

# Who owns Children and does it Matter?\*

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## Abstract

Most countries seem to have experienced a shift in the property rights over children. While two centuries ago parents had extensive control over their children, laws have been implemented in many countries to curb these control rights and thereby essentially led to a self-ownership of children. In this paper, we first document these changes, then argue that the property right allocation alters the incentive to have children, and thereby may have contributed to the decline in fertility observed in the data. Further, we show that the lack of property rights that parents have over children today may indeed lead to inefficiently low fertility levels. This is an interesting break-down of Coase's theorem, and we provide a detailed analysis of the mechanism responsible for the break-down. This mechanism holds in a variety of environments with altruism, overlapping lives, heterogeneity, infinite horizons. Finally, we address two issues. First, we relate our efficiency results to previous efficiency results in OLG models with and without altruism and with and without fertility choice and argue that property rights are key. Second, potential policy options, preserving the sovereignty of children, are analyzed. These include government debt and a PAYG social security system, both of which need to be designed not to distort fertility decisions.

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# 1 Introduction

In this paper we argue that property rights over children matter for fertility choices. The basic observation is that children are a resource for society in the sense that they increase the total labor endowment in the future. Property rights over this resource may therefore affect incentives. We document that over the course of the last two centuries, most developed countries have experienced a shift from a regime where parents had almost perfect control over their offspring (including their adult children) to a regime where children are protected from their parents' exploitation by law, and thus essentially own themselves—in particular, their labor endowment and income. This shift in the ownership of children, essentially from parents to children, has lowered the net return of having a child, and may therefore have been a contributing factor in the fertility decline in many countries. Further, we show that without parental property rights, equilibrium fertility may in fact be inefficiently low. Finally, we explore various policies, including government debt and a PAYG social security system, both of which need to be designed not to distort fertility decisions, as means to restore efficiency.

We start by providing extensive evidence on the ownership of children. We document carefully how over time children have gained rights and parents have lost rights in several developed countries. Examples of laws that shifted the allocation of power from parents to children include the decline in age of majority, the introduction of laws banning child labor, as well as the rise of social welfare programs allowing governments to take (abused) children away from their parents.

Next, we build a model to illustrate how the degree of property rights over children affects the incentives to bear children.<sup>1</sup> This has positive as well as normative implications. From a positive viewpoint, changes in such laws may lead to a fertility decline.

From a normative point of view, using the efficiency concepts proposed by Golosov, Jones, and Tertilt (2007), we show that equilibrium fertility may be inefficiently low,

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<sup>1</sup>More formally, our notion of “property rights” is meant to capture a parent’s legal access to his/her child’s resources, such as labor income. What we call self-ownership of a child (i.e. property rights assigned to the child) is sometimes also called incomplete markets or borrowing constraints – capturing the idea that a parent cannot borrow against a child’s future income. Aiyagari, Greenwood, and Seshadri (2002) analyze the implications of such borrowing constraints for the efficiency of investments in children in a model where fertility is exogenous. Similarly, Fernández and Rogerson (2001) analyze the implications of borrowing constraints for child schooling decisions and long-run inequality in a set-up with exogenous (but stochastic) fertility.

when parents do not have full property rights over their children. This result seems to violate Coase's theorem, and we explore the origin of the inefficiency within the context of an extreme but instructive example. We find that the inefficiency is caused by the non-existence of a market in which parents and unborn children can make trades. Of course such a market can never exist due to the inter-temporal nature of fertility. Concretely, we show that who owns a child's labor is important for efficient fertility choices. If parents have property rights over children, the costs and benefits of producing new people are aligned and equilibria are generally efficient. On the other hand, if property rights are allocated to the children themselves, costs and benefits are borne by different people and inefficiency may result. Because children who are not born yet cannot negotiate and promise compensation, the original Coasian argument breaks down. Thus, when property rights are allocated to children, fertility might be inefficiently low.

The finding that equilibrium fertility may be inefficiently low when property rights rest with children is interesting in light of recent policy debates in many developed countries. Governments in some countries seem extremely concerned about low birth rates and many different policies to increase fertility are being discussed and implemented.<sup>2</sup> Given the possible failure of the first welfare theorem in this context, such policies may indeed be desirable.

Starting with an extreme example where children are a resource only in Section 3, we extend the model to allow for children entering utility as a consumption good and for parents to be altruistic towards children in a two period setting in Section 4. Within this context, we analyze the effects of standard pay-as-you-go social security in Section 5. We show that, if property rights lie with children and parents are therefore constrained, such a pension system alleviates the downward pressures on fertility and increases the desired transfers from parents to children. Once the constraint on parents is no longer binding, fertility starts to decrease as the pension system expands. However, even if the pension system is so large that parents are no longer constrained by their children's rights, the resulting equilibrium is still not efficient. Individual parents do not take into account that they are producing future contributors to the system. Therefore, the costs and benefits of having children are still not aligned. That is, even though taxes are modeled as lump-sum to children, they are distortionary when it

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<sup>2</sup>See for example "Europe, East and West, wrestles with falling birthrates – Long decline threatening economy," *International Herald Tribune* (September 3, 2006) and "Europe: The fertility bust, Charlemagne" *The Economist*, February 11, 2006.

comes to the fertility decision of parents. As opposed to the exogenous fertility literature, operative transfers (bequests) are no longer sufficient for efficiency when fertility is endogenous.

Given this result, it seems intuitive to consider a fertility dependent pension system. We show that fertility dependent pensions also alleviate the downward pressures of fertility when parents are constrained, and increase desired transfers. However, the expansion of such a system never leads to a fertility decrease but rather to an efficient allocation corresponding to parents having property rights. In contrast to the literature with endogenous fertility without altruism, we can use more general efficiency concepts and can pinpoint the source of the inefficiency: property rights. The presence of altruism also leads to different implications regarding the set of policies that implement efficient allocations.

In Section 6 we extend the basic results, that fertility decreases to inefficiently low levels as property rights shift from parents to children, to an infinite horizon dynastic overlapping-generations (OLG) model.

Within this context, we confront our efficiency results to those in other OLG models. In standard OLG models, a necessary and sufficient conditions for optimality/efficiency is that the interest rate is greater than the population growth rate (see Cass (1972) and Balasko and Shell (1980)). Burbidge (1983) argues that when altruism is properly added to the standard OLG model, the interest rate will always be larger than the population growth rate while Conde-Ruiz, Giménez, and Pérez-Nievas (2004) show that an interest rate larger than the population growth rate is not a sufficient condition for (Millian) efficiency when children are a consumption good in a model where parents are not altruistic. In a model with both, endogenous fertility and altruism, we show that the implicit and differing assumptions on property rights are key to these results.

Even when children's rights lead to inefficiently low fertility, these rights might still be desirable. We thus intend to explore further government policies to solve the inefficiency created through children's rights in the dynastic setting. These include government debt and pensions. We expect to find that these policies have to depend on fertility in unexplored ways to avoid distorted fertility choices.

## **2 History of Property Rights over Children**

It is remarkable how much control parents had over their children at one point in time in comparison to the rights that children have today. Starting from the mid-nineteenth

until the mid-twentieth century, reforms were passed in western countries that completely altered the rights that a parent has over his children; children went from a state of total control by their parents, to a state in which they have a lot of freedom to make their own decisions, marry without requiring parental consent and achieve majority at much lower ages. In this section, we document the history of children's rights *vis à vis* their parents and subdivide it into three periods in history: a period of total control, a period of transition, and, finally, the current state of children's rights.

## 2.1 Total parental control

Historically parents were in a position of almost absolute control over the decisions of their children. Parents—or, in most cases, the father—had many legal recourses to assure that their wishes were executed. Examining the common law system of the United States and England along with the roman based legal system in France and Spain before the nineteenth-century, we can see how western countries ensured that parents had control over children.

The most extreme case of parental control over children in the United States has to do with the stubborn child laws of 1646. This law obligated children to be obedient to their parents, and if they failed to do so, parents had the right to take their children into court and their offenses could be punishable by death.<sup>3</sup> Furthermore, the United States passed a law in 1641 which made it illegal to curse or hit one's parents.<sup>4</sup> Even though there is no record of the stubborn child law ever being used to execute someone, it begins to demonstrate the amount of power parents had in the common law systems in the United States and England.<sup>5</sup> Corporal punishment and physical cruelty were common methods for teachers and parents to enforce discipline.<sup>6</sup> In both the United States and England, children were also obligated to support their parents through the

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<sup>3</sup>An act of the General Court of Massachusetts decreed: "If a man have a stubborn or rebellious son, of sufficient years and understanding, viz. sixteen years of age, which will not obey the voice of his Father or the voice of his Mother, and that when they have chastened him will not harken unto them: then shall his Father and Mother being his natural parents, lay hold on him, and bring him to the Magistrates assembled in Court and testify unto them, that their son is stubborn and rebellious and will not obey their voice and chastisement such a son shall be put to death." States that followed: Connecticut 1650, Rhode Island 1668, New Hampshire 1679 (Mason 1994, p. 11).

<sup>4</sup>The child has to be over 16 and the law could be punishable by death (Hawes 1991, p. 4).

<sup>5</sup>Hawes (1991), p. 5: "No young children were ever put to death under the provisions of the stubborn child law, and there were no cases of children winning against their parents in court."

<sup>6</sup>Mason (1994), p. xvii.

Poor Law Act of 1601. This law obligated adult children to care for their parents by law.<sup>7</sup> Parental consent was also necessary for marriage.<sup>8</sup> And, until 1793 in the United States, parents could legally indenture their children as servants,<sup>9</sup> and some states even had laws that banned children from living apart from their families.<sup>10</sup> Finally, child labor was very widespread; families would force their children to work, relying on the child's pay as a source of income.<sup>11</sup>

Similar degrees of parental control are also present in Roman-based legal systems.<sup>12</sup> In both Spain and France, the idea of *patria potestad* underlined all legal decisions regarding children's rights. *Patria Potestad* refers to "the control which a father exercised over his children, a control similar to that over material things and one which permitted a father to sell or pawn a child if necessary and even to eat it in an extreme case" (Sponsler 1982, p. 147-148). In France, the legal system allowed parents to use *lettres de cachet*, letters signed by the king often used to enforce authority and sentence someone without trial; *lettres de cachet* "could be used by parents when their child refused to follow parental direction with respect to a marriage partner or career" (Kertzer 2001, p. 133). Additionally, children in France were forced to obtain parental consent for all marriage decisions for males under the age of 25 and females under the age of 21.<sup>13</sup> The Napoleonic Civil code also had a law, similar to those found in England and the United States at around the same time, that required children to support their elderly

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<sup>7</sup>Elizabethan Poor law: "The family, as a unit, was to be responsible for poverty-stricken kinfolk The Poor Law did not concentrate on the children of elderly, but extended the network of potential support to include the fathers and mothers, and the grandfathers and grandmothers, of the poor When these laws passed over into the American scene, during the seventeenth and eighteenth centuries, the focus was on the responsibilities of children towards their elderly parents" (Callahan 1985, p. 33).

<sup>8</sup>Kertzer and Barbagli (2001), p. 114.

<sup>9</sup>In *Respublica v. Kepple* (1793) the Pennsylvania Supreme Court "held that there were limits to the purposes for which a parent could bind out a child: they could bind him out to learn a trade, but not as a servant Parents could have the benefit of the child's services or relief from their obligation of support, but only as an incident of preparing him for later life." (Marks 1975, p. 81).

<sup>10</sup>Marks 1975, p. 81.

<sup>11</sup>Hawes (1991).

<sup>12</sup>For an interesting description of the powers a father had over his children in roman times, see Arjava (1998), p. 147-165.

<sup>13</sup>Civil Code of 1804 - "children, regardless of age, were bound to seek the consent of their parents (or grandparents if both parents were deceased) (Article 151). However, as a practical matter, consent of parents was only required for the marriage of males under the age of 25, and females under the age of 21; if the parents disagreed, the consent of the father was deemed sufficient (Article 148)." (Kertzer 2001, p. 138).

parents.<sup>14</sup>

## 2.2 Transition

In the mid-nineteenth century and early twentieth century, many reforms were passed in western countries that expanded children's rights at the expense of their parents' rights. Laws were being passed in 'the best interest of the child' instead of in the interests of the parents. Governments created laws that limited child labor, demanded compulsory education, and allowed governments to intervene in the household, completely changing the child-parent power structure.

After the mid-nineteenth century, the United States passed numerous laws that reduced the control that parents had over their children.<sup>15</sup> Strict laws were passed on abuse, cruelty and parental neglect.<sup>16</sup> Agencies were created to protect children from cruelty (for example, the famous Society for the Prevention of Cruelty to Children) and a Child Court was created.<sup>17</sup> Parents also slowly lost the benefit of relying on their adult children for support, as the United States and England slowly began to repeal or ignore the laws that obligated children to support their elderly parents.<sup>18</sup> Additionally, many states passed compulsory education laws;<sup>19</sup> Kertzer and Barbagli (2001) point out that "for many poor parents among the working class, the artisans and the peasantry, the regular school attendance of their children implied enforced withdrawal from work, whether at home or in the workshop" which eliminated a form of income for parents. By 1938, child labor was banned throughout the country.<sup>20</sup> Finally, in the

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<sup>14</sup>Code of Napoleon: art. 205, "Children are liable for the maintenance of their parents and other ascendants in need" (Byrd 1988, p. 88).

<sup>15</sup>A few notable laws were passed before then: By the 1800s, Parents were no longer allowed to indenture their children in many states, laws prohibiting children from leaving the family were "either repealed or ignored" and parents were obligated by law to provide proper care for their children (Marks 1975, p. 81).

<sup>16</sup>For example, in 1846 Michigan enacted a law making it a crime for parents to abandon a child under six years of age (Marks 1975, p. 83).

<sup>17</sup>Hawes (1991).

<sup>18</sup>For a more detailed description on elderly support laws in the 19th and 20th centuries see Thomson (1984) for England and Britton (1990) for the United States.

<sup>19</sup>Massachusetts was the first state to pass a compulsory school attendance law. "The Massachusetts law served as a model for the laws of most other states." All states followed with compulsory education laws between 1871 and 1929, with Alaska being the last state to implement such laws. For more information on what year each state passed compulsory education laws, see Landes 1972, p. 55-58.

<sup>20</sup>By 1938, Every state had passed laws which "effectively banned child labor and enforced compul-

1970s, reforms were passed that reduced the age of majority from 21 to 18,<sup>21</sup> expanded children's medical rights,<sup>22</sup> increased abuse protection,<sup>23</sup> and provided new definitions on children's emancipation rights.<sup>24</sup> Children in the United States gained rights that drastically reduced the amount of control a parent could have over his children.

The same pattern is witnessed in European countries such as France and Spain. These reforms occurred slightly later in these countries, from the late-nineteenth century to about three-quarters of the twentieth century.<sup>25</sup> In France, at this time, "neglected children came under the protection of the courts (1889), children were protected from the physical abuse of their parents by criminal statute (1898) and 21 was established as the age of majority when children were allowed to undertake legal acts and marry without parental consent (1907)" (Kertzer 2001, p. 141). In legal terms, by 1972, "the relationship of parent to child [was] no longer viewed as a power of domination" and instead was "seen as an authority conferred upon parents to protect the child, thus entailing responsibilities as well as rights"(Alexandre 1972, p. 652-653). In both France and Spain, the focus of child custody legislation began to focus on making

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sory schooling" (Margolin 1978, p. 443). As well, in 1938, Fair Labor Standard Acts was passed, "which broadly regulated child labor" (Guggenheim 2005, p. 4)

<sup>21</sup>1970 - "In the 1970s, the age of majority was reduced from twenty-one years [to eighteen years]." As of 1978, "Eighteen is the statutory majority age in forty-four jurisdictions" (Castle 1986, p. 348).

<sup>22</sup>Plotkin (1981) documents changes in laws related to medical consent of minors: In 1972, Mississippi passed a statute that "allows any minor who can understand the consequences of the [medical] treatment to give personal consent, without regard to the youth's age." Previously all forms of medical and health treatment for minors (under the age of majority) required parental consent (p. 123). In 1975, California allowed minors to give consent for "certain types of services, such as birth control and treatment for venereal disease and pregnancy." (p. 123); In 1973, Colorado "enacted legislation that grants immunity from suit to doctors who act on a good-faith belief that parental consent is not required" (p. 124); 1971 New York Public Health Law allows "any minor, whether or not officially emancipated, who is or has been married or who has borne a child may consent to treatment" (p. 126).

<sup>23</sup>1973 - Child Abuse Prevention and Treatment Act passed, which "required states to meet federal standards on custody provisions", including granting child welfare agencies the power to "remove a child from a family for three days if the agency believes the child is in danger". (Hawes 1991, p. 101). 1974 - Congress enacted law calling for separate legal representation for children (and parents) in neglect and abuse cases (Guggenheim 2005, p. 8).

<sup>24</sup>In the 1970s, many states expanded their definition of child emancipation; for more details, see citeCastle1986, p. 388. As well, for a description of the 1975 California Code definition of emancipation, see Plotkin (1981), p.125.

<sup>25</sup>In Spain, many potential children's and women's rights from the Spanish second republic of the early 1930s were completely reversed when Franco came into power in 1938. Most of these gains were only reinstated after his eventual death in 1975.

ing decisions in the best interest of the *child* and children could potentially have their voices heard in custody decisions.<sup>26</sup> In Spain, child labor was abolished in 1908,<sup>27</sup> the age of majority was reduced from 21 to 18 (1978)<sup>28</sup> and restrictions on daughters leaving the household were eliminated (1978).<sup>29</sup> In both countries, parents could no longer control their children as much as they could in the past.

### 2.3 The present state of children's rights

Today, children possess many rights that were once at the discretion of their parents. In almost all the countries in the OECD and/or EU, children reach majority at 18 and marriageable age is now 18 without consent, 15-16 with court consent (parental consent is not strictly required any longer).<sup>30</sup> The UN has established the Convention on the Rights of the Child, legislation ratified by many countries that enforces the passing of laws in countries in order to guarantee the well being of children.<sup>31</sup> Moreover, children have more rights to represent their views (including views against their parents) in child protective proceedings, in part due to the requirements by Article 12(2) of the Convention on the Rights of the Child. In a Yale Law School study examining how children's voices are heard around the world, almost all EU and/or OECD countries' laws require either that the child must be heard directly according to the law or the child must be heard through a representative or a body.<sup>32</sup> This means that, in cases where

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<sup>26</sup>For example, in Spain - 1981: "The Law of 13 May, 1981 has finally done away with preference given to a father in control over his children within a marriage. Articles 154 and 156 of this new law provide for the joint exercise of patria potestad by both the father and the mother rather than the superior authority of the father. If the parents disagree the question can be heard by a judge and the opinion of the child (if old enough) can be taken into account as well" (Sponsler 1982, p. 150)

<sup>27</sup>The Royal Decree of the 29th of January, 1908, prohibits child labor for both sexes under the age of 16, and for underage women in certain industries (Nash 1983, p. 58)

<sup>28</sup>"As of 16 November, 1978 majority is now reached at age 18 (Article 320)." (Sponsler 1982, p. 129)

<sup>29</sup>"According to article 321 of the Civil Code, a daughter reached her majority at the age 21 but was not allowed to leave her parents' home without their permission, except to marry, to enter an order or if mistreated. A son, on the other hand, was under no such restrictions. As of 22 July, 1972 this portion of article 321 has been stricken" (Sponsler 1982, p. 128-129)

<sup>30</sup>In Japan and Korea, majority is reached at 20. In Canada, majority is reached at either 18 or 19 depending on the province. As well, marriageable age in Japan and Korea is 20 without consent, 18 with parental consent for males, 16 for females.

<sup>31</sup>see <http://www.unicef.org/specialsession/about/world-summit.htm>. The US has not ratified the Convention on the Rights of the Child.

<sup>32</sup>Exceptions are Italy, Japan, New Zealand, Poland and the United States (in the United States, the laws vary by state: most do require the child being heard through a representative). For a more detailed

parents attempt to control their children too much, children can testify against their parents and are protected by law to do so. All these changes, including the lowering of the age of majority around the world (in most cases from 21 to 18), demonstrate the large amount of rights and freedoms that children now hold relative to the few rights they possessed before the nineteenth-century.

### 3 The End of the World – An Instructive Example

We start with a very simple example. The example is extreme and thus allows a clear illustration of our main points. In the example we show that who owns children (i.e. the parent or the child itself) matters both for the equilibrium level of fertility and also for the efficiency of the equilibrium allocation. In later sections, we show that the same mechanism is also at work in more complex and realistic settings.

Assume that there are two periods and two generations: parents and, potentially, children. There is a continuum of identical parents who live for two periods. The utility function of a parent is  $U_p = u(c^m) + \beta u(c^o)$ , where  $c^m$  is consumption when middle-aged and  $c^o$  is parent's consumption when old. Each parent is endowed with one unit of labor when middle-aged and can earn a wage,  $w^m$ . When middle-aged, parents can save for old age and have children. It costs  $\theta$  units of the consumption good to produce one child. Let  $n$  denote the number of children a parent has and let  $s$  be savings. Furthermore, parents can make (potentially negative) transfers to children,  $b$ , subject to a minimum constraint determined by law. Hence, the budget constraints when middle-aged and when old are

$$\begin{aligned} c^m + \theta n + s &\leq w^m, \\ c^o + \int_0^n b^i di &\leq r s, \\ b^i &\geq \underline{b} \in \mathbb{R}, \end{aligned}$$

where  $b^i$  is the bequest or transfer the parent chooses to give to each child  $i \in (0, n_0]$ . If this transfer is negative the parent takes resources from the child; if the transfer is positive, the parent gives to the child. Children, if born, value consumption,  $c^{ik}$ , and are endowed with one unit of labor, which earns  $w^k$ , when parents are old. Hence, child  $i$ 's budget constraint is  $c^{ki} \leq w^k + b^i$ . The production function when parents are old is  $F(K, L) = K^\alpha L^{1-\alpha}$ . Savings are invested as capital so that market clearing

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explanation on the Yale school study, see <http://www.law.yale.edu/rcw/index.htm>

requires  $s = K$ .<sup>33</sup> Labor market clearing requires  $L = n$ . Notice that, since every parent is infinitesimal, his individual choice of fertility,  $n$ , does not change aggregate labor supply and hence does not affect prices.

Now, suppose  $\underline{b} = 0$ . Then the competitive equilibrium is characterized by  $b_1^i = \underline{b} = 0$  for all  $i$ ,  $c^{o*} = 0$ ,  $n^* = 0$ ,  $r^* = 0$ , and  $s^* = 0$  and the economy does not survive beyond period one. To verify that this is a competitive equilibrium, one needs to find prices such that the allocation is consistent with consumer optimization and profit maximization. Consider the consumer first. Given  $r^* = 0$ , and the assumption of full depreciation, the return to savings is zero, hence there is no point in saving, and  $s^* = 0$  follows immediately. The return to children is also zero because negative transfers are not possible and since children provide no direct utility benefit. Therefore, since children are costly, the utility-maximizing number of children is  $n^* = 0$ . Since the parent has no wealth when old, it follows that  $c^{o*} = 0$ . Next, consider firm optimization when parents are old and children productive:

$$\max_{L,K} Y - w^{k*}L - r^*K = \max_{L,K} F(K, L) - w^{k*}L$$

The only wage at which the profit-maximizing output is  $Y = 0$  is  $w^* = \infty$ . Given  $w^* = \infty$  the above is indeed the unique competitive equilibrium.

So we have established that in equilibrium the world ends after the first period. People are not bearing children because they don't particularly care about them. If people don't like children and they are costly, it seems obvious that the equilibrium will not involve children. What is unusual in this allocation is that people live for two periods but cannot consume resources in the second period. In principle people can save and there is a production technology in the second period, yet, savings and output are zero. Why is there no production in period two? The answer is simple: to generate output, labor is needed. Since old people don't have a labor endowment and no young people are being produced, production comes to a halt.

Two points are worth making about this example. Note that the equilibrium would look very different, if parents had more control over their children. Full parental property rights in the example corresponds to  $\underline{b} = -\infty$ .<sup>34</sup> As long as parents can at least recoup the cost of producing children when old, then parents would be indifferent between producing children or not and there would be an equilibrium with positive

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<sup>33</sup>Note that we assume full depreciation here, in the sense that whatever capital is around in period 0 determines  $w^m$  but the capital stock in the second period has to be completely rebuilt by savings.

<sup>34</sup>Note that feasibility requires  $b \geq -w^k$

fertility and positive output in period two. The first observation thus is that the allocation of property rights between parents and children matters for equilibrium fertility. Specifically, fertility is lower when children own themselves to a larger extent ( $\underline{b}$  increases). Secondly, most people would agree that the equilibrium allocation when children own themselves is clearly inefficient. This is a bit tricky, since Pareto Efficiency is not well-defined when fertility is endogenous. However, alternative applicable welfare criteria (e.g. maximizing utility for the representative agent in steady state) suggest that the equilibrium allocation is not optimal.

To be more specific, we use the concepts developed in Golosov, Jones, and Tertilt (2007) to analyze efficiency in this paper.<sup>35</sup> An allocation is said to be  $\mathcal{A}$ -efficient if there is no other allocation such that someone alive in both allocations can be made strictly better off without making someone else (also alive in both allocations) strictly worse off. Similarly, an allocation is  $\mathcal{P}$ -efficient if there is no alternative allocation that makes at least one potential person (that is, including unborns) better off, without making anyone else worse off.<sup>36</sup>

It is quite clear then that the equilibrium allocation with  $\underline{b} = 0$  is  $\mathcal{A}$ -inefficient since one could make the first generation better off by having some children who would be the workers in period two and thus provide some consumption goods for old people. If in addition one assumes people in general are better off being born than not being born, then the equilibrium allocation is also  $\mathcal{P}$ -inefficient. The reason for this inefficiency is that when property rights are assigned to children, then the private costs and benefits of child-bearing are in the hands of two different agents and hence there is no reason for marginal costs and benefits of producing new people to be equated in equilibrium. Note that when parents have full property rights over their children, this problem does not occur. This is an interesting break-down of Coase's theorem: the allocation of property rights matters for the efficiency of the equilibrium allocation. The reason is a missing market. Essentially, the market for private contracts between parents and children where children promise to compensate parents for child-bearing expenses does not exist. Clearly, unborn people cannot write such contracts with their

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<sup>35</sup>Conde-Ruiz, Giménez, and Pérez-Nievas (2004) propose a similar concept and call it Millian Efficiency. The main difference is that Millian Efficiency is only defined for allocations that are symmetric across identical people. The symmetry requirement has some advantages but also some disadvantages relative to  $\mathcal{A}$ - and  $\mathcal{P}$ -efficiency. Yet, for many applications the conclusions are the same.

<sup>36</sup>The latter concept requires utility functions to be well-defined for states of the world where a person is not alive. See Appendix A.1 for a brief outline and Golosov, Jones, and Tertilt (2007) for a detailed discussion on this issue.

parents, but once they are born children have no incentive to sign such a contract.

Finally, it is worth pointing out that there are many efficient allocations in this example. We are not trying to say that allocating rights to the parent leads to the unique efficient allocation. In fact, in this extreme example, when parents have full rights, children would consume nothing in equilibrium, which is also extreme. However, there are indeed many other  $\mathcal{A}$ -efficient allocations where children do have positive consumption. One way to find them is to consider the following planning problem (given  $x$ ):

$$\begin{aligned} \max & u(c^m) + \beta u(c^o) \\ & c_m + \theta L + K \leq w^m \\ & c^o = F(K, L) - xL \end{aligned}$$

where  $x$  is the consumption for each child. Any allocation that maximizes this planning problem *and* has the property that  $\theta F_K(K, L) \geq F_L(K, L)$ , where  $F_i$  is the marginal product of input  $i$ , will be  $\mathcal{A}$ -efficient. The second condition guarantees that the parent would not prefer even more children but allocate less resources to those new children. As long as the marginal product of labor is below the return from investing the child-bearing cost, there is no way to make the parent better off by adding additional people. It should be pretty clear that many such allocations exist.

The example above is clearly an extreme one. Analyzing fertility choice in a world where parents do not care at all about children may seem like a strange starting point. Many fertility models model children as a consumption good (e.g. Becker (1960), Eckstein and Wolpin (1985), Conde-Ruiz, Giménez, and Pérez-Nievas (2004), etc.), a utility function in the case of parental altruism (e.g. Barro (1974), Carmichael (1982), Burbidge (1983), etc.) or both (e.g., Razin and Ben-Zion (1975), Pazner and Razin (1980), Becker and Barro (1986, 1988), Barro and Becker (1989), etc.). In the next section we show that the basic problem illustrated above is present also in settings where children are a consumption good and when parents are altruistic. Later we show that the logic is also present in models with infinitely lived dynasties.

## 4 Property Rights, Fertility Decline and Efficiency

We now show that property rights matter for equilibrium fertility choices as well as efficiency even if parents view children as consumption goods and are altruistic. For

ease of exposition we do the analysis within two-period models. Later, we extend the analysis to dynamic settings and show that the basic results go through.

We also show that there is a special case where the problem pointed out above is *not* operative. When parents are altruistic towards their children *and* do not overlap with productive children, then parents always want to leave positive bequests.<sup>37</sup> Hence, whether property rights are assigned to parents or not is irrelevant since in equilibrium parents optimally want to leave resources for their children. This happens only because bequests are the only way to ensure a positive capital stock in the next period. Since parents care about the utility of children a zero capital stock cannot be optimal. However, as soon as parents and children overlap this logic may break down. Alternatively, even when parents and children do not overlap, as long as there is heterogeneity in the types of parents, the logic also breaks down. With heterogeneity, *some* parents wanting to take resources from their children is perfectly consistent with a positive capital stock.

#### 4.1 Model setup and solution

Suppose now the parent also cares about children as a consumption good and about the average utility of children. Otherwise, the example is identical to the one in the previous section. The parent weighs utility from number of children relative to her own consumption when middle-aged at  $\gamma$  and discounts children's average utility at  $\zeta$ . Thus the problem is

$$\begin{aligned} \max \quad & u(c^m) + \beta u(c^o) + \gamma u(n) + \zeta \frac{\int_0^n u(c^{ki}) di}{n} & (1) \\ \text{s.t.} \quad & c^m + \theta n + s \leq w^m & (2) \\ & c^o + \int_0^n b^i di \leq Rs \\ & c^{ki} \leq w^k + b^i \\ & b^i \geq \underline{b} \end{aligned}$$

First, suppose  $\zeta = 0$  so that, besides being a resource children are also a consumption good, but parents are not altruistic. In this case, parents will have children,  $n^* > 0$ , even if  $\underline{b} \geq 0$ . However, it is still the case that parents would take everything they feasibly or legally can from children. That is, if  $\underline{b} \leq -w^k$ ,  $b^{i*} = -w^k$  and  $c^{ki} = 0$  while if  $\underline{b} > -w^k$ ,  $b^{i*} = \underline{b}$ .

<sup>37</sup>This is the case in Barro and Becker (1989) and in Theorem 1 in Golosov, Jones, and Tertilt (2007).

On the other hand, if  $\zeta > 0$  and utility of children,  $u(\cdot)$ , satisfies Inada conditions, then the parent will always choose  $b^i$  such that  $c^{ki} > 0$  and hence the minimal transfer constraint,  $b^i \geq \underline{b}$ , is not necessarily binding. We analyze the two cases below and show that, if the transfer constraint is binding, fertility is (1) decreasing in  $\underline{b}$  and (2) inefficiently low—both in partial and general equilibrium. The concern with general equilibrium is that extra children tend to decrease the marginal product of labor. We show that this pecuniary externality is present but, as usual, does not matter for efficiency.

To start, we can simplify the model as follows. As long as  $u(\cdot)$  is strictly concave, the parent will choose  $c^{ki} = c^k$  for all  $i$ . Hence the problem can be rewritten as

$$\begin{aligned} \max \quad & u(c^m) + \beta u(c^o) + \gamma u(n) + \zeta u(c^k) & (3) \\ \text{s.t.} \quad & c^m + \theta n + s \leq w^m \\ & c^o + nb \leq Rs \\ & c^k \leq w^k + b \\ & b \geq \underline{b} \end{aligned}$$

Taking first order conditions we get the following set of optimality conditions:

$$c^m + \theta n + s = w^m \qquad \gamma u'(n) = \beta u'(c^o)(R\theta + b) \qquad (4)$$

$$c^o + nb = Rs \qquad u'(c^m) = R\beta u'(c^o) \qquad (5)$$

$$c^k = w^k + b^i \qquad \beta u'(c^o)n = \zeta u'(c^k) + \lambda_b \qquad (6)$$

$$\lambda_b(b - \underline{b}) = 0 \qquad (7)$$

These, together with the firm's first order conditions and market clearing if we consider general equilibrium in the second period<sup>38</sup>

$$w^k = F_k(K, L), \qquad L = n,$$

$$R = F_L(K, L), \qquad K = s,$$

fully characterize the unique equilibrium, given parameters, in particular,  $\underline{b}$ . There are two cases: (1) when the constraint is not binding ( $\lambda_b = 0$ ) and (2) when the constraint is binding ( $\lambda_b > 0$  and  $b = \underline{b}$ ). In case (1), the equilibrium allocation is  $\mathcal{A}$ - and  $\mathcal{P}$ -efficient. In case (2), fertility is decreasing in  $\underline{b}$  and is also  $\mathcal{A}$ - and  $\mathcal{P}$ -inefficiently low.

<sup>38</sup>For simplicity, we keep the assumption of full depreciation, but the results are not sensitive to this assumption.

#### 4.1.1 Parents have all property rights

Suppose  $\underline{b} = -\infty$ . That is, there are no legal constraints on transfers and parents have essentially full property rights over their children.<sup>39</sup> Then we have  $b > \underline{b}$  and by complementary slackness, the multiplier has to be zero,  $\lambda_b = 0$ . Using  $u(\cdot) = \log(\cdot)$ , the solution is:

$$c^{m*} = \frac{w^m}{\beta + \gamma + 1} \quad n^* = \frac{(\gamma - \zeta)Rw^m}{(\beta + \gamma + 1)[R\theta - w^k]} \quad (8)$$

$$c^{o*} = \frac{\beta Rw^m}{\beta + \gamma + 1} \quad b^* = \frac{\zeta R\theta - w^k \gamma}{\gamma - \zeta} \quad (9)$$

$$c^{k*} = \frac{\zeta[R\theta - w^k]}{\gamma - \zeta}$$

For the problem to be well-defined in partial equilibrium, we need

$$R\theta > w^k, \quad (10)$$

and

$$\gamma > \zeta. \quad (11)$$

Condition (10) is related to the rate of return condition in models with more than one investment good. If children did not provide utility, the condition would have to hold with equality. Since children also have a direct utility benefit, the pure market benefit of children must be smaller than the return to saving in physical assets. The second assumption is required because otherwise the “optimal” thing for parents to do would be to have  $\epsilon$  children and provide them with a large amount of consumption. By driving  $\epsilon$  to zero, the consumption per child could (feasibly) go to infinity, so clearly such a specification would have no solution. To rule this out, we need to assume that  $\gamma > \zeta$ . Note that condition (10) is automatically satisfied in the general equilibrium version since prices will adjust to satisfy it. Condition (11) still needs to be imposed, however.

Indeed, with a Cobb-Douglas production function, prices are given by:

$$\begin{aligned} w^{k*} &= F_k(K^*, n^*) = (1 - \alpha)k^{*\alpha} \\ R^* &= F_n(K^*, n^*) = \alpha k^{*\alpha-1} \end{aligned}$$

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<sup>39</sup>As far as we know, Pazner and Razin (1980) is the only previous paper that has used the expression “property rights” in this context. However, they only analyze the case where parents have full property rights and also use a different efficiency criterion. Their main finding, namely that when parents have full control, equilibria are efficient, is in the same spirit as our results.

where everything is defined as in the previous section. Solving for the capital stock per worker gives

$$k^* = \frac{\alpha\theta(\beta + \zeta)}{(1 - \alpha)(\beta + \zeta) + (\gamma - \zeta)}. \quad (12)$$

It is then immediate to show that  $R^*\theta > w^{k^*}$  as long as  $\gamma > \zeta$ , which holds by assumption. Transfers are given by:

$$b^* = \frac{k^{*\alpha-1}(\zeta\alpha\theta - \gamma(1 - \alpha)k^*)}{\gamma - \zeta}. \quad (13)$$

In what follows we often use the special case  $\underline{b} = 0$ , for simplicity. That is, the case in which parents are constrained if they want to leave negative transfers. It is therefore useful to see under what condition parents would like to leave negative transfers. These are summarized in the next lemma.

**Lemma 1** *Equilibrium transfers,  $b^*$  are negative if and only if*

$$\begin{cases} \zeta R\theta < w^k\gamma, & \text{in partial equilibrium;} \\ \alpha\zeta < \beta(1 - \alpha), & \text{in general equilibrium.} \end{cases} \quad (14)$$

**Proof.** In partial equilibrium, the condition follows from equation (9) and condition (11). In general equilibrium, the condition can be directly derived using equations (12) and (13). ■

Note that condition (14) is not necessarily ruled out by our assumptions (10) and (11). That is, there are many parameter combinations for which the parent's problem is well-defined and the optimal transfers are negative. For these cases whether negative transfers are allowed by law or not matters.

#### 4.1.2 Parents are constrained by law

More generally, parents are constrained by law if and only if

$$\begin{cases} \underline{b} > \frac{\zeta R\theta - w^k\gamma}{\gamma - \zeta} = b^*, & \text{in partial equilibrium;} \\ \underline{b} > \frac{\alpha\theta(\zeta - (1 - \alpha)(\beta + \zeta))}{[(1 - \alpha)(\beta + \zeta) + (\gamma - \zeta)]^\alpha [(\beta + \zeta)\alpha\theta]^{1 - \alpha}} = b^*, & \text{in general equilibrium.} \end{cases} \quad (15)$$

In these cases,  $\lambda_b > 0$  and the parent (constrained) optimally chooses  $\hat{b} = \underline{b}$ . The

solution is given by:<sup>40</sup>

$$\begin{aligned} \hat{c}^m &= \frac{Rw^m}{R(1 + \gamma + \beta)} & \hat{n} &= \frac{\gamma R w^m}{(1 + \gamma + \beta)(R\theta + \underline{b})} \\ \hat{c}^o &= \frac{\beta}{1 + \gamma + \beta} [Rw^m] & \hat{s} &= w^m - \frac{Rw^m}{R(1 + \gamma + \beta)} - \theta \frac{\gamma [Rw^m]}{(1 + \gamma + \beta)(R\theta + \underline{b})} \\ \hat{c}^k &= w^k + \underline{b} \end{aligned} \quad (16)$$

In general equilibrium, using market clearing in the second period, the solution above and the firm's first-order conditions, we find that  $\hat{k}$  solves

$$\gamma \alpha k^\alpha = \beta \alpha \theta k^{\alpha-1} + (\beta + \gamma) \underline{b}, \quad (17)$$

which has a closed form solution if  $\underline{b} = 0$ . Using the firm's first order conditions in equation (16) to find  $\hat{n}$  gives

$$\hat{n} = \frac{\gamma \alpha \omega^m}{(\beta + \zeta + 1)(\alpha \theta + \underline{b} \hat{k}^{1-\alpha})}. \quad (18)$$

## 4.2 Shift of property rights from parents to children

Our first argument is that property rights matter from a positive point of view. When children gain rights while parents lose rights, fertility declines.

**Proposition 2** *If parents are constrained by law,  $\underline{b} > b^*$ , an increase in children's rights,  $\underline{b}$ , causes fertility to fall.<sup>41</sup> That is,*

$$d\hat{n}/d\underline{b} < 0, \quad \text{both, in partial and general equilibrium.} \quad (19)$$

**Proof.** In partial equilibrium, this follows directly from equation (16). In general equilibrium, we can see that the LHS of equation (17) is increasing in  $k$  while the RHS is decreasing in  $k$ . The LHS shifts up if  $\underline{b}$  increases while the RHS is unchanged. Thus  $\hat{k}$  is increasing in  $\underline{b}$ , i.e.  $d\hat{k}/d\underline{b} > 0$ . Thus, from equation (18), we see that the question

<sup>40</sup>Note that for this problem in partial equilibrium to be well-defined we need  $R\theta + \underline{b} > 0$ . Recall that as long as  $\zeta > 0$  the parent is never constrained if  $\underline{b} < -w^k$ . Under condition (10), we therefore get that a necessary condition for the transfer constraint to bind is  $\underline{b} > -w^k > -R\theta$ . Thus, if the unconstrained problem is well-defined, but  $\underline{b}$  is such that we are in the constrained case, the constrained problem is well-defined.

<sup>41</sup>In the full-blown dynastic model with overlapping lives below, these comparative statics become more complicated depending on whether  $\underline{b}$  changes for one or more than one generation and when the information about the change becomes available.

boils down to whether  $(\underline{b}k^{1-\alpha})$  increases or decreases when  $\underline{b}$  increases. To see that  $d(\underline{b}k^{1-\alpha})/d\underline{b} > 0$ , rewrite equation (18) as

$$\gamma\alpha\hat{k} - \beta\alpha\theta = (\beta + \gamma)\underline{b}\hat{k}^{1-\alpha}.$$

Since we know that  $\hat{k}$  increases when  $\underline{b}$  increases, the LHS of this equation increases. Hence, the RHS must also increase. This establishes the result. ■

We interpret the historical changes in laws described in Section 2 as an increase in  $\underline{b}$ . As  $\underline{b}$  increases over time,  $\underline{b}$  is irrelevant at first. Eventually it enters the region where condition (15) holds and parents are constrained. We then know from Proposition 2 that fertility starts to fall.

So far, we have only analyzed one representative parent. It is straightforward to extend the analysis to allow for heterogeneity, for example in altruism towards children,  $\zeta$ , or in ability,  $w^m$ . It is easy to construct examples where some parents are constrained by  $\underline{b}$  while others are not. We briefly address heterogeneity in Section 4.4.

### 4.3 Property rights and efficiency

As an extension from the extreme example in the previous section we now show that fertility is also inefficiently low in this richer framework. Furthermore, we show that the results go through when general equilibrium effects—which decrease the marginal product of labor as children are added—are taken into account.

**Proposition 3** *If  $b^* > \underline{b}$ , the equilibrium allocation is  $\mathcal{A}$ - (and  $\mathcal{P}$ -)efficient.*

**Proof.** This follows from Theorems 1 and 2 in Golosov, Jones, and Tertilt (2007). ■

Next, we show that in the case where  $\underline{b} > b^*$ , the equilibrium allocation is inefficient and is characterized by inefficiently low fertility. To do this, we first assume that the unconstrained problem is well defined in the log utility case,  $\gamma > \zeta$  (and  $R\theta > w^k$  in partial equilibrium). Also, for simplicity we consider the special case  $\underline{b} = 0$ . That is, parents cannot take any of their children's wage income.<sup>42</sup> Finally, we assume that this constraint is binding, so that  $b^* < 0$  (see Lemma 1). In this case, the equilibrium

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<sup>42</sup>We relax these assumptions in Section 6.

allocation is given by:

*Partial equilibrium*

$$\hat{n} = \frac{\gamma R w^m}{(1 + \gamma + \beta)(R\theta)}$$

$$\hat{c}^m = \frac{R w^m}{R(1 + \gamma + \beta)}$$

$$\hat{c}^o = \frac{\beta}{1 + \gamma + \beta} R w^m$$

$$\hat{c}^k = w^k$$

*General equilibrium*

$$\hat{n} = \frac{\gamma w^m}{(1 + \zeta + \beta)\theta} \quad (20)$$

$$\hat{c}^m = \frac{w^m}{(1 + \zeta + \beta)} \quad (21)$$

$$\hat{c}^o = \left(\frac{\gamma}{\theta}\right)^{1-\alpha} \frac{\beta^\alpha \alpha w^m}{1 + \zeta + \beta} \quad (22)$$

$$\hat{c}^k = (1 - \alpha) \left(\frac{\beta\theta}{\gamma}\right)^\alpha \quad (23)$$

$$\hat{k} = \frac{\beta\theta}{\gamma} \quad (24)$$

Finally, we construct an  $\mathcal{A}$ - (and  $\mathcal{P}$ )-superior allocation to the unconstrained equilibrium. Consider an alternative allocation where  $\varepsilon$  additional children are born and the parent receives  $M$  resources from each additional child. More specifically, the allocation is given by: <sup>43</sup>

$$\begin{aligned} \tilde{n} &= \hat{n} + \varepsilon; & \tilde{s} &= \begin{cases} \hat{s}, & \text{in partial equilibrium;} \\ \hat{s} = \tilde{K} = \hat{K} = \hat{k}\hat{n}, & \text{in general equilibrium;} \end{cases} \\ \tilde{c}^m &= \hat{c}^m - \varepsilon\theta; & & \\ \tilde{c}^o &= \hat{c}^o + \varepsilon M; & \tilde{c}^n &= \begin{cases} w^k - M, & \text{in partial equilibrium;} \\ \frac{F(\tilde{K}, \tilde{n}) - F(\hat{K}, \hat{n})}{\varepsilon} - M, & \text{in general equilibrium.} \end{cases} \\ \tilde{c}^k &= \hat{c}^k; & & \end{aligned}$$

where  $\tilde{c}^n$  denotes consumption of the  $\varepsilon$  new children. That is, we are giving the new-borns an equal fraction of the additional output they produce and take  $M$  away from each to give to the parent.<sup>44</sup>

Let  $\hat{U}_p$  and  $\tilde{U}_p(\varepsilon, M)$  be the utility of the parent under the equilibrium  $(\hat{x})$  and alternative  $(\tilde{x})$  allocation, respectively. Notice that  $\tilde{U}_p|_{\varepsilon=0} = \hat{U}_p$ .

**Proposition 4** Suppose  $U_p = u(c^m) + \beta u(c^o) + \gamma u(n) + \zeta \frac{\int_0^n u(c^{ki}) di}{n}$ ,  $u(\cdot) = \log(\cdot)$ ,  $\gamma > \zeta$  and  $M \leq w^k < \theta R$  and that the appropriate condition in (14) holds (i.e.,  $\underline{b} = 0$  is binding). Then:

<sup>43</sup>Note that we are treating all additional children the same in this allocation.

<sup>44</sup>It is worth noting that the equilibrium allocation when  $\underline{b} = -\infty$  and  $\lambda_b = 0$  described in Section 4.1.1, though  $\mathcal{A}$ - (and  $\mathcal{P}$ -) efficient, is neither  $\mathcal{A}$ - (nor  $\mathcal{P}$ -) superior to the allocation described in Section 4.1.2 where  $\underline{b} > b^*$  and  $\lambda_b > 0$  because the  $\hat{n}$  children alive in both allocations would be worse off.

1. The tilde allocation is feasible.

2. There exists  $\bar{M} > 0$  such that if  $M \in (0, \bar{M})$  where  $\bar{M}$  is the solution to

$$\begin{cases} \bar{M} + \frac{\zeta}{\gamma} \log\left(\frac{w^k - \bar{M}}{w^k}\right) R\theta = 0, & \text{in partial equilibrium;} \\ \bar{M} \beta u'(\hat{c}^o) = \frac{\zeta}{\hat{n}} \left( u(F_L(\hat{K}, \hat{n})) - u(F_L(\hat{K}, \hat{n}) - \bar{M}) \right), & \text{in general equilibrium;} \end{cases}$$

then  $\left. \frac{\partial \hat{U}_P(\varepsilon, M)}{\partial \varepsilon} \right|_{\varepsilon=0} > 0$ .

This implies that, for given  $M \in (0, \bar{M})$ , there exists a positive  $\varepsilon$  which makes the alternative allocation  $\mathcal{A}$ -superior to the equilibrium allocation. Assuming  $u(\tilde{c}^n) > u_{unborn}$ , the alternative allocation is also  $\mathcal{P}$ -superior to the equilibrium allocation.

**Proof.** A detailed proof is available in Appendix A.2. It boils down to noticing that since  $\lambda_b > 0$ , we have  $\beta u'(\hat{c}^o) > \zeta \frac{u'(\hat{c}^k)}{\hat{n}}$ . Hence, the marginal utility of consumption when old is too high, while the marginal utility of children's consumption is too low. Thus there is room for decreasing the average utility of children by having more and transferring some of their additional production to the parent when old—as long as this does not decrease consumption when middle-aged too much ( $\varepsilon$  small).<sup>45</sup> ■

#### 4.4 Negative bequests, overlapping lives and heterogeneity

It is well known that in the standard closed economy growth model with parental altruism such as the model analyzed in Barro and Becker (1989), bequests are always positive. If this is the case, then transfers never run from children to parents and any minimum constraint less than or equal to zero,  $\underline{b} \leq 0$ , will never be binding. In this subsection, we note that if either parents overlap with productive children or are heterogeneous, bequests or transfers may still be negative and hence more easily constrained by law.

To see this, start with the case where parents live for only one period, are homogeneous, care about children's utility and die before children become productive. Whether they care about number of children ( $\gamma > 0$  and  $\theta > 0$ ) or not ( $\gamma = 0, \theta = 0$  and

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<sup>45</sup>All that is needed for this result is that utility is separable and  $u$  has the usual properties  $u' > 0, u'' < 0$ , continuous, twice differentiable and  $F$  satisfies  $F_L > 0$ , continuously differentiable. In particular, this does not depend on log utility or Cobb-Douglas production function. We extend this result to the more general case in the dynastic model in Section 6.

$\bar{n}$  given), is not essential here. The parent's problem is:

$$\begin{aligned}
\max \quad & u(c^m) + \gamma u(n) + \zeta u(c^k) & (25) \\
s.t. \quad & c^m + nb + \theta n \leq w^m + rk \\
& c^k \leq w^k + rb \\
& b \geq \underline{b}
\end{aligned}$$

Prices are determined in general equilibrium as before. In this case net savings in the economy, which are left to children when the parent dies, must be equal to the capital stock,  $k = b$ . For  $k = 0$ , the interest rate would be infinite and since parents care about children, they would want to take advantage of this situation. Hence, parents leave positive bequests,  $k = b > 0$ , in equilibrium. This was not the case in the example where parents overlap with productive children (see Lemma 1).

On the other hand, if there is heterogeneity in the economy, say in ability  $a^i$ , so that  $w^{mi} = a^i \bar{w}^m$ ,<sup>46</sup> or preferences,  $(\zeta/\gamma)^i$ ,<sup>47</sup> then overlapping lives are not necessary to generate negative transfers for some families. Even though average bequests must be positive for a positive capital stock, not all parents need to leave positive bequests. A fraction of the population may be constrained and those are the ones with inefficiently low fertility.<sup>48</sup>

## 5 Policy Analysis

Even when children's rights lead to inefficiently low fertility, these rights might still be desirable. In this section, we explore the effects of intergenerational policies in the presence of binding property rights in the two-period model laid out above.

First, we show that the introduction of a standard pay-as-you-go (PAYG) social security system in which children are taxed lump-sum to finance lump-sum transfers to parents when old, alleviates the downward pressure on fertility and increases the desired transfer when parents are originally constrained. By increasing the PAYG system, holding the property rights fixed, desired transfers may increase so much that parents

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<sup>46</sup>Some authors such as De la Croix and Doepke (2003, 2004, Forthcoming), Doepke (2004) and Zhao (2008) have used ability or "skill" heterogeneity to generate differential fertility.

<sup>47</sup>See Jones, Schoonbroodt, and Tertilt (2008) for examples.

<sup>48</sup>It would be interesting to derive similar comparative statics to determine what type of households are likely to be bequest-constrained as those in Drazen (1978), Laitner (1979) and Cukierman and Meltzer (1989) in models with endogenous fertility.

are no longer constrained. At this point and whenever parents are unconstrained, fertility is decreasing in the size of the PAYG system.

This decrease in fertility is theoretically in line with Boldrin, De Nardi, and Jones (2005). However, timing suggests that, if on average, parents in the U.S. were constrained by property rights in the 1930s, the introduction of the social security system may at first have contributed to the rise in fertility and the Baby Boom, while possibly contributing to the decrease in the late 1960s and early 1970s when parents were less constrained on average.

Further, we show that the standard PAYG system does not lead to an  $\mathcal{A}$ -efficient allocation. The intuition for this result is that, while taxes that finance pension payments may be lump-sum to children, they are distortionary to the parent's fertility decision. That is, since individual parents are not enticed to realize that their fertility produces future contributors to the system, the costs and benefits of having children are still not aligned. This relates back to the "operative bequests/transfers" literature starting with Barro (1974) and followed by Carmichael (1982), Burbidge (1983), Abel (1987) and others, where operative bequests/transfers are sufficient for efficiency.

As pointed out in Eckstein and Wolpin (1985) and Conde-Ruiz, Giménez, and Pérez-Nievas (2004) in models without altruism, payments to old parents should be linearly dependent on the number of children they have. We extend these results as follows. We add altruism, use more general efficiency concepts and pinpoint the source of the inefficiency: misaligned property rights. We show how such a fertility dependent PAYG (FDPAYG) system, like standard PAYG, alleviates the downward pressure on fertility due to binding property rights. Since parents are altruistic in our setup, FDPAYG also generates an increase in the desired transfer. If the FDPAYG system is large enough, the allocation of consumption levels is the same as in the case where parents have all the property rights. Any further increases in the FDPAYG system are completely undone by the parent's transfer choice. Hence, if the goal is to implement this particular  $\mathcal{A}$ -efficient allocation, there is no unique "optimal tax", per se, as in the papers above, but the entire range of large enough FDPAYG taxes implement the same  $\mathcal{A}$ -efficient allocation.

It is important to note, however, that the allocation resulting from a large enough FDPAYG system is not the only efficient allocation. In fact, it is not  $\mathcal{A}$ -superior to the allocation where parents are constrained and taxes are zero because children are worse off.

## 5.1 Introducing PAYG social security

Suppose we introduce a government in the two-period model laid out in Section 4. In period two, the government taxes children lump-sum at  $\tau$  and gives the proceeds as a lump-sum pension,  $T$ , to the old parent. Both, the children and parents take these taxes and pensions as given. Hence, the budget constraints in the second period change as follows:

$$c_o + bn \leq Rs + T$$

$$c_k = w_k + b - \tau$$

Furthermore, a PAYG system requires that the government balances its budget. Hence, we have  $n\tau = T$ . That is, the government chooses one instrument, say  $\tau$ , while the other,  $T$ , is determined in equilibrium, by the fertility choice of all parents. The (infinitesimal) individual parent realizes that his/her fertility choice alone will not affect the average pension,  $T$ . Otherwise, everything remains the same as before. In particular, other than the budget constraints none of the first-order conditions of the household or the firm and none of the feasibility conditions are affected by this change. Throughout this section, we assume that  $u(\cdot) = \log(\cdot)$  and  $F(K, n) = K^\alpha n^{1-\alpha}$ .

First, assume that  $\underline{b}$  is low enough, so that  $b^* > \underline{b}$  given  $\tau$ . Using the first-order condition for  $n$  in equation (4), the budget constraints for the household and the government's budget balance, one can solve for  $n^{*\tau}$  and  $b^*$  to find:

$$n^{*\tau} = \frac{(\gamma - \zeta) [Rw^m]}{(\beta + \zeta + 1)[R\theta - w^k + \tau] + (\gamma - \zeta)(R\theta - w^k)}; \quad (26)$$

$$b^{*\tau} = \frac{\zeta R\theta - \gamma w^k + \gamma\tau}{(\gamma - \zeta)}. \quad (27)$$

In general equilibrium, using the feasibility constraint in the second period, we find that  $k^{*\tau}$  solves:

$$[(1 - \alpha)(\beta + \zeta) + (\gamma - \zeta)]k^\alpha = (\beta + \zeta)(\alpha\theta k^{\alpha-1} + \tau). \quad (28)$$

Finally, using the firm's first-order conditions in equations (26) and (27), we get:

$$n^{*\tau} = \frac{(\gamma - \zeta)\alpha\omega^m}{(1 + \beta + \gamma)(\alpha\theta - (1 - \alpha)k^{*\tau}) + (1 + \beta + \zeta)\tau(k^{*\tau})^{1-\alpha}}; \quad (29)$$

$$b^{*\tau} = \frac{(k^{*\tau})^{\alpha-1}(\zeta\alpha\theta - \gamma(1 - \alpha)k^{*\tau}) + \gamma\tau}{\gamma - \zeta}. \quad (30)$$

The next proposition shows that whenever the parent is unconstrained, fertility is decreasing in the size of the PAYG system, while desired transfers,  $b^{*\tau}$ , and the capital stock per worker,  $k^{*\tau}$ , are increasing.

**Proposition 5** *If, given  $\tau$ , parents are not constrained by law,  $b^* > \underline{b}$ , an increase in the size of the PAYG system,  $\tau$ , causes fertility to fall, the capital stock per worker to increase and transfers to increase. That is,*

$$\begin{aligned} dn^{*\tau}/d\tau &< 0, & \text{both, in partial and general equilibrium;} \\ dk^{*\tau}/d\tau &> 0, & \text{in general equilibrium;} \\ db^{*\tau}/d\tau &> 0, & \text{both, in partial and general equilibrium.} \end{aligned}$$

**Proof.** See Appendix A.3 ■

Now suppose that  $\underline{b} > b^{*\tau}$  so that the parent is constrained under the current PAYG system. Using the first-order condition for  $n$  in equation (4), the budget constraints for the household with  $b = \underline{b}$ , and the government's budget balance, one can solve for  $\hat{n}^\tau$  to find:

$$\hat{n}^\tau = \frac{\gamma R w^m}{(1 + \beta + \gamma)(R\theta + \underline{b}) - \tau\gamma}. \quad (31)$$

In general equilibrium, using the feasibility constraint in the second period, we find that  $\hat{k}^\tau$  solves:

$$\gamma\alpha k^\alpha = \beta\alpha\theta k^{\alpha-1} + (\beta + \gamma)\underline{b} - \gamma\tau. \quad (32)$$

Finally, using the firm's first-order conditions in equation (31), we get:

$$\hat{n}^\tau = \frac{\gamma\alpha\omega^m}{(1 + \beta + \gamma)\alpha\theta + ((1 + \beta + \gamma)\underline{b} - \tau\gamma)(\hat{k}^\tau)^{1-\alpha}}. \quad (33)$$

The next proposition shows that whenever the parent is constrained, fertility is increasing in the size of the PAYG system, while the capital stock per worker decreases.

**Proposition 6** *If, given  $\tau$ , parents are constrained by law,  $\underline{b} > b^*$ , an increase in the size of the PAYG system,  $\tau$ , causes fertility to increase, the capital stock per worker to fall and desired transfers to increase, even though actual transfers remain at  $\underline{b}$ . That is,*

$$\begin{aligned} d\hat{n}^\tau/d\tau &> 0, & \text{both, in partial and general equilibrium;} \\ d\hat{k}^\tau/d\tau &< 0, & \text{in general equilibrium;} \\ db^{*\tau}/d\tau &> 0, & \text{both, in partial and general equilibrium.} \end{aligned}$$

**Proof.** See Appendix A.4.2 ■

Figure 1 shows how the introduction and expansion of the PAYG system may have affected fertility. On the x-axis, we have the size of the PAYG system, as a fraction of children's income,  $(\tau/w^k)$ .<sup>49</sup> The left panel plots the desired (dotted line) and actual transfers (solid line), while the right panel plots  $2 * n$ —as an approximation to the number of children per woman, where  $n$  is first given by  $\hat{n}^\tau$ , because parents are constrained, then by  $n^{*\tau}$  because the constraint is no longer binding. As the figure suggests, the introduction and expansion of PAYG in many western countries may have contributed to the baby boom and the subsequent fertility decline.

Furthermore, the next proposition shows that even though transfers can become operative if the PAYG tax is large enough, the resulting equilibrium is still not  $\mathcal{A}$ -efficient. This is because the tax distorts the fertility decision in this setup.

**Proposition 7** *Suppose the PAYGO tax is large enough, so that  $b^{*\tau} > \underline{b}$ . Then the equilibrium allocation is still characterized by inefficiently low fertility.*

**Proof.** First, we construct an allocation by adding  $\varepsilon$  children who consume  $\tilde{c}^n = c^{k^{*\tau}}$ . That is,

$$\begin{aligned} \tilde{n} &= n^{*\tau} + \varepsilon; & \tilde{s} &= n^{*\tau} k^{*\tau}; \\ \tilde{c}^m &= c^{m^{*\tau}} - \varepsilon\theta; & \tilde{c}^n &= c^{k^{*\tau}}; \\ \tilde{c}^o &= c^{o^{*\tau}} + F(\tilde{K}, \tilde{n}) - F(\hat{K}, \hat{n}) - \varepsilon\tilde{c}^n; & \tilde{c}^k &= c^{k^{*\tau}}. \end{aligned}$$

Using the first-order conditions at the original equilibrium allocation, it is then straightforward to show that

$$\lim_{\varepsilon \rightarrow 0} \frac{\partial \tilde{U}_p}{\partial \varepsilon} = \beta u'(c^{o^{*\tau}}) \tau > 0$$

whenever  $\tau > 0$ . Hence, the parents can be made better off while all children are at least as well-off. ■

This result is in contrast with the exogenous fertility DOLG literature, starting with Barro (1974) and followed by Carmichael (1982), Burbidge (1983), Abel (1987) and others, where operative bequests/transfers are a sufficient condition for optimality or

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<sup>49</sup>The wage is determined in equilibrium here. The range of  $(\tau/w^k)$  corresponds to the size of the system in the U.S.. See the Social Security Online webpage, in particular <http://www.socialsecurity.gov/history/pdf/t2a3.pdf>.

Pareto efficiency.<sup>50</sup> The basic problem with a standard PAYG system is that when parents make fertility choices they do not take into account that they are producing future contributors to the system. Therefore the costs and benefits of producing children are still not aligned.

## 5.2 Fertility dependent PAYG pensions

Given the results above, it seems then that a social security system where the pension payments depend on fertility choices, can do better than the standard PAYG system. That is precisely what we analyze in this section. In particular, we show that a fertility dependent PAYG system (FDPAYG) alleviates the downward pressure on fertility whenever present but does not drive fertility down when the transfer constraint becomes slack and leads to an  $\mathcal{A}$ -efficient allocation.

Again, in period two, the government taxes children lump-sum at  $\tau$  and gives the proceeds as a fertility dependent pension,  $T(n)$ , to the old parent. That is, the parent knows that an increase in his/her own fertility affects his/her pension payment in period two. Hence, the budget constraints in the second period change as follows:

$$\begin{aligned} c^o + bn &\leq Rs + T(n) \\ c^k &= w^k + b - \tau \end{aligned}$$

Again, a PAYG system requires that the government balances its budget. Hence, we have  $n\tau = T(n)$ .

This change alters the first-order condition for  $n$ . That is, equation (4) becomes:

$$\gamma u'(n) = \beta u'(c^o)(R\theta + b - T'(n)) \quad (34)$$

First, assume that  $\underline{b}$  is low enough, so that  $b^* > \underline{b}$  given  $\tau$ . Assuming  $u(\cdot) = \log(\cdot)$ ,  $F(K, n) = K^\alpha n^{1-\alpha}$  and  $T(n) = \tau n$  so that  $T'(n) = \tau$ , it is straightforward to show that the equilibrium allocation for  $n, c^m, c^o$  and  $c^k$  is independent on  $\tau$ . Using this in the feasibility in the second period, one can see that  $k$  is also independent on  $\tau$ . The solution for these three variables is therefore given in Section 4.1.1. Now, since  $b^{*\tau n} = c^{k*} + \tau - w^k$ , it is immediate that  $b^{*\tau n}$  adjust one for one with increases in  $\tau$ . The intuition for this result is that the parent's problem can be rewritten as a problem without taxes

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<sup>50</sup>We intend to address government debt in this context in the infinite horizon dynastic model in Section 6.

Figure 1: Introduction of Social Security,  $\tau$ : 0 to 15% of  $w^k$

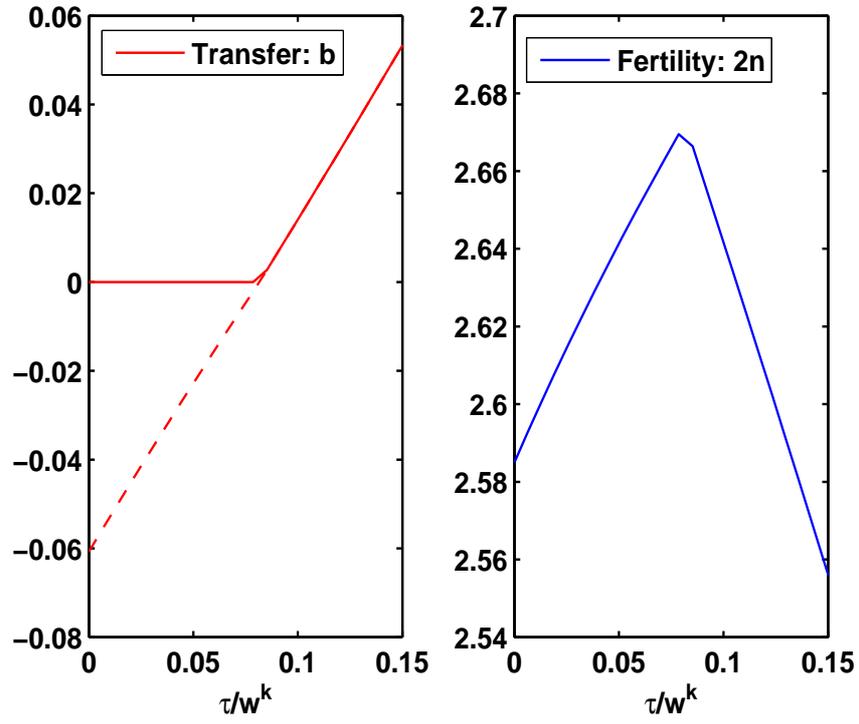
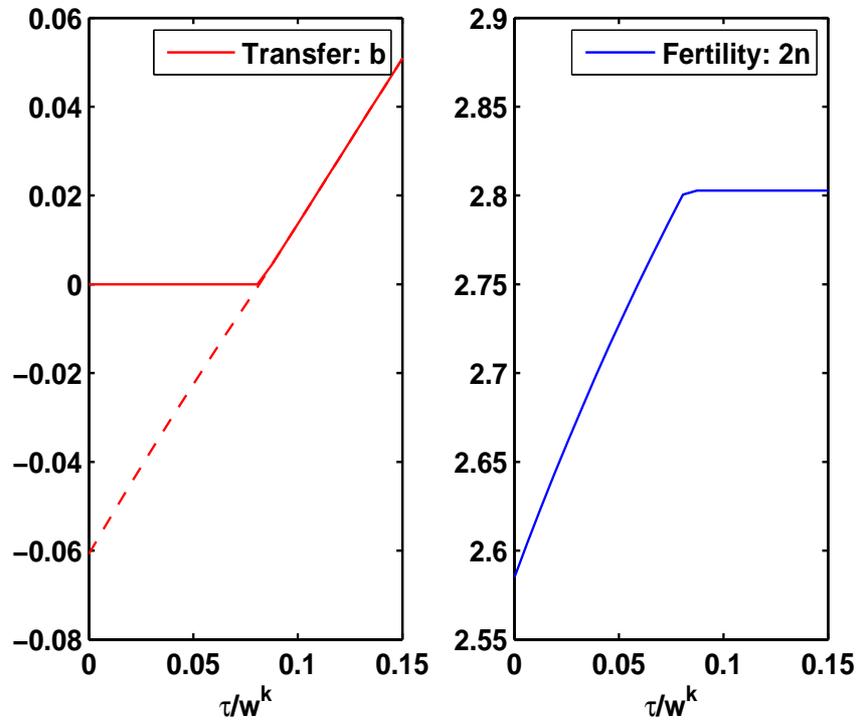


Figure 2: Fertility Dependent Social Security,  $\tau n$ : 0 to 15% of  $w^k$



by defining  $\check{\underline{b}} = \underline{b} - \tau$  and  $\check{\underline{b}} = \underline{b} - \tau$ . So that the constraints become

$$\begin{aligned} c_o + \check{\underline{b}}n &\leq Rs \\ c_k &= w_k + \check{\underline{b}} \\ \check{\underline{b}} &\geq \check{\underline{b}} \end{aligned}$$

Then, if  $\underline{b}$  is not binding, then  $\check{\underline{b}}$  is also not binding. It follows from Proposition 3 that the resulting allocation is  $\mathcal{A}$ -efficient. Vice versa, however, it may be that  $\underline{b}$  is binding but that if  $\tau$  is large enough  $\check{\underline{b}}$  is no longer binding. In this case, fertility stops increasing and stays at the same  $\mathcal{A}$ -efficient allocation.

Now, consider the case where  $\underline{b}$  is binding and  $\tau$  is small so that  $\check{\underline{b}}$  is also binding. Then one can solve for  $\hat{n}^{\tau n}$  in partial equilibrium to find:

$$\hat{n}^{\tau n} = \frac{\gamma R w^m}{(1 + \beta + \gamma)(R\theta + \underline{b} - \tau)}. \quad (35)$$

In general equilibrium, using the feasibility constraint in the second period, we find that  $\hat{k}^{\tau n}$  solves:

$$\gamma \alpha k^\alpha = \beta \alpha \theta k^{\alpha-1} + (\beta + \gamma)(\underline{b} - \tau). \quad (36)$$

Finally, using the firm's first-order conditions in equation (35), we get:

$$\hat{n}^{\tau n} = \frac{\gamma \alpha w^m}{(1 + \beta + \gamma)(\alpha \theta + (\underline{b} - \tau)\hat{k}^{1-\alpha})}. \quad (37)$$

Again, we show that whenever the parent is constrained, fertility is increasing in the size of the FDPAYG system, while the capital stock per worker decreases.

**Proposition 8** *If, given  $\tau$ , parents are constrained by law,  $\underline{b} > b^{*\tau n}$ , an increase in the size of the FDPAYG system,  $\tau$ , causes fertility to increase, the capital stock per worker to fall and desired transfers to increase, even though actual transfers remain at  $\underline{b}$ . That is,*

$$\begin{aligned} d\hat{n}^{\tau n}/d\tau &> 0, & \text{both, in partial and general equilibrium;} \\ d\hat{k}^{\tau n}/d\tau &< 0, & \text{in general equilibrium;} \\ db^{*\tau n}/d\tau &> 0, & \text{both, in partial and general equilibrium.} \end{aligned}$$

**Proof.** See Appendix A.4.3 ■

Hence, this FDPAYG system, like standard PAYG, alleviates the downward pressure on fertility due to binding property rights. However, once the constraint is no

longer binding, fertility stays at the same,  $\mathcal{A}$ -efficient, level. That is, any tax large enough to make transfers operative is sufficient for  $\mathcal{A}$ -efficiency when pension pay-outs depend on individual fertility decisions. Without altruism, transfers never become operative. In this sense our results are in contrast with Eckstein and Wolpin (1985) where due to the optimality concept used (maximize utility of the representative agent), the optimal tax is unique; and Conde-Ruiz, Giménez, and Pérez-Nievas (2004) where different Millian efficient allocation can be decentralized by different sizes of the tax system.<sup>51</sup>

Figure 2 shows how the introduction and expansion of a FDPAYG system could have altered the fertility path in many western countries had a FDPAYG system been introduced as opposed to PAYG. On the x-axis, we have the size of the PAYG system, as a fraction of children's income,  $(\tau/w^k)$ .<sup>52</sup> The left panel plots the desired (dotted line) and actual transfers (solid line), while the right panel plots  $2*n$ —as an approximation to number of children per woman, where  $n$  is first given by  $\hat{n}^{\tau n}$ , because parents are constrained, then by  $n^*$  because the constraint is no longer binding.

This result speaks to the current policy debate that blames too low fertility rates for the insolvency of the standard PAYG systems around the western world. We show that, while a social security system was needed to alleviate the downward pressures on fertility, it should have been designed not to distort fertility decisions so that fertility would not have declined after the baby boom.

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<sup>51</sup>Millian efficiency is somewhat similar to  $\mathcal{A}$ -efficiency but imposes symmetric treatment of identical people. This is a useful condition in their set-up without altruism, since otherwise any efficient allocation would involve some children with zero consumption—a property the authors find undesirable.

<sup>52</sup>The wage is determined in equilibrium here. The range of  $(\tau/w^k)$  corresponds to the size of the system in the U.S.. See the Social Security Online webpage, in particular <http://www.socialsecurity.gov/history/pdf/t2a3.pdf>.

## 6 Infinite Horizon Dynastic Model

In this section we extend the model from the previous section to an infinite horizon dynastic model. This allows a straightforward comparison to results derived in other OLG models. The model we use is a special case of dynastic endogenous fertility model first developed in Razin and Ben-Zion (1975) who assume that utility is separable in  $(c, n)$  versus  $U_{t+1}$ . Ours is also separable in  $c$  versus  $n$ , as above. With log utility, it is also a special case of Becker and Barro (1986, 1988) and Barro and Becker (1989), though extended to two periods overlapping lives.<sup>53</sup>

First, we show that the basic results derived above, namely that property rights matter for equilibrium fertility and efficiency, carry over to the dynastic model.

Second, after characterizing steady states, we confront our efficiency results to those in other OLG models. In standard OLG models, a necessary and sufficient conditions for optimality/efficiency is that the interest rate is greater than the population growth rate (see Cass (1972) and Balasko and Shell (1980)). Burbidge (1983) argues that when altruism is properly added to the standard OLG model, the interest rate will always be larger than the population growth rate while Conde-Ruiz, Giménez, and Pérez-Nievas (2004) show that an interest rate larger than the population growth rate is not a sufficient condition for (Millian) efficiency when children are a consumption good in a model where parents are not altruistic. In a model with both, endogenous fertility and altruism, we show that the implicit and differing assumptions on property rights are key to these results.

Third, we intend to explore different government policies to solve the inefficiency created through children's rights. We consider issuing debt and PAYG social security. We expect to find that both of these need to depend on fertility in particular ways, not addressed in the literature so far.

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<sup>53</sup>In terms of model properties, the Barro-Becker papers assume that parents and children do not overlap while productive. In general equilibrium, we know that this implies that bequests are always positive. Razin and Ben-Zion (1975) start with the same demographic structure, but then extend their framework to allow for one period overlap in adult/productive lives. However, they simply solve the planner's problem and do not derive conditions for when transfers run from parents to children or vice versa. That is, they maximize  $U_0$  in equation (40) subject to the constraints in (39), assuming  $b_t = -\infty$  for all  $t$ . Further, Lucas (2002) shows that if utility in Razin and Ben-Zion (1975) is log in  $c$  and  $n$ , their model with one period adult lives is a special case of the basic Barro-Becker model. The proof is due to Ivan Werning and hinges on an intertemporal(-generational) elasticity of substitution larger than one. We have extended this proof to two-period adult lives and the full parameter space. Details are available upon request.

## 6.1 Model Setup

People go through three periods of life: children, middle-aged/fertile/productive period and retirement. Households care about their own consumption when middle-aged,  $c_t^m$ , and when old,  $c_{t+1}^o$ , the number of children,  $n_t$ , as well as their offspring's average utility. The utility of a middle-aged household in period  $t$  (born in  $t - 1$ ) is given by:

$$U_t = u(c_t^m) + \beta u(c_{t+1}^o) + \gamma u(n_t) + \zeta \frac{\int_0^n U_{t+1}^i di}{n_t} \quad (38)$$

where  $n_t$  is the number of children in period  $t$ . We assume that  $u(\cdot)$  is continuous, strictly increasing, strictly concave and  $u'(0) = \infty$ . Discounting between periods is given by  $\beta$  while children's (average) utility is weighed by  $\zeta$ . The budget constraints are given by

$$\begin{aligned} c_t^m + \theta_t n_t + s_{t+1} &\leq w_t + b_t \\ c_{t+1}^o + \int_0^{n_t} b_{t+1}^i &\leq r_{t+1} s_{t+1} \\ b_{t+1}^i &\geq \underline{b}_{t+1} \\ c_t^m, c_{t+1}^o, n_t, s_{t+1} &\geq 0 \end{aligned} \quad (39)$$

where  $s_{t+1}$  are savings,  $b_{t+1}^i$  is the transfer from parent to child  $i$  if positive, from child  $i$  to the parent if negative, and  $\theta_t$  is the cost of children.<sup>54</sup> Thus the middle-aged adult in period  $s$  chooses  $(c_s^m, c_{s+1}^o, n_s, s_{s+1}, \{b_{s+1}^i\}_{i=0}^{n_s})$  to maximize  $U_s$  in equation (38) subject to the constraints in (39), given  $b_s$ , the transfer from his own parents and prices  $r_{s+1}, w_s$  taking the behavior of all descendants as given. Since we assume that utility,  $u(\cdot)$ , satisfies Inada conditions, the non-negativity constraints on fertility and consumption never bind, while the constraint on transfers,  $b_{t+1}$ , may or may not be binding. We derive parametric conditions in steady state below for the problem to be well-defined. For now, we simply assume that the solution is interior.<sup>55</sup>

<sup>54</sup>In particular,  $\theta_t = a_t^g + (a_t^c - \kappa_t)w_t$  with  $a_t^g$  the goods cost of children,  $a_t^c$  is the fraction of time that has to be spent with every child in raising it and  $\kappa_t$  is the amount of (effective) labor the parent can extract from the child. For example, if a period is 20 years and children can work from age 10 and are half as productive as an adult, then  $\kappa_t \approx 0.25$ . Below we concentrate on parent's property rights over adult children but a change in  $\kappa_t$  could reflect changes in child-labor laws, for example.

<sup>55</sup>In particular, we have to make sure that  $n_t > 0, n_t < \infty$  or  $n_t < \bar{n}_t$  if there is a maximum number of children any household can have, and that utility is bounded. The first condition basically require that  $\gamma > 0$ , otherwise, average utility of children can be made very large by driving fertility towards zero; the second requires that  $r_{t+1}\theta > w_{t+1}$  so that the (unconstrained) parent does not borrow an infinite amount

The representative firm has a neo-classical production function  $Y_t = F(K_t, \gamma_A^t L_t)$ , takes prices  $(r_t, w_t)$  as given when choosing  $(K_t, L_t)$  to maximize profits. For simplicity, we assume full depreciation throughout.

Finally, markets clear. Factor markets clear if the firm's labor demand per old person,  $L_t$ , is equal to the number of middle-aged people per old person,  $n_{t-1}$ , since they are the only ones who are productive and labor is supplied inelastically; the capital stock per old person,  $K_t$ , is equal to savings from last period of currently old people,  $s_t = k_t n_{t-1}$ , where  $k_t$  is the capital stock per worker.

$$L_t = n_{t-1} \quad \text{and} \quad K_t = s_t = k_t n_{t-1}.$$

Goods market clearing in period  $t$  can be expressed in per old person terms as follows:

$$c_t^o + n_{t-1}(c_t^m + \theta_t n_t + s_{t+1}) = F(s_t, n_{t-1}) = n_{t-1} F(k_t, 1).$$

## 6.2 Characterizing equilibria

Assuming that  $u(\cdot)$  is strictly concave and there is no heterogeneity among children, then it is always best for the parent to impose the same transfer on each child,  $b_{t+1}^i = b_{t+1}, \forall i$ . Hence, we can rewrite the utility as:

$$U_t = u(c_t^m) + \beta u(c_{t+1}^o) + \gamma u(n_t) + \zeta U_{t+1}$$

and the budget constraint when old as:

$$c_{t+1}^o + n_t b_{t+1} \leq r_{t+1} s_{t+1}$$

Sequentially substituting utility functions from period  $s$  to  $\infty$ , we get:

$$U_s = \sum_{t=s}^{\infty} \zeta^{t-s} [u(c_t^m) + \beta u(c_{t+1}^o) + \gamma u(n_t)] \quad (40)$$

Thus the young adult in period  $s$  chooses  $(c_s^m, c_{s+1}^o, n_s, s_{s+1}, b_{s+1})$  to maximize  $U_s$  in equation (40) subject to the constraints in (39) for  $t = s$ . Note that, without heterogeneity other than period of birth, savings will always be positive as long as the production function,  $F$ , satisfies Inada conditions.

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and has an infinite amount of children, thereby achieving infinite utility through the transfers from his/her children,  $b$ ; and the third condition is derived in steady state to which the problem converges and requires  $\zeta < 1$ .

The first-order conditions for the household boil down to:

$$\gamma u'(n_t) = u'(c_t^m)\theta_t + \beta u'(c_{t+1}^o)b_{t+1} \quad (41)$$

$$u'(c_t^m) = \beta u'(c_{t+1}^o)r_{t+1} \quad (42)$$

$$\beta u'(c_{t+1}^o)n_t = \zeta u'(c_{t+1}^m) + \lambda_{b,t+1} \quad (43)$$

$$c_t^m + \theta_t n_t + s_{t+1} = w_t + b_t \quad (44)$$

$$c_{t+1}^o + n_t b_{t+1} = r_{t+1} s_{t+1} \quad (45)$$

$$\lambda_{b,t+1}(b_{t+1} - \underline{b}_{t+1}) = 0 \quad (46)$$

The first-order conditions for the firm's problem are given by:

$$w_t = F_L(k_t, 1) \quad (47)$$

$$r_t = F_K(k_t, 1) \quad (48)$$

The following three equations are used in several proofs below. Using equation (48) to substitute out  $r_{t+1}$  in equation (42) gives

$$u'(c_t^m) = \beta u'(c_{t+1}^o)F_K(k_{t+1}, 1). \quad (49)$$

Combining equation (41) and equation (42), again using equation (48), gives

$$\gamma u'(n_t) = u'(c_t^m)\left(\theta_t + \frac{b_{t+1}}{F_K(k_{t+1}, 1)}\right) \quad (50)$$

$$= \beta u'(c_{t+1}^o)(F_K(k_{t+1}, 1)\theta_t + b_{t+1}). \quad (51)$$

And combining equation (43) and equation (46) gives

$$\begin{aligned} \beta u'(c_{t+1}^o)n_t &= \zeta u'(c_{t+1}^m) & \text{if } \lambda_{b,t+1} = 0 \\ &> \zeta u'(c_{t+1}^m) & \text{if } \lambda_{b,t+1} > 0 \end{aligned} \quad (52)$$

Again, we consider two cases: (1) the constraint is not binding for *any* generation, i.e.  $\lambda_{b,t} = 0 \forall t$ ; (2) it is binding for *at least one* generation, i.e.  $\exists t$  s.t.  $\lambda_{b,t+1} > 0$ .

As before, if  $\underline{b}_t = -\infty \forall t$  and parents care about the utility of children,  $\zeta > 0$ , then  $\lambda_{b,t} = 0 \forall t$  and parents take less from children than is feasible,  $b_{t+1}^* > -w_{t+1}$  in order to leave them with positive consumption. We denote the equilibrium allocation in this case by  $\{c_t^{m*}, c_{t+1}^{o*}, n_t^*, s_{t+1}^*, k_t^*, b_{t+1}^*\}_{t=0}^{\infty}$ .

In the case where for some generation,  $s_t, \hat{b}_{s+1} = \underline{b}_{s+1}$  and  $\lambda_{b,s+1} > 0$ , while we are agnostic about whether or not other generations are also constrained for now, we denote the resulting equilibrium allocation by  $\{\hat{c}_t^m, \hat{c}_{t+1}^o, \hat{n}_t, \hat{s}_{t+1}, \hat{k}_t, \hat{b}_{t+1}\}_{t=0}^{\infty}$ .

### 6.3 Shift of property rights from parents to children

In this subsection, we extend the results from Section 4.2, that a shift in property rights from parents to children leads to a decrease in fertility, to the dynastic equilibrium. While we do not necessarily assume any particular functional forms for  $u(\cdot)$  or  $F(\cdot)$  as we did before, some additional assumptions on information are needed. In particular, when we derive comparative statics with respect to  $\underline{b}_{s+1}$  where  $s$  is a constrained generation (born in  $(s - 1)$ ), we assume that the change occurs as an unanticipated shock. That is, the information occurs at the beginning of period  $s$  when  $n_{s-1}$  and  $k_s$  have already been chosen. Thus, for old people in period  $s$ , the only possible adjustment lies in their choice of transfer  $b_s$ , while middle-aged people can reoptimize fully, taking  $b_s$ ,  $n_{s-1}$  and  $k_s$  as given. In the following proposition, we also assume that  $b_s = \underline{b}_s$  so that generation  $(s - 1)$  is also constrained. Then the following proposition holds.

**Proposition 9** *Suppose in an equilibrium allocation,  $\{\hat{c}_t^m, \hat{c}_{t+1}^o, \hat{n}_t, \hat{s}_{t+1}, \hat{k}_t, \hat{b}_{t+1}\}_{t=0}^\infty$ , generations  $(s - 1)$  and  $s$  are constrained by law,  $\lambda_s > 0$  and  $\lambda_{s+1} > 0$ . Then an increase in children's rights that gets to be known in period  $s$ , and applies to period  $(s + 1)$ , i.e. an increase in  $\underline{b}_{s+1}$ , causes fertility to fall in partial equilibrium. That is,*

$$\frac{d\hat{n}_s}{d\underline{b}_{s+1}} < 0 \quad \text{in partial equilibrium.}$$

In general equilibrium, assuming that  $u(x) = \frac{x^{1-\sigma}-1}{1-\sigma}$ , we have

$$\frac{d\hat{k}_{s+1}}{d\underline{b}_{s+1}} > 0, \quad \frac{d\hat{r}_{s+1}}{d\underline{b}_{s+1}} < 0.$$

Further, assuming  $\sigma = 1$ , we have

$$\left. \frac{d\hat{n}_s}{d\underline{b}_{s+1}} \right|_{\sigma=1} < 0 \quad \text{in general equilibrium.}$$

**Proof.** A detailed proof is available in Appendix A.5. Here is an outline. First, consider generation  $(s - 1)$ 's decision. As long as  $\lambda_s > 0$ , we have  $\hat{b}_s = \underline{b}_s$  and hence, a marginal change in  $\underline{b}_{s+1}$  will not change the transfer decision of generation  $(s - 1)$ . Note also that  $w_s = F_L(k_s, 1)$  does not respond to changes in  $\underline{b}_{s+1}$  because  $k_s$  is given.

Second, consider generation  $s$ 's decisions. The proof is straightforward in partial equilibrium. In general equilibrium we make the additional functional form assumptions in order to get

$$\hat{n}_s = \frac{\gamma(\hat{w}_s + \underline{b}_s)}{(1 + \beta + \gamma) \left( \theta_s + \frac{\underline{b}_{s+1}}{F_k(k_{s+1}, 1)} \right)}$$

As in the two-period example, the proof boils down to showing that, while  $\hat{k}_{s+1}$  is increasing in  $\underline{b}_{s+1}$ ,  $\left(\frac{b_{s+1}}{F_k(\hat{k}_{s+1}, 1)}\right)$  is also increasing in  $\underline{b}_{s+1}$ , and the proof is very similar. ■

If generation  $(s-1)$  were unconstrained,  $\lambda_s = 0$ , and generation  $s$  were to decrease  $c_s^m$ , then generation  $(s-1)$ , would want to increase their transfers to bring  $c_s^o$  in line with  $c_s^m$  in equation (43). This has a counterbalancing effect on fertility choice in equation (69). If the information that  $b_{s+1}$  was going to change became available in period  $(s-1)$  then generation  $(s-1)$  may also want to adjust their fertility and savings choices,  $\hat{n}_{s-1}$  and  $\hat{s}_s = \hat{n}_{s-1} \hat{k}_s$ , which would result in ambiguous comparative statics in terms of  $\frac{d\hat{n}_s}{db_{s+1}}$ .

## 6.4 Property rights and efficiency

In this subsection, we extend the results from Section 4.3, that if property rights lie with children the equilibrium allocation is inefficient, to the dynastic equilibrium and general formulations for utility and production functions.

**Proposition 10** *If  $\underline{b}_t = -\infty$  for all  $t$ , then the equilibrium allocation,  $\{c_t^{m*}, c_{t+1}^{o*}, n_t^*, s_{t+1}^*, k_t^*, b_{t+1}^*\}_{t=0}^\infty$ , is  $\mathcal{A}$ - (and  $\mathcal{P}$ -) efficient.*

**Proof.** This follows from Theorems 1 and 2 in Golosov, Jones, and Tertilt (2007). ■

**Proposition 11** *If  $\lambda_{s+1} > 0$  for some generation  $s$ , then the equilibrium allocation,  $\{\hat{c}_t^m, \hat{c}_{t+1}^o, \hat{n}_t, \hat{s}_{t+1}, \hat{k}_t, \hat{b}_{t+1}\}_{t=0}^\infty$  is  $\mathcal{A}$ - (and  $\mathcal{P}$ -) inefficient.<sup>56</sup>*

**Proof.** Here we provide the outline of the proof. Details are available in Appendix A.6. We show that the equilibrium allocation is  $\mathcal{A}$  (and  $\mathcal{P}$ )-inefficient by constructing an alternative,  $\mathcal{A}$ - (and  $\mathcal{P}$ )-superior allocation in which  $\varepsilon$  newborns are added. Consider the alternative allocation in which generation  $s$ , is allocated the following tilde consumption path, savings and number of children:

$$\begin{aligned} \tilde{c}_s^m &= \hat{c}_s^m - \theta\varepsilon & \tilde{n}_s &= \hat{n}_s + \varepsilon \\ \tilde{c}_{s+1}^o &= \hat{c}_{s+1}^o + M\varepsilon & \tilde{s}_{s+1} &= \hat{s}_{s+1} \end{aligned}$$

<sup>56</sup>Notice that if  $\zeta = 0$  (as in Eckstein and Wolpin (1985) or Conde-Ruiz, Giménez, and Pérez-Nievas (2004)), any  $\underline{b}_t > w_t^m$  is binding. This proposition implies that the laissez-faire equilibrium allocation in this case is always  $\mathcal{A}$ - (and  $\mathcal{P}$ -) inefficient.

The  $\varepsilon$  mass of newborn children receives:<sup>57</sup>

$$\begin{aligned}\tilde{c}_n^m &= \frac{F(\tilde{s}_{s+1}, \tilde{n}_s) - F(\hat{s}_{s+1}, \hat{n}_s)}{\varepsilon} - \hat{s}_{s+2} - \theta_{s+2} \hat{n}_{s+1} - M \\ \tilde{c}_n^o &= \hat{c}_{s+2}^o \quad \tilde{n}_n = \hat{n}_{s+1} \quad \tilde{s}_n = \hat{s}_{s+2}\end{aligned}$$

While everyone else receives the same as in the hat equilibrium allocation. That is  $\forall t \neq s$ :

$$\begin{aligned}\tilde{c}_t^m &= \hat{c}_t^m & \tilde{n}_t &= \hat{n}_t \\ \tilde{c}_{t+1}^o &= \hat{c}_{t+1}^o & \tilde{s}_{t+1} &= \hat{s}_{t+1}\end{aligned}$$

That is, we are giving the newborns an equal fraction of the extra output they produce and take  $M$  away from each child when middle-aged to give to the parent when old in period  $s + 1$ . The newborns are otherwise treated equally to their siblings. Let  $\hat{U}_s$  and  $\tilde{U}_s(\varepsilon, M)$  be the utility of the parent under the equilibrium and alternative allocation, respectively. Notice that  $\lim_{\varepsilon \rightarrow 0} \tilde{U}_s(\varepsilon, M) = \hat{U}_s$ . As usual, the proof proceeds in two steps:

1. The alternative tilde allocation is feasible.
2. There exist  $\varepsilon > 0$  and a feasible  $M$  such that generation  $s$  is strictly better off while everyone else, alive in both allocations is just as well off.

To show this, we show that

$$\lim_{\varepsilon \rightarrow 0} \frac{\partial \tilde{U}_s(\varepsilon, M)}{\partial \varepsilon} \Leftrightarrow \beta u'(\hat{c}^o) \hat{n} > \zeta u'(\hat{c}^m).$$

From equation (52), this is true whenever  $\lambda_{b,t+1} > 0$ , i.e. the parent is constrained. By continuity if  $u(\cdot)$ , we get the result.

This establishes that the alternative tilde allocation is  $\mathcal{A}$ -superior to the equilibrium hat allocation. Further, since  $\lim_{\varepsilon \rightarrow 0} \frac{F(\tilde{K}, \tilde{n}_p) - F(\hat{K}, \hat{n})}{\varepsilon} = F_L(\hat{K}, \hat{n})$ , the  $\varepsilon$  children will be better off if

$$u(F_L(\hat{K}, \hat{n}) - \hat{s} - \theta \hat{n} - M) > u(\text{unborn}).$$

Assuming this, the proposed tilde allocation is also  $\mathcal{P}$ -superior to the hat equilibrium allocation. ■

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<sup>57</sup>Note that we are treating all additional children the same in this allocation.

## 6.5 Characterizing steady states

Throughout this subsection we assume  $u(\cdot) = \log(\cdot)$  and  $F(K_t, \gamma_A^t L_t) = AK_t^\alpha (\gamma_A^t L_t)^{1-\alpha}$ , with  $\gamma_A = 1$  and  $\alpha \in (0, 1)$ . We then analyze steady states. That is,  $\underline{b}_t$ 's are such that, either all generations are unconstrained or all are constrained.

First, assume  $\underline{b}_t = \underline{b} = -\infty$ . Then, after some algebra<sup>58</sup>, the steady state allocation is given by:

$$k^* = \frac{\alpha\theta(\beta + \zeta(1 + \beta + \gamma))}{\beta(1 - \alpha) + \gamma - \alpha\zeta(1 + \gamma + \beta)} \quad (53)$$

$$n^* = \zeta A \alpha \left( \frac{\beta(1 - \alpha) + \gamma - \alpha\zeta(1 + \gamma + \beta)}{\alpha\theta(\beta + \zeta(1 + \beta + \gamma))} \right)^{1-\alpha} \quad (54)$$

$$b^* = \frac{A(k^*)^{\alpha-1} [\zeta\theta\alpha(1 + \beta + \gamma) - (1 - \alpha)k^*\gamma]}{(\gamma - \zeta(1 + \gamma + \beta))} \quad (55)$$

For the problem to be well defined when  $\underline{b} = -\infty$ , we need  $n^* > 0$  and utility to be finite in equilibrium. Let  $U^{*ss}$  be the utility from equation (40) in steady state. That is,

$$U^{*ss} = (\log c^{m^*} + \beta \log c^{o^*} + \gamma \log n^*) \sum_{t=0}^{\infty} \zeta^t.$$

Also, to make comparisons to other OLG results readily comparable, we often use the special case where  $\underline{b} = 0$  and all generations are constrained in steady state below. That is,  $b^* < 0$ . Hence the following lemma:

**Lemma 12** *Suppose  $\underline{b} = -\infty$ . Then*

$$n^* \in (0, \infty) \quad \Leftrightarrow \quad \gamma > \zeta(1 + \beta + \gamma); \quad (56)$$

$$|U^{*ss}| < \infty \quad \Leftrightarrow \quad \zeta < 1; \quad (57)$$

$$b^* < 0 \quad \Leftrightarrow \quad \beta(1 - \alpha) > \alpha\zeta(1 + \gamma + \beta). \quad (58)$$

**Proof.** For the individual household not to borrow an infinite amount to produce an infinite number of children and achieve infinite utility, we need  $r^*\theta > w^*$ . Using the first-order conditions for the firm and the solution for  $k^*$ , this inequality leads to  $\gamma > \zeta(1 + \beta + \gamma)$ . This also ensures  $n^* > 0$  from equation (54).  $|U^{*ss}| < \infty$  is true if  $\sum_{t=0}^{\infty} \zeta^t$  converges, i.e.  $\zeta < 1$ . Finally, from equation (55),  $b^* < 0$  if and only if

$$\zeta\theta\alpha(1 + \beta + \gamma) < \gamma(1 - \alpha)k^*.$$

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<sup>58</sup>Available upon request.

Using equation (53) and condition (56) and rearranging this is true if and only if

$$\alpha\zeta(1 + \beta + \gamma) < \beta(1 - \alpha).^{59}$$

■

Next, assume that  $\underline{b}_t = \underline{b} > b^*$ . Then, after some algebra<sup>60</sup>, the steady state allocation is given by:

$$\hat{k} \text{ solves } \beta\theta\hat{k}^{\alpha-1} + \frac{\beta + \gamma}{A\alpha}\underline{b} = \gamma\hat{k}^\alpha \quad (59)$$

$$\hat{n} = \frac{\gamma(A(1 - \alpha)\hat{k}^\alpha + \underline{b})}{(1 + \beta + \gamma)\left(\theta + \frac{\underline{b}}{A\alpha}\hat{k}^{1-\alpha}\right)} \quad (60)$$

$$\hat{c}^m = \frac{A(1 - \alpha)\hat{k}^\alpha + \underline{b}}{1 + \beta + \gamma} \quad (61)$$

## 6.6 Dynamic efficiency revisited

In this section, we revisit results on dynamic efficiency derived in exogenous fertility overlapping generations (OLG) models first developed in Samuelson (1958) and Diamond (1965) and first extended to allow for parental and/or filial altruism by Barro (1974). We further revisit results derived in OLG models where children are a consumption good but parents are not altruistic first developed by Eckstein and Wolpin (1985) and extended to the Millian efficiency concept by Conde-Ruiz, Giménez, and Pérez-Nievas (2004). We argue that implicit assumptions on property rights are key.

We know from Cass (1972) and Balasko and Shell (1980) that in a standard OLG model the equilibrium allocation is Pareto optimal if and only if  $r > n$ . Our model is two steps away from the standard OLG model. First, we add altruism, and second, fertility is endogenous. Both of these extensions have been analyzed separately before.

In OLG models with altruism but exogenous fertility (DOLG, for dynastic OLG models), the result depends on how exactly the altruism is introduced. The specification most closely related to ours is the one in Burbidge (1983) ( $\gamma = \theta = 0$ ,  $\bar{n}$  given exogenously and  $\zeta = \bar{n}/(1 + \rho) < 1$ , where  $\rho$  is the intergenerational rate of time preference). In his model, either transfers from parents to children or from children to

<sup>59</sup>The condition that  $\beta(1 - \alpha) > \alpha\zeta(1 + \gamma + \beta)$  or  $\frac{1-\alpha}{\alpha} > \frac{\zeta(1+\gamma+\beta)}{\beta}$  can be interpreted as: parents want to take resources from children,  $b^* < 0$ , if the labor share in output,  $(1 - \alpha)$ , is sufficiently high and parents value their children's utility little enough,  $\zeta(1 + \beta + \gamma)$ , relative to their own consumption when old,  $\beta$ .

<sup>60</sup>Available upon request

parents are always operative. This model is equivalent to one in which parents have all property rights and are free to leave positive or negative bequests/transfers. It differs from ours only because fertility is not endogenous. In Burbidge (1983)'s setting,  $r = (1 + \rho)$  in equilibrium and, since  $\zeta < 1$ ,  $r > \bar{n}$  always holds. Hence, the inefficiency pointed out in standard OLG models never occurs—under the implicit assumption that property rights lie with parents. In an endowment economy, Pazner and Razin (1980) have shown that Samuelson (1958)'s inefficiency disappears in a DOLG model with endogenous fertility—assuming all property rights parents. We extend this result to an economy with production and use the efficiency criteria in Golosov, Jones, and Tertilt (2007). Extended to our model, we get:

**Proposition 13** *If the economy is  $\mathcal{A}$ -efficient then  $r^* > n^*$ .*

**Proof.** Using equations (53) and (54) and recalling that  $r^* = A\alpha k^{*1-\alpha}$ , we get

$$n^* = \zeta r^*. \quad (62)$$

As shown in condition (57),  $\zeta < 1$ . ■ Since this steady state allocation is  $\mathcal{A}$ -efficient if and only if  $b^* > \underline{b}$ , that is, property rights lie with parents, this assumption is crucial.

On the reverse side, recall that in models without altruism, bequests or transfers have to be restricted because parents would always take everything they feasibly or legally can from children. The typical assumption is  $\underline{b} = 0$ . Now consider an OLG model (without altruism  $\zeta = 0$ ) where fertility is chosen and children are a consumption good,  $\gamma > 0$ ,  $\theta > 0$ , such as the one in Eckstein and Wolpin (1985) and Conde-Ruiz, Giménez, and Pérez-Nievas (2004). In these models, the property rights constraint is always binding and the laissez-faire economy never leads to an  $\mathcal{A}$ -efficient allocation.<sup>61</sup> However, as shown in Conde-Ruiz, Giménez, and Pérez-Nievas (2004) it may very well be that  $r > n$  in equilibrium. Hence, the original OLG criterion is no longer a sufficient condition for (Millian) efficiency. In line with Conde-Ruiz, Giménez, and Pérez-Nievas (2004),  $r > b$ , is also not a sufficient condition for  $\mathcal{A}$ -efficiency.

For simplicity and comparability, consider the special case where  $b^* < \underline{b} = 0$ . This holds under condition (58). Then we have:

**Proposition 14** *If  $b^* < \underline{b} = 0$ , then  $\hat{r} > \hat{n}$  if and only if*

$$\alpha(1 + \beta + \gamma) > (1 - \alpha)\beta. \quad (63)$$

*But, the equilibrium allocation is  $\mathcal{A}$ - and  $\mathcal{P}$ -inefficient independent of condition (63).*

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<sup>61</sup>It may lead to a Millian efficient allocation where unequal treatment of people from the same generation is ruled out.

**Proof.**  $\hat{r} > \hat{n} \Leftrightarrow \alpha \hat{k}^{\alpha-1} > \frac{\gamma(1-\alpha)\hat{k}^\alpha}{\theta(1+\beta+\gamma)}$   
 $\Leftrightarrow \alpha\theta(1+\beta+\gamma) > \gamma(1-\alpha)\hat{k}$

Using  $\hat{k}$  in equation (59) with  $\underline{b} = 0$ ,

$$\Leftrightarrow \alpha(1+\beta+\gamma) > (1-\alpha)\beta.$$

This is compatible with the condition that  $\underline{b} = 0$  is binding in (58) if  $\zeta < 1$  which is assured by condition (57). The fact that the equilibrium allocation is  $\mathcal{A}$ - and  $\mathcal{P}$ -inefficient independently on condition (63) follows directly from Proposition 11. ■

Proposition 14 says that, the economy may look efficient from a standard OLG point of view because  $r > n$ . But as we have shown in Proposition 11, the equilibrium is not  $\mathcal{A}$ -efficient. The reason is that, due to misaligned property rights, the birth rate is too low. That is, compared to standard OLG models where only capital is a variable factor and the potential for over-saving may lead to dynamic inefficiency, here labor is also endogenous and  $r > n$  may not guarantee efficiency since it may well be the case that not enough resources are invested into producing future labor supply.

Some authors (e.g. Abel, Mankiw, Summers, and Zeckhauser (1989)) use the  $r > n$  criterion together with data to argue that the United States and other major OECD nations are in fact dynamically efficient. We argue that they may look efficient from an exogenous fertility model's point of view but they may not be  $\mathcal{A}$ - (and  $\mathcal{P}$ -) efficient when endogenous fertility is taken into account—with or without altruism. The underlying reason are misaligned property rights!

Below, we analyze how policy implications differ across the above-mentioned models and ours.

## 6.7 Policy Implications

Even when children's rights lead to inefficiently low fertility, these rights might still be desirable. In this subsection, we plan to explore different government policies to solve the inefficiency created through children's rights. We consider issuing debt and PAYG social security. We expect to find that both of these need to depend on fertility in particular ways, not addressed in the literature so far.

### 6.7.1 Issuing debt

Here we plan to revisit Barro (1974), Carmichael (1982) and, in particular, Burbidge (1983)'s main result that, as long as current generations are connected to future gen-

erations through operative intergenerational transfers (either from parents to children or vice versa), Ricardian equivalence holds. We abstract from filial altruism and show that issuing debt which has to be repaid by descendants, increases the transfer from parents to children. As in the exogenous fertility DOLG models, issuing debt tends to make bequests or transfers operational when parents are originally constrained by property rights. However, we expect that the tax burden on future generations should be designed in such a way that having more children does not increase the tax burden of the dynasty to avoid distortions in fertility choice. Notice that such a policy has no effect in models without altruism—with or without endogenous fertility—because transfers never become operative.

### 6.7.2 PAYG Social Security

Eckstein and Wolpin (1985), in an OLG model where children are purely a consumption good ( $\zeta = 0$  and  $\underline{b} = 0$ ), show that a voluntary social security system with transfers depending on number of children is needed to lead their competitive allocation to be identical to the allocation corresponding to the maximum of the steady state utility of the representative individual. People would participate because it basically allows them to recover the extra cost per child. Conde-Ruiz, Giménez, and Pérez-Nievas (2004) derive a similar result where the dynamic (Millian) inefficiency can be resolved with a fertility dependent social security system. We expect that similar results to those in Section 5 go through. What we add to the previous literature is that in our setup we can precisely pinpoint the source of the inefficiency: property rights. In addition, parents are altruistic in our model which leads to different implications regarding the set of policies that lead to an  $\mathcal{A}$ -efficient allocation.

## 7 Conclusion

In this paper we analyze the consequences of shifts in property rights over children. We start by documenting that in most developed countries laws were implemented in the last two centuries resulting in an effective reallocation of property rights from parents to children. We show that when the laws are binding (i.e. when parents would prefer to take more resources from their children than they are allowed by law), fertility is lower than when they are not. More generally, equilibrium fertility decreases in the strength of the intergenerational transfer constraint. This mechanism may have

contributed to the historical fertility decline observed in most countries. We further show that when children own themselves, the costs and benefits of having children are not aligned, which can lead to inefficiently low fertility. We show that these results are robust to a variety of different model specifications. We also show how property rights over children interact with other intergenerational policies such as a standard PAYG social security system. As long as parents are constrained, a social security system increases fertility because of the income effect. However, a social security system means also that parents are richer relative to their children and thus eventually the constraint may stop binding, in which case fertility starts falling. Within this context, we show that a standard PAYG system will not lead to an  $A$ -efficient allocation. The reason for this is that, even though taxes when middle-aged are lump-sum to children, they are distortionary from the parent's point of view. We therefore analyze alternative pension systems, in particular one where pension payments are a function of fertility choices. The latter system is able to implement an  $A$ -efficient allocation. Of course such a policy might be problematic when fertility has a stochastic component. Potentially such a system would need to make provisions for involuntary infertility.

At this point, the focus of our work has been on the theoretical properties of fertility models and the characterization of equilibria with and without parental control rights. We believe, however, that the ideas presented in this paper may have something to say about the historical fertility experience in the United States. Assuming that property rights rested safely with parents until the beginning of the 19th century, high fertility would follow naturally. During the 19th and early 20th century, laws were introduced that gave more rights to children, which coincided with the fertility decline. Assuming these laws are binding, stricter laws generate a fertility decline in our theoretical framework. In 1937 the U.S. government introduced a PAYG social security system. Assuming again that children's rights laws were binding at the time, a PAYG system leads to an increase in fertility. So perhaps the introduction of PAYG was partially responsible for the baby boom. As the social security system was expanded, at some point, constraints stop binding and parents want to give more resources to their children and have fewer of them. This logic may explain the baby bust after 1960. Note also that this logic is consistent with Caldwell (1978)'s reversal of net transfers between parents and children. That is, a combination of laws (restrictions on transfers together with a social security system) lead from a state of the world where transfers run from children to parents to a world where private transfers run from parents to children. Whether such a logic indeed played a role for the U.S. fertility experience and whether

it was quantitatively important will be analyzed in future research.

The set-up could easily be extended to analyze many other interesting applications. For example, constraints on bequests are likely to be binding only for some families. Introducing heterogeneity and analyzing the impact of legal changes on differential fertility would clearly be very interesting. Also, it might be interesting to explicitly model the distinction of transfers from minor vs. adult children to parents. For example, child labor laws were concerned with minor children, while the English poor laws were concerned with adult children. Explicitly introducing this distinction into the theoretical set-up may further shed light on the importance of such laws.

An important open question is why laws shifting property rights from parents to children were introduced. This is an interesting political economy question. The answer may be similar to previous analyses of child labor laws. Some people may have benefited while others would have preferred to remain in a world without constraints. Once enough people are in favor of these legal changes, children's rights laws were implemented. It is not clear, however, exactly why people would benefit from imposing constraints on *other* families. This is different from child labor laws where relative wages of unskilled labor rise in response to a ban on child labor, which benefitted unskilled parents (see for example, Doepke and Zilibotti (2005)). It is not exactly clear what the analogue is in the case of laws that restrict parental control rights over children. However, given that these laws were introduced, it is of interest to analyze who were the proponents and opponents and what led to the implementation of laws that essentially shifted control from parents to children.

# A Appendix

## A.1 Formal definitions of $\mathcal{A}$ - and $\mathcal{P}$ -Efficiency

Since Pareto efficiency is not defined in models with fertility choice, Golosov, Jones, and Tertilt (2007) suggest two generalizations of Pareto efficiency that can handle models with fertility choice. We briefly summarize the concepts here and refer the reader to the original paper for details.

Let  $\mathcal{P}$  be the set of potential people. An allocation  $z = \{z_i\}_{i \in \mathcal{P}}$  specifies  $z_i$  the vector of all the goods over which  $i$ 's utility is defined. Let  $A$  be the set of all possible allocations. Further, let  $A(i)$  be the set of all allocations in which  $i$  is born.

To define  $\mathcal{A}$ -efficiency, the following assumption is needed:

**Assumption 15** For each  $i \in \mathcal{P}$ , there is a well defined, real-valued utility function  $u_i : A(i) \rightarrow \mathbb{R}$ .

**Definition 16** A feasible allocation  $z = \{z_i\}_i$  is  $\mathcal{A}$ -efficient if there is no other feasible allocation  $\hat{z}$  such that

1.  $u_i(\hat{z}) \geq u_i(z) \quad \forall i$  alive in both allocations.
2.  $u_i(\hat{z}) > u_i(z)$  for some  $i$  alive in both allocations.

To define  $\mathcal{P}$ -efficiency, the following assumption is needed:

**Assumption 17** For each  $i \in \mathcal{P}$ , there is a well defined, real-valued utility function  $u_i : A \rightarrow \mathbb{R}$ .

**Definition 18** A feasible allocation  $z = \{(c_i, n_i)\}_i$  is  $\mathcal{P}$ -efficient if there is no other feasible allocation  $\hat{z}$  such that

1.  $u_i(\hat{z}) \geq u_i(z)$  for all  $i \in \mathcal{P}$
2.  $u_i(\hat{z}) > u_i(z)$  for at least one  $i \in \mathcal{P}$ .

In this paper, we use these concepts for welfare analysis as follows. In Sections 4 and 5 an allocation specifies parent's consumption when middle-aged, when old, number of children as well as consumption of children,  $\{c^m, c^o, n, c^k\}$ . Parents utility is increasing in all these goods, while children's utility is increasing only in  $c^k$ .

In Section 6 an allocation specifies consumption when middle-aged, when old and number of children for all generations,  $t = 0$  to  $t = \infty$ ,  $\{c_t^m, c_{t+1}^o, n_t\}_{t=0}^{t=\infty}$ . Generation  $s$ 's utility is increasing in  $\{c_t^m, c_{t+1}^o, n_t\}_{t=s}^{t=\infty}$ . Throughout we assume that  $u_i(\text{unborn}) < u_i(z), z \in A(i)$ .

## A.2 Proof of Proposition 4

### A.2.1 Proof of Proposition 4: partial equilibrium

**Proof.**

1. **The tilde allocation is feasible** (identical to above)

In period 1, we must have:

$$\tilde{c}^m + \theta\tilde{n} + \tilde{s} \leq w^m.$$

Substituting we get:

$$\hat{c}^m - \theta\varepsilon + \theta\hat{n} + \theta\varepsilon + \hat{s} \leq w^m.$$

Since the hat allocation is feasible, this holds. In period 2, we must have:

$$\tilde{c}^o \leq w^o + R\tilde{s} + \varepsilon M.$$

Substituting we get:

$$\hat{c}^o + \varepsilon M \leq w^o + R\hat{s} + \varepsilon M.$$

Since the hat allocation is feasible, this holds. Finally, we need  $M \leq w^k$ .

2.  $U_p(\varepsilon, M)$  **is increasing in  $\varepsilon$  at  $\varepsilon = 0$**

$$\begin{aligned} \tilde{U}_P(\varepsilon, M) &= u(\hat{c}^m - \varepsilon\theta) + \beta u(\hat{c}^o + \varepsilon\theta) + \gamma u(\hat{n} + \varepsilon) + \zeta \left( \frac{\hat{n}u(\hat{c}^k) + \varepsilon u(\hat{c}^n)}{\hat{n} + \varepsilon} \right) \\ \frac{\partial \tilde{U}_P(\varepsilon, M)}{\partial \varepsilon} \Big|_{\varepsilon=0} &= -\theta u'(\hat{c}^m) + \beta M u'(\hat{c}^o) + \gamma u'(\hat{n}) - \zeta \frac{u(w^k) - u(w^k - M)}{n} \end{aligned}$$

Using equations (4) and (5)

$$= M \left[ \beta u'(\hat{c}^o) - \zeta \frac{u(w^k) - u(w^k - M)}{\hat{n}} \right]$$

We need to show that this expression is positive for some  $M$ . Notice that if  $\gamma = 0$ , we know that the expression is positive. Also notice that for  $M = 0$ ,  $\frac{\partial \tilde{U}_P(\varepsilon, M)}{\partial \varepsilon} \Big|_{\varepsilon=0} = 0$ , while for  $M > 0$ , both terms are positive. Thus, we need to show that for some range of  $M > 0$ ,

$$LHS(M) = M \beta u'(\hat{c}^o) > \zeta \frac{u(w^k) - u(w^k - M)}{\hat{n}} = RHS(M).$$

The *LHS* is linearly increasing in  $M$ , while the *RHS* is increasing and convex in  $M$ . Thus, we need to show that the slopes at  $M = 0$  satisfy  $\left. \frac{\partial LHS}{\partial M} \right|_{M=0} > \left. \frac{\partial RHS}{\partial M} \right|_{M=0}$ . Since both are continuous, this is enough to show that they must cross at some  $\bar{M} > 0$  and that, for  $M \in (0, \bar{M})$ ,  $\left. \frac{\partial \hat{U}_{P(\varepsilon, M)}}{\partial \varepsilon} \right|_{\varepsilon=0} > 0$ . So, we need a condition for (every step is “if and only if”):

$$\begin{aligned} \left. \frac{\partial LHS}{\partial M} \right|_{M=0} &> \left. \frac{\partial RHS}{\partial M} \right|_{M=0} \\ \beta u'(\hat{c}^o) &> \zeta \frac{u'(w^k)}{\hat{n}} \end{aligned}$$

This is equivalent to saying that  $\lambda_b > 0$  in equation (6).

Using log utility and replacing  $\hat{n}$  and  $\hat{c}^o$ , from equations (20):

$$\frac{\beta(1 + \beta + \gamma)}{\beta(w^o + R w^m)} > \frac{\zeta R \theta (1 + \beta + \gamma)}{\gamma w^k (w^o + R w^m)}$$

Simplifying and multiplying both sides by  $\gamma w^k$ , we get

$$\gamma w^k > \zeta \theta R$$

This is exactly the condition for  $\underline{b} = 0$  to be binding.

Hence, for  $M \in (0, \bar{M})$ , there exists an  $\varepsilon > 0$  such that the parent is strictly better off. Pre-existing children are indifferent and the  $\varepsilon$  children will be better off if  $u(w^k - M) > u(\text{unborn})$ . Summarizing, we have:

- Parent:  $\hat{U}_{P|\varepsilon>0} > U_P^*$
- Children:  $\hat{U}_i = U_i^* \quad \forall i \in [0, n^*]$
- New children:  $\hat{U}_n = u(w^k - M) > u(\text{unborn})$

Hence, the tilde allocation is  $\mathcal{A}$ - and  $\mathcal{P}$ -superior to the hat allocation. ■

## A.2.2 Proof of Proposition 4: general equilibrium

**Proof.**

### 1. The tilde allocation is feasible (identical to above)

In period 1, we must have:

$$\tilde{c}_m + \theta \tilde{n} + \tilde{K} \leq \omega^m.$$

Substituting we get:

$$\hat{c}^m - \theta\varepsilon + \theta\hat{n} + \theta\varepsilon + \hat{K} \leq w_m.$$

Since the hat allocation is feasible, this holds. In period 2, we must have:

$$\tilde{c}_o + \hat{n}\tilde{c}_k + \varepsilon\tilde{c}_n \leq F(\tilde{K}, \tilde{n}).$$

Substituting we get:

$$\hat{c}^o + \varepsilon M + \hat{n}\hat{c}^k + F(\tilde{K}, \tilde{n}) - F(\hat{K}, \hat{n}) - \varepsilon M \leq F(\tilde{K}, \tilde{n}).$$

$$\hat{c}^o + \varepsilon M + \hat{n}\hat{c}^k - \varepsilon M \leq F(\hat{K}, \hat{n}).$$

Since the hat allocation is feasible, this holds. Finally, we need  $M \leq w_k$ .

2.  $U_p(\varepsilon, M)$  is increasing in  $\varepsilon$  as  $\varepsilon \rightarrow 0$

$$\begin{aligned} \tilde{U}_P(\varepsilon, M) &= u(\hat{c}^m - \varepsilon\theta) + \beta u(\hat{c}^o + \varepsilon M) + \gamma u(\hat{n} + \varepsilon) + \zeta \left( \frac{\hat{n}u(\hat{c}^k) + \varepsilon u(\tilde{c}_n)}{\hat{n} + \varepsilon} \right) \\ \frac{\partial \tilde{U}_P(\varepsilon, M)}{\partial \varepsilon} &= -\theta u'(\hat{c}^m - \varepsilon\theta) + \beta M u'(\hat{c}^o + \varepsilon M) + \gamma u'(\hat{n} + \varepsilon) \\ &\quad + \zeta \frac{(\hat{n} + \varepsilon)(u(\tilde{c}_n) + \varepsilon u'(\tilde{c}_n) \frac{\partial \tilde{c}_n}{\partial \varepsilon}) - \hat{n}u(\hat{c}^k) - \varepsilon u(\tilde{c}_n)}{(\hat{n} + \varepsilon)^2} \\ &= -\theta u'(\hat{c}^m - \varepsilon\theta) + \beta M u'(\hat{c}^o + \varepsilon M) + \gamma u'(\hat{n} + \varepsilon) \\ &\quad + \zeta \frac{\varepsilon u'(\tilde{c}_n) \frac{\partial \tilde{c}_n}{\partial \varepsilon}}{(\hat{n} + \varepsilon)} - \zeta \frac{\hat{n}}{(\hat{n} + \varepsilon)^2} (u(\hat{c}^k) - u(\tilde{c}_n)) \\ &= -\theta u'(\hat{c}^m - \varepsilon\theta) + \beta M u'(\hat{c}^o + \varepsilon M) + \gamma u'(\hat{n} + \varepsilon) \\ &\quad + \zeta \frac{u'(\tilde{c}_n)}{(\hat{n} + \varepsilon)} \left( \frac{\varepsilon F_L(\hat{K}, \hat{n} + \varepsilon) - F(\hat{K}, \hat{n} + \varepsilon) + F(\hat{K}, \hat{n})}{\varepsilon} \right) \\ &\quad - \zeta \frac{\hat{n}}{(\hat{n} + \varepsilon)^2} (u(\hat{c}^k) - u(\tilde{c}_n)) \\ \lim_{\varepsilon \rightarrow 0} \frac{\partial \hat{U}_P(\varepsilon, M)}{\partial \varepsilon} &= -\theta u'(\hat{c}^m) + \beta M u'(\hat{c}^o) + \gamma u'(\hat{n}) \\ &\quad - \zeta \frac{u(F_L(\hat{K}, \hat{n})) - u(F_L(\hat{K}, \hat{n}) - M)}{\hat{n}} \end{aligned}$$

Using equations (4) and (5)

$$= M\beta u'(\hat{c}^o) - \zeta \frac{u(F_L(\hat{K}, \hat{n})) - u(F_L(\hat{K}, \hat{n}) - M)}{\hat{n}}$$

We need to show that this expression is positive for some  $M > 0$ . Notice that if  $\zeta = 0$ , we know that the expression is positive. Also notice that for  $M > 0$ , both terms are positive. Thus, we need to show that for some range of  $M > 0$ ,

$$LHS(M) = M\beta u'(\hat{c}^o) > \zeta \frac{u(F_L(\hat{K}, \hat{n})) - u(F_L(\hat{K}, \hat{n}) - M)}{\hat{n}} = RHS(M).$$

The  $LHS$  is linearly increasing in  $M$ , while the  $RHS$  is increasing and convex in  $M$ . Thus, we need to show that the slopes at  $M = 0$  satisfy  $\left. \frac{\partial LHS}{\partial M} \right|_{M=0} > \left. \frac{\partial RHS}{\partial M} \right|_{M=0}$ . Since both are continuous, this is enough to show that they must cross at some  $\bar{M} > 0$  and that, for  $M \in (0, \bar{M})$ ,  $\lim_{\varepsilon \rightarrow 0} \frac{\partial \hat{U}_P(\varepsilon, M)}{\partial \varepsilon} > 0$ . So, we need a condition for (every step is “if and only if”):

$$\begin{aligned} \left. \frac{\partial LHS}{\partial M} \right|_{M=0} &> \left. \frac{\partial RHS}{\partial M} \right|_{M=0} \\ \beta u'(\hat{c}^o) &> \zeta \frac{u'(\hat{c}^k)}{\hat{n}} \end{aligned}$$

This is true whenever  $\lambda_b > 0$ , i.e. the parent is constrained, because from the equation (6) we can see that  $\lambda_b = \beta u'(\hat{c}^o)\hat{n} - \zeta u'(\hat{c}^k)$ . Notice also that  $\bar{M} < F_L(\hat{K}, \hat{n})$  if  $\beta u'(\hat{c}^o) < \zeta \frac{u(F_L(\hat{K}, \hat{n})) - u(0)}{\hat{n}}$ . In particular, if  $u(0) = -\infty$  because then  $RHS$  asymptotes toward  $+\infty$  from the left at  $M \rightarrow F_L(\hat{K}, \hat{n})$  (e.g. log utility)

Hence, for  $M \in (0, \max\{F_L(\hat{K}, \hat{n}), \bar{M}\})$ , there exists an  $\varepsilon > 0$  such that the parent is strictly better off. Pre-existing children are indifferent and the  $\varepsilon$  children will be better off if  $u(F_L(\hat{K}, \hat{n}) - M) > u(\text{unborn})$ . Summarizing, we have:

- Parent:  $\hat{U}_{P|\varepsilon>0} > U_P^*$
- Children:  $\hat{U}_i = U_i^* \quad \forall i \in [0, n^*]$
- New children:  $\hat{U}_n = u(F_L(\hat{K}, \hat{n}) - M) > u(\text{unborn})$

Hence, the tilde allocation is  $\mathcal{A}$ - and  $\mathcal{P}$ -superior to the hat allocation. ■

### A.3 Proof of Proposition 5

**Proof.** Combining all three budget constraints into one gives:

$$c_o + n[c_k - w_k + \tau] = R[w^m - c_m - \theta n] + T$$

Substituting in for  $c^k$  and  $c^m$  from equations (6) and (5) and using  $\lambda_b = 0$ , this becomes:

$$c_o \left( 1 + \frac{\zeta}{\beta} + \frac{1}{\beta} \right) - n(w_k - \tau) = R[w^m - \theta n] + T$$

Solving for  $c_o$ :

$$c_o = \beta \frac{Rw^m + n(w_k - \tau - R\theta) + T}{\beta + \zeta + 1} \quad (64)$$

Using equation (4) and substituting for  $b = c^k + \tau - w^k$  gives:

$$c^o = \frac{\beta n [R\theta + \tau - w^k]}{\gamma - \zeta} \quad (65)$$

Combining (64) and (65), can solve for  $n^{*\tau}$  in equation (26). Using this solution in (65), we find  $c^{o*\tau}$  and  $c^{k*\tau}$  through equation 6. Given that,  $b^{*\tau}$  is determined through the child's budget constraint to find equation (27). From those the comparative statics are immediate in partial equilibrium.

In general equilibrium, to solve for  $k^{*\tau}$ , we use equations (65), the solution outlined above and the firm's first order conditions in the goods market clearing condition in the second period given by

$$\frac{c^{o*\tau}}{n^{*\tau}} + c^{k*\tau} = (k^{*\tau})^\alpha$$

to find that  $k^{*\tau}$  solves equation (28). The LHS of this equation is increasing in  $k$  while the RHS is decreasing in  $k$ . The RHS shifts up if the social security tax increases. Thus  $k^{*\tau}$  is increasing in  $\tau$ .

Using equation (26) and the firm's first-order conditions we get equation (29). Now, from equation (28), we know that

$$(\beta + \zeta)\tau(k^{*\tau})^{1-\alpha} = [(1 - \alpha)(\beta + \zeta) + (\gamma - \zeta)]k^{*\tau} - (\beta + \zeta)\alpha\theta;$$

and

$$\tau(k^{*\tau})^{1-\alpha} = \frac{[(1 - \alpha)(\beta + \zeta) + (\gamma - \zeta)]}{(\beta + \zeta)} k^{*\tau} - \alpha\theta;$$

Adding those two up, we get

$$(1 + \beta + \zeta)\tau(k^{*\tau})^{1-\alpha} = \frac{(1 + \beta + \zeta)[(1 - \alpha)(\beta + \zeta) + (\gamma - \zeta)]}{(\beta + \zeta)} k^{*\tau} - (1 + \beta + \zeta)\alpha\theta;$$

Thus  $n^{*\tau}$  becomes:

$$n^{*\tau} = \frac{(\beta + \zeta)\alpha\omega^m}{(\beta + \zeta)\alpha\theta + (1 + \alpha(\beta + \zeta))k^{*\tau}};$$

Since  $k^{*\tau}$  is increasing in  $\tau$ ,  $n^{*\tau}$  is decreasing in  $\tau$ .

Similarly, from equation (27) and the firm's first-order conditions, we get equation (30). Computing  $\frac{dk^{*\tau}}{d\tau}$  and  $\frac{db^{*\tau}}{d\tau}$  gives

$$\frac{dk^{*\tau}}{d\tau} = \frac{(\beta + \gamma)(k^{*\tau})^{2-\alpha}}{\alpha[(1 - \alpha)(\beta + \zeta)k^{*\tau} + (\gamma - \zeta)k^{*\tau} + (1 - \alpha)\theta(\beta + \zeta)]} > 0$$

$$\frac{db^{*\tau}}{d\tau} = \frac{\gamma - \alpha(1 - \alpha)(k^{*\tau})^{\alpha-2}(\zeta\theta + \gamma k^{*\tau})\frac{dk^{*\tau}}{d\tau}}{\gamma - \zeta}$$

Now,  $\frac{db^*}{d\tau} > 0$  if and only if

$$\gamma > \alpha(1 - \alpha)k^{\alpha-2}(\zeta\theta + \gamma k)\frac{dk}{d\tau}$$

Plug in the expression for  $\frac{dk^{*\tau}}{d\tau}$  from above:

$$\gamma > \alpha(1 - \alpha)k^{\alpha-2}(\zeta\theta + \gamma k)\frac{(\beta + \gamma)k^{2-\alpha}}{\alpha[(1 - \alpha)(\beta + \zeta)k + (\gamma - \zeta)k + (1 - \alpha)\theta(\beta + \zeta)]}$$

Cross-multiply:

$$\gamma\alpha[(1 - \alpha)(\beta + \zeta)k + (\gamma - \zeta)k + (1 - \alpha)\theta(\beta + \zeta)] > \alpha(1 - \alpha)(\zeta\theta + \gamma k)(\beta + \gamma)$$

Separate terms on the RHS:

$$\gamma\alpha[(1 - \alpha)(\beta + \zeta)k + (\gamma - \zeta)k + (1 - \alpha)\theta(\beta + \zeta)] > \alpha(1 - \alpha)\zeta\theta(\beta + \gamma) + \alpha(1 - \alpha)\gamma k(\beta + \gamma)$$

Combine the first term on the LHS with last term on the RHS:

$$\gamma\alpha[(\gamma - \zeta)k + (1 - \alpha)\theta(\beta + \zeta)] > \alpha(1 - \alpha)\zeta\theta(\beta + \gamma) + \alpha(1 - \alpha)\gamma k(\gamma - \zeta)$$

Combine last term on LHS with first term on RHS

$$\gamma\alpha(\gamma - \zeta)k + \gamma\alpha(1 - \alpha)\theta(\beta + \zeta) - \alpha(1 - \alpha)\zeta\theta(\beta + \gamma) > \alpha(1 - \alpha)\gamma k(\gamma - \zeta)$$

$$\gamma\alpha(\gamma - \zeta)k + (\gamma - \zeta)\alpha(1 - \alpha)\theta\beta > \alpha(1 - \alpha)\gamma k(\gamma - \zeta)$$

Since  $(\gamma - \zeta) > 0$ , this is equivalent to:

$$\gamma\alpha k + \alpha(1 - \alpha)\theta\beta > \alpha(1 - \alpha)\gamma k$$

Simplifying:

$$\alpha(1 - \alpha)\theta\beta > -\alpha^2\gamma k$$

Since the *LHS*  $> 0$  while the *RHS*  $< 0$ , this inequality always holds. Hence desired transfers,  $b^{*\tau}$ , are increasing in  $\tau$ . ■

## A.4 Proof of Propositions 6 and 8

### A.4.1 Preliminaries with general $T(n)$

Here we give an outline of the solution with pensions specified as a general function of  $n$ ,  $T(n)$ . We then look at the case where  $T(n) = T$  so that  $T'(n) = 0$  and  $T(n) = n\tau$  so that  $T'(n) = \tau$ , separately.

Combining the budget constraints when middle-aged and when old to eliminate savings gives:

$$c^o + \underline{b}n = R w^m - R\theta n - R c^m + T(n)$$

and plugging in (49) and using the functional form assumptions gives:

$$\left(\frac{1+\beta}{\beta}\right)c^o + \underline{b}n = R w^m - R\theta n + T(n)$$

solving for  $c^o$  and using government's budget balance:

$$c^o = \left(\frac{\beta}{1+\beta}\right) [R w^m - (R\theta + \underline{b} - \tau)n]$$

Using the functional form assumptions in (34), we have:

$$\gamma c^o = \beta n (R\theta + \underline{b} - T'(n))$$

Plugging in for  $c^o$  from above:

$$\gamma \left(\frac{\beta}{1+\beta}\right) [R w^m - (R\theta + \underline{b} - \tau)n] = \beta n (R\theta + \underline{b} - T'(n)) \quad (66)$$

### A.4.2 Proof of Proposition 6

**Proof.** First, assuming that  $T(n) = T$  so that  $T'(n) = 0$  and using the government's budget balance,  $T = \tau n$  we can solve for  $\hat{n}^\tau$  in equation (31). Clearly, fertility is increasing in  $\tau$  in partial equilibrium. As in the proof of proposition 5, we can also solve for the rest of the allocation.

Next, we solve for equilibrium fertility and capital per worker in general equilibrium. To solve for  $\hat{k}^\tau$ , we use the partial equilibrium solution in the goods market clearing condition in the second period to find that  $\hat{k}^\tau$  is the solution to equation 32. The LHS of this equation is increasing in  $k$  while the RHS is decreasing in  $k$ . The RHS shifts down if  $\tau$  increases. Thus  $\hat{k}^\tau$  is decreasing in  $\tau$ .

Using the firm's first-order conditions in equations (31) to find  $\hat{n}^\tau$  gives equation (33). Thus, we see that the question boils down to whether  $((1 + \beta + \gamma)\underline{b} - \tau\gamma)(\hat{k}^\tau)^{1-\alpha}$  increases or decreases when  $\tau$  increases. But we know from equation (32)

$$\gamma\alpha\hat{k} + \underline{b}\hat{k}^{1-\alpha} = \beta\alpha\theta + ((1 + \beta + \gamma)\underline{b} - \tau\gamma)\hat{k}^{1-\alpha}.$$

Since  $\hat{k}^\tau$  decreases when  $\tau$  increases, the LHS of this equation decreases. Hence the RHS must decrease. That is,  $((1 + \beta + \gamma)\underline{b} - \tau\gamma)(\hat{k}^\tau)^{1-\alpha}$  decreases, which implies that  $\hat{n}^\tau$  increases. ■

#### A.4.3 Proof of Proposition 8

**Proof.** If the parent realizes that payments in the future depend on his fertility choice today, e.g.  $T'(n) = \tau$ , then equation (66) can be solved for  $\hat{n}^{\tau n}$  in equation (35) in partial equilibrium. Clearly, fertility is increasing in  $\tau$  in this case.

In general equilibrium, we can use the feasibility constraint in period two to find that  $\hat{k}^{\tau n}$  solves equation (36). The LHS is decreasing in  $k$  and shifts down if  $\tau$  increases, while the RHS is increasing in  $k$ . Hence,  $\hat{k}^{\tau n}$  is decreasing in  $\tau$ .

Using the firms first-order conditions in equation (35) leads to equation (37). We know from equation (36) that

$$\beta\alpha\theta + (\beta + \gamma)(\underline{b} - \tau)(\hat{k}^{\tau n})^{1-\alpha} = \gamma\alpha\hat{k}^{\tau n}.$$

Since  $\hat{k}^{\tau n}$  is decreasing in  $\tau$ , the LHS decreases, thus  $(\underline{b} - \tau)(\hat{k}^{\tau n})^{1-\alpha}$  must also decrease. This implies that  $\hat{n}^{\tau n}$  increases with  $\tau$ .

Hence, even this FDPAYG system increases fertility when the parents are constrained, both in partial and general equilibrium. ■

#### A.5 Proof of Proposition 9

**Proof.** First, consider generation  $(s - 1)$ 's decision. As long as  $\lambda_s > 0$ , we have  $\hat{b}_s = \underline{b}_s$  and hence, a marginal change in  $\underline{b}_{s+1}$  will not change the transfer decision of generation  $(s - 1)$ . Note also that  $w_s = F_L(k_s, 1)$  does not respond to changes in  $\underline{b}_{s+1}$  because  $k_s$  is given.

Second, consider generation  $s$ 's decisions. Consider the partial equilibrium case first, where  $\hat{r}_{s+1} = \bar{r}_{s+1}$  is fixed. From equations (41) and (42), we get

$$\gamma u'(\hat{n}_s) = \beta u'(\hat{c}_{s+1}^o)(\bar{r}_{s+1}\theta_s + \underline{b}_{s+1})$$

Using the budget constraints in equations (44) and (45) to substitute out  $\hat{c}_{s+1}^o$  and  $\hat{s}_{s+1}$ , and inverting equation (50) to substitute out  $\hat{c}_s^m = u'^{-1}\left(\frac{\gamma\bar{r}_{s+1}u'(\hat{n}_s)}{\theta_s\bar{r}_{s+1} + \underline{b}_{s+1}}\right)$ , this equation becomes

$$\gamma u'(\hat{n}_s) = \beta u' \left( \bar{r}_{s+1} \left[ \bar{w}_s + \underline{b}_s - u'^{-1} \left( \frac{\gamma\bar{r}_{s+1}u'(\hat{n}_s)}{\theta_s\bar{r}_{s+1} + \underline{b}_{s+1}} \right) \right] - (\theta_s\bar{r}_{s+1} + \underline{b}_{s+1}) \hat{n}_s \right) (\bar{r}_{s+1}\theta_s + \underline{b}_{s+1}). \quad (67)$$

Note that the only endogenous variable in this equation is  $n_s$ . Since utility is assumed to be strictly concave, both  $u'(\cdot)$  and  $u'^{-1}(\cdot)$  are decreasing functions. The RHS of this equation is increasing in  $\underline{b}_{s+1}$ , holding  $\hat{n}_s$  fixed. It is also increasing in  $\hat{n}_s$  holding  $\underline{b}_{s+1}$  fixed, while the LHS is decreasing in  $\hat{n}_s$ . Hence, holding  $\bar{r}_{s+1}$  fixed,  $\hat{n}_s$  must decrease in response to an increase in  $\underline{b}_{s+1}$ .

Next, consider the general equilibrium case with CES utility. Then from equation (51) we get:

$$\hat{c}_{s+1}^o = \left( \frac{\beta(\theta_s F_K(\hat{k}_{s+1}, 1) + \underline{b}_{s+1})}{\gamma} \right)^{\frac{1}{\sigma}} \hat{n}_s$$

Using this, and capital market clearing in the budget constraint when old, equation (45), we get

$$\left( \frac{\beta(\theta_s F_K(\hat{k}_{s+1}, 1) + \underline{b}_{s+1})}{\gamma} \right)^{\frac{1}{\sigma}} + \underline{b}_{s+1} = F_K(\hat{k}_{s+1}, 1)\hat{k}_{s+1} \quad (68)$$

Note that the only endogenous variable in this equation is  $k_{s+1}$ . The RHS of this equation, the total return to capital, is increasing in  $\hat{k}_{s+1}$ , while the LHS is decreasing in  $\hat{k}_{s+1}$  and shifts up when  $\underline{b}_{s+1}$  increases. Hence, the capital stock per worker next period,  $\hat{k}_{s+1}$ , increases when  $\underline{b}_{s+1}$  increases. This implies that the interest rate,  $\hat{r}_{s+1} = F_K(\hat{k}_{s+1}, 1)$ , decreases when  $\underline{b}_{s+1}$  increases.

Using log utility, equation (67) becomes:

$$\hat{n}_s = \frac{\gamma(\hat{w}_s + \underline{b}_s)}{(1 + \beta + \gamma) \left( \theta_s + \frac{\underline{b}_{s+1}}{F_k(\hat{k}_{s+1}, 1)} \right)} \quad (69)$$

Thus, the question boils down to whether  $\left( \frac{\underline{b}_{s+1}}{F_k(\hat{k}_{s+1}, 1)} \right)$  increases or decreases when  $\underline{b}_{s+1}$  increases. To see that  $\frac{d \frac{\underline{b}_{s+1}}{F_k(\hat{k}_{s+1}, 1)}}{d \underline{b}_{s+1}} > 0$  in the log case, rewrite equation (68) as

$$\beta\theta_s + (\beta + \gamma) \frac{\underline{b}_{s+1}}{F_k(\hat{k}_{s+1}, 1)} = \gamma\hat{k}_{s+1}$$

Since the RHS of this equation increases when  $\underline{b}_{s+1}$  increases, the LHS must increase as well. Hence,  $\frac{d}{db_{s+1}} \frac{\frac{b_{s+1}}{F_k(k_{s+1}, 1)}}{db_{s+1}} > 0$ , and therefore  $\left. \frac{d\hat{n}_s}{db_{s+1}} \right|_{\sigma=1} < 0$ . ■

## A.6 Proof of Proposition 11

**Proof.** We show that the equilibrium allocation is  $\mathcal{A}$  (and  $\mathcal{P}$ )-inefficient by constructing an alternative,  $\mathcal{A}$ - (and  $\mathcal{P}$ )-superior allocation in which  $\varepsilon$  newborns are added. This allocation is denoted with tilde and given in the proof of proposition 11 in the text. As usual, the proof proceeds in two steps:

### 1. Claim 1: The alternative allocation is feasible

First, we need to choose  $M$  such that  $M \in \left( -\underline{b}_{s+1}, \frac{F(\tilde{s}_{s+1}, \tilde{n}_s) - F(\hat{s}_{s+1}, \hat{n}_s)}{\varepsilon} - \hat{s}_{s+2} - \theta_{s+1}\hat{n}_{s+1} \right]$ . Such an  $M$  exists for  $\varepsilon$  small enough. To see this, note that

$$\lim_{\varepsilon \rightarrow 0} \tilde{c}_n^m(\varepsilon) = F_L(\hat{s}_{s+1}, \hat{n}_s) - \hat{s}_{s+2} - \theta_{s+1}\hat{n}_{s+1} - M$$

and since  $\tilde{c}^m > 0$ , we know  $F_L(\hat{s}_{s+1}, \hat{n}_s) - \hat{s}_{s+2} - \theta_{s+1}\hat{n}_{s+1} > -\underline{b}_{s+1}$ .

Second, in the period where the parent is middle-aged,  $s$ , we must have:

$$\tilde{c}_s^o + \tilde{n}_{s-1}(\tilde{c}_s^m + \theta_s \tilde{n}_s + \tilde{s}_{s+1}) \leq F(\tilde{s}_s, \tilde{n}_{s-1}).$$

Substituting in from the allocation above, we get:

$$\hat{c}_s^o + \hat{n}_{s-1}(\hat{c}_s^m - \theta_s \varepsilon + \theta_s(\hat{n}_s + \varepsilon) + \hat{s}_{s+1}) \leq F(\hat{s}_s, \hat{n}_{s-1}).$$

$$\hat{c}_s^o + \hat{n}_{s-1}(\hat{c}_s^m + \theta_s \hat{n}_s + \hat{s}_{s+1}) \leq F(\hat{s}_s, \hat{n}_{s-1}).$$

Since the hat allocation is feasible, this holds.

Third, in the period where the parent is old,  $s + 1$ , while his children, including the newborns  $\varepsilon$  are middle-aged, we must have:

$$\tilde{c}_{s+1}^o + \hat{n}_s \tilde{c}_{s+1}^m + \varepsilon \tilde{c}_n^m + \hat{n}_s \theta_{s+1} \tilde{n}_{s+1} + \varepsilon \theta_{s+1} \tilde{n}_n + \hat{n}_s \tilde{s}_{s+2} + \varepsilon \tilde{s}_n \leq F(\tilde{s}_{s+1}, \tilde{n}_s).$$

Substituting we get:

$$\hat{c}_{s+1}^o + M\varepsilon + \hat{n}_s \hat{c}_{s+1}^m + \varepsilon \left[ \frac{F(\tilde{s}_{s+1}, \tilde{n}_s) - F(\hat{s}_{s+1}, \hat{n}_s)}{\varepsilon} - \hat{s}_{s+2} - \theta_{s+1}\hat{n}_{s+1} - M \right]$$

$$+ (\hat{n}_s + \varepsilon)\theta_{s+1}\hat{n}_{s+1} + (\hat{n}_s + \varepsilon)\hat{s}_{s+2} \leq F(\hat{s}_{s+1}, \hat{n}_s + \varepsilon).$$

$$\hat{c}_{s+1}^o + \hat{n}_s \hat{c}_{s+1}^m + F(\hat{s}_{s+1}, \hat{n}_s + \varepsilon) - F(\hat{s}_{s+1}, \hat{n}_s) + \hat{n}_s \theta_{s+1} \hat{n}_{s+1} + \hat{n}_s \hat{s}_{s+2} \leq F(\hat{s}_{s+1}, \hat{n}_s + \varepsilon).$$

$$\hat{c}_{s+1}^o + \hat{n}_s(\hat{c}_{s+1}^m + \theta_{s+1}\hat{n}_{s+1} + \hat{s}_{s+2}) \leq F(\hat{s}_{s+1}, \hat{n}_s).$$

Since the hat allocation is feasible, this holds. In all other periods the feasibility constraint (per old person) is unaltered.

2. **Claim 2: There exist  $\varepsilon > 0$  and a feasible  $M$  such that generation  $s$  is strictly better off.** We will show this by first showing that there exists

$$\bar{M} \in \left( -\underline{b}, \frac{F(\hat{s}_{s+1}, \hat{n}_s) - F(\hat{s}_{s+1}, \hat{n}_s)}{\varepsilon} - \hat{s}_{s+2} - \theta_{s+1}\hat{n}_{s+1} \right) \text{ that solves}$$

$$(\bar{M} + \underline{b}_{s+1})\beta\hat{n}_s u'(\hat{c}_{s+1}^o) = \zeta \left( u(\hat{c}_{s+1}^m) - u(\hat{c}_{s+1}^m - (\bar{M} + \underline{b}_{s+1})) \right),$$

such that if  $M \in (-\underline{b}, \bar{M})$ , then  $\lim_{\varepsilon \rightarrow 0} \frac{\partial \tilde{U}_s(\varepsilon, M)}{\partial \varepsilon} > 0$ . Then, by continuity of  $U$  the claim is true.

To show this, first notice that:

$$\begin{aligned} \tilde{U}_t &= \hat{U}_t \quad \forall t \neq s \\ \tilde{U}_n(\varepsilon) &= \hat{U}_{s+1} - u(\hat{c}_{s+1}^m) + u(\hat{c}_n^m(\varepsilon)) \\ \tilde{U}_s(\varepsilon, M) &= u(\hat{c}_s^m - \theta_s \varepsilon) + \beta u(\hat{c}_{s+1}^o + \varepsilon M) + \gamma u(\hat{n}_s + \varepsilon) + \zeta \left( \frac{\hat{n}_s \tilde{U}_{s+1} + \varepsilon \tilde{U}_n(\varepsilon)}{\hat{n}_s + \varepsilon} \right) \\ \tilde{U}_s(\varepsilon, M) &= u(\hat{c}_s^m - \theta_s \varepsilon) + \beta u(\hat{c}_{s+1}^o + \varepsilon M) + \gamma u(\hat{n}_s + \varepsilon) \\ &\quad + \zeta \left( \frac{\varepsilon [u(\hat{c}_n^m(\varepsilon)) - u(\hat{c}_{s+1}^m)]}{\hat{n}_s + \varepsilon} \right) + \zeta \hat{U}_{s+1} \\ \frac{\partial \tilde{c}_n^m(\varepsilon)}{\partial \varepsilon} &= \frac{\varepsilon F_L(\hat{s}_{s+1}, \hat{n}_s + \varepsilon) - (F(\hat{s}_{s+1}, \hat{n}_s + \varepsilon) - F(\hat{s}_{s+1}, \hat{n}_s))}{\varepsilon^2} \end{aligned}$$

Hence, we get

$$\begin{aligned}\frac{\partial \tilde{U}_s(\varepsilon, M)}{\partial \varepsilon} &= -\theta u'(\hat{c}_s^m - \theta\varepsilon) + \beta M u'(\hat{c}_{s+1}^o + M\varepsilon) + \gamma u'(\hat{n}_s + \varepsilon) \\ &\quad + \zeta \frac{u(\tilde{c}_n^m(\varepsilon)) - u(\hat{c}_{s+1}^m) + \varepsilon u'(\tilde{c}_n^m(\varepsilon)) \frac{\partial \tilde{c}_n^m(\varepsilon)}{\partial \varepsilon}}{\hat{n}_s + \varepsilon} \\ &\quad - \zeta \frac{\varepsilon (u(\tilde{c}_n^m(\varepsilon)) - u(\hat{c}_{s+1}^m))}{(\hat{n}_s + \varepsilon)^2}\end{aligned}$$

$$\begin{aligned}\frac{\partial \tilde{U}_s(\varepsilon, M)}{\partial \varepsilon} &= -\theta u'(\hat{c}_s^m - \theta\varepsilon) + \beta M u'(\hat{c}_{s+1}^o + M\varepsilon) + \gamma u'(\hat{n}_s + \varepsilon) \\ &\quad + \zeta \frac{\varepsilon u'(\tilde{c}_n^m(\varepsilon)) \frac{\partial \tilde{c}_n^m(\varepsilon)}{\partial \varepsilon}}{\hat{n}_s + \varepsilon} + \zeta \frac{\hat{n}_s (u(\tilde{c}_n^m) - u(\hat{c}_{s+1}^m))}{(\hat{n}_s + \varepsilon)^2}\end{aligned}$$

$$\begin{aligned}\frac{\partial \tilde{U}_s(\varepsilon, M)}{\partial \varepsilon} &= -\theta u'(\hat{c}_s^m - \theta\varepsilon) + \beta M u'(\hat{c}_{s+1}^o + M\varepsilon) + \gamma u'(\hat{n}_s + \varepsilon) \\ &\quad + \zeta \frac{\varepsilon u'(\tilde{c}_n^m(\varepsilon))}{\hat{n}_s + \varepsilon} \left[ \frac{\varepsilon F_L(\hat{s}_{s+1}, \hat{n}_s + \varepsilon) - (F(\hat{s}_{s+1}, \hat{n}_s + \varepsilon) - F(\hat{s}_{s+1}, \hat{n}_s))}{\varepsilon^2} \right] \\ &\quad + \zeta \frac{\hat{n}_s (u(\tilde{c}_n^m(\varepsilon)) - u(\hat{c}_{s+1}^m))}{(\hat{n}_s + \varepsilon)^2}\end{aligned}$$

$$\begin{aligned}\frac{\partial \tilde{U}_s(\varepsilon, M)}{\partial \varepsilon} &= -\theta u'(\hat{c}_s^m - \theta\varepsilon) + \beta M u'(\hat{c}_{s+1}^o + M\varepsilon) + \gamma u'(\hat{n}_s + \varepsilon) \\ &\quad + \zeta u'(\tilde{c}_n^m(\varepsilon)) \frac{[\varepsilon F_L(\hat{s}_{s+1}, \hat{n}_s + \varepsilon) - (F(\hat{s}_{s+1}, \hat{n}_s + \varepsilon) - F(\hat{s}_{s+1}, \hat{n}_s))]}{\varepsilon(\hat{n}_s + \varepsilon)} \\ &\quad + \zeta \frac{\hat{n}_s (u(\tilde{c}_n^m(\varepsilon)) - u(\hat{c}_{s+1}^m))}{(\hat{n}_s + \varepsilon)^2}\end{aligned}$$

Taking limits and using Hôpital's Rule:

$$\begin{aligned}\lim_{\varepsilon \rightarrow 0} \frac{\partial \hat{U}_s(\varepsilon, M)}{\partial \varepsilon} &= -\theta u'(\hat{c}_s^m) + \beta M u'(\hat{c}_{s+1}^o) + \gamma u'(\hat{n}_s) \\ &\quad + \frac{\zeta}{\hat{n}_s} (u(\hat{c}_{s+1}^m - (M + \underline{b}_{s+1})) - u(\hat{c}_{s+1}^m))\end{aligned}$$

Using equations (49) and (51)

$$\lim_{\varepsilon \rightarrow 0} \frac{\partial \hat{U}_s(\varepsilon, M)}{\partial \varepsilon} = \beta(M + \underline{b}_{s+1})u'(\hat{c}_{s+1}^o) - \frac{\zeta}{\hat{n}_s} [u(\hat{c}_{s+1}^m) - u(\hat{c}_{s+1}^m - (M + \underline{b}))]$$

We need to show that

$$LHS(M) = \beta(M + \underline{b})u'(\hat{c}_{s+1}^o) > \frac{\zeta}{\hat{n}_s} [u(\hat{c}_{s+1}^m) - u(\hat{c}_{s+1}^m - (M + \underline{b}_{s+1}))] = RHS(M).$$

for some feasible  $M$ . The  $LHS$  is linearly increasing in  $M$ , with  $LHS(-\underline{b}) = 0$ , while the  $RHS$  is increasing and convex in  $M$ , with  $RHS(-\underline{b}) = 0$ . Since both are continuous, if

$$\left. \frac{\partial LHS}{\partial M} \right|_{M=-\underline{b}} > \left. \frac{\partial RHS}{\partial M} \right|_{M=-\underline{b}},$$

then there exists  $\bar{M} > -\underline{b}$  such that  $LHS(\bar{M}) = RHS(\bar{M})$ . Since  $u'(0) = \infty$ ,  $\bar{M} < \frac{F(\hat{s}_{s+1}, \hat{n}_s) - F(\hat{s}_{s+1}, \hat{n}_s)}{\varepsilon} - \hat{s}_{s+2} - \theta_{s+1}\hat{n}_{s+1}$ . Now,

$$\left. \frac{\partial LHS}{\partial M} \right|_{M=-\underline{b}} > \left. \frac{\partial RHS}{\partial M} \right|_{M=-\underline{b}} \Leftrightarrow \beta u'(\hat{c}^o)\hat{n} > \zeta u'(\hat{c}^m)$$

From equation (52), this is true whenever  $\lambda_{b,t+1} > 0$ , i.e. the parent is constrained. Then, for  $M \in (-\underline{b}, \bar{M})$ , we have  $\lim_{\varepsilon \rightarrow 0} \frac{\partial \hat{U}_P(\varepsilon, M)}{\partial \varepsilon} > 0$ .

This establishes that the alternative tilde allocation is  $\mathcal{A}$ -superior to the equilibrium hat allocation. Further, since  $\lim_{\varepsilon \rightarrow 0} \frac{F(\hat{K}, \hat{n}_p) - F(\hat{K}, \hat{n})}{\varepsilon} = F_L(\hat{K}, \hat{n})$ , the  $\varepsilon$  children will be better off if

$$u(F_L(\hat{K}, \hat{n}) - \hat{s} - \theta\hat{n} - M) > u(\text{unborn}).$$

Assuming this, the proposed tilde allocation is also  $\mathcal{P}$ -superior to the equilibrium hat allocation. ■

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