Inflation Differentials in a Currency Area: Some Facts and Explanations

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Abstract

1 Introduction

The issue of analyzing and explaining consumer price and inflation differentials in a currency area has attracted the attention of a considerable number of researchers in the recent past years.[1] Indeed, persistent inflation differentials are a reality in the recent monetary-union experience among European countries.

This work aims to contribute to the literature along three main dimensions. First we present descriptive statistics on the presence of inflation dispersion based on the experience of European countries in the EMU. We show that there is still a sizeable dispersion of HICP inflation rates across countries. In a sectoral decomposition of this dispersion, we find that most of it occurs in the Service category of HICP, even if also the Energy category has been a relevant source in some historical periods. This suggests that most of the sources of dispersion are in the non-traded components of the HICP.

Second, we use a dynamic factor model to decompose the aggregate and sectoral dispersions between a common component driven by common factors and an idiosyncratic component. The haterogeneous response across countries to a change in the common factor can account for large fraction of the inflation dispersion. This is particularly the case for the Industrial good categories, even thought the cross-country dispersion of the inflation rates of items in this category is quite low. However, it also applies to the Services categories, which is instead characterized by a high level of dispersion.

Along these lines our interpretation of the data generating process is the following: Countries in a currency area might have structural differences: price-setting mechanism, degree of competition in the goods and labour markets, preferences and technologies and others. Moreover, policy-makers might interact in this environment by setting their policy instruments; countries might experience idiosyncratic or common shocks. The interrelation between shocks, structures and policies drive the data generating process and help to explain which factors can be treated as common or idiosyncratic.

Finally, we build a minimalist and stylized model of a currency area to give directions on which shocks, structures and policies can be important in explaining the qualitatively features of the data. In particular we study deviations from consumer-price equalization that arise because of movements in the relative prices of non-traded goods. The flexible-price version of the model by construction cannot capture any dynamic beyond the one implied by the shocks. However we can describe the long-run properties of the model. Contrary to the Balassa-Samuelson

[1] Our bibliography collects most of the recent and past works.

argument, we show analytically that an unbalanced country-specific productivity shock in the traded sector is almost irrelevant for explaining consumer price differential. Instead, a balanced increase in productivity in one country has a negative and sizeable impact on the consumer price of the country relative to the rest of the area; it also reduces the overall mark-up in all the sectors of the country where the shock occurred. Asymmetric demand shocks, driven by government purchases, can be also an important source of consumer price dispersion. There is otherwise no role for monetary policy and for area-wide shocks in explaining price dispersion. These features are introduced and analyzed in the sticky-price version of the model: We show that a modest degree of sticky prices can be sufficient to mute the short-run response to the shocks and to generate persistence well beyond the one implied by the stochastic process followed by the driving force. We find that most often sticky prices induce interesting dynamics and responses that contrast the ones of the flexible-price model. We show that monetary policy can matter in two important dimensions. First, discretionary movements of policy can itself create persistent deviations in consumer prices if there are considerable structural asymmetries across countries. Second, the rules and the targets followed by monetary policy are critical in explaining the response and dynamic of price dispersion and other variables following common and idiosyncratic shocks.

The work is structured as follows: Section 2 presents the descriptive statistics on euro-area inflation differentials and proposes a dynamic factor model decomposition of it. Section 3 presents the structure of the model. Section 4 discusses the flexible-price version of the model while Section 5 analyzes the implications of the model under sticky prices. Finally Section 6 concludes.

2 Euro area inflation differentials

This section motivates our work as well as our modelling strategy by addressing three questions concerning the euro area economy. How sizable are the inflation differentials in the euro area, also compared to other currency areas? Are the inflation differentials in the euro area a sectoral phenomena, i.e. tradable versus non-tradable? Are the inflation differentials in the euro area the result of differentiated responses to common area-wide factors, or of idiosyncratic (sector/country specific) factors?

The understanding of inflation differentials in the euro area has been attracting quite a lot of empirical work in the recent years. Alberola (2000) is one of the first analysis of post-EMU data specifically focusing on cross-country inflation differentials. Recently a more extensive review of empirical evidence and literature has been conducted by the ECB, (ECB, 2003). The latter work surveys a variety of measures of price and cost developments at the national level in EU-12 during the 1999-2002 period and explores different possible macroeconomic determinants. While it is possible to refer to these papers for a broad analysis, here we revisit some of the statistical facts that are more relevant for our analysis, adding also the specific angle provided by dynamic factor models for this purpose.

The empirical analysis is based on the harmonized index of consumption prices (HICP). These data are harmonized final consumer prices for a basket of all goods and services consumed by a country, expenditure-weighted. Seasonally adjusted data for the area as a whole and for ten individual countries (all euro countries but Greece and Luxemburg) ranging from 1990.01 to 2004.02 have been analyzed. Greece has been excluded from the analysis because it has been included in the euro area HICP since 2001 upon entering the euro zone, while data for Luxemburg are available only from 1995. For the euro area and the individual countries, we considered the main index as well as its five main subcomponents: Services, Industrial Good excluding Energy, Energy, Processed Food and Unprocessed Food.[2]
Notation: To set notation, $\pi^i_{j,t}$ is the year on year growth rate of price in subindex j of country i and the aggregate euro area counterpart is $\pi^{euro}_{j,t}$. While π^i_t denotes the overall inflation rate in country i and π^{euro}_t is the overall euro area inflation rate. The inflation differential between country i and the euro area is denoted as:

$$\delta_{i,t} = (\pi^i_t - \pi^{euro}_t);$$

while the dispersion of inflation is measured by the root mean squared around the euro area counterpart, as :

$$\Delta_t = \left(\frac{\sum_{i=1}^{10} \delta^2_{i,t}}{10} \right)^{1/2}.$$

The inflation differential between subindex j in country i and the euro area one is denoted as:

$$\delta_{j,i,t} = (\pi^i_{j,t} - \pi^{euro}_{j,t});$$

while the dispersion of inflation in the sub-component j is measured as:

$$\Delta_{j,t} = \left(\frac{\sum_{i=1}^{10} \delta^2_{j,i,t}}{10} \right)^{1/2}.$$

[2] The weights of the five subindexes in the euro area HICP index in 2004 are 0.41, 0.31, 0.08, 0.12 and 0.08, respectively.

The overall dispersion Δ_t is not the weighted average of the sectoral dispersion given the nonlinearity of the transformation. However, it might be interesting to decompose the overall dispersion into the relative contribution of the different subcomponents. To this end, first, we assume that the sectoral country weights inside the HICP, $w_{j,i,t}$, are equal across countries, i.e. $w_{j,i,t} = w_{j,t}\ \forall i$, then we define the contribution of sector j to the overall inflation dispersion as:

$$\frac{w_j \times \Delta_{j,t}}{\sum_j w_j \times \Delta_{j,t}}.$$

Empirical Findings: The cross-country dispersion in inflation has been declining during the 90s across euro area as documented in Figures 1-2. Dispersion was above 4% in early 90s while it reached its minimum at 0.26% at the end of 1997. After this point, it inverted its trend and started to edge up again to around 1% between 1998 and 2002. In early 2004 it stood at around 0.64%.[3]

Compared with the degree of dispersion observed within some individual euro area countries, inflation dispersion within the euro area remains relatively high. In particular, the recent degree of dispersion within the euro area is around twice the comparable measures computed across the German *Länder*, the Spanish *Comunidades Autónomas* and the Italian cities. On the contrary, the recent euro area inflation dispersion is quite comparable to the one measured among the 14 US metropolitan areas. The inflation divergence among US cities stayed remarkably constant at around 1 percent for many years. However the analogy is probably misleading, given that US cities are much smaller than EU nations, and their price indices tend to be more volatile.

Despite the possible similarities in the level of inflation dispersion between US and euro area, the high persistence of inflation differential is a characterizing feature of the euro area economy, as noted by Cecchetti, Mark and Sonora (2000). Indeed, in our data set the measured persistence of the inflation differential, $\delta_{j,i,t}$, is very close to the one of a unit root process on average across countries and sectors.[4]

The importance of sectoral pattern in explaining the overall dispersion is addressed in Figures 3-6. Figure 3 shows the year-on-year inflation rates in the euro

[3] This finding is slightly different from the one reported in ECB (2003), due to the fact that we considered the dispersion of the 10 indicated countries vis-à-vis the euro area while ECB (2003) considered also Greece and Luxemburg.

[4] If we measure the persistence of the differential process by the largest autoregressive roots of a fitted ARMA model, it turns out that the largest autoregressive root is 0.981 on average across sectors and countries.

area countries in the *Services*, *Industrial Good Excluding Energy* and *Energy* sectors, respectively. Figure 4 and 5 present the dispersion for each of the five sectors, $\Delta_{j,t}$, plotted together with the overall dispersion, Δ_t. Finally, Figure 6 presents the contribution of the sectors to the overall inflation dispersion.

Several conclusions can be drawn. First, all the components of inflation contributed to the very low dispersion observed in 1997-1998. Second, the increase on dispersion between 2000 and 2002 can mainly attributed to the dispersion in Services and Energy sectors. Third, Figures 4 shows that the dispersion in the Service sector has been almost always higher than the overall dispersion and its contribution has been increasing over time as indicated in Figure 6, also in line with the increase of the weight of this subcomponent inside the HICP index. Fourth, differently to the Services, the Industrial Good excluding energy index presents a low degree of dispersion and its overall importance is decreasing. Finally, differently from the common wisdom, the dynamic of the Energy subindex is a major source of overall dispersion. This is due both to the large volatility of this subindex but also to considerable heterogeneity in the countries' response to shocks.

Interpreting those results some cautionary remarks should be made. The HIPC is an index of final consumer prices for a basket of all goods and services consumed; so Industrial Good excluding energy price includes also the prices of (tradable) imported goods and a share of final sale services, such as the prices of any non-tradable marketing and other final consumption services; the services component included in the final Industrial good price might induce to overestimate the dispersion of this sector. On the contrary, this is not the case in the Services sector, where the value added deflator of Services almost entirely accounts for the final price of Services, as indicated by the input-output evidence produced by Sondergaard (2003) for some euro countries.

Finally, regarding the last of the three motivating questions - namely, whether the observed dispersion in inflation across euro area countries is due to different reaction to area-wide factors or the result of country/sectoral developments. To address this question we follow a similar approach to the one employed by Forni and Reichlin (2001) to decompose euro countries GDP growth into a European, a national, and a residual component. We estimate an approximate dynamic factor model[5] on the inflation differential which allows to decompose the differential in each countries and sectors as:

$$\delta_{j,i,t} = c_{j,i} + \Phi_{j,i}(L) \times u_t + \xi_{j,i,t} = \chi_{j,i,t} + \xi_{j,i,t} \tag{2.1}$$

where the first term of the right hand side is the average dispersion over the sample

[5] See Stock and Watson (1999) and Forni, Hallin, Lippi and Reichlin (2000).

(ideally nil), the second term captures the effect of common area wide shocks, u_t, which is allowed to propagate across countries and sectors with a differentiate dynamic, and the last term captures an idiosyncratic dynamic, mainly associated to country or sectoral specific developments. We grouped the first and second term into $\chi_{j,i,t}$.

The approximate dynamic factor models exploits the cross-section dimension of a large panel of time series to identify and estimates the part of the time series driven by few common shocks, $\chi_{j,i,t}$. To this end, in order to have proper estimates of the euro area common components in (2.1), we augmented the sixty time series on differential, $\delta_{j,i,t}$, with 193 monthly macro-economic time series related to the ten euro area countries considered.[6] The estimation has been performed on the sample period 1993.01-2003.06. In line with the finding of similar exercises, the results points to the presence of strong commonalities among the 253 variables and the estimated common factors account on average of around 50% of the variance of the 253 variables.[7]

Having an estimate of the two components in (2.1), we can first ask how much of the historical dynamic of differentials is accounted by the identified area wide factors. The table below reports the average across countries of the share of variance of the differentials, $\delta_{j,i,t}$, accounted by common shocks, both for the overall index and for the individual subindexes.

Overall	Services	Industrial	Energy	Proc. Food	Unproc. Food
0.66	0.58	0.67	0.52	0.65	0.42

It turns out that the Energy and the Services sector are the ones with the large idiosyncratic components, while the Industrial Good excluding Energy and the Processed Food sectors are the most common ones.

The importance of the common factors is also clear from Figure 7-10, where we show how the dispersion in inflation can be decomposed into the part attributed to the common factor, $\chi_{j,i,t}$, and a remaining one, associated to the idiosyncratic part. Figure 7 presents the cross-country dispersion of the overall HICP and its decomposition. The common part is clearly responsible for the large part of the observed dispersion. The idiosyncratic part has a nil contribution to overall dispersion from 1994 to 2000, but it contributes positively to dispersion from 2000

[6] The data are: main and sectoral industrial production indexes, consumer and producer surveys, producer prices index, financial quantities, interest rates, trade statistics and more. The data has been seasonally adjusted and transformed to be stationary.

[7] The estimation procedure is based on the principal componet decomposition of the variance-covariance matrix of the data. The estimation indicates the presence of five static factors in the models.

onward. The Service sector presents a behavior similar to the overall index with large part of its dispersion explained by countries-specific reaction to common shocks. The increase in dispersion since 2000 is mainly associated to the common factor; even if idiosyncratic elements contributed positively both during 2000 and 2002. In the Industrial sector the commonality is even more striking, however it should be noted that the dispersion in this sector is relative low. By contrast, in the case of Energy prices most of the dispersion is due to presence of idiosyncratic factors, see Figure 10.

To conclude, the inflation differentials in the euro area are relevant and persistent. Our proxy of the non-tradable sector, namely the Services sector, seems the main source of dispersion; even if also the heterogeneous dynamic of energy prices is strongly contributing to it. On the contrary, the dispersion in our proxy of the tradable sector, namely the one producing industrial goods excluding energy, is quite limited. Finally, a first attempt to understand the source of those differentials indicates that they are mainly associated to differentiate responses of the ten euro area economies to common area wide shocks.

3 Model

We propose a stylized model of a currency area to investigate which shocks can be most important to drive the price dispersion and which structural features can rationalize the observed persistence. We model a currency area as composed by two countries of equal population size. Consumption preferences depend on non-traded and traded goods. In particular, each country is specialized in the production of a bundle of traded goods. Financial markets are complete within and across countries. Labor is immobile across countries but imperfectly mobile within sectors of a country. There can be price rigidities in all sectors of the economy. The law of one price holds in the traded sector and the price dispersion at the consumer level depends on the movements of the relative prices of non-traded goods.

Our model is closely related to open-economy models, like Obstfeld and Rogoff (2000) who introduces non-traded goods in stochastic models with sticky prices. Benigno and Thoenissen (2003) use a similar model to study the real exchange rate behavior of UK with respect to the Euro area with the purpose of analyzing whether supply shocks can account for the real exchange rate appreciation in the late nineties. In reference to the recent literature on monetary policy in a currency area our model is closely related to Duarte and Wolman (2002, 2003). They further allow for price discrimination in goods that are tradeable. Moreover, they analyze the role of fiscal policy rules in reducing or amplifying inflation differentials. Indeed, in most of their work they assume that lump-sum forms of taxation are unavailable to the government. Our focus here is instead limited to the case in which lump-sum taxes are available. In Duarte and Wolman (2002) they find that their model can deliver more inflation dispersion than in the data following productivity shocks, while negligible dispersion following government spending shocks. Andrés et al. (2003) instead analyze a model with only traded goods, but they allow for price discrimination across countries due to different degrees of market competition. They show that their model can account for sizeable inflation differentials. In particular their findings are that the driving force of the price dispersion in the traded sector originates from the mechanism of price discrimination more than on price rigidity. They also find that the degree of openness of countries can play an important role. Angeloni and Ehrmann (2004) present a more stylized 12-country model of the euro area and in particular they focus on the role of past inflation in the aggregate supply equation. They find that this additional source of inflation persistence is important in driving up inflation dispersion in the currency area. There are other papers that have analyzed monetary models of currency areas, as Benigno (2004), Beetsma and Jensen (2004) and Lombardo (2004).[8] However these models do not allow for price dispersion at the consumer level and mainly focus on the role of the terms of trade and price stickiness in stabilizing asymmetric shocks.

3.1 Households

We consider a model of a currency area composed by two countries, Home (H) and Foreign (F). Each country is populated by a measure one of households. A generic household j belonging to either country H or F maximizes the following utility function:

$$U_t^j \equiv \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s^j) - V(L_s^j) \right],$$

where E_t denotes the expectation conditional on the information set at date t and β is the intertemporal discount factor, with $0 < \beta < 1$. Households derive utility from consumption and disutility from supplying hours of work.

There are two classes of goods in both economies: traded and non-traded goods. In particular, in each country, a measure one of goods is produced of which a

[8] Our work is also related to model of small open economies as Natalucci and Ravenna (2002) that have investigated exchange rate policy for accession countries in the EU. See Soto (2001) for a small open-economy model with non-traded goods.

fraction γ (γ^* in region F), with $0 < \gamma, \gamma^* < 1$, is composed by traded goods. The remaining fractions are non-traded goods. The consumption index C_t^j in region H is defined as a Dixit-Stiglitz aggregator of indexes of traded, C_T^j, and non-traded goods, C_N^j, as it follows

$$C^j \equiv \left[\omega^{\frac{1}{\varphi}}(C_T^j)^{\frac{\varphi-1}{\varphi}} + (1-\omega)^{\frac{1}{\varphi}}(C_N^j)^{\frac{\varphi-1}{\varphi}}\right]^{\frac{\varphi}{\varphi-1}},$$

where φ is the elasticity of substitution between the bundles C_T and C_N with $\varphi > 0$ while ω denotes the share of traded goods in the general consumption basket, with $0 < \omega < 1$. (This share may be different for households in region F and it will be denoted by ω^*.) The traded goods are not homogenous and in particular they are differentiated in consumption preferences. They are also produced with different technologies. In particular the index C_T^j is defined as a Dixit-Stiglitz aggregator of the bundles of home-produced traded goods, C_H^j, and foreign-produced traded goods, C_F^j,

$$C_T^j \equiv \left[n^{\frac{1}{\theta}}(C_H^j)^{\frac{\theta-1}{\theta}} + (1-n)^{\frac{1}{\theta}}(C_F^j)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

where θ, with $\theta > 0$, is the elasticity of substitution between the bundles C_H^j and C_F^j and n, with $0 < n < 1$, denotes the share of home-produced traded goods in the overall index of traded goods. In the foreign economy n^*, with $0 < n^* < 1$, denotes the share of foreign-produced traded goods in the overall basket of traded goods. The consumption bundles C_H^j and C_F^j are composed by the continuum of differentiated traded goods produced respectively in region H and F and are defined as

$$C_H^j \equiv \left[\gamma^{-\frac{1}{\sigma}} \int_0^{\gamma} c^j(h)^{\frac{\sigma-1}{\sigma}} dh\right]^{\frac{\sigma}{\sigma-1}}, \qquad C_F^j \equiv \left[\gamma^{*-\frac{1}{\sigma}} \int_0^{\gamma^*} c^j(f)^{\frac{\sigma-1}{\sigma}} df\right]^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution among the differentiated goods. In the same way C_N^j is the home consumption index of the continuum of differentiated non-traded goods:

$$C_N^j \equiv \left[(1-\gamma)^{-\frac{1}{\sigma}} \int_{\gamma}^{1} c_N^j(h)^{\frac{\sigma-1}{\sigma}} dh\right]^{\frac{\sigma}{\sigma-1}},$$

with the same elasticity of substitution σ. In country F, γ^* replaces γ in the consumption bundles of non-traded goods.

Given the above consumption indices, we can derive the appropriate price indices. With respect to the general consumption index, we obtain the consumption-based price index P

$$P = \left[\omega(P_T)^{1-\varphi} + (1-\omega)(P_N)^{1-\varphi}\right]^{\frac{1}{1-\varphi}},$$

where P_T and P_N are given by

$$P_T = \left[n(P_H)^{1-\theta} + (1-n)(P_F)^{1-\theta}\right]^{\frac{1}{1-\theta}},$$

$$P_N = \left[(1-\gamma)^{-1}\int_{\gamma}^{1} p_N(h)^{1-\sigma}dh\right]^{\frac{1}{1-\sigma}},$$

while P_H and P_F are given by

$$P_H = \left[\gamma^{-1}\int_{0}^{\gamma} p(h)^{1-\sigma}dh\right]^{\frac{1}{1-\sigma}}, \qquad P_F = \left[\gamma^{*-1}\int_{0}^{\gamma^*} p(f)^{1-\sigma}df\right]^{\frac{1}{1-\sigma}},$$

where $p(h)$, $p(f)$, $p_N(h)$ are respectively the prices in the common currency faced by households in country H for a generic home-produced traded good, a foreign-produced traded good and a domestic non-traded good. Similar indices are derived for country F with the appropriate modifications of the respective shares. Prices faced by foreign consumers are denoted with asterisks.

Given the consumption-based price indexes the generic home consumer j has the following demand of each of the home-produced traded goods

$$c^j(h) = \frac{n\omega}{\gamma}\left(\frac{p(h)}{P_H}\right)^{-\sigma}\left(\frac{P_H}{P_T}\right)^{-\theta}\left(\frac{P_T}{P}\right)^{-\varphi}C^j,$$

for $0 \leq h < \gamma$; of each of the foreign-produced traded goods

$$c^j(f) = \frac{(1-n)\omega}{\gamma^*}\left(\frac{p(f)}{P_F}\right)^{-\sigma}\left(\frac{P_F}{P_T}\right)^{-\theta}\left(\frac{P_T}{P}\right)^{-\varphi}C^j,$$

for $0 \leq f < \gamma^*$ and of each of the home-produced non-traded goods

$$c_N^j(h) = \frac{1-\omega}{1-\gamma}\left(\frac{p_N(h)}{P_N}\right)^{-\sigma}\left(\frac{P_N}{P}\right)^{-\varphi}C^j,$$

for $\gamma \leq h \leq 1$. Similar demands hold in the foreign economy.

Households get disutility from supplying labor to all the firms operating in their country of residence. In particular the function $V(.)$ is increasing and convex in an index of labor L^j. Each firm uses a specific labor factor and each household can supply all the varieties of labor used in the country to produce the continuum of traded and non-traded goods. In particular each household supplies a measure one of labor varieties, of which a fraction γ will be employed in the traded sector (γ^* in the foreign economy) and the remaining fractions in the non-traded sector.

In particular we assume that L^j is a Dixit-Stiglitz aggregator of labor indices L_T and L_N in the traded and non-traded sectors, respectively, as it follows

$$L^j \equiv \left[\gamma^{\frac{1}{\phi}}(L_T^j)^{\frac{\phi-1}{\phi}} + (1-\gamma)^{\frac{1}{\phi}}(L_N^j)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}},$$

where $\phi > 0$ is the elasticity of substitution between labor in the traded and non-traded sector and L_T^j and L_N^j are composite index of the continuum of varieties supplied in both sectors

$$L_T^j \equiv \left[\gamma^{-\frac{1}{\upsilon}} \int_0^\gamma l_T^j(i)^{\frac{\upsilon-1}{\upsilon}} di\right]^{\frac{\upsilon}{\upsilon-1}}, \qquad L_N^j \equiv \left[(1-\gamma)^{-\frac{1}{\upsilon}} \int_\gamma^1 l_N^j(i)^{\frac{\upsilon-1}{\upsilon}} di\right]^{\frac{\upsilon}{\upsilon-1}},$$

where $\upsilon > 0$ is the elasticity of substitution between varieties of labor within a sector. In contrast with standard models of traded and non-traded production, we are not necessarily assuming that labor is perfectly substitutable and mobile across sectors and instead we allow for wage differentiation across varieties of labor. The case of perfect substitutability, and perfect labor mobility, is nested under the assumption that both ϕ, $\upsilon \to \infty$. In general, given wages $w_T(i)$ and $w_N(i)$ specific to the generic variety i in the respective traded and non-traded sector, we can write the following wage indexes associated with the above defined labor indices

$$W = \left[\gamma W_T^{1-\phi} + (1-\gamma)W_N^{1-\phi}\right]^{\frac{1}{1-\phi}},$$

$$W_T = \left[\gamma^{-1} \int_0^\gamma w_T^{1-\upsilon}(i)di\right]^{\frac{1}{1-\upsilon}}, \qquad W_N = \left[(1-\gamma)^{-1} \int_\gamma^1 w_N^{1-\upsilon}(i)di\right]^{\frac{1}{1-\upsilon}}.$$

Given the relative wages and the choice of L^j, we can then characterize the household's labor decisions in the following way

$$l_T^j(h) = \left(\frac{w_T(h)}{W_T}\right)^{-\upsilon} \left(\frac{W_T}{W}\right)^{-\phi} L^j, \quad l_N^j(i) = \left(\frac{w_N(i)}{W_N}\right)^{-\upsilon} \left(\frac{W_N}{W}\right)^{-\phi} L^j,$$

for each variety of labor supplied for a generic firm in a traded and non-traded sector, respectively.

Each household faces the following flow budget constraint

$$B_t^j \le A_t^j + W_t L_t^j + \Pi_t^j - P_t C_t^j + T_t^j \tag{3.2}$$

where A_t^j represents the beginning-of-period wealth that includes the bonds carried from the previous period. B_t^j is the end-of period portfolio that includes a wide selection of instruments that pay in each contingency that occurs. In particular they pay A_t in the particular contingency at date t. As of time $t-1$, A_t is a random variable whose realization depends on the state of nature at time t.

Here it is assumed that there are complete financial market which implies that there exists a unique discount factor $Q_{t,t+1}$ with the property that the price in period t of a portfolio with random value A_{t+1} is

$$B_t = E_t[Q_{t,t+1}A_{t+1}],$$

where E_t denotes the expectation conditional on the state of nature at date t. In particular we define the short-term interest rate in the following way, as the price of the portfolio that delivers one unit of currency in each contingency that occur one-period ahead, i.e.

$$\frac{1}{1+i_t} = E_t[Q_{t,t+1}].$$

In (3.2), Π_t are aggregate profits of all the firms within a country. Profits are risk shared across households. T_t^j are transfers from the government to household j. The economy is a cashless-limiting monetary economy as discussed in Woodford (2003). The flow budget constraint of the consumer can be written as

$$E_t[Q_{t,t+1}A_{t+1}^j] \leq A_t^j + W_t L_t^j + \Pi_t^j - P_t C_t^j + T_t^j$$

and the consumer's problem is further subject to the following borrowing limit condition in each contingency and date that the consumer will face.

$$A_{t+1}^j \geq -\sum_{s=t+1}^{\infty} E_{t+1}Q_{t+1,s}\{W_s L_s^j + \Pi_s^j + T_s^j\} > -\infty.$$

The borrowing limit condition together with the flow budget constraint imply the standard intertemporal budget constraint

$$\sum_{s=t}^{\infty} E_t Q_{t,s}\left[P_s C_s^j\right] \leq A_t^j + \sum_{s=t}^{\infty} E_t Q_{t,s}\left[W_s L_s^j + \Pi_s^j + T_s^j\right]. \tag{3.3}$$

Given the above decisions on how to allocate consumption and labor across all the varieties, the household j chooses the optimal path of the consumption index C_t and labor index L_t at all times and contingencies to maximize its utility under the intertemporal budget constraint. In particular the set of optimality conditions can be described by the set of Euler conditions

$$\frac{U_c(C_t^j)}{U_c(C_{t+1}^j)} = \frac{\beta}{Q_{t,t+1}}\frac{P_t}{P_{t+1}} \tag{3.4}$$

for each state of nature at time $t+1$ looking ahead from time t. By choosing appropriately the distribution of initial state-contingent wealth, complete markets assure that consumption is perfectly equalized within households belonging to a country. Moreover, across countries, marginal utilities of consumption are proportional to the consumer-price differential

$$\frac{U_c(C_t)}{U_c(C_t^*)} = \varsigma \frac{P_t}{P_t^*} \tag{3.5}$$

for a positive factor of proportionality $\varsigma > 0$ which again depends on the initial wealth distribution. As shown by Backus and Smith (1993), under the assumption of complete financial markets, consumer-price differentials directly translate into consumption differentials, so that a country which experiences an increase in its consumer price level relative to another country should experience a fall in its own consumption relative to the other country. This complete-market assumption is a convenient simplification but it comes at a cost of neglecting wealth distribution as an important channel through which price and inflation differentials can propagate into the economy and be amplified.

Real wages, computed using the general price and wage indices, are equated to the marginal rate of substitution between the labor index L and the consumption index C as

$$\frac{W_t}{P_t} = \frac{V_l(L_t)}{U_c(C_t)}. \tag{3.6}$$

Finally the last optimality condition is the exhaustion of the intertemporal budget constraint, i.e. (3.3) holds with equality at all times.

3.2 Firms

Regarding the supply side of the economy, we indeed assume that there is a continuum of firms, of measure one, which is producing the continuum of goods. In particular a fraction γ (γ^* in the foreign economy) is producing traded goods, while the remaining fractions are producing non-traded goods. Taking as representative the home economy, a generic firm producing in the traded sector is using the following technology $y^T(h) = A_T f(l(h))$, where A_T is a country- and sector-specific technological shock and $f(.)$ is a standard concave production function in the specific variety of labor used in the production of good h. In the non-traded sector the technology is given by $y^N(h) = A_N f(l(h))$ for a generic firm h in the non-traded sector. Firms in both sectors are monopolist and set their prices considering the overall demand of their goods. In the traded sector, we assume that there is no price discrimination and that all the consumers of the area face the same price for the same variety of goods. In particular in the traded sector a generic firm h faces the following demand

$$y^T(h) = \frac{n\omega}{\gamma}\left(\frac{p(h)}{P_H}\right)^{-\sigma}\left(\frac{P_H}{P_T}\right)^{-\theta}\left(\frac{P_T}{P}\right)^{-\varphi} C + \frac{n^*\omega^*}{\gamma}\left(\frac{p(h)}{P_H}\right)^{-\sigma}\left(\frac{P_H}{P_T^*}\right)^{-\theta}\left(\frac{P_T^*}{P^*}\right)^{-\varphi} C^*$$

In the non-traded sector, a generic firm faces the following demand

$$y^N(h) = \frac{1-\omega}{1-\gamma}\left(\frac{p_N(h)}{P_N}\right)^{-\sigma}\left[\left(\frac{P_N}{P}\right)^{-\varphi} C_N + G\right],$$

where in particular G is a country-specific exogenous government-purchase shock that affects only the demand of non-traded goods. In both sectors, prices are sticky and staggered as in the Calvo's style price-setting behavior.[9] In particular a mass $1-\alpha_T$ of firms in the traded sector ($1-\alpha_N$ in the non-traded sector) with $0 \leq \alpha_T, \alpha_N < 1$, is allowed in each period to reset their prices. (In the foreign economy, we have respectively α_N^* and α_T^* with $0 < \alpha_T^*, \alpha_N^* < 1$.) In this case a generic firm h in the traded sector of country H sets its price in order to maximize the present discounted value of profits, taking in consideration that the price chosen at time t will remain the same at time s, with $s \geq t$, with the probability $(\alpha_T^i)^{s-t}$. The present discounted value of profits is

$$\mathrm{E}_t \sum_{s=t}^{\infty} (\alpha_T)^{s-t} Q_{t,s}[(1-\tau_{T,s})\tilde{p}_t(h)\tilde{y}_{t,s}^T(h) - w_{T,s}(h) l_{T,s}(h)], \tag{3.7}$$

where $\tau_{T,t}$ is a sectoral country-specific time-varying proportional tax on sales in the traded sector, $\tilde{p}_t(h)$ denotes the price of the firm h chosen at date t and $\tilde{y}_{t,s}^T(h)$ is the total demand of firm h at time s conditional on the fact that the price $\tilde{p}_t(h)$ has not changed.

It can be shown that the optimal choice of the price satisfies

$$\mathrm{E}_t \sum_{s=t}^{\infty} (\alpha_T \beta)^{s-t} U_c(C_s) \tilde{y}_{t,s}^T(h) \frac{1}{\mu_{T,s}} \frac{P_{T,s}}{P_s}\left[\frac{\tilde{p}_t(h)}{P_{H,t}} \frac{P_{H,t}}{P_{H,s}} \frac{P_{H,s}}{P_{T,s}} - mc_{t,s}^T(h)\right] = 0 \tag{3.8}$$

where the real marginal cost for firm j at time s conditional on the fact that the price $\tilde{p}_t(j)$ has not changed are defined by

$$\begin{aligned} mc_{t,s}^T(h) &\equiv \mu_{T,t} \frac{P_s}{P_{T,s}} \frac{P_{T,s}}{P_{H,s}} \frac{w_{T,s}(h)}{P_s} \frac{1}{f'(f^{-1}(\tilde{y}_{t,s}^T(h)/A_{T,s}))A_{T,s}} \\ &= \mu_{T,t} \frac{P_s}{P_{T,s}} \frac{P_{T,s}}{P_{H,s}} \frac{W_s}{P_s} \left(\frac{W_{T,s}}{W_s}\right)^{1-\frac{\phi}{\upsilon}} \left(\frac{f^{-1}[(\tilde{y}_{t,s}^T(h)/A_{T,s}]}{L_s}\right)^{-\frac{1}{\upsilon}} \frac{1}{f'(f^{-1}(\tilde{y}_{t,s}^T(h)/A_{T,s}))A_{T,s}} \end{aligned}$$

[9] See Calvo (1983).

where $1/\mu_{T,t}$ is defined as

$$\frac{1}{\mu_{T,t}} \equiv \frac{(1-\tau_{T,t})(\sigma-1)}{\sigma}.$$

Given the Calvo's mechanism, the evolution of the price index $P_{H,t}$ is described by the following law of motion

$$P_{H,t}^{1-\sigma} = \alpha_T P_{H,t-1}^{1-\sigma} + (1-\alpha_T)\tilde{p}_t(h)^{1-\sigma}, \tag{3.9}$$

where indeed $1-\alpha_T$ is the fraction of firms that can reset their prices. Following the same reasoning we can write the first-order condition for a generic firm in the non-traded sector

$$\mathrm{E}_t \sum_{s=t}^{\infty} (\alpha_N \beta)^{s-t} U_c(C_s) \tilde{y}_{t,s}^N(h) \frac{P_{N,s}}{P_s} \frac{1}{\mu_{N,s}} \left[\frac{\tilde{p}_{N,t}(h)}{P_{N,t}} \frac{P_{N,t}}{P_{N,s}} - mc_{t,s}^N(h) \right] = 0 \tag{3.10}$$

where

$$\begin{aligned} mc_{t,s}^N(h) &\equiv \mu_{N,s} \frac{P_s}{P_{N,s}} \frac{w_{N,s}(h)}{P_s} \frac{1}{f'(f^{-1}(\tilde{y}_{t,s}^N(h)/A_{N,s}))A_{N,s}} \\ &= \mu_{N,s} \frac{P_s}{P_{N,s}} \frac{W_s}{P_s} \left(\frac{W_{N,s}}{W_s} \right)^{1-\frac{\phi}{v}} \left(\frac{f^{-1}[(\tilde{y}_{t,s}^N(h)/A_{N,s}]}{L_s} \right)^{-\frac{1}{v}} \frac{1}{f'(f^{-1}(\tilde{y}_{t,s}^N(h)/A_{N,s}))A_{N,s}} \end{aligned}$$

and

$$\frac{1}{\mu_{N,t}} \equiv \frac{(1-\tau_{N,t})(\sigma-1)}{\sigma},$$

where τ_N is a time-varying proportional tax on sales in the non-traded sector. The evolution of the price index $P_{N,t}$ is described by the following law of motion

$$P_{N,t}^{1-\sigma} = \alpha_N P_{N,t-1}^{1-\sigma} + (1-\alpha_N)\tilde{p}_{N,t}(h)^{1-\sigma}, \tag{3.11}$$

where $1-\alpha_N$ is the mass of firms that can change their prices in the non-traded sector. Similar price conditions can be obtained in the sectors of the foreign country.

3.3 Monetary and Fiscal Policies

Each country has its own fiscal authority and there is a common monetary policymaker in the whole area. In each region, the government raises revenues from the distortionary sale taxes to finance the expenditure for domestic non-traded goods.

Moreover, lump-sum taxes are available to balance the budget in each period. It follows that

$$
\begin{aligned}
0 &= \int_{\gamma}^{1} p_{N,t}(h) g_t(h) dh + \int_{0}^{1} T_t^j dj - \tau_{N,t} \int_{\gamma}^{1} p_{N,t}(h) y_t^N(h) dh \\
&\quad - \tau_{T,t} \int_{0}^{\gamma} p_t(h) y_t^T(h) dh
\end{aligned}
$$

for country H while

$$
\begin{aligned}
0 &= \int_{\gamma^*}^{1} p_{N,t}^*(f) g_t^*(f) df + \int_{0}^{1} T_t^{*j} dj - \tau_{N,t}^* \int_{\gamma^*}^{1} p_t^{N*}(f) y_{N,t}^*(f) df \\
&\quad - \tau_{T,t}^* \int_{0}^{\gamma^*} p_t^*(f) y_t^*(f) df
\end{aligned}
$$

for country F. The model is closed with the policy function chosen by the common monetary authority that will be specified later.

4 Price differentials with flexible prices

We can write the ratio of the CPI prices of the two countries in the following way

$$
\frac{P_t}{P_t^*} = \frac{P_{T,t}}{P_{T,t}^*} \frac{[\omega + (1-\omega) T_{N,t}^{1-\varphi}]^{\frac{1}{1-\varphi}}}{[\omega^* + (1-\omega^*) T_{N,t}^{*1-\varphi}]^{\frac{1}{1-\varphi}}}
$$

where we have defined the relative price of non-traded with respect to traded goods in each country as $T_N \equiv P_N/P_T$ and $T_N^* \equiv P_N^*/P_T^*$. The ratio of the tradeable goods prices is instead given by

$$
\frac{P_{T,t}}{P_{T,t}^*} = \frac{[n P_{H,t} + (1-n) P_{F,t}]^{\frac{1}{1-\theta}}}{[n^* P_{F,t}^* + (1-n^*) P_{H,t}^*]^{\frac{1}{1-\theta}}}.
$$

There can be several possible ways through which differences in consumer prices can arise among countries. One obvious reason has to do with the different composition of the consumption indices, due to differences in tastes. This can happen either because there can be home bias in the consumption of traded good, so that $n^* \neq 1 - n$ or because the share of traded goods in the overall consumption basket can vary across countries, $\omega \neq \omega^*$. Heterogeneity in consumption preferences can be a source of consumer-price differentials even if the prices of all the goods, traded and non-traded, are equalized across countries. Conversely, even if tastes are similar, price differentials at the single good level can produce differential at the consumer-price level. The above decomposition shows that this can happen either because of deviations of the law of one price for traded goods or because relative price of non-traded goods can vary across countries.

The issue of explaining consumer price differential in a currency area is closely related to a long-lasting puzzle in the international finance literature: the PPP puzzle and the related literature on real exchange rate behavior.[10] Using the above decomposition, Engel (1999) has studied the US real exchange rate relative to several countries and shown that its variability can be accounted mostly by the variability of the deviations from the law of one price for traded goods relative to the variability of the relative price of non-traded goods. With multiple currencies, firms might set their prices sticky in the currency of the buyer, hence protecting consumers from fluctuations of the exchange rate. In this way deviations from the law of one price for traded goods reflect mostly fluctuations of the nominal exchange rate. This interpretation is naturally absent in a currency area, since there is no reason to protect prices from fluctuations of the nominal exchange rate.[11] However, a unique currency does not exclude the possibility for firms to price discriminate across different markets (countries) due to different degrees of competition in the markets or structural characteristics. Moreover, even if firms do not price discriminate and markets are characterized by similar structures and degrees of competition, there can be deviations of traded goods prices at the consumer level, steming fromthe fact that traded goods usually carry some non-traded components (e.g. distribution costs) before reaching the consumer markets.[12] Sondergaard (2003) using input-output tables for France, Italy and Spain has shown that the traded sector relies more on intermediate inputs from other sectors in the economy; in particular total inputs from other sectors account for 60

[10] See Rogoff (1996).

[11] A puzzling finding of Engel (1999) is that the importance of the relative price of traded goods in explaining the variability of the real exchange rate is robust across the fixed exchange rate period of Bretton Woods and the following floating exchange rate period. This decomposition holds in general also for developing economies, as in the case of Mexico studied by Engel (2001). However in this case Mendoza (2000) has shown that during periods of fixed-exchange rate regime movements in the relative price of non-traded goods are dominant in explaining real exchange rate volatility. In relation to the debate on whether deviations from the law of one price for tradeable goods originate from local currency pricing, works by Obstfeld and Rogoff (2000) and Campa and Goldberg (2002) have emphasized that the pass-through of the exchange rate at the border level can be high.

[12] The importance of distribution sectors in explaining differential at the consumer-price leve has been emphasized by Burstein et al. (2003) and Corsetti and Dedola (2003). Duarte and Wolman (2003) incorporate this feature in their currency-area model.

percent of gross output in the traded sector. This suggests that the distribution sector can be an important factor for explaining inflation differentials in a currency area. Movements in the prices of non-traded goods that enter in the production or transportation of traded goods can be an important source of cross-country deviations of consumer tradeable goods prices. Moreover, taking in consideration intermediate stages of production de-emphasizes the importance of pure traded goods prices in explaining the fluctuations of the consumer price differential. Our evidence shows that inflation dispersion in the traded sector is much lower than in the non-traded sector, which suggests that in any case differences in the relative price of non-traded goods can be much more important in explaining the consumer price differential. In this work, we choose not to model any price differential in the traded sector to get more insight on what we believe can be a stronger channel.[13] We assume that there is no price discrimination and that $P_H = P_H^*$ and $P_F = P_F^*$ at all times. We also assume perfect symmetry across countries and sectors and set $\omega = \omega^* = \gamma = \gamma^*$ and $n = n^* = 1/2$. It further follows that $P_T = P_T^*$. We first focus on the flexible-price allocation. In the appendix, we solve the model by taking a log-linear approximation to the relevant structural equilibrium conditions around a deterministic steady state in which the shocks $\{\mu_{T,t}, \mu_{T,t}^*, \mu_{N,t}, \mu_{N,t}^*, A_{T,t}, A_{T,t}^*, A_{N,t}, A_{N,t}^*, G_t, G_t^*\}$ are such that $\bar{\mu}_T = \bar{\mu}_T^* = \bar{\mu}_N = \bar{\mu}_N^* = \bar{\mu}$ and $\bar{A}_T = \bar{A}_T^* = \bar{A}_N = \bar{A}_N^* = \bar{A}$, $\bar{G} = \bar{G}^* = 0$. In a log-linear approximation around this steady state the consumer-price differential is proportional to the non-traded goods price differential (all in logs)

$$\begin{aligned} \ln P_t - \ln P_t^* &= (1-\gamma)(\hat{T}_{N,t} - \hat{T}_{N,t}) \\ &= (1-\gamma)(\ln P_{N,t} - \ln P_{N,t}^*), \end{aligned}$$

where hat variables represent log-deviations of the respective variable from the steady state.

The most popular and often advocated reason for why there can be long-lasting departures from PPP originating from non-traded goods prices is due to Balassa (1964) and Samuelson (1964). According to this view, countries that experience higher productivity growth in the traded sector will show also higher consumer prices. The reason is that productivity growth in the traded sector will translate into an increase in the overall wage in the economy, since prices of traded goods are tied internationally and there is perfect labor mobility. In the non-traded sector, firm will increase their prices since costs have increased and there are no benefits from productivity gains. There are several assumptions needed to get

[13] Andres et al. (2003) aknowledge the importance of the relative price of non-traded goods but they focus on differentials that arise from price discrimination in the traded-good sector.

this result, as discussed in Froot and Rogoff (1995): mobility of labor and capital across sectors and mobility of capital internationally, constant returns to scale in the mobile factors and exogenous world real interest rate. Indeed, we cannot expect our model to display the Balassa-Samuelson features since capital is constant in each sector and labor is not perfectly mobile. Moreover, as shown in recent works by Benigno and Thoenissen (2003), Cova (2004), Duarte and Wolman (2003), Fitzgerald (2002), MacDonald and Ricci (2002), the assumption of homogenous traded-good market is critical for the result to hold. In our model, each country is specialized in the production of a bundle of traded goods. When the traded sector of a country is subject to a productivity shock, prices of home-produced traded goods can fall with respect to foreign-produced traded goods and terms of trade worsen. In this case movements in terms of trade can absorb the productivity shock and reduce the pressure on wages and then on non-traded goods prices.

Another feature of the Balassa-Samuelson framework is that the consumer-price differential depends only on the supply side. Instead in our framework, as in Rogoff (1992), a model with low labor mobility can also capture additional sources of price differentials driven by demand shocks.[14] This is also the case in a more modern exposition of the Balassa-Samuelson model as in Canzoneri et al. (2001). In our context, we also introduce sectoral mark-up shocks to capture if changes in taxation and competition can account for consumer-price differences.

To get further insight into the model, we assume that the production functions are linear in the only factor of production, labor. In the appendix, we show that under this assumption terms of trade and the log difference of non-traded goods prices are determined by the following two equations

$$\ln P_{N,t}/P_{N,t}^{*} = -\frac{\left(1-\frac{\theta}{\phi}\right)}{\left(1-\frac{b}{\phi}\right)}\hat{T}_t - \frac{1}{\left(1-\frac{b}{\phi}\right)\phi}\hat{G}_t^R + \frac{\left(1-\frac{1}{\phi}\right)}{\left(1-\frac{b}{\phi}\right)}(\hat{A}_{T,t}^R - \hat{A}_{N,t}^R) - \frac{1}{\left(1-\frac{b}{\phi}\right)}(\hat{\mu}_{T,t}^R - \hat{\mu}_{N,t}^R) \tag{4.12}$$

$$\begin{aligned}\left[1+\left(\eta+\frac{1}{\phi}\right)\theta\gamma-\frac{\theta}{\phi}\right]\hat{T}_t &= \left(\eta+\frac{1}{\phi}\right)b(1-\gamma)(\ln P_{N,t}/P_{N,t}^{*}) + \left(1-\frac{1}{\phi}\right)\hat{A}_T^R \\ &\quad -\hat{\mu}_T^R + \left(\eta+\frac{1}{\phi}\right)[\gamma\hat{A}_T^R + (1-\gamma)\hat{A}_N^R] \\ &\quad -\left(\eta+\frac{1}{\phi}\right)(1-\gamma)\hat{G}_t^R \end{aligned} \tag{4.13}$$

[14]De Gregorio et al. (1994) find evidence that demand shocks can be important in explaining deviations from PPP. As well, Froot and Rogoff (1991) find that government spending can be an important explanatory variable for real exchange rate movements of several European countries in the EMS.

where $\hat{T}_t \equiv \ln P_{F,t}/P_{H,t}$ and where $b \equiv \varphi\gamma + (1-\gamma)\rho^{-1}$; η–the inverse of the elasticity of labor supply– is defined as $\eta \equiv \bar{V}_{ll}\bar{L}/\bar{V}_l$, ρ–the risk-aversion coefficient– is given by $\rho \equiv \bar{U}_{cc}\bar{C}/\bar{U}_c$ and an index R denotes the difference between the home and the foreign respective variable. Under perfect labor mobility, $\phi \to \infty$, we obtain, by using (4.12) and (4.13), that

$$\begin{aligned}\ln P_{N,t}/P^*_{N,t} &= \frac{(\theta-1)\gamma\eta}{(1+\theta\gamma\eta+\eta b(1-\gamma))}\hat{A}^R_{T,t} - \frac{[1+\theta\gamma\eta+\eta(1-\gamma)]}{(1+\theta\gamma\eta+\eta b(1-\gamma))}\hat{A}^R_{N,t} + \\ &\quad -\frac{\theta\gamma\eta}{(1+\theta\gamma\eta+\eta b(1-\gamma))}\hat{\mu}^R_{T,t} + \frac{\eta(1-\gamma)}{(1+\theta\gamma\eta+\eta b(1-\gamma))}\hat{G}^R_t \\ &\quad +\frac{(1+\theta\gamma\eta)}{(1+\theta\gamma\eta+\eta b(1-\gamma))}\hat{\mu}^R_{N,t},\end{aligned}$$

which shows that when the intratemporal elasticity of substitution, θ, is close to one then an increase in the productivity in the home traded sector does not increase in a significant way the non-traded good price and thus the consumer price in the home economy. Contrary to the Balassa-Samuelson result, a balanced productivity shock in all the sectors within a country creates a major reduction in the domestic consumer price. A relative increase in government spending in the domestic economy increases price differentials across countries. The reason is that government spending falls on non-traded goods and induces an increase in their prices. Mark-up shocks in the non-traded sector increase price differential; the opposite happens if they originate in the traded sector. A balanced mark-up shock in the domestic economy increases the price differential. In the limiting case in which domestic and foreign traded goods become perfectly substitutable (as in the Balassa-Samuelson hypothesis), i.e. θ goes to infinity, then the above solution collapses to

$$\ln P_{N,t}/P^*_{N,t} = \hat{A}^R_{T,t} - \hat{A}^R_{N,t} + \hat{\mu}^R_{N,t} - \hat{\mu}^R_{T,t}$$

which captures now the Balassa-Samuelson result augmented with mark-up shocks. Note that in this limiting case, demand shocks and balanced shocks do not matter.[15] Our results on the role of the elasticity of substitution between domestic and foreign traded goods in driving the Balassa-Samuelson effect are consistent with the findings of Duarte and Wolman (2003).

[15] As discussed in Froot and Rogoff (1995), balanced productivity shocks can matter when labor intensity differs across sectors.

Another illustrative case is when $\varphi = \theta = \rho = 1$. Under this assumption

$$\begin{aligned}\ln P_{N,t}/P^*_{N,t} &= -\frac{(\phi\eta+1)\gamma}{(\phi-1)(1+\eta)}\hat{\mu}^R_{T,t} - \hat{A}^R_{N,t} + \frac{(1+\phi\eta)(\phi(1-\gamma)-1)-(\phi-1)}{(\phi-1)\phi(1+\eta)}\hat{G}^R_t \\ &\quad + \frac{[\phi+(\phi\eta+1)\gamma-1]}{(\phi-1)(1+\eta)}\hat{\mu}^R_{N,t}\end{aligned}$$

in which the Balassa-Samuelson effect is absent and where we note that values of ϕ below one can change the sign of some of the responses as well as the magnitude.

We now go back to our more general model and calibrate it in order study which of the shocks in the model can be more relevant to explain price differential when prices are flexible.

Table 1: Calibration of the parameters

$\beta = 0.99$	Intertemporal discount factor in consumer preferences
$\rho = -\bar{U}_{cc}\bar{C}/\bar{U}_c = 2$	Risk aversion coefficient in consumer preferences
$\varphi = 0.44$	Elas. of substitution between traded and non-traded goods
$\theta = 1.5$	Elas. of substitution between domestic and foreign traded goods
$\sigma = 7.88$	Elas. of substitution across goods within a sector
$\gamma = 0.5$	Share of traded goods in the consumption bundle
$\eta = \bar{V}_{ll}\bar{L}/\bar{V}_l = 0.25$	Inverse of the Frisch elasticity of labor supply
$\lambda = \bar{f}'\bar{l}/\bar{f} = 0.75$	Labor share
$1-\tilde{\lambda} = -\bar{f}''\bar{l}/\bar{f}' = 0.25$	Curvature of the production function

Table 1 presents the calibration of the parameters. The coefficient of risk aversion in consumer preferences is set to 2 as in Stockman and Tesar (1995) to get an intertemporal elasticity of substitution equal to 0.5. From Stockman and Tesar (1995), we borrow also the elasticity between traded and non-traded goods, $\varphi = 0.44$, and the share of traded goods in the consumption basket, $\gamma = 0.5$. The intratemporal elasticity of substitution between home and foreign traded goods is set such that $\theta = 1.5$ as in Backus et al. (1995). Consistent with several microeconomic studies, the Frisch elasticity of labor supply, $1/\eta$, is set to 4 and the labor share is set such that $\lambda = 0.75$. The discount factor β is assumed to be 0.99 and the elasticity of substitution for goods within a sector, σ, is set to 7.88 to imply a 15% mark-up as in Rotemberg and Woodford (1997). In Table 2, we present the results. We are not calibrating the parameter ϕ which measures the degree of labor mobility across sectors. We discuss three possible cases, $\phi = \infty$ to capture the perfect labor mobility case, an intermediate case of $\phi = 4.5$ and a case of low elasticity below 1, $\phi = 0.5$. In our model, when prices are flexible, the real variables of the model inherits the stochastic properties of the shocks. There is no intrinsic persistence in the model.[16] Our table can then capture short- and long-run components of consumer price differentials. In particular in table 2, we analyze the response of the consumer price differential and the terms of trade to a 1% increase of the shocks that are in the model. We also describe the response of terms of trade to capture the dimension through which movements in terms of trade absorb the usual Balassa-Samuelson effect. And indeed, focusing on the perfect labor mobility case, we see that a 1% increase in the traded-sector productivity in the home country does increase the price differential only of 2 basis point, consistent with the results of Duarte and Wolman (2003). The terms of trade worsen to absorb most of the shock. Instead an increase in productivity in the non-traded sector and a balanced increase in productivity in the domestic economy lower the domestic consumer price with respect to the foreign. In the same way, terms-of-trade movements absorb mark-up shocks in the traded-sector with negligible spillovers on consumer prices, while mark-up shocks in the non-traded sector produce increases in prices. Government-purchase shocks that affect demand of non-traded goods increase the non-tradeable goods price and thus the overall consumer price level. With intermediate values of the labor-mobility parameter, the picture does no change much. On the opposite, when ϕ goes below the unitary value, most of the responses are amplified and some change sign. Most important, the impact of government-spending shocks is of larger magnitude.

[16]By assuming incomplete financial market is instead possible to introduce persistent behavior of the price differential through a unit root, as shown in Rogoff (1992).

Table 2: Price Differential and Terms of Trade under Flexible-Price

	$\phi \to \infty$		$\phi = 4.5$		$\phi = 0.5$	
1% shock to:	$\ln P_t/P_t^*$	$\hat{T}_t$	$\ln P_t/P_t^*$	$\hat{T}_t$	$\ln P_t/P_t^*$	$\hat{T}_t$
$\hat{A}_T^R$	0.02	0.85	0.04	0.89	-0.21	0.55
$\hat{A}_N^R$	-0.60	0.04	-0.55	0.09	-0.98	-0.46
$\hat{A}_T^R$ and $\hat{A}_N^R$	-0.58	0.89	-0.51	0.98	-1.19	0.09
$\hat{\mu}_T^R$	-0.05	-0.57	-0.13	-0.67	0.64	0.35
$\hat{\mu}_N^R$	0.40	0.03	0.44	0.08	0.07	-0.40
$\hat{\mu}_T^R$ and $\hat{\mu}_N^R$	0.35	-0.54	0.31	-0.59	0.71	-0.05
$\hat{G}_t^R$	0.19	-0.07	0.11	-0.18	0.91	0.86

5 Nominal rigidities and price level differentials

While it provides a useful benchmark, there are obvious limitations of the flexible-price model. First, price differentials under flexible prices can arise only if there are asymmetric shocks across countries. This result hinges on the fact that we have imposed symmetry on the model's steady state. Allowing for asymmetries in the steady-state position of countries, like for example different labor shares, would imply that balanced shocks to the whole area would have an effect on the price differential. Second, under flexible prices persistence stems only from the intrinsic persistence of the exogenous shock process, although adding capital accumulation and incomplete financial markets would create additional persistence. Finally, there is no role for monetary policy to affect price differentials.

We show now that introducing sticky prices in the model can account for these three deficiencies: balanced shocks in the whole area create price and inflation dispersion; the model generates persistence; and monetary policy will matter for the determination of persistence and price dispersion. These three aspects of the model are shown below, after introducing the model's log-linear equations under sticky prices.

In a log-linear approximations to the structural equilibrium conditions (in particular equations (3.8), (3.9), (3.10) and (3.11) and the respective conditions for the foreign country) we obtain four aggregate supply (AS) equations for the respective traded and non-traded sector in each economy. These AS equations are familiar forms to the standard New-Keynesian AS equations of the closed-economy models of Galí and Gertler (1999) and Sbordone (2001). The inflation rate in each sector will depend on the real marginal cost in the sector and on the discounted value of the expected future sectoral inflation rate as follows

$$\pi_{H,t} = k_T(\widehat{mc}_{T,t}) + \beta E_t \pi_{H,t+1}$$

$$\pi_{N,t} = k_N(\widehat{mc}_{N,t}) + \beta E_t \pi_{N,t+1}$$

$$\pi^*_{F,t} = k^*_T(\widehat{mc}^*_{T,t}) + \beta E_t \pi^*_{F,t+1}$$

$$\pi^*_{N,t} = k^*_N(\widehat{mc}^*_{N,t}) + \beta E_t \pi^*_{N,t+1}$$

where we have defined $\pi_{H,t} = \ln P_{H,t}/P_{H,t-1}$, $\pi_{N,t} = \ln P_{N,t}/P_{N,t-1}$, $\pi^*_{F,t} = \ln P_{F,t}/P_{F,t-1}$, $\pi^*_{N,t} = \ln P^*_{N,t}/P^*_{N,t-1}$ and $k_j \equiv \frac{1-\alpha_j}{\alpha_j}\frac{1-\alpha_j\beta}{1+\frac{\vartheta\sigma}{\lambda}}$ where α_j can assume different values across sectors and countries (generically we have α_T, α_N, α^*_T, α^*_N) and $\vartheta \equiv 1-\tilde{\lambda}-\frac{1}{\upsilon}$. Moreover the deviations of real marginal costs from the steady-state are given by

$$\widehat{mc}_{T,t} = \mu_{T,t} + (1-\gamma)\hat{T}_{N,t} + \frac{1}{2}\hat{T}_t + \eta\hat{L}_t + \rho\hat{C}_t - \frac{1}{\phi}(\hat{l}_{T,t} - \hat{L}_t) - \hat{A}_{T,t} + (1-\tilde{\lambda})\hat{l}_{T,t},$$

$$\widehat{mc}_{N,t} = \mu_{N,t} - \gamma\hat{T}_{N,t} + \eta\hat{L}_t + \rho\hat{C}_t - \frac{1}{\phi}(\hat{l}_{N,t} - \hat{L}_t) - \hat{A}_{N,t} + (1-\tilde{\lambda})\hat{l}_{N,t},$$

$$\widehat{mc}^*_{T,t} = \mu^*_{T,t} + (1-\gamma)\hat{T}^*_{N,t} - \frac{1}{2}\hat{T}_t + \eta\hat{L}^*_t + \rho\hat{C}^*_t - \frac{1}{\phi}(\hat{l}^*_{T,t} - \hat{L}^*_t) - \hat{A}^*_{T,t} + (1-\tilde{\lambda})\hat{l}^*_{T,t},$$

$$\widehat{mc}^*_{N,t} = \mu^*_{N,t} - \gamma\hat{T}^*_{N,t} + \eta\hat{L}^*_t + \rho\hat{C}^*_t - \frac{1}{\phi}(\hat{l}^*_{N,t} - \hat{L}^*_t) - \hat{A}^*_{N,t} + (1-\tilde{\lambda})\hat{l}^*_{N,t}.$$

We can manipulate the above aggregate supply equations to obtain the following form

$$\pi_{H,t} = k_H[\delta_{T,1}\hat{T}_{N,t} + \delta_{T,2}\hat{T}_t + \delta_{T,3}\hat{T}^R_{N,t} + (\tilde{\eta}+\rho)\hat{Y}^w_t + \delta'_{T,\xi}\xi_t] + \beta E_t\pi_{H,t+1}, \quad (5.14)$$

$$\pi_{N,t} = k_N[\delta_{N,1}\hat{T}_{N,t} + \delta_{N,2}\hat{T}_t + \delta_{N,3}\hat{T}^R_{N,t} + (\tilde{\eta}+\rho)\hat{Y}^w_t + \delta'_{N,\xi}\xi_t] + \beta E_t\pi_{N,t+1}, \quad (5.15)$$

$$\pi^*_{F,t} = k^*_F[\delta^*_{T,1}\hat{T}^*_{N,t} + \delta^*_{T,2}\hat{T}_t + \delta^*_{T,3}\hat{T}^R_{N,t} + (\tilde{\eta}+\rho)\hat{Y}^w_t + \delta^{*\prime}_{T,\xi}\xi_t] + \beta E_t\pi^*_{F,t+1}, \quad (5.16)$$

$$\pi^*_{N,t} = k^*_N[\delta^*_{N,1}\hat{T}^*_{N,t} + \delta^*_{N,2}\hat{T}_t + \delta^*_{N,3}\hat{T}^R_{N,t} + (\tilde{\eta}+\rho)\hat{Y}^w_t + \delta^{*\prime}_{N,\xi}\xi_t] + \beta E_t\pi^*_{N,t+1}, \quad (5.17)$$

where the coefficients δ are functions of the parameters of the model and ξ_t is a vector of all the shocks in the model. Moreover we note that

$$\hat{T}_{N,t} = \hat{T}_{N,t-1} + \pi_{N,t} - (\frac{1}{2}\pi_{H,t} + \frac{1}{2}\pi^*_{F,t}) \quad (5.18)$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t} \quad (5.19)$$

$$\hat{T}_{N,t} - \hat{T}^*_{N,t} = (\hat{T}_{N,t-1} - \hat{T}^*_{N,t-1}) + \pi_{N,t} - \pi^*_{N,t} \quad (5.20)$$

5.1 Monetary policy shocks and price dispersion

Our first objective is to explore the importance of monetary policy shock in explaining consumer-price differentials. There is some evidence in the VAR literature that monetary policy shocks can be an important source of movements in the real exchange rate as discussed in Rogers (1999).[17] We analyze the following issue. Consider the common monetary policymaker in the currency area that moves its instrument of policy in a discretionary way, how much price and inflation dispersion is going to be generated? In our context, we interpret a monetary policy shock as it is usually done in the VAR literature. We consider a one-time negative shock to the instrument of policy, the short-term nominal interest rate. This shock produces usually an hump-shaped positive response of output. We use the system of equations (5.14) to (5.20) to determine the responses of relative prices and inflation rates to this hump-shaped positive response of output.[18] To address this issue, we can then write (5.14) to (5.20) as a system of the form

$$E_t \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} k_{t+1} \\ k_t \end{bmatrix} = \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} \begin{bmatrix} k_t \\ k_{t-1} \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} F_t$$

where $k_t = [\hat{T}_{N,t} - \hat{T}^*_{N,t} \;\; \hat{T}_t \;\; \hat{T}_{N,t}]$ and $F_t = [\hat{Y}^w_t \; \xi_t]$ and A_j, B_j, C_j are matrices. Provided there are 3 stable eigenvalues of the above system, we can solve it obtaining the law of motion of the state variables as

$$k_t = -V_1^{-1}V_2 k_{t-1} - V_1^{-1}\Lambda^{-1}E_t\sum_{T=t}^{\infty}\Lambda^{-(T-t)}V\tilde{C}F_T \quad (5.21)$$

[17]See also Benigno (2004) and Chari et al. (2002) on the issue whether a model with price rigidities and local currency pricing can account for monetary policy shocks to be important in capturing the volatility and persistence of real exchange rate

[18]This analysis is similar to Woodford (2003)'s analysis on the persistence of real wages after monetary policy shocks in sticky wages and prices model.

where $V = [V_1\ V_2]$ is the matrix of left eigenvectors associated with the unstable eigenvalues, where V_1 and V_2 are 3 by 3 matrices, Λ is a 3 by 3 diagonal matrices which contains the unstable eigenvalues and $\tilde{C} \equiv A^{-1}C$. In particular in the vector k_t we are interested to the variable $\hat{T}_{N,t} - \hat{T}^*_{N,t}$ which is proportional to the consumer price differential $\ln P_t/P^*_t$. The eigenvalues of the matrix $V_1^{-1}V_2$ characterize the intrinsic persistence that naturally arises when prices and relative prices are sticky across all sectors. In particular, under the assumption of symmetry in the rigidity across all the sectors in the area ($\alpha_T = \alpha_N = \alpha^*_T = \alpha^*_N$), a shock to the area output $\hat{Y}^w_t$ does not produce any consumer price dispersion. Asymmetries in the structures of the AS equations, across countries and sectors, are critical for monetary policy to have a role in creating price dispersion. This is at the same time good and bad news, since it means that a monetary policy shock, in the way we interpreted it, does not create any consumer price dispersion but on the other side monetary policy is limited in its role to correct any unwanted price dispersion that can arise following other disturbances. In our experiment, we feed in (5.21) the estimated output response to a monetary shock found by Smets and Wouters (2003).[19] As shown in the bottom chart of Figure 11, a discretionary decrease in the interest rate produces an hump-shaped response of output. Output in the union increases up to 0.4% after 5 quarters and then converges back to the steady state. To perform this experiment, we need to calibrate additional parameters on top of the ones used in Table 1. We set all the elasticities in the aggregators of labor to infinity to get perfect labor mobility across sectors within a country. As already mentioned it is crucial to calibrate the degrees of rigidities that characterize the Calvo's mechanism of the four sector. We refer here to some micro and macro studies. On the micro side, Le Bihan and Sevestre (2004), by analyzing CPI micro-data for France, find that the average duration between price adjustment in service sector is 9.66 months while for foods and goods is around 4 to 5 months. Costa Dias and Neves (2003) analyze consumer prices for Portugal and find relatively short "live" of posted prices, around 3 times a year on average with higher duration for services. Stahl (2004) shows that for producer prices in manufacturing sector the average duration can be of 9 months. On the opposite, there are studies that estimate AS equations for European countries, as Galí et al. (2001) and Benigno and Lopez-Salido (2002). They find that reasonable estimates of α for a country can be around 0.78.

In this work, we assume that one country has high rigidity and in particular a rigidity in the traded sector as in the work of Stahl (2004) equal to 0.67 (a duration of nine months) and a correspondingly rigidity in the non-traded sector such that

[19] However, the model estimated by Smets and Wouters (2003) does not include the possibility of heterogeneity across countries.

$\alpha_N = 0.84$ so that the overall rigidity in this country is around 0.78. In the other country, we assume that the average duration in the traded sector as in line with the work of Costa Dias and Neves equal to 5 months and that in the non-traded sector is as in Le Bihan and Sevestre (2004) equal to 9 months. Our benchmark I calibration is then: $\alpha_T = 0.67$, $\alpha_N = 0.84$, $\alpha_T^* = 0.37$, $\alpha_N^* = 0.67$. We also propose two other possible cases: in benchmark II, we assume $\alpha_T = 0.67$, $\alpha_N = 0.84$, $\alpha_T^* = 0.6$, $\alpha_N^* = 0.75$, in benchmark III we assume $\alpha_T = 0.7$, $\alpha_N = 0.4$, $\alpha_T^* = 0.67$, $\alpha_N^* = 0.37$. In benchmark II and III with respect to I the two countries are more similar in terms of rigidities; in benchmark II the rigidities are increased in both sectors of country F, in benchmark III they are reduced in both sectors of country H.

The results are presented in the first three charts of Figure 11, where the impulse responses of the log of the consumer-price differential and of the CPI inflation rates in both countries are plotted following the output shock. The impulse responses of the inflation rates are also consistent with VAR responses to a discretionary monetary policy shock. However, CPI inflation rate increases by a larger amount in the more flexible-price economy, but it is more persistent in the rigid economy. The overall picture that emerges is that the consumer-price differential is highly persistent. In the benchmark I case, the price differential has a peak after 10 quarters (i.e. after 5 quarters the peak of output). The differential reaches 8 basis point. This differential is of much smaller magnitude in the other two benchmarks, however the persistence is not altered by the different assumption. Overall a 'discretionary' monetary shock produces persistent price and inflation differentials although only of a small magnitude.

5.2 Nominal rigidities and price dispersion

Under flexible prices, consumer-price differentials inherit the stochastic properties of the shocks. There is no persistence other than the one implied by the shocks. We now investigate how the introduction of sticky prices affects the response to the shocks and whether it increases persistence.

The answer depends on the policy rule used by the common monetary policymaker as we will further investigate in the next section. In this section, we assume that the policymaker sets its policy in a way to target a weighted average of the CPI inflation rates of the two countries, with weights given by the economic size of each country, in our case 1/2 for each country. The monetary policymaker then sets the average CPI inflation rate in the area equal to zero as

$$\frac{1}{2}\pi_t + \frac{1}{2}\pi_t^* = 0$$

at all times. The second important assumption in this experiment is the one on the persistence of the shocks. We assume that there are three possible degrees of persistence. Indeed we assume that shocks are autoregressive of type AR(1) as

$$s_t = \rho s_{t-1} + \varepsilon_t$$

for a generic shock s_t where ρ can be either 0 (for a white-noise process), or 0.9 for a persistence process or 1 for a unit-root process.[20] We analyze impulse response functions to a one-time unitary increase (1% movement) in ε_t. The analysis of the response to a disturbance with zero correlation will be helpful to capture the persistence implicit in the model given by the combination of the stickiness of prices and policy rule; the analysis of more persistent process will be useful to understand how the persistence of the shock interacts with the persistence intrinsic in the model. Finally the analysis to permanent shocks can allow to investigate whether sticky prices are adding important transitional dynamics. In particular, the long-run response to a permanent shock exactly captures the flexible-price response, with the important caveat that short- and long-run are the same under flexible prices. Having in the same graph the flexible-price response would serve as an important benchmark for comparison.

Figure 12 presents the impulse responses of the consumer-price differential to productivity shocks. In the order (from the left to the right starting from the top), we analyze a one-time shock to the productivity in the home traded sector, to the productivity of the home non-traded sector, to a balanced increase in the productivity in both sectors in the home economy, to a balanced productivity increase in all the sectors of the area. As already discussed, a first presumption is that the stickiness of prices and, in particular, of relative prices can add persistence to the shock. And indeed this is the case when we focus on a white-noise shock. Under flexible prices the effect of this shock would disappear after one period, under sticky prices this happens after more or less 8 quarters. The second presumption is that sticky prices, while adding persistence, dampen the response to the shocks. Interestingly, this is not necessarily a feature of our model. Indeed, by inspecting the impulse response to a productivity shock in the home traded sector, we see that actually sticky prices can revert the sign of the response and sometimes magnify it. Indeed, with our parametrization, the flexible-price response (as in Table 2 and in the long-run response to a permanent shock) would require $\ln P/P^*$ to increase by 2 basis points. With sticky prices, we actually get a complete reversal of the Balassa-Samuelson effect with a relative magnification of the response which can

[20] In the case of a unit-root process, we are allowed to use our log-linear approximation provided the shock lasts for a finite period of time.

achieve 8 basis points when shocks are more persistent. The intuition for this result is that even the terms of trade is very slow in the adjustment when sectoral prices are sticky. We note that under a more persistent shock, the response changes sign after 20 quarters. Indeed, when the shock is permanent it should reach the long-run positive value of 2 basis points. The picture is different when we analyze a shock to productivity in the non-traded sector or a balanced productivity shock in the home economy. These two cases are similar since the shock in the non-traded sector dominates. Indeed, we find that the existence of sticky prices mutes the response to these shocks. In particular the short-run response is negative and below or around -0.1% compared to -0.6% of the flexible-price model. The adjustment towards the long-run equilibrium proceeds slowly as in the previous case. A further interesting question to address is whether area-wide shocks can have important consequences. We remind that under flexible-price these shocks have no effects on price differentials. We find that in the case of a balanced area-wide productivity shock, the deviations from full equalizations of the consumer price level are quantitatively insignificant.[21]

Figure 13 repeats the same analysis following mark-up shocks. The analysis is parallel to the case of productivity shocks with the appropriate qualifications. We find that there is a change in the sign of the short-run response for shocks in the traded sector, a dampening effect for non-traded and balanced shocks and the non-significant effect of area-wide mark-up shock.

Finally in Figure 14, we focus on demand shocks. Namely, we consider a one-time shock to government purchase in the home country and then a common shock to both countries. The response to an asymmetric government purchase shock shows again the dampening effect of sticky prices as well as the additional source of inertia implied. Most interesting, we find that an area-wide government purchase shock can have some non-negligible effect on price differential. Given our parametrization, a symmetric government-purchase shock lowers the consumer price in the home economy relative to the foreign, since the home economy is more rigid. Indeed, a demand shock increases prices by a larger amount in the more flexible-price country. This is why consumer prices in the home economy fall below foreign. As a difference from other area-wide shock, government-purchase shocks affect only the non-traded sector, so it is not surprising that they can create larger asymmetric responses of prices, even if they are common to both countries.

[21] The symmetry imposed in the steady-state obviously plays an important role in this result.

5.3 Monetary policy rules and price dispersion

In a previous section we have shown that a one-time monetary shock produces persistent but small price and inflation differentials. This section addresses the question of how the systematic component of the common monetary policy affects price dispersion and the persistence in responses to shocks. Benigno and López-Salido (2003) report the inflation differential across two regions of a model calibrated for the euro area and for a number of inflation targeting rules, finding that alternative targeting rules have a relatively minor impact on the size and persistence of inflation differentials. Output gap targeting rules however generate in their model a somewhat different pattern for the producer inflation differentials relative to inflation targeting and other rules.[22] In our model with a non-tradable goods sector in each region, systematic monetary policy is prone to have a more important bearing in shaping price dispersion and its persistence. To address this question, we extend the analysis of the previous section by considering four additional policy rules, as described in Table 3.

Table 3: Monetary policy rules

(1)	$\pi_t = \pi_t^*$
(2)	$\pi_t^W = \frac{1}{2}\pi_t + \frac{1}{2}\pi_t^* = 0$
(3)	$r_t = 0.8\ r_{t-1} + 0.2\ \{1.5\ \pi_t^W + 0.5\ y_t^W\}$
(4)	$r_t = 1.5\ \pi_t^W + 0.5\ y_t^W$
(5)	$r_t = r_{t-1} + 0.4\ E_t\{\pi_{t+1}^W\} + 0.4\ y_t^W$

Rule 1 equalises price levels across regions. This rule shows the cost of using monetary policy to stabilise relative prices in terms of area wide inflation and output variability. Rule 2 stabilises area wide inflation, as in the previous section. The three remaining rules are variations of a Taylor-type rule. Rules (3) to (5) are variations of a standard Taylor rule. In particular, rules (3) and (5) introduce different degrees of nominal interest rate inertia and relative weigths of inflation and output. Figures 15, 16 and 17 are the impulse reponses to a shock to home traded sector productivity, home non-traded sector productivity and governement purchases respectively. All shocks are transitory and are assumed to follow an AR(1) with autocorrelation of 0.9.

Comparing Figures 15 to 17, the form of systematic monetary policy has a strong impact on price dispersion and persistence, trivially so for rule (1) but also

[22] Benigno and López-Salido (2003) report impulse responses to terms of trade shocks in a model with only tradable goods and with a hybrid New-Keynesian Phillips curve in one of the regions. A direct comparison with our results is hence not possible.

for other rules. However, the impact of the rule on price dispersion dynamics varies considerably for the different shocks considered. For the case of the transitory shock to home non-trade productivity (Figure 15), the result in the previous Section of a negative impact on the home to foreign price level ratio under the rule of area-wide inflation stabilisation is reversed for all Taylor-type rules. Under the latter, the home traded sector productivity shock has a positive and more persistent effect on home price level relative to foreign, i.e. the Balassa-Samuelson effect is re-established and even amplified relative to the case of rule (2). It is also noted that introducing inertia in the monetary policy rule (i.e. comparing rules (3) and (4)) does not necessarily increase the dispersion and persistence of price levels: with inertial rule (3) price dispersion is somewhat lower and persistence similar than under the standard Taylor rule (3). Finally, it is noted that the rule suppresing inflation differentials (rule (1)) generates a persistently large output gap, although similar to the case of the area-wide inflation stabilisation rule.

Interestingly, a different picture results from the comparison of price dispersion dynamics for the different rules in the case of a home non-traded productivity shock (Figure 16). here all three Taylor-type rules produce a similar pattern in price disperison to the case of the area-wide inflation stabilisation rule. Figure 16 also shows a higher relative cost in terms of variability in inflation and output of the rule that equalises prices across regions. And again a different pattern results in the case of the transitory home governement spending shock (Figure 17). Here the Taylor-type rules do not reverse the sign of the ara-wide inflation stabilisation rule, but the magnitude and of price dispersion and its persistence under the former rules is considerable larger than for the case of the latter. It is also noted that for this shock the effect on the output gap of rules that stabilise area-wide inflation and inflation differentials is higher.

6 Conclusions

[to be added]

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7 Appendix

When prices are fully flexible, all firms within a sector set the same price. In the traded sector of country H, the first-order conditions imply

$$1 = \mu_{T,t} \frac{P_t}{P_{T,t}} \frac{P_{T,t}}{P_{H,t}} \frac{V_l(L_t)}{U_c(C_t)} \left(\frac{l_{T,t}}{L_t} \right)^{-\frac{1}{\phi}} \frac{1}{f'(l_{T,t}) A_{T,t}}, \tag{7.22}$$

in the traded sector of country F, we obtain that

$$1 = \mu^*_{T,t} \frac{P^*_t}{P^*_{T,t}} \frac{P^*_{T,t}}{P_{F,t}} \frac{V_l(L^*_t)}{U_c(C^*_t)} \left(\frac{l^*_{T,t}}{L^*_t} \right)^{-\frac{1}{\phi}} \frac{1}{f'(l^*_{T,t}) A_{T,t}}. \tag{7.23}$$

In the non-traded sectors, respectively in country H and F, we obtain that

$$1=\mu_{N,t}\frac{P_t}{P_{N,t}}\frac{V_l(L_t)}{U_c(C_t)}\left(\frac{l_{N,t}}{L_t}\right)^{-\frac{1}{\phi}}\frac{1}{f'(l_{N,t})A_{N,t}}, \tag{7.24}$$

$$1=\mu^*_{N,t}\frac{P^*_t}{P^*_{N,t}}\frac{V_l(L^*_t)}{U_c(C^*_t)}\left(\frac{l^*_{N,t}}{L^*_t}\right)^{-\frac{1}{\phi}}\frac{1}{f'(l^*_{N,t})A^*_{N,t}}. \tag{7.25}$$

We define $T\equiv P_F/P_H$, $T_N\equiv P_N/P_T$ and $T^*_N\equiv P^*_N/P^*_T$. Note that since the law of one price holds for traded goods it follows that $P_F=P^*_F$ and $P_H=P^*_H$. Using the definition of the price indexes, we obtain that

$$\left(\frac{P_{H,t}}{P_{T,t}}\right)^{\theta-1} = n+(1-n)T_t^{1-\theta}, \tag{7.26}$$

$$\left(\frac{P_{F,t}}{P_{T,t}}\right)^{\theta-1} = nT_t^{\theta-1}+(1-n), \tag{7.27}$$

$$\left(\frac{P_{H,t}}{P^*_{T,t}}\right)^{\theta-1} = n^*+(1-n^*)T_t^{1-\theta}, \tag{7.28}$$

$$\left(\frac{P_{F,t}}{P^*_{T,t}}\right)^{\theta-1} = n^*T_t^{\theta-1}+(1-n^*), \tag{7.29}$$

$$\left(\frac{P_{N,t}}{P_t}\right)^{\varphi-1} = \omega T_{N,t}^{\varphi-1}+(1-\omega), \tag{7.30}$$

$$\left(\frac{P_{T,t}}{P_t}\right)^{\varphi-1} = \omega+(1-\omega)T_{N,t}^{1-\varphi}, \tag{7.31}$$

$$\left(\frac{P^*_{N,t}}{P^*_t}\right)^{\varphi-1} = \omega^*T_{N,t}^{*\varphi-1}+(1-\omega^*), \tag{7.32}$$

$$\left(\frac{P^*_{T,t}}{P^*_t}\right)^{\varphi-1} = \omega^*+(1-\omega^*)T_{N,t}^{*1-\varphi}. \tag{7.33}$$

We finally note that (7.31) and (7.33) imply

$$\frac{P_t}{P^*_t}=\frac{P_{T,t}}{P^*_{T,t}}\frac{[\omega+(1-\omega)T_{N,t}^{1-\varphi}]^{\frac{1}{1-\varphi}}}{[\omega^*+(1-\omega^*)T_{N,t}^{*1-\varphi}]^{\frac{1}{1-\varphi}}}$$

moreover (7.26) and (7.28) imply

$$\frac{P_{T,t}}{P^*_{T,t}} = \frac{[n + (1-n)T_t^{1-\theta}]^{\frac{1}{1-\theta}}}{[n^* + (1-n^*)T_t^{1-\theta}]^{\frac{1}{1-\theta}}}. \tag{7.34}$$

We can finally write that

$$\frac{P_t}{P^*_t} = \frac{[n + (1-n)T_t^{1-\theta}]^{\frac{1}{1-\theta}}}{[n^* + (1-n^*)T_t^{1-\theta}]^{\frac{1}{1-\theta}}} \frac{[\omega + (1-\omega)T_{N,t}^{1-\varphi}]^{\frac{1}{1-\varphi}}}{[\omega^* + (1-\omega^*)T_{N,t}^{*1-\varphi}]^{\frac{1}{1-\varphi}}}$$

which can be substituted in (3.5) to obtain

$$\frac{U_C(C_t)}{U_C(C^*_t)} = \frac{[n + (1-n)T_t^{1-\theta}]^{\frac{1}{1-\theta}}}{[n^* + (1-n^*)T_t^{1-\theta}]^{\frac{1}{1-\theta}}} \frac{[\omega + (1-\omega)T_{N,t}^{1-\varphi}]^{\frac{1}{1-\varphi}}}{[\omega^* + (1-\omega^*)T_{N,t}^{*1-\varphi}]^{\frac{1}{1-\varphi}}}. \tag{7.35}$$

Under flexible prices, we note that $L_{T,t} = \gamma l_{T,t}$ and $L^*_{T,t} = \gamma^* l^*_{T,t}$ and $L_{N,t} = (1-\gamma) l_{N,t}$ and $L^*_{N,t} = (1-\gamma^*) l^*_{N,t}$ so that

$$L_t = \left[\gamma (l_{T,t})^{\frac{\phi-1}{\phi}} + (1-\gamma)(l_{N,t})^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{7.36}$$

$$L^*_t = \left[\gamma (l^*_{T,t})^{\frac{\phi-1}{\phi}} + (1-\gamma)(l^*_{N,t})^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{7.37}$$

The sectorial demand of goods are given by

$$y_{T,t} = \frac{n\omega}{\gamma}\left(\frac{P_H}{P_T}\right)^{-\theta}\left(\frac{P_T}{P}\right)^{-\varphi} C + \frac{n^*\omega^*}{\gamma}\left(\frac{P_H}{P^*_T}\right)^{-\theta}\left(\frac{P^*_T}{P^*}\right)^{-\varphi} C^* \tag{7.38}$$

$$y^*_{T,t} = \frac{(1-n)\omega}{\gamma^*}\left(\frac{P_F}{P_T}\right)^{-\theta}\left(\frac{P_T}{P}\right)^{-\varphi} C + \frac{(1-n^*)\omega^*}{\gamma^*}\left(\frac{P_F}{P^*_T}\right)^{-\theta}\left(\frac{P^*_T}{P^*}\right)^{-\varphi} C^* \tag{7.39}$$

$$y_{N,t} = \frac{1-\omega}{1-\gamma}\left(\frac{P_{N,t}}{P_t}\right)^{-\varphi} C_t + G_t \tag{7.40}$$

$$y^*_{N,t} = \frac{1-\omega^*}{1-\gamma^*}\left(\frac{P^*_{N,t}}{P^*_t}\right)^{-\varphi} C^*_t + G^*_t. \tag{7.41}$$

while

$$l_{T,t} = f^{-1}\left(\frac{y_{T,t}}{A_{T,t}}\right) \tag{7.42}$$

$$l^*_{T,t} = f^{-1}\left(\frac{y^*_{T,t}}{A^*_{T,t}}\right) \tag{7.43}$$

$$l_{N,t} = f^{-1}\left(\frac{y_{N,t}}{A_{N,t}}\right) \tag{7.44}$$

$$l^*_{N,t} = f^{-1}\left(\frac{y^*_{N,t}}{A^*_{N,t}}\right) \tag{7.45}$$

Equations (7.22), (7.23), (7.24), (7.25), (7.26), (7.27), (7.28), (7.29), (7.30), (7.31), (7.32), (7.33), (7.35), (7.36), (7.37), (7.38), (7.39), (7.40), (7.41), (7.42), (7.43), (7.44), (7.45) should be solved for the variables $\{\frac{P_t}{P_{T,t}}, \frac{P_{T,t}}{P_{H,t}}, L_t, C_t, l_{T,t}, \frac{P^*_t}{P^*_{T,t}}, \frac{P^*_{T,t}}{P_{F,t}}, L^*_t, C^*_t, l^*_{T,t}, \frac{P_t}{P_{N,t}}, \frac{P^*_t}{P^*_{N,t}}, l^*_{N,t}, l_{N,t}, T_t, T_{N,t}, T^*_{N,t}, \frac{P_{F,t}}{P_{T,t}}, \frac{P_{F,t}}{P^*_{T,t}}, y_{T,t}, y^*_{T,t}, y_{N,t}, y^*_{N,t}\}$ given the processes $\{\mu_{T,t}, \mu^*_{T,t}, \mu_{N,t}, \mu^*_{N,t}, A_{T,t}, A^*_{T,t}, A_{N,t}, A^*_{N,t}, G_t, G^*_t\}$.

We approximate the previous solution around a deterministic steady state in which the process $\{\mu_{T,t}, \mu^*_{T,t}, \mu_{N,t}, \mu^*_{N,t}, A_{T,t}, A^*_{T,t}, A_{N,t}, A^*_{N,t}, G_t, G^*_t\}$ are such that $\bar{\mu}_H = \bar{\mu}^*_H = \bar{\mu}_F = \bar{\mu}^*_F = \bar{\mu}_N = \bar{\mu}^*_N = \bar{\mu}$ and $\bar{A}_T = \bar{A}^*_T = \bar{A}_N = \bar{A}^*_N = \bar{A}$, $\bar{G} = \bar{G}^* = 0$. We assume for simplicity that $\gamma = \gamma^*$, $n = n^* = 1/2$, $\omega = \omega^* = \gamma$. We show that there is a steady-state in which $\frac{P_t}{P_{T,t}} = \frac{P_{T,t}}{P_{H,t}} = \frac{P^*_t}{P^*_{T,t}} = \frac{P^*_{T,t}}{P_{F,t}} = \frac{P_t}{P_{N,t}} = \frac{P^*_t}{P^*_{N,t}} = \frac{P_{F,t}}{P_{T,t}} = \frac{P_{F,t}}{P^*_{T,t}} = T_t = T_{N,t} = T^*_{N,t} = 1$, $L_t = l_{T,t} = l_{N,t} = \bar{L}$ and $L^*_t = l^*_{T,t} = l^*_{N,t} = \bar{L}^*$, $y_{T,t} = y^*_{T,t} = y_{N,t} = y^*_{N,t} = C_t = C^*_t = \bar{C}$. It is clear that (7.26), (7.27), (7.28), (7.29), (7.30), (7.31), (7.32), (7.33), (7.35), (7.36), (7.37), (7.38), (7.39), (7.40), (7.41) are satisfied by the above steady-state conditions. Moreover (7.42), (7.43), (7.44), (7.45) imply that

$$\bar{L} = f^{-1}\left(\frac{\bar{C}}{\bar{A}}\right) = \bar{L}^*, \tag{7.46}$$

while (7.22), (7.23), (7.24), (7.25) imply

$$1 = \bar{\mu}\frac{V_l(\bar{L})}{U_c(\bar{C})}\frac{1}{f'(\bar{L})\bar{A}}. \tag{7.47}$$

Equations (7.46) and (7.47) can be solved for $\bar{L}$ and $\bar{C}$ given $\bar{A}$. Under standard preference specifications the values $\bar{L}$ and $\bar{C}$ exist uniquely.

Maintaining the assumption that $\gamma = \gamma^*$, $n = n^* = 1/2$, $\omega = \omega^* = \gamma$ we study the solution of the model in a log-linear approximation of the structural equations for small perturbations of the exogenous shocks around the steady state outlined above. We start by log-linearizing equations (7.26) to (7.33). We obtain that

$$\widehat{\frac{P_{H,t}}{P_{T,t}}} = -\frac{1}{2}\hat{T}_t = -\widehat{\frac{P_{F,t}}{P_{T,t}}}$$

$$\frac{\widehat{P_{H,t}}}{P^*_{T,t}} = -\frac{1}{2}\hat{T}_t = -\frac{\widehat{P_{F,t}}}{P^*_{T,t}}$$

$$\frac{\widehat{P_{N,t}}}{P_t} = \gamma\hat{T}_{N,t} = -\frac{\gamma}{1-\gamma}\frac{\widehat{P_{T,t}}}{P_t}$$

$$\frac{\widehat{P^*_{N,t}}}{P^*_t} = \gamma\hat{T}^*_{N,t} = -\frac{\gamma}{1-\gamma}\frac{\widehat{P^*_{T,t}}}{P^*_t}$$

Equation (7.35) yields to

$$-\rho(\hat{C}_t - \hat{C}^*_t) = (1-\gamma)(\hat{T}_{N,t} - \hat{T}^*_{N,t}). \tag{7.48}$$

A log-linear approximation to (7.22), (7.23), (7.24), (7.25) yields to

$$\hat{\mu}_{T,t} + (1-\gamma)\hat{T}_{N,t} + \frac{1}{2}\hat{T}_t + \eta\hat{L}_t + \rho\hat{C}_t - \frac{1}{\phi}(\hat{l}_{T,t} - \hat{L}_t) - \hat{A}_{T,t} + (1-\tilde{\lambda})\hat{l}_{T,t} = 0 \tag{7.49}$$

$$\hat{\mu}^*_{T,t} + (1-\gamma)\hat{T}^*_{N,t} - \frac{1}{2}\hat{T}^*_t + \eta\hat{L}^*_t + \rho\hat{C}^*_t - \frac{1}{\phi}(\hat{l}^*_{T,t} - \hat{L}^*_t) - \hat{A}^*_{T,t} + (1-\tilde{\lambda})\hat{l}^*_{T,t} = 0 \tag{7.50}$$

$$\hat{\mu}_{N,t} - \gamma\hat{T}_{N,t} + \eta\hat{L}_t + \rho\hat{C}_t - \frac{1}{\phi}(\hat{l}_{N,t} - \hat{L}_t) - \hat{A}_{N,t} + (1-\tilde{\lambda})\hat{l}_{N,t} = 0 \tag{7.51}$$

$$\hat{\mu}^*_{N,t} - \gamma\hat{T}^*_{N,t} + \eta\hat{L}^*_t + \rho\hat{C}^*_t - \frac{1}{\phi}(\hat{l}^*_{N,t} - \hat{L}^*_t) - \hat{A}^*_{N,t} + (1-\tilde{\lambda})\hat{l}^*_{N,t} = 0 \tag{7.52}$$

where $\eta \equiv \bar{V}_{ll}\bar{l}/\bar{V}_l$, $\rho \equiv -\bar{U}_{cc}\bar{C}/\bar{U}_c$, $(1-\tilde{\lambda}) \equiv -\bar{f}''\bar{l}/\bar{f}'$ with $\tilde{\lambda} \leq 1$. A log-linear approximation to equations (7.36) and (7.37) yields to

$$\hat{L}_t = \gamma\hat{l}_{T,t} + (1-\gamma)\hat{l}_{N,t}, \tag{7.53}$$

$$\hat{L}^*_t = \gamma\hat{l}^*_{T,t} + (1-\gamma)\hat{l}^*_{N,t}. \tag{7.54}$$

A log-linear approximation to equations (7.38), (7.39), (7.40), (7.41) yields

$$\hat{y}_{T,t} = \frac{\theta}{2}\hat{T}_t + \frac{\varphi}{2}(1-\gamma)\hat{T}_{N,t} + \frac{1}{2}\hat{C}_t + \frac{\varphi}{2}(1-\gamma)\hat{T}^*_{N,t} + \frac{1}{2}\hat{C}^*_t, \tag{7.55}$$

$$\hat{y}^*_{T,t} = -\frac{\theta}{2}\hat{T}_t + \frac{\varphi}{2}(1-\gamma)\hat{T}_{N,t} + \frac{1}{2}\hat{C}_t + \frac{\varphi}{2}(1-\gamma)\hat{T}^*_{N,t} + \frac{1}{2}\hat{C}^*_t, \tag{7.56}$$

$$\hat{y}_{N,t} = -\varphi\gamma\hat{T}_{N,t} + \hat{C}_t + \hat{G}_t, \tag{7.57}$$

$$\hat{y}^*_{N,t} = -\varphi\gamma\hat{T}^*_{N,t} + \hat{C}^*_t + \hat{G}^*_t, \tag{7.58}$$

where $\hat{G}_t = G_t/\bar{Y}$ and $\hat{G}^*_t = G^*_t/\bar{Y}$. Finally a log-linear approximation to (7.42), (7.43), (7.44), (7.45) imply

$$\lambda\hat{l}_{T,t} = \hat{y}_{T,t} - \hat{A}_{T,t} \tag{7.59}$$

$$\lambda \hat{l}^*_{T,t} = \hat{y}^*_{T,t} - \hat{A}^*_{T,t} \quad (7.60)$$

$$\lambda \hat{l}_{N,t} = \hat{y}_{N,t} - \hat{A}_{N,t} \quad (7.61)$$

$$\lambda \hat{l}^*_{N,t} = \hat{y}^*_{N,t} - \hat{A}^*_{N,t} \quad (7.62)$$

where $\lambda \equiv \bar{f}'\bar{l}/\bar{f}$. with $0 < \lambda \leq 1$.

We first solve the model for the aggregate variables and then we solve for relative prices and consumption. First, we take a weighted average of equations (7.49) and (7.51) with weights γ and $1-\gamma$ and obtain

$$\gamma\hat{\mu}_{T,t} + (1-\gamma)\hat{\mu}_{N,t} + \frac{\gamma}{2}\hat{T}_t + \left(\eta + 1 - \tilde{\lambda}\right)\hat{L}_t + \rho\hat{C}_t - \gamma\hat{A}_{T,t} - (1-\gamma)\hat{A}_{N,t} = 0$$

and take a weighted average of equations (7.50) and (7.52) to obtain

$$\gamma\hat{\mu}^*_{T,t} + (1-\gamma)\hat{\mu}^*_{N,t} - \frac{\gamma}{2}\hat{T}_t + \left(\eta + 1 - \tilde{\lambda}\right)\hat{L}^*_t + \rho\hat{C}^*_t - \gamma\hat{A}^*_{T,t} - (1-\gamma)\hat{A}^*_{N,t} = 0$$

We can define $\hat{\mu}_t \equiv \gamma\hat{\mu}_{T,t} + (1-\gamma)\hat{\mu}_{N,t}$, $\hat{\mu}^*_t \equiv \gamma\hat{\mu}^*_{T,t} + (1-\gamma)\hat{\mu}^*_{N,t}$, $\hat{A}_t \equiv \gamma\hat{A}_{T,t} + (1-\gamma)\hat{A}_{N,t}$, $\hat{A}^*_t \equiv \gamma\hat{A}^*_{T,t} + (1-\gamma)\hat{A}^*_{N,t}$ and take a weighted average of the above conditions with weights $1/2$ and obtain

$$\hat{\mu}^W_t + \left(\eta + 1 - \tilde{\lambda}\right)\hat{L}^W_t + \rho\hat{C}^W_t - \hat{A}^W_t = 0, \quad (7.63)$$

where an index W denotes a weigthed average of home and foreign variables. We can as well take a weighted average of (7.55) to (7.58), with weights $\frac{\gamma}{2}$, $\frac{\gamma}{2}$, $\frac{1-\gamma}{2}$, $\frac{1-\gamma}{2}$

$$\frac{\gamma}{2}\hat{y}_{T,t} + \frac{\gamma}{2}\hat{y}^*_{T,t} + \frac{(1-\gamma)}{2}\hat{y}^*_{N,t} + \frac{(1-\gamma)}{2}\hat{y}_{N,t} = \hat{C}^W_t + (1-\gamma)\hat{G}^W_t.$$

We can use (7.59) to (7.62) to obtain

$$\hat{C}^W_t + (1-\gamma)\hat{G}^W_t = \lambda\hat{L}^W_t + \hat{A}^W_t. \quad (7.64)$$

We can use (7.63) and (7.64) to obtain

$$\hat{L}^W_t = \frac{1-\rho}{\lambda(\tilde{\eta}+\rho)}\hat{A}^W_t + \frac{\rho(1-\gamma)}{\lambda(\tilde{\eta}+\rho)}\hat{G}^W_t - \frac{1}{\lambda(\tilde{\eta}+\rho)}\hat{\mu}^W_t$$

and

$$\hat{C}^W_t = \frac{\tilde{\eta}+1}{(\tilde{\eta}+\rho)}\hat{A}^W_t - \frac{\tilde{\eta}(1-\gamma)}{(\tilde{\eta}+\rho)}\hat{G}^W_t - \frac{1}{(\tilde{\eta}+\rho)}\hat{\mu}^W_t$$

$$\hat{Y}^W_t = \frac{\tilde{\eta}+1}{(\tilde{\eta}+\rho)}\hat{A}^W_t + \frac{\rho(1-\gamma)}{(\tilde{\eta}+\rho)}\hat{G}^W_t - \frac{1}{(\tilde{\eta}+\rho)}\hat{\mu}^W_t$$

where we have defined $\tilde{\eta} \equiv (1 - \tilde{\lambda} + \eta)/\lambda$. To solve for the difference variables in the model, we first consider equation (7.48) which can be written as

$$-\rho \hat{C}_t^R = (1-\gamma)\hat{T}_{N,t}^R \tag{7.65}$$

where a variable with a superscript R has been defined as the difference between the respective home and foreign variable.

We can take the difference between equations (7.49) and (7.50) to obtain

$$\hat{\mu}_{T,t}^R + (1-\gamma)\hat{T}_{N,t}^R + \hat{T}_t + c_1(1-\gamma)\hat{l}_{N,t}^R + \rho\hat{C}_t^R + (\gamma c_1 + c_2)\hat{l}_{T,t}^R - \hat{A}_{T,t}^R = 0$$

where we have defined

$$c_1 \equiv \left(\eta + \frac{1}{\phi}\right)$$

$$c_2 \equiv (1-\tilde{\lambda}) - \frac{1}{\phi}$$

Using (7.65) we can obtain

$$\hat{\mu}_{T,t}^R + \hat{T}_t + c_1(1-\gamma)\hat{l}_{N,t}^R + (\gamma c_1 + c_2)\hat{l}_{T,t}^R - \hat{A}_{T,t}^R = 0. \tag{7.66}$$

We can take the difference between (7.51) and (7.52) to obtain

$$\hat{\mu}_{N,t}^R - \gamma\hat{T}_{N,t}^R + \gamma c_1\hat{l}_{T,t}^R + \rho\hat{C}_t^R - \hat{A}_{N,t}^R + [(1-\gamma)c_1 + c_2]\hat{l}_{N,t} = 0,$$

which can be written as

$$\hat{\mu}_{N,t}^R - \hat{T}_{N,t}^R + \gamma c_1\hat{l}_{T,t}^R - \hat{A}_{N,t}^R + [(1-\gamma)c_1 + c_2]\hat{l}_{N,t} = 0, \tag{7.67}$$

We can use equations (7.66) and (7.67) to solve for $\hat{l}_{T,t}^R$ to obtain

$$\lambda\tilde{\eta}c_3\hat{l}_{T,t}^R = -c_4\hat{\mu}_{T,t}^R - c_4\hat{T}_t + c_4\hat{A}_{T,t}^R + c_1(1-\gamma)\hat{\mu}_{N,t}^R - c_1(1-\gamma)\hat{T}_{N,t}^R - c_1(1-\gamma)\hat{A}_{N,t}^R \tag{7.68}$$

where

$$c_3 \equiv (1-\tilde{\lambda}) - \frac{1}{\phi}$$

and

$$c_4 \equiv (1-\gamma)c_1 + c_2$$

We can also use (7.66) and (7.67) to solve for $\hat{l}_{N,t}^R$ to obtain

$$\lambda\tilde{\eta}\xi_\phi\hat{l}_{N,t}^R = \gamma c_1\hat{\mu}_{T,t}^R + \gamma c_1\hat{T}_t - \gamma c_1\hat{A}_{T,t}^R - c_5\hat{\mu}_{N,t}^R + c_5\hat{T}_{N,t}^R + c_5\hat{A}_{N,t}^R \tag{7.69}$$

where

$$c_5 \equiv \gamma c_1 + c_2.$$

We can take the difference between (7.57) and (7.58), after using (7.61) and (7.62) to obtain

$$\hat{l}_{N,t}^R = -\frac{\varphi\gamma}{\lambda}\hat{T}_{N,t}^R + \frac{1}{\lambda}\hat{C}_t^R + \frac{1}{\lambda}\hat{G}_t^R - \frac{1}{\lambda}\hat{A}_{N,t}^R$$

which can be re-written as

$$\hat{l}_{N,t}^R = -c_6\hat{T}_{N,t}^R + \frac{1}{\lambda}\hat{G}_t^R - \frac{1}{\lambda}\hat{A}_{N,t}^R \tag{7.70}$$

where $c_6 =$? We can now take the difference between (7.55) and (7.56) and use the difference between (7.59) and (7.60) to obtain

$$\hat{l}_{T,t}^R = \frac{\theta}{\lambda}\hat{T}_t - \frac{1}{\lambda}\hat{A}_{T,t}^R \tag{7.71}$$

Equations (7.68), (7.69), (7.70), (7.71) can be solved for $\hat{l}_{N,t}^R$, $\hat{l}_{T,t}^R$, $\hat{T}_{N,t}^R$, $\hat{T}_t$. We can write the following matrix form in relation to the vector $x_t = [\hat{l}_{T,t}^R \;\; \hat{l}_{N,t}^R \;\; \hat{T}_t \;\; \hat{T}_{N,t}^R]$

$$Ax_t = Bs_t$$

where $s_t = [\hat{\mu}_{T,t}^R \;\; \hat{\mu}_{N,t}^R \; \hat{A}_{T,t}^R \; \hat{A}_{N,t}^R \; \hat{G}_t^R]$

$$A = \begin{bmatrix} \lambda\tilde{\eta}c_3 & 0 & c_4 & c_1(1-\gamma) \\ 0 & \lambda\tilde{\eta}c_3 & -\gamma c_1 & -c_5 \\ 1 & 0 & -\frac{\theta}{\lambda} & 0 \\ 0 & 1 & 0 & c_6 \end{bmatrix}$$

$$B = \begin{bmatrix} -c_4 & c_1(1-\gamma) & c_4 & -c_1(1-\gamma) & 0 \\ \gamma c_1 & -c_5 & -\gamma c_1 & c_5 & 0 \\ 0 & 0 & -\frac{1}{\lambda} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\lambda} & \frac{1}{\lambda} \end{bmatrix}$$

We can obtain a solution

$$x_t = A^{-1}Bs_t.$$

Consider the case in which $\lambda = 1$ and $\tilde{\lambda} = 1$. The relevant equations for the determination of $(\hat{T}_t, \hat{T}_{N,t}^R, \hat{L}_t^R, \hat{l}_{T,t}^R, \hat{l}_{N,t}^R)$ are

$$\hat{\mu}_{T,t}^R + \hat{T}_t + \eta\hat{L}_t^R - \frac{1}{\phi}(\hat{l}_{T,t}^R - \hat{L}_t^R) - \hat{A}_{T,t}^R = 0 \tag{7.72}$$

$$\hat{\mu}^R_{N,t} - \hat{T}^R_{N,t} + \eta \hat{L}^R_t - \frac{1}{\phi}(\hat{l}^R_{N,t} - \hat{L}^R_t) - \hat{A}^R_{N,t} = 0 \tag{7.73}$$

$$\hat{l}^R_{N,t} = -b\hat{T}^R_{N,t} + \hat{G}^R_t - \hat{A}^R_{N,t} \tag{7.74}$$

$$\hat{l}^R_{T,t} = \theta \hat{T}_t - \hat{A}^R_{T,t} \tag{7.75}$$

$$\hat{L}^R_t = \gamma \hat{l}^R_{T,t} + (1-\gamma)\hat{l}^R_{N,t} \tag{7.76}$$

where $b \equiv \varphi\gamma + (1-\gamma)\,\rho^{-1}$. I left ρ generic. We first solve for $\hat{T}_t, \hat{T}^R_{N,t}$. To do this we consider the difference between (7.72) and (7.73) and obtain

$$\hat{\mu}^R_{T,t} - \hat{\mu}^R_{N,t} + \hat{T}_t + \hat{T}^R_{N,t} - \frac{1}{\phi}(\hat{l}^R_{T,t} - \hat{l}^R_{N,t}) - (\hat{A}^R_{T,t} - \hat{A}^R_{N,t}) = 0 \tag{7.77}$$

as well as the difference between (7.74) and (7.75) and obtain

$$\hat{l}^R_{T,t} - \hat{l}^R_{N,t} = \theta \hat{T}_t + b\hat{T}^R_{N,t} - \hat{G}^R_t - (\hat{A}^R_{T,t} - \hat{A}^R_{N,t}) \tag{7.78}$$

We can substitute (7.78) into (7.77) to obtain

$$\hat{\mu}^R_{T,t} - \hat{\mu}^R_{N,t} + \left(1 - \frac{\theta}{\phi}\right)\hat{T}_t + \left(1 - \frac{b}{\phi}\right)\hat{T}^R_{N,t} + \frac{1}{\phi}\hat{G}^R_t - \left(1 - \frac{1}{\phi}\right)(\hat{A}^R_{T,t} - \hat{A}^R_{N,t}) = 0$$

which implies a relation between $\hat{T}^R_{N,t}$ and $\hat{T}_t$ of the form

$$\hat{T}^R_{N,t} = -\frac{\left(1-\frac{\theta}{\phi}\right)}{\left(1-\frac{b}{\phi}\right)}\hat{T}_t - \frac{1}{\left(1-\frac{b}{\phi}\right)\phi}\hat{G}^R_t + \frac{\left(1-\frac{1}{\phi}\right)}{\left(1-\frac{b}{\phi}\right)}(\hat{A}^R_{T,t} - \hat{A}^R_{N,t}) - \frac{1}{\left(1-\frac{b}{\phi}\right)}(\hat{\mu}^R_{T,t} - \hat{\mu}^R_{N,t}) \tag{7.79}$$

We need another relation to determine $\hat{T}_N$ and $\hat{T}$. Consider a weighted average of (7.74) and (7.75), we obtain

$$\hat{L}^R_t = \theta\gamma\hat{T}_t - b(1-\gamma)\hat{T}^R_{N,t} - \gamma\hat{A}^R_{T,t} - (1-\gamma)\hat{A}^R_{N,t} + (1-\gamma)\hat{G}^R_t$$

which can be substituted together with (7.75) into (7.72) to obtain

$$\begin{aligned} 0 \;=\;& \hat{\mu}^R_{T,t} + \left(1 - \frac{\theta}{\phi}\right)\hat{T}_t + \left(\eta + \frac{1}{\phi}\right)\theta\gamma\hat{T}_t - \left(\eta + \frac{1}{\phi}\right)b(1-\gamma)\hat{T}^R_{N,t} + \\ & \left(\eta + \frac{1}{\phi}\right)(1-\gamma)\hat{G}^R_t - \left(\eta + \frac{1}{\phi}\right)[\gamma\hat{A}^R_T + (1-\gamma)\hat{A}^R_N] - \left(1 - \frac{1}{\phi}\right)\hat{A}^R_T \end{aligned}$$

from which we can get that

$$
\begin{aligned}
\left[1+\left(\eta+\frac{1}{\phi}\right)\theta\gamma-\frac{\theta}{\phi}\right]\hat{T}_t \quad = \quad & \left(\eta+\frac{1}{\phi}\right)b(1-\gamma)\hat{T}^R_{N,t}+\left(1-\frac{1}{\phi}\right)\hat{A}^R_{T,t} \\
& -\hat{\mu}^R_{T,t}+\left(\eta+\frac{1}{\phi}\right)[\gamma\hat{A}^R_{T,t}+(1-\gamma)\hat{A}^R_{N,t}] \\
& -\left(\eta+\frac{1}{\phi}\right)(1-\gamma)\hat{G}^R_t \qquad (7.80)
\end{aligned}
$$

We can combine (7.79) and (7.80) to solve for $(\hat{T}^R_{N,t}, \hat{T}_t)$.

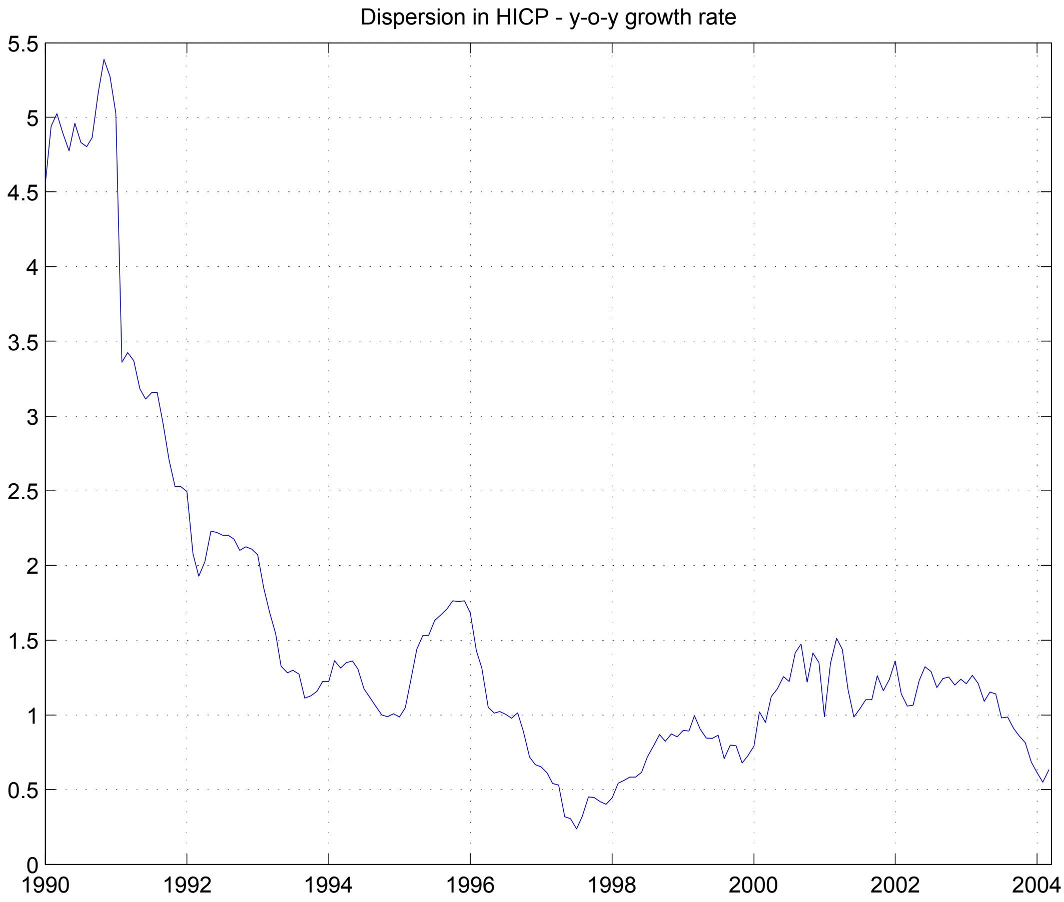

Fig. 1: Dispersion in HICP - y-o-y growth rate

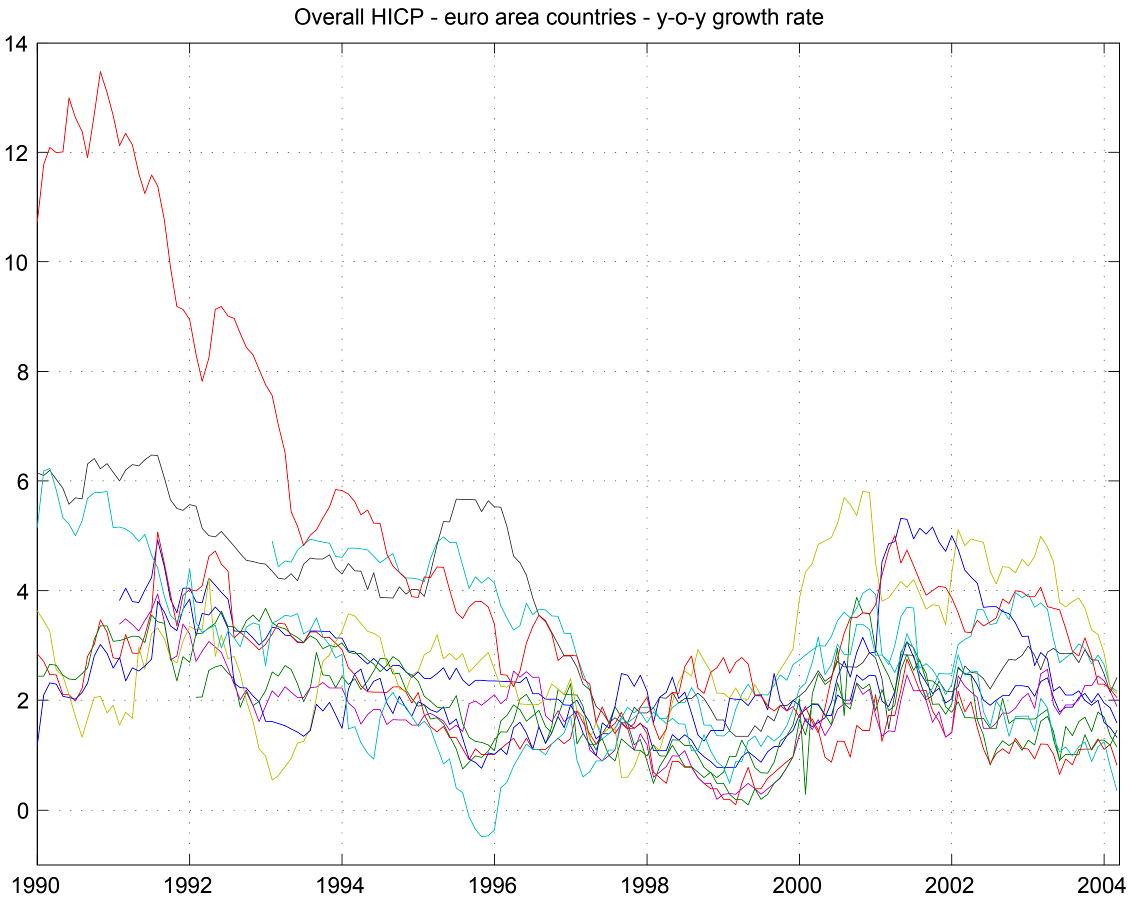

Fig. 2: Overall HICP - euro area countries - y-o-y growth rate

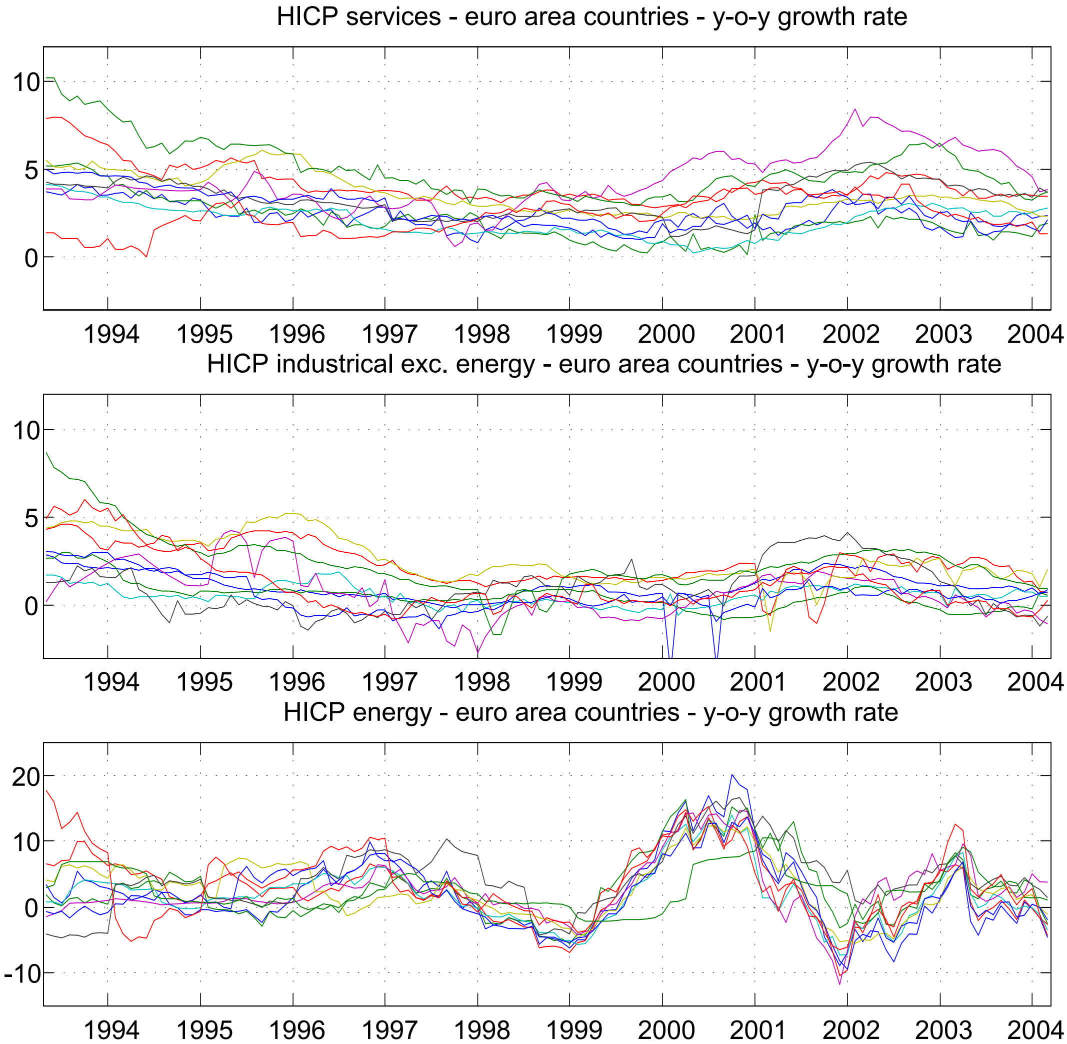

Fig. 3: HICP main sectors - euro area countries - y-o-y growth rate

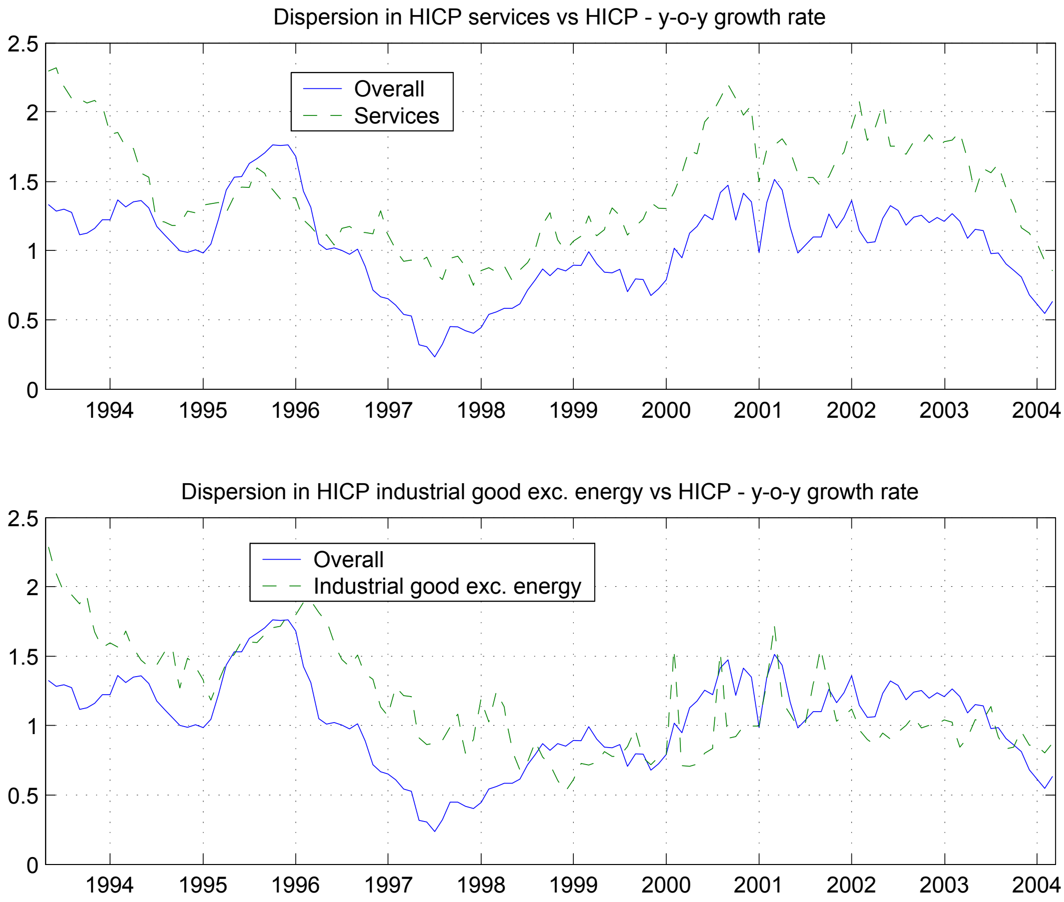

Fig. 4: Dispersion by sector.

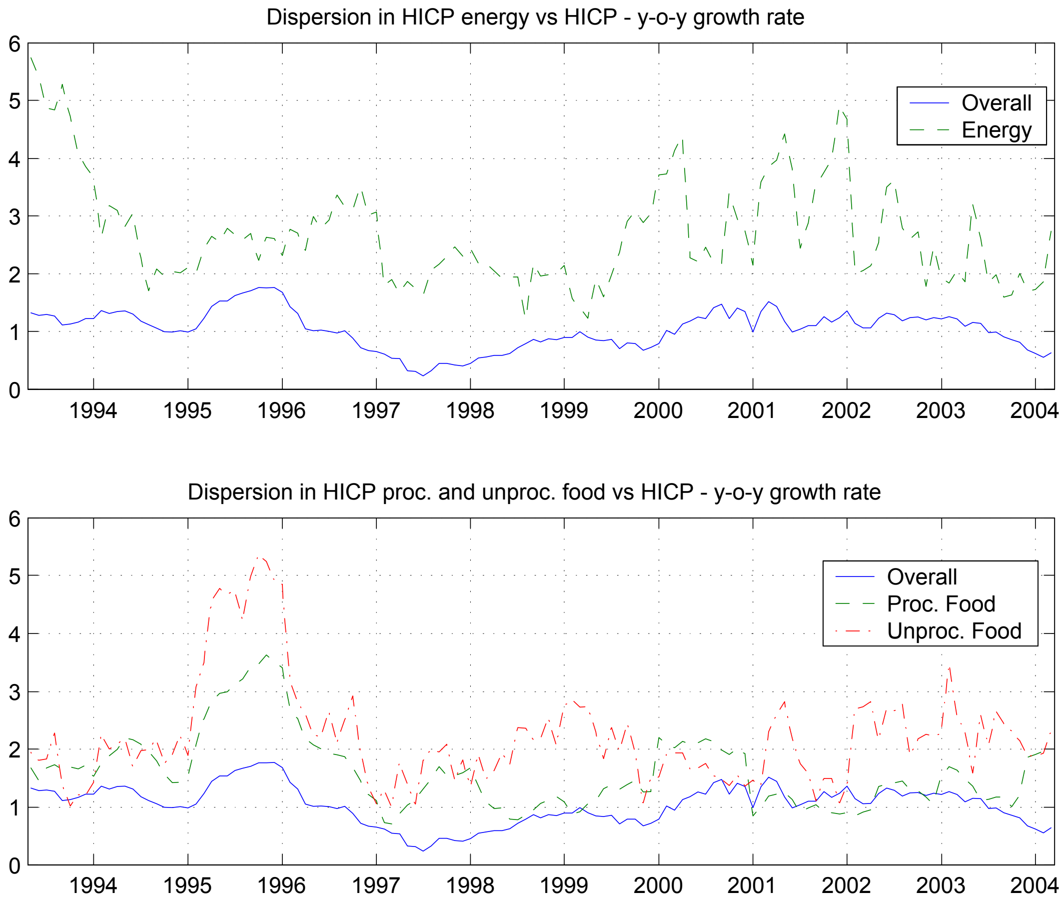

Fig. 5: Dispersion by sector.

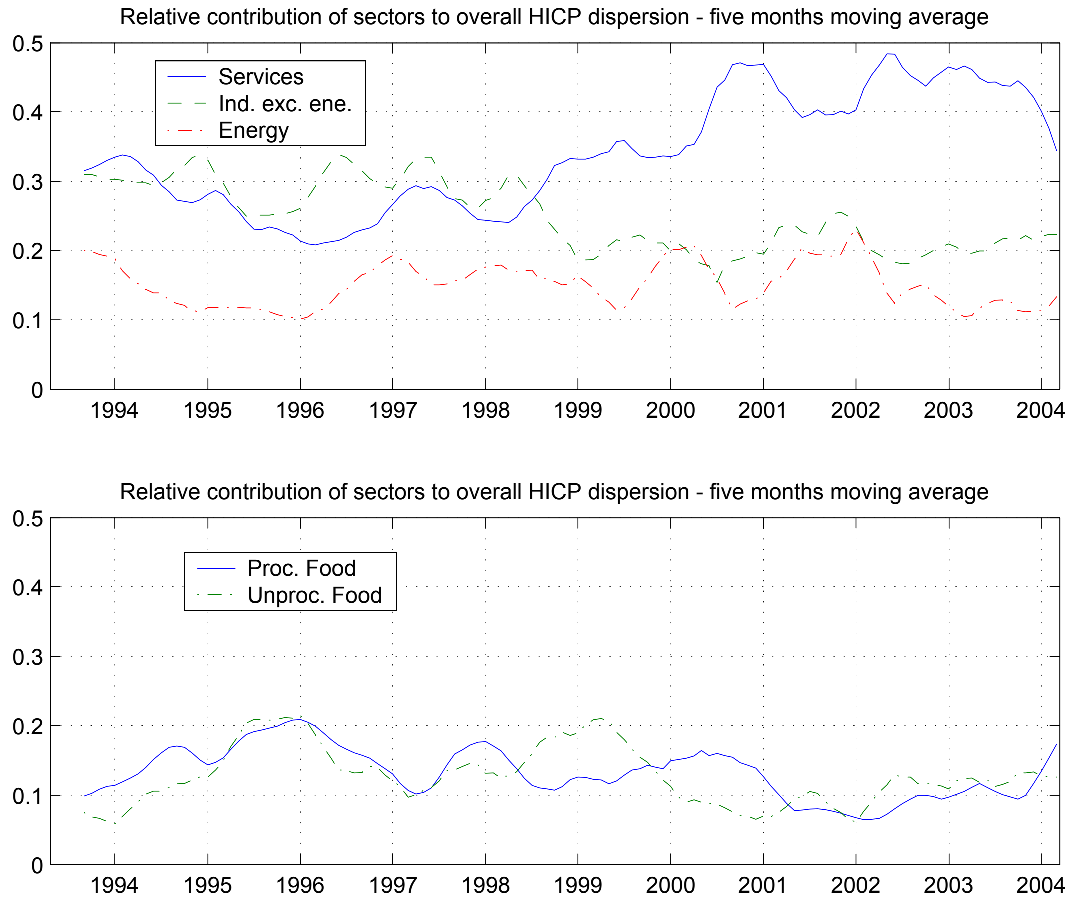

Fig. 6: Dispersion decomposition.

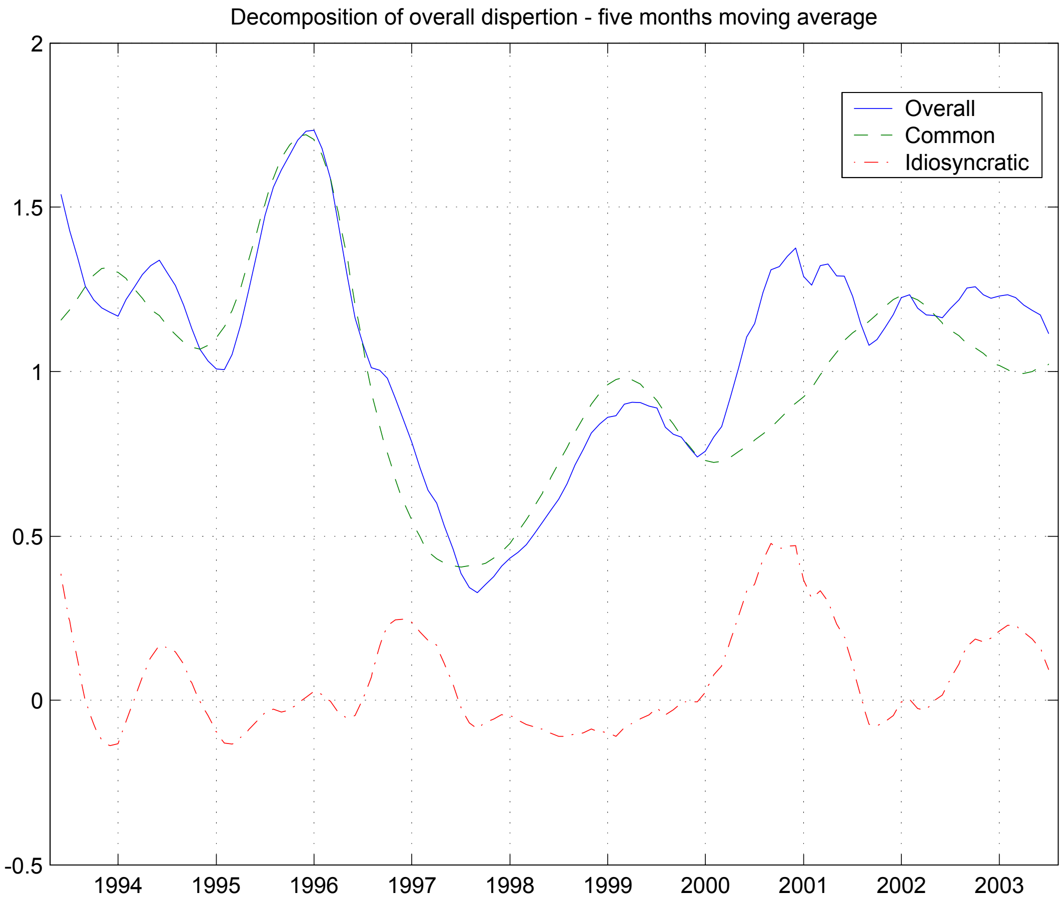

Fig. 7: Decomposition of Overall dispersion

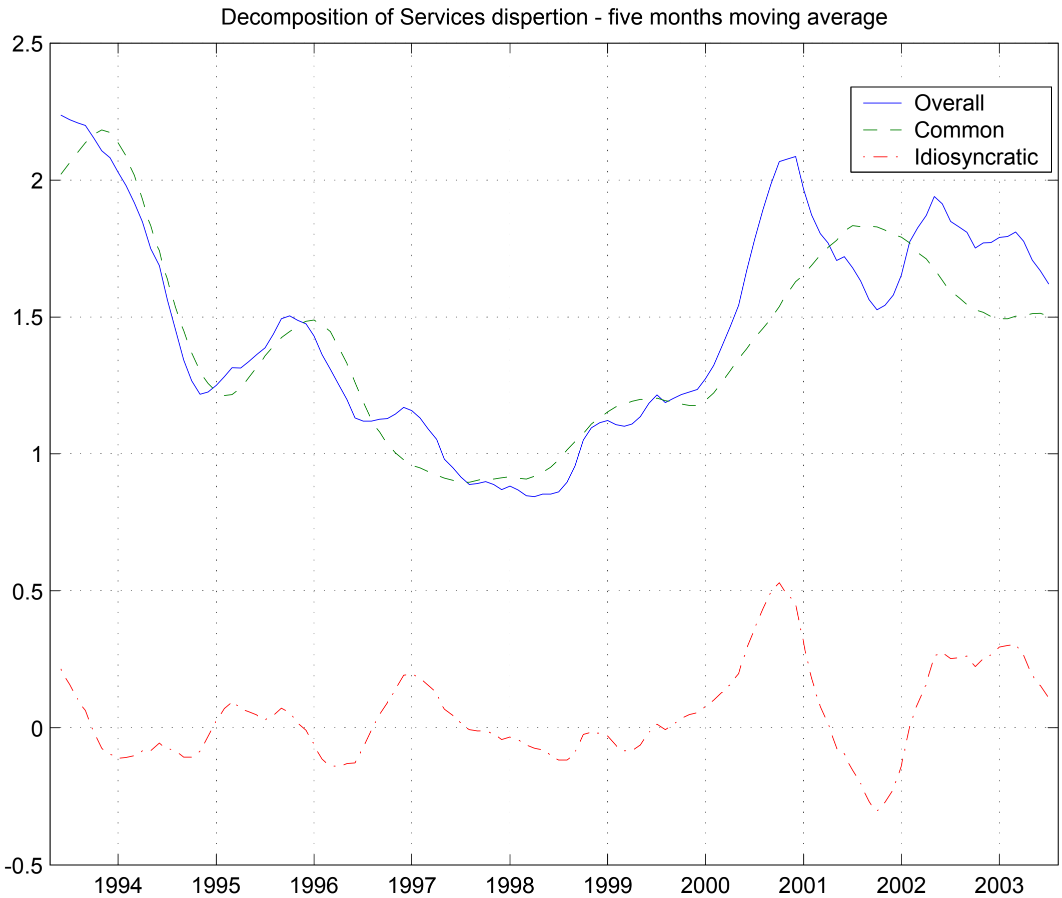

Fig. 8: Decomposition of Services dispersion

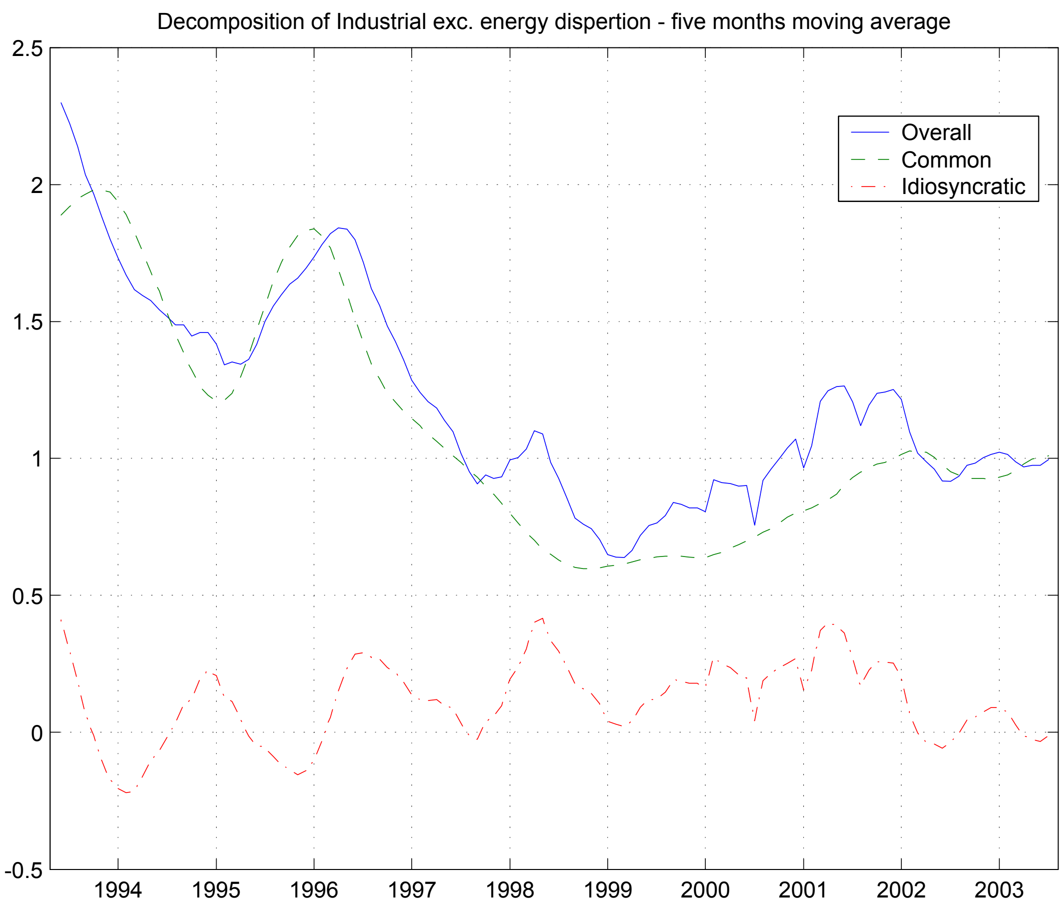

Fig. 9: Decomposition of Industrial Goods exc. energy dispersion

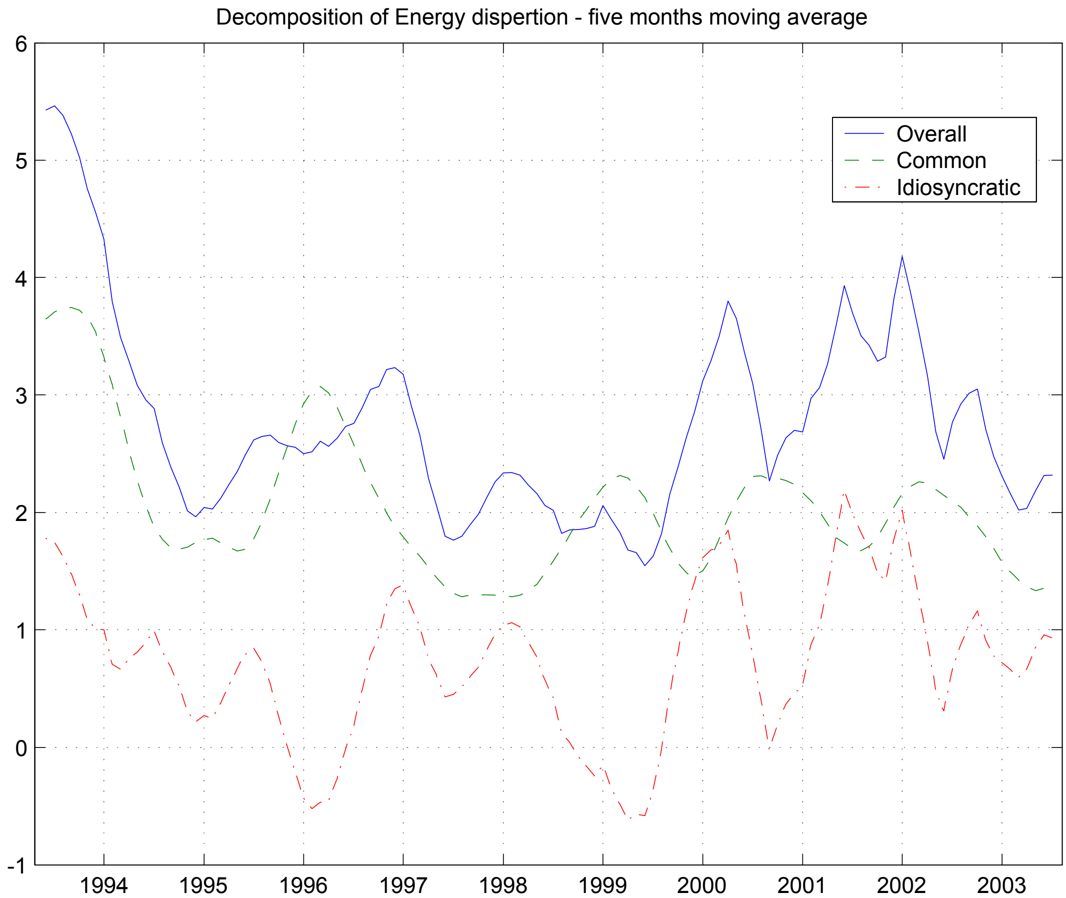

Fig. 10: Decomposition of Energy dispersion

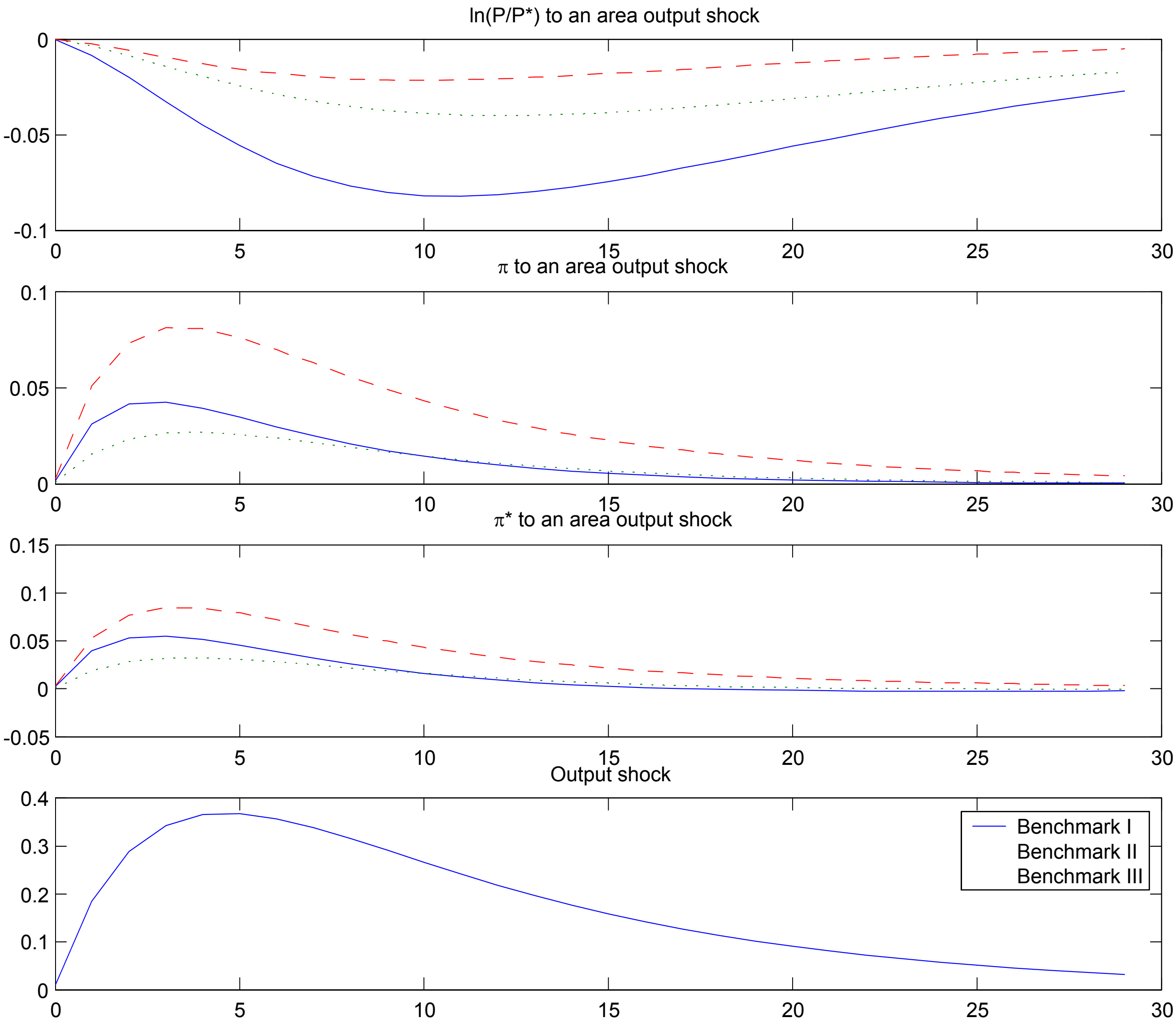

Fig. 11: monetary policy shock

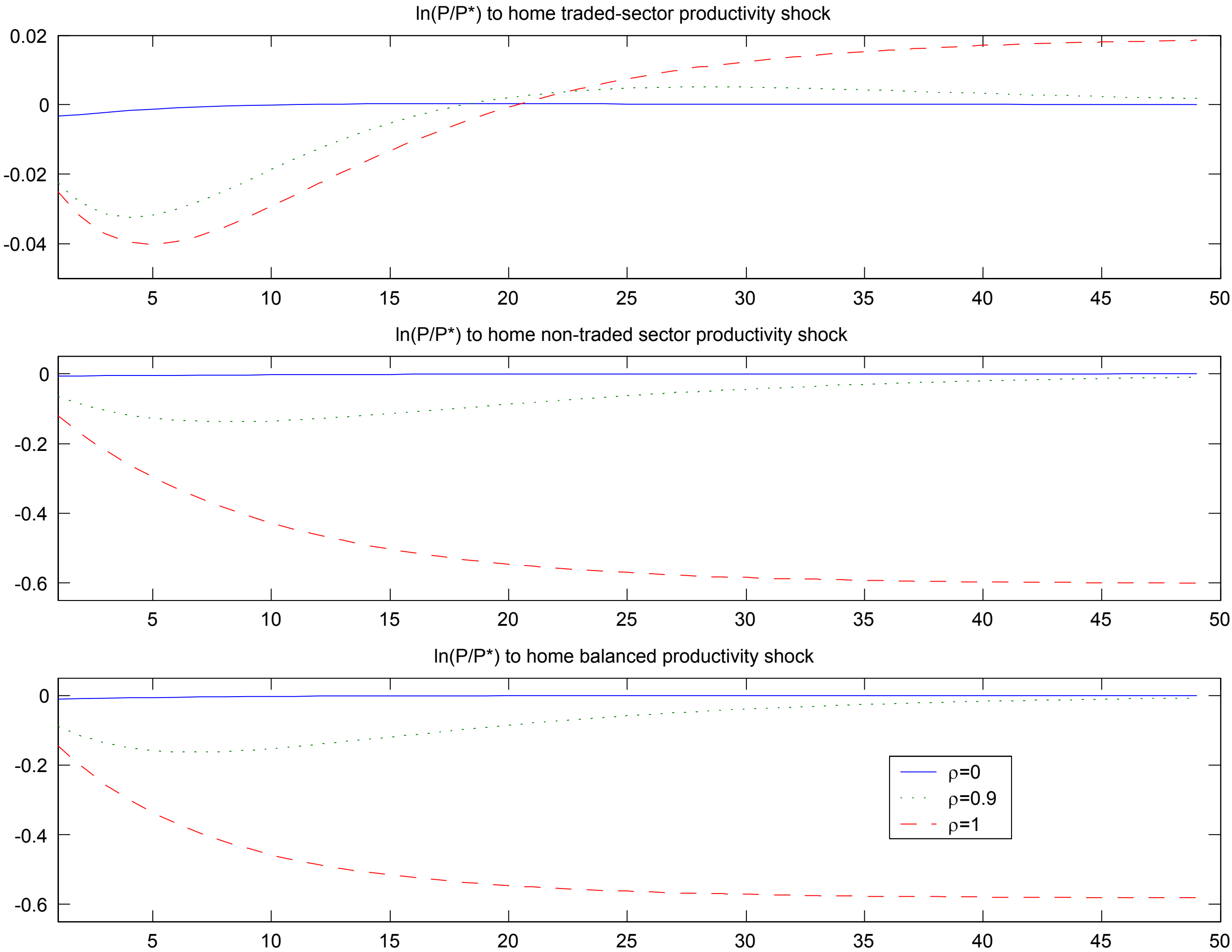

Fig. 12: Productivity shocks

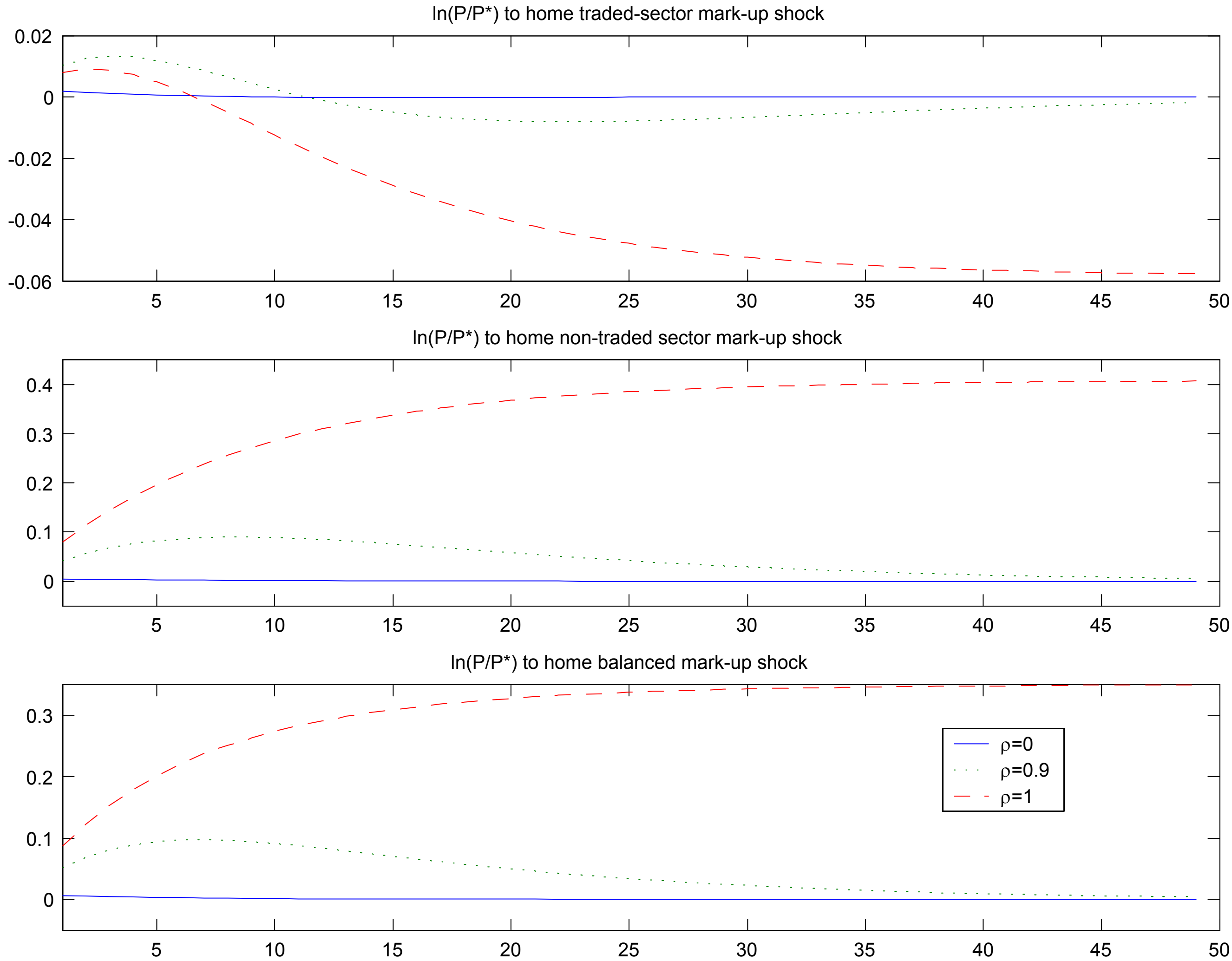

Fig.13: Mark-up shocks

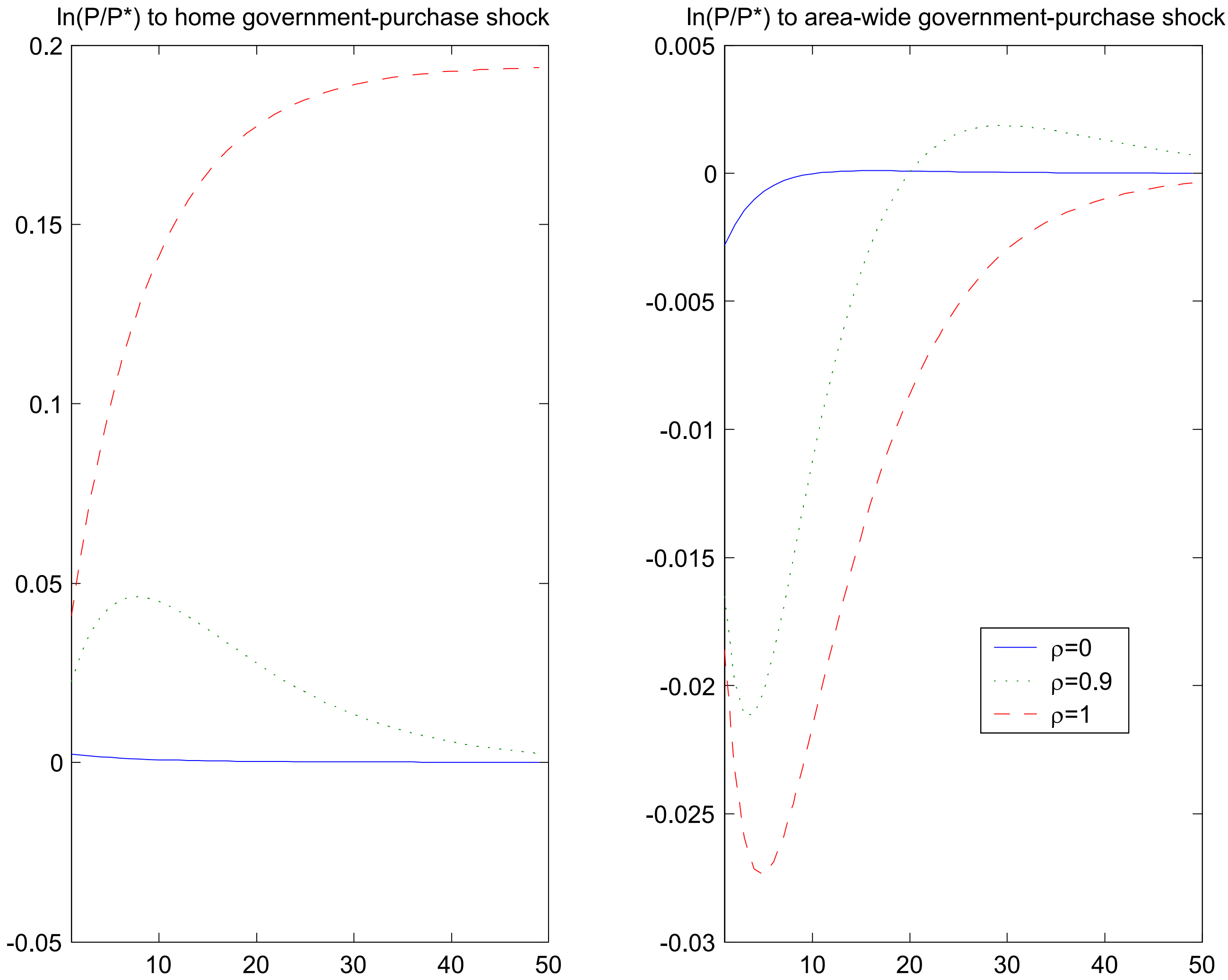

Fig. 14: Government purchase shocks

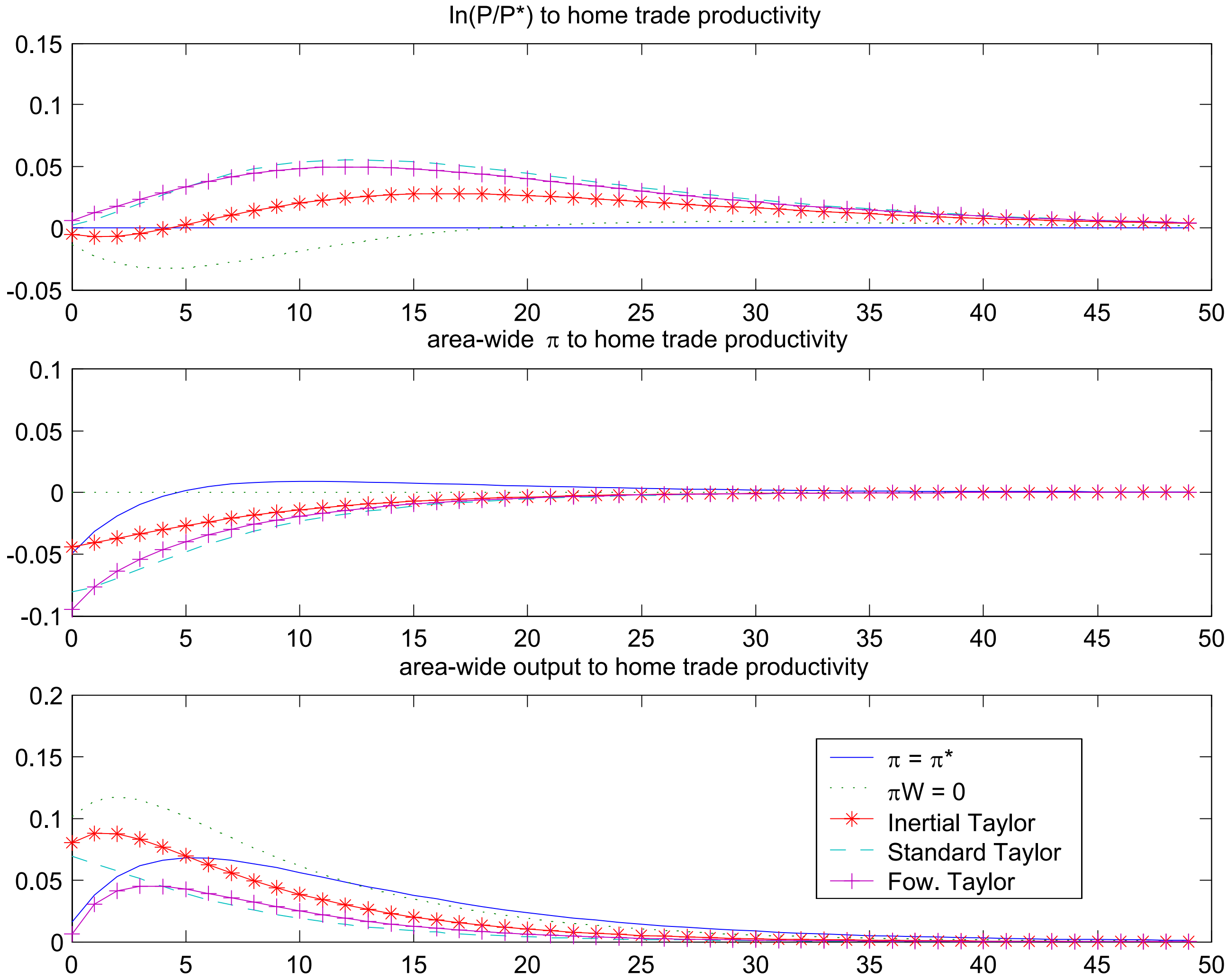

Fig. 15: Monetary policy and home trade sector productivity

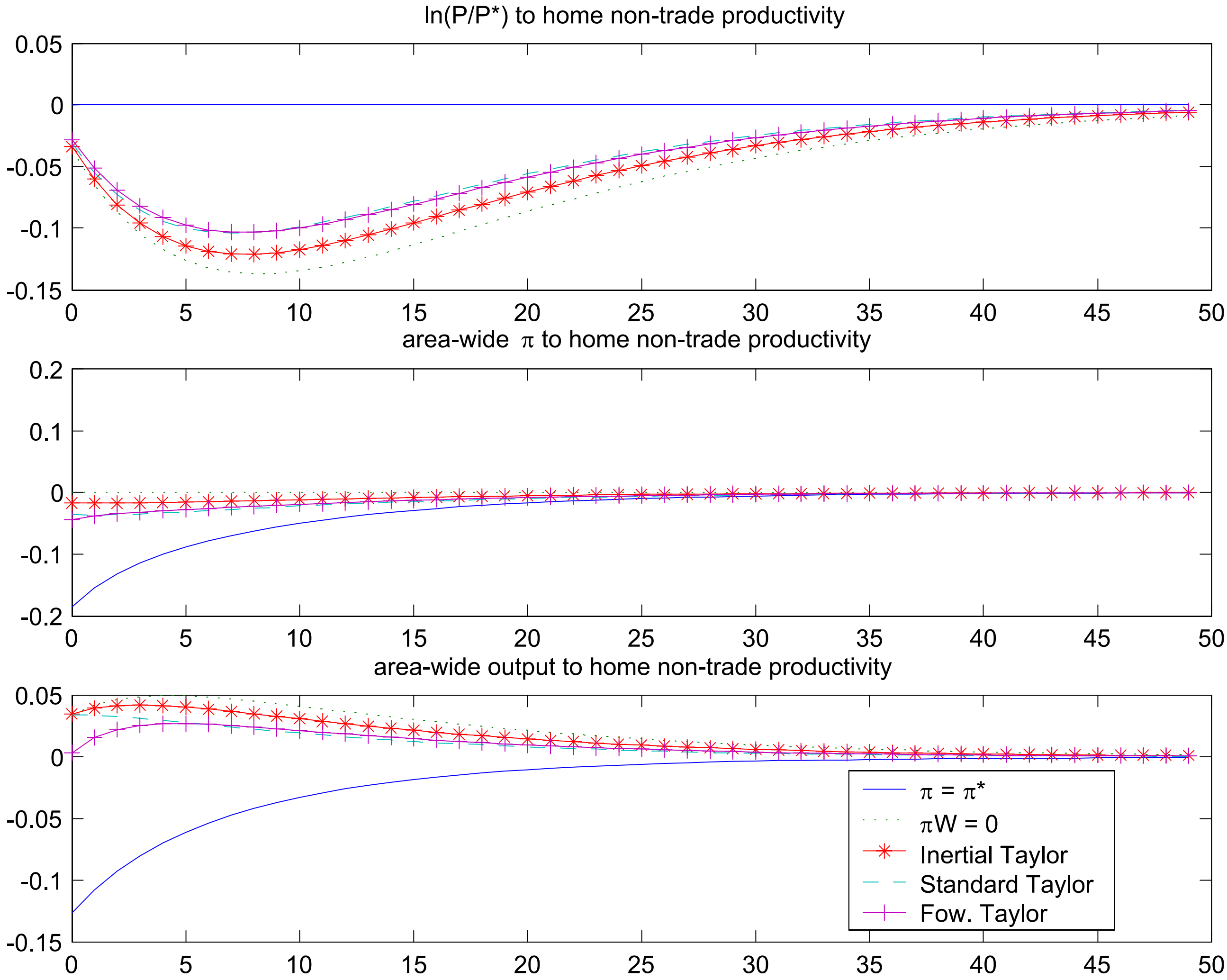

Fig. 16: Home non-trade sector productivity shock

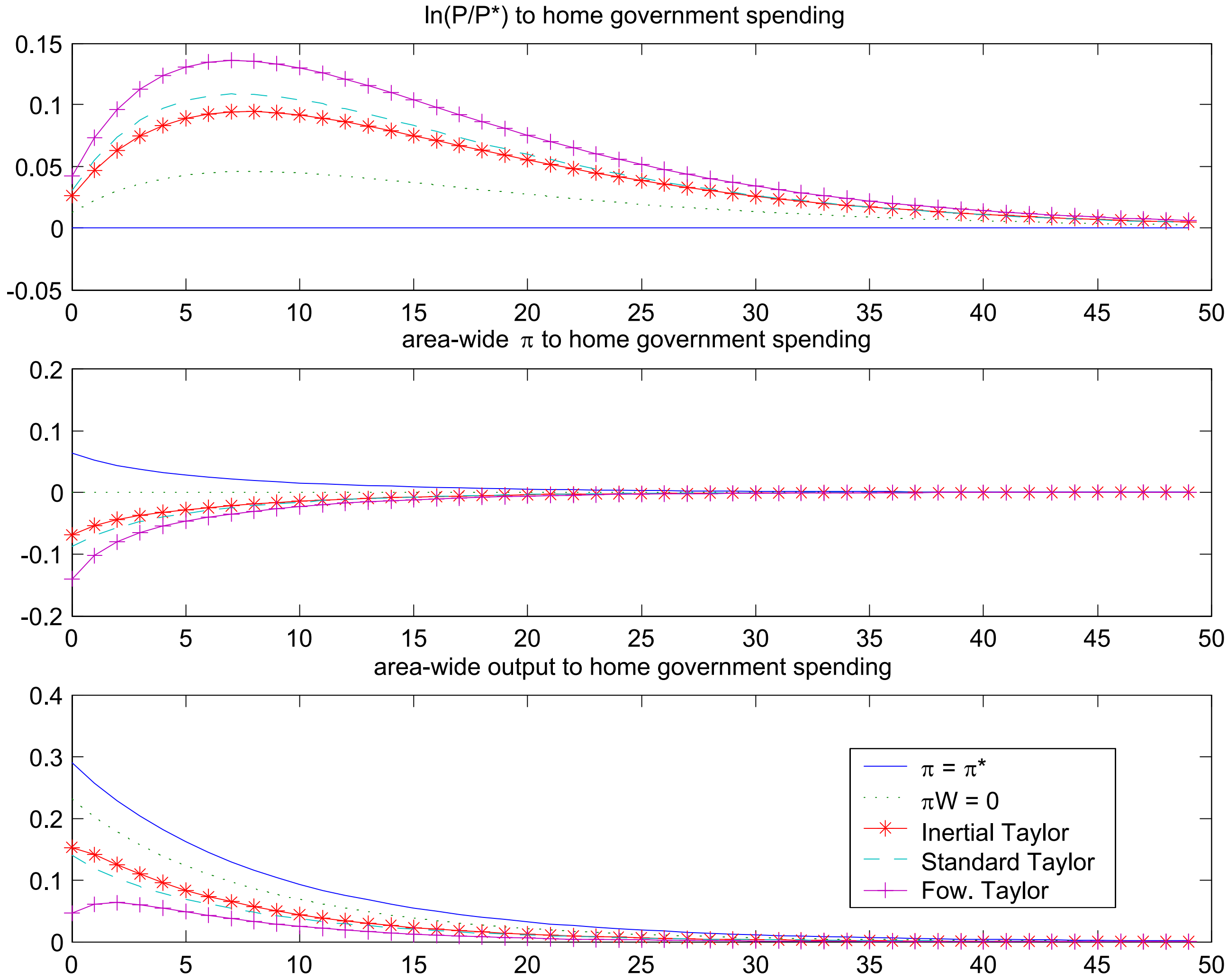

Fig. 17: Government purchase productivity shock