Rank reduction and sources of fluctuations: implications for business cycle analysis

Luca Sala*
Universitá Bocconi and IGIER

November 6, 2006

VERY PRELIMINARY AND INCOMPLETE

Abstract

It is shown that comovements between real macroeconomic variables can be modelled as generated by a single shock and that such a restricted representation delivers forecasts comparable with those obtained from VARs or BVARs. A Real Business Cycle model driven by only one shock is then estimated under various assumption regarding the unique source of fluctuations. We find that the shock that best fit the data is the shock to the disutility of labor supply. An implication of our result is that DSGE models should have a richer propagation mechanism and not a larger set of structural shocks, and that contrary to what is claimed in the recent literature a large set of structural shocks is not necessary to obtain a good fit and good forecasts.

JEL Classification:

Keywords: Factor models, DSGE models, sources of fluctutation, loss function-based evaluation.

*Correspondence to: Luca Sala - IGIER, Universitá Bocconi, Via Salasco 5, 20136 Milano, Italy; E-mail: luca.sala@uni-bocconi.it

"Business cycles are "all alike" in many ways [...]. The need to produce roughly similar dynamics severely constrains the dynamic structure of the shocks and hence argues for a common source" - Cochrane, 1994

1 Introduction

One of the distinctive features of macroeconomic data is comovement. As noted by Cochrane, 1994, (see the citation above) business cycles are all alike and this calls for theories in which macro data are generated by few sources of fluctuation.

First generation Real Business Cycle models were driven by a single shock, random fluctuations to the rate of technological progress. Researchers soon noticed that the basic RBC model failed in explaining some features of the data and the literature moved on adding more shocks (Christiano and Eichenbaum, 1991) and more frictions to the basic neoclassical model.

State-of-the-art DSGE models (Smets and Wouters, 2001, Christiano, Eichenbaum and Evans, 2006) are the final result of this modeling effort: they are characterized by a large number of frictions and a large number of structural shocks.

The addition of structural shocks has been often justified by statistical reasons. A model driven by less shocks than observables is stochastically singular: it has the counterfactual implication that some linear combinations of the observable variables are deterministic and therefore will always be rejected by the data (see Ingram, Kocherlakota and Savin, 1994). In order to be confronted with the data, the number of shocks must be at least as large as the number of observables. As the models became larger, additional structural shocks were added, often with not much theoretical foundations (Geweke, calibration and reality). Structural VAR models, often used to empirically validate economic models, implicitly support the same view: by construction, a VAR explains the comovements between a set of n variables by the combined effect of exactly n innovations.

Other researchers developed empirical methods that allowed to remain consistent with the one (or few) driving force assumption by assuming the presence of additional random elements often interpreted as measurement errors (Altug, 1989, Sargent, 1989) or as misspecification errors (Ireland, 2004).

In this paper we first document that real variables, consistently with the idea that business cycles are "all alike", are well represented by a dynamic factor model, in which all the covariation and a substantial part of the variance is generated by a single shock (see Geweke, 1977, Sargent and Sims, 1977 and Altug, 1989). The statistical model will imply restrictions on the likelihood function of the data and we will show that these restrictions are not rejected. The one-factor model will also deliver forecasts in line with those obtained from widely used VAR or BVAR.

The lack of economic interpretation of the common shocks has limited the usefulness of factor models for understanding the functioning of the economy. In order to overcome this limitation, in the second part of the paper we add economic structure to the purely statistical factor model by estimating an extended RBC model driven by a single shock (for papers in which common shocks in factor models are identified as in the VAR tradition, see Forni and Reichlin (1998), Sala (2003) or Giannone, Reichlin and Sala (2004)).

The economic model will have a number of exogenous shocks, but we will estimate it by restricting only one shock at a time to have positive variance, so to be consistent with the statistical evidence.

By comparing the fit of the model as driven by different shocks, we will be able to see under what shock the model best reproduces the behavior of the data.
An advantage of the approach we follow (which draws on previous work by Watson, 1993 and Diebold, Ohanian and Berkowitz, 1998) is that the economic model is not assumed to be the true data generating process, an heroic assumption that often characterize empirical macro modelling. The model will be best characterized as an approximation of reality and will be evaluated only in its ability to match the data in selected dimensions considered relevant by the researcher

The loss function we use will penalize discrepancies between the spectral density of the component of the data driven by the common shock and the spectral density of the economic model (Schorfheide, 2001, proposes a related Bayesian approach based on loss-functions for estimating DSGE models).

Another advantage is that it provides an immediate visual assessment of model's fit and of what component in the model displays the largest misspecification.

Results show that the economic shock that best fits the data is the shock to the disutility of labor supply (as in Hall, 1998 and Chari, Kehoe and McGrattan, 2006), while the investment-specific shock and the discount factor shock generate counterfactual negative correlations between consumption and output. The forecasts generated by the estimated economic model are comparable with those obtained from VARs or BVARs.

Our results imply that extensions to the model to correct for the inevitable misspecification require a richer propagation mechanism and not a richer set of structural shocks and that a large set of structural shocks is not necessary to obtain good fit and good forecasts.

The rest of the paper is organized as follows. Section 2 presents the statistical evidence on rank reduction. Section 3 compares the forecasting performance of various statistical models. Section 4 introduces the economic model, Section 5 presents the estimation results, while Section 8 concludes.

2 Rank reduction: Evidence

In this section we show that real variables can be well represented as the sum of two distinct components. One component is driven by a single shock, explains the comovement between series, a large part of their variance and almost all the dynamics. The other is of full rank, explains a small part of the variance of the series, does not generate any comovement and it is basically white noise.

2.1 Some Background

The model we consider is a dynamic factor model (see Geweke (1977), Sargent and Sims (1977) and Altug, 1989) in which the series x_t are represented as:

$$x_t = B(L)u_t + \xi_t = \chi_t + \xi_t \tag{2.1}$$

where x_t is a n-dimensional vector, u_t is a q-dimensional ($q < n$) vector white noise with identity covariance matrix, $B(L)$ is a $n \times q$ matrix in the lag operator L and ξ_t is a n-dimensional process whose components are orthogonal at all leads and lags and where u_t and ξ_t are orthogonal at all leads and lags as well.

All comovements between x_t is assumed to be generated by the common shocks u_t.

Estimation of this model can be easily carried on by combining spectral methods and classical factor analysis for i.i.d. observations.

Let us write the spectral density of x_t as:

$$S_x(\theta) = S_\chi(\theta) + S_\xi(\theta) = B(e^{-i\theta})B(e^{-i\theta})' + S_\xi(\theta) \tag{2.2}$$

where $S_\chi(\theta)$ is of reduced rank q and $S_\xi(\theta)$ is of full rank n and diagonal. The property of orthogonality of spectral densities at different frequencies implies that the spectral density $S_x(\theta) = B(e^{-i\theta})B(e^{-i\theta})' + S_\xi(\theta)$ at every frequency θ is the classical factor model for i.i.d observations for which a well developed theory exists (see, among others, Anderson, 2003). The only difference with respect to classical factor models is that $S_x(\theta)$ is a complex matrix.

The transformation from the time domain to the frequency domain has therefore transformed the original time-series model into a sequence of factor models for i.i.d. observations [1].

If x_t is assumed to be normally distributed, the Fourier trasform[2] $\tilde{x}(\theta)$ of x_t has asymptotically a complex normal distribution with zero mean and covariance $S_x(\theta)$:

$$L(\tilde{x}(\theta)) = \frac{1}{\pi^n |S_x(\theta)|} exp(-\tilde{x}(\theta)' S_x(\theta)^{-1} \tilde{x}(\theta)) \tag{2.3}$$

[1] Another way to estimate model (2.1) is to make parametric assumptions on both χ_t and ξ_t, write the model in state space form and estimate it with ML and Kalman filtering techniques

[2] The Fourier transform is defined $\tilde{x}(\theta_j) = (2\pi T)^{-1/2} \sum_{t=1}^{T} x_t e^{it\theta_j}$, where $\theta_j = 2\pi j/T$, and T is the sample size

Suppose to divide the interval $(0, \pi]$ in P subintervals, each of them indexed by p, $p = 1, ..., P$, and that each subinterval contains $m + 1$ frequencies, $[\theta_p^0, \theta_p^2, ..., \theta_p^m]$.

If $S_x(\theta)$ is constant at frequencies $\theta_p^0, ..., \theta_p^m$ (that is, the $\tilde{x}$ are identically distributed within each frequency band), it is possible to show (Hannan, 1973) that the joint distribution of $\tilde{x}(\theta_p^0), ..., \tilde{x}(\theta_p^m)$ is complex normal:

$$f(\tilde{x}(\theta_p^0), ..., \tilde{x}(\theta_p^m)) = \frac{1}{\pi^{n(m+1)}|S_x(\theta_p)|^{m+1}} exp(-\sum_{j=0}^{m} \tilde{x}(\theta_p^j)' S_x(\theta_p^j)^{-1} \tilde{x}(\theta_p^j)) \tag{2.4}$$

and that the maximum likelihood estimator[3] of $S_x(\theta_p)$ is

$$\hat{S}_x(\theta_p) = \frac{1}{m+1} \sum_{j=0}^{m} \tilde{x}(\theta_p^j)' \tilde{x}(\theta_p^j) \tag{2.5}$$

In practice, in order to make more credible the assumption of constancy of the Fourier ordinates in each frequency bands, series are individually prefiltered (this step is known as *"prewhitening"*) with autoregressions before the computation of the Fourier trasform and are *"recolored"* after the estimation of the model.

Within each frequency band, estimation is based on the representation $S_r = BB' + S_\xi$. The log-likelihood under the factor model restriction is thus:

$$L(S_r) = (m + 1)(-nln(\pi) - ln|S_x| - trace(\hat{S}_x S_r^{-1})) \tag{2.6}$$

where: $\hat{S}_x = \frac{1}{m+1} \sum_{j=1}^{m} \tilde{x}'\tilde{x}$ and it is maximized with respect to B and S_ξ.

Having estimated $B(\theta)$ and $S_\xi(\theta)$, the null hypothesis that the data are generated by the factor model in a frequency band can be tested with a likelihood ratio test. The statistic $LR(\theta_p) = -2(L(\hat{S}_x(\theta_p)) - L(\hat{S}_r(\theta_p)))$ is asymptotically distributed as a χ^2 with $(n - q)^2 - n$ degrees of freedom.

As the estimates on non-overlapping bands are asymptotically independent, the overall fit of the model can be tested by summing the individual statistics: $LR_{all} = \sum_{p=1}^{P} LR(\theta_p)$. Being the sum of R independent χ^2 distributions with $(n-q)^2-n$ degrees of freedom, the statistic LR_{all} is distributed as a χ^2 with $R[(n - q)^2 - n]$ degrees of freedom.

The time-domain vector autoregressive representation of the model can be obtained by estimating the factor model (B and S_ξ) at all frequencies θ_j (and not only on non-overlapping bands) and this turns out to be very important in forecasting. The cost is that estimates at different frequencies are not independent anymore.

Having obtained $\hat{S}_r(\theta) = \hat{B}(\theta)'\hat{B}(\theta) + \hat{S}_\xi(\theta)$, it is possible to compute the autocovariance function $\hat{\Gamma}(k) = x_t x_{t+k}'$ for $k = 0, \pm 1, \pm 2, ...$ under the factor model restrictions via the inverse Fourier transform:

$$\hat{\Gamma}(k) = \int_{-\pi}^{\pi} \hat{S}_r(v) e^{ivk} dv$$

[3]The assumption of normality can be relaxed and replaced with less stringent assumptions , see Hannan, 1973. In this case, the estimator $\hat{S}_x(\theta_p)$ will be a quasi-ML estimator

It is then easy to build the implied projection:

$$Proj(x_t|x_{t-1}) = \hat{E}(x_t x_t')^{-1}\hat{E}(x_t x_{t-1})x_{t-1} = \hat{\Gamma}(0)^{-1}\hat{\Gamma}(1)x_{t-1} \tag{2.7}$$

and generate forecasts for x_t, conditional on the reduced rank restriction[4].

It is also possible to obtain the time domain representation of the common component, χ_t, by computing the inverse Fourier transform of the projection of $\tilde{x}(\theta)$ on the common factor in the frequency domain[5]:

$$\chi_t = IFT(Proj(\tilde{x}(\theta)|f(\theta))) = IFT(B'(BB'+S)^{-1}\tilde{x}(\theta))$$

where $IFT(.)$ denotes the inverse Fourier operator.

With the common component at hand, the common shocks and a one-sided (in the past) and fundamental representation for χ_t can also be obtained, by fitting a VAR on χ_t, and extracting the first q principal component from the VAR residuals (see Giannone, Reichlin and Sala, 2003).

2.2 Empirical results

The data we use are the growth rates of per capita real private GNP, defined as gross national product less government expenditures (Y), per capita private fixed investment (I), per capita private consumption (C) and per capita total hours worked (L).

The data are quarterly, the sample goes from 1953:2 to 2002:3, and the total civilian non-institutional population series (P16) has been used to obtain per-capita variables.

The spectral density is estimated by smoothing the periodogram with a Daniell (rectangular) window of width 13 (the Daniell window is compatible with the log-likelihood in equation 2.6 in which arithmetic averages are used as estimators of $S_x(\theta)$).

The likelihood function of the dynamic factor model is maximized with the EM algorithm (see Rubin and Thayer, 1982).

In Figure 1 we report the spectral density of the data, of the common component and of the idiosyncratic component.

It is immediate to see that the one-factor model explains a large fraction of the variance (the commonality ratio, the ratio between the variance of the common component and the variance of the data is 0.58 for C, 0.83 for Y, 0.76 for I and 0.77 for L; the commonality ratio at business cycle frequencies (frequencies with period between 2 and 8 years) is even higher: 0.79 for C, 0.97 for Y, 0.89 for I and 0.86 for L). In addition, it is interesting to notice that the idiosincratic component is basically white noise (flat) for all series: the common component driven by the single shock explains the dynamic properties of the data.

[4]For simplicity, we show the projection of x_t on x_{t-1}. The projection of x_t on more lags can be derived similarly. In the empirical part, we will project x_t on 4 lags

[5]To see this, note that: $Cov(x_t) = BB' + S$, $Cov(f_t) = I_q$ and $Cov(x_t, f_t) = B$ and that $Proj(\tilde{x}(\theta)|f(\theta)) = Cov(f(\theta), \tilde{x}(\theta))Var(\tilde{x}(\theta))^{-1}\tilde{x}(\theta) = B'(BB'+S)^{-1}\tilde{x}(\theta)$

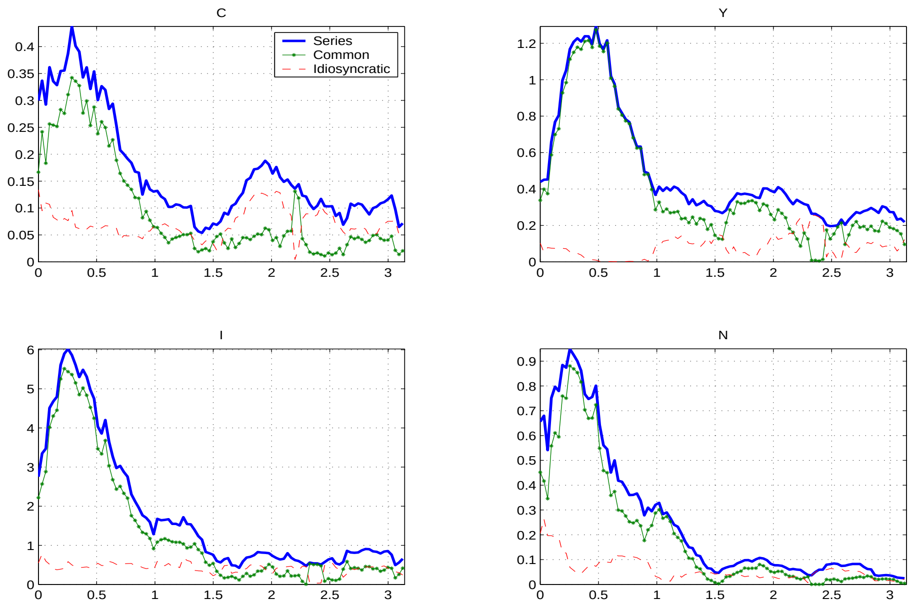

Figure 1: Spectra of data, common and idiosyncratic component

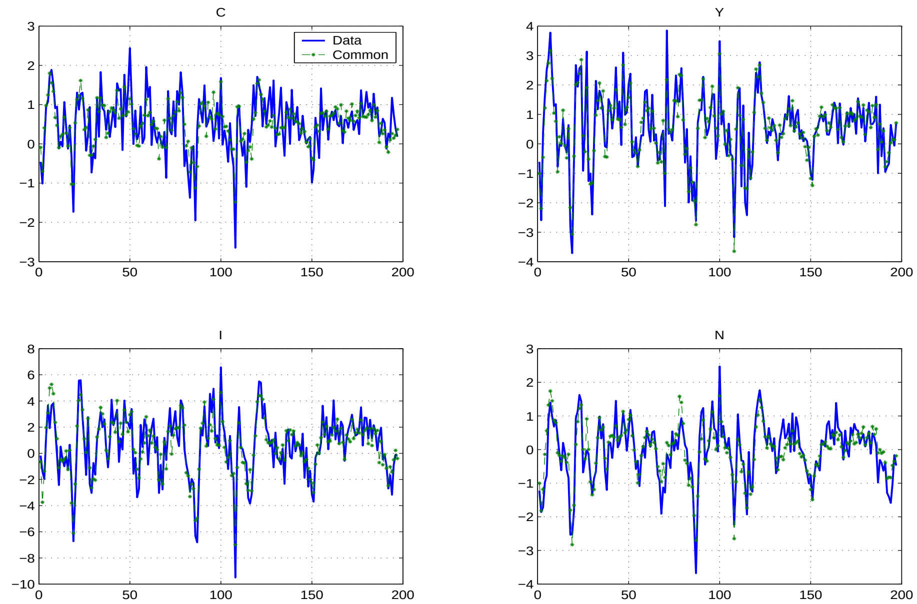

Figure 2: Time series: data and common component

Table 1: LR test for the one shock assumption

Period (quarters)	P-value
28	.63
8	.27
5	.17
4	.46
3	.18
2	.26
overall	.19

The LR test on non overlapping bands is displayed in Table 1[6]: the one-factor model is not rejected.

The conclusion we draw is that a one-shock dynamic factor model is a good representation for real macro data.

This result can be interpreted as offering a discipline device for the construction of theoretical model of the business cycle: good models must be able to reproduce the observed comovement with few exogenous shocks (see Giannone, Reichlin and Sala, 2006), because this is what the data are telling us.

In Section 5 we will show in what sense the common factor representation can be used to sharpen our inference on the sources of business cycle fluctuations.

3 Forecasting performance under Rank Reduction

In this Section, we study the forecasting performance of the common factor model estimated above.

We compare the performance of two widely used forecasting models, a VAR and a BVAR with Minnesota priors (with tightness $\theta = .1$; weighting matrix W with .5 out-of-diagonal and 1s on the diagonal and decay factor $\tau = 1$ [7] for the four variables of interest, with the forecasting performance of the factor model (each model is estimated with 4 lags).

The experiment we perform is the following. We estimate the three models from 1953:2 to 1993:4. We then compute the forecasts h step ahead, ($h = 1, 2, ...12$) and compute the Mean Squared Forecast Error (MSFE). We then add one observation,

[6] In computing the statistics, we leave out zero frequency, as it is characterized by a different asymptotic distribution (see Brockwell and Davis, 1987); in addition, as independence between frequency bands is only asymptotic, we skip 2 frequencies between bands, so to make the assumption of independence more credible

[7] Minnesota priors for a VAR with K lags are specified as follows: $\beta_{ij,k} \sim N(1, \sigma^2_{\beta_{ij,k}})$ for the first lag $k = 1$ and $i = j$ and $\beta_{ij,k} \sim N(0, \sigma^2_{\beta_{ij,k}})$ for all the other coefficients, where: $\sigma^2_{\beta_{ij,k}} = \theta W(i,j)k^{-\tau}\frac{\hat{\sigma}_{uj}}{\hat{\sigma}_{ui}}$, where $\hat{\sigma}_{ui}$ is the estimated standard error from a univariate regression of variable i

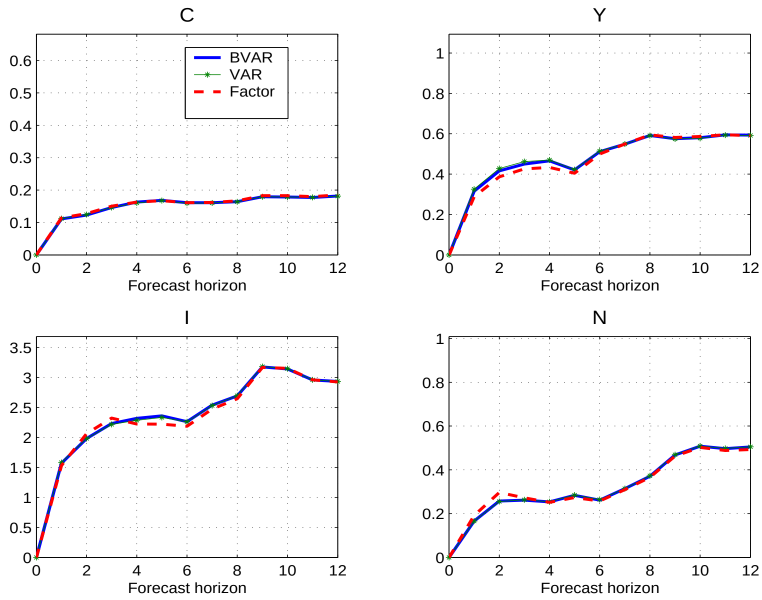

Figure 3: Mean squared forecast errors

reestimate the three models, compute the forecasts h step ahead, $(h = 1, 2, ...12)$, compute the MSFE and so on. We repeat this 24 times, estimating the last model up to 1999:2 (13 points before the end of the sample), so to have 24 forecasts for each forecasting horizon. We then average the MSFE for each forecasting horizon.

Figure 2 reports the MSFE as a function of the forecasting horizon, while Figure 3 reports the ratio between the MSFEs of the VAR and of the factor model with respect to that of the BVAR.

The factor model produces a forecasting performance in line with VARs and BVARs. The over-identifying restrictions that the factor model imposes on a vector autoregression are not rejected and deliver forecasts in line with other empirical models.

4 A Singular Real Business Cycle Model

As we have seen in the previous Section, comovements are well characterized as driven by a single shock. In this Section we will try to understand what is the source of fluctuations analyzing the data through the lenses of an RBC model. By estimating the RBC model under different assumptions on the origin of the shock, we will be able to shed light on the question "what drives business cycle fluctuations?".

The parameter estimates will be those values that minimize the distance between the singular spectral density estimated in the previous Section and the singular spectral density of the model driven by a single shock.

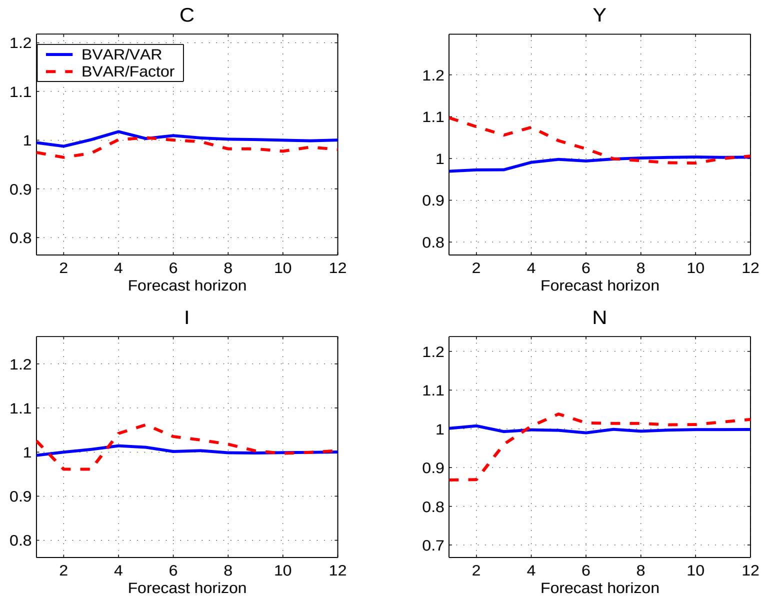

Figure 4: Relative mean squared forecast errors

The main advantage of our approach is that we are not taking the economic model as the data generating process. We explicitly recognize that the model is not a null hypothesis in a statistical sense[8]. On the contrary, we assume that the model is an approximation of reality as in Diebold, Ohanian and Berkowitz, 1998. Our inference will therefore be based on a minimization of a particular loss-function. In a setup in which misspecification is an issue, as recently highlighted by Schorfheide, 2000, loss-function based inference has many appealing features.

Another advantage of this approach, as opposed to standard maximum likelihood estimation of singular economic models as in Altug (1989) or Sargent (1989) is that we do not have to specify a parametric and often restrictive form for the idiosyncratic component and at the same time we reduce the number of parameters to be estimated. Our method has the flavor of a semi-parametric approach, in which parametric assumptions are made only on some components of the model, while others are left unspecified.

4.1 The model

The model is a standard RBC model, extended in several directions. We add habit formation in leisure and an an employment externality in the production function, as in Wen (1998) and habit formation in consumption an adjustment cost in investment following the specification used in Smets and Wouters (2003) and Christiano, Eichenbaum

[8]If we interpreted the model as a null hypothesis, any divergences between data and model must be caused by sampling error

and Evans (2006).

We add four shocks to the model, but in the estimation we will consider only one of them at a time, so to be consistent with the empirical results in Section 2. By analyzing the fit of the model under each of the 4 shocks, we will be able to conclude which shock is more likely to have generated the data.

The representative agent maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t b_t [\log(C_t) + \mu_t \log(h(L)L_t)]$$

where b_t is a discount factor shock, following:

$$b_t = (1-\rho_b) + \rho_b b_{t-1} + \epsilon_t^b \qquad \epsilon_t^b \sim i.i.d.(0, \sigma_b^2)$$

and μ_t is a shock to the relative utility of consumption and leisure, following:

$$\mu_t = (1-\rho_\mu)\bar{\mu} + \rho_\mu \mu_{t-1} + \epsilon_t^\mu \qquad \epsilon_t^\mu \sim i.i.d.(0, \sigma_\mu^2)$$

The production function is:

$$Y_t = \tilde{N}_t^\phi K_t^{1-\alpha} (e^{X_t} N_t)^\alpha$$

where $\tilde{N}_t$ represents the average hours worked in the economy taken as given by the representative agent and where X_t is technological progress, evolving according to: $X_t = A_t + \gamma_X t$, where γ_X is the average growth rate of the economy[9]. A_t follows:

$$A_t = \rho A_{t-1} + \epsilon_t^A \qquad \epsilon_t^A \sim i.i.d.(0, \sigma_A^2)$$

The capital accumulation equation is:

$$\left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) v_t I_t = K_{t+1} - (1-\delta)K_t$$

where δ is the rate of depreciation of capital and v_t is an investment specific technology shock (see Hercowitz, Greenwood and Krusell, 1997) evolving as

$$v_t = (1-\rho_v) + \rho_v v_{t-1} + \epsilon_t^v \qquad \epsilon_t^v \sim i.i.d.(0, \sigma_v^2)$$

The function $S(.)$ represent the cost of adjusting investment, with $S(e^{\gamma_x}) = 0$ and $S^{''}(.) > 0$. The market clearing condition is: $Y_t = C_t + I_t$ and the time endowment is normalized to 1: $N_t + L_t = 1$

The model can be rewritten in terms of variables that do not grow in steady state. In order to to do so, the transformation $x_t = \frac{X_t}{e^{\gamma_X t}}$ must be applied to Y_t, C_t, I_t and K_t.

[9] Ireland (2001) has shown that the trend stationary specification for the technology process delivers better forecasts than the unit root specification. See also Smets and Wouters (2007)

It is interesting to see how different shocks enter in the first order conditions in the case in which $h(L) = 1 - h_1 L$.

$$\frac{b_t \mu_t}{L_t - h_1 L_{t-1}} - \frac{\beta b_{t+1}\mu_{t+1} h_1}{L_{t+1} - h_1 L_t} = \frac{b_t}{c_t}(N_t^{\phi+\alpha-1} k_t^{1-\alpha} \alpha e^{A_t \alpha}) \tag{4.8}$$

$$\frac{b_t}{c_t}\frac{1}{v_t} = \frac{b_{t+1}\beta}{e^{\gamma_x} c_{t+1}}(N_{t+1}^{\phi+\alpha}(1-\alpha)k_{t+1}^{-\alpha} e^{\alpha A_{t+1}} + \frac{1-\delta}{v_{t+1}}) \tag{4.9}$$

$$v_t[N_t^{\phi+\alpha} k_t^{1-\alpha} e^{\alpha A_t} - c_t] - k_{t+1} e^{\gamma_x} + (1-\delta)k_t = 0 \tag{4.10}$$

Equation (4.8) is the intratemporal condition, capturing the optimal choice between consumption and leisure. Equation (4.9) is the consumption Euler equation and equation (4.10) is the capital accumulation equation.

We log-linearize the relevant conditions, obtain the unique rational expectation solution (using the Anderson and Moore algorithm), and the state-space representation for the vector ΔZ_t:

$$\Delta \ln Z_t = [\Delta \ln C_t \;\; \Delta \ln Y_t \;\; \Delta \ln I_t \;\; \Delta \ln N_t]'$$

which is given by the dynamic evolution of the states:

$$s_t = A s_{t-1} + B \epsilon_{it} \tag{4.11}$$

and the evolution of the observables as a function of the states:

$$\Delta \ln Z_t = C s_t \tag{4.12}$$

For each of the four model-shock combinations, the matrices A, B, C are function of the structural parameters $\lambda = [\gamma_x, \delta, \beta, \alpha, \bar{\mu}, \phi, h_1, h_2, h_3, \rho_i, \sigma_i]$, where ρ_i and σ_i (for $i = b, \mu, A, v$)are respectively the AR parameter and the standard error of the 4 shock processes.

5 Estimation of the Economic Model

Given the estimated spectral density of the common component, $\hat{S}_\chi$, we will estimate the parameters λ by minimizing the quadratic distance between $\hat{S}_\chi$ and the singular spectral density implied by the solution of the model in equations 4.11 and 4.12:

$$S_Z(\theta) = CPB\frac{\sigma_\epsilon^2}{2\pi}B'P'C'$$

where: $P = (I - Ae^{-i\theta})^{-1}$

The quadratic distance is defined as:

$$D(\lambda) = \int_0^\pi trace(Q(\theta)'Q(\theta))d\theta \tag{5.13}$$

where: $Q(\theta) = S_Z(\theta) - \hat{S}_\chi(\theta)$
The estimator for λ is defined as

$$\hat{\lambda} = \underset{\lambda}{argmin}\ D(\lambda)$$

5.1 Results

Table 2: Bounds

	γ_x	δ	β	α	$\bar{\mu}$	ϕ	h_1	h_2	h_3	ρ_i	σ_i
lower	.002	.01	.988	.2	.5	0	0	-1	-1	0	0
upper	.006	.05	.998	.8	4	1	1	1	1	1	5

Table 3 reports parameter values for the four model.

Table 3: Parameters' estimates

shock	A_t	b_t	μ_t	v_t
γ_x	.004	.004	.004	.004
δ	.0162	.01	.01	.01
β	.9921	.999	.999	.999
α	.3082	.5886	.6059	.4692
$\bar{\mu}$	1.5222	2.9857	1.7958	1.8131
ϕ	.4751	.6455	.7139	.9982
a	.5147	.6686	.8624	.2731
h_1	.6454	.8785	.8239	.585
h_2	.1058	.3933	.3154	.5491
h_3	-.7397	-.4074	-.2986	-.9862
S''	1.282	10.4707	12.2252	8.3157
ρ_i	.9305	.4458	.5005	.5231
σ_i	.139	.2073	.3451	.7539
distance	.0537	.0664	.0523	.1016

The plots on the diagonal elements of Figure 6 display the target spectral density S_χ and the spectral densities of the four models. The off-diagonal elements report the cohesion[10] for $\hat{S}_\chi$ and the cohesion from the 4 estimated models. The graphs also report 95% confidence bands for the target spectral density S_χ, obtained with the Berkowitz and Diebold (1998) bootstrap method.

[10]The cohesion is a spectral decomposition of the correlation coefficient and was introduced by Croux, Forni and Reichlin, 2001. It is a function with domain [0; π] and it is defined as: $\rho_{xy}(\theta) = \frac{Re(S_{xy}(\theta))}{\sqrt{S_{xx}(\theta)S_{yy}(\theta)}}$

It is evident that the shocks that best fits the data are the labor supply shock μ_t and the technology shock A_t, while the shocks that fits the least are the discount factor shock, b_t and the investment specific shock v_t.

It is impressive how well these models match the variance of output, investment and hours. From the graphs, it is clear that the variable that displays the largest misspecification is consumption. The shocks A_t and mu_t captures reasonably well the correlations in the data, while the shocks b_t and v_t generate counterfactual negative correlations between consumption and output, hours and investment.

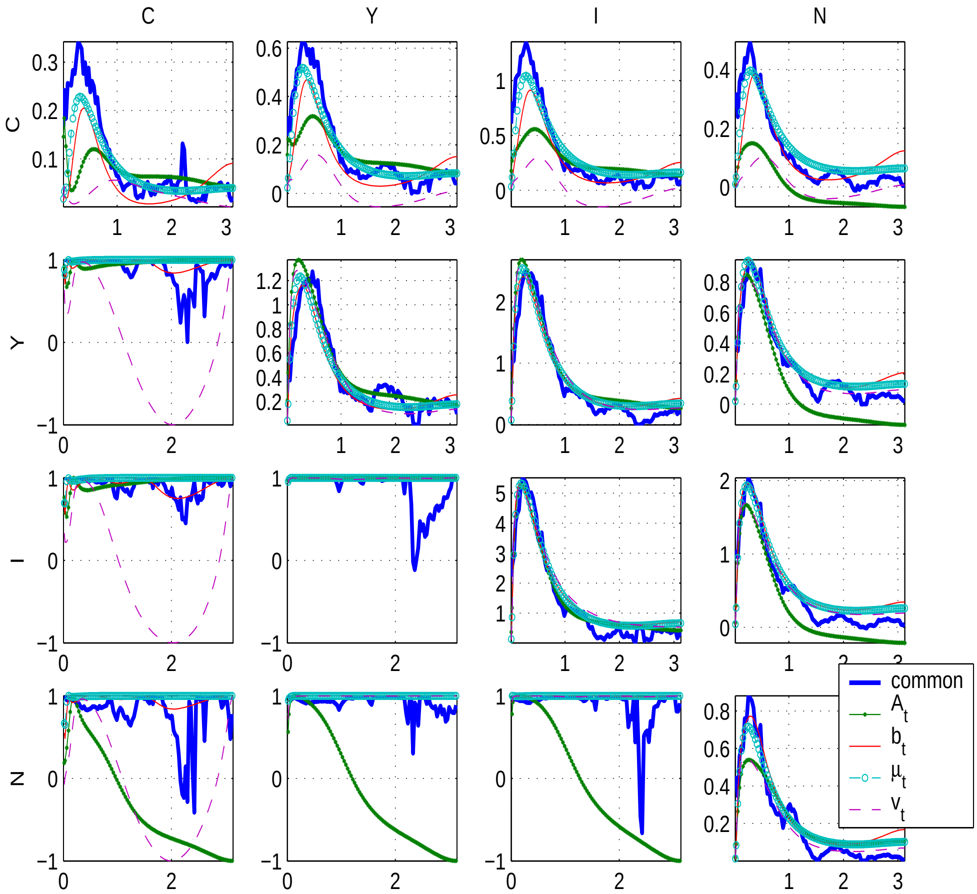

Figure 5: Cohesion (below diagonal) - Spectra (diagonal) - Cospectra (above diagonal)

6 Structural Forecast

In this section we repeat the forecasting exercise of Section 3, with the only difference that now we use the autocovariances generated by the 4 RBC models estimated above.

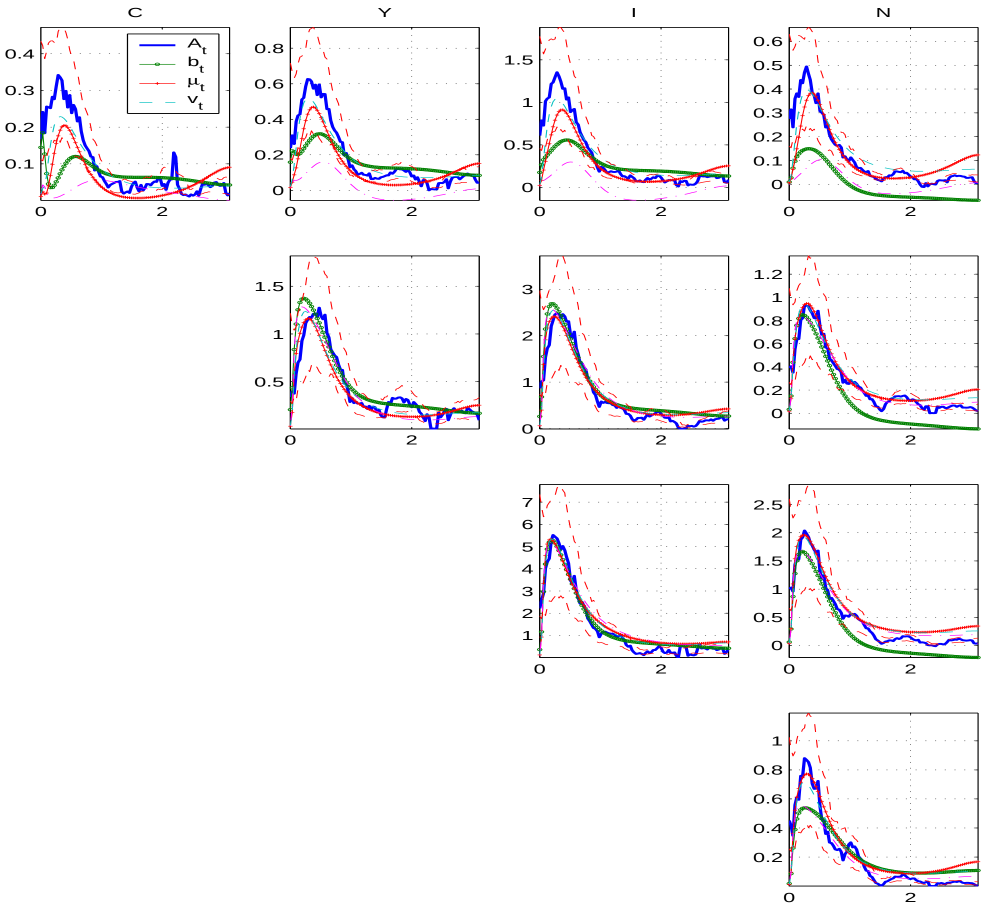

Figure 6: Spectra (diagonal) and Cospectra (above diagonal) with Confidence Intervals

The autocovariances implied by the four models (respectively indexed by $i = b, \mu, A, v$) will be obtained by "inverse Fourier trasforming" the model spectral density $S_Z^{\lambda}(\theta)$:

$$\Gamma(k)_{RBC_i} = \int_{-\pi}^{\pi} S_Z^{\lambda}(k) e^{ivk} dv$$

and selecting for each of them the elements corresponding to $\Delta \ln Z_t = [\Delta \ln C_t \; \Delta \ln Y_t \; \Delta \ln I_t \; \Delta \ln N_t]'$.

Forecasts will then be obtained by computing the coefficient of the VAR projections[11]:

$$Proj(\Delta \ln Z_{t+1} | \Delta \ln Z_t, RBC_i) = \Gamma(0)^{-1}_{RBC_i} \Gamma(1)'_{RBC_i} \Delta \ln Z_t$$

Figure 7 reports the MSFEs associated with different RBC models, together with MSFE

[11] We show here the projection using only one lag. In the application we trid 1 to 4 lags. Results are basically unchanged.

obtained from the factor model and from the BVAR. The RBC-based forecasts are in line with those obtained from the BVAR models and the factor model.

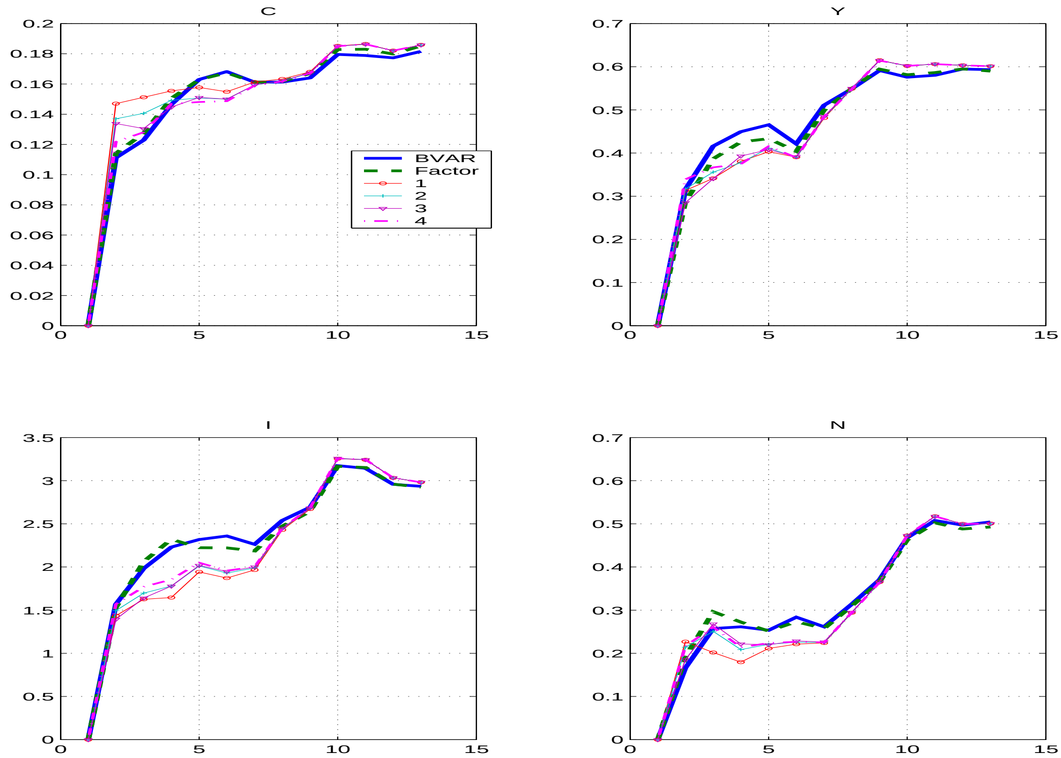

Figure 7: Forecasts from the economic models

7 Discussion

To be completed

8 Conclusion

To be completed

9 Bibliography

Altug, Sumru, 1989. "Time-to-Build and Aggregate Fluctuations: Some New Evidence," *International Economic Review*, vol. 30(4), pages 889-920

Anderson, T. W. (2003) "An Introduction to Multivariate Statistical Analysis", 3rd Edition, *John Wiley & Sons*.

Berkowitz J. and Francis X. Diebold (1998), Bootstrapping Multivariate Spectra, *The Review of Economics and Statistics*, vol. 80(4), pages 664-666

Chow, G.C., Kwan, Y.K., 1998. How the basic RBC model fails to explain US time series. *Journal of Monetary Economics* 41, 301318.

Christiano, L.J., 1988. Why does inventory investment fluctuate so much? *Journal of Monetary Economics* 21, 247280.

Christiano L.J., Martin Eichenbaum and Charles Evans, Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *Journal of Political Economy*, forthcoming.

Cochrane J. Shocks *Carnegie-Rochester Conference Series on Public Policy* 41, (1994) 295-364.

Cogley, T., Nason, J.M., 1995. Output dynamics in real-business-cycle models. *American Economic Review* 85, 492511.

Corana, A., M. Marchesi, C. Martini, and S. Ridella, 1987, Minimizing multimodal functions of continuous variables with the simulated annealing algorithm, *ACM Transactions on Mathematical Software* 13, 262-280.

Diebold, F. X., Ohanian, L. E., Berkowitz, J., 1998. Dynamic Equilibrium Economies: A Framework for Comparing Models and Data, *Review of Economic Studies*, vol. 65(3), pages 433-51, July.

Geweke John, The Dynamic Factor Analysis of Economic Time Series Models (1977) in D. Aigner and A. Goldberger (eds.), *Latent Variables in Socioeconomic Models*, 365-383. Amsterdam: North-Holland.

Geweke John and K. Singleton (1981) Maximum Likelihood Confirmatory Factor Analysis of Economic Time Series, *International Economic Review* 22, 37-54.

Goffe, William L., Ferrier, Gary D. and Rogers, John, 1994. "Global optimization of statistical functions with simulated annealing," *Journal of Econometrics*, vol. 60(1-2), pages 65-99.

Hall, Robert E, 1997. "Macroeconomic Fluctuations and the Allocation of Time", *Journal of Labor Economics*, vol. 15(1), pages S223-50, January

Hall, G.J., 1996. "Overtime, effort, and the propagation of business cycle shocks", *Journal of Monetary Economics* 38, 139160.

Hercowitz Z., Jeremy Greenwood and Per Krusell (1997), "Long-Run Implications of Investment-Specific Technological Change", *American Economic Review*, (87), pp. 342-362

Hercowitz Z., Jeremy Greenwood and Per Krusell (2000), "The Role of Investment-Specific Technological Change in the Business Cycle", *European Economic Review*, (44). 91-115.

Ingram, B.F., Kocherlakota, N.R., Savin, N.E., 1994. Explaining business cycles: a multiple-shock approach. *Journal of Monetary Economics* 34, 415428.

Ireland, Peter N., 2004. "A method for taking models to the data" *Journal of Economic Dynamics and Control*, vol. 28(6), pages 1205-1226, March.

McGrattan, E.R., 1994. The macroeconomic effects of distortionary taxation. *Journal of Monetary Economics* 33, 573601.

McGrattan, E.R., Rogerson, R., Wright, R., 1997. An equilibrium model of the business cycle with household production and fiscal policy. *International Economic Review* 38, 267290.

Sargent, Thomas J, 1989. "Two Models of Measurements and the Investment Accelerator," *Journal of Political Economy*, vol. 97(2), pages 251-87, April.

Smets F. and R. Wouters, 2003. "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, vol. 1(5), pages 1123-1175, September

Watson, M.W., 1993. Measures of Fit for Calibrated Models. *Journal of Political Economy* 101, 10111041.