

What if the UK had Joined the Euro in 1999? An Empirical Evaluation using a Global VAR*

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Abstract

This paper attempts to provide a conceptual framework for the analysis of counterfactual scenarios using macroeconomic models. As an application we consider UK entry to the euro. Entry involves a long-term commitment to restrict UK nominal exchange rates and interest rates to be the same as those of the euro area. We derive conditional probability distributions for the difference between the future realisations of variables of interest (e.g UK and euro area output and prices) subject to UK entry restrictions being fully met over a given period and the alternative realisations without the restrictions. The robustness of the results can be evaluated by also conditioning on variables deemed to be invariant to UK entry, such as oil or US equity prices. Economic interdependence means that such policy evaluation must take account of international linkages and common factors that drive fluctuations across economies. In this paper this is accomplished using the Global VAR recently developed by Dees, di Mauro, Pesaran and Smith (2005). The paper briefly describes the GVAR which has been estimated for 25 countries and the euro area over the period 1979-2003. It reports probability estimates that output will be higher and prices lower in the UK and the euro area as a result of entry. It examines the sensitivity of these results to a variety of assumptions about when and how the UK entered and the observed global shocks and compares them with the effects of Swedish entry.

Keywords: Global VAR (GVAR), Counterfactual Analysis, UK and Sweden entry to euro.

JEL Classification: C32, C35, E17, F15, F42.

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1 Introduction

In economic analysis it is usually assumed that agents are capable of evaluating alternative potential outcomes: to answer ‘what if’ questions. Agents often make decisions by conducting a counterfactual analysis, judging what would happen under alternative states of the economy. This ability is taken for granted in strategic games, and in arriving at arbitrage conditions in financial markets. Similarly in economic policy-making, a monetary authority will ask what will be the outcome if interest rates are left unchanged compared to the outcome if they are raised or lowered by a certain number of basis points. In policy-making, such scenarios are usually constructed using an econometric model and the outcomes may be described in terms of point, interval or probability forecasts. This paper attempts to provide a conceptual framework for the analysis of such counterfactuals. The approach is illustrated with a specific example that has caused considerable controversy, the debate as to whether the UK or Sweden should adopt the euro. A natural question to ask in this context is whether output in the UK (Sweden) and euro area (EA), would be higher and prices lower if the UK had joined the euro in 1999. Given a global econometric model this is a well defined question that has not, to our knowledge, been given a quantified answer. The model we shall use is the Global Vector Autoregression, GVAR, described in Dees, di Mauro, Pesaran and Smith (2005), DdPS, which is estimated for 25 countries plus the euro area over the period 1979Q4 to 2003Q4.

Our approach to the counterfactual analysis involves deriving the conditional probability distributions for the differences between the future realisations of variables of interest subject to the restrictions implied by the counterfactual under consideration and the alternative realisations without such restrictions. Given a complete econometric model, i.e. with no exogenous variables, with a vector of endogenous variables \mathbf{x}_t , one can calculate the multi-period probability distribution for \mathbf{x}_{T+h} conditional on information at time $T : f(\mathbf{x}_{T+h} | \mathcal{I}_T)$, for $h = 1, 2, \dots, H$, where H is the forecast horizon. The counterfactual scenario, say the UK or Sweden euro entry, can be viewed as imposing restrictions on the space of the endogenous variables, $\Psi \mathbf{x}_{T+j} = \mathbf{d}_{T+j}$, $j = 1, 2, \dots, \bar{H}$, where Ψ and \mathbf{d}_{T+j} are specified such as to ensure that UK (Sweden) and EA nominal exchange rates and interest rates are equalised for a certain number of periods after entry, and \bar{H} is the commitment horizon (typically assumed to be much larger than H). One can then calculate the probability distribution of \mathbf{x}_{T+h} subject to the long-term entry restrictions, $f(\mathbf{x}_{T+h} | \mathcal{I}_T, \Psi \mathbf{x}_{T+j} = \mathbf{d}_{T+j}, j = 1, 2, \dots, \bar{H})$, for $h = 1, 2, \dots, H < \bar{H}$.

The effect of the entry can then be derived as the probability distribution of the difference between the two counterfactuals. Our baseline case looks at the effect of UK entry in 1999Q1 at par, namely the exchange rate prevailing in 1998Q4, on output and prices in the UK and EA over the five years 1999Q1-2003Q4. To model long-term commitment we impose the entry restrictions for 10 years, and consider the effects of the UK entry over the period 1999Q1-2003Q4 (five years). We examine the robustness of our results to a variety of other assumptions about how the UK joined, when it joined and to the

realisations of oil prices and US equity and interest rates. A similar analysis is also carried out for Sweden.

There is a large literature on conditional probability forecasts, e.g. Doan, Litterman and Sims (1984) and Waggoner and Zha (1999), but our approach is rather different. For instance, when a Central Bank fixes a future path for interest rates to investigate a particular scenario, it usually does this by fixing the errors in the model so that the required interest path is achieved, but ignores the covariances between those assumed errors and the errors of the other equations, which will change other variables. In contrast, our approach allows for the possibility of non-zero covariances. It differs from impulse response function analysis because it puts the restrictions on the observable variables not the unobservable shocks, whose interpretation can be problematic. Like conditional forecasting, the counterfactual analysis can be carried out using the reduced form model, which renders the analysis invariant to the different schemes discussed in the literature for exact identification of the structural shocks.

The issue we are dealing with is similar to that dealt with by models of potential outcomes introduced in the design of experiments by Neyman (1935) and used in the treatment effect literature. The UK did not join the euro in 1999 and we cannot observe what would have happened had it done so any more than we can observe what would have happened to Italy had it not joined. The relation between models of potential outcomes and econometric models is discussed by Heckman (2000). However, our approach differs in that it examines alternative distributions generated by a parametric model rather than making non-parametric comparisons between the observations on those treated and those not treated. The very small sample means that it is not sensible to compare the outcomes of the twelve EU members who joined the euro with the three who stayed out.

Our approach also differs from the type of counterfactuals that are often contemplated in historical discourses. For example, Fogel (1964) asked: what might have happened to the US economy if the railroad had not been brought into use? The standard counterfactual compares the outcome of the hypothetical alternative scenario to what actually happened. The former can be viewed as restricted forecasts, and as with all forecasts can be subject to significant errors, thus making it difficult to distinguish the effects of the restrictions (the counterfactuals) from the forecasts errors associated with the econometric model. Our approach avoids this problem by evaluating the probability distribution of the difference between two counterfactuals, one with and one without the restrictions imposed. The particular aspect of the probability distribution that we look at is the probability that GDP in the UK and the EA would have been higher and prices lower had the UK joined the euro at the beginning of 1999. We phrase this question in terms of probabilities rather than point forecasts because measurement of such effects is inherently uncertain.

There is a vast literature on various aspects of the euro, including the massive research project conducted by the UK Treasury involving many independent academics published as HMT (2003), but none of it seems to address the particular question we pose. Much of the analysis has been preoccupied with the

effects on the UK. We look at the effects on both the UK and the EA, since they are very interdependent. The EA is the UK's largest trading partner accounting for 53.7% of its trade, compared to 18% for the US, but the UK is also the EA's largest trading partner, accounting for 23.8% of its trade as compared with 22.7% for the US, see Table 1. Thus UK entry could have important effects on the EA.

Table 1: Trade Weights Based on Direction of Trade Statistics

Country/ Region	Rest of W. Europe								Rest*
	US	EA	China	Japan	UK	Sweden	Switz.	Norway	
US	0.000	0.155	0.073	0.124	0.052	0.008	0.012	0.004	0.571
EA	0.227	0.000	0.056	0.072	0.238	0.057	0.090	0.028	0.232
China	0.236	0.164	0.000	0.248	0.029	0.010	0.007	0.003	0.304
Japan	0.319	0.132	0.128	0.000	0.032	0.007	0.009	0.003	0.371
UK	0.180	0.537	0.020	0.042	0.000	0.027	0.028	0.023	0.143
Sweden	0.104	0.517	0.025	0.035	0.115	0.000	0.017	0.099	0.088
Switz.	0.113	0.670	0.015	0.039	0.066	0.015	0.000	0.004	0.079
Norway	0.090	0.449	0.020	0.030	0.181	0.132	0.008	0.000	0.089

Note: Trade weights are computed as shares of exports and imports displayed in rows by region such that a row, but not a column, sums to one. *"Rest" gathers the remaining countries. This Table has been reproduced from Table 2 in Dees, di Mauro, Pesaran and Smith (2005).

Economic interdependence means that policy evaluation of alternative scenarios must take account of international linkages and common factors which drive business cycle fluctuations. Thus, to conduct the counterfactual that we have in mind requires a global macroeconomic model which contains specific models for the euro area and the UK, allows for nominal exchange rates and interest rates to be equalised and can calculate the probability distributions for the endogenous variables multiple periods ahead, with and without the restrictions implied by the counterfactuals. The GVAR described in detail in DdPS allows us to do this. It is estimated over 1979Q4 to 2003Q4, so the parameter estimates embody information after the introduction of the euro. It is exactly the version described there. It has not been adjusted for this particular exercise. An earlier version of the GVAR is described in Pesaran, Schuermann and Weiner (2004).

To examine the effect of entry on levels of output and prices in the UK and the EA, a model which distinguishes between the EA and other countries is essential since we need to fix the UK nominal interest rate and exchange rate relative to the ones in EA, while letting the EA (now including the UK) interest rate and exchange rate to vary against other economies. Models which are based on indices of effective exchange rates are not suitable for this purpose. The experience of the EA would have been different under UK entry, because

the UK has strong trading links with the EA. Therefore changes in UK economic performance would have feedback effects on the EA. The GVAR allows for this, so the impact on the EA of UK entry can be computed.

The approach developed in this paper could be used to examine a range of other counterfactuals, e.g. the new entrants to the EU joining the euro, the possibility of China pegging her currency to the yen rather than to the US dollar, or the effects of a policy committed to holding interest rates unchanged over a given period.

In the rest of the paper we briefly describe the historical background to the creation of the euro, provide an overview of the GVAR, present a framework for the analysis of counterfactuals starting with a simple illustration. We explain how we model entry and derive the probability distribution of the effects of entry under different scenarios, and finally present the main results for the effects of UK and Sweden entries, separately.

2 Historical Background

A central theme in European economic policy in the last quarter of the twentieth century was the need to reduce exchange rate volatility. After the breakdown of Bretton Woods, there were various attempts to reduce exchange rate fluctuations between European countries, culminating in June 1988, when the European Council confirmed the objective of Economic and Monetary Union, EMU. It established the Delors Committee which proposed a three stage process to EMU. Stage 1 started on 1 July 1990, Stage 2, with an embryonic Central Bank, the European Monetary Institute, started on 1 January 1994. Stage 3 started with the adoption of the euro as a virtual currency on 1 January 1999. Eleven countries fixed their exchange rates to the euro and adopted common interest rates. Greece joined on 1 January 2001. Euro notes and coins entered circulation in January 2002.

During the same period, a central theme in UK economic policy was the control of inflation. Money supply targeting was tried in the late 1970s and early 1980s but failed. The target then switched to the exchange rate, first informally under Chancellor Lawson who shadowed the DM, then formally after October 1990 through membership of the European Exchange Rate Mechanism, ERM. The UK and Italy were ejected from the ERM by a large speculative attack in September 1992. Italy later returned to the ERM and joined the euro while the UK began a period of inflation targeting.

The Labour Party was elected in 1997. In opposition, it knew that if elected to government it would have to establish credibility. Keegan (2003) provides an account of the development of Labour party economic policy. During the election campaign, Labour had tried to establish that it would follow a prudent fiscal policy by committing itself to the public expenditure plans of the previous Conservative government. It also needed to establish that it would follow a prudent monetary policy. There were two obvious routes: rejoining the ERM and subsequently the euro or making the Bank of England independent. Neither

could have been put in the election manifesto and either could in economic terms be regarded as providing a plausible nominal anchor. In the event, it chose Bank of England independence. To join the euro the UK would have had to meet a set of five convergence criteria. Four of them it easily met: government deficit below 3% of GDP, government debt below 60% of GDP, inflation less than 1.5% above the three best performing states, and long interest rates less than 2% above the three best performing states. The fifth, membership of the exchange rate mechanism without severe tensions for at least two years, would have been more difficult to meet, but might have been waived by the other members.

To determine policy towards the euro, the Labour Chancellor of the Exchequer, Gordon Brown, in a Statement on Economic and Monetary Union on October 27 1997 (available on <http://www.hm-treasury.gov.uk>) proposed five economic tests:

- “1. Whether there can be sustainable convergence between Britain and the economies of a single currency;
2. Whether there is sufficient flexibility to cope with economic change;
3. The effect on investment;
4. The impact on our financial services industry, and;
5. Whether it is good for employment.”

Some questioned the economic coherence, as distinct from the political rationale, of these tests and in any event they were about conditions for entry, not about what the outcome would be if the UK did enter, which is the question that concerns us in this paper.

3 An Overview of the GVAR

Suppose there are $N + 1$ countries indexed by $i = 0, 1, 2, \dots, N$, with country 0, the US, as the reference country. The DdPS version of the GVAR covers 33 countries, 25 treated separately, the remainder grouped into the euro area. For simplicity we will refer to the EA as a country. A second-order country-specific VARX*(2,2) model with deterministic trends can be written as

$$\hat{\mathbf{x}}_{it} = \alpha_i + \beta_i t + \Phi_{i1} \hat{\mathbf{x}}_{i,t-1} + \Phi_{i2} \hat{\mathbf{x}}_{i,t-2} + \Psi_{i0} \hat{\mathbf{x}}_{it}^* + \Psi_{i1} \hat{\mathbf{x}}_{i,t-1}^* + \Psi_{i2} \hat{\mathbf{x}}_{i,t-2}^* + \hat{\mathbf{u}}_{it}, \quad (1)$$

for $i = 0, 1, \dots, N$, where $\hat{\mathbf{x}}_{it}$ is a $k_i \times 1$ (usually five or six) vector of country-specific variables, $\hat{\mathbf{x}}_{it}^*$, a $k_i^* \times 1$ vector of foreign variables specific to country i , Φ_{i1} and Φ_{i2} are $k_i \times k_i$ matrix of lagged coefficients, Ψ_{i0} , Ψ_{i1} and Ψ_{i2} are $k_i \times k_i^*$ matrix of coefficients associated with the foreign-specific variables, and $\hat{\mathbf{u}}_{it}$ is a $k_i \times 1$ vector of idiosyncratic country-specific shocks. All the 26 country-specific models in the GVAR are estimated over the period 1979Q4-2003Q4.

The variables $\hat{\mathbf{x}}_{it}$ are a $k_i \times 1$ subset of the logarithm of real output, y_{it} ; inflation, $\pi_{it} = p_{it} - p_{it-1}$, where p_{it} is the logarithm of a price index; the real exchange rate, which we define as $e_{it} - p_{it}$, where e_{it} is the logarithm of the nominal exchange rate against the dollar; a short interest rate, $r_{it}^S = 0.25 \log(1 + R_{it}^S/100)$, where R_{it}^S is a short interest rate measured in percent per annum; a long interest rate, $r_{it}^L = 0.25 \log(1 + R_{it}^L/100)$; and the logarithm of

real equity prices, q_{it} . The variables included in the different country models are not always the same, e.g. there are no equity price or long-term interest rate data for some.

The lag orders are selected by AIC separately for each country up to a maximum of 2. The US, the reference country, is treated differently. Oil prices are included in the US model as an endogenous variable but included in other country models as weakly exogenous. In contrast, exchange rates (in terms of US dollars) are included as endogenous variables in all country models except for the US model. Also all foreign variables are included in the non-US models as weakly exogenous variables, but only foreign real output and foreign inflation are included as weakly exogenous in the US model.

The country specific models in the GVAR could be derived from structural models. Most theoretical models have VARs as reduced forms. For example, solution of Dynamic Stochastic General Equilibrium (DSGE) models can be approximated by VAR's with restrictions on the coefficients. But a formal link up of the GVAR with multi-country DSGE models will be beyond the scope of the present paper.

The foreign variables $\hat{\mathbf{x}}_{it}^*$ are calculated as country specific trade weighted averages of the corresponding variables of the other countries

$$\hat{\mathbf{x}}_{it}^* = \sum_{j=0}^N w_{ij} \hat{\mathbf{x}}_{jt}, \text{ with } w_{ii} = 0$$

where w_{ij} is the share of country j in the trade (exports plus imports) of country i . The $\hat{\mathbf{x}}_{it}^*$ are treated as weakly exogenous, an assumption found acceptable, when tested. A summary of the trade-weight matrix is given in Table 1. The VARX* models can be estimated separately for each country, taking into account the possibility of cointegration between $\hat{\mathbf{x}}_{it}$ and $\hat{\mathbf{x}}_{it}^*$. Including the country specific world averages, $\hat{\mathbf{x}}_{it}^*$, also reduces between-groups dependence in the errors. In the VARX* only the just identifying restrictions are imposed on the cointegrating vectors. It is worth noting that the outcome does not depend on the precise form of the just identifying restrictions.

Although estimation is done on a country by country basis, the GVAR model is solved for the world as a whole, taking account of the fact that all the variables are endogenous to the system as a whole. To do this (1) can be written as

$$\mathbf{A}_{i0} \hat{\mathbf{z}}_{it} = \mathbf{h}_{i0} + \mathbf{h}_{i1} t + \mathbf{A}_{i1} \hat{\mathbf{z}}_{it-1} + \mathbf{A}_{i2} \hat{\mathbf{z}}_{it-2} + \hat{\mathbf{u}}_{it}, \quad (2)$$

for $i = 0, 1, 2, \dots, N$ where $\hat{\mathbf{z}}_{it}$

$$\hat{\mathbf{z}}_{it} = \begin{pmatrix} \hat{\mathbf{x}}_{it} \\ \hat{\mathbf{x}}_{it}^* \end{pmatrix},$$

and

$$\mathbf{A}_{i0} = (\mathbf{I}_{k_i}, -\mathbf{\Psi}_{i0}), \quad \mathbf{A}_{i1} = (\mathbf{\Phi}_{i1}, \mathbf{\Psi}_{i1}), \quad \mathbf{A}_{i2} = (\mathbf{\Phi}_{i2}, \mathbf{\Psi}_{i2}).$$

The dimensions of \mathbf{A}_{i0} , \mathbf{A}_{i1} and \mathbf{A}_{i2} are $k_i \times (k_i + k_i^*)$ and \mathbf{A}_{i0} has full column rank, namely $\text{Rank}(\mathbf{A}_{i0}) = k_i$. Also note that

$$\dot{\mathbf{z}}_{it} = \mathbf{W}_i \dot{\mathbf{x}}_t,$$

where $\dot{\mathbf{x}}_t = (\dot{\mathbf{x}}'_{0t}, \dot{\mathbf{x}}'_{1t}, \dots, \dot{\mathbf{x}}'_{Nt})'$ is the $k \times 1$ vector which collects all the endogenous variables of the system, and \mathbf{W}_i is the $(k_i + k_i^*) \times k$ matrix defined by the trade weights w_{ij} . Using this (2) can be written as

$$\mathbf{A}_{i0} \mathbf{W}_i \dot{\mathbf{x}}_t = \mathbf{h}_{i0} + \mathbf{h}_{i1}t + \mathbf{A}_{i1} \mathbf{W}_i \dot{\mathbf{x}}_{t-1} + \mathbf{A}_{i2} \mathbf{W}_i \dot{\mathbf{x}}_{t-2} + \dot{\mathbf{u}}_{it}, \text{ for } i = 0, 1, 2, \dots, N, \quad (3)$$

and the systems stacked to yield the model for $\dot{\mathbf{x}}_t$

$$\mathbf{H}_0 \dot{\mathbf{x}}_t = \mathbf{h}_0 + \mathbf{h}_1 t + \mathbf{H}_1 \dot{\mathbf{x}}_{t-1} + \mathbf{H}_2 \dot{\mathbf{x}}_{t-2} + \dot{\mathbf{u}}_t,$$

where

$$\mathbf{H}_j = \begin{pmatrix} \mathbf{A}_{0j} \mathbf{W}_0 \\ \mathbf{A}_{1j} \mathbf{W}_1 \\ \vdots \\ \mathbf{A}_{Nj} \mathbf{W}_N \end{pmatrix}, \quad \mathbf{h}_j = \begin{pmatrix} \mathbf{h}_{0j} \\ \mathbf{h}_{1j} \\ \vdots \\ \mathbf{h}_{Nj} \end{pmatrix}, \quad \dot{\mathbf{u}}_t = \begin{pmatrix} \dot{\mathbf{u}}_{0t} \\ \dot{\mathbf{u}}_{1t} \\ \vdots \\ \dot{\mathbf{u}}_{Nt} \end{pmatrix},$$

for $j = 0, 1, 2$. Since \mathbf{H}_0 is a known non-singular matrix that depends on the trade weights and parameter estimates, we can obtain the GVAR

$$\dot{\mathbf{x}}_t = \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{G}_1 \dot{\mathbf{x}}_{t-1} + \mathbf{G}_2 \dot{\mathbf{x}}_{t-2} + \dot{\mathbf{v}}_t, \quad (4)$$

where $\mathbf{G}_j = \mathbf{H}_0^{-1} \mathbf{H}_j$, $\mathbf{a}_j = \mathbf{H}_0^{-1} \mathbf{h}_j$, for $j = 0, 1, 2$, and $\dot{\mathbf{v}}_t = \mathbf{H}_0^{-1} \dot{\mathbf{u}}_t$. The GVAR can be solved recursively and used for a variety of purposes.

There are no restrictions on the covariance matrix $\Sigma = \mathbf{E}(\dot{\mathbf{v}}_t \dot{\mathbf{v}}_t')$. For each country we have a $k_i \times 1$ vector of estimated residuals $\hat{\mathbf{u}}_{it}$ from which can be calculated $\hat{\mathbf{v}}_{it}$ and the elements of the covariance matrix are estimated freely by the $k_i \times k_j$ matrix $\hat{\Sigma}_{ij} = \sum_t \hat{\mathbf{v}}_{it} \hat{\mathbf{v}}_{jt}' / T$.

The model has 134 endogenous variables 71 stochastic trends and 63 cointegrating relations. A full description of the model properties including an analysis of its stability is given in DdPS. Although linear with a simple overall structure, this is a large and complicated model which allows for a large degree of interdependence. There are two routes for between country interdependence, through the impact of the $\dot{\mathbf{x}}_{it}^*$ variables and through the covariances. The effects through the $\dot{\mathbf{x}}_{it}^*$ are generally large, shocks to one country have marked effects on other countries. The between country covariances are quite small, with the exception of those for the real exchange rate equations, perhaps because of the base-country effect, since they are all expressed against the US dollar.

4 A Framework for Counterfactual Analysis

4.1 A Simple Illustration

Before presenting our counterfactual framework for the analysis of UK entry it might be helpful to consider a simple illustration where we focus only on two

variables, x_1 and x_2 in the context of a one-period commitment problem. Suppose that in the absence of interventions the variables (x_1, x_2) have a bivariate normal distribution with means (μ_1, μ_2) and variance matrix

$$\begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}.$$

Denote the corresponding random variables after the intervention by x_1^* and x_2^* , and suppose that the objective of the intervention is to ensure $x_1^* = x_2^*$,¹ with the post-intervention variables being no more volatile than the pre-intervention ones, namely $V(x_1^*) \leq V(x_1)$ and $V(x_2^*) \leq V(x_2)$.² To clarify the role of these optimality conditions in deriving our proposed solution we introduce the intervention variables η_1 and η_2 so that

$$\begin{aligned} x_1^* &= \mu_1 + \eta_1 + u_1, \\ x_2^* &= \mu_2 + \eta_2 + u_2, \end{aligned}$$

and η_1 and η_2 are such that

$$\eta_1 - \eta_2 = \mu_2 - \mu_1 + u_2 - u_1. \quad (5)$$

Clearly, without further restrictions, there are many possible forms of interventions that could satisfy (5). For example, intervention could only affect x_2 and not x_1 , with x_1 realised first and then x_2 set to equal x_1 . Under this set up $\eta_1 = 0$, and $\eta_2 = \mu_1 - \mu_2 + u_1 - u_2$ so that x_1^* and x_2^* both will be distributed as $N(\mu_1, \sigma_1^2)$. But such an intervention need not be optimal since it does not necessarily ensure that $V(x_1^*) \leq V(x_1)$ and $V(x_2^*) \leq V(x_2)$.

Under the above set up and noting that x_1 and x_2 are jointly normally distributed, the mean and variance of x_1^* (or x_2^*) are given by³

$$E(x_1^*) = E(x_1 | x_1 - x_2 = 0) = \mu_1 + \frac{\text{Cov}(x_1, x_1 - x_2)}{V(x_1 - x_2)} [0 - (\mu_1 - \mu_2)],$$

and

$$V(x_1^*) = V(x_1 | x_1 - x_2 = 0) = V(x_1) - \frac{\text{Cov}(x_1, x_1 - x_2)\text{Cov}(x_1 - x_2, x_1)}{V(x_1 - x_2)},$$

which can be simplified to

$$E(x_1^*) = E(x_2^*) = \frac{\mu_1(\sigma_2^2 - \rho\sigma_1\sigma_2) + \mu_2(\sigma_1^2 - \rho\sigma_2\sigma_1)}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2},$$

¹Note that by construction $x_1^* = x_2^* = x^*$, where x^* is also normally distributed under our assumptions.

²In the context of more structured problems, one could also employ utility-based optimality conditions. However, to the extent that lower volatilities are welfare enhancing our statistical approach can be reconciled with alternative utility-based procedures.

³Recall that for normally distributed random variables we have

$$E(\mathbf{y} | \mathbf{x} = \mathbf{d}) = E(\mathbf{y}) + \mathbf{Cov}(\mathbf{y}, \mathbf{x})\mathbf{V}(\mathbf{x})^{-1}(\mathbf{d} - E(\mathbf{x})).$$

and

$$V(x_1^*) = V(x_2^*) = \frac{\sigma_1^2 \sigma_2^2 (1 - \rho^2)}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}.$$

It is also easily verified that $V(x_i^*) \leq V(x_i)$ for $i = 1, 2$, as required.

Finally, for future reference we note that $Cov(x_1, x_1^*) = Cov(x_2, x_1^*) = V(x_1^*)$. To prove this result consider the following convex combination of the random variables x_1 and x_1^*

$$z = \lambda x_1 + (1 - \lambda) x_1^*,$$

where λ is a scalar in the range $0 \leq \lambda \leq 1$. Since under our intervention rule $V(x_1^*) \leq V(x_1)$, hence the value of λ that minimizes $V(z)$ must be zero. From the first order condition for the minimization of $V(z)$ with respect to λ , the optimal value of λ , say λ^* , is given by

$$\lambda^* = \frac{V(x_1^*) - Cov(x_1, x_1^*)}{V(x_1 - x_1^*)} = 0,$$

and hence $Cov(x_1, x_1^*) = V(x_1^*)$, as required.

4.2 Modeling of Entry

The illustrative model of the above section is static and focuses on a single period where the forecast and the commitment periods are the same. But in the case of dynamic models, as noted in the introduction, it is important that a distinction is made between the forecast horizon and the commitment period. Let H denote the forecast horizon, $h = 1, 2, \dots, H$, and \bar{H} the commitment horizon, $j = 1, 2, \dots, \bar{H}$, such that $H < \bar{H}$. The choice of \bar{H} would typically depend on the nature and the expected durability of the commitment. In what follows we take H and \bar{H} as given. In the empirical applications we ensure that \bar{H} is sufficiently large relative to H .

Also the GVAR includes as endogenous variables in $\hat{\mathbf{x}}_t$ the real exchange rates and rates of inflation of the various countries. To analyse the effect of UK entry we need to transform the model to explain a new set of endogenous variables \mathbf{x}_t which include the levels of prices and nominal exchange rates. We treat a counterfactual as a restriction on the space of endogenous variables, \mathbf{x}_t . In this case the restriction is the equality between UK and EA nominal exchange rates and interest rates. For entry at the end of period T (e.g. 1998Q4), the restrictions can be written in the form: $\Psi \mathbf{x}_{T+j} = \mathbf{d}_{T+j}$, $j = 1, 2, \dots, \bar{H}$. where the restrictions embodied in Ψ and \mathbf{d}_{T+j} are defined by the counterfactual. UK entry imposes two restrictions on the endogenous variables. We can then calculate the unrestricted expected values $\boldsymbol{\mu}_h = E(\mathbf{x}_{T+h} | \mathcal{I}_T)$ and the restricted expected values $\boldsymbol{\mu}_h^* = E(\mathbf{x}_{T+h} | \mathcal{I}_T, \Psi \mathbf{x}_{T+j} = \mathbf{d}_{T+j}, j = 1, 2, \dots, \bar{H})$, for $h = 1, 2, \dots, H < \bar{H}$. The difference in these expected values is our point estimate

of the effects of the counterfactual, UK entry. We also derive the variance-covariance matrix of the difference for each horizon. Assuming joint normality, this allows us to make probability statements about the effects of entry on events, for example that UK GDP will be higher, and joint probability statements, e.g. that UK GDP will be higher and UK prices lower across the different horizons.

The advantage of the GVAR in this exercise is that it can consistently allow for a large degree of interdependence through a variety of channels within a coherent structure. One disadvantage for exposition is that the linkages in the GVAR are complex. We assume that the parameters of the model remain constant at the values estimated from the historical sample. The stochastic trends in the model mean that the variance of the forecast \mathbf{x}_{T+h} increases at the rate, h , thus the forecasts themselves are likely to have large standard errors which will increase with the horizon. The estimates of the effect of entry, the difference between two forecasts, will have smaller variances than the forecasts, but the variance of the effect of entry will also increase at the rate, h . To allow for this uncertainty we will focus on the probability of particular events. Specifically, we consider the probability that output in the UK and EA will be higher with entry, that prices will be lower and the joint probability of higher output and lower prices.

With the above considerations in mind, suppose that UK joins euro at the end of date T with the commitment (agreed by the euro area and UK authorities jointly) that UK's nominal exchange rate, $E_{2,T+j}$, and short term interest rate, $r_{2,T+j}^S$, will be fixed at the corresponding euro values over the commitment period $j = 1, 2, \dots, \bar{H}$.⁴ Since in the GVAR all exchange rates are measured with respect to the US dollar, the alignment of nominal exchange rates between UK and euro can be achieved by setting

$$\frac{E_{2,T+j}}{E_{1,T+j}} = \gamma \frac{E_{2,T}}{E_{1,T}}, \quad (6)$$

where γ denotes the £/euro rate at which UK joins the euro. The case of $\gamma = 1$ implies entry at par, i.e nominal £/euro rate being equivalent to that at the end of period T . Clearly entry at a different exchange rate can also be entertained. In the case, for example, of a UK entry with a rate which is 10% (say) below the £/euro at the end of period T , $\gamma = 0.90$. Taking logs, (6) may also be written

$$e_{2,T+j} - e_{1,T+j} = e_{2T} - e_{1T} + \ln(\gamma), \text{ for } j = 1, 2, \dots, \bar{H}. \quad (7)$$

For short term interest rates we have

$$r_{2,T+j}^S = r_{1,T+j}^S \text{ for } j = 1, 2, \dots, \bar{H}. \quad (8)$$

To work out the effects of UK entry we shall consider two multi-period forecast scenarios: (i) the forecasts of \mathbf{x}_{T+h} , $h = 1, 2, \dots, H$ in the absence of a UK entry, (ii) forecasts in the presence of a UK entry. This could be done either in terms of point or probability forecasts that allow for parameter

⁴In what follows we denote UK and euro variables by the subscripts '2' and '1', respectively.

and/or model uncertainty. Under entry at 1999Q1, we further consider forecasts conditional on the realisations of four variables namely, oil prices (endogenous to the US model), US equity prices and US short and long term interest rates. As mentioned at the outset, it is unlikely that UK entry would have any effect on these particular variables.

As noted earlier, the GVAR uses real exchange rates as a variable, but to model entry we need to fix nominal exchange rates. The solution of the GVAR given by (4) can be expressed in terms of the price level and the nominal exchange rate yielding the equation

$$\mathbf{x}_t = \mathbf{b}_0 + \mathbf{b}_1 t + \mathbf{F}_1 \mathbf{x}_{t-1} + \mathbf{F}_2 \mathbf{x}_{t-2} + \mathbf{F}_3 \mathbf{x}_{t-3} + \mathbf{v}_t, \quad (9)$$

where

$$\mathbf{b}_0 = \mathcal{B}\mathbf{a}_0, \mathbf{b}_1 = \mathcal{B}\mathbf{a}_1,$$

and

$$\mathbf{F}_1 = (\mathcal{C} + \mathcal{B}\mathbf{G}_1\mathcal{B}^{-1}), \mathbf{F}_2 = \mathcal{B}(\mathbf{G}_2\mathcal{B}^{-1} - \mathbf{G}_1\mathcal{B}^{-1}\mathcal{C}), \mathbf{F}_3 = -\mathcal{B}\mathbf{G}_2\mathcal{B}^{-1}\mathcal{C}.$$

The details of this procedure and the precise relationship between \mathbf{x}_t and $\hat{\mathbf{x}}_t$, defined by (1), can be found in the Appendix where for expositional convenience the first three countries in the $k \times 1$ vector $\mathbf{x}_t = (\mathbf{x}'_{0t}, \mathbf{x}'_{1t}, \dots, \mathbf{x}'_{Nt})'$ are ordered as US, EA and UK respectively, followed by the remaining. Similarly, in all country models, \mathbf{x}_{it} , the first variable is the price level p_{it} and, with the exception of the US, the second and third entries in \mathbf{x}_{it} are the nominal exchange rate, e_{it} , and the short term interest rate, r_{it}^S , followed by the remaining.

Using (9), the point forecasts under (i) (in the absence of UK entry) are given recursively by

$$\boldsymbol{\mu}_h = \hat{\mathbf{b}}_0 + \hat{\mathbf{b}}_1(T+h) + \hat{\mathbf{F}}_1 \boldsymbol{\mu}_{h-1} + \hat{\mathbf{F}}_2 \boldsymbol{\mu}_{h-2} + \hat{\mathbf{F}}_3 \boldsymbol{\mu}_{h-3}, \text{ for } h = 1, 2, \dots, H \quad (10)$$

with the initial values $\boldsymbol{\mu}_0 = \mathbf{x}_T$, $\boldsymbol{\mu}_{-1} = \mathbf{x}_{T-1}$ and $\boldsymbol{\mu}_{-2} = \mathbf{x}_{T-2}$, where

$$\begin{aligned} \hat{\mathbf{F}}_1 &= (\mathcal{C} + \mathcal{B}\hat{\mathbf{G}}_1\mathcal{B}^{-1}), \hat{\mathbf{F}}_2 = \mathcal{B}(\hat{\mathbf{G}}_2\mathcal{B}^{-1} - \hat{\mathbf{G}}_1\mathcal{B}^{-1}\mathcal{C}), \hat{\mathbf{F}}_3 = -\mathcal{B}\hat{\mathbf{G}}_2\mathcal{B}^{-1}\mathcal{C}, \\ \hat{\mathbf{b}}_0 &= \mathcal{B}\hat{\mathbf{a}}_0, \hat{\mathbf{b}}_1 = \mathcal{B}\hat{\mathbf{a}}_1 \end{aligned}$$

and $\hat{\mathbf{a}}_0$, $\hat{\mathbf{a}}_1$, $\hat{\mathbf{G}}_1$ and $\hat{\mathbf{G}}_2$ are obtained from the individual country estimates using the GVAR modeling framework (described in Section 3).

It is also convenient to work with the companion form of (9) and so we have

$$\begin{pmatrix} \mathbf{x}_t \\ \mathbf{x}_{t-1} \\ \mathbf{x}_{t-2} \end{pmatrix} = \begin{pmatrix} \mathbf{F}_1 & \mathbf{F}_2 & \mathbf{F}_3 \\ \mathbf{I}_k & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_k & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{x}_{t-1} \\ \mathbf{x}_{t-2} \\ \mathbf{x}_{t-3} \end{pmatrix} + \begin{pmatrix} \mathbf{b}_0 + \mathbf{b}_1 t \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} + \begin{pmatrix} \mathbf{v}_t \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix},$$

or

$$\mathbf{X}_t = \mathbf{F}\mathbf{X}_{t-1} + \mathbf{D}_t + \mathbf{V}_t.$$

Hence

$$\mathbf{X}_{T+h} = \mathbf{F}^h \mathbf{X}_T + \sum_{s=0}^{h-1} \mathbf{F}^s \mathbf{D}_{T+h-s} + \sum_{s=0}^{h-1} \mathbf{F}^s \mathbf{V}_{T+h-s}, \quad (11)$$

and

$$\mathbf{x}_{T+h} = \mathbf{E}_1 \mathbf{X}_{T+h},$$

where $\mathbf{E}_1 = \begin{pmatrix} \mathbf{I}_k & \mathbf{0}_{k \times k} & \mathbf{0}_{k \times k} \end{pmatrix}$.

Therefore, conditional on the initial values, \mathbf{X}_T , and in the absence of entry

$$\mathbf{x}_{T+h} | \mathcal{I}_T \sim N(\boldsymbol{\mu}_h, \boldsymbol{\Omega}_{hh}), \quad (12)$$

where $\boldsymbol{\mu}_h$ is given by⁵

$$\boldsymbol{\mu}_h = \mathbf{E}_1 \mathbf{F}^h \mathbf{X}_T + \sum_{s=0}^{h-1} \mathbf{E}_1 \mathbf{F}^s \mathbf{D}_{T+h-s}, \quad (13)$$

and

$$\boldsymbol{\Omega}_{hh} = \mathbf{E}_1 \sum_{s=0}^{h-1} \mathbf{F}^s \tilde{\boldsymbol{\Sigma}} \mathbf{F}'^s \mathbf{E}_1', \quad (14)$$

in which

$$\tilde{\boldsymbol{\Sigma}} = \begin{pmatrix} \boldsymbol{\Sigma} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}, \quad (15)$$

and $\boldsymbol{\Sigma} = \mathcal{B}Cov(\hat{\mathbf{v}}_{T+h-i})\mathcal{B}'$. The covariance of $\hat{\mathbf{v}}_i$ can be estimated using the residuals of the global model, (4).

4.3 Point and Probability Forecasts Conditional on UK Entry

Under UK entry, forecasts of the global model must be obtained subject to the entry restrictions (7) and (8) which can also be written more generally as

$$\boldsymbol{\Psi} \mathbf{x}_{T+j} = \mathbf{d}_{T+j}, \quad j = 1, 2, \dots, \bar{H} \quad (16)$$

where $\boldsymbol{\Psi}$ is a suitably defined $l \times k$ matrix and \mathbf{d}_{T+j} is a $l \times 1$ vector of known constants. In the case of the entry restrictions (7) and (8) we have $l = 2$,

$$\mathbf{d}_{T+j} = \begin{pmatrix} e_{2T} - e_{1T} + \ln(\text{rate of appreciation/depreciation}) \\ 0 \end{pmatrix} = d_T \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

for $j = 1, 2, \dots, \bar{H}$, where $d_T = e_{2T} - e_{1T} + \ln(\text{rate of appreciation/depreciation})$, and

$$\boldsymbol{\Psi} = \begin{pmatrix} \mathbf{0}_{1 \times k_0} & 0 & -1 & 0 & \mathbf{0}_{1 \times (k_1-3)} & 0 & 1 & 0 & \mathbf{0}_{1 \times (k_2-3)} & \mathbf{0}_{1 \times (k-k_0-k_1-k_2)} \\ \mathbf{0}_{1 \times k_0} & 0 & 0 & -1 & \mathbf{0}_{1 \times (k_1-3)} & 0 & 0 & 1 & \mathbf{0}_{1 \times (k_2-3)} & \mathbf{0}_{1 \times (k-k_0-k_1-k_2)} \end{pmatrix}. \quad (17)$$

⁵ $\boldsymbol{\mu}_h$ can also be computed using the recursions (10).

Point forecasts of \mathbf{x}_{T+h} under UK entry restrictions (16) are given by, for $h = 1, 2, \dots, H$,

$$\mu_h^* = E(\mathbf{x}_{T+h} | \mathcal{I}_T, \Psi \mathbf{x}_{T+j} = \mathbf{d}_{T+j}, j = 1, 2, \dots, \bar{H}).$$

In deriving these expectations we assume that the announcement of the entry restrictions does not affect the GVAR parameters, \mathbf{F}_i , $i = 1, 2, 3$ and Σ .⁶

First note that, using (11), we have

$$\mathbf{x}_{T+h} = \boldsymbol{\mu}_h + \boldsymbol{\xi}_{T+h}, \quad (18)$$

where

$$\boldsymbol{\xi}_{T+h} = \sum_{s=0}^{h-1} \mathbf{E}_1 \mathbf{F}^s \mathbf{V}_{T+h-s}.$$

Also

$$\boldsymbol{\xi}_{T+h} | \mathcal{I}_T \sim N(\mathbf{0}, \boldsymbol{\Omega}_{hh}),$$

where $\boldsymbol{\Omega}_{hh}$ is given by (14). The entry restrictions now imply that

$$\Psi \boldsymbol{\xi}_{T+j} = \mathbf{d}_{T+j} - \Psi \boldsymbol{\mu}_j \text{ for } j = 1, 2, \dots, \bar{H}. \quad (19)$$

Setting $\mathbf{g}_j = \mathbf{d}_{T+j} - \Psi \boldsymbol{\mu}_j \forall j$, (19) can be written as

$$(\mathbf{I}_{\bar{H}} \otimes \Psi) \tilde{\boldsymbol{\xi}}_{\bar{H}} = \tilde{\mathbf{g}}_{\bar{H}},$$

where $\tilde{\boldsymbol{\xi}}_{\bar{H}} = (\boldsymbol{\xi}'_{T+1}, \boldsymbol{\xi}'_{T+2}, \dots, \boldsymbol{\xi}'_{T+\bar{H}})'$ and $\tilde{\mathbf{g}}_{\bar{H}} = (\mathbf{g}'_1, \mathbf{g}'_2, \dots, \mathbf{g}'_{\bar{H}})'$.

Under joint normality of the shocks, we have, for $h = 1, \dots, H < \bar{H}$

$$\begin{aligned} & E(\boldsymbol{\xi}_{T+h} | \mathcal{I}_T, \Psi \mathbf{x}_{T+j} = \mathbf{d}_{T+j}, j = 1, \dots, \bar{H}) \\ &= E\left(\boldsymbol{\xi}_{T+h} | \mathcal{I}_T, (\mathbf{I}_{\bar{H}} \otimes \Psi) \tilde{\boldsymbol{\xi}}_{\bar{H}} = \tilde{\mathbf{g}}_{\bar{H}}\right) \\ &= (\mathbf{s}'_{h\bar{H}} \otimes \mathbf{I}_k) \tilde{\boldsymbol{\Omega}}_{\bar{H}} (\mathbf{I}_{\bar{H}} \otimes \Psi)' [(\mathbf{I}_{\bar{H}} \otimes \Psi) \tilde{\boldsymbol{\Omega}}_{\bar{H}} (\mathbf{I}_{\bar{H}} \otimes \Psi)']^{-1} \tilde{\mathbf{g}}_{\bar{H}}, \end{aligned}$$

where $\mathbf{s}_{h\bar{H}}$ is a $\bar{H} \times 1$ selection vector with unity as its h^{th} element and zeros elsewhere, and $\tilde{\boldsymbol{\Omega}}_{\bar{H}}$ is the $k\bar{H} \times k\bar{H}$ matrix

$$\tilde{\boldsymbol{\Omega}}_{\bar{H}} = \begin{pmatrix} \boldsymbol{\Omega}_{11} & \boldsymbol{\Omega}_{12} & \cdots & \boldsymbol{\Omega}_{1\bar{H}} \\ \boldsymbol{\Omega}_{21} & \boldsymbol{\Omega}_{22} & \cdots & \boldsymbol{\Omega}_{2\bar{H}} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\Omega}_{\bar{H}1} & \boldsymbol{\Omega}_{\bar{H}2} & \cdots & \boldsymbol{\Omega}_{\bar{H}\bar{H}} \end{pmatrix}.$$

The diagonal elements of $\tilde{\boldsymbol{\Omega}}_{\bar{H}}$, that is $\{\boldsymbol{\Omega}_{ii}\}_{i=1}^{\bar{H}}$, are given by (14), while the off-diagonal elements can be expressed as

$$\boldsymbol{\Omega}_{ij} = \begin{cases} \mathbf{E}_1 (\sum_{s=0}^{i-1} \mathbf{F}^s \tilde{\Sigma} \mathbf{F}'^s) \mathbf{F}'^{(j-i)} \mathbf{E}'_1, & i < j \\ \mathbf{E}_1 \mathbf{F}^{(i-j)} (\sum_{s=0}^{j-1} \mathbf{F}^s \tilde{\Sigma} \mathbf{F}'^s) \mathbf{E}'_1, & i > j \end{cases}$$

⁶Other possibilities can also be entertained. But we do not believe there are sufficient grounds to make *a priori* assumptions about the way some of the parameters might change as a result of the UK entry. The general issue of parameter and model uncertainty does, however, deserve further attention.

where $\tilde{\Sigma}$ is defined by (15).

Hence, point forecasts under UK entry restrictions are given by

$$\boldsymbol{\mu}_h^* = \boldsymbol{\mu}_h + (\mathbf{s}'_{h\bar{H}} \otimes \mathbf{I}_k) \tilde{\boldsymbol{\Omega}}_{\bar{H}}(\mathbf{I}_{\bar{H}} \otimes \boldsymbol{\Psi}') [(\mathbf{I}_{\bar{H}} \otimes \boldsymbol{\Psi}) \tilde{\boldsymbol{\Omega}}_{\bar{H}}(\mathbf{I}_{\bar{H}} \otimes \boldsymbol{\Psi}')]^{-1} \tilde{\mathbf{g}}_{\bar{H}}. \quad (20)$$

It is easily seen that $\boldsymbol{\mu}_h^*$ satisfy the entry restrictions, namely $\boldsymbol{\Psi} \boldsymbol{\mu}_h^* = \mathbf{d}_{T+h}$. To see this note that

$$\boldsymbol{\Psi}(\mathbf{s}'_{h\bar{H}} \otimes \mathbf{I}_k) = (\mathbf{1} \otimes \boldsymbol{\Psi})(\mathbf{s}'_{h\bar{H}} \otimes \mathbf{I}_k) = (\mathbf{s}'_{h\bar{H}} \otimes \boldsymbol{\Psi}) = (\mathbf{s}'_{h\bar{H}} \otimes \mathbf{I}_2)(\mathbf{I}_{\bar{H}} \otimes \boldsymbol{\Psi}),$$

and $(\mathbf{s}'_{h\bar{H}} \otimes \mathbf{I}_2) \tilde{\mathbf{g}}_{\bar{H}} = \mathbf{g}_h = \mathbf{d}_{T+h} - \boldsymbol{\Psi} \boldsymbol{\mu}_h$. Recall that under UK entry restrictions $\boldsymbol{\Psi}$ is $2 \times k$.

For future reference also note that the mean effects of UK entry are given by

$$\boldsymbol{\mu}_h^* - \boldsymbol{\mu}_h = (\mathbf{s}'_{h\bar{H}} \otimes \mathbf{I}_k) \tilde{\boldsymbol{\Omega}}_{\bar{H}}(\mathbf{I}_{\bar{H}} \otimes \boldsymbol{\Psi}') [(\mathbf{I}_{\bar{H}} \otimes \boldsymbol{\Psi}) \tilde{\boldsymbol{\Omega}}_{\bar{H}}(\mathbf{I}_{\bar{H}} \otimes \boldsymbol{\Psi}')]^{-1} \tilde{\mathbf{g}}_{\bar{H}}. \quad (21)$$

4.4 Probability Distribution of the Effects of UK Entry

Let \mathbf{x}_{T+h}^* be the values of \mathbf{x}_{T+h} under UK entry

$$\mathbf{x}_{T+h}^* = \boldsymbol{\mu}_h + \boldsymbol{\xi}_{T+h}^*, \quad h = 1, 2, \dots, H$$

where $\boldsymbol{\mu}_h$ is given by (13) and $\boldsymbol{\xi}_{T+h}^*$ is the random variable defined by the probability distribution of $\boldsymbol{\xi}_{T+h}$ conditional on $(\mathbf{I}_{\bar{H}} \otimes \boldsymbol{\Psi}) \tilde{\boldsymbol{\xi}}_{\bar{H}} = \tilde{\mathbf{g}}_{\bar{H}}$. More specifically,

$$\boldsymbol{\xi}_{T+h}^* = \sum_{s=0}^{h-1} \mathbf{E}_1 \mathbf{F}^s \mathbf{V}_{T+h-s} \Big|_{(\mathbf{I}_{\bar{H}} \otimes \boldsymbol{\Psi}) \tilde{\boldsymbol{\xi}}_{\bar{H}} = \tilde{\mathbf{g}}_{\bar{H}}}, \quad (22)$$

for $h = 1, 2, \dots, H < \bar{H}$.⁷ The effects of UK entry in this case are given by

$$\boldsymbol{\delta}_{T+h} = \mathbf{x}_{T+h}^* - \mathbf{x}_{T+h} = \boldsymbol{\xi}_{T+h}^* - \boldsymbol{\xi}_{T+h}$$

The mean effects are

$$E(\boldsymbol{\delta}_{T+h} | \mathcal{I}_T) = \boldsymbol{\mu}_h^* - \boldsymbol{\mu}_h.$$

The variance $\mathbf{V}(\boldsymbol{\delta}_{T+h} | \mathcal{I}_T)$ is given by

$$\begin{aligned} \mathbf{V}(\boldsymbol{\delta}_{T+h} | \mathcal{I}_T) &= \mathbf{V}(\boldsymbol{\xi}_{T+h}^* | \mathcal{I}_T) + \mathbf{V}(\boldsymbol{\xi}_{T+h} | \mathcal{I}_T) \\ &\quad - Cov(\boldsymbol{\xi}_{T+h}^*, \boldsymbol{\xi}_{T+h} | \mathcal{I}_T) - Cov(\boldsymbol{\xi}_{T+h}, \boldsymbol{\xi}_{T+h}^* | \mathcal{I}_T) \end{aligned} \quad (23)$$

where⁸

$$\mathbf{V}(\boldsymbol{\xi}_{T+h}^* | \mathcal{I}_T) = \boldsymbol{\Omega}_{hh}^* \quad (24)$$

with

$$\boldsymbol{\Omega}_{hh}^* = \boldsymbol{\Omega}_{hh} - (\mathbf{s}'_{h\bar{H}} \otimes \mathbf{I}_k) \tilde{\boldsymbol{\Omega}}_{\bar{H}}(\mathbf{I}_{\bar{H}} \otimes \boldsymbol{\Psi}') [(\mathbf{I}_{\bar{H}} \otimes \boldsymbol{\Psi}) \tilde{\boldsymbol{\Omega}}_{\bar{H}}(\mathbf{I}_{\bar{H}} \otimes \boldsymbol{\Psi}')]^{-1} (\mathbf{I}_{\bar{H}} \otimes \boldsymbol{\Psi}) \tilde{\boldsymbol{\Omega}}_{\bar{H}}(\mathbf{s}_{h\bar{H}} \otimes \mathbf{I}_k), \quad (25)$$

⁷To simplify the notations we are suppressing the dependence of $\boldsymbol{\xi}_{T+h}^*$ on \bar{H} . Strictly speaking, we should write $\boldsymbol{\xi}_{T+h, \bar{H}}^*$.

⁸Similarly, we suppress the dependence of $\boldsymbol{\Omega}_{hh}^*$ on \bar{H} .

and Ψ defined by (17).

To obtain $Cov(\xi_{T+h}^*, \xi_{T+h} | \mathcal{I}_T)$, recall that conditional on \mathcal{I}_T the $k \times 1$ random vector ξ_{T+h}^* defined by (22) is dependently distributed with mean μ_h^* and variance $\mathbf{V}(\xi_{T+h}^* | \mathcal{I}_T) = \Omega_{hh}^*$. Let $\tilde{\Psi}'_{\bar{H}} = \tilde{\Omega}_{\bar{H}}^{1/2}(\mathbf{I}_{\bar{H}} \otimes \Psi')$ and note that Ω_{hh}^* defined by (25) can alternatively be expressed as

$$\Omega_{hh}^* = \Omega_{hh} - (\mathfrak{s}'_{h\bar{H}} \otimes \mathbf{I}_k) \tilde{\Omega}_{\bar{H}}^{1/2} \mathbf{P}_{\bar{H}} \tilde{\Omega}_{\bar{H}}^{1/2} (\mathfrak{s}_{h\bar{H}} \otimes \mathbf{I}_k),$$

where

$$\mathbf{P}_{\bar{H}} = \tilde{\Psi}'_{\bar{H}} \left(\tilde{\Psi}_{\bar{H}} \tilde{\Psi}'_{\bar{H}} \right)^{-1} \tilde{\Psi}_{\bar{H}},$$

is a projection matrix which is symmetric, idempotent and positive semi-definite. Hence,

$$\Omega_{hh} - \Omega_{hh}^* = (\mathfrak{s}'_{h\bar{H}} \otimes \mathbf{I}_k) \tilde{\Omega}_{\bar{H}}^{1/2} \mathbf{P}_{\bar{H}} \tilde{\Omega}_{\bar{H}}^{1/2} (\mathfrak{s}_{h\bar{H}} \otimes \mathbf{I}_k).$$

is a positive semi-definite matrix.

As illustrated in Section 4.1, by considering the convex combination of the random variables $\mathbf{q}'\xi_{T+h}$ and $\mathbf{q}'\xi_{T+h}^*$ for any non-zero $k \times 1$ vector of constants \mathbf{q} we obtain

$$Cov(\xi_{T+h}, \xi_{T+h}^* | \mathcal{I}_T) = \mathbf{V}(\xi_{T+h}^* | \mathcal{I}_T). \quad (26)$$

Using this result in (26) the variance of the effects of UK entry is derived as

$$\mathbf{V}(\delta_{T+h} | \mathcal{I}_T) = \Omega_{hh} - \Omega_{hh}^* \geq 0$$

which is a positive semi-definite matrix. Hence,

$$\delta_{T+h} | \mathcal{I}_T \sim N(\mu_h^* - \mu_h, \Omega_{hh} - \Omega_{hh}^*). \quad (27)$$

4.5 Conditioning on US variables

When conditioning on the realisations of the four variables in the US model, a new $4 \times k$ restriction matrix Ψ_a is defined and imposed under both the case of no entry and UK entry. If the rest of the variables in the US are ordered as $\mathbf{x}_{0t}^{rest} = (y_{0t}, q_{0t}, r_{0t}^L, oil_t)'$, then we have ($k_0 = 4$)

$$\Psi_a = \begin{pmatrix} 0 & 1 & \mathbf{0}_{1 \times (k_0 - 2)} & \mathbf{0}_{1 \times (k - k_0)} \\ \mathbf{0}_{(k_0 - 3) \times (k_0 - 3)} & \mathbf{I}_{k_0 - 3} & \mathbf{0}_{(k_0 - 3) \times (k - k_0)} \end{pmatrix}. \quad (28)$$

Unlike the entry restrictions that are committed to extend beyond the forecast horizon, H , conditioning on the realisations of \mathbf{x}_{0t}^{rest} is only possible when such realisations are in fact available. Therefore, the analysis of this section is only applicable when the entry date is assumed to be prior to the end of the available sample.

Assuming that the observations \mathbf{x}_{0t}^{rest} , $t = T + 1, \dots, T + h$ are given, the associated restrictions can be written as

$$(\mathbf{I}_H \otimes \Psi_a) \tilde{\xi}_H = \tilde{\mathfrak{g}}_{aH},$$

where $\tilde{\boldsymbol{\xi}}_H = (\boldsymbol{\xi}'_{T+1}, \boldsymbol{\xi}'_{T+2}, \dots, \boldsymbol{\xi}'_{T+H})'$ and $\tilde{g}_{aH} = (\mathbf{g}'_{a1}, \mathbf{g}'_{a2}, \dots, \mathbf{g}'_{aH})'$, where $\mathbf{g}_{ah} = \mathbf{d}_{a,T+h} - \boldsymbol{\Psi}_a \boldsymbol{\mu}_h$, $\mathbf{d}_{a,T+h} = (r_{0,T+h}^S, q_{0,T+h}^S, r_{0,T+h}^L, oil_{T+h})'$. For these set of restrictions the point forecasts under no entry are given by

$$\boldsymbol{\mu}_{ah}^* = \boldsymbol{\mu}_h + (\mathbf{s}'_{hH} \otimes \mathbf{I}_k) \tilde{\boldsymbol{\Omega}}_H (\mathbf{I}_H \otimes \boldsymbol{\Psi}'_a) [(\mathbf{I}_H \otimes \boldsymbol{\Psi}_a) \tilde{\boldsymbol{\Omega}}_H (\mathbf{I}_H \otimes \boldsymbol{\Psi}'_a)]^{-1} \tilde{\mathbf{g}}_{aH},$$

where \mathbf{s}_{hH} is a $H \times 1$ vector of zeros with its h^{th} element set to unity.

Point forecasts under UK entry plus the US restrictions are given by

$$\boldsymbol{\mu}_{bh}^* = \boldsymbol{\mu}_h + (\mathbf{s}'_{h\bar{H}} \otimes \mathbf{I}_k) \tilde{\boldsymbol{\Omega}}_{\bar{H}} \tilde{\boldsymbol{\Psi}}'_b (\tilde{\boldsymbol{\Psi}}_b \tilde{\boldsymbol{\Omega}}_{\bar{H}} \tilde{\boldsymbol{\Psi}}'_b)^{-1} \tilde{\mathbf{g}}_b, \quad (29)$$

where $\tilde{\mathbf{g}}_b = (\mathbf{g}'_{b1}, \mathbf{g}'_{b2}, \dots, \mathbf{g}'_{bH}, \mathbf{g}'_{H+1}, \mathbf{g}'_{H+2}, \dots, \mathbf{g}'_{\bar{H}})'$,

$$\tilde{\boldsymbol{\Psi}}_b = \begin{pmatrix} \mathbf{I}_H \otimes \boldsymbol{\Psi}_b & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{\bar{H}-H} \otimes \boldsymbol{\Psi} \end{pmatrix}, \quad (30)$$

and

$$\boldsymbol{\Psi}_b = \begin{pmatrix} \boldsymbol{\Psi}_a \\ \boldsymbol{\Psi} \end{pmatrix}, \quad \mathbf{g}_{bh} = \begin{pmatrix} \mathbf{g}_{ah} \\ \mathbf{g}_h \end{pmatrix}. \quad (31)$$

The mean effects of committed UK entry are then given by the difference $\boldsymbol{\mu}_{bh}^* - \boldsymbol{\mu}_{ah}^*$. Conditional on the realisations of the variables in the US model, denoting $\mathbf{x}_{a,T+h}^*$ and $\mathbf{x}_{b,T+h}^*$ the values of \mathbf{x}_{T+h} under no entry and UK entry, respectively, we have

$$\mathbf{x}_{i,T+h}^* = \boldsymbol{\mu}_h + \boldsymbol{\xi}_{i,T+h}^*, \quad \text{for } i = a, b,$$

where $\boldsymbol{\mu}_h$ is given by (13), $\boldsymbol{\xi}_{a,T+h}^*$ is the random variables defined by the probability distribution of $\boldsymbol{\xi}_{T+h}$ conditional on

$$\{\boldsymbol{\Psi}_a \boldsymbol{\xi}_{T+j} = \mathbf{d}_{a,T+j} - \boldsymbol{\Psi}_a \boldsymbol{\mu}_j = \mathbf{g}_{aj}, \text{ for } j = 1, 2, \dots, H\},$$

and $\boldsymbol{\xi}_{b,T+h}^*$ is the vector of random variables defined by $\boldsymbol{\xi}_{T+h}$ conditional on $\tilde{\boldsymbol{\Psi}}_b \tilde{\boldsymbol{\xi}}_{\bar{H}} = \tilde{\mathbf{g}}_b$, where $\tilde{\boldsymbol{\Psi}}_b$ and $\tilde{\mathbf{g}}_b$ are as defined above.

The difference between the two counterfactuals, is given by $\boldsymbol{\delta}_{T+h} = \boldsymbol{\xi}_{b,T+h}^* - \boldsymbol{\xi}_{a,T+h}^*$ and we have

$$E(\boldsymbol{\delta}_{T+h} | \mathcal{I}_T) = \boldsymbol{\mu}_{bh}^* - \boldsymbol{\mu}_{ah}^*.$$

Also

$$\begin{aligned} \mathbf{V}(\boldsymbol{\delta}_{T+h} | \mathcal{I}_T) &= \mathbf{V}(\boldsymbol{\xi}_{b,T+h}^* | \mathcal{I}_T) + \mathbf{V}(\boldsymbol{\xi}_{a,T+h}^* | \mathcal{I}_T) \\ &\quad - Cov(\boldsymbol{\xi}_{b,T+h}^*, \boldsymbol{\xi}_{a,T+h}^* | \mathcal{I}_T) - Cov(\boldsymbol{\xi}_{a,T+h}^*, \boldsymbol{\xi}_{b,T+h}^* | \mathcal{I}_T), \end{aligned} \quad (32)$$

where

$$\mathbf{V}(\boldsymbol{\xi}_{i,T+h}^* | \mathcal{I}_T) = \boldsymbol{\Omega}_{ih}^*, \quad \text{for } i = a, b,$$

with

$$\boldsymbol{\Omega}_{ah}^* = \boldsymbol{\Omega}_{hh} - (\mathbf{s}'_{hH} \otimes \mathbf{I}_k) \tilde{\boldsymbol{\Omega}}_H \tilde{\boldsymbol{\Psi}}'_a (\tilde{\boldsymbol{\Psi}}_a \tilde{\boldsymbol{\Omega}}_H \tilde{\boldsymbol{\Psi}}'_a)^{-1} \tilde{\boldsymbol{\Psi}}_a \tilde{\boldsymbol{\Omega}}_H (\mathbf{s}_{hH} \otimes \mathbf{I}_k),$$

where $\tilde{\Psi}_a = (\mathbf{I}_H \otimes \Psi_a)$, and

$$\Omega_{bh}^* = \Omega_{hh} - (\mathbf{s}'_{h\bar{H}} \otimes \mathbf{I}_k) \tilde{\Omega}_{\bar{H}} \tilde{\Psi}'_b (\tilde{\Psi}_b \tilde{\Omega}_{\bar{H}} \tilde{\Psi}'_b)^{-1} \tilde{\Psi}_b \tilde{\Omega}_{\bar{H}} (\mathbf{s}_{h\bar{H}} \otimes \mathbf{I}_k).$$

Following similar lines of reasoning as above, we also have

$$\mathbf{V}(\delta_{T+h} | \mathcal{I}_T) = \Omega_{ah}^* - \Omega_{bh}^*,$$

which can be shown to be a positive semi-definite matrix. Hence,

$$\delta_{T+h} | \mathcal{I}_T \sim N(\boldsymbol{\mu}_{bh}^* - \boldsymbol{\mu}_{ah}^*, \Omega_{ah}^* - \Omega_{bh}^*). \quad (33)$$

5 Estimates of the Effects of UK Entry

5.1 Interest and Exchange Rate Co-movements

Euro entry involves restricting UK and EA interest rates and the rate of change of their exchange rate to be equal. To judge the feasibility of such a scenario it is worth examining the co-movements of interest rates and exchange rates of UK and the EA, particularly before the euro was formed. Figures 1 and 2 plot UK and EA short interest rates (in percent per annum) and exchange rate changes (in percent) from 1979Q1 to 2003Q4. Before 1999 the EA measures are PPP GDP weighted average of values for the largest eight EA members. The exchange rate measures are quarterly percentage changes of the EA and UK exchange rate against the US dollar. The UK and the EA series follow each other quite closely. The rate of change of exchange rates are I(0). But using a variety of tests, including the GLS and Weighted Symmetric versions of the ADF, one cannot reject a unit root in the UK and EA interest rates. However, they cointegrate, one can reject a unit root in the difference between interest rates. There has clearly been strong historical relationships between the interest rates and exchange rates of the two economies, and the patterns of these co-movements do not indicate any fundamental difficulty with equalization of either interest rates or exchange rates.

Figure 1: UK and EA Nominal Short-Term Interest Rates at Annual Percentage Rates

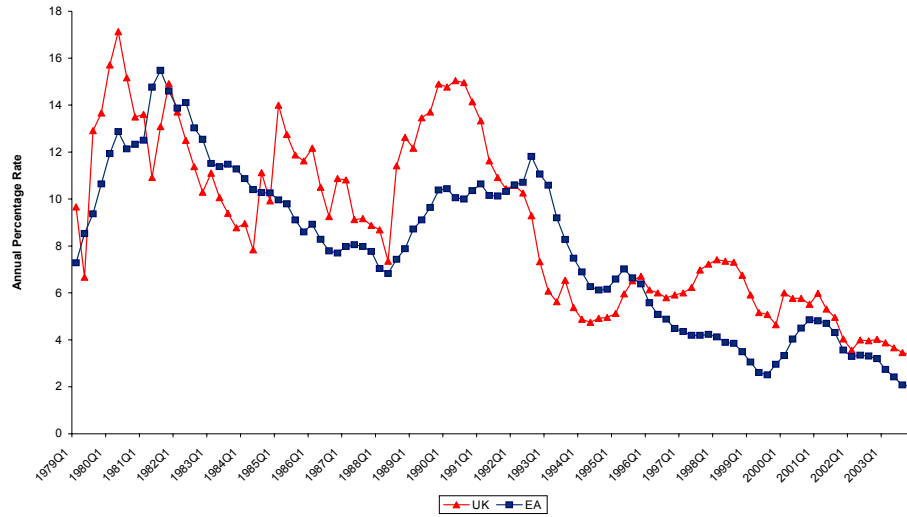
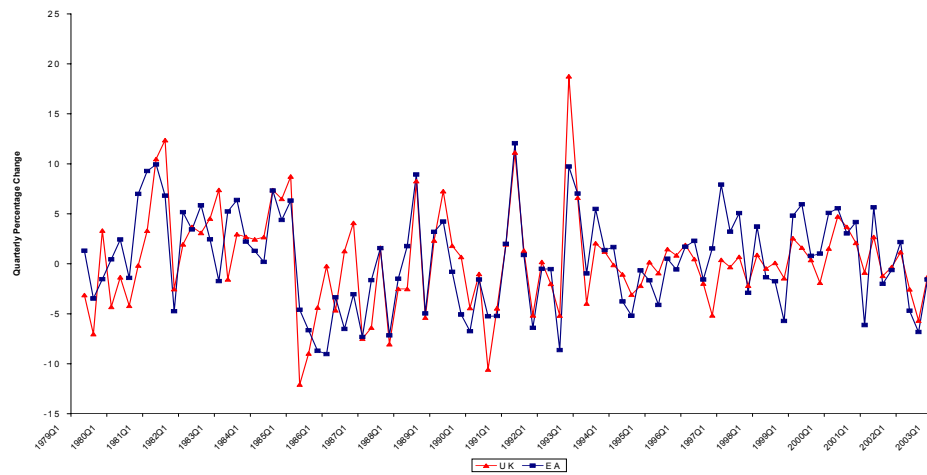


Figure 2: Quarterly Percentage Changes in UK and EA Exchange Rates



As can be seen from the graphs UK short-term interest rates were above euro area rates from the establishment of the euro, ranging from above 3% higher

at the start of the euro, dropping to a quarter of a percent higher in 2002Q4 increasing to one percent higher by 2003Q4, and 2.75% higher in 2005. The euro-sterling rate in January 1999 was 1.4, and stayed above this rate for the whole of the subsequent period; exceeding 1.7 in parts of 2000, then falling back towards 1.4.

5.2 Alternative Scenarios for the UK

Our base-line case considers entry in 1999Q1 and uses the GVAR to calculate probability distributions for the 5 years ahead to 2003Q4, with and without UK entry. We focus on output and the price level in the EA and UK. EA always refers to the existing euro area. Entry is assumed to be at par, the exchange rate prevailing in 1998Q4, with a long-term commitment to euro membership. We model the long-term commitment by imposing the restrictions for ten years, 40 quarters, at least twice the forecast horizon. Extending it to 56 quarters (the maximum number of quarters our computer codes can run on our pc) makes virtually no difference to the results. For practical considerations, as a first approximation, it is assumed that the parameters of the model are invariant to the nature of the commitment. The nature of the commitment does, however, affect the conditional forecasts (and expectations). Given that joining is such a major decision, it is not something that one would commit to on a period by period basis.⁹

We describe our results in terms of the probability of the effect of entry, $\delta_{T+h} = x_{T+h}^* - x_{T+h}$, having a particular sign. This takes account both of the size of the effect and its standard error. We are examining the probability distribution of the difference between two counterfactuals from the GVAR. We are not comparing our estimates with what actually happened over the period 1999Q1-2003Q4 to the UK and EA. Although we can estimate the shocks to the UK and EA without UK entry from the residuals of the model, we do not know what they would be with entry. There is no reason to suppose that they would be the same as without entry. This exercise is quite different from standard approaches such as impulse response analysis, and prior expectations based on such approaches may not be appropriate.

We compare this base-line case with various other scenarios. The first alternative considers entry at par in 1999Q1, but conditions on observed values of oil prices, US interest rates and US equity prices over the period 1999Q1-2003Q4. The second alternative has the UK entering the euro in 2004Q1 at par, the exchange rate in 2003Q4, and calculates the effects from 2004Q1 to 2008Q4 to examine the effect of initial conditions. The third alternative considers the effects different patterns of appreciation or depreciation have on entry in 1999Q1. Finally, for comparison we consider Swedish entry.

⁹As noted earlier these results are invariant to identification, in the sense that they would be exactly the same whatever set of just identifying set of restrictions one chose to impose on the underlying country-specific models.

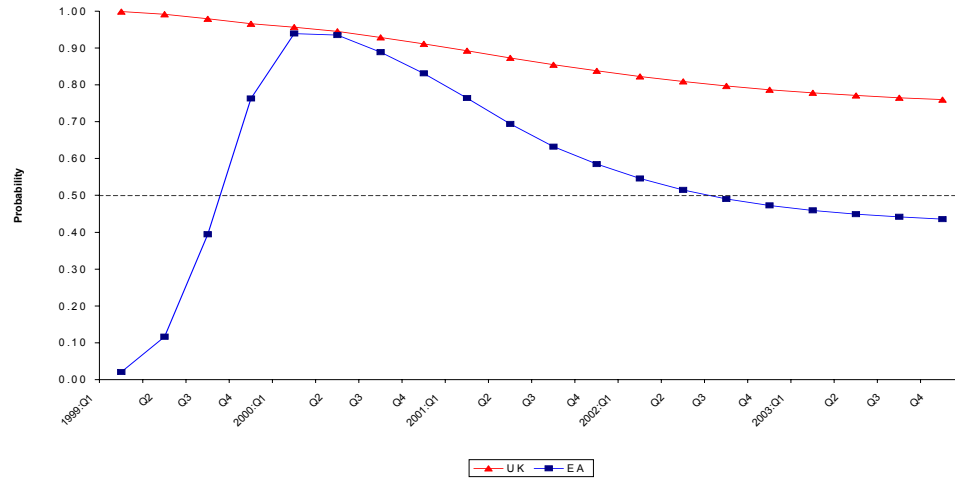
5.3 UK Entry at Par

Our base-line case has the UK entering the euro from the start at parity, the exchange rate prevailing in 1998Q4. In 1998Q4 UK short interest rates were 6.75% per annum compared to euro area short rates of 3.5% per annum. Table 2 and Figure 3 show the probability that UK and EA interest rates would have been lower with entry than without entry. The probability that UK short interest rates are lower with entry than without starts at almost exactly one and drifts down steadily to 76% in 2003Q4. The probability that EA interest rates are lower as a result of entry starts at zero (interest rates are certain to have been higher), rises sharply to peak at 94% in 2000Q1, then falls steadily to 44% in 2003Q4. The lower UK interest rates would tend to cause output to rise but this is offset by the exchange rate effect. Under no entry the UK is expected to depreciate against the euro, so relative to no entry sterling appreciates against the euro. We examine the exchange rate effects further below.

Table 2: Probability Estimates that UK and EA Short-Term Interest Rates are Lower under Entry at par at Beginning of 1999Q1

Year	Probability	
	UK	EA
1999Q1	1.00	0.02
Q2	0.99	0.12
Q3	0.98	0.39
Q4	0.97	0.76
2000Q1	0.96	0.94
Q2	0.95	0.94
Q3	0.93	0.89
Q4	0.91	0.83
2001Q1	0.89	0.76
Q2	0.87	0.69
Q3	0.86	0.63
Q4	0.84	0.58
2002Q1	0.82	0.55
Q2	0.81	0.51
Q3	0.80	0.49
Q4	0.79	0.47
2003Q1	0.78	0.46
Q2	0.77	0.45
Q3	0.77	0.44
Q4	0.76	0.44

Figure 3: Probability Estimates that UK and EA Short-Term Interest Rates are Lower under Entry at par at Beginning of 1999Q1



The effects on GDP and prices are shown in Table 3. Figure 4 shows estimates of the probability that UK and EA GDP are higher, Figure 5 the probability that prices are lower. The probability that UK GDP is higher as a result of entry starts very small, then increases peaking after four years at 62%, then starting to fall again to 58% in 2003Q4. The probability that EA GDP is higher as a result of entry also starts very small then increases steadily, reaching 46% in 2003Q4. The probability that UK prices will be lower with entry starts high, then falls steadily, reaching 66% in 2003Q4. The probability that EA prices will be lower starts at 47%, falls to 7% after a year, then rises to 17% by 2003Q4. The joint probability of higher GDP and lower prices in the UK starts close to zero and rises rapidly at first and then more slowly, finishing at 53%. The joint probability of higher output and lower prices is close to zero in the EA throughout.

Table 3: Probability Estimates that Output is Higher and Prices are Lower, separately and jointly in UK and EA under Entry at par at Beginning of 1999Q1

Year	Output		Prices		Output & Prices	
	UK	EA	UK	EU	UK	EU
1999Q1	0.03	0.00	0.99	0.47	0.03	0.00
Q2	0.00	0.00	0.99	0.22	0.00	0.00
Q3	0.02	0.01	0.95	0.10	0.01	0.00
Q4	0.06	0.03	0.94	0.08	0.04	0.00
2000Q1	0.11	0.07	0.93	0.07	0.08	0.00
Q2	0.15	0.11	0.90	0.07	0.12	0.00
Q3	0.24	0.16	0.87	0.08	0.20	0.00
Q4	0.37	0.21	0.85	0.09	0.32	0.00
2001Q1	0.49	0.26	0.83	0.10	0.43	0.01
Q2	0.56	0.30	0.80	0.10	0.49	0.01
Q3	0.60	0.34	0.78	0.11	0.53	0.01
Q4	0.62	0.37	0.76	0.12	0.54	0.01
2002Q1	0.62	0.39	0.74	0.13	0.55	0.01
Q2	0.62	0.41	0.73	0.14	0.55	0.01
Q3	0.62	0.43	0.71	0.15	0.55	0.01
Q4	0.61	0.44	0.70	0.15	0.55	0.01
2003Q1	0.60	0.44	0.69	0.16	0.54	0.02
Q2	0.60	0.45	0.68	0.16	0.54	0.02
Q3	0.59	0.45	0.67	0.17	0.54	0.02
Q4	0.58	0.46	0.66	0.17	0.53	0.02

Figure 4: Probability Estimates that UK and EA Output is Higher under Entry at par at Beginning of 1999Q1

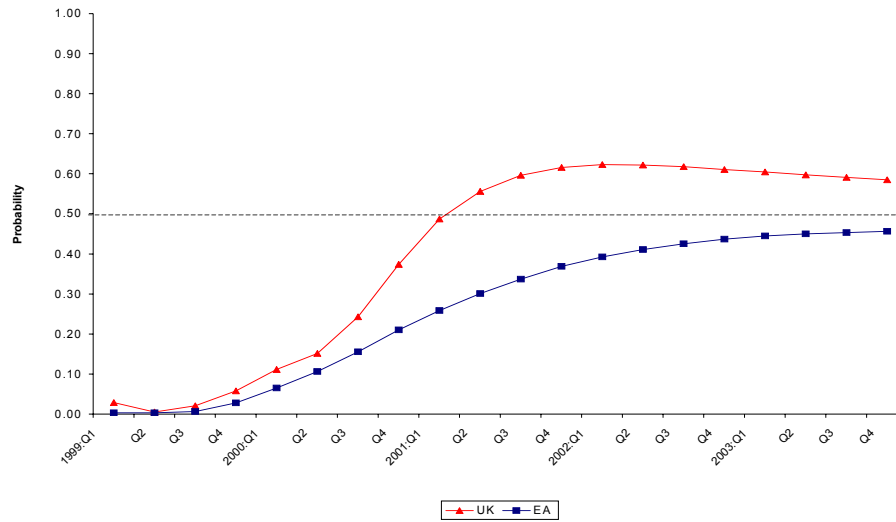
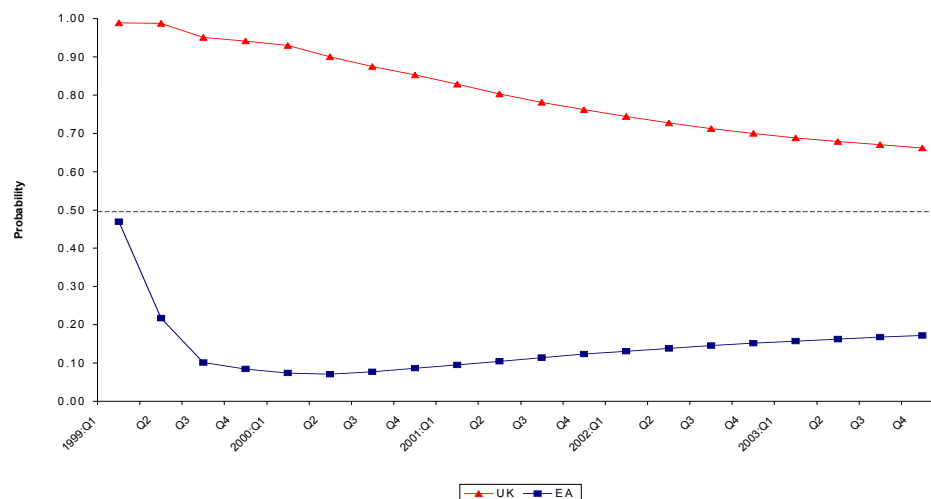


Figure 5: Probability Estimates that UK and EA Prices are Lower under Entry at par at Beginning of 1999Q1



In terms of output the UK initially would have lost from the entry, but gained in the medium term with higher output levels and lower prices. The EA would have experienced lower output and higher prices throughout the period. Thus UK entry seems likely to have generated welfare losses for the EA. For the UK the welfare effects are ambiguous depending on the relative weights given to output and inflation and the discount rate.

To check the robustness of this conclusion we considered a more general base line model where the forecasts are obtained conditional on the observed values of the oil price, US interest rates and US equity prices over the period 1999Q1-2003Q4. Since entry imposes restrictions on the model, the effect of these variables can be different in the restricted and unrestricted cases. This will be reflected in the measure of the effect of entry. Table 4 gives the probabilities that interest rates are lower as a result of entry. In the base-line case, the probability of UK interest rates being lower declines smoothly. With the global shocks included, the pattern is quite similar, but with some extra volatility. The probability that the EA interest rate is lower conditional on observed variables is similar for the first two years, rising from zero to a maximum of 98% then falling, but more slowly in the conditional case than the unconditional case being 67% in 2003Q4.

Table 4: Conditional Probability Estimates that UK and EA Short-Term Interest Rates are Lower under Entry at par at Beginning of 1999Q1

Year	Probability	
	UK	EA
1999Q1	1.00	0.01
Q2	0.99	0.36
Q3	0.97	0.87
Q4	0.94	0.96
2000Q1	0.92	0.98
Q2	0.91	0.97
Q3	0.88	0.96
Q4	0.88	0.94
2001Q1	0.85	0.94
Q2	0.81	0.93
Q3	0.82	0.91
Q4	0.79	0.89
2002Q1	0.75	0.86
Q2	0.75	0.80
Q3	0.72	0.78
Q4	0.68	0.77
2003Q1	0.66	0.76
Q2	0.68	0.75
Q3	0.64	0.73
Q4	0.69	0.67

Note: The probability estimates are obtained conditional on the realised values of oil prices, US equity prices and US short and long-term interest rates over the period 1999Q1-2003Q4.

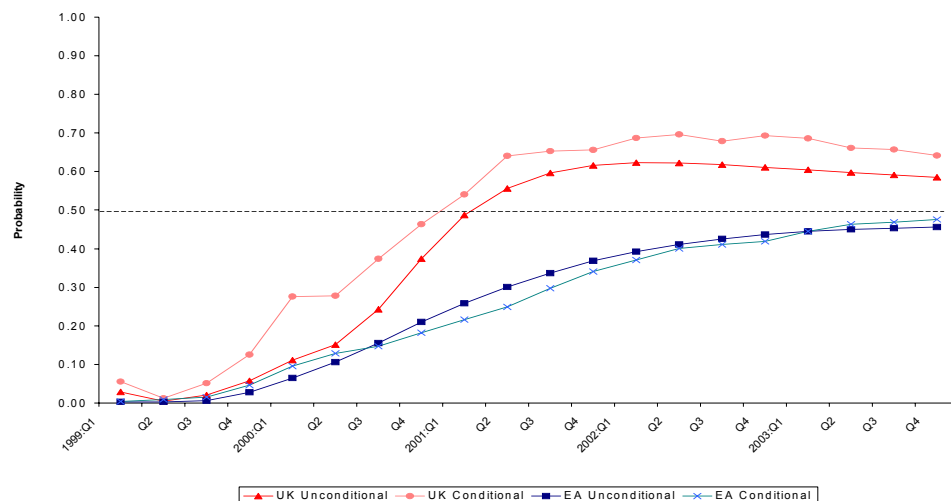
Table 5 shows the probabilities that GDP is higher or prices lower in the two areas as a result of UK entry at par, conditional on the observed values of oil prices, US equity prices and interest rates over the period 1999Q1-2003Q4. Figure 6 plots the unconditional and conditional probabilities that GDP is higher for the EA and UK. Conditioning on the observed values introduces some volatility and the UK conditional probabilities are rather higher than the unconditional probabilities. The EA conditional probabilities are somewhat lower, but the time profile of the effects are rather similar. These results suggest that we do not need to be able to forecast these four variables in order to measure the overall effects of the UK joining the euro.

Table 5: Conditional Probability Estimates that Output is Higher and Prices are Lower, separately in UK and EA under Entry at par at Beginning of 1999Q1

Year	Output		Prices	
	UK	EA	UK	EA
1999Q1	0.06	0.00	1.00	0.81
Q2	0.01	0.01	1.00	0.59
Q3	0.05	0.02	0.98	0.34
Q4	0.13	0.05	0.96	0.28
2000Q1	0.28	0.10	0.94	0.25
Q2	0.28	0.13	0.91	0.23
Q3	0.37	0.15	0.89	0.21
Q4	0.46	0.18	0.86	0.20
2001Q1	0.54	0.22	0.83	0.20
Q2	0.64	0.25	0.80	0.20
Q3	0.65	0.30	0.78	0.21
Q4	0.66	0.34	0.76	0.23
2002Q1	0.69	0.37	0.74	0.23
Q2	0.70	0.40	0.72	0.24
Q3	0.68	0.41	0.71	0.25
Q4	0.69	0.42	0.68	0.25
2003Q1	0.69	0.44	0.67	0.26
Q2	0.66	0.46	0.66	0.27
Q3	0.66	0.47	0.65	0.28
Q4	0.64	0.48	0.64	0.29

Note: The probability estimates are obtained conditional on the realised values of oil prices, US equity prices and US short and long-term interest rates over the period 1999Q1-2003Q4.

Figure 6: Conditional and Unconditional Probability Estimates that UK and EA Output is Higher under Entry at par at Beginning of 1999Q1



Note: The probability estimates are obtained conditional on the realised values of oil prices, US equity prices and US short and long-term interest rates over the period 1999Q1-2003Q4.

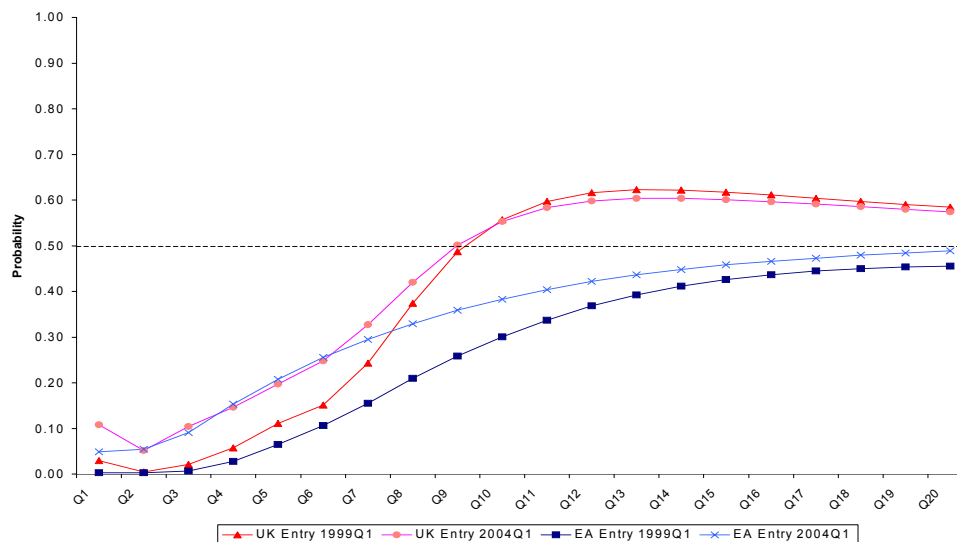
Table 6 shows the results for entry in 2004. Figure 7 plots the probability that GDP is higher for the EA and UK for 1999 entry and 2004 entry, superimposed. For GDP the time patterns are very similar, with the differences being larger at the beginning of the period, when the probability of both EA and UK output being higher is larger with 2004 entry than with 1999 entry, but the differences are very small by the end. For prices, the patterns over time are again similar, with the probability that prices are lower being rather smaller for the UK and, except for the first two periods, rather higher for the EA. The qualitative conclusions are thus almost identical whether one starts from 1999 or 2004. This may be partly because sterling and euro rates against the dollar were similar in 1998Q4 and 2003Q4 and the UK euro interest differential was quite similar, but other initial conditions were very different at the two entry dates. Thus our conclusions seem to be robust to initial conditions.

Conditioning on the realisations of observed variables and different initial conditions has quite small effects on the quantitative results and does not change the qualitative conclusions.

Table 6: Probability Estimates that Output is Higher and Prices are Lower, separately in UK and EA under Entry at par at Beginning of 2004Q1

Year	Output		Prices	
	UK	EA	UK	EA
2004Q1	0.11	0.05	0.84	0.25
Q2	0.05	0.05	0.79	0.15
Q3	0.10	0.09	0.74	0.11
Q4	0.15	0.15	0.74	0.10
2005Q1	0.20	0.21	0.73	0.10
Q2	0.25	0.26	0.71	0.11
Q3	0.33	0.29	0.69	0.12
Q4	0.42	0.33	0.68	0.13
2006Q1	0.50	0.36	0.67	0.14
Q2	0.55	0.38	0.66	0.16
Q3	0.58	0.40	0.64	0.17
Q4	0.60	0.42	0.63	0.18
2007Q1	0.60	0.44	0.63	0.19
Q2	0.60	0.45	0.62	0.20
Q3	0.60	0.46	0.61	0.21
Q4	0.60	0.47	0.60	0.21
2008Q1	0.59	0.47	0.60	0.22
Q2	0.59	0.48	0.59	0.23
Q3	0.58	0.48	0.59	0.23
Q4	0.57	0.49	0.58	0.24

Figure 7: Probability Estimates that UK and EA Output is Higher under Entry at par at Beginning of 1999Q1 and at Beginning of 2004Q1, by Quarters after Entry



5.4 UK Entry with Appreciation or Depreciation

In much of the discussion of euro entry a central issue has been what would be the right rate at which to join. To examine this issue we return to the baseline case, entry in 1999Q1 without conditioning on the four US variables. However, rather than assuming that entry was at par, the exchange rate prevailing in 1998Q4, we assume that the UK was able to appreciate or depreciate on entry and lock itself into the euro at a higher or lower rate. One might question whether the other members would have allowed such a move, but analysis of the effects allows us to judge the significance of the joining rate. We consider four cases: an appreciation of 10%, parity, depreciation of 10% and a depreciation of 30%.

Table 7 shows the probability that interest rates are lower as a result of entry in the UK and EA for the four cases. In the UK the probability of lower interest rates starts out at one and declines roughly linearly in all four cases. The speed of decline is fastest in the case of 10% appreciation, slowest in the case of 30% depreciation, with probabilities of lower interest rates between 70% (10% appreciation) and 88% (30% depreciation). In the EA the probability of lower interest rates as a result of entry starts at zero in all four cases. In the 10% appreciation and parity cases, these probabilities rise to a maximum of 73%

and 94%, respectively, in 2000Q1, then decline to 28% and 44% by 2003Q4. In the two depreciation cases, the probability of lower interest rates starts at zero reaching a maximum of 99% (10% depreciation) 100% (30% depreciation) in 2001Q1 then decline to 59% and 81% by the end of 2003. Thus for both the EA and UK entry with depreciation makes lower interest rates more likely.

Table 7: Probability Estimates that UK and EA Short-Term Interest Rates are Lower under Entry at par at Beginning of 1999Q1 for Alternative Exchange Rates

Year	UK				EA			
	Appreciation	At par	Depreciation		Appreciation	At par	Depreciation	
	10%	0%	10%	30%	10%	0%	10%	30%
1999Q1	1.00	1.00	1.00	1.00	0.05	0.02	0.01	0.00
Q2	0.99	0.99	0.99	1.00	0.04	0.12	0.25	0.59
Q3	0.97	0.98	0.98	0.99	0.17	0.39	0.63	0.92
Q4	0.96	0.97	0.97	0.98	0.54	0.76	0.90	0.99
2000Q1	0.94	0.96	0.97	0.98	0.73	0.94	0.99	1.00
Q2	0.92	0.95	0.96	0.98	0.68	0.94	0.99	1.00
Q3	0.90	0.93	0.95	0.97	0.60	0.89	0.98	1.00
Q4	0.88	0.91	0.94	0.97	0.53	0.83	0.96	1.00
2001Q1	0.85	0.89	0.92	0.96	0.46	0.76	0.93	1.00
Q2	0.82	0.87	0.91	0.95	0.40	0.69	0.88	0.99
Q3	0.80	0.86	0.90	0.94	0.36	0.63	0.83	0.98
Q4	0.78	0.84	0.88	0.94	0.33	0.58	0.79	0.96
2002Q1	0.76	0.82	0.87	0.93	0.31	0.55	0.74	0.95
Q2	0.75	0.81	0.86	0.92	0.30	0.51	0.71	0.93
Q3	0.73	0.80	0.85	0.91	0.29	0.49	0.68	0.90
Q4	0.72	0.79	0.84	0.90	0.28	0.47	0.65	0.88
2003Q1	0.72	0.78	0.83	0.90	0.28	0.46	0.63	0.86
Q2	0.71	0.77	0.82	0.89	0.28	0.45	0.61	0.85
Q3	0.70	0.77	0.81	0.88	0.28	0.44	0.60	0.83
Q4	0.70	0.76	0.81	0.88	0.28	0.44	0.59	0.81

Table 8 shows the probability that GDP would be higher under the different entry rates. Figure 8 plots the four cases for UK GDP. One might expect that UK entry with appreciation would reduce the probability that entry increases UK GDP and depreciation increase it. One gets the expected ordering before 2000Q1 and after 2002Q2, but in between the ordering is reversed. Figure 9 plots the probability EA GDP would be higher for the four cases. The EA probabilities show a single crossing point in late 1999, and after that the probabilities are in the expected order with UK appreciation making higher EA GDP most likely and UK 30% depreciation making it least likely. With a 10% UK appreciation the probability that EA GDP is higher is over 50% from 2002 onwards.

Table 8: Probability Estimates that UK and EA Output is Higher under Entry at par at Beginning of 1999Q1 for Alternative Exchange Rates

Year	UK				EA			
	Appreciation	At par	Depreciation		Appreciation	At par	Depreciation	
	10%	0%	10%	30%	10%	0%	10%	30%
1999Q1	0.00	0.03	0.32	0.98	0.00	0.00	0.02	0.27
Q2	0.00	0.00	0.03	0.27	0.00	0.00	0.01	0.07
Q3	0.01	0.02	0.03	0.08	0.01	0.01	0.00	0.00
Q4	0.05	0.06	0.07	0.09	0.06	0.03	0.01	0.00
2000Q1	0.11	0.11	0.12	0.13	0.12	0.07	0.04	0.01
Q2	0.17	0.15	0.14	0.11	0.18	0.11	0.06	0.02
Q3	0.29	0.24	0.21	0.15	0.25	0.16	0.09	0.03
Q4	0.43	0.37	0.33	0.25	0.32	0.21	0.13	0.05
2001Q1	0.54	0.49	0.44	0.37	0.38	0.26	0.17	0.07
Q2	0.59	0.56	0.52	0.46	0.42	0.30	0.21	0.09
Q3	0.62	0.60	0.57	0.53	0.46	0.34	0.24	0.12
Q4	0.63	0.62	0.60	0.58	0.49	0.37	0.27	0.14
2002Q1	0.63	0.62	0.62	0.61	0.51	0.39	0.30	0.16
Q2	0.62	0.62	0.62	0.62	0.52	0.41	0.32	0.18
Q3	0.61	0.62	0.62	0.63	0.53	0.43	0.33	0.20
Q4	0.60	0.61	0.62	0.63	0.54	0.44	0.35	0.21
2003Q1	0.59	0.60	0.62	0.63	0.54	0.44	0.36	0.22
Q2	0.58	0.60	0.61	0.63	0.55	0.45	0.37	0.23
Q3	0.57	0.59	0.61	0.63	0.55	0.45	0.37	0.24
Q4	0.57	0.58	0.60	0.63	0.55	0.46	0.37	0.25

Figure 8: Probability Estimates that UK Output is Higher under Entry at Beginning of 1999Q1 for Alternative Exchange Rates

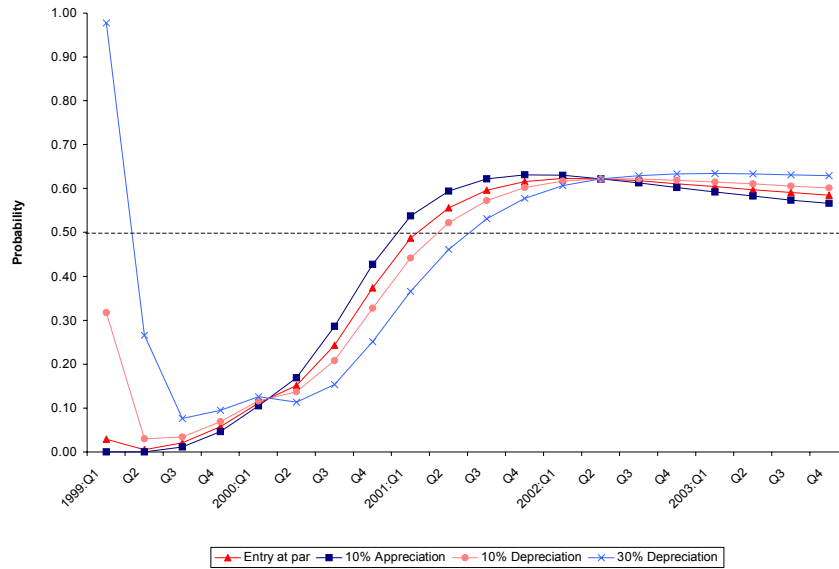


Figure 9: Probability Estimates that EA Output is Higher under Entry at Beginning of 1999Q1 for Alternative Exchange Rates

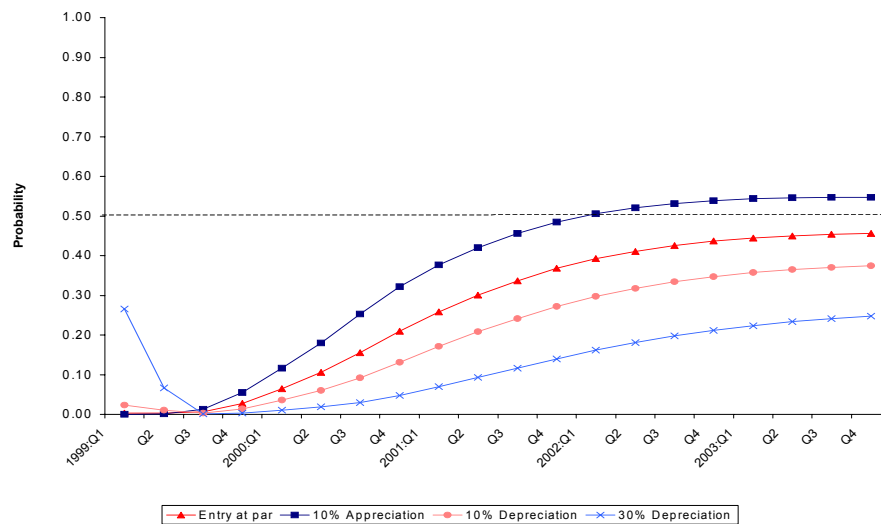


Table 9 shows the probability that prices are lower. For the UK the probabilities that prices will be lower start large and decline, but are always greater than 50%. The ordering is the reverse of what one might expect. UK prices are most likely to be lower with entry with a 30% depreciation and least likely with a 10% appreciation. For the EA the probability of prices being lower first declines and then rises, except for the 30% depreciation where the probability falls continually. The EA shows the same ordering as the UK, prices are most likely to be lower with an initial depreciation and least likely to be lower with an initial appreciation.

Table 9: Probability Estimates that UK and EA Prices are Lower under Entry at par at Beginning of 1999Q1 for Alternative Exchange Rates

Year	UK				EA			
	Appreciation	At par	Depreciation		Appreciation	At par	Depreciation	
	10%	0%	10%	30%	10%	0%	10%	30%
1999Q1	0.82	0.99	1.00	1.00	0.02	0.47	0.96	1.00
Q2	0.89	0.99	1.00	1.00	0.01	0.22	0.73	1.00
Q3	0.85	0.95	0.99	1.00	0.01	0.10	0.42	0.96
Q4	0.86	0.94	0.98	1.00	0.01	0.08	0.36	0.92
2000Q1	0.83	0.93	0.97	1.00	0.01	0.07	0.31	0.88
Q2	0.78	0.90	0.96	0.99	0.01	0.07	0.27	0.82
Q3	0.75	0.87	0.94	0.99	0.01	0.08	0.27	0.79
Q4	0.72	0.85	0.93	0.98	0.01	0.09	0.27	0.76
2001Q1	0.70	0.83	0.91	0.98	0.02	0.10	0.27	0.74
Q2	0.67	0.80	0.89	0.97	0.02	0.10	0.28	0.72
Q3	0.64	0.78	0.87	0.96	0.03	0.11	0.28	0.70
Q4	0.63	0.76	0.86	0.95	0.04	0.12	0.28	0.68
2002Q1	0.61	0.74	0.84	0.94	0.04	0.13	0.29	0.66
Q2	0.59	0.73	0.82	0.93	0.05	0.14	0.29	0.65
Q3	0.58	0.71	0.81	0.92	0.05	0.15	0.29	0.63
Q4	0.57	0.70	0.80	0.91	0.06	0.15	0.29	0.62
2003Q1	0.56	0.69	0.79	0.91	0.06	0.16	0.29	0.61
Q2	0.56	0.68	0.77	0.90	0.07	0.16	0.30	0.60
Q3	0.55	0.67	0.76	0.89	0.07	0.17	0.30	0.59
Q4	0.55	0.66	0.76	0.88	0.08	0.17	0.30	0.58

5.5 Point Estimates of the Effects of UK Entry

We have emphasised probabilities because we think they provide a more useful way of conveying results that can be quite uncertain. However, to give some indication of the orders of magnitude of the effects, Table 10 shows the effect

of entry (the point estimate with entry less the associated point estimate without entry) for UK and EA interest rates (percent annualised), the sterling-euro exchange rate and the UK and EA effective rates, expressed as percentage differences between the case with entry and without entry. These effects are for the case of entry at par in 1999 conditioning on oil prices, US interest rates and US equity prices. Entry makes UK short rates over 2% lower initially though the size of the reduction varies over time. Entry initially raises EA rates and then lowers them by around half a percentage point. Entry causes sterling to be generally stronger relative to the euro than it would have been without entry. UK entry causes the UK effective rate to appreciate relative to what it would have been without entry and the EA effective rate to depreciate. Towards the end of the period the EA effective exchange rate depreciates by about 15% as a result of the UK entry.

Table 10: Conditional Point Estimates of the Effects of UK Entry on UK and EA Short-Term Interest Rate, the £/Euro Rate and Effective Exchange Rate under Entry at par at Beginning of 1999Q1

Year	Short-Term		Exchange Rate (%)		
	Interest Rate (%)		Spot Rate £/Euro	Effective Rate	
	UK	EA		UK	EA
1999Q1	-2.35	0.26	-0.21	0.13	0.72
Q2	-2.49	0.06	-0.97	0.54	2.90
Q3	-2.55	-0.20	-0.26	1.67	3.80
Q4	-2.36	-0.35	1.41	3.04	3.50
2000Q1	-2.33	-0.55	1.57	3.34	3.82
Q2	-2.45	-0.65	-1.62	1.37	5.76
Q3	-2.35	-0.67	-4.37	-0.16	7.78
Q4	-2.49	-0.68	-8.69	-2.94	10.25
2001Q1	-2.33	-0.73	-11.74	-4.87	12.07
Q2	-2.07	-0.77	-13.21	-5.76	13.03
Q3	-2.20	-0.73	-15.60	-7.51	14.03
Q4	-2.04	-0.70	-15.43	-7.43	13.90
2002Q1	-1.75	-0.64	-13.95	-6.35	13.31
Q2	-1.83	-0.53	-15.51	-7.41	14.11
Q3	-1.60	-0.51	-18.06	-9.13	15.40
Q4	-1.32	-0.52	-18.62	-9.49	15.76
2003Q1	-1.22	-0.51	-20.44	-10.96	16.27
Q2	-1.45	-0.49	-21.16	-11.71	16.18
Q3	-1.10	-0.47	-18.31	-9.78	14.77
Q4	-1.61	-0.35	-18.61	-10.12	14.70

Note: The point estimates are obtained conditional on the realised values of oil prices, US equity prices and US short and long-term interest rates over the period 1999Q1-2003Q4. The figures for the interest rate are percent per annum.

Table 11 shows the effect on UK and EA output and prices as percentage changes between the with entry and without entry cases. The effects are quite small. The maximum reduction in UK output per quarter as a result of entry is -0.54% in 1999Q2, the maximum increase is less than 1%. The reductions in EA GDP as a result of entry are less than 1%. Entry causes UK prices to be 3% lower and EA prices 3% higher after 5 years; the effects on annual inflation rates would be barely noticeable.

Table 11: Conditional Point Estimates of the Effects of UK Entry on UK and EA Output and Prices under Entry at par at Beginning of 1999Q1

Year	Output (%)		Prices (%)	
	UK	EA	UK	EA
1999Q1	-0.25	-0.26	-0.54	-0.09
Q2	-0.54	-0.55	-1.71	-0.05
Q3	-0.48	-0.81	-1.91	0.15
Q4	-0.35	-0.84	-2.23	0.30
2000Q1	-0.19	-0.79	-2.67	0.45
Q2	-0.19	-0.80	-2.87	0.63
Q3	-0.11	-0.83	-3.08	0.84
Q4	-0.04	-0.78	-3.20	1.04
2001Q1	0.05	-0.73	-3.34	1.26
Q2	0.22	-0.67	-3.39	1.50
Q3	0.30	-0.55	-3.40	1.66
Q4	0.36	-0.45	-3.56	1.79
2002Q1	0.52	-0.37	-3.64	1.96
Q2	0.63	-0.30	-3.65	2.11
Q3	0.64	-0.27	-3.67	2.29
Q4	0.78	-0.25	-3.53	2.50
2003Q1	0.83	-0.18	-3.49	2.66
Q2	0.78	-0.12	-3.43	2.78
Q3	0.83	-0.11	-3.64	2.91
Q4	0.80	-0.08	-3.58	3.01

Note: The point estimates are obtained conditional on the realised values of oil prices, US equity prices and US short and long-term interest rates over the period 1999Q1-2003Q4.

6 Swedish Entry

For comparison we conducted exactly the same set of exercises for Sweden. The tables and figures, which match those for the UK are available on request. Starting with the baseline case of entry at par in 1999, Sweden like the UK would probably have had lower interest rates, though the probabilities are a little higher in Sweden. However, the EA trajectory for the probability of lower interest rate is very different in the two cases. With UK entry it started at

zero and rose to 94% after a year, then fell back to 44%. With Swedish entry it starts at 85%, falls to 34% then rises to 48%. The probability that output is higher in Sweden with entry starts at 65%, rises to 84% and then declines to 69%. The probability that prices are lower starts at 27%, rises to 48%, then drops to 32% before finishing at 35%. Thus Sweden is likely to have had higher output and prices with entry. For the EA, Swedish entry gives an initial probability of higher output of 69%, which rises to 80% and falls back to 53%. The EA probability that prices are lower is between 10% and 20% throughout the sample. Whereas UK entry would probably lower EA GDP, Swedish entry would raise it. Conditioning on observed values of oil prices, US interest rates and US equity prices adds a lot of volatility to the Swedish output probability estimates, but raises the probability of the effect of entry having a positive effect on output towards the end of the sample. In the case of the UK starting in 2004Q1 made very little difference, in the case of Sweden it does make a difference. The probability that output is higher with Swedish entry in 2004 starts at 46% rising to 61% at the end of the sample, a little below the 69% at the end of the 1999 entry sample. For the EA it starts at 30% and rises to 53%, the same value as at the end of the 1999 sample. Once the effect of the initial conditions has worn off, the probabilities of the two become very similar. In terms of the effect on output, 1999 would have been a better time for Sweden to enter than 2004, both from the Swedish and EA points of view. However, in terms of prices, the probability that Swedish entry would lower prices is higher for both Sweden and the EA for 2004 entry than for 1999 entry. So later entry would tend to lower both output and prices relative to earlier entry.

We now consider the four entry exchange rate cases. The effects on interest rates is very different from the UK. In the UK the probability of lower interest rates starts out at one and declines roughly linearly in all four cases, with final probabilities of lower interest rates between 70% (10% appreciation) and 88% (30% depreciation). In Sweden initially, like the UK, the 30% depreciation case has the highest probability of lower interest rates (71%) and 10% appreciation the lowest (66%), but the probabilities cross in 1999Q3 and the order is reversed with final probabilities between 68% (10% appreciation) and 62% (30% depreciation). At the beginning entry with a 30% depreciation gives the highest probability of lower interest rates (100%) and 10% appreciation the lowest (45%). At the end of the sample appreciation gives the highest probability of lower interest rates (52%) and 30% depreciation the lowest (37%). For the UK, entry with depreciation made lower EA interest rates more likely, for Sweden entry with appreciation makes lower EA interest rates more likely. For GDP appreciation reduces the probability that entry will increase Swedish output and depreciation increases the probability. At the beginning the differences are very large with a probability of higher Swedish output 15% with a 10% appreciation and 100% for a 30% depreciation, but by the end of the sample the differences are smaller, ranging from 65% to 79%, respectively. Exactly the same pattern holds for the EA. Swedish entry with depreciation increases the probability EA output will be higher. This is the reverse of the pattern for the UK, where appreciation increases the probability that EA output will be higher. Except

for the first period, when the order is reversed, entry with appreciation has the highest probability of giving lower prices (51% at the end of the period) and 30% depreciation the lowest (8% at the end of the period). This is the order one would expect, rather than the reverse order found in the UK. The same order holds for the EA: entry with appreciation has the highest probability of lower prices, entry with depreciation the lowest. However, even with appreciation the highest probability of lower prices is only 27%.

Looking at the size of the effects rather than the probabilities, the effect of entry on Swedish interest rates is much larger than the effect on UK interest rates, the maximum reduction, in 2003Q2, is 7.38 percentage points. The effect on EA interest rates is much smaller for Swedish entry than for UK entry. Whereas entry causes appreciation relative to no entry for the UK, for Sweden entry causes depreciation relative to no entry. Swedish entry, like UK entry causes the EA effective rate to depreciate in the medium term. The size of the output effects of entry are much larger in Sweden than the UK, reaching a maximum of 3.76% higher in 2003Q3. The effects on EA output are small as are the effects on Swedish prices. The effect of Swedish entry on EA prices is rather larger than the effect of UK entry, EA prices are about 8% higher at the end of the sample, compared to only 3% in the case of the UK.

7 Conclusion

This paper has a methodological and a substantive objective. The methodological objective is to provide a consistent approach to estimating the probability distribution for the effects of a regime change which can take account of international interdependence and observed global shocks. The substantive objective is to estimate the probability that output would be higher and prices lower in the UK and the EA if the UK had joined the euro.

UK entry to the euro in 1999Q1 at 1998Q4 exchange rates would probably have reduced UK GDP in the short term and raised it in the longer term, but the effects are small. UK entry would have probably caused EA GDP to be lower. Entry would have probably caused UK prices to be lower and EA prices to be higher. UK entry would have been bad for the EA, because of lower output and higher prices, though the effects are small. For the UK the welfare effect is ambiguous, depending on the discount rate used, and on the relative weights given to output and inflation. This conclusion seems quite robust to alternative scenarios, including conditioning on observed global shocks over the period 1999Q1 to 2003Q4 and entering with different initial conditions in 2004Q1.

Much of the discussion has emphasised the importance of the exchange rate at which the UK joined the euro and our experiments confirm the importance of this. The probabilities change when one allows for a permanent appreciation or depreciation of the sterling-euro rate on entry. With a large depreciation on entry, the probability of entry having a positive effect on UK output becomes greater.

We also made a comparison with Sweden, where entry to the euro in 1999 would probably have increased output and raised prices, so again no unambiguous welfare conclusion is possible. However, the Swedish results differed in a number of important respects from the UK results. This is not surprising as they are rather different economies.

There are a number of other issues that might merit investigation. These include the effect of parameter and model uncertainty. Our conclusion about the effect of UK entry can only be tentative until the robustness of the result to parameter and model uncertainty has been analysed, using techniques such as those applied in Garratt et al. (2003) in the context of a macroeconometric model of the UK. There are also other channels such as national debt and budget deficits that could be important for the analysis of the UK entry. The GVAR model used in this paper assumes that the fiscal implications of the UK entry will, to a large extent, be picked up by the term premium and their differences across the UK and the EA. But a more explicit analysis of the fiscal effects of the UK entry within the GVAR framework would be of interest.

APPENDIX

Transforming the GVAR: Prices and Nominal Exchange Rates

Assume that inflation, $p_{it} - p_{i,t-1}$, is the first variable in $\hat{\mathbf{x}}_{it}$, in all country models and, with the exception of the US, the second and third entries in $\hat{\mathbf{x}}_{it}$ are the real exchange rate, $e_{it} - p_{it}$, and the short term interest rate, r_{it}^S . The rest of the variables in $\hat{\mathbf{x}}_{it}$ will be denoted by the $(k_i - 3) \times 1$ vector \mathbf{x}_{it}^{rest} for the non-US models, and by the $(k_0 - 2) \times 1$ vector \mathbf{x}_{0t}^{rest} for the US model. More specifically we have

$$\hat{\mathbf{x}}_{0t} = \begin{pmatrix} p_{0t} - p_{0,t-1} \\ r_{0t}^S \\ \mathbf{x}_{0t}^{rest} \end{pmatrix}, \text{ and } \hat{\mathbf{x}}_{it} = \begin{pmatrix} p_{it} - p_{i,t-1} \\ e_{it} - p_{it} \\ r_{it}^S \\ \mathbf{x}_{it}^{rest} \end{pmatrix} \text{ for } i = 1, 2, \dots, N.$$

The GVAR model in these variables as set above

$$\hat{\mathbf{x}}_t = \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{G}_1 \hat{\mathbf{x}}_{t-1} + \mathbf{G}_2 \hat{\mathbf{x}}_{t-2} + \hat{\mathbf{v}}_t, \quad (\text{A.1})$$

where $\hat{\mathbf{x}}_t = (\hat{\mathbf{x}}'_{0t}, \hat{\mathbf{x}}'_{1t}, \dots, \hat{\mathbf{x}}'_{Nt})'$, and \mathbf{a}_0 , \mathbf{a}_1 , \mathbf{G}_1 , \mathbf{G}_2 are defined in terms of the country-specific models as in Section 3.

Since our aim is to fix UK's nominal exchange rate and short term interest rate to the ones in the euro area, we need to work with a GVAR in the levels of these variables. Assume that $\hat{\mathbf{x}}_{1t}$ and $\hat{\mathbf{x}}_{2t}$ refer to the endogenous variables of the models for the euro area and the UK, respectively, and denote the $k_i \times 1$ vector of the endogenous variables in p_{it} and e_{it} by \mathbf{x}_{it} . More specifically

$$\mathbf{x}_{0t} = \begin{pmatrix} p_{0t} \\ r_{0t}^S \\ \mathbf{x}_{0t}^{rest} \end{pmatrix}, \text{ and } \mathbf{x}_{it} = \begin{pmatrix} p_{it} \\ e_{it} \\ r_{it}^S \\ \mathbf{x}_{it}^{rest} \end{pmatrix} \text{ for } i = 1, 2, \dots, N.$$

The relationships between \mathbf{x}_{it} and $\hat{\mathbf{x}}_{it}$ are given by the following dynamic identities

$$\mathbf{x}_{it} = \mathcal{B}_i \hat{\mathbf{x}}_{it} + \mathcal{C}_i \mathbf{x}_{i,t-1}, \text{ for } i = 0, 1, \dots, N,$$

where

$$\mathcal{B}_0 = \mathbf{I}_{k_0}, \mathcal{C}_0 = \begin{pmatrix} 1 & \mathbf{0}_{1 \times (k_0-1)} \\ \mathbf{0}_{(k_0-1) \times 1} & \mathbf{0}_{(k_0-1) \times (k_0-1)} \end{pmatrix},$$

and

$$\mathcal{B}_i = \begin{pmatrix} 1 & 0 & \mathbf{0}_{2 \times (k_i-2)} \\ 1 & 1 & \mathbf{0}_{2 \times (k_i-2)} \\ \mathbf{0}_{(k_i-2) \times 2} & \mathbf{I}_{k_i-2} & \mathbf{0} \end{pmatrix},$$

$$\mathcal{C}_i = \begin{pmatrix} 1 & 0 & \mathbf{0}_{2 \times (k_i-2)} \\ 1 & 0 & \mathbf{0}_{2 \times (k_i-2)} \\ \mathbf{0}_{(k_i-2) \times 2} & \mathbf{0}_{(k_i-2) \times (k_i-2)} & \mathbf{0} \end{pmatrix}.$$

Hence

$$\mathbf{x}_t = \mathcal{B}\hat{\mathbf{x}}_t + \mathcal{C}\mathbf{x}_{t-1}, \quad (\text{A.2})$$

where

$$\mathcal{B} = \begin{pmatrix} \mathbf{I}_{k_0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathcal{B}_1 & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & & \mathcal{B}_{N-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathcal{B}_N \end{pmatrix},$$

and

$$\mathcal{C} = \begin{pmatrix} \mathcal{C}_0 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathcal{C}_1 & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & & \mathcal{C}_{N-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathcal{C}_N \end{pmatrix}.$$

It is clear that \mathcal{B} is non-singular and its inverse is given by

$$\mathcal{B}^{-1} = \begin{pmatrix} \mathbf{I}_{k_0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathcal{B}_1^{-1} & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & & \mathcal{B}_{N-1}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathcal{B}_N^{-1} \end{pmatrix},$$

where

$$\mathcal{B}_i^{-1} = \begin{pmatrix} 1 & 0 & \mathbf{0}_{2 \times (k_i - 2)} \\ -1 & 1 & \\ \mathbf{0}_{(k_i - 2) \times 2} & \mathbf{I}_{k_i - 2} & \end{pmatrix}, \text{ for } i = 1, 2, \dots, N,$$

and

$$\hat{\mathbf{x}}_t = \mathcal{B}^{-1}(\mathbf{x}_t - \mathcal{C}\mathbf{x}_{t-1}).$$

Using this result in (A.1) we have

$$\mathcal{B}^{-1}(\mathbf{x}_t - \mathcal{C}\mathbf{x}_{t-1}) = \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{G}_1 \mathcal{B}^{-1}(\mathbf{x}_{t-1} - \mathcal{C}\mathbf{x}_{t-2}) + \mathbf{G}_2 \mathcal{B}^{-1}(\mathbf{x}_{t-2} - \mathcal{C}\mathbf{x}_{t-3}) + \hat{\mathbf{v}}_t,$$

or

$$\begin{aligned} \mathbf{x}_t &= \mathcal{B}(\mathbf{a}_0 + \mathbf{a}_1 t) + (\mathcal{C} + \mathcal{B}\mathbf{G}_1\mathcal{B}^{-1})\mathbf{x}_{t-1} - \\ &\quad (\mathcal{B}\mathbf{G}_1\mathcal{B}^{-1}\mathcal{C} - \mathcal{B}\mathbf{G}_2\mathcal{B}^{-1})\mathbf{x}_{t-2} - \mathcal{B}\mathbf{G}_2\mathcal{B}^{-1}\mathcal{C}\mathbf{x}_{t-3} + \mathbf{v}_t, \end{aligned} \quad (\text{A.3})$$

where

$$\mathbf{v}_t = \mathcal{B}\hat{\mathbf{v}}_t.$$

Note that (A.3) can be written as

$$\mathbf{x}_t = \mathbf{b}_0 + \mathbf{b}_1 t + \mathbf{F}_1 \mathbf{x}_{t-1} + \mathbf{F}_2 \mathbf{x}_{t-2} + \mathbf{F}_3 \mathbf{x}_{t-3} + \mathbf{v}_t,$$

where

$$\mathbf{b}_0 = \mathcal{B}\mathbf{a}_0, \mathbf{b}_1 = \mathcal{B}\mathbf{a}_1,$$

and

$$\mathbf{F}_1 = (\mathcal{C} + \mathcal{B}\mathbf{G}_1\mathcal{B}^{-1}), \mathbf{F}_2 = \mathcal{B}(\mathbf{G}_2\mathcal{B}^{-1} - \mathbf{G}_1\mathcal{B}^{-1}\mathcal{C}), \mathbf{F}_3 = -\mathcal{B}\mathbf{G}_2\mathcal{B}^{-1}\mathcal{C}.$$

Suppose that the characteristic equation of the model in inflation and real exchange rates, defined by (A.1), lie on or inside the unit circle. Let λ be the characteristic root of the model in price levels and nominal exchange rates as specified by (A.3) and note that

$$|\mathbf{I}_k - (\mathcal{C} + \mathcal{B}\mathbf{G}_1\mathcal{B}^{-1})\lambda + (\mathcal{B}\mathbf{G}_1\mathcal{B}^{-1}\mathcal{C} - \mathcal{B}\mathbf{G}_2\mathcal{B}^{-1})\lambda^2 + \mathcal{B}\mathbf{G}_2\mathcal{B}^{-1}\mathcal{C}\lambda^3| = 0, \quad (\text{A.4})$$

where $k = \sum_{i=0}^N k_i$. Since \mathcal{B} is non-singular we also have

$$\begin{aligned} |\mathbf{I}_k - (\mathcal{B}^{-1}\mathcal{C}\mathcal{B} + \mathbf{G}_1)\lambda + (\mathbf{G}_1\mathcal{B}^{-1}\mathcal{C}\mathcal{B} - \mathbf{G}_2)\lambda^2 + \mathbf{G}_2\mathcal{B}^{-1}\mathcal{C}\mathcal{B}\lambda^3| &= \\ |(\mathbf{I}_k - \mathbf{G}_1\lambda - \mathbf{G}_2\lambda^2)(\mathbf{I}_k - \mathcal{B}^{-1}\mathcal{C}\mathcal{B}\lambda)| &= \\ |(\mathbf{I}_k - \mathbf{G}_1\lambda - \mathbf{G}_2\lambda^2)| |(\mathbf{I}_k - \mathcal{B}^{-1}\mathcal{C}\mathcal{B}\lambda)| &= 0. \end{aligned}$$

Hence, the characteristic roots of (A.4) are given by the solutions to

$$|(\mathbf{I}_k - \mathbf{G}_1\lambda - \mathbf{G}_2\lambda^2)| = 0, \quad (\text{A.5})$$

which is the characteristic equation of (A.1), and

$$|(\mathbf{I}_k - \mathcal{B}^{-1}\mathcal{C}\mathcal{B}\lambda)| = 0. \quad (\text{A.6})$$

However, it is easily seen that

$$\mathcal{B}^{-1}\mathcal{C}\mathcal{B} = \begin{pmatrix} \mathcal{C}_0 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathcal{B}_1^{-1}\mathcal{C}_1\mathcal{B}_1 & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & & \mathcal{B}_{N-1}^{-1}\mathcal{C}_{N-1}\mathcal{B}_{N-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathcal{B}_N^{-1}\mathcal{C}_N\mathcal{B}_N \end{pmatrix},$$

with

$$\mathcal{B}_i^{-1}\mathcal{C}_i\mathcal{B}_i = \begin{pmatrix} 1 & 0 & \mathbf{0}_{2 \times (k_i-2)} \\ -1 & 1 & \mathbf{I}_{k_i-2} \end{pmatrix} \begin{pmatrix} 1 & 0 & \mathbf{0}_{2 \times (k_i-2)} \\ 1 & 0 & \mathbf{0}_{(k_i-2) \times (k_i-2)} \end{pmatrix} \begin{pmatrix} 1 & 0 & \mathbf{0}_{2 \times (k_i-2)} \\ 1 & 1 & \mathbf{I}_{k_i-2} \end{pmatrix},$$

or

$$\mathcal{B}_i^{-1}\mathcal{C}_i\mathcal{B}_i = \begin{pmatrix} 1 & 0 & \mathbf{0}_{2 \times (k_i-2)} \\ 0 & 0 & \mathbf{0}_{(k_i-2) \times (k_i-2)} \end{pmatrix}.$$

The eigenvalues of $\mathcal{B}^{-1}\mathcal{C}\mathcal{B}$ are given by those of \mathcal{C}_0 and $\mathcal{B}_i^{-1}\mathcal{C}_i\mathcal{B}_i$ for $i = 1, 2, \dots, N$, which are zero or unity. Therefore, the characteristic roots of (A.3) are the same as the characteristics of (A.1) plus the roots of (A.6) which are either zero or unity.

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