Fiscal-Monetary Policy Interactions and Macroeconomic Stability

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Abstract

This paper studies interactions of interest rate and fiscal policy feedback rules in a sticky price model where public debt influences aggregate demand. In particular, we present two means by which the stock of government bonds affects its rate of return: bonds as an argument of a transaction cost function and partial debt repayments. Under both assumptions, a rise in public debt reduces its rate of return for a given nominal interest rate and therefore induces households to reduce savings in favor of current consumption. Steady state uniqueness and saddle path stability requires inflation reactions of interest rate policy not to exceed a threshold that rises with the fraction of government expenditures financed with lump-sum taxes. Under cost push shocks, monetary policy faces the usual trade-off between stabilization of inflation and output, whereas both are stabilized through high shares of tax financing under an aggressive anti-inflationary monetary policy regime. Output and inflation volatility are further found be be lower than in the corresponding model with debt neutrality.

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1 Introduction

This paper studies the role of interactions between the central bank's interest rate setting and government deficit policy for macroeconomic stability. The academic literature on business cycle theory has (with exceptions to be noted below) mainly concentrated on the design of monetary stabilization policies in the recent past. Yet public policy debates in Europe show a large and growing concern about the possible (de-)stabilizing properties of government deficits. How seriously the role of government debt is taken can be seen from recent discussions surrounding the 'Stability and Growth Pact', which intends to restrict fiscal deficits on EMU member countries in order to ensure the independence of the European Central Bank.

Theoretically, it is clear that government debt does not matter for stabilization policy as long as the conditions of the Ricardian equivalence theorem are satisfied and the government is solvent. Thus, a large number of current dynamic general equilibrium models where monetary policy has real effects through the assumption of price stickiness (labelled 'New Keynesian' models by Clarida et al., 1999) avoids the discussion of fiscal-monetary interaction altogether by assuming that taxes are lump-sum and the planning horizon of the representative agent is infinite.[3] In this case, Ricardian equivalence holds and public debt has no bearing on aggregate demand and inflation (as long as the public sector respects its solvency constraint; see e.g. Leeper, 1991, Woodford, 2001a, or Benhabib et al., 2001, for results on 'non-Ricardian' policies). Recently, several studies have examined optimal monetary and fiscal policy within the sticky price framework where government debt matters because taxes are distortionary (see Benigno and Woodford, 2003, Schmitt-Grohe and Uribe, 2004). In these models, public debt has an aggregate supply effect since it is negatively related to labor income taxes in the short run. This mechanism is different from the one studied by Leith and Wren-Lewis (2000). In their Blanchard (1985)-type model of finite horizons Ricardian equivalence does not hold because the possibility that an agent will not be around to pay future taxes creates a positive effect of government debt on aggregate consumption demand.

The present paper also studies aggregate demand effects of public debt, and analyzes the consequences for equilibrium uniqueness and stability, and the policy trade-offs associated with stabilization of inflation and output. Specifically, fiscal-monetary interactions are introduced through a model setup that implies non-neutrality of public debt by having the rate of return on government bonds depend on its current stock. We consider two modifications of an otherwise conventional New Keynesian model which both have in common that the (total) rate of return on government bonds decreases with the real value of its outstanding stock for a given nominal interest rate set by the central bank. The first modification is inspired by Canzoneri and Diba's (2003) idea to resolve price level determinacy under interest rate policy by allowing public debt to provide transac-

[3] Some articles have used such a framework to analyze the interaction of (lump-sum financed) government spending policies with monetary policy (e.g. Beetsma and Jensen, 2002).

tion services.[4] This property is modeled in this paper by allowing transaction costs to be decreasing and convex in both types of government liabilities, money and bonds. The total rate of return on bonds, therefore, consists of interest rate payments and of a real return from lowering transaction costs. As the marginal return from transaction services is decreasing in households' government bond holdings, real public debt is negatively related to its rate of return. Thus, an increase in public debt induces – for a given nominal interest rate – substitution of consumption from the future to the present. As an alternative modelling strategy that leads to an equivalent intertemporal substitution effect of debt, the fiscal authority is assumed to repay only a fraction of debt obligations in each period. The aforementioned effect then also emerges, when this fraction decreases with the current stock of public debt, which can interpreted as an ad-hoc specification of a sovereign default probability.[5]

Both model variants lead to an identical reduced form presentation, which is used to study the interaction of fiscal and monetary policies.[6] We assume the fiscal authority to levy lump-sum taxes as a percentage of its expenditures, while central bank sets the nominal interest rate contingent on current inflation. Monetary policy influences, on the one hand, aggregate demand through price stickiness and has, on the other hand, a bearing on the government budget, since the real interest rate determines the government's cost of servicing outstanding bonds. Reactiveness of interest rate setting thus has an influence on government deficits, while the fiscal stance decides on the magnitude of the change in public debt, which feeds back on private expenditures (by intertemporal sbustitution), inflation, and, therefore, on interest rate adjustments of the monetary authority.

The main results are as follows. The fact that government debt is a determinant of aggregate demand makes the conditions for steady state uniqueness and saddle path stability depend on the interaction between the monetary authority's interest rate reaction to inflation and the fiscal authority's tax reaction to budget deficits. The greater the part of the interest payments on outstanding public debt that the government is willing to finance by primary surpluses, i.e. by raising taxes, the less critical are changes in the real interest rate for the government budget. There is a stabilizing mechanism that originates in a negative feedback from rising inflation on future real public debt that tends to reduce aggregate consumption demand, which finally rules out self-fulfilling (inflation) expectations.[7] An aggressive interest rate policy that raises the real interest rate in response to inflation, however, runs the risk to interfere with and overturn this stabilizing effect; a policy of a

[4] Similar assumptions can, for example, also be found in Bansal and Coleman (1996) and Lahiri and Vegh (2003).

[5] Arbitrage freeness then demands the interest rate on government bonds to exceed a risk-free interest rate by a (risk) premium. For a thorough analysis of sovereign default risk in a general equilibrium framework, see Uribe (2002).

[6] Similar structures also arise from an overlapping generations framework (see Leith and Wren-Lewis, 2000), or when households face convex adjustment costs for bond holdings, as for example in the "limited participation" literature (see, e.g., Christiano and Eichenbaum, 1992).

[7] By implication, the well-known Taylor-principle, which ensures uniqueness of local equilibrium paths when public debt is neutral (see Woodford, 2001b), does not apply.

fixed nominal interest rate, by contrast, is sufficient for the existence of a unique stable equilibrium path regardless of fiscal behavior. On the other extreme, in the case of a fiscal budget that is balanced period by period, all interest payments are financed by taxes, so that local stability and uniqueness of equilibrium paths prevail for any interest rate rule. The condition for the uniqueness of the steady state is closely related to the condition for saddle path stability.

Second, we show by deriving the impulse responses to interest rate and tax shocks that the model behaves in an intuitive way for simple specifications of the monetary and fiscal policy feedback rules. An unexpected increase in the nominal interest rate and a temporary rise in taxes are contractionary, i.e., cause output and inflation to decline. However, a high degree of debt financing (for the former shock) or a very aggressive monetary policy (for the latter shock) can counteract these effects since they imply large increases in public debt which tend to raise output due to the intertemporal substitution effect.

Third, we analyze the possible contributions of monetary and fiscal policy to the stabilization of fluctuations induced by cost push shocks, which lead to an immediate rise in inflation, and a decline in output and public debt in this model. A more aggressive interest rate setting reduces (raises) inflation (output) fluctuations, as usual. We further find that a high degree of tax financing always reduces the inflation variance, while it may raise or reduce the output variance depending on the monetary stance. The reason is that a rise in inflation due to a cost push shock tends to lower the real value of public debt, which reinforces via the intertemporal substitution channel the output contraction. When this channel is more pronounced, the inflation variance declines, while the output variance rises for moderate interest rate policies. This, however, changes under aggressive anti-inflationary monetary policy regimes, where higher shares of tax financing also reduce output fluctuations. We further find that both, the variance of output and of inflation are lower in the present model than in a standard New Keynesian model, where, other things equal, public debt is neutral. These findings are broadly supported by simulation results conducted for an alternative – more general – fiscal policy (debt) rule.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 derives analytical results and provides numerical examples for the variances of output and inflation. Section 4 concludes.

2 A sticky price model with non-neutral debt

In this section a sticky price model where the stock of government bonds affects its total rate of return is presented. We separately provide two modelling strategies which are to introduce this feedback mechanism into an otherwise conventional New Keynesian model. First, government bonds are – analogously to money – an argument of a convex transaction costs function. Alternatively, the fiscal authority is assumed to repay only a fraction of its debt obligations, and the size of the fraction is assumed to be decreasing in the total amount of outstanding debt.

2.1 Version A: Transaction services of bonds

Throughout the paper, nominal variables are denoted by upper-case letters, while real variables are denoted by lower-case letters. There is a continuum of households indexed with $j \in [0,1]$. Households exhibit identical asset endowments and identical preferences. Household j maximizes the expected sum of a discounted stream of instantaneous utilities u :

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{jt}, l_{jt}\right), \tag{1}$$

where E_0 is the expectation operator conditional on the time 0 information set, and $\beta \in (0,1)$ is the subjective discount factor. The instantaneous utility u is assumed to be increasing in consumption c, decreasing in working time l, strictly concave, twice continuously differentiable, and to satisfy the usual Inada conditions. For analytical simplicity, instantaneous utility u is further restricted to be separable in private consumption c and working time $l : u(c_{jt}, l_{jt}) = \upsilon(c_{jt}) - \mu(l_{jt})$.

At the beginning of period t household j is endowed with holdings of money M_{jt-1}, government bonds B_{jt-1}, which are carried over from the previous period, and a portfolio of state contingent claims on other households yielding a (random) payment Z_{jt}. Let $q_{t,t+1}$ denote the period t price to one unit of currency in a particular state of period $t+1$ normalized by the probability of occurrence of that state conditional on the information available in period t. Then, the price of a random payoff Z_{jt+1} in period $t+1$ is given by $E_t[q_{t,t+1}Z_{jt+1}]$.

Purchases of the consumption good are assumed to be associated with real transaction costs. While it is commonly assumed that only money provides transaction services, here also holdings of government bonds do reduce transaction costs. We view this assumption, which is, for example, also applied Lahiri and Vegh (2003) and, particularly, in Canzoneri and Diba (2003) for a related purpose, as quite reasonable, since securities serve as collateral for many types of transactions. Our specification of transaction costs h further implies that private debt, which can be freely issued by the households, is different, as it does not provide transaction services. This assumption can for example be rationalized by the asset acquisition policy of many central banks, by which public debt but not unbacked private debt is eligible in open market operations. We assume that the goods market opens before the asset market, such that households rely on the beginning of period holdings of government liabilities to reduce transaction costs.[8]

Assumption 1 *The transaction cost function $h(c_{jt}, M_{jt-1}/P_t, B_{jt-1}/P_t)$ satisfies: i) h is non-negative, increasing in c, decreasing in M_{jt-1}/P_t and in B_{jt-1}/P_t, and twice continuously differentiable in all arguments, ii) $h_{cc} \geq 0$, $h_{mm} > 0$, $h_{bb} > 0$, $\lim_{m\to 0} h_m =$*

[8]Note that the partial derivative of h with respect to the real value of beginning-of-period t money (bond) holdings M_{jt-1}/P_t (B_{jt-1}/P_t) is denoted by h_m (h_b).

$-\infty$, $\lim_{b\to 0} h_b = -\infty$, *and iii)* $h_{cm} = h_{cb}\, (= h_{mb}) = 0.$

Part *iii)* implies that the transaction cost function is separable in all arguments (see Lahiri and Vegh, 2003), which is convenient for analytical tractability. We further assume that transaction costs are private costs that are paid to a particular sector whose only function is to rebate its receipts immediately to the household sector through lump-sum transfers, such that transaction costs do not show up in the aggregate resource constraint. Both assumptions ensure that there is no direct (wealth) effect of assets holdings on consumption. Nonetheless, there is an effect of government bonds holdings on consumption that operates exclusively through intertemporal substitution.

In order to introduce supply side disturbances, we assume that households' monopolistically supply differentiated labor services as, for example, in Clarida et al. (2002). Differentiated labor services l_j are transformed into one type of labor input l, which can be employed for the production of the final good. The transformation is conducted via the aggregator $l_t^{1-1/\eta_t} = \int_0^1 l_{jt}^{1-1/\eta_t} dj$. The elasticity of substitution between differentiated labor services η_t varies exogenously over time. Cost minimization with respect to differentiated labor services then leads to the following demand for differentiated labor services l_j,

$$l_{jt} = \left(\frac{w_{jt}}{w_t}\right)^{-\eta_t} l_t, \qquad \text{with} \qquad w_t^{1-\eta_t} = \int_0^1 w_{jt}^{1-\eta_t} dj, \tag{2}$$

where l denotes aggregate labor services. Household j faces a lump-sum tax $P_t\tau_t$ (where P is the aggregate price level), labor income $P_t w_{jt} l_{jt}$ and dividends $D_{j,it}$ from monopolistically competitive firms indexed by $i \in [0,1]$. After the goods market is closed, the financial market opens where households can either invest in nominal bonds B_{jt} at the price $1/R_t$, in money holdings M_{jt}, or in nominal state contingent claims. Household j's flow budget constraint reads

$$\begin{aligned} & M_{jt} + B_{jt}/R_t + E_t\left[q_{t,t+1} Z_{jt+1}\right] + P_t c_{jt} + P_t h(c_{jt}, M_{jt-1}/P_t, B_{jt-1}/P_t) \\ & \leq P_t w_{jt} l_{jt} + Z_{jt} + B_{jt-1} + M_{jt-1} - P_t\tau_t + \int_0^1 D_{j,it} di. \end{aligned} \tag{3}$$

It maximizes (1) subject to its flow budget constraint (3), (2) and a borrowing constraint $\lim_{s\to\infty} E_t q_{t,t+s}\left(M_{jt+s} + B_{jt+s} + Z_{jt+1+s}\right) \geq 0$, for given initial values $M_{j(-1)} = M_{-1}$, $B_{j(-1)} = B_{-1}$, and $Z_{j0} = Z_0$. The first order conditions for the household's problem are given by

$$\lambda_{jt}(1 + h_c(c_{jt})) = \upsilon'(c_{jt}), \tag{4}$$

$$\varphi_t^{-1} w_{jt} \lambda_{jt} = \mu'(l_{jt}), \tag{5}$$

$$\beta E_t\left[\lambda_{jt+1}\pi_{t+1}^{-1}\left(1 - h_b(b_{jt}\pi_{t+1}^{-1})\right)\right] R_t = \lambda_{jt}, \tag{6}$$

$$\beta\left(\lambda_{jt+1}/\lambda_{jt}\right)\pi_{t+1}^{-1} = q_{t,t+1}, \tag{7}$$

and $\beta E_t\left[\lambda_{jt+1}\pi_{t+1}^{-1}\left(1-h_m(m_{jt}\pi_{t+1}^{-1})\right)\right]=\lambda_{jt}$, where λ is the Lagrange multiplier on the budget constraint, $\pi_t=P_t/P_{t-1}$ is the gross inflation rate, and $\varphi_t\equiv\eta_t/(\eta_t-1)$ denotes the wage mark-up, which is assumed to vary stochastically according to

$$\varphi_t=\varphi_{t-1}^{\rho_c}\exp(\varepsilon_{ct}),$$

with ε_{ct} being a white noise shock. Further, the transversality condition $\lim_{s\to\infty}E_tq_{t,t+s}(M_{jt+s}+B_{jt+s}+Z_{jt+1+s})=0$ is required to hold. It should be noted that (7) holds for each state in period $t+1$, and determines the price of one unit of currency for a particular state at time $t+1$ normalized by the conditional probability of occurrence of that state in units of currency in period t. Since all households face the same stochastic discount factor $q_{t,t+1}$, they can completely share consumption risks, i.e., their growth rates of $\lambda_{jt+1}/\lambda_{jt}$ are identical (see 7). Further, given identical initial asset endowments, it follows that shadow prices equalize across households $\lambda_{jt}=\lambda_t$. By (4) and (6), consumption and bond holdings are, therefore, also identical between households. We thus omit the indices j in what follows, except for the idiosyncratic working time l_j, which is monopolistically supplied to firms at a wage rate w_j.

The interest rate on a risk-free portfolio between t and $t+1$ is given by $R_t^f\equiv 1/E_tq_{t,t+1}$. Arbitrage freeness then implies the risk-free interest rate R_t^f to exceed the interest rate on government bonds R_t, for strictly positive bond holdings (see 6). It should be noted that the central bank will be assumed to set R_t, while R_t^f will be irrelevant in a competitive equilibrium.

Firms The final consumption good is an aggregate of differentiated goods produced by monopolistically competitive firms indexed with $i\in[0,1]$. The CES aggregator of differentiated goods is defined as follows: $y_t^{\frac{\epsilon-1}{\epsilon}}=\int_0^1 y_{it}^{\frac{\epsilon-1}{\epsilon}}di$, with $\epsilon>1$,where y is the number of units of the final good, y_i the amount produced by firm i, and ϵ the constant elasticity of substitution between these differentiated goods. Let P_i and P denote the price of good i set by firm i and the price index for the final good. The demand for each differentiated good is $y_{it}=(P_{it}/P_t)^{-\epsilon}y_t$, with $P_t^{(1-\epsilon)}=\int_0^1 P_{it}^{(1-\epsilon)}di$. A firm i produces good y_i employing a technology which is linear in the labor input: $y_{it}=l_{it}$. Hence, labor demand satisfies: $mc_{it}=w_t$, where mc denotes real marginal costs. We introduce a nominal stickiness in form of staggered price setting as developed by Calvo (1983). Each period firms may reset their prices with the probability $1-\phi$ independent of the time elapsed since the last price setting. The fraction $\phi\in(0,1)$ of firms are assumed to adjust their previous period's prices according to the following simple rule: $P_{it}=\overline{\pi}P_{it-1}$, where $\overline{\pi}$ denotes the average inflation rate. Firms are assumed to maximize their market value, which equals the expected sum of discounted dividends $E_t\sum_{s=0}^{\infty}q_{t,t+s}D_{it+s}$, where $D_{it}\equiv(P_{it}-P_tmc_{it})\,y_{it}$ and we assumed that firms also have access to contingent claims. In each period a measure $1-\phi$ of randomly selected firms set new prices $\widetilde{P}_{it}$ according to $\max_{\widetilde{P}_{it}}E_t\sum_{s=0}^{\infty}(\beta\phi)^s q_{t,t+s}(\overline{\pi}^s\widetilde{P}_{it}y_{it+s}-P_{t+s}mc_{t+s}y_{it+s})$, s.t. $y_{it+s}=(\overline{\pi}^s\widetilde{P}_{it})^{-\epsilon}P_{t+s}^{\epsilon}y_{t+s}$. The first order condition for the optimal price setting of re-optimizing producers is given by

$$\widetilde{P}_{it} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta\phi)^s \left[q_{t,t+s} y_{it+s} P_{t+s}^{\epsilon+1} \overline{\pi}^{-\epsilon s} mc_{t+s}\right]}{E_t \sum_{s=0}^{\infty} (\beta\phi)^s \left[q_{t,t+s} y_{it+s} P_{t+s}^{\epsilon} \overline{\pi}^{(1-\epsilon)s}\right]},$$

where we used $mc_{it} = mc_t$. The linear approximation of this first order condition and $P_t^{1-\epsilon} = \phi \left(\overline{\pi} P_{t-1}\right)^{1-\epsilon} + (1 - \phi) \widetilde{P}_t^{1-\epsilon}$ at the steady state for a given initial price level P_{-1} is known to lead to $\widehat{\pi}_t = \chi \widehat{mc}_t + \beta E_t \widehat{\pi}_{t+1}$, with $\chi > 0$ (see Yun, 1996), where for any variable x_t, $\widehat{x}$ denotes the percent deviation from the steady state value $x : \widehat{x} = \log(x_t) - \log(x)$.

Public sector The public sector consists of the fiscal authority and the central bank. The fiscal authority issues risk-less one-period bonds B_t at the price $1/R_t$ paying B_t units of currency in period $t + 1$, receives a transfer τ_t^c from the central bank, and collects lump-sum taxes τ from households

$$B_{t-1} = B_t/R_t + P_t\tau_t + P_t\tau_t^c. \tag{8}$$

Note that we abstract from any government expenditures on goods, such that the services on outstanding debt are the only flow that needs to be financed, either by issuing new debt or by raising taxes. The fiscal authority is assumed to be committed to a simple feedback rule. To facilitate the derivation of analytical results, we specify the level of taxes as a fraction of debt service costs $\frac{i_t}{1+i_t} B_{t-1}$ net of central bank transfers:

$$P_t\tau_t = \kappa_t \frac{i_t B_{t-1}}{1 + i_t} - P_t\tau_t^c, \tag{9}$$

where $1 + i_t = R_t$ and κ_t satisfies $\kappa_t = \kappa \exp(\varepsilon_{kt})$, where $\kappa \in (0, 1]$ and ε_{kt} is an i.i.d. random variable with mean zero. The stochastic feedback parameter κ_t specifies how actively the government seeks to collect funds from the private sector to finance its debt burden. Using (8) and (9), the evolution of government debt can be summarized by

$$B_t = (1 + (1 - \kappa_t)i_t) B_{t-1}. \tag{10}$$

Note that $\kappa_t = 1$ is the case of a budget that balances in every instant, such that nominal government bonds are constant over time: $B_{t-1} = B_t$. Thus, κ can be seen as an indicator of the proportion to which government expenditures are financed through taxation as opposed to debt; we will sometimes refer to κ as the share of tax finance for short. More general fiscal policy rules are considered in the last part of the paper, where we present simulations of a calibrated model; the analytical results derived for the specific rule (9) carry over to this case for all parameter sets we investigated.

The central bank transfers seigniorage to the fiscal authority, $P_t\tau_t^c = M_t - M_{t-1}$, and is assumed to control the nominal interest rate R on government bonds. Following the literature on local determinacy under interest rate rules (see, e.g., Benhabib et al., 2001, or, Carlstrom and Fuerst, 2001), we assume for simplicity that the central bank sets R

contingent on current inflation, subject to a monetary policy shock,

$$R_t = R(\pi_t, \varepsilon_{rt}) = \mathcal{R}\pi_t^{\rho_\pi} \exp(\varepsilon_{rt}), \qquad \rho_\pi > 0,\ R_t \geq 1, \tag{11}$$

where ε_{rt} is assumed to be i.i.d. with a mean equal to zero. We further restrict the support of ε_{rt} to be small enough such that the central bank can choose $\mathcal{R}$ to ensure that $R_t \geq 1$. Hence, the nominal interest rate is non-negatively related to the inflation rate; further restrictions that need to be fulfilled with respect to existence and uniqueness of a steady state are discussed below.

Using that all households are identical, their asset holdings entirely consist of government liabilities and the indices for the labor market variables can be omitted. Further, using that transaction costs are private ($y_t = c_t$), the aggregate version of the production function $y_t = l_t$, and that money is irrelevant for the determination of the remaining variables, a competitive equilibrium of the model can be defined as follows.

A *rational expectations equilibrium* is a set of sequences $\{y_t, \pi_t, w_t, b_t, R_t\}_{t=0}^{\infty}$ satisfying the firms' optimal price setting and labor demand conditions, the households' first order conditions $\mu'(y_t)\upsilon'(y_t)^{-1}(1 + h_c(y_t)) = \varphi_t^{-1} w_t$, and

$$\beta E_t \left[\left[\upsilon'(y_{t+1})(1 + h_{c,t+1}) \right]^{-1} \pi_{t+1}^{-1} \left(1 - h_b(b_t \pi_{t+1}^{-1})\right) \right] R_t = \left[\upsilon'(y_t)(1 + h_{c,t}) \right]^{-1}, \tag{12}$$

and the transversality condition, for fiscal and monetary policy satisfying $b_t = (1 + (1 - \kappa_t) i_t) b_{t-1} \pi_t^{-1}$ and (11), and given sequences of $\{\kappa_t\}$, $\{\varepsilon_{rt}\}$, and $\{\varphi_t\}$ and initial values B_{-1} and $P_{-1} > 0$.

2.2 Version B: Partial debt repayment

The reduced version of the model with non-neutral public debt can also be derived from a second approach where we abstain from assuming that bonds provide transaction services $h_b = 0$. In order to induce public debt non-neutrality, assume that the fiscal authority only partially repays its debt obligations. Specifically, only a fraction $1 - \delta$ with $\delta \in (0, 1]$ per unit of public debt is repaid in every period, where δ is an increasing function of the real value of the beginning of period (aggregate) stock of public debt $\delta'(B_{t-1}/P_t) > 0$. This assumption can either be interpreted as a deterministic debt repayment rule of the fiscal authority, which aims at alleviating the households' tax burden in states where public indebtedness if high, or as a probability of sovereign default which rises with the stock of public debt. The latter interpretation, which we view as more appealing, relates to a modelling strategy applied to induce stationarity of open economy models, by introducing a domestic default probability increasing with foreign debt (see, e.g., Turnovsky, 1997, or Schmitt-Grohe and Uribe, 2001). It should, however, be noted that a default probability interpretation is not exactly consistent with our specification of public policy, which ensures government solvency.[9] The flow budget constraint of household j is then given by

$$M_{jt} + B_{jt}/R_t + E_t\left[q_{t,t+1}Z_{jt+1}\right] + P_t c_{jt} + P_t h(c_{jt}, M_{jt-1}/P_t)$$
$$\leq P_t w_{jt} l_{jt} + Z_{jt} + \left(1 - \delta(B_{t-1}/P_t)\right) B_{jt-1} + M_{jt-1} - P_t \tau_t + \int_0^1 D_{j,it} di.$$

The first order conditions for government bonds changes to $\lambda_t = R_t \beta E_t(\lambda_{t+1} \frac{1-\delta(b_t/\pi_{t+1})}{\pi_{t+1}})$, where we used that $\lambda_{it} = \lambda_t$. Further, the public sector budget constraint now reads $P_t g_t + M_{t-1} + (1-\delta) B_{t-1} = P_t \tau_t + B_t/R_t + M_t$. Suppose that the fiscal authority rebates the savings from partial debt repayment in a lump-sum way, such that taxes satisfy $P_t \tau_t = \kappa_t (i_t \frac{B_{t-1}}{1+i_t}) - (P_t \tau_t^c + \delta B_{t-1})$. Then public debt again evolves according to (10). A rational expectations equilibrium is then a set of sequences $\{y_t, \pi_t, w_t, b_t, R_t\}_{t=0}^{\infty}$ satisfying the firms' first order conditions, the households' first order conditions

$$\beta E_t \left[\left[\upsilon'(y_{t+1}) \left(1 + h_{c,t+1}\right) \right]^{-1} \pi_{t+1}^{-1} \left(1 - \delta(b_t/\pi_{t+1})\right) \right] R_t = \left[\upsilon'(y_t) \left(1 + h_{c,t}\right) \right]^{-1}, \quad (13)$$

and the transversality condition, for a fiscal and monetary policy satisfying (10) and (11), and given sequences of $\{\kappa_t\}$, $\{\varepsilon_{rt}\}$, and $\{\varphi_t\}$ and initial values B_{-1} and $P_{-1} > 0$. In contrast to the former version, the risk free interest rate R_t^f cannot exceed the nominal interest rate on government bonds R_t. However, the dynamic behavior of the equilibrium sequences in version A and B are qualitatively identical for $b_t > 0$, given that $\delta' > 0$ in version B accords to $h_{bb} > 0$ in version A.

3 Results

In the first two parts of this section we examine the steady state properties and the conditions for saddle path stability. In the last two parts we assess the role of fiscal and monetary policy for the transmission of shocks and their impact on inflation and output fluctuations under cost push shocks. To facilitate comparisons with corresponding models where debt is neutral, we restrict our attention on case where the effect of public debt on the intertemporal substitution of consumption is small.

3.1 Deterministic steady state

In a deterministic steady state ($\varepsilon_{rt} = \varepsilon_{ct} = \varepsilon_{kt} = 0$), where bars indicate steady state values of endogenous variables, real marginal cost is constant and equal to the inverse of the price-marginal cost markup that would obtain under price flexibility, $1/\overline{mc} = \epsilon/(\epsilon - 1)$. Thus, the first order conditions on consumption and labor, and the aggregate resource constraint uniquely determine steady state output by

$$\mu'(\overline{y})\upsilon'(\overline{y})^{-1}(1 + h_c(\overline{y})) = (\eta - 1)(\epsilon - 1)/(\eta\epsilon). \quad (14)$$

[9] For an endogenous derivation of a probability of sovereign default, see Uribe (2002).

A steady state further requires $b_t = \overline{b}$ and $\pi_t = \overline{\pi}$ (see 10 and 12). The fiscal and monetary policy specification leads to the restriction

$$\overline{\pi} = (1 + (1-\kappa)(\mathcal{R}\overline{\pi}^{\rho_\pi} - 1)), \tag{15}$$

on the steady state inflation rate. Whether condition (15) has a unique or multiple solutions for the steady state inflation rate depends on the fiscal and the monetary policy. The equilibrium condition on bond holdings (12) for version A can then be used to uniquely determine the steady state level of government bonds for a given steady state inflation rate

$$h_b(\overline{b}/\overline{\pi}) = 1 - \overline{\pi}/\left[\beta\mathcal{R}\overline{\pi}^{\rho_\pi}\right]. \tag{16}$$

The steady state inflation rate and, thus, of the steady state level of government bonds, is determined by (15). As policy satisfies $\kappa \in (0,1]$ and $\overline{R} > 1$, we know that $G(\overline{\pi}) \equiv (1 + (1-\kappa)(\mathcal{R}\overline{\pi}^{\rho_\pi} - 1)) - \overline{\pi}$ is strictly positive for $\overline{\pi} = 0$. Hence, $G(\overline{\pi}) = 0$ has a unique solution if $G'(\overline{\pi}) < 0 \Leftrightarrow \rho_\pi < \left[\left(\mathcal{R}\overline{\pi}^{\rho_\pi - 1}\right)(1-\kappa)\right]^{-1}$. Using that (16) and $h_b < 0$ imply $\mathcal{R}\overline{\pi}^{\rho_\pi - 1} < 1/\beta$, a sufficient condition for the existence of a unique steady state inflation rate for version A is given by

$$\rho_\pi < \beta/(1-\kappa). \tag{17}$$

If (17) is satisfied, the model further exhibits a unique steady state level of government bonds. If, $G'(\overline{\pi}) > 0$, there exists no steady state. For version B, the condition for existence and uniqueness of the steady state is slightly different, as the risk-free interest rate cannot exceed the interest rate on government bonds. In particular, the necessary and sufficient condition for the existence of a unique steady state inflation for version B is given by $\rho_\pi < \frac{\beta(1-\overline{\delta})}{1-\kappa}$, which is evidently more restrictive than (17). The properties of the steady state for version A are summarized in the following proposition.

Proposition 1 (Steady state) *Consider that monetary policy satisfies (17). Then a steady state of version A exists and is uniquely determined. It is characterized by i)* $\overline{y} > 0$, $\overline{\pi} \geq 1$, *and* $\overline{b} > 0$*; ii)* $\partial\overline{y}/\partial\kappa = 0, \partial\overline{\pi}/\partial\kappa < 0$, *and* $\partial\overline{b}/\partial\kappa \lesseqgtr 0 \Leftrightarrow \rho_\pi \gtreqless 1 - \Psi$, *where* $\Psi \equiv \frac{\overline{h_{bb}}}{1-\overline{h_b}}\frac{\overline{b}}{\overline{\pi}} > 0$*; and iii)* $\partial\overline{y}/\partial\rho_\pi = 0, \partial\overline{\pi}/\partial\rho_\pi > 0$, *and* $\partial\overline{b}/\partial\rho_\pi \gtreqless 0 \Leftrightarrow \rho_\pi \gtreqless 1 - (\Psi + \Upsilon)$, *where* $\Upsilon \equiv \frac{\overline{\pi}(1+\ln\overline{\pi})}{\partial\overline{\pi}/\partial\rho_\pi} > 0$.

Proof. The steady state condition (14) determines $\overline{y}$ independently of the policy parameters, such that $\partial\overline{y}/\partial\kappa = \partial\overline{y}/\partial\rho_\pi = 0$. Condition (15) implies that $\partial\overline{\pi}/\partial\kappa = (\overline{R} - 1)/G'(\overline{\pi}) < 0$ and $\partial\overline{\pi}/\partial\rho_\pi = -[(1-\kappa)\overline{R}\ln\overline{\pi}]/G'(\overline{\pi}) > 0$, given that (17) ensures $G'(\overline{\pi}) < 0$. Condition (16), can then be used to derive the impact on $\overline{b}$. As $\partial\overline{b}/\partial\kappa = \left(\partial\overline{b}/\partial\overline{\pi}\right)(\partial\overline{\pi}/\partial\kappa)$ and $\left(\partial\overline{b}/\partial\overline{\pi}\right) = \overline{b}/\overline{\pi} - \overline{\pi}(1-\rho_\pi)/\left(\overline{h_{bb}}\beta\overline{R}\right) \gtreqless 0 \Leftrightarrow \frac{\overline{h_{bb}}}{1-\overline{h_b}}\frac{\overline{b}}{\overline{\pi}} \gtreqless (1-\rho_\pi)$, we can conclude that $\partial\overline{b}/\partial\kappa \lesseqgtr 0 \Leftrightarrow \rho_\pi \lesseqgtr 1 - \Psi$. From (16), we obtain $\partial\overline{b}/\partial\rho_\pi = \overline{b}\overline{\pi}^{-1}(\partial\overline{\pi}/\partial\rho_\pi) + \overline{\pi}\left(\overline{h_{bb}}\beta\overline{R}\right)^{-1}\left[\overline{\pi}(1+\ln\overline{\pi}) - (1-\rho_\pi)(\partial\overline{\pi}/\partial\rho_\pi)\right] \gtreqless 0 \Leftrightarrow \rho_\pi \gtreqless 1 - (\Psi + \Upsilon)$. ■

Output and (equivalently) consumption are not affected by monetary or fiscal policy measures in the steady state, which is due to the separability of the transaction cost function. Given that (17) is satisfied, steady state inflation unambiguously rises with the reactiveness of monetary policy and declines with permanent rise in the fiscal policy parameter κ governing the proportion of tax financing. The effects on real public debt are not unambiguous. A rise in κ leads to a decline in public debt if and only if the inflation elasticity of the interest rate rule is sufficiently aggressive, $\rho_\pi > 1 - \Psi$. While the fiscal authority can reduce nominal debt by raising the share of tax financing (see 10), its influence on the real value of outstanding debt crucially relies on the stance of monetary policy. The latter has a further (direct) impact on public debt via bond demand (16), which is responsible for real public debt to increase with ρ_π for $\rho_\pi > 1 - (\Psi + \Upsilon)$, and vice versa. It should be noted that the results summarized in proposition 1 also apply for version B, where $\rho_\pi < \frac{\beta(1-\overline{\delta})}{1-\kappa}$ replaces condition (17) and the composite parameter Ψ is defined as $\Psi \equiv \frac{\overline{\delta'}}{1-\overline{\delta}} \frac{\overline{b}}{\overline{\pi}} > 0$.

3.2 Local dynamics

In this subsection, the model's behavior in the neighborhood of a steady state is analyzed. The purpose is to find the conditions under which the rational expectations equilibrium path is locally unique and stable, such that the steady state is a saddle point. In the context of models with staggered price setting, it is well-known that if the timing of debt and taxes is irrelevant due to Ricardian equivalence, then saddle path stability is ensured if monetary policy is active in the sense of using the real interest rate to fight inflation. The reason is that with sticky prices, inflation is above normal if aggregate demand, hence consumption, is higher than in the steady state. By raising the real interest rate in the face of inflation, the central bank induces intertemporal substitution on the part of consumers who will shift consumption to the future, which reduces demand and stabilizes inflation. Obviously, the central bank cannot control the real interest rate directly, so it must adjust the nominal rate according to the Taylor-principle, i.e., aggressively enough to let it rise by more than one for one with inflation (see Woodford, 2001b).

In the context of the present model, the situation is different as Ricardian equivalence does not hold and debt policy does matter; in the following, it will be shown that there is fiscal-monetary interaction in the sense that the fiscal stance fundamentally affects the conditions for monetary policy to ensure saddle path stability. For this purpose, we log-linearize the equilibrium conditions at the steady state, leading to the following set of equilibrium conditions in $(\widehat{y}_t, \widehat{\pi}_t, \widehat{b}_t, \widehat{R}_t)$:

$$\sigma\widehat{y}_t = \sigma E_t\widehat{y}_{t+1} - \widehat{R}_t + (1-\Psi) E_t\widehat{\pi}_{t+1} + \Psi\widehat{b}_t, \qquad \Psi > 0, \tag{18}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \omega\widehat{y}_t + \widehat{\varphi}_t, \tag{19}$$

$$\widehat{b}_t = \widehat{b}_{t-1} + \eta\widehat{R}_t - \widehat{\pi}_t - \varepsilon_{kt}, \qquad \eta \in [0,1) \text{ and } \partial\eta/\partial\kappa < 0, \tag{20}$$

$$\widehat{R}_t = \rho_\pi\widehat{\pi}_t + \varepsilon_{rt}, \tag{21}$$

and $\widehat{\varphi}_t = \rho_c \widehat{\varphi}_{t-1} + \varepsilon_{ct}$, where $\sigma \equiv -\frac{\overline{u_{cc}c}}{\overline{u_c}} + \frac{\overline{h_{cc}c}}{1+\overline{h_c}} > 0$, $\omega \equiv \chi(\sigma + \frac{\overline{u_{ll}l}}{\overline{ul}}) > 0$, $\eta \equiv \frac{(1-\kappa)\overline{R}}{1+(1-\kappa)(\overline{R}-1)}$, for given sequences of $\{\varepsilon_{\kappa t}\}$, $\{\varepsilon_{rt}\}$, and $\{\varepsilon_{ct}\}$. Recall that $\Psi \equiv \frac{\overline{h_{bb}}}{1-\overline{h_b}} \frac{\overline{b}}{\overline{\pi}} > 0$ in version A and $\Psi \equiv \frac{\overline{\delta'}}{1-\overline{\delta}} \frac{\overline{b}}{\overline{\pi}} > 0$ in version B. Equation (18) specifies the evolution of real aggregate demand as a function of the nominal interest rate and inflation. In the New Keynesian model, consumption growth depends on the real interest rate only; crucially, this is different here as real debt $\widehat{b}_t$ enters the demand equation. Equation (20) is the law of motion of real debt, i.e. the log-linearized flow budget constraint of the composite government sector. Note that the composite parameter $\eta(\kappa)$ is strictly decreasing in κ. Finally, equation (21) gives the log-linearized nominal interest rate setting rule of the central bank. The following proposition states the qualitative local dynamic properties of the model, for a small weight of debt in the aggregate demand constraint (18), $\Psi \leq 2$.

Proposition 2 (Saddle path stability) *Suppose that there exists a steady state and that $\Psi \leq 2$. Then, the model exhibits a non-oscillatory and unique rational expectations equilibrium path converging to the steady state if and only if $\rho_\pi < 1 + \frac{\kappa}{(1-\kappa)\overline{R}}$. If (17) is satisfied for version A, the steady state is unique and saddle path stable.*

Proof. The deterministic version of the model (18)-(21) can be summarized and rewritten as

$$\begin{pmatrix} \widehat{y}_{t+1} \\ \widehat{\pi}_{t+1} \\ \widehat{b}_t \end{pmatrix} = \begin{pmatrix} -(\sigma\beta)^{-1}\omega(\Psi-1)+1 & \Xi & -\Psi/\sigma \\ -\frac{1}{\beta}\omega & \frac{1}{\beta} & 0 \\ 0 & \eta\rho_\pi - 1 & 1 \end{pmatrix} \begin{pmatrix} \widehat{y}_t \\ \widehat{\pi}_t \\ \widehat{b}_{t-1} \end{pmatrix} = A \begin{pmatrix} \widehat{y}_t \\ \widehat{\pi}_t \\ \widehat{b}_{t-1} \end{pmatrix},$$

where $\Xi \equiv \frac{1}{\sigma}(\rho_\pi - \Psi(\eta\rho_\pi - 1) + (\Psi-1)/\beta)$. The characteristic polynomial of A reads $H(X) = X^3 - (\sigma\beta)^{-1}(\sigma + \omega + 2\sigma\beta - \Psi\omega)X^2 - (\sigma\beta)^{-1}(\Psi\eta\omega\rho_\pi - \omega - \sigma\beta - \omega\rho_\pi - 2\sigma)X - (\sigma\beta)^{-1}(\sigma + \omega\rho_\pi)$. As the determinant of A is strictly larger than one, $\det(A) = -H(0) = (\sigma\beta)^{-1}(\sigma + \omega\rho_\pi) > 1$, A exhibits at least one unstable eigenvalue. Given that $H(1) = (\sigma\beta)^{-1}(1 - \eta\rho_\pi)\Psi\omega$, there is at least one stable (and positive) eigenvalue lying between zero and one if $1 - \eta\rho_\pi > 0$. As $H(-1) = \omega(\sigma\beta)^{-1}[\Psi(1+\eta\rho_\pi) - 2(1+\rho_\pi) - 4\frac{\sigma}{\omega}(1+\beta)]$, we know that $H(-1) < 0$ for $\Psi \leq 2$ and $\eta < 1$, and that the third eigenvalue is unstable. Hence, the model exhibits exactly one stable and positive eigenvalue if and only if $1 - \eta\rho_\pi > 0 \Leftrightarrow \rho_\pi < 1 + \frac{\kappa}{(1-\kappa)\overline{R}}$. From (15) and (16), we further know that $\overline{R} < \frac{\kappa}{\kappa-(1-\beta)}$. Hence, $\rho_\pi < \beta/(1-\kappa)$, which ensures steady state uniqueness, is sufficient for a saddle path configuration of the model (exactly one eigenvalue with modulus strictly less than one). ■

The result stated in the proposition departs in two fundamental ways from the New Keynesian result: in order to act stabilizing, the central bank must care not to raise the nominal interest rate too much in response to inflation, and the precise meaning of what is too much depends on the fiscal parameter κ. To clarify the intuition for this result, consider the case of a sudden increase in inflation. If the central bank obeyed the Taylor-principle, it would raise the nominal interest rate so much as to raise the real rate.

While this tends to reduce consumption, and thus tends to counteract the possibility of self-fulfilling (inflation) expectations, this is only a partial effect in the present model. The second effect here is that a higher real rate would let the burden of public debt weigh more heavily on the fiscal budget, such that the government would have to finance additional interest payments on existing debt. If $\kappa < 1$, not all of this additional expenditure is financed through taxation, but a fraction is covered by issuance of new debt. Since these add to existing debt holdings, they trigger the intertemporal substitution effect of real debt: a higher stock of debt reduces the marginal return from transaction services such that optimality requires that consumption is ceteris paribus shifted to the present. Thus, for $\kappa < 1$ there is a stabilizing and a destabilizing partial effect of real interest rate increases, and the strength of the latter depends on the composition of government finance. There is an interesting special case that is defined by $\kappa = 1$, in which case the budget is balanced such that government debt is constant in nominal terms. The model's properties for $\kappa = 1$, which can immediately be derived from proposition 1 and 2, are summarized in the following corollary.

Corollary 1 (Balanced Budget) *If the government budget is balanced in the sense* $B_t = B_{t-1}$, *then the steady state is unique, saddle path stable, and satisfies* $\overline{\pi} = 1$.

What is remarkable of the balanced budget case is that it renders the equilibrium determinate without any restrictions on the monetary policy rule.[10] In other words, budget balance eliminates the interaction between fiscal and monetary policy with respect to ensuring macroeconomic stability in that it makes monetary policy reactiveness irrelevant in this respect. To see why, note from (20) that with $\kappa = 1 \Rightarrow \eta = 0$ the evolution of real government liabilities is driven by inflation alone: as a balanced budget implies that nominal debt is constant, any surge in inflation reduces its real value. Hence, higher than normal inflation automatically reduces real debt, increases (by $h_{bb} < 0$) the return from transaction services, induces postponement of consumption to the future, and, therefore, rules out inflation expectations to become self-fulfilling. This stabilizing intertemporal substitution effect on demand overturns any countervailing influence of real interest rate changes. However, this mechanism ensures saddle path stability only in the case of a fully balanced budget. In any other case, interest rate adjustments of the central bank are relevant. The smaller is the degree of tax financing of the fiscal budget, the less aggressive must nominal interest rate increases in response to inflation be in order not to threaten (saddle path) stability. Interestingly, one novel result of the present model is that it is always a safe job for the central bank to do nothing at all by pegging the nominal interest rate to an arbitrary constant.[11]

[10] This finding closely relates to Canzoneri and Diba's (2003) result that the price level is determined under an interest rate peg when bonds provide transaction services.

[11] It should be noted that for a non-Ricardian fiscal policy, the result from the New Keynesian model carries over. A non-Ricardian regime can be represented by a fiscal policy that gives no feedback from debt to taxes ($\kappa = 0$). Thus, any increase in debt requires more borrowing, eventually leading to an explosive

3.3 Impulse responses

In this subsection we aim at revealing the impact of shocks on the endogenous variables $\widehat{b}_t$, $\widehat{\pi}_t$, and $\widehat{y}_t$. It is assumed that (17) is satisfied, implying that the fundamental solution with the state variables $\widehat{b}_{t-1}$, ε_{rt}, ε_{kt}, and $\widehat{\varphi}_t$ is the unique solution of the model. The model is analytically solved applying the method of undetermined coefficients. Let δ_{xs} be the solution coefficient describing the impact of state variable s on the endogenous variable x. The following proposition summarizes the qualitative properties of the coefficients given (17).

Proposition 3 (Impulse responses) *Suppose that* $\Psi < 1$ *and (17) is satisfied. Then*

1. $\partial\widehat{b}_t/\partial\varepsilon_{\kappa t} = \delta_{b\kappa} < 0$, $\partial\widehat{\pi}_t/\partial\varepsilon_{\kappa t} = \delta_{\pi\kappa} < 0$, and if $\rho_\pi < \widetilde{\rho_\pi} : \partial\widehat{y}_t/\partial\varepsilon_{\kappa t} = \delta_{y\kappa} < 0$,
2. $\partial\widehat{b}_t/\partial\varepsilon_{rt} = \delta_{br} > 0$, and if $\eta < \widetilde{\eta} : \partial\widehat{\pi}_t/\partial\varepsilon_{rt} = \delta_{\pi r} < 0$ and $\partial\widehat{y}_t/\partial\varepsilon_{rt} = \delta_{yr} < 0$,
3. $\partial\widehat{b}_t/\partial\widehat{\varphi}_t = \delta_{bc} < 0$, $\partial\widehat{\pi}_t/\partial\widehat{\varphi}_t = \delta_{\pi c} > 0$ and $\partial\widehat{y}_t/\partial\widehat{\varphi}_t = \delta_{yc} < 0$,

and $\partial\widehat{b}_t/\partial\widehat{b}_{t-1} = \delta_b \in (0,1)$, $\partial\widehat{\pi}_t/\partial\widehat{b}_{t-1} = \delta_{\pi b} > 0$, *and* $\partial\widehat{y}_t/\partial\widehat{b}_{t-1} = \delta_{yb} > 0$, *where* $\widetilde{\rho_\pi} \equiv (1-\Psi)/\beta > 0$ *and* $\widetilde{\eta} \equiv \left(\Psi + \rho_\pi + (1-\delta_b)\left[\frac{\sigma}{\omega}(1+\beta-\beta\delta_b) + 1 - \Psi\right]\right)^{-1} > 0$.

Proof. See appendix 5.1.

Proposition 3 shows that the model behaves in an intuitive way. In response to a temporary rise in taxes ($\varepsilon_{\kappa t} > 0$), public debt and, by the intertemporal substitution effect of debt, inflation declines (see part 1.). As the central bank reacts to the latter by lowering the nominal interest rate, the output response crucially depends on monetary policy reactiveness. If the central bank is not too aggressive, for which $\rho_\pi < \widetilde{\rho_\pi}$ is sufficient, the expansionary impact of the decline in the nominal interest rate is dominated by the contractionary effect of a lower debt, such that output decreases.

A contractionary monetary policy shock ($\varepsilon_{rt} > 0$), raises the interest rate burden on outstanding debt, which leads to a future rise in debt. The response of output and inflation is generally ambiguous (see part 2.). The reason is that if a sufficiently large portion of government expenditures are tax financed ($\eta < \widetilde{\eta}$), then inflation and output decline in response to a monetary contraction. Otherwise, with heavy deficit finance the implied large rise in public debt can cause an increase in inflation and output due the positive intertemporal substitution effect of debt on private consumption. Finally, part 3 of the proposition states that a cost push shock leads to a decline in output and a rise in inflation, while the latter causes a reduction in real public debt.

Summing up, for monetary and fiscal policy feedback rules which do not feature extreme parameter values, in the sense that ρ_π is not too high and κ is not too low, the model's predictions about responses to interest rate and tax shocks qualitatively accord to the empirical (VAR) evidence provided by Christiano et al. (1999) for federal funds rate shocks and by Mountford and Uhlig (2002) for tax cut shocks.

path of government liabilities. This scenario can be avoided if the central bank does not allow the real interest rate to rise in response to inflation ($\rho_\pi < 1$).

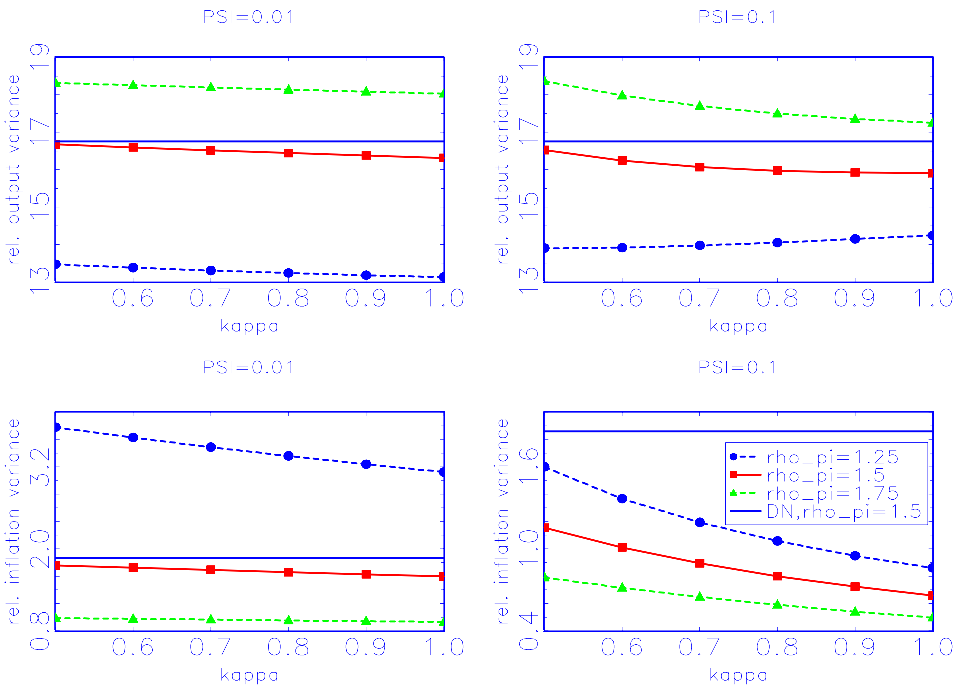

Figure 1: Output and inflation variances

3.4 Output and inflation variances

In this subsection, the impact of public debt non-neutrality and of fiscal and monetary policies on the variances of output and inflation under autocorrelated cost push shocks is studied. The purpose is to investigate whether there are any new policy trade-offs emerging when government debt influences aggregate demand.

Using the coefficients of the model's solution derived above to find the impulse response functions (see the proof to proposition 3 for the exact definitions), the variances of the model's endogenous variables output (var_y), inflation (var_π) and government debt (var_b) can easily be expressed in relation to the exogenous variance of the cost push shock, $var_\varphi = (1-\rho_c^2)^{-1} var_{\varepsilon_c}$, where var_{ε_c} is the variance of the white noise innovation to the shock. The resulting expressions are $var_b/var_\varphi = (1-\delta_b^2)^{-1}\delta_{bc}^2$, $var_\pi/var_\varphi = \left(\delta_{bc}^2\delta_{\pi b}^2\right)/\left(1-\delta_b^2\right) + \delta_{\pi c}^2$, and $var_y/var_\varphi = \left(\delta_{bc}^2\delta_{yb}^2\right)/\left(1-\delta_b^2\right) + \delta_{yc}^2$. For convenience, we present the results in graphical form relying on numerical simulations of the model under a set of deep parameters in accordance with values often found in the literature, intended to roughly match their empirical counterparts. In particular, we set $\sigma = \vartheta = 2$, $\beta = 0.99$, $\overline{R} = 1.01$, $\phi = 0.8$, and $\rho_c = 0.9$ (the autocorrelation of the cost push shock process), and simulate the model for various values of the policy parameters.

Figure 1 shows var_y/var_φ (first row) and var_π/var_φ (second row), each for various values of the fiscal feedback parameter κ and the inflation elasticity ρ_π of the nominal interest rate on government bonds. The first column is for $\Psi = 0.01$ and the second for $\Psi = 0.1$; thus, the first column is intended to show the case of a low impact of government debt on aggregate demand, and the second a higher one. To facilitate comparisons with the previous literature, we further show the special case where public debt is neutral, i.e. $\Psi = 0$, for reference in figure 1. This case is labelled DN (for *d*ebt *n*eutral) in the figure; it corresponds to the prototype New Keynesian model of Clarida et al. (1999). Their model can be summarized by (21), $\sigma\widehat{y}_t = \sigma E_t\widehat{y}_{t+1} - \rho_\pi\widehat{R}_t + E_t\widehat{\pi}_{t+1}$, and $\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \omega\widehat{y}_t + \widehat{\varphi}_t$. Given that public debt is irrelevant in their model, the fundamental solution exhibits no endogenous state and is characterized by the following coefficients on cost push shocks: $\widetilde{\delta}_{yc} = -\frac{\rho_\pi - \rho_c}{\omega(\rho_\pi - \rho_c) + (1-\beta\rho_c)(1-\rho_c)\sigma}$ and $\widetilde{\delta}_{\pi c} = \frac{\sigma(1-\rho_c)}{\omega(\rho_\pi - \rho_c) + (1-\beta\rho_c)(1-\rho_c)\sigma}$. The relative variances, which are given by $\widetilde{\delta}^2_{yc}$ and $\widetilde{\delta}^2_{\pi c}$, are displayed for $\rho_\pi = 1.5$ by the solid lines in figure 1.

The figure shows that for a given monetary policy stance, in this case for the example value $\rho_\pi = 1.5$, the variances of both output and inflation are generally lower for $\Psi > 0$ than in the DN case (compare the solid lines marked with squares). Thus, the role that government debt has in the determination of aggregate demand in our model appears to be stabilizing in itself. The reason is that if a cost push shock hits the economy, output declines while inflation rises. As has been shown above, the inflation increase reduces the real value of public debt (despite the positive partial effect from a higher real interest rate). The debt reduction exerts a negative intertemporal substitution effect on consumption, such that the output decline is exacerbated, but the inflation increase is mitigated. In equilibrium, the inflation dampening effect is strong enough to limit the real interest rate increase (which is inevitable with active monetary policy) so much that, in the end, the output variance is even lower than in the DN case.

The variances obviously depend on the monetary policy stance parameterized through ρ_π. Not surprisingly, higher ρ_π values reduce the inflation variance and raise the output variance; this effect is already well known from the DN case. A comparison of the columns of figure 1 shows that the effects attributable to the non-neutrality of debt are stronger when the intertemporal substitution effect of debt is quantitatively more pronounced (i.e. with higher Ψ).

Further, figure 1 shows that the inflation variance is always declining in the tax financing share κ, while the output variance is ambiguously linked to κ. In fact, the inflation variance reaches a minimum in the balanced budget case $\kappa = 1$. The reason is that with a nominally balanced budget the negative influence of inflation on the real value of debt is strongest, and the mechanism described above is maximal. The effects on the output variance are ambiguous, since debt reduction on the one hand reduces output partially, but the resulting inflation decrease makes room for lower real interest rates. Given an aggressive monetary policy (high ρ_π), however, there is no trade-off involved in fiscal policy: both the output and inflation variance decrease in κ and are minimized by a balanced budget policy ($\kappa = 1$).

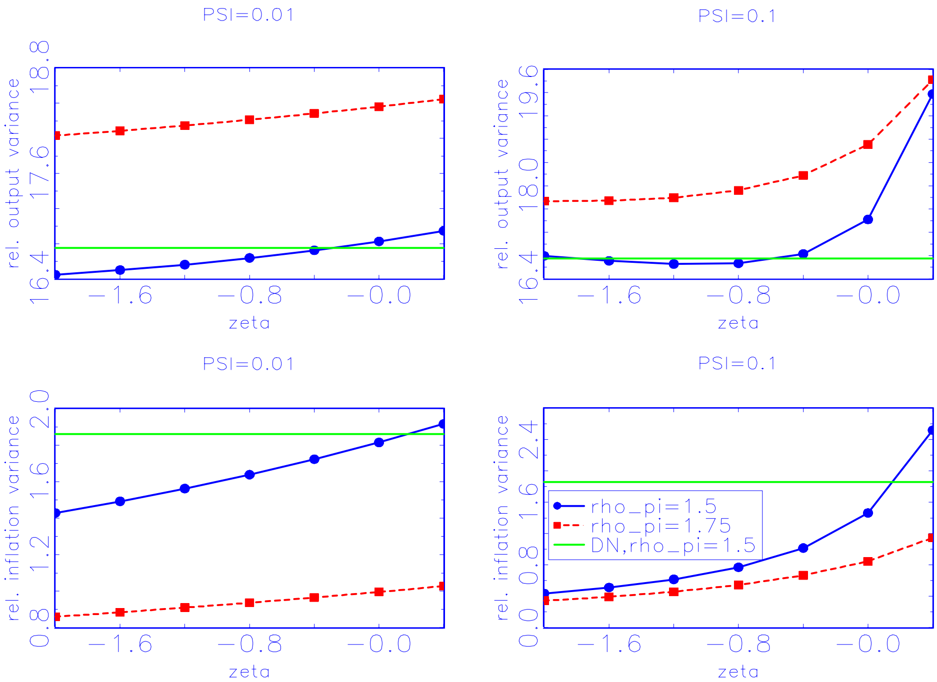

Figure 2: Output and inflation variances under the alternative fiscal rule

The above discussion has stressed that the model's essential properties depend on the link between inflation and government debt. Through the assumed fiscal rule, this link has been negative and decreasing in κ in the simulations shown in figure 1 above. Below, figure 2 aims at revealing the generality of the results for an ad-hoc fiscal rule that is not formulated as a tax reaction function, but postulated directly as a reduced form specification of real government debt of the form

$$\widehat{b}_t = \rho_b \widehat{b}_{t-1} + \zeta \widehat{\pi}_t, \quad \text{with} \quad \rho_b \in (0,1). \tag{22}$$

Here, government debt is simply an autoregressive process driven by inflation with the parameter ζ. Figure 2 shows relative output and inflation variances (organized in the same way as above in figure 1) as a function of ζ when government debt is determined through (22). Here, ρ_b is set to 0.9 and the policy responses to inflation are varied for $\rho_\pi \geq 1.5$ and for $\zeta \leq 1$, which is sufficient to ensure saddle path stability (see appendix 5.2).

Obviously, the message of figure 1 carries over to the setup used in figure 2, namely that an absolutely large negative link between inflation and real government debt generally acts stabilizing (the caveat with respect to the output variance in certain cases made above in the discussion of figure 1 also applies to figure 2). Figure 2 also shows that for a positive influence of inflation on debt ($\zeta > 0$), the variances are actually larger than in the corresponding DN case. Thus, summing up, if government debt matters for the determination of aggregate demand, and if monetary policy is aggressive on inflation, the most stabilizing fiscal policy appears to be the one that establishes the strongest negative influence of inflation on real government debt.

4 Conclusion

In this paper, we developed a sticky price model where public debt is non-neutral, by allowing the rate of return on government bonds to depend on its outstanding stock. We provide two means of modelling this mechanism: transaction services of bonds and partial debt repayment. A rise public debt leads in both cases to a decline in the rate of return on bonds for a given nominal interest rate, which is set by the central bank according to a simple feedback rule. As a consequence, households' are willing to reduce savings in favor of current consumption, such that public debt exerts a temporary expansion due to intertemporal substitution of consumption. While debt policy has no impact on real output in the long run, it does affect inflation, as stationarity of public debt requires steady state inflation to increase with ratio of debt to tax financing. It is further shown that uniqueness and saddle path stability of the steady state can be ensured through an interest rate rule by which the nominal interest rate on government bonds does not increase too strongly in response to changes in inflation, while the respective threshold increases with the degree to which government expenditures are tax financed. Thus, for example a balanced budget or an interest rate peg ensure steady state uniqueness and saddle path stability.

Regarding the responses to aggregate shocks, it is shown that the model's predictions with respect to responses to interest rate and tax cut shocks qualitatively accord to empirical findings, as long as interest rates and taxes are not set highly reactive to the state. Cost push shocks are further shown to increase inflation and to decrease output and public debt. Monetary policy faces the usual trade-off between stabilization of inflation versus output fluctuations under cost push shocks. By raising the share of tax financing the fiscal authority can, due to the aforementioned feedback from debt to aggregate demand, also stabilize the inflation variance and – under an aggressive anti-inflationary monetary policy regime – even the output variance. For the applied parametrization, we finally found that the both variances never exceed the variances of a corresponding model without a debt feedback mechanism, which accords to a standard New Keynesian model.

5 Appendix

5.1 Proof of proposition 3

To derive qualitative properties of the impulse response of the endogenous variables $X_t = (\widehat{b}_t,\ \widehat{\pi}_t,\ \widehat{y}_t)'$ to policy and cost push shocks, we apply the fundamental solution of the model which features the state variables $S_t = (\widehat{b}_{t-1},\ \varepsilon_{rt},\ \varepsilon_{kt},\ \widehat{\varphi}_t)'$. In what follows we assume that (17) is satisfied, such that the fundamental solution is the unique solution to (18)-(21). The model is then solved applying the method of undetermined coefficients for the elements of Δ defined by

$$X_t = \begin{pmatrix} \delta_b & \delta_{br} & \delta_{bk} & \delta_{bc} \\ \delta_{\pi b} & \delta_{\pi r} & \delta_{\pi k} & \delta_{\pi c} \\ \delta_{yb} & \delta_{yr} & \delta_{\pi k} & \delta_{\pi c} \end{pmatrix} \cdot S_t = \Delta \cdot S_t,$$

Given that (17) is assumed to be satisfied, we already know from proposition 2 that $\delta_b \in (0, 1)$. Hence, we aim at deriving the solutions for the remaining elements of Δ as functions of δ_b. The two other coefficients describing the structural part of the solution are given by

$$\delta_{\pi b} = \frac{1-\delta_b}{1-\eta\rho_\pi} > 0, \qquad \delta_{yb} = \frac{1-\beta\delta_b}{\omega}\frac{1-\delta_b}{1-\eta\rho_\pi} > 0,$$

which are unambiguously positive as (17) ensures $\eta\rho_\pi < 1$ (see proof of proposition 2). The coefficients governing the impact responses to the fiscal policy shocks (ε_{kt}) are

$$\delta_{bk} = -\frac{\rho_\pi\omega + \sigma}{(1-\eta\rho_\pi)\,\Gamma + \rho_\pi\omega + \sigma} \in (-1, 0), \qquad \delta_{\pi k} = -\frac{\Gamma}{(1-\eta\rho_\pi)\,\Gamma + \rho_\pi\omega + \sigma} < 0,$$
$$\delta_{yk} = -\frac{\Psi + \delta_{\pi b}(1-\Psi-\rho_\pi\beta) + \delta_{yb}\sigma}{(1-\eta\rho_\pi)\,\Gamma + \rho_\pi\omega + \sigma},$$

where $\Gamma \equiv \omega(\Psi + \delta_{yb}\sigma) + \delta_{\pi b}\left[\omega(1-\Psi) + \beta\sigma\right] > 0$. Inspecting the solution for δ_{yk}, immediately reveals that $\rho_\pi < (1-\Psi)/\beta$ is sufficient for $\delta_{yk} < 0$. The coefficients on the monetary policy shock (ε_{rt}), are given by

$$\delta_{br} = \frac{\omega\,(1-\eta\rho_\pi) + (\rho_\pi\omega + \sigma)\,\eta}{(1-\eta\rho_\pi)\,\Gamma + \rho_\pi\omega + \sigma} > 0, \qquad \delta_{\pi r} = -\frac{\omega - \eta\Gamma}{(1-\eta\rho_\pi)\,\Gamma + \rho_\pi\omega + \sigma},$$
$$\delta_{yr} = -\frac{1}{\Gamma}\frac{(\omega-\eta\Gamma)\left[\delta_{\pi b}(1-\Psi) + \Psi + \delta_{yb}\sigma\right] + \beta\delta_{\pi b}\,(\Gamma+\sigma)}{(1-\eta\rho_\pi)\,\Gamma + \rho_\pi\omega + \sigma},$$

Thus, a low value for η (high κ) satisfying $\eta < \widetilde{\eta}$, where $\widetilde{\eta} \equiv (\Psi + \rho_\pi + (1-\delta_b)\left[\frac{\sigma}{\omega}(1+\beta-\beta\delta_b) + 1 - \Psi\right])^{-1} > 0$, is sufficient to ensure $\omega - \eta\Gamma > 0 \Leftrightarrow \delta_{\pi r} < 0$ and also guarantees $\delta_{yr} < 0$. Finally, the coefficients on the cost push shocks ($\widehat{\varphi}_t$) are given by

$$\delta_{bc} = -\frac{(1-\eta\rho_\pi)(1-\rho_c)\sigma}{\Theta} < 0, \qquad \delta_{\pi c} = \frac{(1-\rho_c)\sigma}{\Theta} > 0,$$
$$\delta_{yc} = -\frac{\omega\,(\rho_\pi(1-\eta\Psi) + 1 - (\delta_b+\rho_c)(1-\Psi)) + \sigma(1-\delta_b)(1-\beta\delta_b)}{\omega\Theta} < 0,$$

where $\Theta \equiv -\omega[(\delta_b+\rho_c)(1-\Psi) + \rho_\pi(\eta\Psi - 1) - 1] + \sigma[(1-\beta\rho_c)(2-\delta_b-\rho_c) + \beta(1-\delta_b)^2] > 0$, given that $1 < \rho_\pi < 1/\eta$ and $\Psi < 2$. These properties of the solution coefficients are summarized in proposition. ■

5.2 Saddle path stability for an alternative fiscal rule

When the fiscal policy rule (20) is replaced by (22), the requirements for saddle path stability change in the following way.

Proposition 4 *Suppose that $\Psi \leq 1$ and that fiscal policy satisfies (22), with $\rho_b \in (0,1)$ and $\zeta \leq 1$. Then, the following conditions are sufficient for a non-oscillatory and saddle stable equilibrium path*

$$\rho_b > \frac{\beta}{\frac{\omega}{\sigma}\rho_\pi + 1} \text{ and } \rho_\pi > (1-\Psi) + \frac{\Psi\zeta}{1-\rho_b}.$$

Proof. The characteristic polynomial of the model under (22), which is given by

$$G(X) = X^3 + \frac{1}{\sigma\beta}\left(-\sigma\rho_b - \omega\rho_\pi\rho_b\right) + \frac{X^2}{\sigma\beta}\left(\Psi\omega - \omega - \sigma\beta - \sigma - \sigma\beta\rho_b\right)$$
$$+\frac{X}{\sigma\beta}\left(\sigma + \omega\rho_\pi + \sigma\rho_b + \omega\rho_b - \Psi\omega\zeta + \sigma\beta\rho_b - \Psi\omega\rho_b\right).$$

is characterized by $G(0) = -\frac{\rho_b}{\sigma\beta}(\sigma + \omega\rho_\pi)$ and $G(1) = -\frac{1}{\beta\sigma}\omega(1 - \Psi - \rho_\pi + \Psi\zeta - \rho_b + \Psi\rho_b + \rho_\pi\rho_b)$. Thus, for $\rho_b > \beta\left(\frac{\omega}{\sigma}\rho_\pi + 1\right)^{-1} \Leftrightarrow \det(\mathcal{A}) > 1$, there is at least one unstable root. The existence of a stable and positive eigenvalue requires $\rho_\pi > (1-\Psi) + \Psi\zeta(1-\rho_b)^{-1}$. This condition further ensures $G(-1) = \frac{1}{\sigma\beta}[-(1+\rho_b)(\rho_\pi\omega + \omega(1-\Psi) + 2\sigma(1+\beta)) + \Psi\omega\zeta] < 0$ for $\zeta \leq 1$ and $\Psi \leq 1$, and that the last eigenvalue is also unstable. ■

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