



The Growth and Welfare Effects of Macroeconomic Volatility

Universitat Pompeu Fabra, Barcelona
17-18 March 2006

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We would like to thank CREI for their generosity in sponsoring and hosting this meeting, and The World Bank for the financial support they have provided.

The views expressed in this paper are those of the author(s) and not those of the funding organization(s) nor of CEPR, which takes no institutional policy positions.

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Abstract

Why is GDP so much more volatile in poor countries than in rich ones? To answer this question, we propose a theory of technological diversification. Production makes use of different input varieties, which are subject to imperfectly correlated shocks. As in endogenous growth models, technological progress increases the number of varieties, raising average productivity. In our model, the expansion in the number of varieties provides diversification benefits against variety-specific shocks and it hence lowers the volatility of output. Technological complexity evolves endogenously in response to profit incentives. Complexity (and hence output stability) is positively related with the development of the country, the comparative advantage of the sector, and the sector's skill and technology intensity. Using sector-level data for a broad sample of countries, we provide extensive empirical evidence confirming the cross-country and cross-sectoral predictions of the model.

*April 29, 2005. E-mail addresses: koren@fas.harvard.edu and s.tenreyro@lse.ac.uk. We thank John Campbell, Francesco Caselli, Elhanan Helpman, Nobuhiro Kiyotaki, Borja Larrain, Marc Melitz, Esteban Rossi-Hansberg, Ádám Szeidl, Ákos Valentinyi, and seminar participants at Harvard, the Central European University, the Hungarian Academy of Sciences, the Boston Fed, and the European Winter Meeting of the Econometric Society for comments. Part of this paper was written while Koren was visiting the Federal Reserve Bank of Boston, whose hospitality he gratefully acknowledges. Koren also thanks financial support from the Lamfalussy Research Fellowship of the European Central Bank. Any remaining errors are ours.

1 Introduction

Economies at early stages of the development process are often shaken by abrupt changes in growth rates. In his influential paper, Lucas (1988) brings attention to this fact, noting that “within the advanced countries, growth rates tend to be very stable over long periods of time,” whereas within poor countries “there are many examples of sudden, large changes in growth rates, both up and down.” This negative relationship between the volatility of growth rates and the level of development is illustrated in Figure 1, which plots the standard deviation of annual growth rates against the level of real GDP per capita for a large cross section of countries.

In an attempt to understand the sources of volatility, Koren and Tenreyro (2004) quantify the contribution of various factors at different stages of development, finding that the high volatility in poor countries is due to 1) higher levels of sectoral concentration, 2) higher levels of sectoral risk (that is, poor countries not only specialize in few sectors, but those sectors also tend to bear particularly high risk), and 3) higher country-specific macroeconomic risk. A volatility accounting exercise carried out by these authors indicates that approximately 50 percent of the differences in volatility between rich and poor countries can be accounted for by differences in the sectoral composition of the economy (higher concentration and sectoral risk), whereas the other 50 percent is due to country-specific risk. These characteristics of the development process, as we later explain, are inconsistent with previous theoretical explanations of the dynamics of volatility and development. The purpose of this paper is to provide an alternative theory that is in line with the empirical evidence.

To that end, we develop an endogenous growth model of technological diversification. The key idea of the model is that firms using a larger variety of inputs can mitigate the impact of shocks affecting the productivity of individual inputs. This takes place through two channels. First, with a larger variety of inputs, each individual input matters less in production, and productivity becomes less volatile by the law of large numbers. Second, whenever a shock hits a particular input, firms can adjust the use of the other inputs to partially offset the shock. This second channel operates even if production exhibits an extreme form of complementarity (as in Kremer (1993)’s O-ring technology). Both channels make the productivity of firms using more sophisticated technologies less volatile.

The idea can be illustrated with an example from agriculture: Growing wheat with only land and labor as inputs renders the yield vulnerable to idiosyncratic shocks (for example, weather shocks such as a severe drought). In contrast, using land and labor together with artificial irrigation, fertilizers, pesticides, etc., might make wheat-growing not only more productive but also less risky, because farmers have more options to react to external shocks. Figure 2 provides a graphical illustration of this example. It displays the volatility of wheat yield (calculated as the standard deviation of percentage deviations from the country’s av-

erage yield) of the 20 biggest wheat producers against their level of GDP per capita.¹ Yield volatility falls sharply with development. This remains true if we control for differences in climate across countries, including the volatility of rainfall and temperature (see Table 1).

The shocks affecting individual inputs or individual production techniques may come from various sources. Another example of such a shock could be a sudden change in the price of a major input of a production technique. Countries with a diverse set of available techniques can cope better with the shock. For instance, the types of power plants that countries rely on to generate electricity vary with development. Small and less-developed countries have only a few plants very highly concentrated on one particular technique of electricity production (employing either traditional thermal or hydroelectric plants). Developed countries, on the other hand, have access to nuclear and renewable-resource plants and are typically more diversified. Firms in these countries will react differently to oil price shocks. Table 2 reports how the electricity production of countries responds to oil price changes. The electricity production of less-developed and small countries concentrated on few types of power plants is significantly more sensitive to oil price shocks than that of countries with a diverse set of plants. More specifically, while the electricity production of countries concentrated on a single energy source drops by about 1 percent after a 30 percent oil price hike, there is no such drop for diversified countries. Firms in countries with diverse sources of electricity can mitigate the negative impact of an oil price shock by substituting away from oil. The share of oil in total energy consumption falls by 0.3 percent after a 30 percent oil price hike, whereas no substitution takes place in concentrated countries.

We next turn to the questions of what determines technological diversification and why poorer countries specialize in less sophisticated sectors. We extend the model to allow for international mobility of goods and for cross-country differences in endowments. Much as in models of endogenous growth and directed technical change, the technological complexity of a sector in a given country evolves endogenously in response to the incentives of the creators and users of new technologies. In particular, more input varieties will be directed towards sectors in which the country has a comparative advantage, making them more complex and less volatile. The stage of development of the country will also matter, because inventing and/or using the new inputs is subject to increasing returns to scale. Countries accumulate new inputs as they develop, which brings about a gradual decline in their volatility. The speed of development, and hence the speed with which volatility declines, may be influenced by the initial level of volatility. If investment risk is harmful for growth, which is the case for a range of plausible parameter values in our model, then poor and volatile countries will develop slower and will remain highly volatile for long periods.²

¹Note that agricultural technology varies substantially with development. For example, of the top 20 wheat producers, India uses 2.3 tractors per 1,000 acres of arable land; this number is 128.8 for Germany. Fertilizer use also varies hugely. India uses 21.9 tons of nitrogenous fertilizers per acre; Germany uses 183.8 tons. We take the level of development as an overall indicator of agricultural sophistication.

²See Angeletos and Calvet (2001) and Angeletos (2004) for a discussion of how volatility affects investment. Note, however, that in these papers there is no explanation for why volatility is higher in the first place.

The model delivers clear-cut predictions about the relationship among technological diversification, volatility, and productivity. Using sector-level data for a broad sample of countries, we provide empirical support for these predictions. First, any given sector is less volatile in developed countries. This result holds if we control for the quality of institutions which may facilitate a smoother response to external shocks, such as financial development and the flexibility of the labor market. Second, within a given country, large, skill intensive sectors using complex technologies are less volatile. This is consistent with our model which says that new inputs/technologies will be directed towards such sectors, thus reducing volatility. These two mechanisms lead to a decline in aggregate volatility as a country develops: The economy experiences less volatility in each sector and resources move towards relatively safer sectors.

The link between volatility and development has been studied before by Acemoglu and Zilibotti (1997), Greenwood and Jovanovic (1990), Saint-Paul (1992), and Obstfeld (1994), who describe the technology choice as a portfolio decision: In order to reap the benefits of high productivity and high growth, an economy has to bear more risk. The risk tolerance typically relates to the level of development and the financial structure of the economy. Acemoglu and Zilibotti (1997)'s model also features increasing returns to scale: Early in the development process diversification opportunities are limited, owing to the scarcity of capital and the indivisibility of investment projects. This feature can explain the high levels of sectoral concentration observed in poor countries. However, all these models predict that at early stages of development countries will tend to specialize in safer (even if less productive) sectors as a way of seeking insurance. This prediction is not borne out by the data: Koren and Tenreyro (2004) document that poor countries are highly concentrated in sectors that bear particularly high volatility. In addition, these authors find that most developing countries are inside the “mean-variance frontier,” being highly prone to specialize in high-variance, low-mean sectors. These findings contradict the predictions of the portfolio-based models and suggest that important constraints must be at play, preventing developing countries from investing in safer and, at the same time, more productive assets.³

Our model departs from the portfolio view of the world that features a necessary trade-off between volatility and performance at the sector level. It can then naturally accommodate the fact that poor countries tend to exhibit high sectoral concentration and also that the high concentration falls mainly on high-risk sectors. In addition, unlike in previous contributions,

Empirically, Ramey and Ramey (1995) find that high-volatility countries grow slower. Imbs (2004) studies the link between growth and volatility at the sectoral level, at which he finds a positive correlation between the two. He argues that the negative relation at the aggregate level is indicative of the harmful effects of *macroeconomic* volatility.

³Kalemli-Ozcan, Sørensen and Yosha (2003) and Imbs and Wacziarg (2003) document that, for highly developed countries, industrial specialization tends to increase with development. However, as we later show, this does not result in higher aggregate volatility because these sectors tend to be technologically diversified and are hence more stable than the rest of the economy. The fact that the higher specialization of rich countries does not increase their aggregate risk has also been shown by Koren and Tenreyro (2004).

the volatility of individual sectors in our model is endogenous: It depends on the level of development and the comparative advantage of the country.

Our paper is related to previous work by Kraay and Ventura (2001). As in their paper, the open-economy version of our model features the prediction that rich countries have a comparative advantage in less-volatile sectors. The difference lies in the way this result is achieved. In Kraay and Ventura (2001), high-skill sectors, which are prevalent in developed countries, enjoy less-elastic product demand. Markups can then serve as a buffer against productivity shocks, reducing the volatility of high-skill sectors. For example, a drop in output of a differentiated product makes that product more expensive in the world market. This terms of trade improvement partly offsets the original shock. On the other hand, no such “terms-of-trade insurance” is taking place for homogenous products that poor countries specialize in.

There are, however, empirical objections to the mechanism proposed by Kraay and Ventura (2001) and its implications. The model predicts a negative relationship between productivity shocks and terms-of-trade fluctuations (particularly negative for developed countries). That is, negative productivity shocks should be associated with an improvement in the terms of trade. In the data, however, the relationship between fluctuations in labor productivity and the terms of trade is somewhat positive, and there is no difference between rich and poor countries in terms of this relationship.⁴

Finally, our model builds on a vast literature on endogenous growth models in which the development of new varieties of goods enhances productivity. (See for example, Romer (1990) and Grossman and Helpman (1991).) The contribution of our paper is to provide a unified framework for the explanation of development and volatility. We provide sectoral evidence for a broad cross-section of countries that confirms the predictions of the model.

The remainder of the paper is organized as follows. In Section 2 we present the model. In Section 3 we discuss the empirical implications and offer novel evidence in support. We summarize and conclude in Section 4.

2 A Model of Technological Diversification

2.1 A static model of technological diversification, productivity, and volatility

In this section, we introduce a production process that features technological diversification: Input varieties contribute not only to higher productivity but also, because inputs are subject to imperfectly correlated shocks, to lower volatility.

⁴It is possible that other factors are at play, blurring the predicted relationship; at this point, nonetheless, we can say that the extent of countercyclicality in the terms of trade is not the *prima facie* mechanism behind the negative relationship between development and volatility.

Output Y is produced using a composite of “machine varieties” with a constant-elasticity-of-substitution (CES) technology,

$$Y = \left[\sum_{i=1}^N X_i^\sigma \right]^{1/\sigma}, \quad (1)$$

where X_i is capital services from capital variety i , N denotes the number of installed machines and $1/(1 - \sigma) \in (0, \infty)$ is the elasticity of substitution across varieties.⁵

Machines can fail randomly, in which case they irreversibly cease to contribute to production. We assume that failure occurs independently across machines with probability γ . We take the extreme assumption of independence for expositional clarity, but our argument goes through as long as failures are imperfectly correlated. The assumption that random failures turn the machine completely useless makes the model more tractable []; however, technological diversification would still take place with less terminal shocks: Appendix A considers an example where there is only a partial drop in productivity after a machine failure.

We assume that the machine can be used with different intensities by employing “operators.” Machine i can provide twice as much service if operated by twice as many workers.⁶ Producing a unit of capital service requires one unit of labor (by appropriate definition of labor units).

Formally, the services of machine i at time t are:

$$X_i = \begin{cases} l_i, & \text{if machine } i \text{ is working;} \\ 0, & \text{otherwise;} \end{cases} \quad (2)$$

where l_i is the number of operators.

Consider the output of a firm, in which $n \leq N$ machines are working, each providing \bar{X} units of services,

$$Y = n^{1/\sigma} \bar{X}. \quad (3)$$

As is apparent from (3), productivity is increasing in the number of varieties holding the amount of each individual variety fixed. This is the usual “love of variety” effect of many endogenous growth models (Romer 1990, Grossman and Helpman 1991). The effect is stronger

⁵As usual in endogenous growth models, we assume that $\sigma > 0$, that is, machines are gross substitutes. Appendix A considers an example when this is not the case. Introducing additional (scarce) factors of production would not change our qualitative results, it would just make the returns to variety more decreasing.

⁶This is a way of capturing endogenous capacity utilization which is recently emphasized in business cycle studies. Allowing for capacity constraints or decreasing returns to capacity utilization would not alter our setting qualitatively. First, capacity constraints would not bind in equilibrium. Economic growth takes place via the expansion of machine varieties while the services of an individual variety shrink. Second, investors will be interested in the *total*, not the *marginal* profit when deciding whether to build a machine. This will remain positive even with decreasing returns to scale. Moreover, if the cost function were isoelastic, the share of profit in total revenue would be constant, just as in the present formulation.

the lower is σ , that is, the less substitutable machines are. Intuitively, if machines are highly substitutable, any additional variety is less needed.

The overall number of machine operators working at the firm is $L = \sum l_i = n\bar{X}$, since each working machine requires \bar{X} operators and broken machines require none. Hence (3) can be rewritten as

$$Y = n^{1/\sigma-1}L. \quad (4)$$

Productivity is also increasing in the number of machines if we hold the total number of operators (L) constant (since $\sigma < 1$). The dependence is weaker than in (3), because any new machine requires operators taken away from old machines.

This implies that we have two alternative definitions of productivity, one holding the operators *per machine* constant, the other holding the *total* number of operators constant. We think both measures are useful, since the adjustment across different machine varieties can take place relatively fast within the firm (in particular, no hiring or firing of workers or capital installation is needed).⁷ In what follows, however, we will focus on the second measure, which better captures the labor productivity measures used in empirical work.

Given that the number of machines is a random variable (individual machines fail at random and there is a finite number of machines), productivity will be random, too. The number of working machines follows a binomial distribution with parameters N and $(1 - \gamma)$, where N is the number of *installed* machines, and γ is the probability of failure:

$$\Pr(n = k) = \binom{N}{k} \gamma^{N-k} (1 - \gamma)^k.$$

The number of working machines has a mean of $(1 - \gamma)N$ and a variance of $\gamma(1 - \gamma)N$.

Let a denote the log of productivity when the total number of operators (L) is held constant.

$$a = y - l = \left(\frac{1}{\sigma} - 1 \right) \ln n \equiv \phi \ln n.$$

Lower-case letters denote logarithms and we have introduced the notation $\phi = 1/\sigma - 1 \in (0, 1)$.

Using a first-order Taylor approximation around $\ln E(n)$, the log number of machines can be written as:

$$\ln n \approx \ln[(1 - \gamma)N] + \frac{n - (1 - \gamma)N}{(1 - \gamma)N},$$

⁷The effectiveness of this margin depends on how quickly and how efficiently machine operators can switch between different machines. Our assumption that any worker can operate any machine captures the extreme case when such a switch is immediate and fully efficient. In reality, of course, we would see less than perfect flexibility. However, as the skills needed to work with advanced technology are very diverse (for example, Autor, Levy and Murnane (2003) document that computerization increased the demand for non-routine cognitive tasks), we believe that such adjustment is important in practice.

and hence the variance of $\ln n$ can be approximated by:

$$\text{Var}(\ln n) \approx \frac{\text{Var}(n)}{(1-\gamma)^2 N^2} = \frac{\gamma}{(1-\gamma)N}.$$

Therefore:

$$\text{Var}(a) = \phi^2 \text{Var}(\ln n) \approx \frac{\phi^2 \gamma}{(1-\gamma)N}$$

The volatility of the log productivity declines with the existing number of machines. Even though as N gets big, a failure gets more and more likely, the proportional (that is, log) drop in the number of machines it induces is less and less important. As is standard in statements of the law of large numbers, the second effect outweighs the first one. In other words, diversification across several machines makes log productivity less volatile.

Given that N measures the number of inputs subject to different shocks, we take it as an index of *technological complexity*. It is clear from (3) and the discussion above that technological complexity both increases average productivity and reduces the volatility of productivity. In the next section, we endogenize the investment in new machines, and consequently, the resulting level of technological complexity.

2.2 The dynamic model with endogenous number of varieties

What determines the level of technological complexity in the long run? In this section we endogenize the decision to invest in machines. Much as in models of endogenous growth (Romer 1990, Grossman and Helpman 1991, Aghion and Howitt 1992), machine owners will be attracted by greater profit opportunities.

We first look at a one-sector economy to bring out the relationship between volatility and development clearly. Later on (Section 2.3), we introduce multiple sectors and investigate how the relative complexity of sectors evolve endogenously.

Technology will be the same as in (1), which results in the following aggregate production function for the final good (4):

$$Y = n^\phi L.$$

Using machines in production involves increasing returns to scale: Machines are indivisible. This means that anyone operating a machine has to buy one unit of the machine beforehand. This minimum scale requirement limits the scope of diversification across machine varieties.⁸

Since we are interested in the inner workings of a sector and how technology choice affects volatility, we posit increasing returns at the input level. Indivisibility and minimum scale requirements are inherent characteristics of many an input used in technologically advanced

⁸Note that there is no incentive to install two or more units of a single machine variety, both because the production function features a “love of variety” and because machines are subject to idiosyncratic shocks. A similar assumption is made by Acemoglu and Zilibotti (1997) who work with minimum scale requirements at the industry level.

sectors. Note that increasing returns are also a feature of the *use* of the machines, not only their invention or production. That is, we assume that machines can be produced and bought in any quantity but only a full unit is productive.

The setup of a machine requires κ units of the final good. Once the machine is set up, the owner gains monopoly power over its services. This monopoly lasts until the machine (exogenously) becomes obsolete. We adopt a continuous-time formulation and assume that the random lifetime of the machine is exponentially distributed with parameter γ .

If at time t investors devote I_t units of the final good to build new machines, then the number of machines is assumed to evolve according to the following Itô process:

$$dn_t = (I_t/\kappa - \gamma n_t) dt + \sqrt{\gamma n_t} dz, \quad (5)$$

where dz is the increment of a standard Wiener process. During a dt period I_t/κ new machines are built and γn_t fail. Since failure is random, we include a diffusion term with instantaneous variance γn_t .⁹

Modelling the number of machines as a diffusion process involves two major simplifications. First, we approximate the number of machines with a continuous real variable to avoid integer problems. Second, we assume that the change in the number of machines over a dt period of time is normally distributed. Both simplifications are only meant to ease the exposition and they are not crucial for any of our results.

Without the diffusion approximation, the number of machines follows a discrete-state Markov process known as the birth and death process. In this case, all endogenous variables follow jump processes, requiring a somewhat different apparatus than is presented here. The discrete-state derivations and a formal proof of how the birth and death process can be approximated by the above diffusion are available from the authors upon request.

The economic environment is characterized as follows. The final good sector is perfectly competitive, that is, firms take output and input prices as given. In contrast, machine providers act as monopolistic competitors, that is, they are price setters for their own machine but take the overall price of the composite machine varieties as given.¹⁰

2.2.1 Consumers

There is a continuum of symmetric consumers/investors with unit mass. Each consumer has access to the well-diversified mutual fund of all machines. They can trade the share of this

⁹Recall that in the static case, the variance of n was $\gamma\Delta t(1-\gamma\Delta t)N$. As we approach the continuous-time limit, $\Delta t \rightarrow 0$, the variance becomes $\gamma N dt$.

¹⁰Note that this is a valid assumption even if there is a finite number of different machine varieties. First, the market share of each machine owner falls at the rate $1/n$, whereas the standard deviation of output is of order $1/\sqrt{n}$. That is, even if n is large enough to make monopolistic competition a realistic assumption, we still have positive aggregate volatility. Second, volatility falls with the number of *independent* machine varieties, which may be smaller than the number of machine owners if some of the machines are subject to common shocks or if there are interactions across machine-operating firms.

mutual fund freely, instantly, and in any quantity (even shorting is allowed). This will ensure that the mutual fund is priced by the consumer's stochastic discount factor. In other words, we assume no frictions in the domestic financial market. Note that there is no positive supply of a riskless asset in the economy, in other words, every production technology is risky.¹¹ Each consumer supplies L units of labor inelastically in the labor market. Consumers decide how much to consume and how much to invest in the mutual fund of machines, taking the rate of return and wage rate as given.

Time is continuous and consumers maximize lifetime expected utility over consumption with constant relative risk aversion, θ ,

$$\mathcal{U}_t = E_t \int_t^\infty e^{-\rho(s-t)} \frac{C_s^{1-\theta}}{1-\theta} ds, \quad (6)$$

subject to a standard intertemporal budget constraint,

$$da_i = [(D/P)a_i + wL - C_i] dt + a_i \frac{dP}{P}. \quad (7)$$

The change in the asset holding of consumer i , da_i , comes from dividend yield (D/P) on the asset and labor income (wL) minus consumption (C_i). There are (possibly random) capital gains contributing to a change wealth, $a_i dP/P$.

Stock prices follow a diffusion process with drift μ_P and instantaneous variance σ_P^2 ,

$$dP/P = \mu_P dt + \sigma_P dz. \quad (8)$$

In general equilibrium, the stochastic properties of the stock price depend on the state of the economy, as we will show in section 2.2.3. For simplicity, we suppress this dependence in notation.

The investor chooses a consumption plan $C(n, t)$, determining a level of consumption for each time t and state n . Assuming that this plan is twice continuously differentiable in n , consumption will follow a diffusion process

$$dC/C = \mu_c dt + \sigma_c dz. \quad (9)$$

The drift and the instantaneous variance depend on the optimal policy function and the evolution of n . Note that the diffusion term (dz) is the same in (8) and (9) because we assume that asset markets are complete. In other words, stock returns and consumption are *instantaneously* perfectly correlated.

We assume that there exists no arbitrage. This implies that there exists a unique stochastic process ξ_t for which

$$\xi_t P_t = E_t \left[\xi_T P_T + \int_t^T \xi_s D_s ds \right] \quad (10)$$

¹¹Alternatively, the rate of return on a riskless asset (for example, storage) is so low that investors do not demand a positive amount.

for all t and $T > t$. Intuitively, ξ_T/ξ_t is the time- t price of one dollar delivered at time T (a stochastic discount factor). The value of any asset today is the sum of expected discounted future price and the discounted sum of dividends. This is analogous to the discrete-time asset pricing equation $p_t = E_t(m_T P_T + \sum m_s D_s)$.

Equation (10) can be rewritten as

$$E_t [\xi_t D_t dt + d(\xi_t P_t)] = 0. \quad (11)$$

Given the the stochastic process for the state prices, the investor can sell her claims on future labor income and purchase the relevant Arrow-Debreu securities to finance her consumption. That is, her optimization problem is equivalent to maximizing (6) subject to a single budget constraint (see Cox and Huang (1989)),

$$E_0 \int_0^\infty \xi_t C_t dt \leq \xi_0 a_0 + E_0 \int_0^\infty \xi_t w_t L dt. \quad (12)$$

The first-order condition of this simple maximization problem is

$$\exp(-\rho t) C_t^{-\theta} = \lambda \xi_t, \quad (13)$$

for all t and state of the world, that is, marginal utility is proportional to the state price.

Applying Itô's lemma for marginal utility, $\xi_t = e^{-\rho t} C_t^{-\theta} / \lambda$,

$$d\xi/\xi = \left[-\rho - \theta\mu_c + \frac{\theta(\theta+1)}{2}\sigma_c^2 \right] dt - \theta\sigma_c dz, \quad (14)$$

where μ_c and σ_c^2 refer to the proportional drift and diffusion of C , respectively. Marginal utility declines with impatience and with the mean consumption growth rate and increases with consumption volatility (that is, the convexity of marginal utility gives rise to precautionary saving motives).

We can now reformulate the asset pricing equation (11). The expected change in the discounted asset price (inclusive of dividends) can be zero only if the sum of all drift terms is zero. Formally,

$$0 = \mu_\xi + D/P + \mu_P + \sigma_\xi \sigma_P, \quad (15)$$

and using equation (14),

$$\mu_c = \frac{1}{\theta} [r - \rho] - \sigma_c \sigma_P + \frac{\theta+1}{2} \sigma_c^2, \quad (16)$$

where r refers to the sum of dividend yield and mean price decrease of stocks.

Equation (16) is a continuous-time Euler equation. The mean growth rate of consumption depends positively on the mean rate of return, with a coefficient equal to the elasticity of intertemporal substitution, $1/\theta$. At the same time, because the consumer's portfolio is risky,

its covariance with consumption will make saving less attractive and will hence result in lower consumption growth. Given that we have complete markets, in other words, there is only one source of uncertainty in the economy, the instantaneous correlation between stock prices and consumption is 1, so the covariance is $\sigma_c \sigma_P$. Finally, since future consumption is risky, prudent consumers have precautionary savings depending on the volatility of consumption and the degree of prudence, $(\theta + 1)/2$.

2.2.2 Firms

To derive the equilibrium of the economy, let us consider the pricing decisions of firms next. The demand for machine variety i is

$$X_i = \frac{\chi_i^{-1/(1-\sigma)}}{\sum_{j=1}^n \chi_j^{-\sigma/(1-\sigma)}} Y,$$

which is decreasing in the variety's own price (χ_i), increasing in competitors' prices ($\sum_{j=1}^n \chi_j^{-\sigma/(1-\sigma)}$) and the final good output (Y). Taking the price index $\sum_{j=1}^n \chi_j^{-\sigma/(1-\sigma)}$ and final demand Y as given, the machine owner faces a constant elasticity demand curve with elasticity $1/(1-\sigma)$.

She will hence follow a constant markup rule when pricing its services. The optimal monopoly price of each capital service will be

$$\chi_i = w/\sigma,$$

where w is the wage rate. Final good prices are, in turn, determined from the price of the services of an individual machine and the number of machines.

Profit maximization implies that price equals marginal cost in the final good sector. We normalize the final good price to unity:

$$1 = \left(\sum_{i=1}^n \chi_i^{\sigma/(\sigma-1)} \right)^{1-1/\sigma} = n^{-\phi} w.$$

This implies that wages increase in productivity:

$$w = n^\phi.$$

Labor market equilibrium requires that the number of operators exhausts the (exogenous) labor supply,

$$\sum_{i=1}^n X_i = nX_i = L.$$

The markup rule also implies that profits are a constant, $(1/\sigma - 1) = \phi$, share of wages. Total wages are wL , hence the wage costs of a single machine operating firm are wL/n (by symmetry), implying that profits are

$$\pi_i = \phi wL/n. \tag{17}$$

The owner of a machine uses this profit flow to calculate the lifetime cash-flow of the machine. Investors take the number of installed capital varieties, the wage rate, capital prices, and the return on equity as given.

2.2.3 Equilibrium

In an equilibrium of the economy (i) consumers optimally set their consumption path ((16) holds), (ii) firms maximize their profit in each period (giving (17)), (iii) dividends equal profits and the financial wealth of households equals the total value of machines, and (iv) aggregate output is either consumed or invested into new machines.

We assume free entry into the machine market. This means that any investor can buy κ units of final good and install a new machine variety. As long as there is positive entry, this pins down the value of a machine at $V = \kappa$.¹² We further assume that the sunk cost required to install a machine is falling with the level of technological progress, n , because of spillovers or learning-by-doing externalities. In particular, it falls at a rate $\phi - 1$ to ensure balanced growth. Expanding variety growth models usually make a similar assumption to ensure long-run growth (Grossman and Helpman 1991, Chapter 3). Alternatively, one could set $\phi = 1$ by restricting the elasticity of substitution across varieties to be 2. Barro and Sala-i-Martin (1995, Chapter 6) put restrictions on the elasticity of substitution across input varieties. Either assumption delivers balanced growth and qualitatively similar results.

Also, we assume that fixed costs are proportional to the overall size of the economy, L . This assumption ensures that the growth rate is not dependent on country size. (See Jones (1995) on the “scale effect” of endogenous growth models.) Recall that κ measures the unit of a machine *variety* that is subject to variety specific shocks. Arguably, bigger countries use more capital of each variety and therefore require a bigger investment. Our main results are not sensitive to this assumption. The only result that would change without this assumption is that large countries would develop faster.

Formally,

$$\kappa = \kappa_0 L n^{\phi-1}. \quad (18)$$

The dividend yield on a machine is

$$\frac{\pi_i}{V} = \phi \frac{wL}{n\kappa} = \frac{\phi}{\kappa_0}, \quad (19)$$

where we have used $w = n^\phi$ and $\kappa = \kappa_0 L n^{\phi-1}$. The dividend yield is higher the higher the profit rate and the lower the fixed cost of installing a machine. The assumption of falling fixed costs ensured that the dividend yield does not vanish as n increases. If the dividend yield tended to zero as n became large, we would obtain a steady-state distribution of n instead of an ever-growing economy.

¹²If $V < \kappa$, no new machines are built and the growth rate is zero. We will later verify that this is not an equilibrium.

Note that even if the dividend yield on a machine is constant, the rate of return is random, because there are random capital losses due to machine failures. This results in an average depreciation rate (and hence capital loss) $\gamma\Delta t$ over a period Δt , but this capital loss is random even as we take $\Delta t \rightarrow 0$. We next turn to characterizing the stochastic process driving the value of machines.

Let $A = \int_i a_i di$ denote the aggregate financial wealth in the economy. Aggregating across the budget constraints of individual households, (7), yields

$$\frac{dA}{A} = \frac{D}{P} + \frac{wL - C}{A} + \frac{dP}{P} \quad (20)$$

In equilibrium financial wealth equals the value of all machines, $A = nV = n\kappa$. We can hence characterize the evolution of prices, dP/P , as

$$\frac{dP}{P} = \frac{dn}{n} - \frac{Y - C}{n\kappa} = -\gamma dt + \sqrt{\gamma/n_t} dz, \quad (21)$$

where we have made use of the facts that the sum of dividends and wages is equal to total output and that saving is equal to investment into new machines.

We find the equilibrium as follows. We posit a policy function, $C = v(n)$, that describes the optimal amount of consumption given the number of machines in the economy. We then substitute it in the Euler equation (16). By Itô's lemma,

$$dC = \left[v'(n)n\mu_n + \frac{1}{2}v''(n)n^2\sigma_n^2 \right] dt + v'(n)n\sigma_n dz,$$

and the Euler equation becomes

$$\mu_n \frac{v'n}{v} + \frac{1}{2}\sigma_n^2 \frac{v''n^2}{v} = \frac{1}{\theta}(r - \rho) - \sigma_n^2 \frac{v'n}{v} + \frac{\theta + 1}{2}\sigma_n^2 \left(\frac{v'n}{v} \right)^2, \quad (22)$$

where we have omitted the argument n from $v(n)$ for brevity.

The growth rate of machines, μ_n , depends on investment, which, in turn, depends on the consumption policy. By equilibrium in the final good market,

$$Y = n^\phi L = v(n) + (\mu_n + \gamma)n\kappa = v(n) + (\mu_n + \gamma)n^\phi \kappa_0 L. \quad (23)$$

Total output has to equal the value of consumption plus investment. Investment is the sum of net investment (μ_n) and the replacement of broken machines (γ).

Equations (22) and (23) together with $\sigma_n^2 = \gamma/n$ define a second-order ordinary differential equation for v , which has two linearly independent solutions. We therefore need two boundary conditions to pin down the optimal policy function, $v(n)$. One is that no consumption takes place without capital, $v(0) = 0$. The other one comes from the fact

that as n becomes arbitrarily large, σ_n^2 becomes zero and the economy resembles a non-stochastic Ramsey model. Consumption growth in the non-stochastic Ramsey model is given by $\tilde{\mu}_c = (r - \rho)/\theta$, so we should have

$$\lim_{n \rightarrow \infty} \mu_n \frac{v'n}{v} = \tilde{\mu}_c = (r - \rho)/\theta.$$

To obtain an analytical solution, we put restrictions on the CRRA and the elasticity of substitution across machine varieties and assume that $\theta\phi = 1$.¹³ Whether this is plausible depends on how broadly we interpret machine varieties. If these are different intermediate inputs necessary to produce a particular good, the inputs may be strong complements, in which case the elasticity is less than one. This would lead to a negative ϕ which we have ruled out (but see Appendix A for an example of such a production function). However, if we think of machine varieties as representing alternative production techniques that can highly substitute each other, then higher elasticities are more plausible. For example the elasticity of substitution across goods produced in different countries (within a narrow product category) is estimated to be around 4–7 (Hummels 2001). Estimates based on the time series of U.S. imports are usually lower, in the range of 1–2 (Gallaway, McDaniel and Rivera 2003). For an intermediate range of 3–4, the value of ϕ is 1/2–1/3, resulting in a θ of 2–3. This is plausible both as a measure of relative risk aversion and as an inverse elasticity of intertemporal substitution (Vissing-Jørgensen 2002).

Proposition 1. If $\theta\phi = 1$, the optimal policy function, $v(n)$ takes the form $v_0 n^\phi$, where v_0 is given by

$$v_0 = (1 - \phi)L + \rho\kappa_0 L. \quad (24)$$

Proof. Direct substitution reveals that whenever v_0 satisfies (24), $v_0 n^\phi$ satisfies (22). For this policy function, $v''n^2/v = \phi(\phi - 1)$, $v'n/v = \phi$, and μ_n is independent of n . Since $v_0 n^\phi$ also satisfies the boundary conditions, it is a unique solution. \square

Defining the value of all the machines as $K = n\kappa$, equation (24) can be rewritten in terms of aggregate variables as

$$Y - C = \phi Y - \rho K = (\phi - \rho\kappa_0) Y,$$

since the capital output ratio in this economy is $n\kappa_0 L n^{\phi-1}/(n^\phi L) = \kappa_0$. Investors save (and invest) a constant fraction of current output. The saving rate is increasing in the profit rate (ϕ) and decreasing in the degree of impatience (ρ) and sunk cost of investment (κ_0).

¹³For other values of relative risk aversion, numerical techniques can be applied to solve (22). If $\theta < 1/\phi$, then the saving rate is increasing in n . Intuitively, low- θ consumers are less prudent, so the precautionary motive is relatively small. This means that risk aversion dominates and saving declines with volatility. In this case, poor countries develop slowly because their excess volatility discourages investment. The reverse is true for $\theta > 1/\phi$. Similarly to Angeletos (2004), we have the cutoff at an IES less than one (RRA greater than one) because capital does not exhaust all income as long as $\phi < 1$.

From (23), we can express the growth rate of the number of machines as

$$\mu_n = \phi/\kappa_0 - \gamma - \rho,$$

resulting in an output growth rate of

$$\phi\mu_n = \phi(\phi/\kappa_0 - \gamma - \rho).$$

This completes the characterization of the dynamic equilibrium of this economy. Countries with high profit rates and low investment costs will develop faster, implying both a faster growth of output and a faster fall of volatility. In the next section, we extend the model in two directions in order to account for the differences in specialization patterns between rich and poor countries.

2.3 Extension to multiple sectors

As we have documented in Koren and Tenreyro (2004), intrinsic volatility differences across sectors together with countries' different patterns of specialization are responsible for an important portion of the difference in output volatility between rich and poor countries. In this section, we allow for a richer characterization of the economy, by extending the model to a multi-sector economy. The sectors differ in the extent of skill intensity. We introduce a multi-country setup, allowing for cross-country differences in endowments and compare the results for closed and open economies. Allowing for international trade, as we later show, can explain the observation that poor countries specialize in less sophisticated sectors. In fact, comparative advantage magnifies the differences in volatility between poor and rich countries through its effect on the patterns of specialization. As in other multi-sector models of endogenous technology, we will have directed technical change (Acemoglu 2002, Caselli and Coleman 2000). Profits per machine variety will depend on the size of the sector (number of available operators), its relative wage, the degree of competition (number of existing machines), and trade openness.

Suppose that there are S sectors, each using the same CES technology but requiring different levels of skill for machine operation. In particular, sector s requires that each operator possesses at least h_s amount of human capital, and we order sectors such that $h_S > h_{S-1} > \dots > h_1$. The output of machine i in sector s is

$$X_{is} = \begin{cases} h_s l_{it}, & \text{if machine } i \text{ is working;} \\ 0, & \text{otherwise,} \end{cases}$$

where l_{it} is the number of workers on machine i who are “qualified” to operate the machine in the sense that they have a level of human capital higher than h_s .

We assume that the country is a small open economy freely trading the output of all final good sectors at an exogenously given world price, \tilde{p}_s . Note that, as standard in small open-economy models with free trade, the production structure is independent of demand

considerations. Relative demand for the output of the sectors (in our case, the consumption/investment decision) will matter only for the patterns of trade. Appendix B discusses the differences between a closed and an open economy in more detail.

We assume that the individual machine varieties cannot be traded. In other words, investors can buy foreign capital goods and install them in their own country as machines, but the physical machines installed abroad cannot contribute to production.¹⁴ This assumption ensures that countries cannot circumvent the fixed costs of machine operation by importing machine services from abroad and hence cannot fully diversify instantly. The number of machines in the country will hence be a state variable that can only be adjusted gradually. At any given point in time, the number of available machines and hence overall technological complexity is given. In the long run, investment in new machines will determine technological complexity, economic development, and volatility.

Trade is balanced at any point in time, ruling out international borrowing and lending. This also means that investment is finite (growth in the number of machines is gradual) in every instant, because the country has only finite flow output to offer in exchange for foreign capital goods. In contrast, if we allow for borrowing, investors can immediately borrow to replace a broken machine, smoothing out some of the shock to productivity. We assume away such consumption smoothing behavior because the current accounts of countries (especially those of less-developed ones) do not seem to act as buffers against productivity shocks.¹⁵

There are altogether L workers in the economy, and their human capital endowment is distributed according to a cumulative distribution function $F(h)$. The number of workers capable of operating machines in sector s is hence $[1 - F(h_s)]L$.

Given the number of machines in each sector, (n_1, n_2, \dots, n_S) , labor market equilibrium requires that each worker be employed on machines that require the highest skill level that this worker can supply.¹⁶

This implies that a fraction $1 - F(h_S)$ of workers is employed in sector S , and a fraction $F(h_{s+1}) - F(h_s)$ in sector s . The output of sector s is hence

$$Y_s = n_s^\phi h_s \alpha_s L,$$

where α_s is defined as the share of workers in sector s , $F(h_{s+1}) - F(h_s)$ (defined for all s with $h_0 = 0$, $h_S = \infty$). Profits per machine are a constant, $(1 - \sigma)$, fraction of revenues per

¹⁴If we interpret machine varieties as different techniques of production, this amounts to very costly imitation and no technology spillovers across countries. Comin and Hobijn (2004) document a relatively slow adoption of leading technologies developed elsewhere. A positive but finite cost of technology adoption could be modelled such that machine varieties already in use abroad have a lower installation cost $\tilde{\kappa} < \kappa$. A $\tilde{\kappa} > 0$ would be sufficient to deliver qualitatively similar results.

¹⁵Kalemli-Ozcan et al. (2003) show that the beta coefficient of consumption response to output shocks of countries is close to one.

¹⁶To prove this, suppose there exists a worker with human capital level $h_j \geq h_{s+1}$ (that is, capable of working in sector $s + 1$) working in sector s . This worker is not willing to switch to sector $s + 1$ because $w_{s+1} < w_s$. But all workers in sector $s + 1$ are capable of operating sector s machinery, and they would earn higher wages in that sector. Hence this cannot be an equilibrium.

machine,

$$\pi_s = (1 - \sigma)\tilde{p}_s n_s^{\phi-1} h_s \alpha_s L,$$

where \tilde{p}_s is the price of product s determined in world markets.

Directed technical change will equate per-machine profits across sectors, $\pi_s = \pi_z$, so the relative number of machines in any two sectors is given by

$$\frac{n_s}{n_z} = \left\{ \frac{\tilde{p}_s h_s \alpha_s}{\tilde{p}_z h_z \alpha_z} \right\}^{1/(1-\phi)}. \quad (25)$$

A sector will use relatively more machines if it is producing an expensive good, it is skill intensive, or has a bigger pool of workers with matching skills. Such sectors are also more productive and less volatile. In other words, given the overall number of machines, $n = n_1 + n_2 + \dots + n_S$, technological complexity and productivity are increasing, while volatility is decreasing in the sector's skill intensity and its share in total employment.

The variance of sector s in country i is $\phi^2 \gamma / n_{is}$, so we can write the log variance as

$$\ln \text{Var}_{is} = 2 \ln \phi + \ln \gamma - \ln n_{is} = \nu_i - [\ln \tilde{p}_s + \ln h_s + \ln(L_{is}/L_i)] / (1 - \phi), \quad (26)$$

where ν_i is a country fixed effect.

This is a key equation for our empirical exercise. While we can measure a sector's skill intensity and its share in employment, we do not observe \tilde{p}_s , the price of the sector's output in world markets. Instead, we interpret it broadly as an unobserved sector-specific variable that affects the level of complexity, capturing not only variations in the value of output but also, for example, technological differences across sectors. Note that this variable is common across countries within a given sector, so we can control for it using either sector fixed effects or observing technological complexity in any given country.

For interested readers, Appendix B discusses in depth an example with two sectors.

3 Productivity, Volatility, and Technological Complexity: The Empirical Evidence

The model developed in the previous sections leads to a set of predictions concerning the relationships among productivity, volatility, and technological diversification. We discuss these predictions in light of the empirical evidence.

Prediction 1. GDP volatility declines with development.

This is one of the stylized facts in the literature and the main motivation of this paper. There are large cross-country differences in volatility. The standard deviation of annual GDP growth during the period 1970 through 2000 ranges from 1.4 percent to 21.8 percent (a factor of 15) across 167 countries. The most volatile decile of countries had a standard deviation of GDP growth of 12.9 percent. This is seven times as high as the volatility of the least volatile decile (1.8 percent). This cross-country variation in volatility is highly correlated with the

cross-country variation in the level of development, gauged by real GDP per capita. More specifically, as shown in Table 4, the elasticity of GDP variance with respect to GDP per capita is -0.326 (with a robust standard error of 0.066).¹⁷

In the model, investment in new machines brings about development and a gradual decline in volatility. Countries that have few machines are both less developed and more volatile. In the multi-sector version, our model proposes two channels to explain this negative association. First, a within-sector channel, whereby a given sector exhibits higher technological complexity in more-developed countries. This, in turn, implies that a given sector is both more productive and less volatile in developed countries. Second, a compositional channel, whereby poor countries specialize in relatively less complex sectors. This implies that poor countries concentrate in sectors with (absolute) lower productivity and higher variance. In what follows, we check the empirical consistency of the predictions associated with these two channels.

Prediction 2. For any given sector, poor countries utilize less complex technologies. This implies that for any sector poor countries are both less productive and more volatile.

- For a given sector, poor countries utilize less complex technologies.

Various studies have explored the process of technology diffusion across countries. For example, Caselli and Coleman (2000), document that the adoption of computers depends heavily on the level of development of the country, and, more specifically, on the level of human capital. Caselli and Wilson (2004) show that this result extends to a broader set of high-technology equipment (where the extent of technology embodied in capital equipment is measured as the R&D content).

Our model implies that these cross-country differences in technology are also present *within* sectors. Since directed technical change equates the rates of return on machines in all sectors, poor countries will use proportionately fewer machines in all sectors, holding comparative advantage patterns constant.

The two examples mentioned in the introduction suggest important cross-country technological differences for a given sector: Developed countries tend to use more agricultural machinery, fertilizers, and pesticides in agriculture and have access to more types of power plants in the energy sector. Recent empirical studies provide additional support for this observation. For example, Comin and Hobijn (2004) document how specific technological innovations have spread across countries. Many of these innovations are relevant only to certain sectors (for example, mule spindle, blast oxygen furnace, internal combustion engine, aviation). The authors show that most innovations originated in developed countries and spread gradually to less-developed countries. This implies that in any given year, in all relevant sectors, poor countries use less sophisticated production techniques than rich ones.

¹⁷Table 3 presents the list of countries included in the computation.

- For a given sector, poor countries are both less productive and more volatile.

In the context of our model, the previous finding, in turn, implies that a given sector is both less productive and more volatile in poor countries. We test this prediction using sectoral data from the United Nations Industrial Development Organization (UNIDO, 2002). The UNIDO data set covers all manufacturing at the 3-digit level of aggregation from 1963 to 1998 for a sample of 64 countries, providing information on employment and value added on an annual basis. Table 3 indicates the countries for which the data are available and Table 5 reports the index of technological diversification for each sector, with the corresponding (average) size of the sector in manufacturing. We compute the sample average of labor productivity for each country and sector. As a measure of volatility, we use the 5-year variance of labor productivity (value added per worker) growth.

To check the consistency of the prediction, we first regress the (log of) sectoral labor productivity on the level of development, proxied by the (log of) real GDP per capita of the country, controlling for sector-specific dummies. The regression yields a positive and significant coefficient: As shown in the first column of Table 6, the point estimate for the elasticity is 0.70 (with a country-clustered standard error of 0.07). This means that, on average, any given sector is significantly less productive in poor countries.

Similarly, we regress the (log of) sectoral variance on the level of development, including sector-specific dummies. We obtain a negative and significant coefficient, displayed in the second column of Table 6. The estimated elasticity is -0.30 (with a country-clustered standard error of 0.10), implying that, on average, every sector is significantly more volatile in poor countries.

Prediction 3. More complex sectors are both more productive and less volatile. A mean-variance frontier might not exist.

- More complex sectors are both more productive and less volatile.

This is a direct prediction of production with “technological diversification.” To test this prediction, we use the measures of labor productivity and volatility computed from the UNIDO data set we referred to before.

Central to this test is the construction of a measure of technological complexity. Following Clague (1991), we measure the technological complexity of a sector by the diversity of inputs it uses. A sector is more complex if it uses more varieties of capital goods. There are two practical shortcomings with this measure of complexity. First, there are no comprehensive data on sector-level input usages for most countries in the sample. Second, even if the data were available, the actual extent of complexity observed would respond endogenously to the level of development of the country and the relative abundance of skilled labor.

To address these issues, we use the approach followed by Clague (1991) and Rajan and Zingales (1998) and calculate the complexity measures for sectors in the U.S. There are two

key assumptions for the validity of the test we will perform: First, there are technological reasons why some industries demand a relatively larger number of capital goods than others. Second, these technological differences persist across countries, leading to a positive correlation between the rankings of technological complexity in the United States and any other given country.¹⁸ More formally, as discussed after equation (26), we treat these complexity measures as a proxy for unobserved technological complexity that is not explained by the sector’s skill intensity and relative size.

To calculate our measure of technological diversity, we use the 1997 Capital Flow Tables of the Bureau of Economic Analysis. This table distinguishes 180 capital good categories (structures, equipment, and software), each usually corresponding to a 6-digit 1997 NAICS category. We then measure technological diversification as the inverse the Herfindahl index of investment expenditure shares. Table 5 reports the (log) technological diversification index for each of the sectors in our sample.

The simple correlation between (log of) labor productivity and our index of technological diversity is positive and statistically significant (without and with country-specific dummies). However, one might argue that this strong positive correlation might be driven by other determinants. For example, capital intensity is likely to be correlated with the level of technological diversification and also to influence productivity. Incidentally, our model also predicts that the skill intensity of the sector also influences the productivity of the sector. The first column in Table 7 shows the within-country regression results, after controlling for the additional potential determinants of labor productivity. We control for the share of materials in the sector, its skill and capital intensity (measured by the share of skilled or semi-skilled workers in production and the value of equipment per worker, respectively), and the relative size of the sector. The regression shows that technological diversification is significantly and positively correlated with the level of labor productivity. A one-standard-deviation increase in the measure of technological diversification is associated with a 3 percent increase in the level of productivity. Also in line with our predictions, skill intensity raises productivity.

The same considerations stated before lead us to include a similar set of controls in the regression of (log) variance on the extent of technological diversification. The results are summarized in the second column of Table 7. Technological diversification is significantly and negatively associated with sectoral volatility. A one-standard-deviation increase in technological diversification is associated with a 4 percent decrease in the volatility of the sector. Volatility also decreases with skill intensity and, as we later document in more detail, the size of the sector.

- There is no evidence of a mean-variance frontier.

¹⁸A measure of technological complexity based on the U.S. is a noisy measure of the complexity of a sector in other countries. As Raddatz (2003) argues, to the extent that the noise corresponds to classical measurement error, the coefficients we are interested in will be biased towards zero, against the hypothesis of our study.

As discussed before, portfolio-view models predict a positive correlation between mean productivity and variance. However, in the data, the simple correlation between volatility and productivity is negative (-0.10 and significantly different from zero). Controlling for sectoral size, and country- and sector-specific effects yields no positive relationship between the two variables. Using a different approach, Koren and Tenreyro (2004) also reject the notion that countries move along a mean-variance frontier in the data.

Our model is consistent and, in fact, predicts the absence of a mean-variance frontier: As countries develop, they use more sophisticated technologies, which leads both to higher productivity and lower variance.

Prediction 4. Poor countries have a comparative advantage in less complex and hence riskier sectors. Consequently, poor countries specialize in less technologically complex sectors. This also implies that poor countries specialize in more volatile sectors.

- Poor countries have a comparative advantage in less complex and hence riskier sectors. Consequently, poor countries specialize in less technologically complex sectors.

As seen in Sections B and 2.3, skill intensive sectors will endogenously become more complex. This implies that skill abundant countries have a comparative advantage in complex sectors. Note that even a small difference in skill abundance can result in a large comparative advantage because of directed technical change.

That poor countries have a comparative advantage in less complex sectors was first documented by Clague (1991). He finds that poor countries are relatively less efficient in industries with a lower index of technological complexity (where complexity is measured similarly to the method employed in the present paper).

This pattern of comparative advantage, according to the model, implies that poor countries should specialize in less complex sectors. We checked this implication, by examining the sectoral composition of the economy. Using the UNIDO data set, we regressed the (log) average sectoral shares on *a*) the index of technological diversification of the sector; *b*) the level of development, proxied by the (log) level of GDP of the country; and *c*) the interaction between sectoral variance and the level of development. According to the model, the interaction term should be positive: As countries develop, they should move to more complex sectors. The results are displayed in Table 8*a*. The interaction term is positive and significantly different from zero, consistent with the model.

- Poor countries specialize in more volatile sectors.

To check whether the pattern of comparative advantage might also imply that poor countries specialize in relatively more volatile sectors, we regress the (log) average sectoral shares on *i*) the variance of the sector; *b*) the (log) of GDP of the country; and *c*) the interaction between sectoral variance and the level of development. As the model predicts, the regression yields a negative and significant coefficient for the interaction term, implying

that more developed countries tend to specialize in lower-variance sectors. The results are displayed in Table 8b, which shows the regressions without and with country-fixed effects.

Prediction 5. Larger sectors, in which the country has a comparative advantage are less volatile.

Profits for an individual machine owner are larger in large sectors (with more machine operators), *ceteris paribus*. Hence more machines will be attracted toward large sectors, making them less volatile. (See equation (26).) The size of the sector, in turn, is determined by its comparative advantage, implying that sectors with a comparative advantage are less volatile than sectors with comparative disadvantage.

Table 9 shows that sectors with a larger share of employment are less volatile even when controlling for country and industry fixed effects. This remains true if we control for other sectoral characteristics such as capital and skill intensity, and technological complexity (Table 7).

Canning, Amaral, Lee, Meyer and Stanley (1998) explored the relationship between GDP volatility and the size of the economy, finding that variance falls with size with an elasticity of about 1/6. We find very similar elasticities for the size of a *sector*. Note that if all risks are idiosyncratic to individual workers or machines, the fall in volatility should be faster, with an elasticity of 1. Canning et al. argue that interactions across economic actors magnify the aggregate importance of idiosyncratic risk. An alternative explanation for why idiosyncratic shocks are important in the aggregate is provided by Gabaix (2004). He shows that if the size distribution of firms has an infinite variance (such as, for example, a Pareto distribution), the decay of idiosyncratic risk with respect to size is slower. In our model, we can account for the slow decay of volatility with size if we assume that each machine has a random productivity drawn from a Pareto distribution. Then, we will have few very productive machines employing many operators. Idiosyncratic shocks to these machines then have a large effect on aggregate productivity.

3.1 Robustness

In this section, we conduct a number of robustness checks for our empirical results. First, some institutions may facilitate the response to external sectoral shocks. Since rich countries have better institutions, this may contribute to lower output volatility. We therefore look at the role of financial development and labor market flexibility in reducing volatility.

Financial development makes raising capital cheaper and faster. Hence if firms are hit by liquidity shocks, they can borrow the necessary funds without significantly disrupting production. This can make the productivity of firms smoother, especially over shorter horizons. Aghion, Angeletos, Banerjee and Manova (2004) show how the liquidity needs of long-term investments make output volatile in financially underdeveloped countries. Empirically, Braun and Larrain (forthcoming) and Larrain (2004) have shown that financial development makes output less volatile, especially in highly finance dependent sectors.

Our model can easily incorporate the pattern that volatility declines with financial development. The development of new inputs requires financing, because initial development/installation costs have to be covered up front. The value of new machines will hence be higher in financially developed countries where the cost of capital is lower, making these countries less volatile. Across sectors, differences in financing needs (“external finance dependence”) lead to similar predictions.

Column 2 of Table 10 reports the regression of sectoral variance on the level of GDP and the degree of financial development, gauged by private credit over GDP. We control for sector-specific fixed effects. Financial development leads to significantly lower volatility, but the effect of general economic development also remains significantly negative.

Our measure of volatility is the variance of labor productivity (value added per worker) growth. This may be higher in countries with rigid labor markets, because firms are less able to react to demand shocks. For example, if the demand for the product of a particular firm falls, optimally it would downsize its workforce. However, firing costs and regulations make this costly, so the firm retains its workers in the hope that the shock is transitory. In the data we would observe this shock as if it were a negative productivity shock; less output is produced with the same number of workers.

To see whether this measurement problem contaminates our results, we control for the costs of modifying and terminating employment contracts across countries, as compiled by Botero, Djankov, La Porta, Schleifer and Lopez-de-Silanes (2004). As Column 3 of Table 10 shows, labor market rigidities significantly increase the volatility of labor productivity within any given sector. However, this does not alter our prediction that volatility declines with development; in fact, the point estimate of the coefficient of GDP is greater in absolute value. Intuitively, some highly developed countries have rather rigid labor markets (notably European countries) but are still highly stable in terms of labor productivity.

In Column 4, we control for both financial development and labor market rigidities. The effect of overall economic development is still highly significant with a coefficient very similar to our benchmark estimates.

An alternative explanation for the decline of volatility with development is that high-income countries specialize in differentiated products, which are subject to idiosyncratic demand and supply shocks. This could result in lower volatility because idiosyncratic shocks wash out when aggregated over many products. Also, if sectors producing multiple differentiated products use a wider variety of inputs, then “output diversification” is correlated with “input diversification,” which potentially biases our results on technological diversification.

To test for the presence of output diversification, the use of firm-level data would be desirable. Lacking such data, however, we can use data on the number of establishments reported by UNIDO. If products are differentiated by producer firms and these products are subject to idiosyncratic demand or supply shocks, the volatility of a sector should decline with the number of firms.

Our model also predicts that larger sectors should be less volatile. The distinction between the two theories relies in the margin through which this takes place. Output diversification takes place across firms, hence volatility declines with the number of firms (extensive margin) but not with the average size of firms (intensive margins). In our model, larger firms attract proportionately more machines, and hence both margins are equally important.

To test the relative importance of the extensive and intensive margins, we decompose the size of sector s in country i , L_{is} , into the number of firms and the average size of firms,

$$\ln L_{is} = \ln N_{is} + \ln(L_{is}/N_{is}).$$

We then regress the log variance of a sector on the log number of firms and the log of their average size, controlling for both country- and industry-specific fixed effects. The output diversification would suggest that the number of firms decreases volatility while firm size does not. Table 11 reports our results. Both the extensive and the intensive margins of sector size contribute to lower volatility, and, in line with our theory, there is no significant difference in the importance of the two. Moreover, when we only focus on “complex” sectors, where product differentiation may be more prevalent, there are still no significant differences. This suggests that “output diversification” does not significantly contribute to the decline in volatility.

4 Conclusion

This paper proposes a model in which the production process makes use of different input varieties, which are subject to imperfectly correlated shocks. As in other expanding variety growth models, technological progress takes place as an expansion in the number of input varieties, increasing productivity. The new insight we develop is that the expansion in varieties also leads to lower volatility of production via two channels. First, as each individual variety matters less and less in production, the contribution of idiosyncratic fluctuations to overall volatility declines. Second, each additional input provides a new adjustment margin in response to external shocks, making productivity less volatile.

In the model, the number of varieties evolves endogenously in response to profit incentives. The consequent change in volatility associated with changes in the number of varieties feeds back into the investment and savings decisions of producers.

The model yields empirical predictions concerning the relationships among productivity, volatility, technological complexity, and comparative advantage. We discuss these predictions in light of the existing empirical evidence and provide novel findings supporting the results.

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A An Example with Fixed-Coefficients Technology

In the benchmark model we assume that $\sigma \in (0, 1)$, that is, the elasticity of substitution across machine varieties is bigger than 1 (the machines are gross substitutes). This is a standard assumption in the expanding variety literature and is needed to ensure that the varieties not yet invented (or installed) are not essential in production.

However, complementarities across different inputs (or tasks) may be an important feature of the development process. As Kremer (1993) points out, many production processes feature an “O-ring” technology: even if a single input fails, it may jeopardize the whole outcome. (Also see Young (1993) and Grossman and Maggi (2000)) on the importance of complementarities for productivity patterns.) We hence consider an example in which all the machine varieties are essential in production. We show that even in the extreme form of complementarity (O-ring), technological diversification may still take place via variable capacity utilization.

In particular, the production function takes the Leontief form:

$$Y = \min_{i=1, \dots, n} \{x_i\}.$$

The services of individual machine varieties are produced as before, with skilled operators, $x_i = h_i$. We assume, however, that the failure of the machine does not render it completely useless (otherwise log output would become minus infinity), but, rather, makes it more expensive to operate. In particular, while good machines require 1 unit of skill labor, broken machines require $\delta > 1$ units.

Let us first focus on the case without variable capacity utilization, that is, when the number of operators per machines is constant at h . This implies that when the first machine fails, output drops from h to h/δ . (Further failures have no impact on output.) So the change in log output is

$$\Delta \ln Y = -\ln \delta.$$

Since the first failures arrives after an exponentially distributed working time with an arrival rate of γn , the instantaneous variance is

$$\text{Var}(\text{dln } Y) / dt = (\ln \delta)^2 \gamma n.$$

This is in fact increasing in n ; the more complex the technology, the more likely a machine failure is. We do not have an offsetting effect from the law of large numbers because the working machines do not substitute for the broken one.

Consider now the case with variable capacity utilization. In this case, we let firms reshuffle operators across machine varieties, only holding the total number of operators fixed at $H = nh$. If a machine “fails,” the firm allocates more operators to that machine to partially offset this negative productivity shock. With free reallocation of operators, it is optimal to equalize the services of each machine variety at, say, x . This requires δx operators on the broken

machine and $(n - 1)x$ operators on the rest. The total number of operators is unchanged, so

$$\delta x + (n - 1)x = nh.$$

The change in log output is hence

$$\Delta \ln Y = \ln n - \ln(n - 1 + \delta),$$

which is negative, that is, output still drops but it drops by less than without VCU. The firm can successfully mitigate some of the impact of the shock with VCU.

The instantaneous variance in this case is

$$\text{Var}(d \ln Y) / dt = [\ln n - \ln(n - 1 + \delta)]^2 \gamma n.$$

The first part decreases with n . The more machine varieties there are, the more possibilities there exist to reshuffle operators without affecting output too much. The second part is increasing in n because more complex technologies fail more often. In general the effect of technological complexity on volatility is hence ambiguous. Nonetheless, as complexity increases without bound ($n \rightarrow \infty$), the first effect dominates and volatility goes to zero,

$$\lim_{n \rightarrow \infty} \text{Var}(d \ln Y) / dt = 0.$$

To see this, use the intermediate value theorem to rewrite $[\ln n - \ln(n - 1 + \delta)]$ as $(\delta - 1) / [n + \xi_n(\delta - 1)]$, where $\xi_n \in [0, 1]$. Since ξ_n is bounded, $[\ln n - \ln(n - 1 + \delta)]^2 = O(n^{-2})$ and $\gamma n = O(n)$.

In summary, the ability to vary capacity utilization can make more complex technologies less volatile even in the case of fully complementary inputs.

B A two-sector model of technological diversification

In 2.3, we presented the supply side of a multi-sector model. This section solves for the general equilibrium in such a model. The demand for individual goods will only matter for the pattern of specialization if the economy is closed to trade; we will discuss this case first.

Let us assume that there are two sectors, one producing a capital good Y_k , the other producing a consumption good Y_c :

$$Y_k = \left[\sum_{i=1}^{n_k} X_i^\sigma \right]^{1/\sigma},$$

$$Y_c = \left[\sum_{i=n_k+1}^{n_k+n_c} X_i^\sigma \right]^{1/\sigma}.$$

Both sectors use the same CES technology, but they have access to a different set of machines. In particular, the number of machines in the two sectors will (endogenously) be different.

The owner of each machine will decide which sector to operate in. The total number of machines $n_k + n_c$ is denoted by n .

We assume that machines in the capital good sector are operated by skilled labor, whereas those in the consumption good sector are operated by unskilled labor.¹⁹ Note that machines are a metaphor for technology in our model so this amounts to assuming that some technologies are skilled labor intensive, whereas others are unskilled labor intensive. Autor, Katz and Krueger (1998) show that computerization has increased the demand for skilled labor. However, previous technological advances such as the industrial revolution and the introduction of the production line relied more on unskilled rather than skilled workers (James and Skinner 1985, Goldin and Katz 1998). In this paper, we think of the skilled-labor intensive sector as a technologically advanced sector, and the unskilled-labor intensive sector as a less technologically complex one.²⁰

More formally,

$$X_{it} = \begin{cases} h_{it}, & \text{if } i \leq n_k, K_{i0} = \kappa \text{ and } t < T_i; \\ l_{it}, & \text{if } n_k < i \leq n, K_{i0} = \kappa \text{ and } t < T_i; \\ 0, & \text{otherwise.} \end{cases} \quad (27)$$

Here h_{it} denotes the number of skilled operators and l_{it} the number of unskilled operators of variety i , and κ and T_i are defined, as previously, as the fixed cost of building a machine and the random lifetime of machine i .

Let H denote the overall stock of skilled labor in the economy and L the stock of unskilled labor. If the labor markets are in equilibrium, the production functions can be rewritten as

$$\begin{aligned} Y_k &= n_k^\phi H, \\ Y_c &= n_c^\phi L. \end{aligned}$$

Similarly to the one-sector case, the productivity of a firm in sector i will be increasing in the number of machines. This also causes the volatility of productivity to decline with the number of machines:

$$\text{Vol}_k = \text{Var}(\text{dln } Y_k | n_k, H) = \phi^2 \gamma / n_k, \quad (28)$$

$$\text{Vol}_c = \text{Var}(\text{dln } Y_c | n_c, L) = \phi^2 \gamma / n_c. \quad (29)$$

Both sectors are perfectly competitive. Each producer takes the wage rate and the set of machine varieties available to the sector as given.

What determines the allocation of machines across the two sectors? Again, investors will maximize profits and move toward sectors with better profit opportunities. The price

¹⁹Any positive difference in skill intensity is sufficient for our results; we assume this extreme difference in skill intensity for tractability.

²⁰In Section 3 we discuss how we identify technological complexity in the data.

of machine service i will be marked up over the skilled wage in the capital good sector and over the unskilled wage in the unskilled sector:

$$\chi_i = \begin{cases} w_H/\sigma, & \text{if } i \leq n_k; \\ w_L/\sigma, & \text{if } n_k < i \leq n, \end{cases}$$

where w_H denotes skilled, w_L denotes unskilled wages. This makes profit per machine a constant ϕ fraction of wages per machine:

$$\pi_i = \begin{cases} \phi w_H H/n_k, & \text{if } i \leq n_k; \\ \phi w_L L/n_c, & \text{if } n_k < i \leq n. \end{cases}$$

In long-run equilibrium, the rate of return on machines in the two sectors have to be equal. Since the values of a machine are the same in each sector, so is the dividend (that is, profit per machine), implying

$$\frac{n_k}{n_c} = \frac{\omega H}{L}, \quad (30)$$

with $\omega = w_H/w_L$ denoting the skill premium.

The number of machines in a sector is proportional to the total wage bill of the sector. Whenever (30) fails to hold, that is, one of the sectors has relatively few machines, that sector has more profits per machine than the other one. Investors will then move machines across sectors (if machines are movable), or invest only in the more profitable sector until the equality is resolved.

We assume that machines are freely and instantly movable across sectors. This assumption ensures that $r_k = r_c$ at any time in any state of the world, implying that (30) holds at any point in time.²¹ Since (30) would always hold in the long run, we only need this assumption to simplify the transitional dynamics: If machines can instantly adjust across sectors then the economy will immediately jump to its balanced-growth path. Because the machines are movable across sectors, the single state variable is the total number of machines, n .

Equation (30) describes a version of *directed technical change*, as in Acemoglu (2002) or Caselli and Coleman (2000): Machine varieties are directed towards the sector with a higher share in employment. On the one hand, this is a size effect: if there are many operators in a sector, it is more profitable to operate machines there and hence more machines will move towards this sector. On the other hand, there is also a relative price effect: If the skill premium is high, the relative price of the capital good is high, so profits are higher in the capital good sector.

²¹Formally, the present discount value of a machine is $p_k \kappa$ in both sectors. If the dividend yield in one sector is higher than in another, an investor could buy one unit of high-dividend stock and short one unit of low-dividend stock. Since the prices of the two stocks always move in parallel, this strategy presents an arbitrage opportunity with positive net dividends in all future periods and no price risk. Absence of arbitrage hence implies the equalization of dividend yields.

Since the final good sectors are competitive, the relative price of the capital good will equal the relative marginal cost:

$$p_k = \omega \left(\frac{n_k}{n_c} \right)^{-\phi}. \quad (31)$$

If there are more machines allocated to the capital sector, it becomes more productive, and its relative price falls.

B.1 The closed economy

In a closed economy, the relative price is determined by the relative supply and demand of the two goods. Suppose that the demand for the two goods take the isoelastic form,

$$\frac{Y_k}{Y_c} = A p_k^{-\varepsilon},$$

where A is a constant and ε is the elasticity of substitution between capital and consumption goods. Then, the skill premium is endogenously determined as

$$\omega = p_k \left(\frac{n_k}{n_c} \right)^\phi = A^{1/\varepsilon} \left(\frac{H}{L} \right)^{-\phi/\varepsilon} \left(\frac{n_k}{n_c} \right)^{\phi(1-1/\varepsilon)}, \quad (32)$$

$$\frac{n_k}{n_c} = A^{\frac{1}{\varepsilon(1-\phi)+\phi}} \left(\frac{H}{L} \right)^{\frac{\varepsilon-\phi}{\varepsilon(1-\phi)+\phi}}. \quad (33)$$

As we will show, the nominal saving rate in the economy is constant, which means that $p_k Y_k / Y_c$ is constant. This implies that the relative demand for the consumption and the capital good has unitary elasticity, $\varepsilon = 1$. Then equation (33) simplifies to

$$\frac{n_k}{n_c} = A \left(\frac{H}{L} \right)^{1-\phi}. \quad (34)$$

Since we have assumed $0 < \phi < 1$, the relative number of machines is increasing in the relative amount of skilled labor in the economy. However, a 1 percent increase in skill abundance induces a less than 1 percent increase in the relative number of machines in the capital good sector. This is because the abundant factor (more precisely, the good that uses the abundant factor intensively) becomes cheaper and hence less profitable for machine owners. This relative price effect has been pointed out by Acemoglu (2002).

The fixed cost required to build a machine is assumed to arise in capital goods. In terms of consumption goods, the fixed cost is $p_k \kappa$. The mean rate of return on machines is the same in both sectors, so let us take the capital good sector:

$$r = \frac{\pi_i}{V} - \gamma = \phi \frac{w_H H / n_k}{p_k \kappa} - \gamma = \phi \left(\frac{n_k}{n} \right)^{\phi-1} \frac{H/L}{\kappa_0} - \gamma, \quad (35)$$

where we have made use of the facts that capital good prices equal marginal costs ($w_H n_k^{-\phi}$) and that the sunk cost is falling at the rate $\phi - 1$, $\kappa = \kappa_0 L n^{\phi-1}$. Relative to (19), the

important difference is that the rate of return is falling in the relative complexity of the capital good sector. The number of machines affects profitability in two ways. First, more machines make the capital sector more productive and hence more profitable (productivity is proportional to n_k^ϕ). Second, competition increases in the number of machines, because machines compete for a scarce supply of operators (H). This second effect lowers profits proportionally with n , so it dominates the first for $\phi < 1$.

The relative demand for capital and consumption goods will be determined by the consumption-saving decision. Since the mutual fund holds all machines in both sectors, the Euler equation of the consumer is the same as in (22), but the growth rate of machines and the return on machines may be different.

The mean growth rate of the number of machines, n is

$$\mu_n = \frac{Y_k}{n\kappa} - \gamma = \frac{H/L}{\kappa_0} \left(\frac{n_k}{n} \right)^\phi - \gamma. \quad (36)$$

As before, we conjecture that optimal consumption is given by $v_0 n^\phi$, which, by the equilibrium in the consumption good market, implies

$$\frac{n_c}{n} = \left(\frac{v_0}{L} \right)^{1/\phi}, \quad (37)$$

$$\frac{n_k}{n} = 1 - \left(\frac{v_0}{L} \right)^{1/\phi}. \quad (38)$$

The Euler equation simplifies to $\mu_n = r - \rho$. Substituting from (35) and (36), we see that $v_0 n^\phi$ is indeed a solution as long as

$$\frac{H/L}{\kappa_0} \left[1 - \left(\frac{v_0}{L} \right)^{1/\phi} \right]^\phi = \phi \frac{H/L}{\kappa_0} \left[1 - \left(\frac{v_0}{L} \right)^{1/\phi} \right]^{\phi-1} - \rho. \quad (39)$$

Since $\phi \in (0, 1)$, the left-hand side of (39) is monotonically declining, the right-hand side is monotonically increasing in v_0 . At $v_0 = 0$ the left-hand side $H/(\kappa_0 L)$, the right-hand side is $\phi H/(\kappa_0 L) - \rho < H/(\kappa_0 L)$. At $v_0 = L$, the left-hand side is zero, whereas the right-hand side tends to infinity. That is, there exists a unique $v_0 \in (0, L)$ such that (39) holds.

Note that v_0 does not depend on n , implying that the allocation of machines across sectors as well as the relative prices are independent of development. In other words, the economy exhibits balanced growth. This also means that the saving rate is constant as previously claimed.

B.2 A small open economy

If the economy is open to trade then the relative price of the two goods is determined in world markets. Let \tilde{p}_k denote the world price of capital. We then have from (31)

$$\tilde{p}_k = \omega \left(\frac{n_k}{n_c} \right)^{-\phi},$$

which results in *conditional factor price equalization*,

$$\omega = \tilde{p}_k \left(\frac{n_k}{n_c} \right)^\phi. \quad (40)$$

Conditional on the levels of productivity in the two sectors, the world relative price of the two goods completely determines the relative wage. All else equal, a higher relative price of the capital good (high \tilde{p}_k) leads to a higher relative wage of the factor which is used intensively in that sector (high ω). This is the FPE part. At the same time, the more productive the capital good sector is relative to the consumption good sector, the higher the relative wage of skilled labor.

Note that, as standard in small open-economy models with free trade, the production structure is independent of demand considerations. Relative demand for the two sectors (in our case, the consumption/investment decision) will matter only for the patterns of trade. Since \tilde{p}_k is exogenously given, we can substitute (40) into (30):

$$\frac{n_k}{n_c} = \tilde{p}_k^{1/(1-\phi)} \left(\frac{H}{L} \right)^{1/(1-\phi)}. \quad (41)$$

Notice that, similarly to the closed economy case, (34), the relative number of machines in the capital sector increases in skill abundance. However, the dependence on skill abundance is stronger, $1/(1-\phi)$, because we no longer have an offsetting relative price effect. This is just the Rybczynski theorem applied to directed technical change.

The impact is also greater than in the case of pure factor price equalization. The reason for this is that machines flow towards the sector that already has a comparative advantage, making it relatively more productive. This becomes an additional source of comparative advantage. In other words, the initial comparative advantage gets magnified by directed technical change. Our model says that even small human capital differences can account for large differences in specialization patterns and, hence, in the relative volatilities of sectors.

C Data Appendix

Variable Definitions

GDP per capita GDP per capita of the country in 1997, measured in 1995 international dollars. [WDI, PWT]

Population Population of the country in 1997. [WDI]

Yield volatility Variance of the log of annual wheat yield. [FAOSTAT]

Rainfall volatility Variance of cumulated log changes in precipitation. Precipitation data are recorded monthly at several meteorological stations within a country. Because many stations do not report data in all months, we take the average of year-on-year

changes for all months and all stations within the country. We cumulate these changes to obtain the country's deviation from long-run precipitation trends. [Global Historical Climatology Network]

Temperature volatility Variance of cumulated changes in temperature, calculated in the same way as rainfall volatility. [Global Historical Climatology Network]

Change in oil price Two-year change in the U.S. CPI-deflated price of West Texas Intermediate oil.

Diverse powerplants dummy Takes the value of one if the concentration of powerplants by type (conventional, hydroelectric, nuclear, renewable) in the country is below the median. [International Energy Annual]

Technological Diversification The log of the inverse of the Herfindahl index of concentration of equipment purchases across different varieties of capital goods. A sector has a high Technological Diversification index if it purchased many different capital goods. [1997 Capital Flow Tables]

Average Share in Manufacturing The sector's share in manufacturing employment, averaged across the sample period, 1963–1998. [UNIDO]

Labor Productivity Value added per worker in 1995 dollars, averaged across the sample period, 1963–1998. [UNIDO]

Variance of Productivity The variance of 5-year growth of value added per worker in 1995 dollars across the sample period, 1963–1998. [UNIDO]

Skill Intensity The fraction of production workers in the 3-digit ISIC sector that are employed in skilled or semi-skilled occupations. [Occupational Employment Statistics]

Share of Materials The share of intermediate inputs in total sales. [NBER-CES Manufacturing Industry Database]

Equipment per Worker, Structure per Worker [NBER-CES Manufacturing Industry Database]

Revealed Comparative Advantage The share of sector s in country i 's manufacturing export relative to the world average. [Trade and Production Database]

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- (11) UNIDO Industrial Statistics Database 2002, 3-digit ISIC, Revision 2. United Nations Industrial Development Organization, 2002.
- (12) World Development Indicators, World Bank, 2002.

Table 1. Yield Volatility and Development

	Variance of wheat yield		
GDP per capita	-0.4004*** (0.1189)	-0.3731*** (0.1291)	-0.3862*** (0.1325)
Population		0.0634 (0.1041)	
Temperature volatility			-0.2413 (0.1480)
Rainfall volatility			0.0319 (0.1159)
Observations	20	20	20
Adjusted R-squared	0.3524	0.329	0.377

Notes: Standard errors are in parentheses. *, **, *** denote significance at 10, 5, 1%. All variables are in logs. Dependent variable is the variance of log wheat yield per acre. (Source: FAOSTAT, Food and Agriculture Organization of the UN.) Temperature volatility is the variance of annual temperature changes. Rainfall volatility is the variance of percentage annual rainfall changes. (Source: Global Historical Climatology Network.)

Table 2. Power Plant Diversity and Energy Production

	Electricity production	Share of oil in energy
Change in oil price	- 0.0235* (0.0121)	0.0003 (0.0038)
Change in oil price × Diverse power plant types	0.0400** (0.0169)	- 0.0103** (0.0053)
Time trend	- 0.0021*** (0.0005)	0.0007*** (0.0002)
Observations	4169	4164
Country Fixed Effects	217	217
Adjusted R-squared	0.089	0.017

Notes: Standard errors are in parentheses. *, **, *** denote significance at 10, 5, 1%. Dependent variables are (1) the log change in electricity production in kWh, (2) the %point change in the share of oil consumption in total energy consumption (in British thermal units). Oil price change is the 2-year change in U.S. CPI-deflated price of West Texas Intermediate oil. The concentration of power plants is measured as the Herfindahl index of shares of power generation techniques (conventional, nuclear, hydroelectric, renewable) in total electricity production of the country. Countries with “diverse power plant types” are those with below-median concentration. (Source: International Energy Annual 2002, Energy Information Administration.)

Table 3. List of Countries. Development and Size

Country	Real GDP per capita	Population (thousands)	Sectoral Data	Country	Real GDP per capita	Population (thousands)	Sectoral Data	Country	Real GDP per capita	Population (thousands)	Sectoral Data
Albania	2789	3339	No	Gambia, The	1489	1186	No	Niger	729	9770	No
Algeria	4798	29000	No	Georgia	2130	5431	No	Nigeria	830	118000	No
Angola	1853	12000	No	Germany	22636	82100	Yes	Norway	27782	4404	Yes
Antigua and Barbuda	9624	66	No	Ghana	1767	18000	No	Pakistan	1802	128000	Yes
Argentina	12342	35700	Yes	Greece	14382	10500	No	Panama	5478	2719	Yes
Armenia	2124	3786	Yes	Grenada	5823	96	No	Papua New Guinea	2379	4761	No
Australia	22967	18500	Yes	Guatemala	3549	10500	Yes	Paraguay	4625	5085	No
Austria	23471	8072	Yes	Guinea	1862	6922	No	Peru	4667	24400	Yes
Azerbaijan	2103	7838	No	Guinea-				Philippines	3872	71300	Yes
Bahamas	15190	289	No	Bissau	926	1126	No	Poland	7703	38700	Yes
Bahrain	14561	620	No	Guyana	3969	749	No	Portugal	15103	9945	Yes
Bangladesh	1388	124000	No	Haiti	1420	7492	No	Romania	6512	22600	Yes
Barbados	13590	265	No	Honduras	2471	5939	Yes	Russian Federation	7184	147000	No
Belarus	6073	10100	No	Hong Kong	23734	6502	Yes	Rwanda	798	7895	No
Belgium	24151	10200	No	Hungary	10244	10200	Yes	Samoa	4213	167	No
Belize	4810	217	No	Iceland	25557	272	No	Saudi Arabia	11033	19200	No
Benin	897	5794	No	India	2034	962000	Yes	Senegal	1341	8777	No
Bhutan	1239	737	No	Indonesia	3217	200000	Yes	Sierra Leone	511	4726	No
Bolivia	2309	7767	Yes	Iran	5416	60900	Yes	Singapore	20560	3794	Yes
Botswana	6552	1533	No	Ireland	21287	3670	Yes	Slovak Republic	9902	5383	No
Brazil	7065	164000	No	Israel	18382	5836	No	Slovenia	14586	1986	No
Brunei	17086	313	No	Italy	21499	57500	Yes	Solomon Islands	2279	404	No
Bulgaria	4860	8312	Yes	Jamaica	3563	2554	No	South Africa	8978	40700	Yes
Burkina Faso	874	10500	No	Japan	25405	126000	Yes	Spain	16623	39300	Yes
Burundi	575	6418	No	Jordan	3899	4459	Yes	Sri Lanka	3041	18600	Yes
Cambodia	1345	11200	No	Kazakhstan	4713	15300	No	St. Kitts and Nevis	10937	41	No
Cameroon	1535	13900	Yes	Kenya	1038	28000	Yes	St. Lucia	5358	150	No
Canada	24557	30000	Yes	Korea, Rep.	15295	46000	Yes	St. Vincent	4779	112	No
Cape Verde	4091	404	No	Kuwait	15748	1809	Yes	Sudan	1465	29300	No
Central African Rep.	1077	3529	No	Kyrgyzstan	2403	4681	No	Suriname	3669	412	No
Chad	839	7086	No	Lao PDR	1384	4919	No	Swaziland	4038	960	No
Chile	8740	14600	Yes	Latvia	5793	2469	Yes	Sweden	21231	8849	Yes
China	3152	1230000	Yes	Lebanon	4276	4146	No	Switzerland	25973	7088	No
Colombia	5891	40000	Yes	Lesotho	1865	1945	No	Syria	3265	15000	No
Comoros	1618	518	No	Lithuania	6530	3706	No	Tajikistan	941	6017	No
				Luxembourg	36355	422	No				

Table 3 continued.

Country	Real			Sectoral Data	Country	Real			Sectoral Data	Country	Real			Sectoral Data
	GDP per capita	Population (thousands)				GDP per capita	Population (thousands)				GDP per capita	Population (thousands)		
Congo, Dem. Rep.	791	46800	No		Macao, China	18970	418	No		Tanzania	484	31300	No	
Congo, Rep.	997	2767	No		Macedonia	4433	1997	No		Thailand	6591	59400	Yes	
Costa Rica	6993	3577	Yes		Madagascar	784	14100	No		Togo	1526	4135	No	
Cote d'Ivoire	1617	14700	No		Malawi	575	9665	Yes		Trinidad and Tobago	7417	1278	Yes	
Croatia	7140	4447	No		Malaysia	8562	21700	Yes		Tunisia	5481	9215	No	
Cyprus	17560	744	Yes		Maldives	6316	256	No		Turkey	6626	62500	Yes	
Czech Republic	13265	10300	No		Mali	723	10100	No		Turkmenistan	2620	4779	No	
Denmark	24897	5284	Yes		Malta	14355	383	No		Uganda	1078	20400	No	
Dominican Republic	4903	7968	No		Mauritania	1601	2415	No		Ukraine	3408	50700	No	
Ecuador	3273	11900	Yes		Mauritius	8425	1148	No		United Arab Emirates	20561	2580	No	
Egypt, Arab Rep.	3141	60400	Yes		Mexico	7839	93900	Yes		United Kingdom	21006	59000	Yes	
El Salvador	4135	5911	No		Moldova	2261	4312	No		United States	30123	272000	Yes	
Equatorial Guinea	3478	421	No		Mongolia	1619	2331	No		Uruguay	8850	3265	Yes	
Eritrea	821	3773	No		Morocco	3335	27300	Yes		Uzbekistan	2137	23700	No	
Estonia	8087	1427	No		Mozambique	705	16600	No		Vanuatu	2778	179	No	
Ethiopia	629	59800	Yes		Namibia	5778	1648	No		Venezuela	6211	22800	Yes	
Fiji	4716	783	No		Nepal	1221	21400	No		Vietnam	1711	75500	No	
Finland	20979	5140	Yes		Netherlands	22464	15600	Yes		Yemen	773	16100	No	
France	21647	58200	Yes		New Caledonia	22241	202	No		Zambia	784	9443	No	
French Polynesia	20541	224	No		New Zealand	18256	3761	Yes		Zimbabwe	2810	11900	No	
Gabon	6535	1137	No		Nicaragua	2138	4680	No						

The list includes all countries used in the analysis. The sectoral– data column indicates the countries for which sectoral data are available

Table 4. Volatility and Development

	Variance of GDP growth (log)	Variance of GDP growth (log)
GDP per capita (1995 international dollars, log)	- 0.2952*** (0.0699)	- 0.3259*** (0.0655)
Population (log)		- 0.1353*** (0.0376)
Constant	- 3.6316*** (0.5968)	- 1.2573 (0.8663)
Observations	167	167
Adjusted R-squared	0.10	0.15

Notes: Robust standard errors are in parentheses. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 5. List of Sectors. Technological Diversification Index and Average Share in Manufacturing

ISIC code	Description	Index of Technological Diversification	Average Share in Manufacturing
311	Food products	2.898	0.1581
313	Beverages	2.975	0.0282
314	Tobacco	2.666	0.0148
321	Textiles	1.733	0.1210
322	Wearing apparel, except footwear	2.303	0.0818
323	Leather products	3.278	0.0089
331	Wood products, except furniture	2.368	0.0345
332	Furniture, except metal	2.909	0.0223
341	Paper and products	2.433	0.0242
342	Printing and publishing	2.340	0.0371
351	Industrial chemicals	2.835	0.0235
352	Other chemicals	2.808	0.0342
353	Petroleum refineries	2.726	0.0099
354	Miscellaneous petroleum and coal products	2.726	0.0023
355	Rubber products	2.217	0.0167
356	Plastic products	1.847	0.0251
361	Pottery, china, earthenware	3.006	0.0065
362	Glass and products	3.006	0.0093
369	Other non– metallic mineral products	3.006	0.0438
371	Iron and steel	2.618	0.0297
372	Non– ferrous metals	3.111	0.0117
381	Fabricated metal products	2.849	0.0589
382	Machinery, except electrical	2.817	0.0662
383	Machinery, electric	2.487	0.0637
384	Transport equipment	2.722	0.0548
385	Professional and scientific equipment	2.999	0.0116

Notes: Sectors correspond to the 3-digit manufacturing sectors from Revision 2 of the International Standard Industrial Classification of all Economic Activities (ISIC). Technological diversification measures the diversity of capital goods a sector purchases in the U.S. Average share is the sector's share in manufacturing employment averaged across countries. (See Data Appendix.)

Table 6. Productivity, Volatility and Development

	Log Productivity	Log Variance
GDP per capita (log)	0.7047*** (0.0690)	- 0.3005*** (0.1020)
Sector Fixed Effects	Yes	Yes
Observations	1429	1429
Adjusted R-squared	0.73	0.14

Notes: The regressions use sectoral data at the 3-digit level and include sector-specific effects. Mean labor productivity and variance of labor productivity growth rates correspond to the period 1963–1998. Robust standard errors are adjusted for country clustering. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 7. Productivity, Volatility and Technological Diversification

	Log Productivity	Log Variance
Technological Diversification	0.0660*** (0.0234)	- 0.0999** (0.0448)
Sectoral Skill Intensity (log)	0.2351*** (0.0441)	- 0.3005*** (0.0919)
Share of Materials in Production (log)	0.2155*** (0.0634)	0.0289 0.0939
Equipment per Worker (log)	0.4613*** (0.0184)	0.0062 0.0211
Labor Share (Log)	- 0.0532*** (0.0151)	- 0.1727*** (0.0240)
Country Fixed Effects	Yes	Yes
Observations	1535	1535
Adjusted R-squared	0.81	0.56

Notes: The equations use sectoral data at the 3-digit level and include country-specific effects. Mean labor productivity and variance of labor productivity growth rates correspond to the period 1963–1998. Technological diversification measures the diversity of capital goods a sector purchases in the U.S. Skill intensity is the share of skilled and semi-skilled workers. Material share is the ratio of material costs to total shipments. Labor share is the sector's share in manufacturing employment. (See Data Appendix.) Robust standard errors are adjusted for country clustering. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 8a. Sectoral Shares, Technological Diversification, and Development

	Sectoral Share	Sectoral Share
Technological Diversification × Real GDP per capita (Log)	0.0083* (0.0044)	0.0089** (0.0044)
Technological Diversification	- 0.1015** (0.0404)	- 0.1075*** (0.0408)
Real GDP per capita (Log)	- 0.0248** (0.0115)	
Country Fixed Effects	No	Yes
Observations	1429	1429
Adjusted R-squared	0.05	0.12

Notes: The equations use sectoral data at the 3– digit level. The dependent variable is the share of the sector in total manufacturing employment, averaged over 1963–1998. The second column includes country– specific effects. Technological diversification measures the diversity of capital goods a sector purchases in the U.S. (See Data Appendix.) Robust standard errors clustered by country are shown in parentheses. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 8b. Sectoral Shares, Volatility, and Development

	Sectoral Share	Sectoral Share
Sectoral Variance × Real GDP per capita (Log)	- 0.0006*** (0.0002)	- 0.0015*** (0.0004)
Sectoral Variance	- 0.0075 (0.0061)	- 0.0018 (0.0060)
Real GDP per capita (Log)	- 0.0064*** (0.0014)	
Country Fixed Effects	No	Yes
Observations	1429	1429
Adjusted R-squared	0.04	0.10

Notes: The equations use sectoral data at the 3– digit level. The dependent variable is the share of the sector in total manufacturing employment, averaged over 1963–1998. The second column includes country– specific effects. Variance of labor productivity growth rates correspond to the period 1963– 1998. Robust standard errors clustered by country are shown in parentheses. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 9. Relative Size and Volatility

	Log Variance	Log Variance	Log Variance
Labor Share	-0.1554*** (0.0256)	-0.1743*** (0.0233)	-0.1893*** (0.0293)
Country Fixed Effects	No	Yes	Yes
Sector Fixed Effects	No	No	Yes
Observations	1521	1521	1521
Adjusted R-squared	0.03	0.55	0.59

Notes: The equations use sectoral data at the 3-digit level. Labor share is the sector's share in total manufacturing employment, averaged over 1963–1998. Robust standard errors are adjusted for country clustering. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 10. Other Institutions Reducing Volatility

	Log Variance			
Real GDP per capita (log)	-0.264*** (0.082)	-0.163* (0.092)	-0.350*** (0.111)	-0.256** (0.120)
Private credit / GDP		-0.523* (0.289)		-0.473 (0.330)
Labor market rigidities			0.846** (0.420)	0.819* (0.431)
Sector Fixed Effects	Yes	Yes	Yes	Yes
Observations	1607	1607	1319	1319
Adjusted R-squared	0.11	0.12	0.16	0.17

Notes: Robust standard errors (in parentheses) allow for clustering within countries. * Significant at 10%; ** significant at 5%; *** significant at 1%. Complex sectors are those with above median investment good diversification. Labor market rigidities are measured by an index that combines the costs of firing workers and changing employment terms (Botero, Djankov, La Porta, Schleifer and Lopez-de-Silanes, 2004).

Table 11. Output versus Input Diversification

	Log Variance (all sectors)	Log Variance (complex sectors)
Number of firms (log)	-0.1140*** (0.0382)	-0.1357*** (0.0487)
Average size of firms (log)	-0.1550*** (0.0516)	-0.1887*** (0.0603)
Country Fixed Effects	Yes	Yes
Sector Fixed Effects	Yes	Yes
Observations	1586	932
Adjusted R-squared	0.48	0.43

Notes: Robust standard errors (in parentheses) allow for clustering within countries. * Significant at 10%; ** significant at 5%; *** significant at 1%. Complex sectors are those with above-median investment good diversification.

Figure 1. GDP Volatility and Development

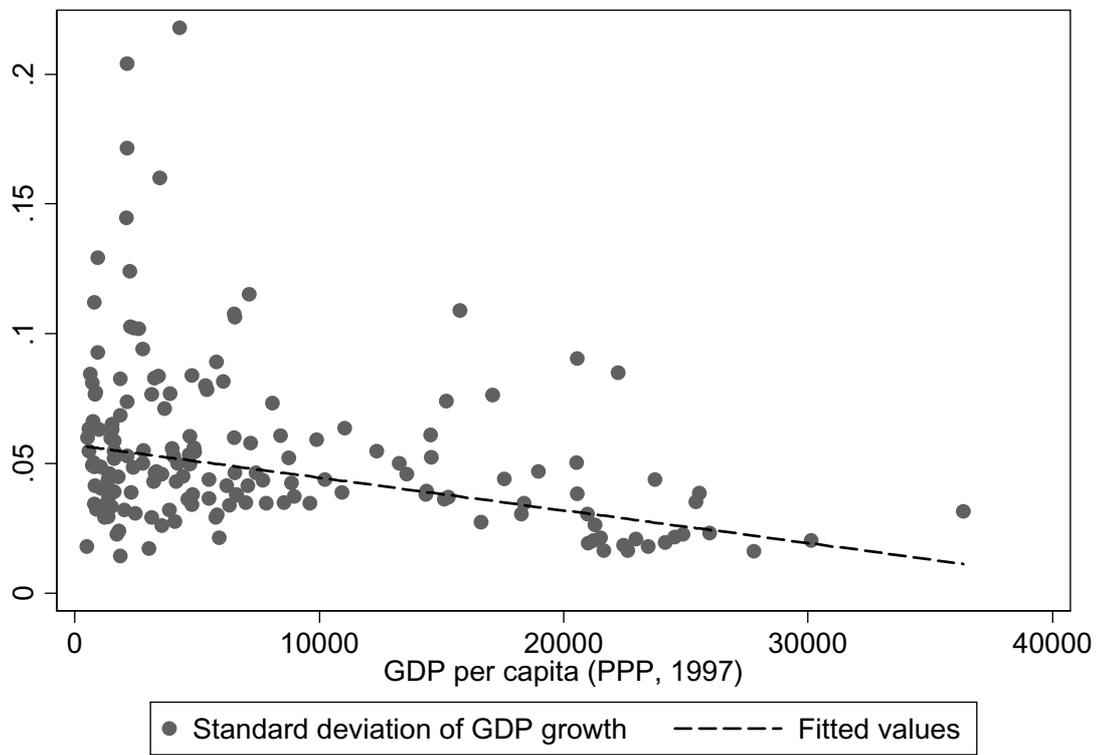


Figure 2. Wheat Yield Volatility and Development

