MODERN FORECASTING MODELS IN ACTION: IMPROVING MACRO ECONOMIC ANALYSES AT CENTRAL BANKS

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Abstract. There are many indications that formal methods based on economic research are not used to their full potential by central banks today. For instance, Christopher Sims published a review in 2002 where he argued that central banks use models that "are now fit to data by ad hoc procedures that have no grounding in statistical theory". There is no organized resistance against formal models at central banks, but the proponents of such models have not always been able to present convincing evidence of the models' advantages. In this paper we demonstrate how BVAR and DSGE models can be used to shed light on questions that policy makers deal with in practice. We compare the forecast performance of BVAR and DSGE models with the Riksbank's official, more subjective forecasts. We also use the models to interpret the low inflation rate in Sweden in 2003 - 2004.

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JEL classification: E52, E37, E47.

1. Introduction

Since the early 1980s monetary policies in many countries have been focused on achieving a more narrow list of objectives than earlier. This has inspired researchers in macroeconomics to develop new tools for analyses of monetary policy, and the gap between academic research and policy implementation has decreased. Nevertheless, there are many indications that the use of formal methods based on academic research is far from its full potential at central banks. Sims' (2002) review of the analyses at four central banks suggests that there is still room for considerable improvements in the basis for policy decisions by applying well-established formal methods in economics, statistics and econometrics.[1]

It is difficult to explain why formal methods have not achieved a higher status at central banks. Possibly, it may be that policy makers and researchers often differ both with respect to the kind of questions they are primarily interested in, and to the manner in which they go about answering them. Policy makers tend to ask questions like: What has happened in the economy recently? Do we know why this has happened? What is likely to happen in the next one to two years? Occasionally, the last question may be conditioned on specific conditions, i.e., policy makers wish to know the likely course of events for the most relevant macro economic variables under a limited set of alternative scenarios. However, policy makers seldom ask their advisers

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[1]We would like to stress that we see a larger potential for improvement of the basis for policy decisions than for the policy decisions themselves. It is quite likely that policy makers today use their own skills and intuition to compensate for deficiencies in the material they get from their economists. This does not however weaken the arguments for improving that material.

what an appropriate monetary policy under current conditions should be. This may be due to a conviction on their part that, given the answers to the questions posed about the state of the economy, their own experience will suffice in inferring what an appropriate monetary policy is.[2] It should be added that the answers to the kinds of questions that policy makers do ask are usually mainly based on judgments from a large numbers of experts with detailed knowledge about different sectors of the economy; formal macro economic models (that tie the statements about sectors together, and to theory) are also used, but play a smaller role than researchers outside central banks probably believe.

Academic researchers, on the other hand, tend to ask questions like: What is an optimal monetary policy – not right now, but more generally, i.e., in terms of some rule for central bank behavior? How does monetary policy, on average, influence inflation, production and employment? What types of formal methods are likely to produce the most reliable answers to these kinds of questions, according to some specific criteria for model evaluation? Obviously, the answers to these questions should be highly relevant also for the questions regarding the state of the economy more frequently asked by policy makers. In order to explain what has happened recently and what is likely to happen in the future, an idea of how monetary policy is designed and how it influences the economy is required. Perhaps it is not necessary (as long as we do not claim to give policy advice) to know what an optimal monetary policy is, but we still need to have an idea about how we can at least approximately describe actual policy – because the public's perception of the design of monetary policy influences both the current state of the economy and its future development.

In his review of "the way data relate to decisionmaking" in central banks, Sims (2002) concludes that "the models are now fit to data by ad hoc procedures that have no grounding in statistical theory". Sims' conclusion was based on the observation that the formal macro models apparently used by central banks were either based on parameters that had been calibrated (rather than estimated), or on parameter estimates from a large number of different sector models, whose joint implications had not been thoroughly investigated. Judgments from experts, that are not based on formal models, of course also involve "ad hoc procedures that have no grounding in statistical theory".

Thus, there appears to be a gap between macroeconomics and reality, in the sense that researchers have developed, and used, methods to study macroeconomic data that have had rather little influence on how policy advisors at central banks view the world. If this is to change, the benefits of formal methods have to be clearly demonstrated in situations that policy advisors recognize. But we also have to understand why formal methods sometimes are met by scepticism at central banks. Sims (2002) suggests that "academic research in these areas has paid very little attention to the central problems of modeling for macro-economic policy in real time", but there may be other reasons for policy advisors' scepticism as well.

In this paper we hope to take this discussion a step further by demonstrating how various formal models can and, we argue, must be used to shed light on practical policy questions. We present results from models currently being developed and used at Sveriges Riksbank (the central bank of Sweden). Unlike the work criticized by Sims (2002), these models have not been "fit to data by ad hoc procedures that have no grounding in statistical theory".

The outline of the paper is as follows. Section 2 compares official forecasts of inflation made by the Riksbank with forecasts from a Bayesian VAR model.[3] The purpose of these exercises

[2]Another reason is probably that policy makers have been overexposed to bad models. Sims (2002) suggests that "by and large, the changes in these models over time have been more regress than progress".

[3]Appendix A presents the details of the BVAR model.

relates mainly to the first type of questions asked by policy makers: what has happened to the economy, and what is likely to happen in the future? These types of questions can be addressed with forecasting models with good statistical properties but using very little economic theory. As pointed out by Sims (1980), and in many of his and his co-authors' papers since then, such models can also provide some information about why certain things have happened or are likely to do so. But as soon as we are interested in predictions conditioned on certain alternative scenarios we need models with more structure from economic theory. We turn to such analyses in Section 3, where we present forecasts from a dynamic stochastic general equilibrium (DSGE). This model extends the closed economy models developed by Christiano, Eichenbaum and Evans (2005) and Altig, Christiano, Eichenbaum and Lindé (2003) to the small open economy setting in a way similar to Smets and Wouters (2002). Following Smets and Wouters (2003), the model is estimated by Bayesian methods. In addition to comparing the forecasting performance of the BVAR and DSGE models, we also contrast their different predictions about the effects of monetary policy.

In order to demonstrate how these models may be used in settings that policy makers and advisors recognize, we let the BVAR and DSGE models in Section 4 interpret the low inflation in Sweden during 2004, when inflation was below the Riksbank's lower tolerance line (1%).

Section 5 summarizes our views on the advantages of formal methods and on the reasons why such methods have not been more influential at central banks. This also involves some suggestions for future research and policy analysis.

2. Forecasting performance

In this Section we first compare the Riksbank's official forecasts of CPI inflation, since 1999, with forecasts from a Bayesian VAR model. Since the Riksbank's forecasts are intended to be conditioned on the assumption of a constant short term interest rate, we also look at the interest rate forecasts from the BVAR model. These are compared with implicit interest rate forecasts calculated from forward interest rates.

The Riksbank publishes official forecasts in quarterly Inflation Reports. The forecasts are not the outcome of any single formal model, but rather the result of a complex procedure with input both from many different kinds of models and judgements from sector experts and from the Riksbank's Board. The Inflation Report only contains tables with yearly data, but monthly forecasts are presented in graphs on the Riksbank's web page. In this paper chose to look at quarterly averages of monthly observations.

The BVAR model contains quarterly data for the following seven variables: trade-weighted measures of foreign GDP growth (y_f), CPI inflation (π_f) and a short term interest rate (i_f), the corresponding domestic variables (y, π and i), and the level of the real exchange rate defined as $q = 100(s + p_f - p)$, where p_f and p are the foreign and domestic CPI levels (in logs) and s is the (log of the) trade-weighted nominal exchange rate. More details on the BVAR are provided in Appendix A.

2.1. **Inflation forecasts.**

Figure 1 presents the outcome of CPI inflation in Sweden 1998 – 2004 (the solid line) together with the Riksbank's official forecasts for 1999Q2 – 2004Q4.[4] The corresponding forecasts from the BVAR models are presented in Figure 2.[5] The visual

[4]The data in Figure 1 refer to annual inflation rates ($p_t - p_{t-4}$), just like the inflation published in the Inflation Reports, whereas the BVAR model is specified in terms of quarterly rates of change ($p_t - p_{t-1}$). The BVAR forecasts are summed up to fourth differences.

[5]No genuine real time data set has been put together, which implies that the BVAR model possibly has an information advantage relative to the official forecasts because the BVAR forecasts are based on ex post data on GDP. (Preliminary GDP data are available with a delay of around one quarter, but they are subsequently

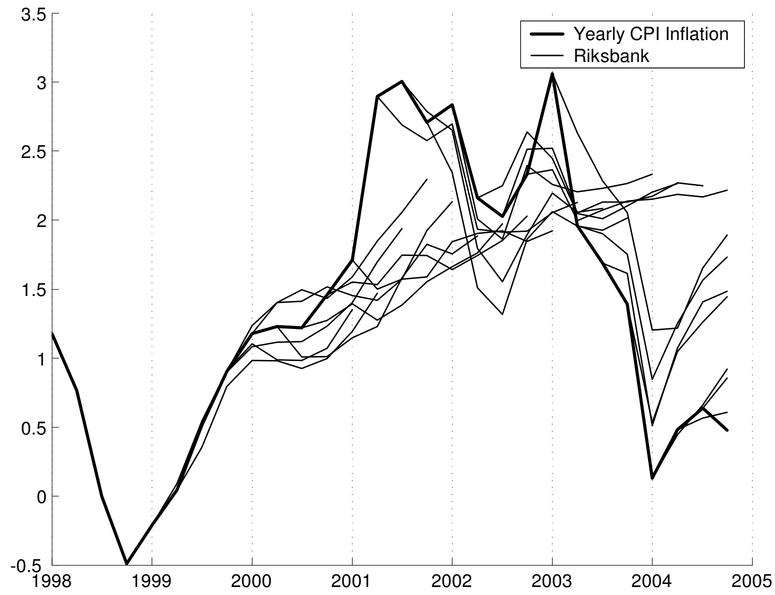

FIGURE 1. Sequential Riksbank forecasts of yearly CPI inflation 1999Q2-2004Q3.

impression is that the official and BVAR forecasts have somewhat different properties. The Riksbank's forecasts appear to be rather "conservative"; most often they predict a very smooth development of inflation, and the changes in the forecasts paths are relatively small from quarter to quarter. The BVAR model seems to view the inflation process as more persistent, since forecast errors have a larger influence on subsequent forecasts. The Riksbank's forecast at the two years horizon are generally closer to the 2 percent inflation target compared to the BVAR. The posterior median of the long-run inflation rate in the BVAR is only slightly below the target (1.95 percent on average in the sequential forecasts), but the forecast horizon is in most periods not long enough for the BVAR forecasts to reach its long-run value. Figure 3 shows the RMSEs for different forecast horizons ($1-8$ quarters ahead). The BVAR and official forecasts have about the same precision, and both are better than a naïve forecast of constant inflation, at least two quarters ahead and beyond.[6] Figure 3 also shows the RMSEs forecast from the DSGE model which we will comment in Section 3.

To get a better understanding of the properties of various approaches to forecasting it is interesting to take a closer look at some specific episodes. Such an exercise sheds light on some questions about "the role of subjective forecasting" raised by Sims (2002). In his comment on Sims' paper, Steven Durlauf suggested that "an additional useful way of comparing two forecasts is to attempt to identify periods when the two forecasts diverge relatively sharply and compare their behavior at those times" (p. 42). Jeffrey Fuhrer also commented that "an important unresolved question is to what extent the expertise of the Federal Reserve

revised.) This is not a problem with the other variables in the BVAR model, since data on prices, interest rates and exchange rates are available on a monthly basis and not revised. The official forecasts have a small information advantage in some quarters, when the Inflation Reports have been published towards the end of the quarter and thus can be based on data on prices, interest rates and exchange rates from the early part of the same quarter.

[6]The BVAR forecasts are significantly better than the random walk for all horizons but the first. We use a Bayesian test of the hypothesis that the squared forecast errors from the No Change forecast are on average larger than those from the BVAR model.

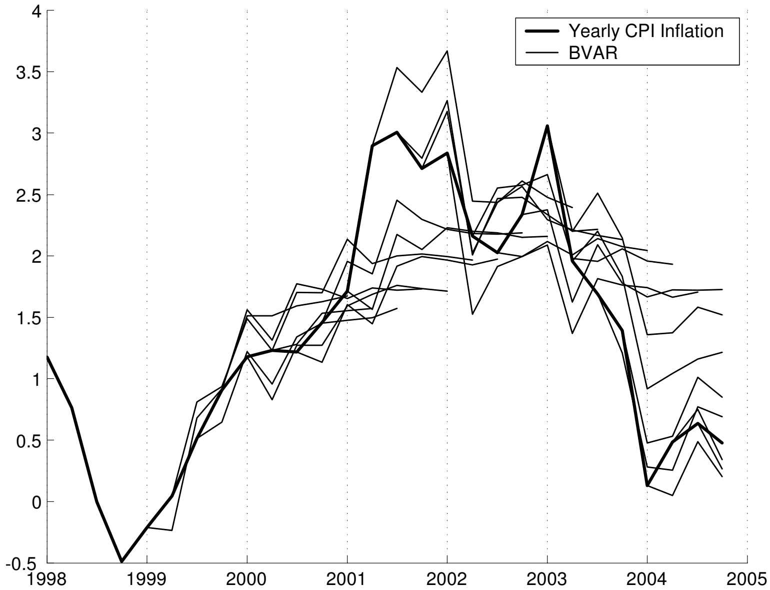

FIGURE 2. Sequential BVAR forecasts of yearly CPI inflation 1999Q2-2004Q3.

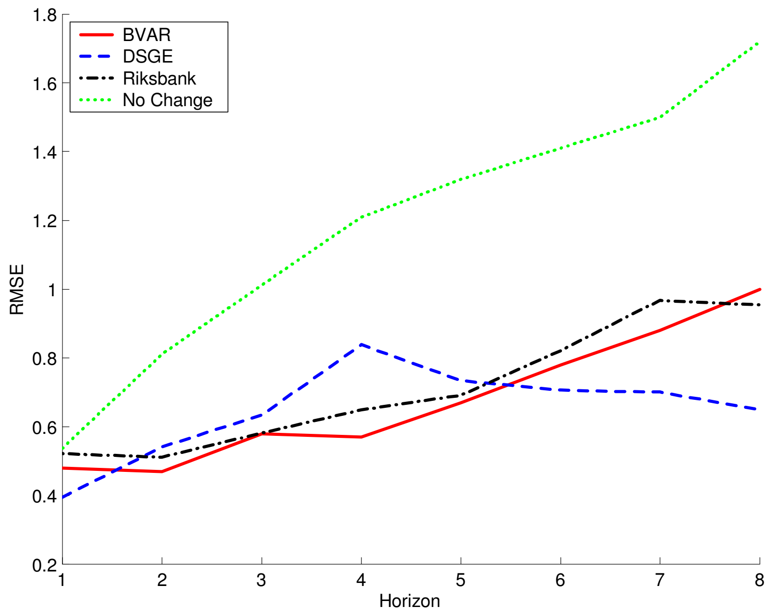

FIGURE 3. Root Mean Squared Error (RMSE) of yearly CPI inflation forecasts 1999Q2-2004Q3.

specialists results in better understanding of transient or persistent influences on inflation in this sample" (p. 56). A closer look at two Swedish episodes provides useful information about such issues.

In Figure 4 and 5 we compare the Riksbank's official inflation forecasts and the corresponding BVAR forecasts on four different occasions (i.e., Figure 4 and 5 contains a subset of the information in Figures 1 and 2). The first episode concerns forecasts made immediately before

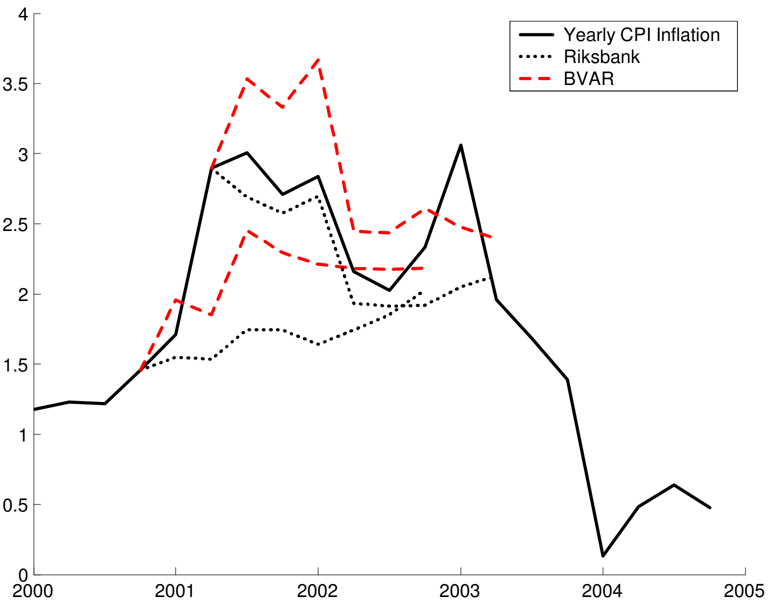

Figure 4. Riksbank and BVAR forecasts of yearly CPI inflation made on two specific occasions: 2000Q4 and 2001Q2.

and after the sudden increase in inflation in 2001Q2. One factor behind this increase was an increase in food prices brought about by the mad cow and foot-and-mouth diseases. The inflation rate had however started to increase already during 1999, possibly because of a general improvement of the business cycle. In 2000Q4, the Riksbank's official forecasts implied a slowly increasing inflation rate over the next two years. The BVAR model suggested a somewhat stronger increase in inflation. This may reflect that the model forecasts attributed a larger weight to recent increases in inflation, while the subjective procedure, leading up to the Riksbank's official forecast, underestimated the persistence in changes in inflation. Because of the food price shock, both approaches exhibited large prediction errors. The subjective approach, involving judgments from sector experts, proved to be very useful once this inflation shock had become apparent. At that time, 2001Q2, sector experts expected the food price shock to involve a persistent shock to the price level but with small further effects on annual inflation from 2002Q2 and onwards. This proved to be the right conclusion, and the Riksbank's forecasts from 2001Q2 were almost identical to the actual outcome for the next five quarters. The BVAR model treated the food price shock as any other inflation shock and overestimated its effects on inflation, mainly during 2001.

The second episode, described in Figure 5, concerns inflation forecasts made in 2003Q1. Cold and dry weather had brought about extreme increases in electricity prices during the winter 2002/2003. This was a temporary shock to the price level, but it had persistent effects on annual inflation which became unusually low when electricity prices declined. Both the Riksbank's official forecasts and the forecasts from the BVAR model underestimated the decline in inflation, possibly because it was difficult to separate the effects of changes in energy prices (the oil price also fluctuated heavily) from the downward pressure on inflation from the relatively weak business cycle. Once the decline in inflation had started, the BVAR model more correctly predicted that inflation would be very low for the next 1 – 2 years (cf. the forecasts from 2003:3 in Figure 5). It is possible that the Riksbank underestimated a general

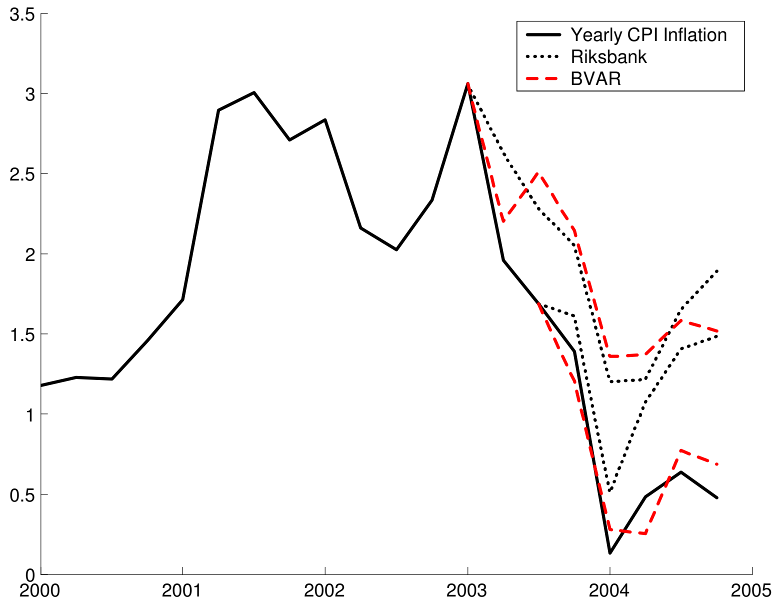

FIGURE 5. Riksbank and BVAR forecasts of yearly CPI inflation made on two specific occasions: 2003Q1 and 2003Q3.

downward trend in inflation associated with the business cycle, because the bank put too much weight on fluctuations in energy prices that were believed to be temporary.

These episodes nicely illustrate how formal statistical models and judgments by sector experts can complement each other. Subjective forecasts may sometimes be too myopic and pay too little attention to systematic inflation dynamics related to the business cycle or other historically important regularities. Model forecasts, on the other hand, cannot take enough account of specific unusual but observable events. As Sims (2002, p. 22) puts it: "Analysis of such historically unusual disturbances – including the determination of whether they really are historically unusual – will inevitably involve an element of subjective judgment. That is, because they are unique or unusual, extrapolating their effects must rely on more than historical statistical patterns of variation".

2.2. **Interest rate forecasts.** One difficulty when it comes to comparing official inflation forecasts from the Riksbank with model forecasts or forecasts made by other institutions is that the Riksbank's forecasts are intended to be conditioned on the assumption that the short term interest rate (more specifically the bank's instrument, the repo rate) stays constant throughout the forecasting period.[7] For our purposes here, this does not seem to be a problem that should affect our conclusions in any significant way. First, it is extremely difficult to ensure that the subjective, official forecasts really are conditioned on a constant interest rate path rather than some more likely path. Second, during the sample period we consider the interest rate level has actually been rather stable.

[7]It is less clear, given the subjective nature of the forecast, what assumptions are made about the interest rate beyond the forecast horizon. The implicit assumption most often seems to have been that the interest rate gradually returns to a level determined by some interest rate equation. The Inflation Report from March 2005 contains both a discussion of the problems with constant interest rate forecasts, and forecasts of inflation and GDP growth that are not based on that assumption.

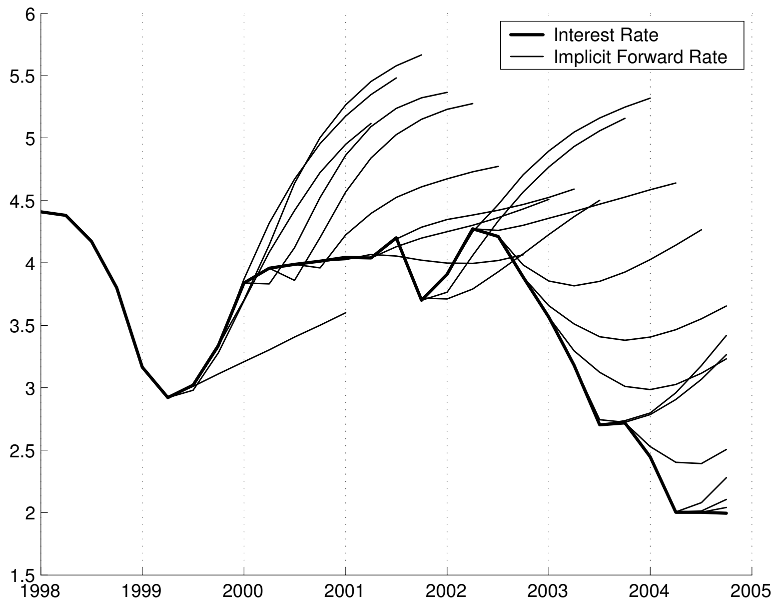

FIGURE 6. Sequential implicit forward rate forecasts of the interest rate 1999Q2-2004Q3.

Figure 6 shows the three month interest rate (quarterly averages) along with expected interest rate paths derived from forward interest rates, and in Figure 7 we show the interest rate forecasts from the BVAR model[8]. Throughout our sample, forward interest rates have systematically overestimated the future interest rate level. In principle, this could be due to (possibly time-varying) term and risk premia, i.e., forward rates do not provide direct estimates of expectations. In Figure 8 we compare the RMSEs (just like we did with inflation forecasts in Figure 3; the RMSE for the DSGE model will be discussed in Section 3). It can be seen that forward interest rates, a naive constant interest rate forecast and the BVAR model all have about the same precision for forecasts 3 – 4 quarters ahead. Forward rates have better precision 1 – 2 quarters ahead, while the BVAR model makes better forecasts for longer horizons.[9] The existence of risk and term premia can probably explain part of the forecast "errors" from forward rates.

One hypothesis examined by Sims (2002) is that the central bank (the Fed) can make good inflation forecasts because it has good information about its own future interest rate decisions. It is interesting to note that the Riksbank's official inflation forecasts are about as good as those from the BVAR model (cf. Figure 3), despite the fact that the bank's forecasts are intended to be conditioned on the assumption of a constant interest rate, which is not a good forecast, at least not for longer horizons than 3 – 4 quarters. There are different possible explanations for this finding. One is that it is difficult to implement the constant interest rate assumption in a largely subjective forecasting approach, in particular when it comes to forecasts one to two years ahead and beyond. That is, it may have been the case that the Riksbank's forecasts have not been fully consistent with the constant interest rate assumption. Another possibility is that the effects of monetary policy shocks on inflation are quite small,

[8]The method used to derive the expected paths is described by Svensson (1995).

[9]The BVAR forecasts are significantly better than the random walk at the longer horizons. See also footnote 7.

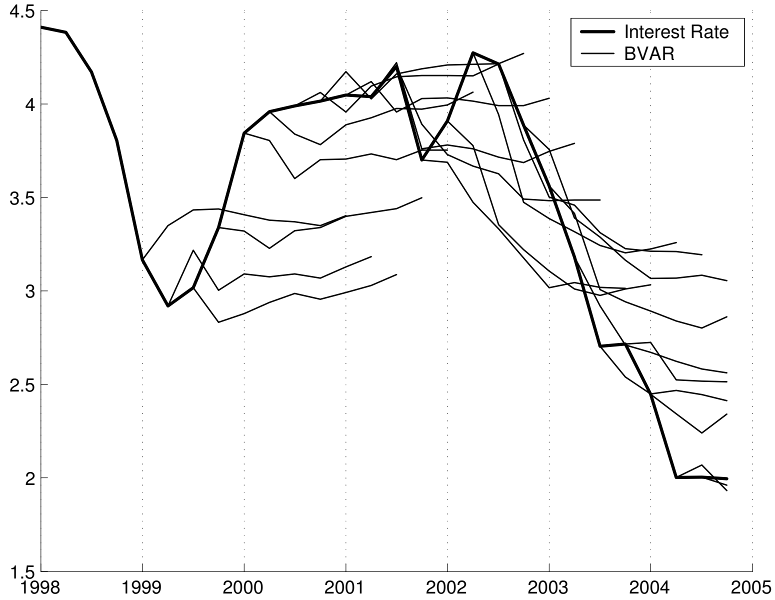

FIGURE 7. Sequential BVAR forecasts of the interest rate 1999Q2-2004Q3.

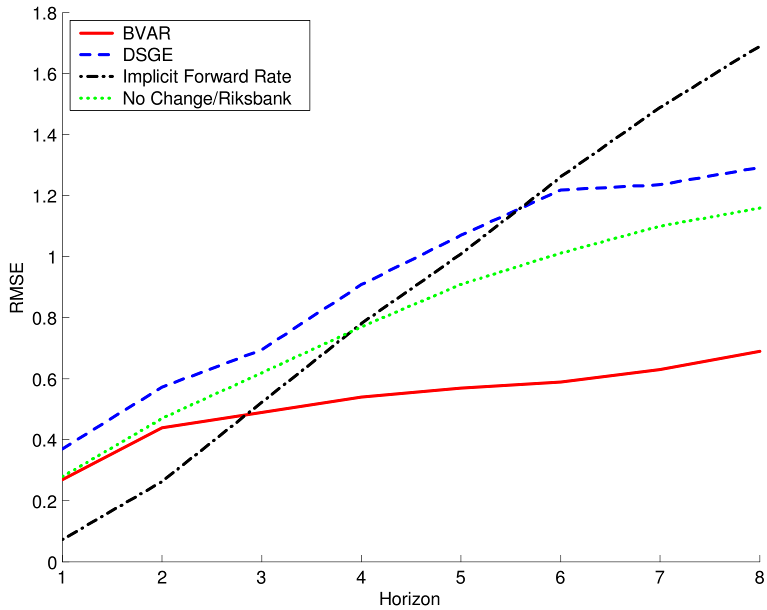

FIGURE 8. Root Mean Squared Error (RMSE) of annualized interest rate forecasts 1999Q2-2004Q3.

so that the interest rate assumption is not crucial for the inflation forecast. We will return to this question in Section 3 below, where we compare the properties of the BVAR model with those of a DSGE model.

Sveriges Riksbank has recently (March 2005) started to publish inflation forecasts conditioned on the assumption that the interest rate follows some exogenously given path, more specifically the path implied by forward interest rates (as in Figure 6). The Bank of England and Norges Bank (the central bank of Norway) have earlier made the same move. The results presented in this section suggest that this should have a larger impact on the inflation forecasts 1 – 2 years ahead than on the forecasts of inflation for the next two quarters. However, forward interest rates seem to provide a more realistic assumption about short term interest rates mainly for the near future. Our BVAR model provides a more realistic picture at longer horizons, where forward rates even seem to have lower predictive power than the constant interest rate assumption. This may be interpreted as arguments against the forecasts conditioned on forward rates. It should be emphasized, however, that these results have been obtained from a sample where the short term interest rate level has relatively stable. It is possible that forward rates are more informative in periods when the interest rates changes more.

2.3. **Subjective vs model forecasts: some conclusions.** In this section we have compared the Riksbank's official inflation and interest rate forecasts with forecasts from a Bayesian VAR model. The inflation forecasts have had about the same precision, which we interpret as an encouraging result for both approaches.[10] BVAR forecasts are relatively easy to produce and could therefore be used routinely as a benchmark or starting point for the rather subjective, complicated and non-transparent forecasting process at the Riksbank. As is well known and supported by our results, BVAR models summarize regularities in the data that forecasters should pay attention to. At the same time, judgments from sector experts based on their detailed knowledge about economy can be extremely useful, in particular when unusual shocks have hit the economy. It should be useful both to develop sets of BVAR models that could regularly provide policy makers with updated forecasts, and also to try to include information from sector experts formally in these BVAR models. The latter question is indeed an important issue for future research, as suggested by Sims (2002, p. 37): "It might be worth exploring a structure in which judgmental forecasters focus almost entirely on the current and the next quarter. Their forecasts could be treated as noisy observations on the data that enter a Bayesian structural model. Model forecasts could then incorporate these observations, as well as generate measures of how implausible they are".

3. BVAR vs DSGE

3.1. **Forecasts.** Figures 9 and 10 show inflation and interest rate forecasts from the DSGE model. An interesting observation regarding the inflation forecasts in Figure 9, that is also supported by the comparisons of RMSEs in Figure 3, is that the DSGE model has a forecasting performance that is remarkably good, given that the model imposes a large number of theoretical restrictions. It is only at the one year horizon that the forecasts from the BVAR model and the Riksbank's official forecasts have a higher precision. The DSGE model makes smaller forecast errors than the other two approaches 7 – 8 quarters ahead. The latter result is perhaps not so surprising, if we think that economic theory is useful primarily for understanding long run tendencies in the economy. That the DSGE model performs relatively well also in terms of short run forecasts is more surprising.[11]

[10] This result is also similar to what Sims (2002) found about the Fed's forecasts.

[11] We have also compared the performance of the DSGE model in the cases of the same two episodes that were examined in section 2.1, Figure 4a – b, above. The results are available upon request. The predictions from the DSGE model are close to those of the BVAR model in 2000Q4, 2001Q2 and 2003Q3, but not in 2003Q1, when the DSGE model predicts much higher inflation for the next two years.

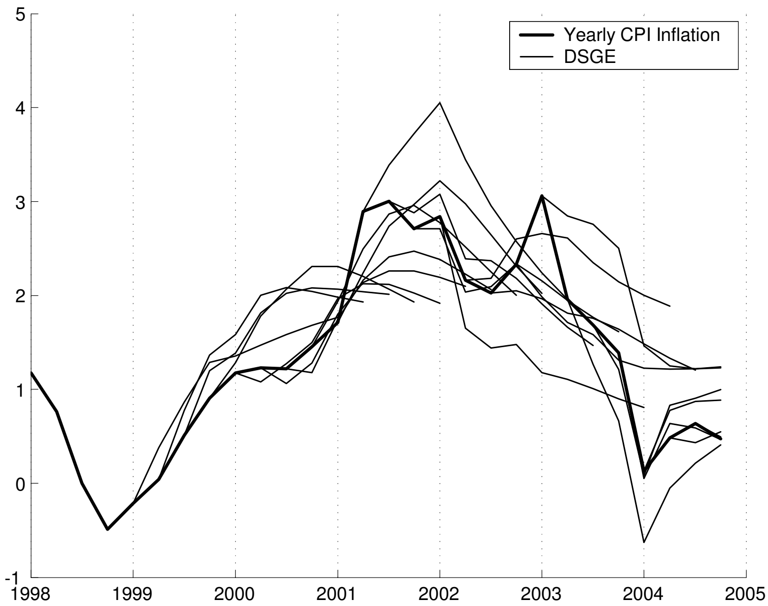

Figure 9. Sequential DSGE forecasts of yearly CPI inflation 1999Q2-2004Q3.

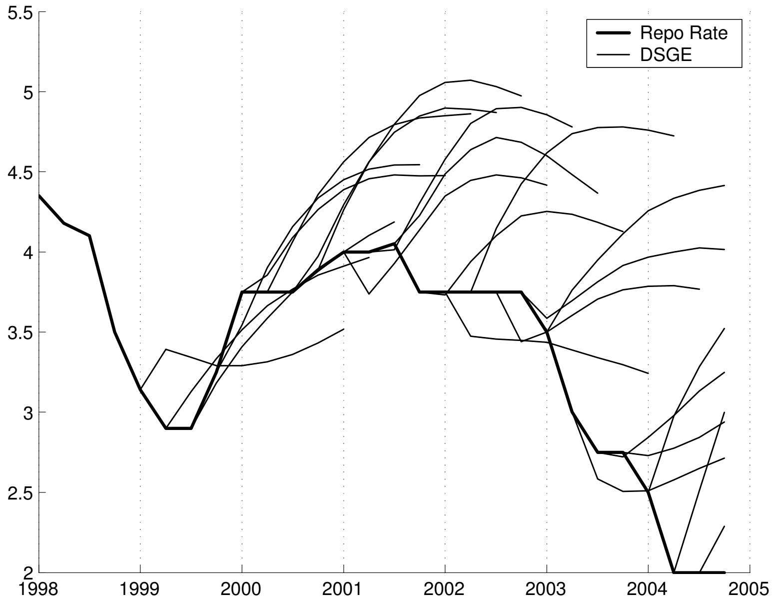

Figure 10. Sequential DSGE forecasts of the interest rate 1999Q2-2004Q3.

The interest rate forecasts from the DSGE model, depicted in Figure 10, are less impressive.[12] Future interest rates have been systematically overestimated. Comparing Figure 10 with Figures 6 and 7 it appears as if the DSGE model's interest rate forecasts have been closer

[12]One difficulty here is that the short term interest rate in the BVAR model is the three month rate, while the DSGE model uses the Riksbank's instrument rate (the repo rate). The differences between the two rates are not large, however, compare Figures 7 and 10.

to forward interest rates than to the BVAR model's forecasts. The RMSEs in Figure 8 show that during the last five years it has been easier to make good interest rate forecasts based on "reality" (the BVAR model) than on "macroeconomics" (the DSGE model). This of course raises some questions about how monetary policy is described in the DSGE model.[13] Perhaps the central banker in the DSGE model is "more conservative" than the Riksbank's Executive Board. These observations also give another example of an issue raised in Section 2.2 above: Apparently it is possible to make relatively good inflation forecast (as the subjective forecasts from the Riksbank, or from the DSGE model) without particularly good interest rate forecasts.

3.2. **The effects of monetary policy shocks.** One way to examine the links between the interest rate and inflation in the BVAR and DSGE models is to look at so called impulse response functions which show the responses to interest rate shocks in the different models.[14] Figure 11 shows the effects of an interest rate shock on the interest rate itself, inflation, output growth and the real exchange rate in the BVAR model. Figure 12 shows the corresponding effects in the DSGE model.[15]

Qualitatively, the effects of an interest rate shock are similar in the BVAR and DSGE models, and consistent with typical prejudices. An increase in the interest rate gives rise to a decline in output growth, inflation and the real exchange rate (i.e., the real exchange rate appreciates). The quantitative differences are very large, however. The DSGE model supports conventional wisdom: if the interest rate is raised by 0.5 percentage points for about a year (and then gradually lowered), the maximum (expected) effect on inflation is around 0.3 percentage points and is recorded after about $1\ \frac{1}{2}$ year. The effects in the BVAR model are much smaller and typically insignificant.

Although there are reasons to expect that effects of monetary policy shocks are more credibly identified in the DSGE model, the differences between the two models may make it difficult to convince policy makers that the models' predictions are useful (even if each model as such rests on solid methodological grounds and has not been fit to data by ad hoc procedures).

To illustrate this point, we show the differences between recent forecasts from the BVAR and DSGE models in Figure 13. If we compare the "unconditional" inflation forecasts in Figure 13, the differences may not seem very large. Both models predict that inflation will increase over the next two years, from around 0.6 per cent to around 1.2 – 1.3 per cent. The DSGE and BVAR models however make quite different predictions about the development of the short term interest rate, see Figure 14.[16] The DSGE model expects the interest rate to

[13]It should be noticed that the DSGE model used here is estimated without allowing for a break in the coefficients in the policy rule pre- and post- the inflation targeting regime in Sweden. Adolfson et al. (2005a) are currently working on a version of the model which allow for a break in the policy rule. In all likelihood, this will improve the DSGE models forecasting performance for the nominal interest rate.

[14]In the BVAR model, this is implemented through exogenous shocks to the interest rate in a Choleski decomposition where the interest rate is ordered after all other variables than the real exchange rate (see Appendix A). Other non-recursive identifying restrictions (e.g. allowing the central bank to react to changes in the real exchange rate within the period, but not to the two GDP variables) gave similar results. In the DSGE model, exogenous shocks are added to the central bank's reaction function.

[15]We have results for two different BVAR models, one where inflation is measured in terms of CPI and another where inflation is instead measured in terms of "core" (or "underlying") inflation, which excludes the effects on CPI from changes in indirect taxes, subsidies and interest rate costs on mortgage loans. Results from the latter are shown in the right hand graphs of Figure 8. The CPI effects show a temporary "price puzzle" which may depend on the response of mortgage costs, although no such puzzle arises for CPI in the DSGE model.

[16]The solid line is the three month interest rate which is the short term interest rate in the BVAR model, while the DSGE model uses the Riksbank's repo rate, given by the dashed line.

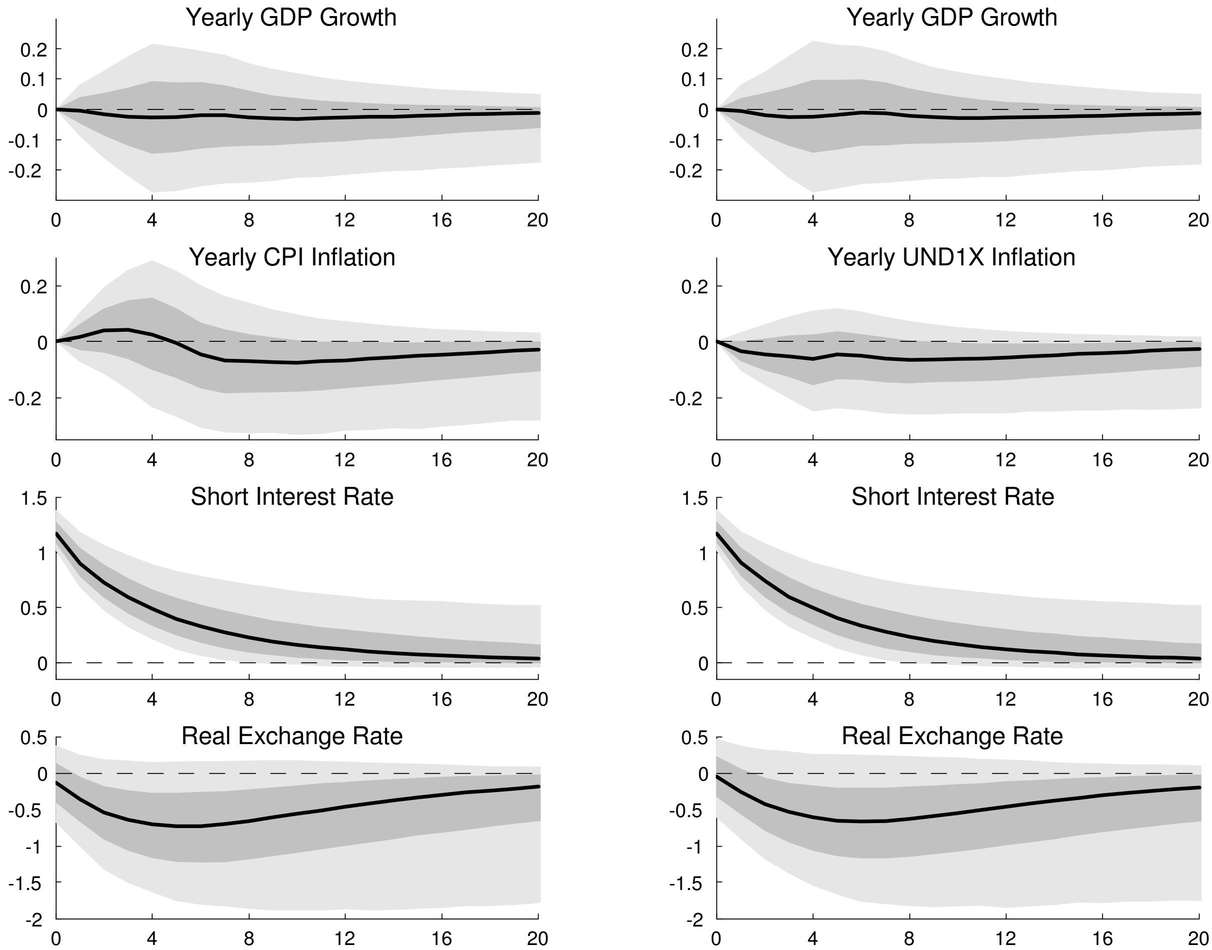

FIGURE 11. Posterior median impulse response functions for the domestic variables in the BVAR model with 68% and 95% probability bands. The graphs to the left is for a system with CPI inflation. The graphs to the right is for a system with UND1X inflation (a measure of core inflation).

increase from 2 per cent to around 3.75 per cent, while the BVAR model expects the interest rate to stay below 3.0 per cent. In order to make the inflation forecasts more comparable, and useful for policy analysis and advice, it makes sense to compute forecasts that are conditioned on the same development of the interest rate. We chose to condition on the interest rate path implied by forward rates, which during the first year is very close to the BVAR forecast, but during the second year rapidly approaches the DSGE model's interest rate path (see Figure 14). Relative to the "unconditional" forecasts of inflation these "conditional" forecast thus imply a more expansionary monetary policy (lower interest rate) in the DSGE model, and a less expansionary policy (higher interest rate) in the BVAR model. This implies, in turn, that there is a larger difference between the "conditional" inflation forecasts than between the "unconditional" forecasts. The DSGE model now predicts that inflation will be only slightly below 2 per cent two years ahead, while the BVAR model still predicts an inflation rate of around 1.3 per cent. Given the large uncertainty which these forecasts are associated with, this may not seem like a very large difference, but for policy purposes, when the inflation

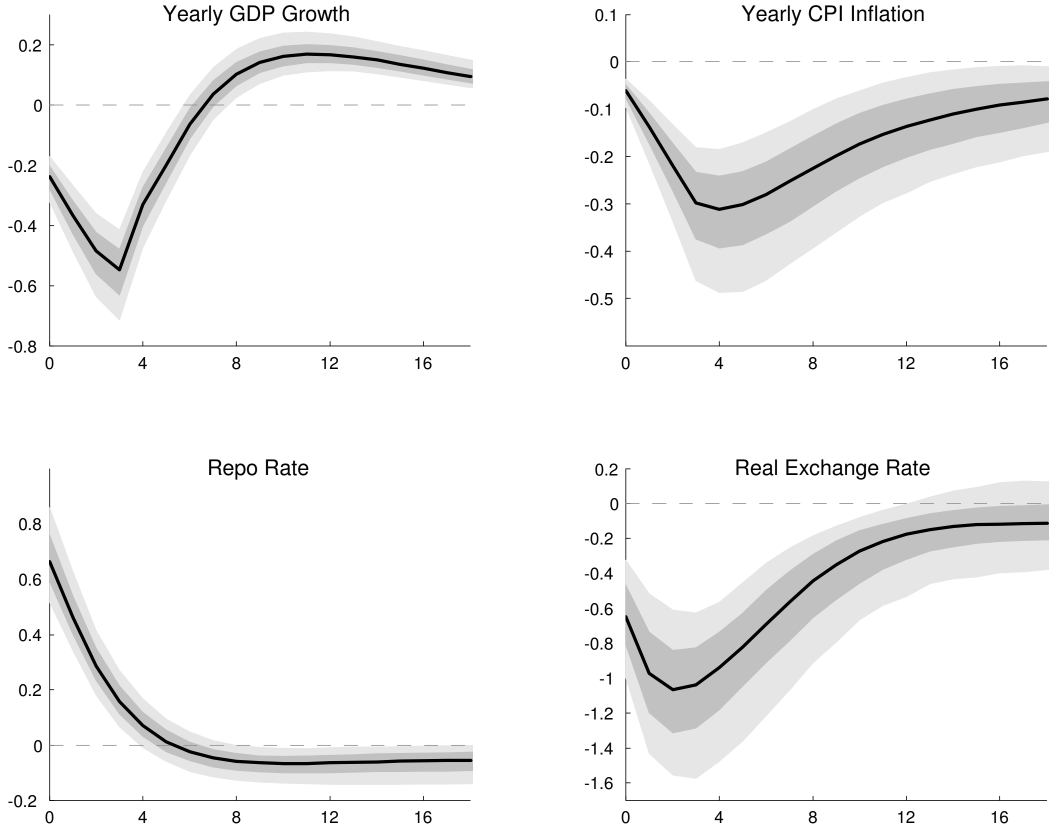

FIGURE 12. Posterior median impulse response functions for the domestic variables in the DSGE model with 68% and 95% probability bands.

target is 2.0 percent, the difference is large enough to discredit both models in the policy process.

The difference is not surprising, given our earlier findings. The DSGE model has a "more conservative" central banker than the BVAR model, and also believes in larger effects of monetary policy shocks. But there are of course many other possible explanations why the BVAR and DSGE models give different predictions of inflation and of the effects of monetary policy shocks. One is that both models are, at this stage, not yet fully developed and tested, and further work may bring the models closer together.

4. Understanding the low rate of inflation in Sweden 2003 – 2004

In Sections 2 and 3 we have looked at the usefulness of BVAR and DSGE models for discussions about inflation forecasts, interest rate forecasts and the effects of monetary policy shocks. These are indeed important questions for policy makers and advisers at central banks: What is likely to happen with inflation in the next two years, and how does the answer to this question depend on the interest rate path? But policy makers are also very interested in understanding the factors that have brought the economy to where it is right now. For instance, it has been necessary for the economists at the Riksbank recently to look more closely at the reasons for the low inflation rate in Sweden during 2003 and 2004.[17]

[17] See, e.g., one of the boxes in the Inflation Report from March 2005.

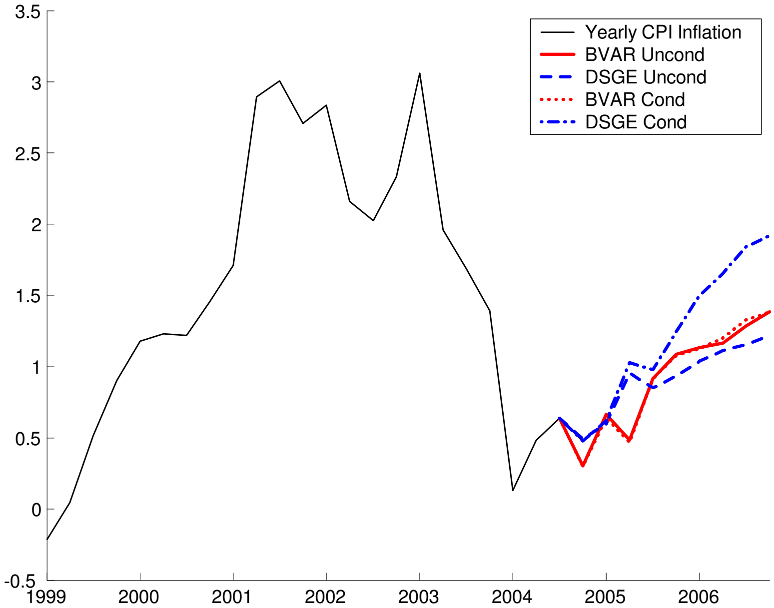

FIGURE 13. Yearly CPI inflation forecasts produced in 2004Q3 from the BVAR and DSGE models. Both unconditional forecasts and forecasts conditional on an interest rate path equal to the implicit forward rate path are presented. Monetary policy shocks are injected to keep the interest rate at its intended path.

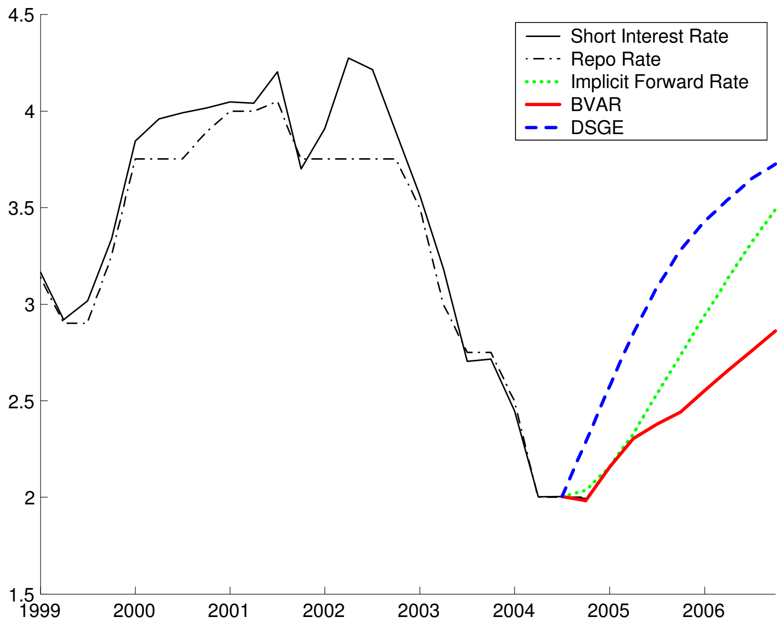

FIGURE 14. Unconditional forecasts of the interest rate (BVAR), the repo rate (DSGE) and the implicit forward rate produced in 2004Q3.

Since the BVAR and DSGE models make reasonably good forecasts of inflation ex ante, they can also be used ex post to interpret the forecast errors. In the DSGE model all shocks are given an economic interpretation, while the BVAR model requires additional identifying restrictions.

In Figure 15 we report the actual outcome of GDP growth, inflation and the short term interest rate together with projections from the BVAR model. The first row of Figure 15 reports forecasts made in 2003Q1 under the assumption that no shocks would hit the Swedish economy during 2003 or 2004. It can be seen that parts of the increase in GDP growth and the decreases in inflation and the interest rate were expected. (Some reasons for the expected fall in inflation were discussed in Section 2.1.) Note that GDP growth was lower than expected in 2003 and higher than expected in 2004.

In the second row of Figure 15 we have added the "foreign" shocks identified by the BVAR model ex post to the BVAR model's no-shock (expected) scenario. "Foreign" shocks are identified through the assumption that foreign GDP growth, foreign inflation and the foreign interest rate are strictly exogenous.[18] No attempt is made to identify individual "foreign" shocks. It can be seen that "foreign" shocks account for most of the changes in the domestic variables during 2003 (lower GDP growth and inflation than expected).

The last row of Figure 15 shows the effects of "domestic" shocks, which are simply identified residually as the parts of the forecast errors that are not accounted for by "foreign" shocks. It can be seen that "domestic" shocks are an important source behind the forecast errors in 2004 (higher GDP growth and lower inflation).[19]

In principle, it is possible to get much more information about the shocks from the BVAR model than this, if we are willing to make other identifying assumptions.[20] An alternative route which we can follow here, since we are equipped also with a DSGE model, is to look at decompositions of the forecast errors based on that specific model. In Figure 16, the first row shows the outcome of GDP growth, inflation and the interest rate along with the predictions from the DSGE model, i.e., the information corresponding to the first row of Figure 15 which was based on the BVAR model. Both models overestimated GDP growth (in particular in 2003), inflation and the interest rate. The other rows in Figure 16 show the "ex post forecasts" from the DSGE model when we add the model's estimates of different kinds shocks 2003 – 2004 to the original forecasts from 2003Q1: monetary policy shocks (second row), technology shocks (third row), mark-up shocks (fourth row), foreign shocks (fifth row), preference shocks (sixth row), and fiscal policy shocks (last row). It can be seen that when the estimated technology shocks and foreign shocks are taken into account, the "ex post forecasts" of inflation from the DSGE model are rather close to the outcome. The DSGE model thus supports the finding from the BVAR model, that a large part of the forecast errors during 2003 were due to foreign shocks. In 2004, when the BVAR model suggested that domestic shocks were more important, the DSGE model attributes a large part of the low inflation to domestic technology shocks.

[18] Formally, this implies, among other things, that the foreign variables are ordered before all domestic variables in the Choleski decomposition.

[19] The forecast error decompositions in Figure 15 is based on the assumption that the real exchange rate is treated as a "domestic" variable (it is ordered last in the Choleski decomposition). The results are however not much affected if we instead assume that real exchange rate shocks are entirely "foreign". The results are available upon request.

[20] Other identifying restrictions may involve, e.g., restrictions on long run impulses as in King, Plosser, Stock and Watson (1991). Jacobson, Jansson, Vredin and Warne (2001) present some results based on such restrictions from a VAR model using similar data as this paper. Alternatively, restrictions may be imposed directly on the impulse response functions as suggested by Canova (xxxx) and Uhlig (xxxx).

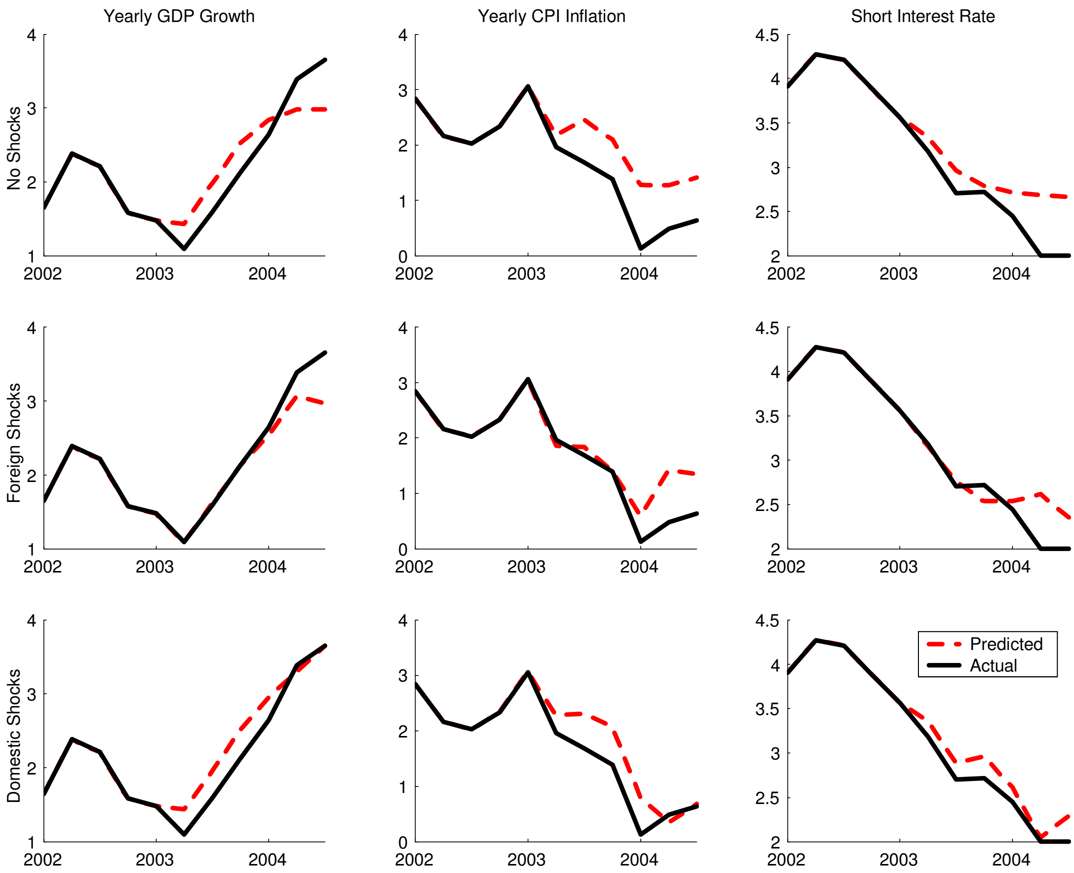

Figure 15. Predictions for 2003Q2-2004Q3 from the BVAR model with only subsets of the shocks active during the forecasting period.

We find the results from these exercises very promising. Not only can the BVAR and DSGE models make forecasts that have, on average, about the same precision as the Riksbank's official more subjective forecasts. They can also, ex post, decompose the forecast errors in ways that should be informative for policy makers and advisers.

5. Improving macro economic analyses at central banks through formal models: advantages and problems

Section 5.2 summarizes our conclusions about how formal models that are not fit to data by ad hoc procedures can improve macro economic analyses at central banks. To academic researchers it is an entirely trivial statement that formal models are beneficial, but in order to influence analyses at central banks more than general arguments are probably needed. Section 5.2 is thus primarily aimed at policy advisers at central banks. The conclusions are based on models of Swedish data recently developed at Sveriges Riksbank, but we think our results are of general interest and relevance. In Section 5.3 we describe, based on our own experiences, some problems that may explain why formal methods, despite all their advantages, have not been optimally explored by policy advisors at central banks. We believe that Section 5.3 is useful for academic researchers who want to influence central banks' analyses. We begin, however, in Section 5.1, with a fundamental prerequisite for good analysis.

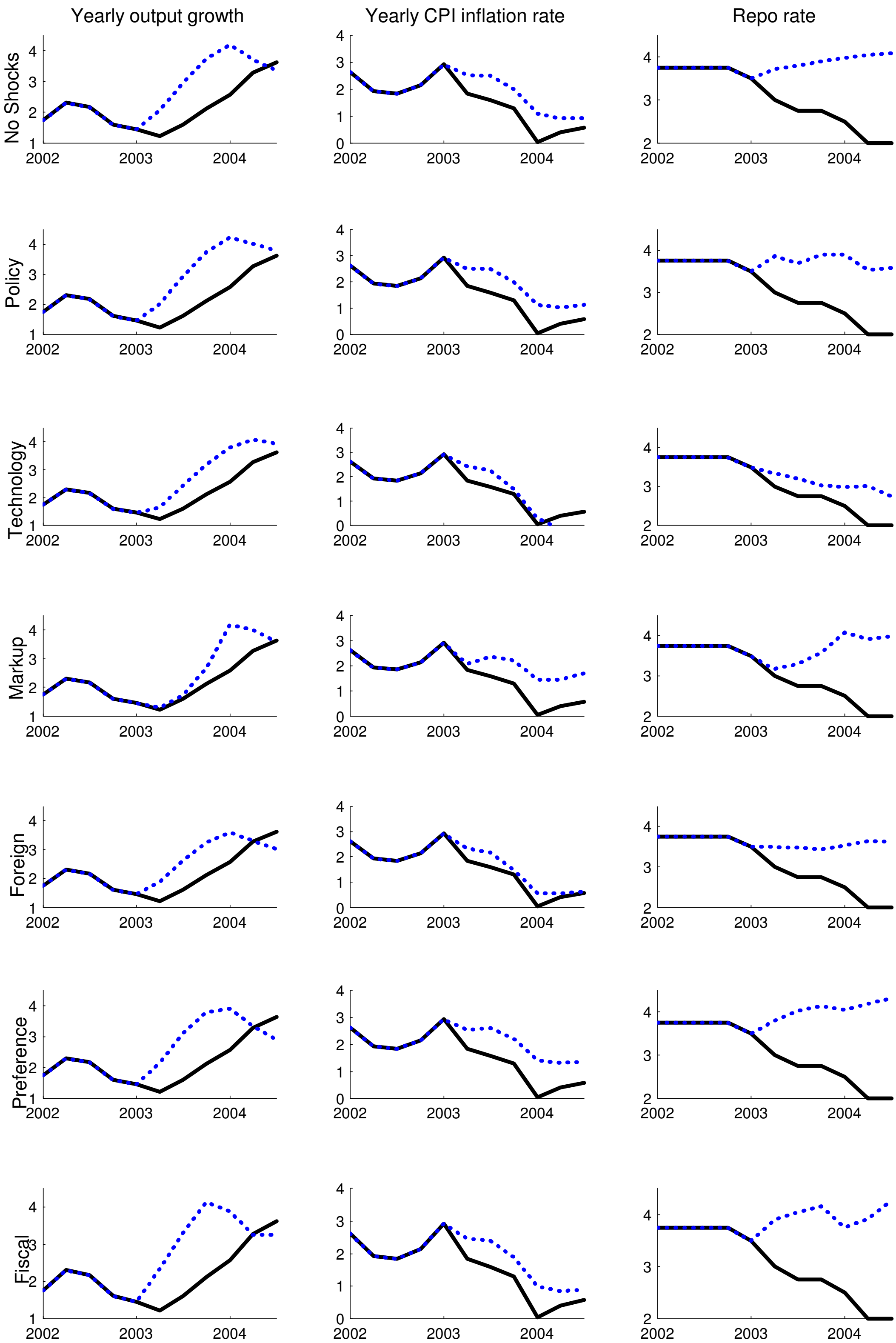

FIGURE 16. Predictions for 2003Q2-2004Q3 from the DSGE model with only subsets of the shocks active during the forecasting period.

5.1. **The importance of publishing all relevant data.** Not all central banks publish their forecasts of inflation and other macro variables that are important for monetary policy, and few central banks have good data bases where real-time data have been stored. This makes it difficult to reconstruct and evaluate forecasts ex post. It also makes it difficult to distinguish and evaluate the influence of formal models versus judgments. This in an important obstacle to progress, because, as noted by Sims (2002, p. 36), "When the role of judgment is kept under the table, it becomes more difficult to discuss it and more difficult to recognize bad judgment".

Although the exercises in this paper are not exclusively based on data that have been published earlier, the Riksbank's experience is that the decision to publish the inflation forecasts has forced us to improve our own analyses. The recent move to publishing forecasts based on two different interest rate assumptions has involved a further check on the underlying logic of the forecasting process. If you only publish one set of forecasts, it is impossible to check whether it makes sense or not. If you publish two sets, that should differ only with respect to one fundamental assumption, the implied "impulse responses" reveal your underlying "model".

5.2. **Advantages of using formal models.**

A.1. It is possible to produce and use BVAR and DSGE models that make about as good forecasts as the much more complicated judgmental procedures typically used at central banks. Formal models can be used as benchmarks for forecasts and policy advice, and to summarize the implications of the continuous flow of new information about the state of the economy that central bank economists are exposed to. It is not impossible to gather this information and to present it in an accessible and fruitful way without the use of formal models, but this work easily becomes extremely time-consuming.

A.2. Formal models make it possible to *decompose forecast errors* into different surprising events, something which is practically impossible with judgmental forecasts. This information about forecast errors can be used to *interpret the current state of the economy.* It can also be used to characterize the uncertainty involved in statements about its future development. An argument that has been used against publishing forecasts from central banks – and against basing policy on such forecasts – is that the forecasts give a false impression of high precision in forecasting and policy making. Formal models make it possible to make credible quantifications of the imprecision and uncertainties involved. Although we have not discussed such issues in this paper, we intend to use the models presented here to produce "fan charts" of the type currently published in e.g. Inflation Reports from Sveriges Riksbank and the Bank of England.

A.3. Formal models can serve as a *learning mechanism* for central bank economists. The models make it possible to accumulate the lessons learnt from mistakes and improvements in earlier analyses. Our results suggest, e.g., that subjective forecast often may be too myopic and not take enough account of important historical regularities in the data. Our present version of the DSGE model may suffer from a model of the central banker that is "too conservative". When properties of various forecast approaches are discussed in this way, the models used also become devices for communication, perhaps primarily within central banks. When the economists work with some common models they believe in, it is easier to avoid being trapped in inefficient "battles of anecdotes".[21]

[21] We are grateful to Guy Debelle for this characterization of what we want to avoid, which he made when commenting Jansson and Vredin (2004).

A.4. Formal models can produce *consistency* in central banks' analyses, both across analyses of different sectors of the economy at a given point in time, and across the analyses made over time. Consistency is important for credibility. This is probably even more important for central bank economists when they are dealing with the large number of questions about how the economy works that they get from policy makers, than in forecasting exercises.

5.3. **Problems in implementing formal models at central banks.**

P.1. Policy makers are often interested in *details* about the state of the current economy. Formal models cannot possibly cover all details within a consistent framework. Neither can sector experts, but their insights about details often lead policy makers to rely on advice and forecasts from the experts rather than from the models. BVAR models could however be developed in order to provide more structure also to the discussions about details: small BVAR models could e.g. be used to summarize information in "indicators"; BVAR models of monthly data could be used to identify relevant new information; etc.

P.2. There are *gaps between different models*. Different models give quite different forecasts and imply different policy recommendations. Researchers are not bothered by that, as long as the models are considered good. Policy makers are of course bothered. It has to be explained why such differences must be expected and why they do not imply that the models are useless. But we also have to work to try to bridge the gap between different models. Perhaps structural VARs that impose more restrictions or identifying assumptions about long run relations can bridge the gap between BVAR and DSGE models. Another possibility is to use Bayesian prior distributions which incorporates identical prior information on features which are common to all models, such as the steady state of the system or impulse response functions, see e.g. Schorfheide and Del Negro (2004) and Villani (2005).

In principle, it makes sense to work with a large number of different models designed for different purposes. BVAR models are relatively easy to construct and evaluate for forecasting purposes. This approach is also flexible in the sense that one can develop a set of BVAR models that can be designed to handle different questions. They are however of limited use when it comes to analyses of different alternative scenarios because the economic relations behind the BVAR's equations are not easily identified. DSGE models are easier to use for such purposes but also, because of their more complex structure, more difficult to adjust to the short-run variations in policy makers' questions. In principle, BVARs and DSGEs can be seen as complements, but when using different models for different purposes one of course runs a risk of inconsistency. This risk is however even larger when forecasts and policy advice are based on subjective judgments.

P.3. There is a *gap between models and sector experts*. There often seems to be a lack of good communication within central banks between those who favour more use of formal aggregated models and those who would like to continue to rely primarily on judgments and sector experts. The list of advantages of formal models provided above may itself be uncontroversial, but unless it is demonstrated how the models can be used to shed light on questions that policy makers ask in practice, such lists with theoretical arguments do not lead to any changes. There is no organized resistance against formal models at central banks, but the proponents of such models have not always been able to present convincing evidence of the models' advantages. It has to be demonstrated how the models can be applied in practical policy work.

It should be emphasized that *governors are not the problem*. They should not be expected to decide what type of forecasting procedures or models that should be used. Governors expect the advice they get to be based on as good information as possible. It is up to their staff to decide what type of information. It should also be emphasized that the results in this paper support the idea that *formal models and sector experts are complements*, not substitutes. A major challenge for future research is the development of techniques to combine judgement and formal models. It seems reasonable to approach this in a Bayesian manner, but it is less clear how to translate the particular form of judgement provided by the sector experts to a usable prior distribution.

P.4. *Formal models can produce consistency in central banks' analyses.* Although this is an important advantage of using formal models, it also imposes discipline on the analyses and discussions within central banks. Some policy advisers may see this as a disadvantage.

P.5. *Resource constraints.* Although central banks have soft budget constraints, there is a limit on how much that can be spent on model development. Researchers have to work also on other problems, and there is also a need for economists that are not active in research. The exercises we have presented above (evaluations of forecasts, conditional forecasts, estimates of the roles of different macroeconomic shocks) are all very useful for policy purposes, but would not have been meaningful without models with good foundations in economic and statistic methodology. This, in turn, requires having good research departments at central banks. The exercises presented above are difficult to perform. Standard software packages are of limited use.

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Appendix A. The BVAR model

The BVAR model contains quarterly data on the following seven variables: trade-weighted measures of foreign GDP growth (y_f), CPI inflation (π_f) and a short term interest rate (i_f), the corresponding domestic variables (y, π and i), and the level of the real exchange rate defined as $q = 100(s + p_f - p)$, where p_f and p are the foreign and domestic CPI levels (in logs) and s is the (log of the) trade-weighted nominal exchange rate.

The BVAR model used in this paper is of the form

$$\Pi(L)(x_t - \Psi d_t) = A\varepsilon_t, \tag{A.1}$$

where $x = (y_f, \pi_f, r_f, y, \pi, r, q)'$ is a p-dimensional vector of time series, $\Pi(L) = I_p - \Pi_1 L - ... - \Pi_k L^k$, L is the usual back-shift operator with the property $Lx_t = x_{t-1}$. The structural disturbances $\varepsilon_t \sim N_p(0, I_p)$, $t = 1, ..., T$, are assumed to be independent across time. A is the lower-triangular (Choleski) contemporaneous impact matrix, such that the covariance matrix Σ of the reduced form disturbances decomposes as $\Sigma = AA'$. We have also experimented with non-recursive identifying restrictions, in which case the equations are normalized with the Waggoner-Zha rule (Waggoner and Zha, 2003b) and the Gibbs sampling algorithm in Waggoner and Zha (2003b) is used to sample from the posterior distribution. The determinstic component is $d_t = (1, d_{MP,t})'$, where

$$d_{MP,t} = \begin{cases} 1 & \text{if } t < 1993Q1 \\ 0 & \text{if } t \geq 1993Q1 \end{cases},$$

is a shift dummy to model the abandonment of the fixed exchange rate and the introduction of an explicit inflation target in $1993Q1$. Since the data are modelled on a quarterly frequency we use $k = 4$ lags in the analysis. Larger lag lengths gave essentially the same results, with a slight increase in parameter uncertainty.

The somewhat non-standard parametrization of the VAR model in (A.1) is non-linear in its parameters, but has the advantage that the unconditional mean, or steady state, of the process is directly specified by Ψ as $E_0(x_t) = \Psi d_t$. This allows us to incorporate prior beliefs directly on the steady state of the system, *e.g.* the information that the steady state inflation is likely to be close to the Riksbank's inflation target. To formulate a prior on Ψ, note that the specification of d_t implies the following parametrization of the steady state

$$E_0(x_t) = \begin{cases} \psi_1 + \psi_2 & \text{if } t < 1993Q1 \\ \psi_1 & \text{if } t \geq 1993Q1 \end{cases},$$

where ψ_i is the ith column of Ψ. The elements in Ψ are assumed to be independent and normally distributed *a priori.* The 95% prior probability intervals are given in Table A.1.

Table A.1. 95% prior probability intervals of Ψ

	y_f	π_f	r_f	y	π	r	q
ψ_1	$(2, 3)$	$(1.5, 2.5)$	$(4.5, 5.5)$	$(2, 2.5)$	$(1.7, 2.3)$	$(4, 4.5)$	$(385, 400)$
ψ_2	$(-1, 1)$	$(1.5, 2.5)$	$(1.5, 2.5)$	$(-0.5, 0)$	$(4.3, 5.7)$	$(3, 5.5)$	$(-50, 0)$

The prior proposed by Litterman (1986) will be used on the dynamic coefficients in Π, with the default values on the hyperparameters in the priors suggested by Doan (1992): overall tightness is set to 0.2, cross-equation tightness to 0.5 and a harmonic lag decay with a hyperparameter equal to one. See Litterman (1986) and Doan (1992) for details. Litterman's prior was designed for data in levels and has the effect of shrinking the process toward the univariate random walk model. We therefore set the prior mean on the first own lag to zero for all the variables in growth rates. The two interest rates and the real exchange rate are assigned a prior which centers on the $AR(1)$ process with a dynamic coefficient equal to 0.9. The usual random walk prior is not used here as it is inconsistent with having a prior on the steady state. We also introduce an additional 'small open economy'-hyperparameter in the prior which shrinks the coefficients on the domestic variables toward zero in the equations for the foreign variables, see Villani and Warne (2003) for details. This hyperparameter is here set to 0.01 which imposes the small open economy restrictions with a very large prior probability. Finally, the usual non-informative prior $|\Sigma|^{-(p+1)/2}$ is used for Σ.

The posterior distribution of the model's parameters and the forecast distribution of the seven endogenous variables were computed numerically by sampling from the posterior distribution with the Gibbs sampling algorithm in Villani (2005).

Appendix B. The estimated DSGE model

In this appendix, we first give a brief presentation of the DSGE model, see Section B.1. In Section B.2, the key log-linearized equations are presented. In Section B.3, the calibrated parameters are reported, along with a table containing the priors and posteriors for the estimated parameters. For more details, we refer the reader to Adolfson et al. (2005b).

B.1. **Overview.** The model extend the closed economy DSGE model of Christiano, Eichenbaum and Evans (2005) and Altig et al. (2003) by incorporating open economy aspects into it in a way similar to Smets and Wouters (2002). Consumption preferences are subject to habit formation and the households attain utility from consuming a basket consisting of domestically produced goods and imported products. These products are supplied by domestic and importing firms, respectively. There is a continuum of households which attain utility from consumption, leisure and real cash balances. The preferences of household j are given by

$$\text{(B.1)} \qquad \mathrm{E}_0^j \sum_{t=0}^{\infty} \beta^t \left[\zeta_t^c \ln\left(C_{j,t} - bC_{j,t-1}\right) - \zeta_t^h A_L \frac{(h_{j,t})^{1+\sigma_L}}{1+\sigma_L} + A_q \frac{\left(\frac{Q_{j,t}}{z_t P_t^d}\right)^{1-\sigma_q}}{1-\sigma_q} \right],$$

where $C_{j,t}$, $h_{j,t}$ and $Q_{j,t}/P_t^d$ denote the j^{th} household's levels of aggregate consumption, work effort and real cash holdings, respectively. To make the real balances stationary when the economy is growing they are scaled by z_t, the unit root technology shock. We allow for internal habit persistence by including $bC_{j,t-1}$. Aggregate consumption is assumed to be given by a basket of domestically produced and imported goods according to the following constant elasticity of substitution (CES) function:

$$\text{(B.2)} \qquad C_t = \left[(1-\omega_c)^{1/\eta_c} \left(C_t^d\right)^{(\eta_c-1)/\eta_c} + \omega_c^{1/\eta_c} \left(C_t^m\right)^{(\eta_c-1)/\eta_c} \right]^{\eta_c/(\eta_c-1)},$$

where C_t^d and C_t^m are consumption of the domestic and imported good, respectively. ω_c is the share of imports in consumption, and η_c is the elasticity of substitution across consumption goods.

The households can save in domestic bonds and foreign bonds, and also hold cash. Following Benigno (2001), we assume that there is a premium on the foreign bond holdings which depends on the aggregate net foreign asset position of the domestic households. This ensures a well defined steady-state in the model.

The households invest in a basket of domestic and imported investment goods to form the physical capital stock, and decide how much capital services to rent to the domestic firms, given certain capital adjustment costs. These are costs to adjusting the investment rate as well as costs of varying the utilization rate of the physical capital stock. Total investment is assumed to be given by a CES aggregate of domestic and imported investment goods (I_t^d and I_t^m, respectively) according to

$$I_t = \left[(1-\omega_i)^{1/\eta_i}\left(I_t^d\right)^{(\eta_i-1)/\eta_i} + \omega_i^{1/\eta_i}\left(I_t^m\right)^{(\eta_i-1)/\eta_i}\right]^{\eta_i/(\eta_i-1)}, \tag{B.3}$$

where ω_i is the share of imports in investment, and η_i is the elasticity of substitution across investment goods.

Further, along the lines of Erceg, Henderson and Levin (2000), each household is a monopoly supplier of a differentiated labour service which implies that they can set their own wage. This gives rise to a wage equation with Calvo (1983) stickiness.

There is a continuum of intermediate domestic firms that each produce a differentiated good. These intermediate goods are sold to a retailer which transforms the intermediate products into a homogenous final good that in turn is sold to the households. The domestic firms determine the capital services and labour inputs used in their production which is exposed to unit root technology growth as in Altig et al. (2003). Production of the domestic intermediate good i follows

$$Y_{i,t} = z_t^{1-\alpha}\epsilon_t K_{i,t}^{\alpha} H_{i,t}^{1-\alpha} - z_t\phi, \tag{B.4}$$

where z_t is a unit-root technology shock, ϵ_t is a covariance stationary technology shock, and $H_{i,t}$ denotes homogeneous labour hired by the i^{th} firm. Notice that $K_{i,t}$ is not the physical capital stock, but rather the capital services stock, since we allow for variable capital utilization in the model. Note also that a fixed cost is included in the production function to ensure that profits are zero in steady state.

The domestic firms, the importing and exporting firms all produce differentiated goods and set prices according to an indexation variant of the Calvo model. By including nominal rigidities in the importing and exporting sectors we allow for short-run incomplete exchange rate pass-through to both import and export prices, following for example Smets and Wouters (2002).

To simplify the analysis we adopt the assumption that the foreign prices, output (HP-detrended) and interest rate are exogenously given by an identified VAR(4) model. The fiscal policy variables - taxes on capital income, labour income, consumption, and the pay-roll, together with (HP-detrended) government expenditures - are assumed to follow an identified VAR(2) model.

B.2. **Key equations in the log-linearized model.** The first-order conditions of the households and the firms are log-linearized around the steady state, according to the following. The domestic (d), importing consumption (mc), importing investment (mi) and exporting (x) firms operating in this economy each have a particular Phillips curve:

$$\begin{aligned}\left(\widehat{\pi}_t^j - \widehat{\bar{\pi}}_t^c\right) &= \frac{\beta}{1+\kappa_j\beta}\left(\mathrm{E}_t\widehat{\pi}_{t+1}^j - \rho_\pi\widehat{\bar{\pi}}_t^c\right) + \frac{\kappa_j}{1+\kappa_j\beta}\left(\widehat{\pi}_{t-1}^j - \widehat{\bar{\pi}}_t^c\right) \\ &\quad -\frac{\kappa_j\beta\left(1-\rho_\pi\right)}{1+\kappa_j\beta}\widehat{\bar{\pi}}_t^c + \frac{(1-\xi_j)(1-\beta\xi_j)}{\xi_j\left(1+\kappa_j\beta\right)}\left(\widehat{mc}_t^j + \widehat{\lambda}_t^j\right),\end{aligned} \tag{B.5}$$

where $j=\{d,mc,mi,x\}$, $\widehat{\pi}_t^j = (\hat{P}_t^j - \hat{P}_{t-1}^j)$ denotes inflation in sector j, and $\widehat{\bar{\pi}}_t^c$ a time-varying inflation target of the central bank.[22] The ξ:s are the Calvo price stickiness parameters in each

[22] A hat denotes log-linearized variables throughout the paper (i.e, $\widehat{X}_t = dX_t/X$), and variables without time-subscript steady-state values. Variables denoted with small letters have been stationarized with the unit root technology shock.

sector, and the κ:s are the indexation parameters.[23] The time-varying markups in the four intermediate markets, $\widehat{\lambda}_t^d$, $\widehat{\lambda}_t^{mc}$, $\widehat{\lambda}_t^{mi}$, and $\widehat{\lambda}_t^x$ are assumed to be i.i.d. $N\left(0,\sigma^2_{\lambda^j}\right)$. The firms' marginal costs are defined as $\widehat{mc}_t^d=\alpha\left(\hat{\mu}_{z,t}+\hat{H}_t-\hat{k}_t\right)+\widehat{\bar{w}}_t+\hat{R}_t^f-\hat{\epsilon}_t$, $\widehat{mc}_t^{mc}=\widehat{P}_t^*+\widehat{S}_t-\widehat{P}_t^{mc}$, $\widehat{mc}_t^{mi}=\widehat{P}_t^*+\widehat{S}_t-\widehat{P}_t^{mi}$, and $\widehat{mc}_t^x=\widehat{P}_t^d-\widehat{S}_t-\widehat{P}_t^x$, respectively. $\hat{\mu}_{z,t}$ is the stochastic growth rate of the unit root technology shock, $\hat{H}_t$ hours worked, $\hat{k}_t$ the capital services stock, $\widehat{\bar{w}}_t$ the real wage, and $\hat{R}_t^f$ the effective nominal interest rate paid by the firms, reflecting the assumption that a fraction ν of the firms' wage bill has to be financed in advance (throughout the paper, we set $\nu=1$). $\hat{\epsilon}_t$ is a stationary technology shock, $\widehat{P}_t^*$ the foreign price level and $\widehat{S}_t$ is the nominal exchange rate. The processes for $\mu_{z,t}$, $\hat{\epsilon}_t$ and $\bar{\pi}_t^c$ are given by

$$\text{(B.6)}\qquad \mu_{z,t} = (1-\rho_{\mu_z})\mu_z+\rho_{\mu_z}\mu_{z,t-1}+\varepsilon_{\mu_z,t},\ ,\ \varepsilon_{\mu_z,t}\sim i.i.d.N\left(0,\sigma^2_{\mu_z}\right),$$

$$\text{(B.7)}\qquad \hat{\epsilon}_t = \rho_\epsilon\hat{\epsilon}_{t-1}+\varepsilon_{\epsilon,t},\ ,\ \varepsilon_{\epsilon,t}\sim i.i.d.N\left(0,\sigma^2_\epsilon\right),$$

$$\text{(B.8)}\qquad \bar{\pi}_t^c = (1-\rho_\pi)\,\bar{\pi}^c+\rho_\pi\bar{\pi}_{t-1}^c+\varepsilon_{\bar{\pi}^c,t},\ \varepsilon_{\bar{\pi}^c,t}\sim i.i.d.N\left(0,\sigma^2_{\bar{\pi}^c}\right).$$

Under the assumption that those households that are not allowed to reoptimize their nominal wage in the current period instead update it according to the indexation scheme $W_{t+1}=(\pi_t^c)^{\kappa_w}\left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w)}\mu_{z,t+1}W_t$, the real wage equation can be written

$$\text{(B.9)}\qquad \mathrm{E}_t\left[\begin{array}{c}\eta_0\widehat{\bar{w}}_{t-1}+\eta_1\widehat{\bar{w}}_t+\eta_2\widehat{\bar{w}}_{t+1}+\eta_3\left(\hat{\pi}_t^d-\widehat{\bar{\pi}}_t^c\right)+\eta_4\left(\hat{\pi}_{t+1}^d-\rho_{\widehat{\bar{\pi}}^c}\widehat{\bar{\pi}}_t^c\right)\\ +\eta_5\left(\hat{\pi}_{t-1}^c-\widehat{\bar{\pi}}_t^c\right)+\eta_6\left(\hat{\pi}_t^c-\rho_{\widehat{\bar{\pi}}^c}\widehat{\bar{\pi}}_t^c\right)\\ +\eta_7\hat{\psi}_{z,t}+\eta_8\hat{H}_t+\eta_9\hat{\tau}_t^y+\eta_{10}\hat{\tau}_t^w+\eta_{11}\hat{\zeta}_t^h\end{array}\right]=0,$$

where $\hat{\pi}_t^c$ denotes CPI inflation, $\hat{\psi}_{z,t}$ the marginal utility of one additional income unit, $\hat{\tau}_t^y$ a labour income tax, $\hat{\tau}_t^w$ a pay-roll tax assumed to be paid by the households, and $\hat{\zeta}_t^h$ a labour supply shock. The η:s are composite parameters determined by the Calvo wage stickiness ξ_w, the pay-roll tax τ^w, the labour income tax τ^y, the labour supply elasticity σ_L, the wage markup λ_w, the wage indexation κ_w, and the discount factor β.

The households' consumption preferences are subject to internal habit formation, which yields the following Euler equation for consumption expenditures:

$$\text{(B.10)}\qquad \mathrm{E}_t\left[\begin{array}{c}-b\beta\mu_z\hat{c}_{t+1}+\left(\mu_z^2+b^2\beta\right)\hat{c}_t-b\mu_z\hat{c}_{t-1}+b\mu_z\left(\hat{\mu}_{z,t}-\beta\hat{\mu}_{z,t+1}\right)+\\ +\left(\mu_z-b\beta\right)\left(\mu_z-b\right)\hat{\psi}_{z,t}+\frac{\tau^c}{1+\tau^c}\left(\mu_z-b\beta\right)\left(\mu_z-b\right)\hat{\tau}_t^c\\ +\left(\mu_z-b\beta\right)\left(\mu_z-b\right)\hat{\gamma}_t^{c,d}-\left(\mu_z-b\right)\left(\mu_z\hat{\zeta}_t^c-b\beta\hat{\zeta}_{t+1}^c\right)\end{array}\right]=0,$$

where $\hat{c}_t$ is consumption, $\hat{\tau}_t^c$ a consumption tax, $\hat{\gamma}_t^{c,d}$ the relative price between consumption and domestically produced goods, $\hat{\zeta}_t^c$ a consumption preference shock, b the habit persistence parameter, and μ_z is the steady-state growth rate. The exogenous processes for the preference shocks $\hat{\zeta}_t^c$ and $\hat{\zeta}_t^h$ in (B.1) are given by

$$\text{(B.11)}\qquad \hat{\zeta}_t^c = \rho_{\zeta^c}\hat{\zeta}_{t-1}^c+\varepsilon_{\zeta^c,t},\ \varepsilon_{\zeta^c,t}\sim i.i.d.N\left(0,\sigma^2_{\zeta^c}\right),$$

$$\text{(B.12)}\qquad \hat{\zeta}_t^h = \rho_{\zeta^h}\hat{\zeta}_{t-1}^h+\varepsilon_{\zeta^h,t},\ \varepsilon_{\zeta^h,t}\sim i.i.d.N\left(0,\sigma^2_{\zeta^h}\right).$$

[23]For the firms that are not allowed to reoptimize their price, we adopt the indexation scheme $P_{t+1}^j=\left(\pi_t^j\right)^{\kappa_j}\left(\bar{\pi}_{t+1}^c\right)^{1-\kappa_j}P_t^j$ where $j=\{d,mc,mi,x\}$.

By combining the first order conditions for the domestic and foreign bond holdings we obtain the following modified uncovered interest rate parity condition:

$$\widehat{R}_t - \widehat{R}_t^* = \left(1 - \widetilde{\phi}_s\right) \mathrm{E}_t \Delta \widehat{S}_{t+1} - \widetilde{\phi}_s \Delta \widehat{S}_t - \widetilde{\phi}_a \widehat{a}_t + \widehat{\widetilde{\phi}}_t, \tag{B.13}$$

where $\widehat{R}_t$ is the domestic nominal interest rate, $\widehat{R}_t^*$ the foreign nominal interest rate, $\widehat{a}_t$ the net foreign asset position, and $\widehat{\widetilde{\phi}}_t$ a shock to the risk premium. Because of our assumption of imperfect integration in the international financial markets, the net foreign asset position and the change in the current nominal exchange rate enters.[24] The process for the exogenous source of variation in the risk-premium is given by

$$\widehat{\widetilde{\phi}}_t = \rho_{\widetilde{\phi}} \widehat{\widetilde{\phi}}_{t-1} + \varepsilon_{\widetilde{\phi},t}, \; \varepsilon_{\widetilde{\phi},t} \sim i.i.d.N\left(0, \sigma^2_{\widetilde{\phi}}\right). \tag{B.14}$$

The households' first order conditions for the physical capital stock ($\widehat{\bar{k}}_t$), investment ($\hat{\imath}_t$), and the utilization rate ($\widehat{u}_t = \widehat{k}_t - \widehat{\bar{k}}_t$, where $\widehat{k}_t$ denotes capital services) can be written:

$$\begin{aligned} &\hat{\psi}_{z,t} + \mathrm{E}_t \hat{\mu}_{z,t+1} - \mathrm{E}_t \hat{\psi}_{z,t+1} - \tfrac{\beta(1-\delta)}{\mu_z} \mathrm{E}_t \hat{P}_{k',t+1} + \hat{P}_{k',t} \\ &\quad - \tfrac{\mu_z - \beta(1-\delta)}{\mu_z} \mathrm{E}_t \hat{r}^k_{t+1} + \tfrac{\tau^k}{(1-\tau^k)} \tfrac{\mu_z - \beta(1-\delta)}{\mu_z} \mathrm{E}_t \hat{\tau}^k_{t+1} = 0, \end{aligned} \tag{B.15}$$

$$\hat{P}_{k',t} + \hat{\Upsilon}_t - \hat{\gamma}^{i,d}_t - \mu_z^2 \tilde{S}'' \left[(\hat{\imath}_t - \hat{\imath}_{t-1}) - \beta (\hat{\imath}_{t+1} - \hat{\imath}_t) + \hat{\mu}_{z,t} - \beta \mathrm{E}_t \hat{\mu}_{z,t+1} \right] = 0, \tag{B.16}$$

$$\widehat{u}_t = \frac{1}{\sigma_a} \widehat{r}^k_t - \frac{1}{\sigma_a} \frac{\tau^k}{(1-\tau^k)} \widehat{\tau}^k_t, \tag{B.17}$$

where $\hat{P}_{k',t}$ is the price of capital, $\widehat{r}^k_t$ the firms' real rental rate of capital services given by $\hat{r}^k_t = \hat{\mu}_{z,t} + \widehat{\bar{w}}_t + \hat{R}^f_t + \hat{H}_t - \hat{k}_t$, $\hat{\Upsilon}_t$ an investment specific technology shock, $\hat{\gamma}^{i,d}_t$ the relative price between investment and domestically produced goods, $\hat{\tau}^k_t$ a capital income tax, $\tilde{S}''$ the adjustment cost of changing investments, δ the depreciation rate, and σ_a the cost of varying the capital utilization rate. The process for the investment-specific technology shocks is given by

$$\hat{\Upsilon}_t = \rho_{\widetilde{\phi}} \hat{\Upsilon}_{t-1} + \varepsilon_{\Upsilon,t}, \; \varepsilon_{\Upsilon,t} \sim i.i.d.N\left(0, \sigma^2_{\Upsilon}\right). \tag{B.18}$$

The log-linearized law of motion for the physical capital stock is given by

$$\widehat{\bar{k}}_{t+1} = (1-\delta)\frac{1}{\mu_z}\widehat{\bar{k}}_t - (1-\delta)\frac{1}{\mu_z}\hat{\mu}_{z,t} + \left(1 - (1-\delta)\frac{1}{\mu_z}\right)\hat{\Upsilon}_t + \left(1 - (1-\delta)\frac{1}{\mu_z}\right)\hat{\imath}_t. \tag{B.19}$$

[24] We adapt the follow specification of the risk-premium, $\Phi(a_t, S_{t+1}/S_{t-1}, \widehat{\widetilde{\phi}}_t) = e^{-\widetilde{\phi}_s(S_{t+1}/S_{t-1}-1) - \widetilde{\phi}_a(a_t - \overline{a}) + \widehat{\widetilde{\phi}}_t}$, which differ to the one considered by Benigno (2001) in that it includes the expected gross change in the nominal exchange rate. There is, however, strong evidence according to Duarte and Stockman (2005) that the risk-premium is negatively correlated with the expected change in the nominal exchange rate. By allowing this possibility in the estimated DSGE model, we provide an alternative assessment of their claim by comparing in- and out-of-sample performance of the model where $\widetilde{\phi}_s = 0$ and $\widetilde{\phi}_s \neq 0$.

The evolution of net foreign assets at the aggregate level satisfies

$$
\begin{aligned}
\hat{a}_t &= -y^*\widehat{mc}_t^x - \eta_f y_t^* \widehat{\gamma}_t^{x,*} + y^*\hat{y}_t^* + y^*\widehat{\tilde{z}}_t^* + (c^m + i^m)\widehat{\gamma}_t^f \\
&\quad -c^m\left[-\eta_c(1-\omega_c)\left(\gamma^{c,d}\right)^{-(1-\eta_c)}\widehat{\gamma}_t^{mc,d} + \widehat{c}_t\right] \\
&\quad -i^m\left[-\eta_i(1-\omega_i)\left(\gamma^{i,d}\right)^{-(1-\eta_i)}\widehat{\gamma}_t^{mi,d} + \widehat{i}_t\right] + \frac{R}{\pi\mu_z}\hat{a}_{t-1},
\end{aligned} \tag{B.20}
$$

where $\hat{y}_t^*$ denotes foreign output, $\widehat{\gamma}_t^{x,*}$,$\widehat{\gamma}_t^f$, $\widehat{\gamma}_t^{mc,d}$ and $\widehat{\gamma}_t^{mi,d}$ are relative prices defined as $\widehat{\gamma}_t^{x,*} = \hat{P}_t^x - \hat{P}_t^*$, $\widehat{\gamma}_t^f = \hat{P}_t^d - \hat{S}_t - \hat{P}_t^*$, $\widehat{\gamma}_t^{mc,d} = \hat{P}_t^{mc} - \hat{P}_t^d$ and $\widehat{\gamma}_t^{mi,d} = \hat{P}_t^{mi} - \hat{P}_t^d$, respectively. $\widehat{\tilde{z}}_t^*$ is a stationary shock which measures the degree of asymmetry in the technological progress in the domestic economy versus the rest of the world, assumed to follow

$$
\widehat{\tilde{z}}_t^* = \rho_{\tilde{z}^*}\widehat{\tilde{z}}_{t-1}^* + \varepsilon_{\tilde{z}^*,t},\ \varepsilon_{\tilde{z}^*,t} \sim i.i.d.N\left(0,\sigma_{\tilde{z}^*}^2\right). \tag{B.21}
$$

The log-linearized first order conditions for money balances and the households cash holdings are, respectively:

$$
\mathrm{E}_t\left[-\mu\hat{\psi}_{z,t} + \mu\hat{\psi}_{z,t+1} - \mu\hat{\mu}_{z,t+1} + \left(\mu - \beta\tau^k\right)\hat{R}_t - \mu\hat{\pi}_{t+1} + \frac{\tau^k}{1-\tau^k}(\beta-\mu)\hat{\tau}_{t+1}^k\right] = 0, \tag{B.22}
$$

$$
q_t = \frac{1}{\sigma_q}\left[\frac{\tau^k}{1-\tau^k}\widehat{\tau}_t^k - \widehat{\psi}_{z,t} - \frac{R}{R-1}\widehat{R}_{t-1}\right]. \tag{B.23}
$$

The log-linearized aggregate resource constraint is

$$
\begin{aligned}
&(1-\omega_c)\left(\gamma^{c,d}\right)^{\eta_c}\frac{c}{y}\left(\hat{c}_t + \eta_c\widehat{\gamma}_t^{c,d}\right) + (1-\omega_i)\left(\gamma^{i,d}\right)^{\eta_i}\frac{i}{y}\left(\hat{\imath}_t + \eta_i\widehat{\gamma}_t^{i,d}\right) \\
&+\frac{g}{y}\hat{g}_t + \frac{y^*}{y}\left(\hat{y}_t^* - \eta_f\widehat{\gamma}_t^{x,*} + \widehat{\tilde{z}}_t^*\right) \\
= \\
&\lambda_f\left(\hat{\epsilon}_t + \alpha\left(\hat{k}_t - \hat{\mu}_{z,t}\right) + (1-\alpha)\hat{H}_t\right) - \left(1-\tau^k\right)r^k\frac{\bar{k}}{y}\frac{1}{\mu_z}\left(\hat{k}_t - \widehat{\bar{k}}_t\right),
\end{aligned} \tag{B.24}
$$

where $\widehat{\gamma}_t^{i,d}$ is the relative price between investment and domestically produced goods.

To clear the loan market, the demand for liquidity from the firms (which are financing their wage bills) must equal the supplied deposits of the households plus the monetary injection by the central bank:

$$
\nu\bar{w}H\left(\hat{\nu}_t + \widehat{\bar{w}}_t + \hat{H}_t\right) = \frac{\mu\bar{m}}{\pi\mu_z}\left(\hat{\mu}_t + \widehat{\bar{m}}_t - \hat{\pi}_t - \hat{\mu}_{z,t}\right) - q\hat{q}_t, \tag{B.25}
$$

We also need to relate money growth to real balances, domestic inflation and real growth, i.e.

$$
\mu_t = \frac{M_{t+1}}{M_t} = \frac{\bar{m}_{t+1}z_tP_t}{\bar{m}_tz_{t-1}P_{t-1}} = \frac{\bar{m}_{t+1}\mu_{z,t}\pi_t}{\bar{m}_t},
$$

and log-linearizing, we have

$$
\hat{\mu}_t - \widehat{\bar{m}}_{t+1} - \hat{\mu}_{z,t} - \hat{\pi}_t + \widehat{\bar{m}}_t = 0. \tag{B.26}
$$

Following Smets and Wouters (2003), monetary policy is approximated with the instrument rule

$$
\begin{aligned}
\widehat{R}_t &= \rho_R\widehat{R}_{t-1} + (1-\rho_R)\left[\widehat{\bar{\pi}}_t^c + r_\pi\left(\hat{\pi}_{t-1}^c - \widehat{\bar{\pi}}_t^c\right) + r_y\hat{y}_{t-1} + r_x\hat{x}_{t-1}\right] \\
&\quad + r_{\Delta\pi}\left(\hat{\pi}_t^c - \hat{\pi}_{t-1}^c\right) + r_{\Delta y}\Delta\hat{y}_t + \varepsilon_{R,t},
\end{aligned} \tag{B.27}
$$

where

$$\varepsilon_{R,t} \sim i.i.d.N\left(0, \sigma_{\varepsilon_R}^2\right).$$

Thus, the central bank is assumed to adjust the short term interest rate in response to deviations of CPI inflation from the time-varying inflation target $\left(\hat{\pi}_t^c - \widehat{\bar{\pi}}_t^c\right)$, the output gap ($\hat{y}_t$, measured as actual minus trend output), the real exchange rate $\left(\hat{x}_t = \hat{S}_t + \hat{P}_t^* - \hat{P}_t^c\right)$ and the interest rate set in the previous period. In addition, note that the nominal interest rate adjusts directly to the inflation target $\widehat{\bar{\pi}}_t^c = \bar{\pi}_t^c - \bar{\pi}^c$ (measured as the deviation from the steady state target rate, see B.8).

To solve the log-linearized model we use the AIM algorithm developed by Anderson and Moore (1985). As a first step, we cast the model on matrix form as

$$\text{(B.28)} \qquad \mathrm{E}_t \left\{\alpha_0 \tilde{z}_{t+1} + \alpha_1 \tilde{z}_t + \alpha_2 \tilde{z}_{t-1} + \beta_0 \theta_{t+1} + \beta_1 \theta_t\right\} = 0,$$

where the parameter matrices α_0, α_1, α_2, β_0 and β_1 corresponds to the equations (B.5 − B.27). In (B.28), $\tilde{z}_t$ is a $n_{\tilde{z}} \times 1$ vector with endogenous variables and θ_t is a $n_\theta \times 1$ vector with exogenous variables which follows

$$\text{(B.29)} \qquad \theta_t = \rho\theta_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \Sigma).$$

Note that since some of the processes for the exogenous variables are given by more than one lag, we expand θ_t with lags of the relevant exogenous variable.

The solution of the fundamental difference equation can then be written as

$$\text{(B.30)} \qquad \tilde{z}_t = A\tilde{z}_{t-1} + B\theta_t$$

where A is the "feedback" matrix and B is the "feed-forward" matrix.

B.3. **Calibrated and estimated parameters.** The solution of the model given by (B.30) and (B.29) can be transformed to the following state-space representation for the unobserved state variables ξ_t in the model

$$\text{(B.31)} \qquad \xi_{t+1} = F_\xi \xi_t + v_{t+1}, \ \mathrm{E}\left(v_{t+1}v_{t+1}'\right) = Q,$$

and the observation equation can be written

$$\text{(B.32)} \qquad \tilde{Y}_t = A_X' X_t + H'\xi_t + \zeta_t,$$

where $\tilde{Y}_t$ is a vector of observed variables, X_t a vector with exogenous or predetermined variables (e.g., a constant) and where we assume the measurement errors ζ_t to be normally distributed with zero mean and $\mathrm{E}\left(\zeta_t\zeta_t'\right) = R$. We set $R = 0.20$ for all "domestic" variables and $R = 0$ for the foreign variables. We allow the entries on the diagonal of R to be non-zero for the domestic variables because we know that the data series at hand are only at best good approximations of the true series. However, it should be emphasized that our chosen values of 0.20 are very small, so the fluctuations driven by the measurement errors are tiny.

In order to ensure proper identification of the various shocks and parameters that we estimate (we estimate 7 shocks that follows AR(1) processes, and 6 shocks that are assumed to be i.i.d.), we include the following set of observable variables in (B.32):

$$\text{(B.33)} \qquad \tilde{Y}_t = \begin{array}{l} [\ \pi_t^d \quad \Delta\ln(W_t/P_t) \quad \Delta\ln C_t \quad \Delta\ln I_t \quad \hat{x}_t \quad R_t \quad \hat{H}_t \quad \Delta\ln Y_t \ldots \\ \qquad \Delta\ln\tilde{X}_t \quad \Delta\ln\tilde{M}_t \quad \pi_t^{def,c} \quad \pi_t^{def,i} \quad \Delta\ln Y_t^* \quad \pi_t^* \quad R_t^* \ ]'. \end{array}$$

Despite the fact that the foreign variables are exogenous, we still include them as observable variables as they enable identification of the asymmetric technology shock.

A subset of the parameters are calibrated (strict priors), whereas another subset of parameters are estimated using Bayesian techniques. We choose to calibrate those parameters which we think are weakly identified by the variables that we include in (B.33) and that we have rather good information about using other sources of information. The remaining parameters are estimated. The estimated parameters pertains mostly to the nominal and real frictions in the model as well as the exogenous shock processes described above. The sample period is $1980Q1 - 2004Q3$, and we use the first 6 years to obtain an prior on the unobserved state, and use the subsample $1986Q1 - 2004Q3$ for inference.

In Table B.1, the calibrated parameters are reported along with the implied steady state nominal interest rate and inflation rates, and the investment-output, consumption-output, government expenditures-output and import/export-output ratios. In addition to calibrating those parameters, the parameters related to the exogenous VARs for the fiscal and foreign variables are kept constant and equal to their point estimates (estimated separately prior to estimating the DSGE model) throughout the estimation.

Table B.1: Calibrated parameters

Parameter	Description	Calibrated value
β	Households' discount factor	0.999
α	Capital share of income	0.25
σ_a	Capital utilization cost parameter	0.05
η_c	Substitution elasticity between C_t^d and C_t^m	5.00
μ	Money growth rate (quarterly rate)	1.01
σ_L	Labor supply elasticity	1.00
δ	Depreciation rate	0.01
λ_w	Wage markup	1.05
ω_i	Share of imported investment goods	0.70
ω_c	Share of imported consumption goods	0.40
ν	Share of wage bill financed by loans	1.00
τ^y	Labor income tax rate	0.30
τ^c	Value added tax rate	0.24
τ^w	Pay-roll tax rate	0.30
$\rho_{\bar{\pi}}$	Inflation target persistence	0.975
	Implied steady state relationships[a]	
$\bar{\pi}^c$	Steady state inflation rate (percent)	1.80
R	Nominal interest rate (percent)	4.66
C/Y	Consumption output ratio	0.50
I/Y	Investment output ratio	0.20
G/Y	Government expenditures output ratio	0.30
$\tilde{X}/Y = \tilde{M}/Y$	Export/Import output ratio	0.24
$S_{t+1} = S_t$	Nominal exchange rate	1.00
A	Net foreign assets	0.00
X	Real exchange rate	1.00

Notes: [a]As some of the parameters affecting the steady state are estimated (see Table B.2), e.g. μ_z, λ_d, $\lambda_{m,c}$ and $\lambda_{m,i}$, the steady state values for some of the variables in the Table will differ somewhat between for the prior/posterior. What is reported in the Table are values implied by our prior.

Table B.2: Prior and posterior distributions in the estimated DSGE model

Parameter		Prior distribution			Posterior distribution				
		type	mean*	std.dev. /df	mode	std. dev. (Hess-ian)	mean	5%	95%
Calvo wages	ξ_w	beta	0.750	0.050	0.691	0.051	0.716	0.627	0.800
Calvo domestic prices	ξ_d	beta	0.750	0.050	0.787	0.051	0.767	0.672	0.862
Calvo import cons. prices	$\xi_{m,c}$	beta	0.750	0.100	0.900	0.016	0.893	0.862	0.922
Calvo import inv. prices	$\xi_{m,i}$	beta	0.750	0.100	0.931	0.011	0.927	0.905	0.947
Calvo export prices	ξ_x	beta	0.750	0.100	0.853	0.022	0.856	0.814	0.892
Indexation wages	κ_w	beta	0.500	0.150	0.270	0.073	0.356	0.156	0.591
Indexation domestic prices	κ_d	beta	0.500	0.150	0.180	0.079	0.248	0.101	0.444
Index. import cons. prices	$\kappa_{m,c}$	beta	0.500	0.150	0.357	0.099	0.375	0.199	0.570
Index. import inv. prices	$\kappa_{m,i}$	beta	0.500	0.150	0.565	0.110	0.522	0.282	0.748
Indexation export prices	κ_x	beta	0.500	0.150	0.386	0.080	0.371	0.197	0.562
Markup domestic	λ_d	trunc. normal	1.200	0.05	1.201	0.049	1.193	1.112	1.275
Markup imported cons.	$\lambda_{m,c}$	trunc. normal	1.200	0.05	1.357	0.032	1.347	1.290	1.403
Markup.imported invest.	$\lambda_{m,i}$	trunc. normal	1.200	0.05	1.258	0.042	1.251	1.179	1.323
Investment adj. cost	$\tilde{S}''$	Normal	7.500	1.500	8.655	1.357	8.416	6.195	10.686
Habit formation	b	Beta	0.650	0.100	0.676	0.053	0.673	0.578	0.768
Subst. elasticity invest.	η_i	inv. Gamma	1.500	4	2.505	0.275	2.538	2.039	3.058
Subst. elasticity foreign	η_f	inv. Gamma	1.500	4	1.495	0.131	1.570	1.348	1.904
Technology growth	μ_z	trunc. normal	1.006	0.0005	1.005	0.000	1.005	1.004	1.005
Risk premium – nfa	$\tilde{\phi}_a$	inv. gamma	0.010	2	0.026	0.012	0.047	0.018	0.097
Risk premium – nom exch	$\tilde{\phi}_s$	beta	0.500	0.150	0.482	0.037	0.522	0.423	0.649
Unit root tech. shock	ρ_{μ_z}	beta	0.850	0.100	0.770	0.030	0.774	0.612	0.927
Stationary tech. shock	ρ_ε	beta	0.850	0.100	0.920	0.034	0.880	0.756	0.954
Invest. spec. tech shock	ρ_Υ	beta	0.850	0.100	0.800	0.047	0.726	0.565	0.845
Asymmetric tech. shock	$\rho_{\tilde{z}*}$	beta	0.850	0.100	0.959	0.018	0.948	0.910	0.979
Consumption pref. shock	ρ_{ζ_c}	beta	0.850	0.100	0.894	0.051	0.817	0.618	0.940
Labour supply shock	ρ_{ζ_h}	beta	0.850	0.100	0.243	0.049	0.272	0.164	0.392
Risk premium shock	$\rho_{\tilde{\phi}}$	beta	0.850	0.100	0.887	0.045	0.837	0.674	0.946
Unit root tech. shock	σ_z	inv. gamma	0.200	2	0.118	0.021	0.126	0.092	0.169
Stationary tech. shock	σ_ε	inv. gamma	0.700	2	0.767	0.043	0.802	0.640	0.987
Invest. spec. tech. shock	σ_Υ	inv. gamma	0.200	2	0.619	0.082	0.746	0.546	0.986
Asymmetric tech. shock	$\sigma_{\tilde{z}*}$	inv. gamma	0.400	2	0.190	0.027	0.197	0.154	0.248
Consumption pref. shock	σ_{ζ_c}	inv. gamma	0.200	2	0.183	0.038	0.192	0.133	0.266
Labour supply shock	σ_{ζ_h}	inv. gamma	0.200	2	0.370	0.035	0.387	0.323	0.462
Risk premium shock	$\sigma_{\tilde{\phi}}$	inv. gamma	0.050	2	0.375	0.069	0.540	0.308	0.917
Domestic markup shock	σ_λ	inv. gamma	0.300	2	0.736	0.067	0.751	0.636	0.883
Imp. cons. markup shock	$\sigma_{\lambda_{m,c}}$	inv. gamma	0.300	2	0.937	0.090	0.955	0.799	1.137
Imp. invest. markup shock	$\sigma_{\lambda_{m,i}}$	inv. gamma	0.300	2	1.014	0.105	1.057	0.868	1.281
Export markup shock	σ_{λ_x}	inv. gamma	0.300	2	0.999	0.132	1.012	0.779	1.276
Monetary policy shock	σ_R	inv. gamma	0.150	2	0.210	0.024	0.220	0.180	0.266
Inflation target shock	$\sigma_{\bar{\pi}^c}$	inv. gamma	0.050	2	0.059	0.027	0.098	0.041	0.185
Interest rate smoothing	ρ_R	beta	0.800	0.050	0.906	0.020	0.908	0.873	0.937
Inflation response	r_π	normal	1.700	0.100	1.709	0.052	1.722	1.554	1.885
Diff. infl response	$r_{\Delta\pi}$	normal	0.300	0.100	0.179	0.033	0.181	0.127	0.237
Real exch. rate response	r_x	normal	0.000	0.050	-0.053	0.021	-0.053	-0.096	-0.009
Output response	r_y	normal	0. 250	0.050	0.133	0.053	0.158	0.075	0.248
Diff. output response	$r_{\Delta\pi}$	normal	0.065	0.050	0.179	0.033	0.181	0.127	0.237
Log marginal likelihood							-2370.1		

*Note: For the inverse gamma distribution. the mode and the degrees of freedom are reported. Also. for the parameters $\lambda_d, \eta_i, \eta_f, \lambda_{m,c}, \lambda_{m,i}$ and μ_z the prior distributions are truncated at 1.