need to be divided among more people. The problem can be represented as a choice of the size of the incumbent army and taxes to maximize the payoff of an army member, subject to the constraint of avoiding a rebellion by producers. This "no-rebellion" constraint acts as a participation constraint for the producers.

Do the equilibrium institutions lead to the incumbent army acting as a "government" in any meaningful sense of the term? That is, are institutions that are designed to maximize the payoffs of those in power ever congruent with the interests of producers? In some cases, the answer turns out to be yes. For example, suppose there is a technology that transforms the output of the producers into a public good that benefit everyone, and which has no impact on any other aspect of the environment. Such a public good will be optimally provided in equilibrium, as if it had been chosen by a benevolent government.

A natural application of the model is to the taxation of investment proceeds. The model is extended so that once institutions are formed and after no rebellion has occurred, producers have access to an investment technology. However, the fruits of this investment are realized only after a lag, during which time there is another round of opportunities for rebellion. If a rebellion were to occur at this point, the group in power would have incentives to expropriate individuals' investments because the cost of investing is sunk, so it does not affect individuals' incentives to rebel.

Although institutions could in principle prescribe any level of capital taxation, whatever is chosen must be "rebellion-proof" after investment decisions have been made. The problem is that investment increases incentives for attacks on the current institutions that lead to new rules permitting full capital expropriation. No individual can commit not to rebel, and both producers and incumbent army members can participate in rebellions. An implication of the conflict technology is that rebellions against the current institutions are costless if all members of the incumbent army take part by defecting. Thus, if no one in the incumbent army has an incentive to defend the "rules of the game" then those rules cannot be rebellion-proof.

In equilibrium, to ensure there will be no incentives to rebel against the existing institutions — leading inevitably to full expropriation — the incumbent army has to be large so that if a rebellion were to take place after investment decisions had been made, the equilibrium size of the subsequent army would be smaller than the incumbent. This is the only way to prevent members of the incumbent army simply launching a costless rebellion from within that maintains intact the composition of the army. Following a rebellion, there would be insufficient places for all existing army members in the subsequent equilibrium army. Thus, some army members will oppose changing the institutions, and so conflict with them makes it costly to expropriate capital.

Adding the possibility of investment to the model thus gives rise to a larger equilibrium army. Sharing power among a wider group of individuals allows the incumbent army to act as a government committed to a certain set of policies that would otherwise be time inconsistent. The model highlights the importance of sharing power as a way to guarantee stability of institutions and thus incentives for investment. This resonates with Montesquieu's doctrine of the separation of powers, which is now accepted and followed in all well-functioning systems of government. It is important to note that in the model, power is not shared among those individuals who are actually investing. The extra army members in no sense represent or care about those who invest — but they do care about their own rents under the status quo. Thus, this group of self-interested individuals acts a government that commits to some protection of property rights.

Although it is possible to sustain protection against expropriation in equilibrium, capital taxation is not set efficiently. While in general it would be optimal from the point of view of society to have a larger group in power in order to guarantee that the fruits of investment would not be expropriated, the equilibrium army chooses taxes on investment that are too high and too little sharing of power.[2]

There are two reasons for the inefficiently high level of captital taxation. The first, and more interesting, follows from the distributional effects of protecting against expropriation. Since people in power can weaken the incumbent army if they take part in a rebellion, they must receive rents. As lower capital taxes require more power sharing, they also imply more rent sharing. This goes against the interests of each individual in the incumbent army. The second is because the incumbent army cannot extract all surplus from investors as the effort required to invest is private information.

While the model is quite abstract, it is congruent with a number of historical examples, some of which are discussed later in the paper: the disappearance of private corporations (the *societas publicanorum*) when power was concentrated under the Roman emperors; the need for a militarily strong leader (*podestà*) to guarantee stability in a society (medieval Genoa) where other strong groups could seize power; the tenacious resistance of the Stuart Kings of England to sharing power with Parliament.

[2] This is in accordance with the empirical literature that highlights the importance of institutions guaranteeing protection against government expropriation (for example Acemoglu and Johnson, 2005).

Section 2 presents the model, with the extension to the case of private investment in section 3. Section 4 draws some conclusions.

1.1 Related literature

Since Downs (1957) emphasized the importance of studying governments composed of self-interested agents, a vast literature on political economy has developed (see, for example, Persson and Tabellini, 2000). Most of this literature focuses on democracies, so institutions are not themselves explained in terms of the decisions of self-interested agents. But in much of the developing world and during most of human history, political regimes have differed greatly from democracies.

Recently, some models have been developed aiming at explaining institutions. Greif (2006) combines a rich historical analysis of trade and institutions in medieval times with economic modelling, part of which focuses on the form of government and political institutions that emerged in Genoa. Acemoglu and Robinson (2006, 2008) analyse conditions leading to democracy or dictatorship in an environment where an elite is trying to maintain its power, while citizens prefer a more egalitarian state. In Besley and Persson (2009a,b, 2010), society comprises two groups of agents that alternate in power, and make investments in two technologies that respectively allow the state to tax people and to enforce contracts. The exogenous parameters are the extent of political turnover and institutional (or demographic) features that determine how much one group cares about the other. They obtain predictions consistent with the data on state capacity and fiscal capacity, civil wars, and different forms of taxation.

This paper shares important similarities with the literature on coalition formation, as analysed by Ray (2007).[3] As in that literature, the process of establishing rules is non-cooperative, but it is assumed that such rules are followed. Moreover, the modelling of rebellions here is related to the idea of blocking in coalitions (Part III of Ray, 2007) in the sense that there is no explicit game-form, as in cooperative game theory. The distinguishing feature of this model is the "rebellion technology" that must be used in order to replace existing institutions by new ones.

This paper is also related to the literature on social conflict and predation, surveyed by Garfinkel and Skaperdas (2007).[4] It is easy to envisage how conflict could be important in a state of nature: individuals could devote their time to fighting and stealing from others. However, when there are fights, there are deadweight losses. Thus, people would be better off if they could agree on transfers to avoid conflict. This paper presupposes such deals are

[3] Baron and Ferejohn (1989) analyse bargaining in legislatures using this approach. Levy (2004) studies political parties as coalitions. Recent contributions include Acemoglu, Egorov and Sonin (2008) Piccione and Razin (2009).

[4] For instance, see Grossman and Kim (1995), Hirshleifer (1995), and also Hafer (2006), Dal Bó and Dal Bó (2010) for some recent contributions.

possible: producers pay taxes to the group in power, which allocates resources according to some predetermined rules. Here, differently from the literature on conflict, individuals fight to be part of the group that sets the rules, not over what has been produced. Moreover, they fight in groups, not as isolated individuals.

There are theoretical models focused on political issues that lead to inefficiencies in protection of property rights. Examples include Glaeser, Scheinkman and Shleifer (2003), Acemoglu (2008), Guriev and Sonin (2009) and Myerson (2010). Here, the possibility of capital expropriation and the consequent need to protect property rights is just a natural consequence of the possibility of investment and the "rebellion technology" that allows institutions to be destroyed and replaced.

Lastly, it is possible to draw an analogy between this paper and models of democracy (Persson and Tabellini, 2000) in the sense that the "election technology" there is replaced by a "rebellion technology" here.

2 The model

2.1 Environment

There is an area containing a measure-one continuum of ex-ante identical individuals. In equilibrium, an individual will either be a member of the incumbent army or a producer. Producers have access to a production technology that yields an exogenous endowment of q units of a homogeneous good. Members of the incumbent army do not produce, but they set up institutions — the "rules of the game". Once they are created, those institutions cannot be modified, unless a rebellion occurs. A rebellion destroys existing institutions, making way for the creation of new ones. After institutions are formed, there are opportunities for rebellions, and everyone has the opportunity to rebel.

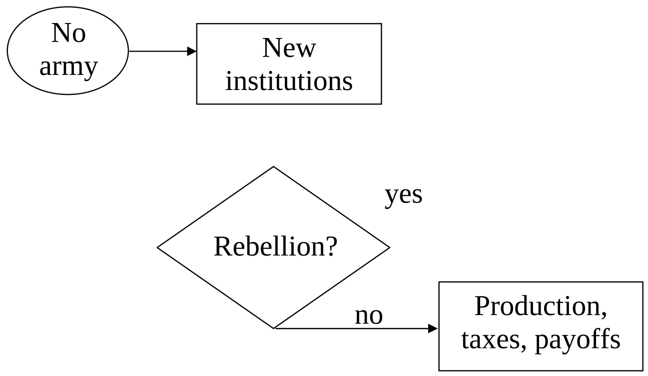
The sequence of events is as shown in Figure 1. The first army takes power and establishes institutions. There are then opportunities for rebellion. If a rebellion occurs, new institutions are established, with these also being subject to the threat of rebellion. When no rebellions occur, all individuals outside the incumbent army become producers, production takes place, rules are followed according to the prevailing institutions, and payoffs are received.

2.1.1 Establishing institutions

Institutions determine who belongs to the incumbent army, and how resources in society are distributed.

It is assumed that the size of the incumbent army a is such that it maximizes the average

Figure 1: *Sequence of events*



utility of army members.[5] Thus the incumbent army cannot have incentives to expel or draft new members. That captures the idea that the distribution of power reflects the interests of those in power, not the welfare of society as a whole. Given the size a, the composition of the incumbent army depends on a predetermined ordering set at the time of the previous rebellion.[6]

The incumbent army maximizes over the distribution of taxes $\tau(\cdot)$ levied on producers, and the distribution of consumption of its members $C_{\text{a}}(\cdot)$. No exogenous restrictions are imposed on taxation, beyond the natural bound of what producers possess.[7]

The choices of a and the distribution of resources are constrained by the threat of rebellion, as described below. Once institutions are established, each producer gets to know the tax $\tau(\imath)$ he faces and each army member gets to know his consumption $C_{\text{a}}(\imath)$. Further rebellions are then considered.

2.1.2 A rebellion

Understanding incentives for rebellion requires knowing individuals' beliefs about what institutions would be created, and what payoffs would be received, following a rebellion. Let a'^e and U'^e_{a} be beliefs about the size of the incumbent army and the average utility of an army member, respectively, that will characterize the next incumbent army if the current institutions are destroyed. Let $U(\imath)$ denote the utility of individual $\imath$ if no rebellion occurs. The

[5]Moving away from the assumption that the army maximizes the average payoff of its members would require modelling its command structure, which is beyond the scope of this paper. For a model of this kind, see Myerson (2008).

[6]Starting from no pre-existing institutions, a random ordering is used.

[7]Taxes can be contingent on any observable variables, but this does not play any role in the basic version of the model. Taxes cannot be contingent on agents' identities. This would imply identities would become state variables in the model, making the analysis much more complicated. In equilibrium, though, the government has no incentives to use identity-contingent taxes.

difference $U'^{e}_{\mathrm{a}} - U(\imath)$ corresponds to the gain individual $\imath$ obtains if the current institutions are destroyed and he belongs to the subsequent incumbent army. So the maximum fighting cost (in utility units) that an individual $\imath$ who expects to have a place in the subsequent incumbent army is willing to make is

$$F(\imath) = U'^{e}_{\mathrm{a}} - U(\imath). \qquad [2.1]$$

A rebellion is an ordering of the whole population. The corresponding rebel army $\mathcal{R}$ is the subset of size a'^{e} of individuals ordered first. This might include members of the incumbent army. Denote by d the measure of individuals in the rebel army that also belong to the incumbent army. This rebellion is profitable if:

$$\int_{\mathcal{R}} F(\imath) d\imath > \delta(a - d). \qquad [2.2]$$

The right-hand side is the total fighting effort required to destroy the current institutions. It is proportional to the measure of individuals the rebels have to fight against: the size of the incumbent army a minus the measure d of incumbent army members in the rebel army. The parameter δ measures the power of an incumbent army. It is assumed that individuals' utility is linear in the amount of fighting effort exerted. Furthermore, total fighting effort is the integral of the effort put in by all individuals in the rebel army $\mathcal{R}$, which is the left-hand side of the inequality above.

Thus for a rebellion to be viable the total fighting effort required to displace the current regime must be smaller than the total effort those in the rebel army would be willing to make. This quantity of effort depends on beliefs about the subsequent institutions, and these beliefs must be consistent with the insitutions that will emerge if the rebellion succeeds. Note that if all incumbent army members defect ($d = a$), there is no cost whatsoever of destroying the current institutions.

This approach to modelling the threat of conflict allows for a simple representation of the constraints faced by those choosing institutions, without accounting explicitly for the punches and sword thrusts. By assumption, all the fighting is between those in the incumbent army (who do not defect) and those in the rebel army, which comprises only those individuals who expect a place in the subsequent incumbent army. This means that those who will be producers after the success of a particular rebellion would not fight in the rebel army. The rationale for this assumption is that armies face a problem of incentivizing their members to fight. This incentive problem is likely to be much more severe for those who will not have a place in the incumbent army once the fighting is over. The basic problem is that those designing the new institutions after a rebellion have no interest in honouring past inducements to fight to the extent that these would reduce their own payoffs. Promises to pay producers for fighting (the carrot) are costly by definition, while punishments (the stick) would also be costly because they create disgruntlement with new institutions and increase incentives for another rebellion. But denying a place in the incumbent army to those who do not exert enough effort does not need to be costly owing to the presence of a pool of identical replacements that would like to join.[8]

2.1.3 Production, taxation and payoffs

Once there is no profitable rebellion against the existing institutions, production takes place, taxes are collected, and payoffs are received.

Let the set of individuals in the incumbent army be $\mathcal{A}$ and the set of producers be $\mathcal{P}$. Producer $\imath$ subject to institutions imposing a tax $\tau(\imath)$ on him receives consumption

$$C_{\mathrm{p}}(\tau(\imath)) = q - \tau(\imath). \qquad [2.3]$$

Total consumption of all army members must be equal to the amount collected in taxes:

$$\int_{\mathcal{A}} C_{\mathrm{a}}(\imath) d\imath = \int_{\mathcal{P}} \tau(\imath) d\imath. \qquad [2.4]$$

An individual who receives consumption C obtains utility $U = u(C)$, net of any past fighting costs that have been incurred, and where $u(C)$ is a strictly increasing and weakly concave function.

2.2 Equilibrium

Equilibrium institutions are those that maximize the average utility of an incumbent army member subject to no profitable rebellion for any composition of the rebel army.

The size of the incumbent army a maximizes the average utility of army members. Those belonging to the incumbent army choose a tax distribution $\tau(\cdot)$ specifying a lump-sum tax for each producer $\imath \in \mathcal{P}$ to maximize their utility. These choices are constrained by the threat of rebellions. The equilibrium institutions are the result of the following problem:

$$\max_{a,\tau(\cdot)} \frac{1}{a} \int_{\mathcal{A}} U_{\mathrm{a}}(\imath) d\imath \;\text{ s.t. }\; \int_{\mathcal{R}\cap\mathcal{P}} \left(U_{\mathrm{a}}'^{e} - U_{\mathrm{p}}(\tau(\imath))\right) d\imath + \int_{\mathcal{R}\cap\mathcal{A}} \left(U_{\mathrm{a}}'^{e} - U_{\mathrm{a}}(\imath)\right) d\imath \leq \delta(a - d), \qquad [2.5]$$

for all sets $\mathcal{R}$ such that $\mathbb{P}(\mathcal{R}) = a'^{e}$, where a'^{e} is the belief about the size of the incumbent army that will be established if a rebellion succeeds, $U_{\mathrm{a}}'^{e}$ is the expected utility of an army member under the institutions that would be established in case of a successful rebellion, and

[8] The formulation necessarily abstracts from coordination issues, and it might be thought that these would be a more pressing problem for the rebels than the incumbent. If this were the case, the power parameter δ could be adjusted to account for such differences.

$d = \mathbb{P}(\mathcal{R} \cap \mathcal{A})$ is the measure of defectors from the incumbent army. The solution to [2.5] depends on the beliefs about the size of the subsequent incumbent army a'^e and about the average utility U'^e_{a} its members receive.

Once the current institutions are destroyed by a rebellion, new ones are formed to maximize the average utility of those who will be in the new incumbent army. The choice of army size a' is not constrained by the size of the rebel army $\mathbb{P}(\mathcal{R})$, the assumption being that the extension of power to an additional individual must be in the interests of all other members of the incumbent army. The order in which membership of the incumbent army is extended to individuals is set down by the predetermined ordering that characterizes the rebellion. The new incumbent army also maximizes over a tax distribution $\tau'(\cdot)$. In equilibrium, the earlier beliefs a'^e and U'^e_{a} must be equal to the actual values of a and U'_{a} given the absence of uncertainty.

The maximization problem characterizing a' and $\tau'(\cdot)$ is of an identical form to that in [2.5] for a and $\tau(\cdot)$ owing to the irrelevance of history: the cost of conflict is sunk and additive; and the choice of new institutions is not constrained by the size of the rebel army. Thus, there are no state variables, and hence no fundamental reason why different institutions would be chosen at each point. It is therefore natural to focus on Markovian equilibria.

A Markovian equilibrium is a solution to the maximization problem [2.5] with $a = a' = a'^e$; an identical distribution of taxes over producers: $\mathbb{P}(\tau(\imath) \leq \uptau) = \mathbb{P}(\tau'(\imath) \leq \uptau)$ for all $\uptau$; an identical distribution of consumption to army members: $\mathbb{P}(C_{\mathrm{a}}(\imath) \leq C) = \mathbb{P}(C'_{\mathrm{a}}(\imath) \leq C)$ for all C. The following result demonstrates some features of any Markovian equilibrium.

Proposition 1 *Any Markovian equilibrium must have the following properties:*

(i) Equalization of producers' payoffs: $U_{\mathrm{p}}(\imath) = U_{\mathrm{p}}$ *for all* $\imath$ *(with measure one)*

(ii) Sharing power implies sharing rents: $U_{\mathrm{a}}(\imath) = U_{\mathrm{a}}$ *for all* $\imath$ *(with measure one)*

(iii) The set of constraints in [2.5] *is equivalent to a single "no-rebellion" constraint:*

$$U_{\mathrm{p}}(\tau) \geq U_{\mathrm{a}'} - \updelta \frac{a}{a'}. \qquad [2.6]$$

(iv) Power determines rents: $U_{\mathrm{a}} - U_{\mathrm{p}} = \updelta$

Proof See appendix A.1. ■

These results do not rely on risk aversion: they also hold for a linear utility function. Since all producers receive the same endowment here, payoff equalization is equivalent to tax equalization. The intuition for the equalization of producer payoffs is that as only a subset of agents takes part in a rebellion, the army wants to maximize the utility of the subset with the minimum utility. This is achieved by equalizing producers' utility. Introducing inequalities in payoffs reduces the average utility of a subset of size a' with minimum utility, so it necessarily makes it harder to avoid a rebellion.[9]

An analogous argument implies that heterogeneity in army member payoffs is undesirable because it makes it harder to avoid rebellions from within at the same time as ensure an overall high payoff for army members. Because the cost of rebelling is proportional to the measure of incumbent army members who do not defect, any inequality will lead the least satisfied army members to participate in a rebellion.

The single "no-rebellion" constraint in the third part of the proposition is the effective constraint faced by the incumbent army when the rebellion includes no defectors and payoffs of producers are equalized. Once this is satisfied, all other constraints are redundant. An immediate consequence of this is that the power parameter δ determines the size of the rents received by members of the incumbent army in a Markovian equilibrium.

Therefore, the maximization problem characterizing the equilibrium institutions has the following recursive form

$$\max_{a,\tau} U_{\mathrm{a}}(a,\tau) \;\text{ s.t. }\; U_{\mathrm{a}}(a',\tau') - \delta\frac{a}{a'} \leq U_{\mathrm{p}}(\tau), \tag{2.7}$$

where a' and τ' solve an identical problem taking a'' and τ'' as given, and so on.

As the revenue of the army $(1-a)\tau$ is equally shared among the army members, utility is

$$U_{\mathrm{a}}(a,\tau) = u\left(\frac{1-a}{a}\tau\right).$$

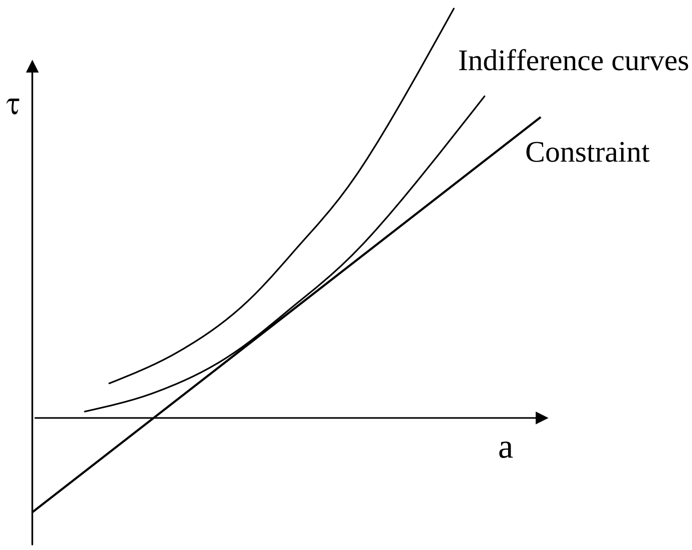
The payoff of an army member is increasing in the tax τ and decreasing in the army size a. The tradeoff between the two is represented by the convex indifference curves in Figure 2. The army has two margins to ensure that it avoids rebellions. It can reduce taxes, or increase its size (the "carrot" or the "stick"). This corresponds to the upward-sloping no-rebellion constraint. The maximum is at the tangency point. With linear preferences, the constraint is a straight line, as depicted in Figure 2. With risk aversion, the constraint implies τ is a concave function of a.

2.3 Examples

There are two exogenous variables: δ and q. The following examples illustrate the workings of the model for some particular utility functions.

[9] This result is different from those found in some models of electoral competition such as Myerson (1993). In the equilibrium of that model, politicians offer different payoffs to different agents. But there is a similarity with the model here because in neither case will agents' payoffs depend on their initial endowments.

Figure 2: *Trade-off between army size and taxation*



2.3.1 The model with linear utility

When the utility function $U(C) = \log(C)$, the problem of the army is

$$\max_{a,\tau} \frac{1-a}{a}\tau \text{ s.t. } C_{\mathrm{a}}' - \delta\frac{a}{a'} \leq q - \tau, \qquad [2.8]$$

Substituting τ from the constraint into the objective function yields:

$$C_{\mathrm{a}} = \frac{1-a}{a}\left(q - C_{\mathrm{a}}' + \delta\frac{a}{a'}\right)$$

The following first-order condition with respect to a is obtained:

$$\frac{C_{\mathrm{a}}}{1-a} = (1-a)\frac{\delta}{a'}$$

Moreover, imposing equilibrium in the equation for C_{a} ($a = a'$ and $C_{\mathrm{a}}' = C_{\mathrm{a}}$) yields:

$$C_{\mathrm{a}} = (1-a)(q+\delta)$$

Combining both yields the solution:

$$a^{lin} = \frac{\delta}{q+2\delta}$$

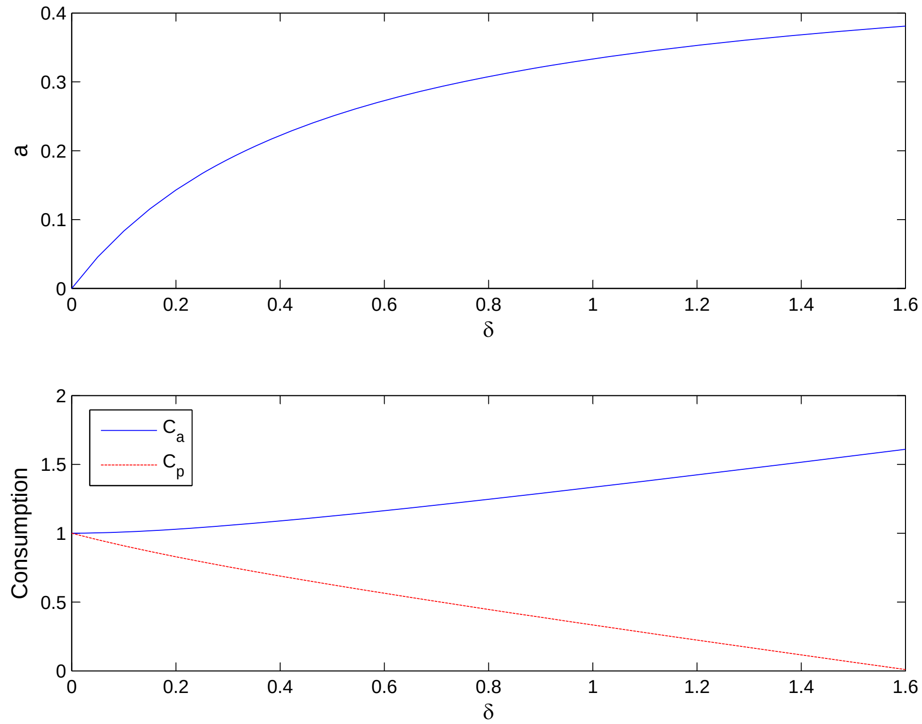
So,

$$C_{\mathrm{a}}^{lin} = \frac{(q+\delta)^2}{q+2\delta} \ , \ C_{\mathrm{p}}^{lin} = \frac{(q+\delta)^2}{q+2\delta} - \delta$$

With linear utility function, the army size is a function of q/δ. GDP is given by $(1-a)q$, and $C_{\mathrm{a}} - C_{\mathrm{p}} = \delta$.

The relationship between the power parameter δ and the key endogenous variables of the model, for $q = 1$, is shown in Figure 3.

Figure 3: *Model with linear utility*



If $\delta/q \geq (1+\sqrt{5})/2$, consumption of producers is equal to 0. With linear utility in consumption, if δ is high enough, consumers get zero consumption and the army chooses its size to prevent producers from rebelling. To make the problem interesting, it is sensible to restrict the parameters to ensure producers obtain positive consumption, which requires $\delta/q < (1+\sqrt{5})/2$.

The power parameter δ affects the equilibrium in three ways. First, an increase in δ makes the current government stronger because the rebels have to pay a higher cost to overthrow it. This leads to an increase in τ and a decrease in a. Second, the payoff that the rebels will receive once in power increases as their position would also be stronger once they have supplanted the government, making rebellion more attractive. This effect makes the government weaker, leading it to decrease τ, and increase a. Third, an increase in δ raises the effectiveness of the marginal fighter in the army, leading the army to increase its size in order to extract higher taxes. As long as $\tau > 0$, the last effect dominates and the army size is increasing in δ.

2.3.2 The model with log utility

When the utility function $U(C) = \log(C)$, the problem of the army is

$$\max_{a,\tau} \ \log\left(\frac{1-a}{a}\tau\right) \quad \text{s.t.} \quad \log\left(C'_{\mathrm{a}}\right) - \delta\frac{a}{a'} \leq \log(q-\tau), \qquad [2.9]$$

Substituting τ from the constraint into the objective function, the following first-order condition is obtained:

$$\frac{1}{1-a} + \frac{1}{a} = \frac{\delta C'_{\mathrm{a}} e^{-\delta a/a'}}{a'\tau}$$

Imposing equilibrium ($a = a'$ and $C'_{\mathrm{a}} = (1-a)\tau/a$):

$$a = (1-a)^2 \delta e^{-\delta}$$

which yields the solution:

$$a^{log} = \frac{2\delta e^{-\delta}}{1 + 2\delta e^{-\delta} + \sqrt{1 + 4\delta e^{-\delta}}}$$

The size of the army is independent of the producers' endowment q. With log utility, as consumption of an individual increases, so does the amount of goods he is willing to give up in order to avoid conflict.

It is also instructive to note how the output of the economy is distributed among individuals. The following condition is obtained by imposing equilibrium and using the no-rebellion constraint:

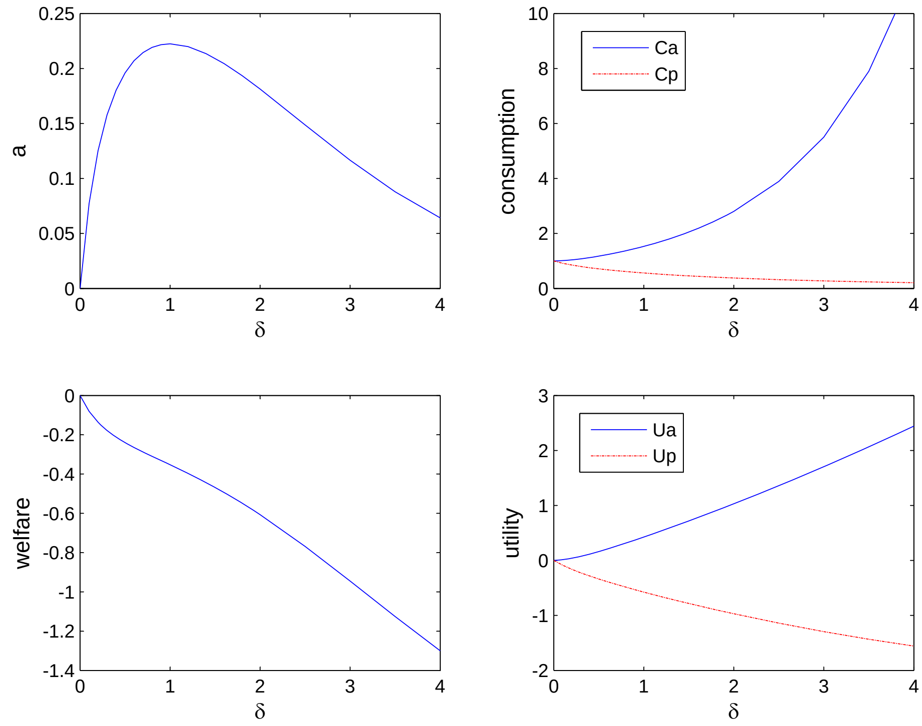
$$C^{log}_{\mathrm{a}} = \frac{(1-a)q}{a + (1-a)e^{-\delta}} \ , \ C^{log}_{\mathrm{p}} = \frac{e^{-\delta}(1-a)q}{a + (1-a)e^{-\delta}} \qquad [2.10]$$

The consumption of army members and producers is proportional to q. The output of the economy $(1-a)q$ is divided among all individuals in the economy, with producers getting a fraction $e^{-\delta}$ of what an army member gets. The value of q (the size of the pie) has no influence on the shares each individual gets, because of the log utility function in consumption.

The relationship between the power parameter δ and the key endogenous variables of the model is shown in Figure 4.

The army size a is positively related to δ for $\delta < 1$ and decreasing in δ otherwise. An increase in δ raises the effectiveness of the marginal fighter in the army. While δ is relatively small, that leads to an increase in the army size and higher taxes. But as δ gets larger, reducing the consumption of producers leads to greater incentives for them to fight, making

Figure 4: *Model with log utility*



the army choose a smaller a and smaller τ.

Output in the economy is negatively related to a. So, it reaches its lowest level at $\delta = 1$, when a reaches its maximum. However, welfare is everywhere decreasing in δ, owing to the negative distributional effects of increases in δ.

2.4 The provision of public goods

The army in power and its policies could be seen as a "government", but in the previous section there was no scope for this "government" to do what they are customarily thought to do, such as the provision of public goods. This section considers a technology that allows for production of public goods. It is then natural to ask whether those would be provided, since unlikely atomistic individuals, the incumbent army can set up institutions that determine the taxes and the spending on provision of public goods.

Suppose there is a technology that converts units of output into public goods. If g units of goods per capita are converted into public goods then everyone receives $f(g)$ units of goods. The utility function of an individual is $U(C + f(g))$.

Per-capita consumption is

$$(1 - a)q - g + f(g).$$

A benevolent social planner would choose g such that

$$f'(g) = 1, \qquad [2.11]$$

to maximize the amount available for consumption. Note that the choice of g is irrespective of a.

The assumptions of section 2 are now modified so that the "institutions" chosen by the army include its provision of public goods g. The agents observe the choice of g and take it into account when determining how much fighting effort they are willing to make.

The consumption of a producer is now

$$C_{\mathrm{p}}(\tau, g) = q - \tau + f(g), \qquad [2.12]$$

and the consumption of an army member becomes

$$C_{\mathrm{a}}(a, \tau, g) = \frac{\tau(1-a) - g}{a} + f(g). \qquad [2.13]$$

So the equilibrium is the solution of the following problem:

$$\max_{a,\tau,g} U\left(\frac{\tau(1-a) - g}{a} + f(g)\right) \;\; \text{s.t. } U'_{\mathrm{a}} - \delta\frac{a}{a'} \leq U\left(q + f(g) - \tau\right), \quad \text{with } a = a' \;,\; \tau = \tau' \text{ and } g = g'. \qquad [2.14]$$

Writing the Lagrangian and taking the first order condition with respect to τ and g yields:

$$\begin{aligned} U'(C_{\mathrm{a}})\left(-\frac{1}{a} + f'(g)\right) &= \lambda U'(C_{\mathrm{p}})f'(g) \\ U'(C_{\mathrm{a}})\left(\frac{1-a}{a}\right) &= -\lambda U'(C_{\mathrm{p}}) \end{aligned}$$

Combining both:

$$f'(g^*) = 1$$

The public good is optimally provided.

Who benefits from the public good provision? The distribution of output among the individuals depends on the utility function. For example, with log utility, an expression similar to [2.10] is obtained:

$$C_{\mathrm{a}} = \frac{(1-a)q + f(g^*) - g^*}{a + (1-a)e^{-\delta}} \; , \; C_{\mathrm{p}} = \frac{e^{-\delta}((1-a)q + f(g^*) - g^*)}{a + (1-a)e^{-\delta}}$$

Thus, even though the incumbent army is extracting rents from producers, this does not preclude it acting as if it were benevolent in other contexts. In this particular setting, the public good is optimally provided, "government" incentives are aligned with the interests of society. This means that compared to a world where no-one had the ability to force others to do something against their will, the welfare of producers and consumption per capita could be either larger or smaller. This reflects a trade-off that having a "government" (or a king) entails.

This result is far from surprising and can be obtained in different models. In the context of voting and elections, this is discussed by Persson and Tabellini (2000) before they analyse common-pool problems that invalidate this result. Hold-up or credibility problems would also turn it down. In the context of this model, it provides a benchmark of public service provision with no distortion.

3 Investment and expropriation

Recent empirical work has highlighted the importance of institutions that prevent the government from expropriating individuals' resources. According to Acemoglu and Johnson (2005), such institutions are more important than those that facilitate contracting among private agents. The framework of this paper can be used to analyse this issue.

For the issue of expropriation to be interesting, it is necessary to allow producers to invest and consider some span of time between when an investment is made and its fruition. During this potentially long period of time, there is no reason to suppose that rebellions cannot take place. These features are captured by a two-stage model: producers can exert effort in the first stage to raise output at the second stage, when all consumption takes place.

This section adds no extra assumptions on the conflict technology. Rebellions are costly and this cost is lower for army members than for producers. In particular, members of the incumbent army can rebel and destroy the current institutions at zero cost (case of $d = a$), as long as all its members agree to do so. When that happens, new institutions are chosen, and are set to maximise the average payoff of the new incumbent army. The only difference from autorefsec:model is that a rebellion is possible once sunk investment decisions have been made and their fruits can be expropriated.

In equilibrium, the possibility of expropriating capital generates extra incentives for rebellions — from producers and also from members of the current army willing to change the rules. How are then the rules of the game in such a world? Naturally, producers' decisions to invest depend on how much they expect to keep, so taxes on capital cannot be too large. But the chosen policies must also prevent individuals (inside and outside the incumbent army) from rebelling.

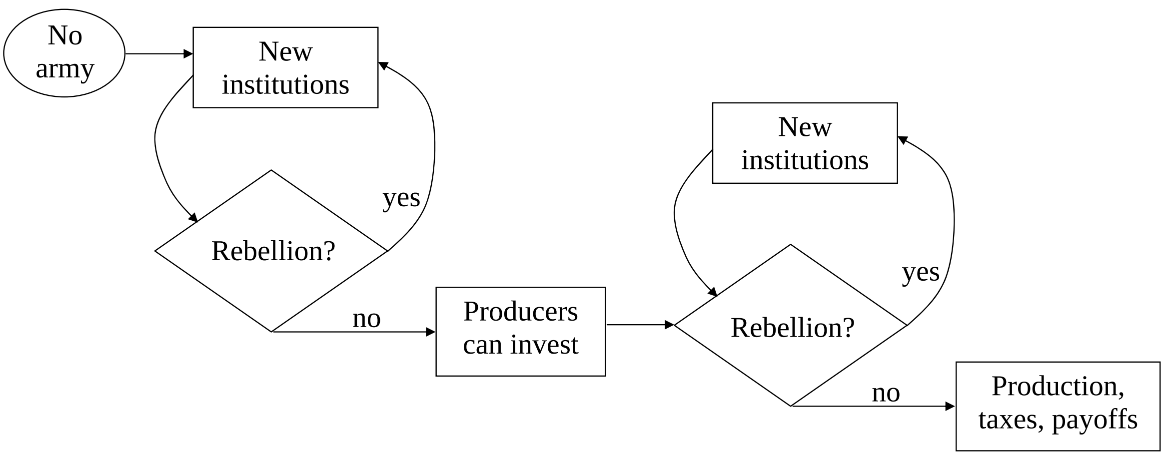
Second, will those institutions be efficiently chosen? Will the incumbent army have incentives to establish rules protecting the fruits of producers' sunk efforts up to its efficiency point? Section 2.4 analysed a case where there was no distortion in the provision of public goods. Will there be any distortion in this case, and if so, why?

3.1 Environment

For analytical tractability, agents' preferences are linear in consumption: $U(C) = C$. This allows for a relatively simple closed-form solution. With such preferences, only lump-sum taxes would be used by a benevolent government that didn't have to worry about rebellions.

Figure 5 shows the sequence of events in the model. There is an opportunity to invest after institutions are established. A rebellion, that destroys the current rules and leads to new ones, might occur after investment decisions have been made.

Figure 5: *Sequence of events*



The first stage begins as in section 2. The rules of the game are announced, taxes on producers can be contingent on observables. Before producers need to decide on investment decisions, there is as usual scope for rebellions that establish new institutions.

When no rebellions occur, producers receive an investment opportunity. Each producer gets to know his own idiosyncratic effort requirement, ϵ. The effort cost ϵ is uniformly distributed, $U[\epsilon_L, \epsilon_H]$. Producers making this effort start the second stage with κ units of output (with $\kappa > \epsilon_H$), referred to as capital. Let $k \in \{0, \kappa\}$ denote the capital holdings of a producer.

At the beginning of the second stage, there is another opportunity for producers and army members to rebel and destroy the current institutions. There are no exogenous restrictions on what the following rules could be, any taxes contingent on observables could be chosen (with capital k as well as q being observable). Similarly to the first stage, any new institutions are also subject to a further threat of rebellion. Once there are no profitable rebellions, each producer receives q and the rules are implemented.

3.2 Equilibrium

Characterizing the equilibrium requires working backwards from the second stage, determining the actions of a rebel army, and then analysing what would be chosen at the first stage.

3.2.1 Second stage

Suppose first that there is a rebellion at the second stage, and that a new army of size a_2 takes power. Determining the equilibrium army size and taxes is done in a similar way to that in section 2.

An argument similar to Proposition 1 shows that an army that takes power at the second stage chooses taxes in order to equalize consumption of producers. The effort cost is sunk, so it does not influence the fighting effort of an individual. As a rebellion comes from the individuals with the lowest consumption, the army chooses taxes in order to equalize consumption so that, for a given total tax revenue, the incentive for rebellions is minimized.

Let U_{p_2} denote consumption of producers and U_{a_2} denote the consumption of army members in the second stage (effort exerted in the first stage is not taken into account). The army size is a_2 and $\tilde{s}$ denotes the number of producers who had invested. The total amount of capital is then $\tilde{s}\kappa$ and the payoff of an army member would be:

$$U_{\mathrm{a}_2}(a_2, U_{\mathrm{p}_2}; \tilde{s}) = \frac{\tilde{s}\kappa + (1-a_2)(q - U_{\mathrm{p}_2})}{a_2}.$$

Maximizing the army members' payoff subject to the constraint

$$U_{\mathrm{p}_2} \leq U'_{\mathrm{a}_2} - \delta \frac{a_2}{a'_2}$$

leads to

$$\hat{a} = \frac{\delta}{q+2\delta}, \quad \hat{U}_{\mathrm{a}_2}(\tilde{s}) = \frac{(q+\delta)^2}{q+2\delta} + \tilde{s}\kappa, \quad \text{and} \quad \hat{U}_{\mathrm{p}_2}(\tilde{s}) = \frac{(q+\delta)^2}{q+2\delta} - \delta + \tilde{s}\kappa. \qquad [3.1]$$

as long as δ/q is smaller than $(1+\sqrt{5})/2$, which is assumed to hold — otherwise $\tilde{U}_{p_2} = 0$.

The army size a_2 is the same as in section 2.3.1. It is independent of $\tilde{s}\kappa$, which is analytically convenient. Total capital $\tilde{s}\kappa$ is confiscated and equally distributed among the whole population.

3.2.2 First stage

For the same reason as in Proposition 1, the incumbent army will not want to generate heterogeneity in producers' consumption. Institutions will be defined by an army size a, a tax τ_q that incides on all producers and a tax on capital τ_κ on everyone that has κ units of capital.

The choice of capital tax determines a threshold for investment. Producers will invest if their effort cost ϵ is smaller than ϵ^*, where

$$\epsilon^* = \kappa - \tau_\kappa$$

That determines the fraction s of producers that will make the effort

$$s = (\epsilon^* - \epsilon_L)/(\epsilon_H - \epsilon_L)$$

and the capital stock at the second stage, $s(1-a)\kappa$.

The incumbent army makes its choices subject to facing no rebellion at either stage. The average payoff of an army member is:

$$\frac{(1-a)\left(\tau_q + s\tau_\kappa\right)}{a}$$

At the first stage, producers still do not know their effort cost ϵ. So their expected utility, net of effort cost, is:

$$q - \tau_q + sE\left(\kappa - \tau_\kappa - \epsilon | \epsilon \leq \epsilon^*\right) = q - \tau_q + s\frac{\epsilon^* - \epsilon}{2}$$

Therefore, similar to what is obtained in section 2, the first stage no-rebellion constraint is:

$$q - \tau_q + s\frac{\epsilon^* - \epsilon_L}{2} \geq U'_a - \delta\frac{a}{a'} \qquad [3.2]$$

Consumption of producers that had not invested is $q - \tau_q$. Consumption of producers that have invested is $q - \tau_q + \kappa - \tau_\kappa$, which is larger than consumption of those without capital ($\kappa - \tau_\kappa > \epsilon^*$). Therefore, producers that had invested are willing to make less fighting effort to take power. Consequently, their no-rebellion constraint is not binding. This is exactly what brings incentives for expropriating their capital.

Substituting the equilibrium equations for the second stage, and assuming there are enough producers that would choose not to invest, equation [2.2] becomes:[10]

$$\hat{a}\hat{U}_{\mathrm{a}_2}(s(1-a)) - (\hat{a}-d)(q-\tau_q) - dU_{\mathrm{a}} \leq \delta(a-d) \qquad [3.3]$$

It has to hold for all values of d. The left hand side is the total utility gain the rebel army would get if they took power. The right hand side is the cost to destroy the current institutions.

Due to the linearity of the constraint on d, it can be expressed by 2 constraints, one for $d = 0$, and the other for $d = \hat{a}$:

$$q - \tau_q \geq \frac{(q+\delta)^2}{q+2\delta} + (1-a)s\kappa - \delta\frac{a}{\hat{a}} \qquad [3.4]$$

$$U_{\mathrm{a}} \geq \frac{(q+\delta)^2}{q+2\delta} + (1-a)s\kappa - \delta\frac{a}{\hat{a}} + \delta \qquad [3.5]$$

Proposition 2 *In a Markovian equilibrium with $s > 0$,*

- *Constraints [3.4] and [3.5] are binding. Constraint [3.2] is slack.*
- *U_{a} can be written as:*

$$U_{\mathrm{a}} = (1-a^*)(s\kappa + q + 2\delta) - \frac{1-a^*}{a^*}\left(\frac{\delta^2}{q+2\delta} + s\epsilon^*\right) \qquad [3.6]$$

- *where*

$$a^* = \frac{\delta\hat{a} + s\epsilon^*}{\delta + s\epsilon^*} \qquad [3.7]$$

Proof See appendix A.2. ■

The second-stage constraints for army members and those who did not invest are binding. Because of the stock of capital from first-stage investment, members of the army have stronger incentives to rebel. The first-stage constraint is slack.

The army size a^* is equal to $\hat{a}$ if $s = 0$ and is strictly larger than $\hat{a}$ if $s > 0$. In an equilibrium with some investment, the size of the army has to be larger. An army that wants low taxes on capital to stimulate investment faces two constraints: it must provide incentives for producers not to rebel; and it must also provide incentives for its members of the not to change its policies once investment decisions have been made. The constraint [3.4] can be satisfied either by a larger army size or lower taxes. The constraint [3.5] requires either a larger army size or *higher* taxes.

[10] If the size of the rebel army $\hat{a}$ is larger than the measure of producers that did not invest $((1-s)(1-a))$, a rebellion with producers only would have to include some that had invested. However, that rebellion is less profitable than a rebellion involving only army members and producers with no capital. For the sake of simplifying exposition, we ignore this point here, which is dealt with in the appendix.

Once investment decisions have been made, individuals have more incentives to rebel and rewrite the rules, so to expropriate capital from those who took an investment opportunity. To rewrite the rules, they need to pay the cost of defeating the incumbent army. This cost however would be zero for the army in power if they could all agree to “rebel” against the current institutions and establish new ones. Thus, the only way to prevent them from doing so is by having an army in power that is larger than the second-stage equilibrium size, because then they will not all agree on removing the old regime.

If power is shared among a large number of people (a large army size a is chosen), then, if there is a rebellion at the second stage, there will not be a place in the new incumbent army for all current army members. All army members know that, if the current institutions are destroyed, some of them will lose their positions. So, there can be no rebellion with the support of all incumbent army members. Some of them will oppose it, and it will be costly to defeat them. Hence, an equilibrium with no change in policies at the second stage is feasible — as long as a is large enough — and that will be chosen if investment opportunities are sufficiently profitable.

Note that the extra army members $(a^* - \hat{a})$ have no special function, except for fighting against rebels (off the equilibrium path). They have power to defend the current institutions from outsiders and, especially, to fight against army members should they defect. In equilibrium, they avoid conflict, share rents, and do not change the rules. Any attack to the current institutions would be opposed by enough people — who simply want to preserve the status quo because they would not have a place in the new incumbent army.

The result relies on the assumption that people cannot agree before a rebellion to throw away some of the old rules but keep others intact (in this case that they will change taxes but keep a fixed). Once a rebellion occurs, the new institutions have to maximize the payoffs of those in power. Allowing for such “partial reoptimization” would imply that there is some external check that prevents power being exercised in certain areas, but not others, which makes no sense in the context of this model.

Going back to the analogy with Myerson (2009), the rules of the game provide the focal point for the allocation of resources once production has taken place. Theorizing about government and institutions is then making assumptions about how those focal points are established. Now, the sheer existence of a government pressuposes a certain set of policies will be in effect. For example, the army cannot simply change taxes on production one it has taken place, although there is no scope for conflict then. It is then natural to assume that the rules of the game can only be changed with a rebellion (of course the rules could be contingent).

Substituting the value of a^* on [3.6] and doing loads of algebra yields

$$U_{\mathrm{a}} = \frac{\delta(q+\delta)}{q+2\delta}\left(\frac{q+\delta+s(\kappa-\epsilon^*)}{\delta+s\epsilon^*}\right) \qquad [3.8]$$

The first order condition with respect to ϵ^* (remember that s is also a function of ϵ^*) yields:

$$(\epsilon_H - \epsilon_L)\kappa s^2 + 2(q + 2\delta)(\epsilon_H - \epsilon_L)s + [\epsilon_L(q + 2\delta) - \delta\kappa] = 0$$

One root of this equation is negative. The other one might be positive and is the only economically meaningful root. It is:

$$s^* = \sqrt{\left(\frac{q+2\delta}{k}\right)^2 + \frac{1}{\epsilon_H - \epsilon_L}\left(\delta - (q+2\delta)\frac{\epsilon_L}{k}\right)} - \frac{q+2\delta}{k} \qquad [3.9]$$

(as long as $s^* \in [0, 1]$). This is positive if

$$\delta\kappa - \epsilon_L(q + 2\delta) > 0 \Rightarrow \frac{\kappa}{\epsilon_L} - 1 > 1 + \frac{q}{\delta}$$

As mentioned above, the no-rebellion constraint for producers with good investment opportunities is not binding. Thus, the distribution of income does not correspond exactly to the distribution of power. Economic prosperity is related to having more citizens with slack no-rebellion constraints. This requires institutions that protect their investment. The model allows the incumbent army to (costlessly) build those institutions. As in equilibrium agents will only invest if those institutions are there, it is in the best interest of the army to build them. Or is it?

3.3 The efficient choice of capital taxes

As long as $\kappa > \epsilon_H$, if property could be directly protected from expropriation at no cost, investing would always be efficient. However, in this model, that might not be true. If there is investment, more people have to be in the army in order to prevent a rebellion, which entails an opportunity cost.

The efficient benchmark in this case is the following: taxes on capital τ_κ are chosen by a benevolent agent, knowing that everything else (a and τ_q) will be determined in equilibrium, from the problem of maximising payoffs of the group in power subject to the threats of rebellion discussed above.

In the case of section 2.4, such a benevolent agent could not improve the economy with a different choice of g. In that case, the public good would be efficiently provided. Here, is protection of property rights efficiently provided? If not, is it underprovided or overprovided and why?

For a given τ_κ, restrictions determine a and τ_q. The capital tax τ_κ determines s, which pins down the incumbent army size, a^*, as any smaller a would lead to a rebellion in the second stage. Then, taxes on q would be such that the constraint in [3.4] would be binding.

Welfare in the economy is given by:

$$W = aU_{\mathrm{a}} + (1-a)U_{\mathrm{p}} + (1-a)s\frac{\epsilon^* - \epsilon_L}{2}$$

As in equilibrium $U_{\mathrm{a}} = U_{\mathrm{p}} + \delta$,

$$W = U_{\mathrm{a}} - (1-a)\delta + (1-a)s\frac{\epsilon^* - \epsilon_L}{2}$$

There are 2 differences between the expression for W and U_{a}. The second term, $-(1-a)\delta$, is related to the distribution of resources in the economy. The third term is the investor surplus.

Substituting $a = a^*$ and rearranging:

$$W = \frac{\delta(q+\delta)}{q+2\delta}\left(\frac{q + s(\kappa - \epsilon)}{\delta + s\epsilon^*} + \frac{s^2(\epsilon_H - \epsilon_L)}{2(\delta + s\epsilon^*)}\right)$$

The last term in brackets corresponds to the investor's surplus. Now consider the welfare function W_c, which ignores the investor surplus:

$$W_c = \frac{\delta(q+\delta)}{q+2\delta}\left(\frac{q + s(\kappa - \epsilon)}{\delta + s\epsilon^*}\right)$$

The difference between them is

$$\frac{\delta(q+\delta)}{q+2\delta}\left(\frac{s^2(\epsilon_H - \epsilon_L)}{2(\delta + s\epsilon^*)}\right)$$

Taking derivative with respect to s (remember ϵ^* is a function of s) shows that this is always positive. Thus

$$\frac{\partial W}{\partial s} > \frac{\partial W_c}{\partial s}$$

That is the first distortion. The constrained welfare-maximizing choice of s leads to more investment than the equilibrium because it takes into account the investor's surplus.

The no-rebellion constraint for those who invested is slack. That is because ϵ is not observable. So it is impossible for the incumbent army to get rents from them at the margin. Therefore the incumbent army does not consider their utility in their decisions, and capital taxes are too high.

The political Coase theorem does not hold here because of the non-observability of effort, ϵ, which makes it impossible writing rules contingent on it.

There is another distortion, as W_c is different from U_a in [3.8] — there is an extra δ in the numerator in the later. Going through the same steps as above, we find that the s that maximizes W_c is:

$$s_c = \sqrt{\left(\frac{q+\delta}{\kappa}\right)^2 + \frac{\delta\kappa - \epsilon_L(q+\delta)}{\kappa(\epsilon_H - \epsilon_L)}} - \frac{q+\delta}{\kappa}$$

This is positive if

$$\delta\kappa - \epsilon_L(q+\delta) > 0 \Rightarrow \frac{\kappa}{\epsilon_L} - 1 > \frac{q}{\delta}$$

This condition is much weaker than the one before. Indeed, as long as either s^* or s_c is strictly between 0 and 1, $s^* < s_c$.

This distortion is related to sharing power. Protection of property rights requires a larger army, thus it requires sharing power. But sharing power requires sharing rents, because a rebellion that includes government insiders requires less fighting effort. Thus even if protection leads to higher output, it also implies that a lower fraction of output goes to an army member. Hence, the army might choose higher taxes on capital even if it leads to lower output, because of its impact on the distribution of power, and consequently, the distribution of income.

Because sharing power requires sharing rents, public goods or services that have to be provided by people with power will be underprovided. Hence, the threat of expropriation from the government turns out to be a serious issue in the model, reflected in capital taxes that are too high from the society's point of view. That is in accordance to the empirical evidence in Acemoglu and Johnson (2005).

Last, a note on the effect of the power parameter, δ. In section 2.4, welfare of producers in an economy with institutions and a self-interested incumbent army might be larger than welfare of producers in an economy where everyone gets q and no transfers can be enforced. That is because the possibility of establishing rules allows for the provision of public goods. However, in that case, $\delta \to 0$ is ideal. Intuitively, it is important to have power to collect and enforce taxes, but then, power is not good, as it tends to lead to larger a and worse income distribution.

However, this section shows that in an economy with a very small δ, there cannot be investment. Any investment would be expropriated (capital taxes would be 100%). If δ is low enough, it is not even efficient to protect property – it would require too many people in the army. A larger δ allows the incumbent army to keep its power. That will be used for their own good, but might also allow them to (partly) protect private property, which would not be possible with a lower δ.

3.4 Analogies with historical examples

The results show the importance of sharing power to estimulate investment and prevent changes in the rules of the game that lead to expropriation. They also show the rulers will not share power as much as would be efficient. Although the model is too abstract to match precisely historical examples, this idea squares out right with our reading of history.

Malmendier (2009) studies what he calls the earliest predecessor of modern business corporation, the Roman *societas publicanorum*. Their demise occurred with the transition from the Roman Republic to the Roman Empire, around two thousand years ago. One possible reason for their demise is that "the Roman Republic was a system of checks and balances. But the emperors centralized power and could, in principle, bend law and enforcement in their favor". In other words, while power was decentralized, it was possible to have rules that guarantee the property rights of the *societas publicanorum*, presumably because changing the rules would require some of the powerful people to launch an attack on others, which was costly. But once power is centralized, protection agaisnt expropriation is not possible anymore.

Greif (2006) analyses in detail the building of a State in Genoa, in Medieval times. As in many cities in medieval Italy, Genoa employed a *podestà*, someone coming from another city that would govern Genoa for a year. Greif analyses the importance of the *podesteria* system for interclan cooperation. The *podestà* was generously paid, but it was worth having him there for it allow cooperation and investment. Interestingly, Greif argues that the *podestà* had to be sufficiently strong because otherwise if one clan had defeated the other, it could easily defeat the *podestà* as well. Translating it into the language of the model, the strength of the *podestà* $(a^* - \hat{a})$ had to be large enough to prevent a clan to kick him out and change the rules.[11]

Glorious revolution. Main thing: power sharing between king and parliament. By accepting the Declaration of Rights and the Bill of Rights, King William is accepting that power is shared between him and the parliament. Result: secure property rights, elimination of confiscatory government. One issue in this case is that people in the parliament were either directly interested or very related to those who were interested in the protection of their own wealth against the crown. But North and Weingast (1989) show that after the glorious revolution England could borrow much more, at substantially lower rates. That was certainly in the interest of the king, but was not possible without sharing power between king and parliament, as that would make too easy for the king to rewrite the rules.

Although that was very good to the king in terms of financing its debt, the kings did not really wanted to do so, they resisted quite a bit. Presumably the existence of the parliament implied that rents would have to be shared between the king and the parliament, so even if the pie was larger, the share of the king could be smaller. That is clearly related to the result that the incumbent army will refrain from sharing power and the level of investment will be sub-optimal.

Broadly speaking, the extra people in the army $(a^* - \hat{a})$ might perhaps be interpreted as a "parliament" or any other group of people with power to deter changes in government – from outsiders and, especially, from insiders. Parliaments usually are thought of as representing

[11] Greif (2006), page 239-240.

the people that elected their members, but then so are democractically elected presidents, and parliaments are not only useful to defend minorities. Power sharing makes the institutions more stable because it makes costly for some members in power to alter the institutions that establish how power is distributed and limit how much can be taken from producers.

4 Conclusions

Governments and institutions play a key role in development by making it possible to direct resources to the provision of public goods and by enforcing collective choices. However, what is called a government in some of the poorest countries bears little resemblance to its counterpart in the most developed nations. As noted by the bible passage in the book of Samuel, those with power to choose the "rules of the game", guided by their own interests, might choose policies that are not necessarily good from the point of view of society. Indeed, in far too many cases, the warnings of Samuel remain as relevant as ever.[12]

This paper provides a model where institutions are set to maximise the payoffs of the group in power. That assumption and the threat of rebellions are the main elements of the model. When private investment is considered, the risk of expropriation emerges. In order to protect investment, sharing power is needed. That is the only way to prevent those in power to tear up the old rules and expropriate capital. However, sharing power requires sharing rents, because those with power can use it against the other members of the ruling group. Hence, there is not enough sharing power and too high capital taxes — too little protection of investment.

This framework could be used in a number of ways. First, it can be explored to understand which factors allow for more or less power sharing — and thus investment. It would also be interesting to analyse other sources of inefficiencies. Also, agents are homogeneous ex ante, with heterogeneity as an equilibrium outcome, but ex-ante heterogeneity of various forms could be incorporated into the model (for example, ethnic differences, as in Caselli and Coleman (2006)). There are no rebellions in equilibrium, but they could arise if the model had some stochastic elements.

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[12]For a model on the trade-off that having a government (or a "king") entails, see Grossman (2002).

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A Technical appendix

A.1 Proof of Proposition 1

Start by fixing a particular choice of institutions, that is, a choice of army size a and the distribution of consumption across agents. These choices also determine the composition of $\mathcal{A}$ and $\mathcal{P}$. This particular choice of institutions does not lead to any rebellion if and only if

$$\max_{\mathcal{R}} \left\{ \int_{\mathcal{R}\cap\mathcal{P}} (U'_{\mathrm{a}} - U_{\mathrm{p}}(\imath))d\imath + \int_{\mathcal{R}\cap\mathcal{A}} (U'_{\mathrm{a}} - U_{\mathrm{a}}(\imath))d\imath + \delta d \right\} \leq \delta a \;\; \text{s.t.} \;\; \mathbb{P}(\mathcal{R}) = a',$$

where the size of the subsequent incumbent army a', as well as U'_{a}, is taken as given. For a given $\mathcal{R}$, d is the measure of the set $\mathcal{R}\cap\mathcal{A}$. Now define disjoint sets $\mathcal{R}_p \equiv \mathcal{R}\cap\mathcal{P}$ and $\mathcal{R}_a \equiv \mathcal{R}\cap\mathcal{A}$, and note that the maximization problem above is equivalent to

$$\max_{\mathcal{R}_p,\mathcal{R}_a} \left\{ (a'-d)U'_{\mathrm{a}} - \int_{\mathcal{R}_p} U_{\mathrm{p}}(\imath)d\imath + d(U'_{\mathrm{a}}+\delta) - \int_{\mathcal{R}_a} U_{\mathrm{a}}(\imath)d\imath \right\} \leq \delta a \;\; \text{s.t.} \;\; \mathbb{P}(\mathcal{R}_{\mathrm{p}}\cup\mathcal{R}_{\mathrm{a}}) = a'. \quad \text{[A.1.1]}$$

Now make the following definitions:

$$\bar{U}_{\mathrm{p}} \equiv \frac{1}{1-a}\int_{\mathcal{P}} U_{\mathrm{p}}(\imath)d\imath, \quad \bar{U}_{\mathrm{a}} \equiv \frac{1}{a}\int_{\mathcal{A}} U_{\mathrm{a}}(\imath)d\imath, \quad \text{and} \qquad \text{[A.1.2a]}$$

$$\underline{U}_{\mathrm{p}}(r_{\mathrm{p}}) = \min_{\mathcal{R}_{\mathrm{p}} \text{ s.t. } \mathbb{P}(\mathcal{R}_{\mathrm{p}})=r_{\mathrm{p}}} \left\{ \frac{1}{r_{\mathrm{p}}}\int_{\mathcal{R}_{\mathrm{p}}} U_{\mathrm{p}}(\imath)d\imath \right\}, \quad \underline{U}_{\mathrm{a}}(r_{\mathrm{a}}) = \min_{\mathcal{R}_{\mathrm{a}} \text{ s.t. } \mathbb{P}(\mathcal{R}_{\mathrm{a}})=r_{\mathrm{a}}} \left\{ \frac{1}{r_{\mathrm{a}}}\int_{\mathcal{R}_{\mathrm{a}}} U_{\mathrm{a}}(\imath)d\imath \right\}. \qquad \text{[A.1.2b]}$$

These definitions imply

$$\underline{U}_{\mathrm{p}}(r_{\mathrm{p}}) \leq \bar{U}_{\mathrm{p}}, \tag{A.1.3a}$$

with equality if and only if (i) $C_{\mathrm{p}}(\imath) = C_{\mathrm{p}}$ for all $\imath$ (with measure one), or (ii) $u''(\cdot) = 0$ and $r_{\mathrm{p}} = 1-a$. Analogously,

$$\underline{U}_{\mathrm{a}}(r_{\mathrm{a}}) \leq \bar{U}_{\mathrm{a}}, \tag{A.1.3b}$$

with equality if and only if (i) $C_{\mathrm{a}}(\imath) = C_{\mathrm{a}}$ for all $\imath$ (with measure one), or (ii) $u''(\cdot) = 0$ and $r_{\mathrm{a}} = a$.

The definitions in [A.1.2a] imply that the constraint [A.1.1] is equivalent to

$$\max_{d \in \mathscr{D}(a,a')} \left\{ (a'-d)U'_{\mathrm{a}} - (a'-d)\underline{U}_{\mathrm{p}}(a'-d) + d(U'_{\mathrm{a}} + \delta) - d\underline{U}_{\mathrm{a}}(d) \right\} \leq \delta a, \tag{A.1.4}$$

where $\mathscr{D}(a,a') \equiv [\max\{0, a+a'-1\}, \min\{a,a'\}]$. Rearranging terms yields

$$a'(U'_{\mathrm{a}} - \bar{U}_{\mathrm{p}}) + \max_{d \in \mathscr{D}(a,a')} \left\{ d(\bar{U}_{\mathrm{p}} - \bar{U}_{\mathrm{a}} + \delta) + (a'-d)(\bar{U}_{\mathrm{p}} - \underline{U}_{\mathrm{p}}(a'-d)) + d(\bar{U}_{\mathrm{a}} - \underline{U}_{\mathrm{a}}(d)) \right\} \leq \delta a, \tag{A.1.5}$$

making use of the definitions in [A.1.2a] again.

Now consider a Markovian equilibrium with $a = a'$ and $U'_{\mathrm{a}} = \bar{U}_{\mathrm{a}}$. Let

$$D \equiv \max_{d \in [\max\{0,2a-1\},a]} \left\{ d(\bar{U}_{\mathrm{p}} - \bar{U}_{\mathrm{a}} + \delta) + (a-d)(\bar{U}_{\mathrm{p}} - \underline{U}_{\mathrm{p}}(a-d)) + d(\bar{U}_{\mathrm{a}} - \underline{U}_{\mathrm{a}}(d)) \right\}, \tag{A.1.6}$$

where d^* is the smallest value (without loss of generality) of d that maximizes the above expression. The single constraint [A.1.4] to which the equilibrium institutions are subject must be binding. Thus, by substituting the solution of the problem in [A.1.6] into [A.1.4], and after some rearrangement:

$$a\bar{U}_{\mathrm{a}} - d^*\underline{U}_{\mathrm{a}}(d^*) = (a-d^*)(\delta + \underline{U}_{\mathrm{p}}(a-d^*). \tag{A.1.7}$$

First, note that $d^* < a$, otherwise higher taxes on producers would be feasible. Substituting the inequalities in [A.1.3a] and [A.1.3b] into [A.1.7] yields

$$\bar{U}_{\mathrm{p}} - \bar{U}_{\mathrm{a}} + \delta \geq 0,$$

with equality if payoffs are equalized among army members and among producers. Thus, if payoffs are equalized, then the value of D in [A.1.6] is necessarily zero.

Now suppose that payoffs among the producers are not equalized. Then $(\bar{U}_{\mathrm{p}} - \underline{U}_{\mathrm{p}}(a-d))$ is strictly positive for $d > 2a-1$ since this ensures $a - d < 1 - a$, the total measure of producers. Moreover, for $d < a$, the coefficient multiplying this term in [A.1.6] is positive as well. As the other terms in [A.1.6] would be non-negative, and since such a choice of d is feasible (as $a < 1$), the value of D would be positive. As has been seen, $D = 0$ is possible if institutions equalize payoffs within groups. This can be achieved without affecting the other terms in the constraint [A.1.5], and without decreasing the objective function. Hence, in a Markovian equilibrium, payoffs of producers must be equalized.

Analogously, if payoffs among army members are not equalized, then $(\bar{U}_{\mathrm{a}} - \underline{U}_{\mathrm{a}}(d))$ is strictly positive for $d < a$. For $d > 0$, this term has a positive coefficient in [A.1.6]. Such a choice is feasible ($a < 1$), and the other terms appearing in [A.1.6] would be non-negative. Again, D would hence be positive. Since $D = 0$ is feasible, a similar argument to that above shows that payoffs of army members must be equalized.

A.2 Proof of Proposition 2

0. If $s > 0$, $a^* > \hat{a}$.

If $s > 0$, consumption at the second stage is not the same for all producers: those who invested have to get more than the others (to compensate for effort). So, the no-rebellion constraint of those producers who invested cannot be binding. Now, that implies that the army would benefit from expropriating their capital, so that their consumption would be exactly the same as the consumption of the other producers. But if $a^* \leq \hat{a}$, a rebellion in the second stage is costless for the members of the incumbent army: they can expropriate capital from those who invested at no cost. But that cannot happen in equilibrium.

1. If $s > 0$, [3.2] and [3.5] cannot both bind.

Suppose they do, then imposing equilibrium, they imply:

$$q - \tau_q + s\frac{\epsilon^* - \epsilon_L}{2} = U_{\text{a}} - \delta$$

$$U_{\text{a}} = \frac{(q+\delta)^2}{q+2\delta} + (1-a)s\kappa - \delta\frac{a}{\hat{a}} + \delta$$

Plugging the value of U_{a} in the first one,

$$q - \tau_q + s\frac{\epsilon^* - \epsilon_L}{2} = \frac{(q+\delta)^2}{q+2\delta} + (1-a)s\kappa - \delta\frac{a}{\hat{a}}$$

which contradicts [3.4].

2. [3.5] cannot be the only one that binds.

Trivial.

3. [3.2] cannot be the only one that binds.

Suppose is the only one that binds. Getting τ_q from equation [3.2] and substituting it at the objective function, we get:

$$U_{\text{a}} = \frac{1-a}{a}\left(q - U'_{\text{a}} + \delta\frac{a}{a'} + s\frac{2\kappa - \epsilon^* - \epsilon_L}{2}\right)$$

The first order condition with respect to a yields

$$a^* = \frac{\delta}{q + 2\delta + s\frac{2\kappa - \epsilon^* - \epsilon_L}{2}} < \hat{a}$$

But $a^* < \hat{a}$ is a contradiction.

4. So [3.4] binds. Using the constraint to get an expression for τ and substituting it in the objective function, we obtain

$$U_{\text{a}} = \frac{1-a}{a}\left(as\kappa + \delta\frac{a}{\hat{a}} - s\epsilon^* - \frac{\delta^2}{q+2\delta}\right) \qquad \text{[A.2.1]}$$

5. Substituting the expression for U_{a} from [A.2.1] into [3.5] and doing algebra, we get:

$$a^* \geq \frac{\delta\hat{a} + s\epsilon^*}{\delta + s\epsilon^*} \qquad \text{[A.2.2]}$$

6. Now need to show that [3.5] is binding. To show that, (a) we show a contradiction when assuming that only [3.4] is binding and (b) we show a contradiction when assuming that [3.2] and [3.4] are binding.

6a. If [3.4] is the only constraint binding, the solution is given by the unconstrained maximization of [A.2.1]. Taking the first order condition with respect to a and rearranging yields

$$a^* = \frac{\delta\hat{a} + s\epsilon^*}{a^*(s\kappa + q + 2\delta)}$$

Condition [A.2.2] implies

$$\frac{\delta\hat{a} + s\epsilon^*}{a^*(s\kappa + q + 2\delta)} \geq \frac{\delta\hat{a} + s\epsilon^*}{\delta + s\epsilon^*}$$

Thus

$$a^*(s\kappa + q + 2\delta) \leq \delta + s\epsilon^* \Rightarrow a^* \leq \frac{\delta + s\epsilon^*}{s\kappa + q + 2\delta}$$

Using again condition [A.2.2],

$$\frac{\delta + s\epsilon^*}{s\kappa + q + 2\delta} \geq \frac{\delta\hat{a} + s\epsilon^*}{\delta + s\epsilon^*}$$

Rearranging shows a contradiction.

6b. The Lagrangian of the problem of maximizing U_{a} subject to [3.2] and [3.4] is:

$$\mathcal{L} = \frac{1-a}{a}\left(\tau_q + s(\kappa - \epsilon^*)\right) + \aleph_1\left(q - \tau_q + s\frac{\epsilon^* - \epsilon_L}{2} - U'_{\mathrm{a}} + \delta\frac{a}{a'}\right) + \aleph_2\left(-\tau_q - \frac{\delta^2}{q + 2\delta} - (1-a)s\kappa + \delta\frac{a}{\hat{a}}\right)$$

The first order condition with respect to τ_q implies

$$\aleph_2 = \frac{1-a}{a} - \aleph_1$$

The first order condition with respect to a implies:

$$-\frac{1}{a^2}\left(\tau_q + s(\kappa - \epsilon^*)\right) + \aleph_1\frac{\delta}{a'} + \aleph_2\left(s\kappa + \frac{\delta}{\hat{a}}\right) = 0$$

Substituting the value of τ_q implied by the binding constraint [3.4] and the value of $\aleph_2$ from the first order with respect to τ_q and rearranging yields

$$\frac{1}{a^2}\left(\delta\hat{a} + s\epsilon^*\right) = s\kappa + q + 2\delta + \aleph_1\left(s\kappa + \frac{\delta}{\hat{a}} - \frac{\delta}{a}\right)$$

As in equilibrium $a \geq \hat{a}$ and $\aleph_1 \geq 0$, the last term is positive. Hence

$$\frac{1}{a^2}\left(\delta\hat{a} + s\epsilon^*\right) > s\kappa + q + 2\delta \Rightarrow \frac{\delta\hat{a} + s\epsilon^*}{a^*(s\kappa + q + 2\delta)} > a^*$$

Now using the same steps as 6a, we get the proof.

As [A.2.2] is a translation of [3.5] using the objective function and the binding constraint [3.4], it is binding and yields the equilibrium value of a^*.

Rearranging [A.2.1] yields [3.6].