On debt optimality under borrowing constraints*

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Abstract

This paper presents an optimal fiscal policy response to address the basic trade-off between the automatic stabilisation properties of budgets and the long run fiscal positions of governments. The framework we use is an overlapping generations model à la Weil (1989), extended to account for stochastic endowments and borrowing constrained households. Steady-state results are the outcome of a constrained optimisation problem in which a benign government chooses over the optimal degree of responsiveness of net taxes to individual incomes, an optimal measure of long-run public debt, or both, in order to smooth households' consumption across states of nature. The importance of an institutional rule framing public finances in obtaining a joint solution for optimal budgetary responsiveness and public debt is underscored. For sensible parameterisations, the results unambiguously point to the desire for lower debt levels than those currently prevailing in order to enable a more effective hedging of personal income uncertainty by means of more active fiscal stabilisers. Those countries facing more pronounced economic cycles will always favour less automatic stabilisation combined with a more aggressive policy of debt reduction in the short run as well as the long run. As an illustration, we calibrate our theoretical model to several European Union countries, making use of the deficit limit imposed by the Stability and Growth Pact. Results of this exercise show an appetite for the elimination of public debt in the very long run in exchange for a heightened degree of automatic stabilisation.

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Key words: Public debt; Automatic stabilisation; Borrowing constraints; Consumption

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1 Introduction

Electorates in mature industrialised economies – notably in Europe – are at the crossroads of difficult decisions. International organisations repeatedly call upon them to repudiate once and for all the course of fiscal profligacy that many of them have displayed over most of their past, and to take various combinations of the following policy actions. (i) Design an entirely new fiscal paradigm solidly built on the principles of sustainability and intergenerational responsibility; (ii) Espouse the cause of "rules rather than discretion" in public finances – as they successfully did in monetary affairs – and commit to a credible path for future fiscal policy; (iii) Resuscitate the role of public budgets as shock absorbers, which had been suffocated by years of high interest spending and recurrent spells of deficit and debt crises; (iv) And finally seize the opportunity offered by a growing economy to pursue a policy of accelerated debt abatement to the advantage of future generations of taxpayers. The adoption of explicit policy rules such as deficit limits and balanced budget legislation by governments in some of these countries has made a decisive contribution to the enactment of this ambitious reform agenda. By setting a clear limit upon the amount that governments can borrow on a year-by-year basis, these rules have generally laid the foundations for a new fiscal regime capable of reconciling the first and the second objectives outlined above. But reconciling the third and the fourth objectives is a more difficult undertaking.

In order to examine the above issues, this paper presents a simple partial equilibrium optimising model, using as a backbone a dynastic overlapping generations framework à la Weil (1989). We consider an open non-monetary economy in which individual gross incomes are *stationary* and subject to *aggregative* (non-idiosyncratic) random shocks drawn from a known distribution, and in which population is expected to grow by virtue of a steady inflow of immigrants from the outside world. We formalise a situation in which households are prevented from freely borrowing against their future income and are required to observe a positive or zero limit to the amount of wealth that they can carry over from one period to the next.

This approach sidesteps the determinants of market prices and asks directly how a primitive agent would allocate resources through its own consumption choices and the policies of a benevolent fiscal authority. At time zero, a single infinitely lived "native household", whose only goal is to optimise its intertemporal utility with *no* concern for the welfare of any future generations of newcomers, is called upon to decide what share of current public debt should be repaid by itself and what share could be offloaded onto future generations of taxpayers in whose welfare it has no interest. In other words, we imagine that at date zero a benign government asks the single native household what is the level at which debt per capita should be stabilised. If the native household decides to stabilise at the debt-per-household ratio that it has inherited from history – the amount of liabilities that are outstanding at time zero, call it b_0 – then the native household and each new immigrant will receive that same amount of debt to bear into eternity – that is, to sustain with phased tax payments. If, conversely, the native inhabitant decides to stabilise at a *lower* debt-per-taxpayer level, say $b^* < b_0$, this decision would imply: (i) that the native household and each new generation of immigrants would be assigned, upon entry, a lower level of per-capita debt to sustain, b^*, than outstanding at time zero; (ii) that the native household would have to shoulder a higher debt than any other future generation of households and would consequently have to pay an excess tax – above that paid by all future generations of immigrants – to cover the difference between b_0 and b^*.

Upon first thought, one might conclude that the native household would never choose option (ii). In other words, it appears that it would never hesitate to stabilise at the *maximum* debt level allowed by the policy exercise. Indeed, why shouldn't the native household seek to share as much of the country's inherited liabilities as possible with future generations of immigrants, whose welfare is *not* internalised in its decision problem? After all, this debt-bias is a well-known feature of overlapping generations frameworks à la Weil (1989), in which infinitely lived agents nonetheless behave in a non-Ricardian manner, because population is steadily increased by new generations of immortal households to which the living taxpayers have no utility connections.

What follows shows that this conclusion would not be correct. Imagine that households were not free to borrow against their future income and thus were to be liquidity constrained in certain, sufficiently "bad", states of nature – i.e. when confronted by low realisations of gross income. Imagine also that, were the native household to decide to bequeath some $b^* < b_0$ to future generations and to hold itself responsible for a higher debt burden b_0, the excess tax that it would be requested to pay against its extra liability $(b_0 - b^*)$, would be designed in a way not to excessively penalise its disposable resources in times of low-endowment conditions.

Then the policy choice would entail a basic trade-off. Stabilising at b_0 would imply an excess burden of zero – which the native household would appreciate because it would entail a zero extra tax. But it would also boost the "core" tax needed to sustain per-capita-debt at that (high) level in the long run in *all* states of nature. This would cause disutility, in expected terms, as having to pay a high tax is painful when liquidity is scarce and credit restrictions bind.

Now, imagine instead that the native household were to choose to stabilise per-capita-debt at some $b^* < b_0$. On the one hand, it would have to pay more than any future resident, which it would dislike. But on the other hand, stabilising at some "low" b^* would reduce the "core" tax required to keep debt at that lower steady state value for ever in all states of nature. And this would be good news for the household attaching a positive probability to liquidity restrictions in the future, as a lower "core" tax would free spendable resources at times in which borrowing would be desirable – but prohibited – and the marginal propensity to consume out of a tax cut would thus be unity. Again, this depends on the scheme designed to run down the native household's excess liability $(b_0 - b^*)$ through time. In particular, it is conditional on devising a repayment scheme that does not penalise consumption during "bad" income conditions. This issue of smoothing consumption across states of nature via government involvement is also examined by Flodén (2001), Aiyagari and McGrattan (1998) and Woodford (1990), although the context is slightly different. Whereas they examine the role of government debt in easing liquidity constraints in an economy with heterogeneous agents, we examine the role of automatic stabilisation in insuring consumers against income shortfalls in an economy with purely aggregative shocks. In doing so, we exploit a different tradeoff. The balancing of the pros attached to a low-debt stabilisation option – the fact that the native household would in this way insure itself against income shortfalls in periods of distress – against the relative cons – the fact that it would have to pay an excess tax in good times compared to future generations – constitutes the analytical basis for our methodological approach to finding the "optimal" fiscal structure.

By an optimal fiscal structure we mean two related features: an "optimal" amount of debt – the b^* chosen by the government on behalf of the household at the dawn of history; and an optimal degree of response of taxes to realised income conditions – what we call a, which is a measure of the automatic stabilisation built into fiscal policy in our model. We provide both a *local* and a *global* measure of these two variables – a and b^*– by which we mean the values that the government would choose *conditional* on an initial inherited debt level b_0 or, alternatively, in a situation in which the initial conditions have faded and the government operates in a sort of "timeless" perspective.

We prove that, if no exogenous deficit limits are enforced, there is a high complementarity between these two dimensions of policy, optimal debt and optimal built-in stabilisation. The higher a the higher the optimal debt b^* at which the government – acting on behalf of the household – would choose to stabilise debt. In the limit, if taxes were designed to adjust in a way to entirely cushion household's income against adverse shocks – i.e. if a tended to unity – in the absence of a deficit limit the government would choose to stabilise the debt at the level inherited from history and the native household would thus bear no excess liability. The intuition we provide is simple. Cutting debt is one way to insure the household against adverse income realisations, because it amounts to reducing the tax that is paid *on average* to sustain it. However, boosting the degree to which the tax structure can act to absorb realised income shortfalls *ex post*, no matter how large the tax is on average, is another – and more efficient – way to provide insurance via the fiscal structure. If asked, households would always prefer the latter over the former. Indeed we show that our native household does have an appetite for debt reduction, but only if prevented from choosing the optimal degree of budgetary responsiveness, a. By contrast, if allowed to choose optimal automatic stabilisation a along with the optimal amount of debt b^*, agents would set a equal to one and never agree to debt reduction.

By adding an exogenous limit to the amount of government borrowing, we can gain some insights into the economics of deficit-constraining regimes such as Europe's Stability and Growth Pact. Here we show that the native household, while still perceiving a and b^* as strict complements, is faced with a trade-off. The higher a – that is, the stronger the automatic decline in taxes in the midst of an economic slump – the larger the deficit in bad income conditions. However, a higher cyclical component in the primary surplus requires a lower interest bill, i.e. rb^*, if the deficit limit is to be respected in all circumstances. A lower interest bill, in turn, implies a lower average level of debt b^*. Hence, we conclude that a deficit limit provides a technological device by which higher private income protection by means of automatic budgetary stabilisation can be purchased only by paying a price in terms of debt reduction. If the native household wants more of the former, it has to agree to a debt abatement scheme that reduces the amount of debt held on average. This trade-off induced by the deficit limit allows – contrary to the no-deficit limit case – for the optimisation problem of the government to lead to interior solutions for a and b^*. In other words, under a deficit limit there generally exists a combination of $a < 1$ and $b^* < b_0$ that can make the native household better off with respect to its initial conditions.

We argue that the 3 per cent deficit limit of the Stability and Growth Pact has indeed priced automatic stabilisation in terms of debt reduction. We notice that this interpretation is consistent with some anecdotal evidence over the rate at which debt has been reduced in the course of the last decade across various European countries. According to our model, countries facing larger fluctuations in output have a strong incentive to choose a large a to cushion income shortfalls. At the same time, given the deficit limit, they are forced to choose a lower b^*, that is to agree to possibly draconian fiscal retrenchments. Indeed, this is what we observe in the data. European countries which historically experienced ampler than normal output fluctuations (say, Finland and Denmark) have recently embarked on very courageous policies of debt reduction, while countries on the other end of the spectrum in terms of output volatility (say, France and Austria) could afford a more measured approach to fiscal stabilisation.

In sum, we believe that the model we use can contribute to substantiating the calls for debt reduction and for a decisive revitalisation of the role of automatic stabilisation in the contemporary public finance debate: points (ii) and (iii) of the first paragraph of this Introduction. Both debt reduction and a higher degree of built-in automatic stabilisation are consistent with optimising agents living in mature economies where capital markets are incomplete.

The rest of the paper is organised as follows. In Section 2 we present the individual household's problem and its solution. Section 2.1 sets the fiscal institutions at the core of our approach. The policy problem that we attribute to the government is stated and solved in Sections 2.2, in the case with no deficit limits, and 2.3, in the case with deficit limits. In Section 3 we discuss an application of our theoretical framework to Europe's Stability and Growth Pact. We show the main empirical results of the paper in Section 3.2, and some concluding remarks in Section 4.

2 The model

We consider a non-monetary open economy originally populated by just one infinitely lived household – or, equivalently, a continuum of such households with measure unity. This household is indicated in what follows as the "native" household. The economy is expected to receive a steady inflow of equally immortal immigrants from time one onwards, with each newcomer identical to the native household – in measure, preferences and initial wealth. Households do not grow in measure once settled in the economy and, at the start of each period, are endowed with the same amount of a non-durable consumption good, which fluctuates randomly around a stationary level y^*. Hence, resident population growth is entirely driven by the constant rate of immigration, n, which is also equal to the growth rate of trend output. Since the model is one of infinitely lived agents, following Weil (1989), a "household" in the model can probably be best thought of as consisting of members of all the different generations of a dynasty. Implicitly, the model assumes that the different generations of a dynasty are linked by an operative chain of bequests and, therefore, focuses only on total family income, wealth, consumption and welfare – where the "household" is obviously more broadly defined than in the data.

Time is discrete and at the beginning of each period the endowment of each resident household receives the same shock as anybody else's, so that aggregate endowment fluctuations are the only source of private income uncertainty within the economy.[1]

Preferences are defined over consumption and described by the following utility measure:

$$E_t \sum_{j=t}^{\infty} \left(\frac{1}{1+\delta}\right)^j U[c_j(g_j)] \;=\; \left[U[c_t(g_t)] + \sum_{j=t+1}^{\infty} \left(\frac{1}{1+\delta}\right)^j \int_{-\infty}^{\infty} U[c_j(g_j)]dG^j\right] \qquad (1)$$

where c_t is real consumption, δ the individual net rate of time preference, g_j the random discrepancy between the actual realisation of individual endowment at time j, y_j, and its time-invariant "normal" level, y^*. $G^j \equiv G(g_j \mid g_0, g_1, ..., g_{j-1})$ denotes the (known) distribution function of the history of such random discrepancies between any future time j and present time t. The integral on the right-hand side of (1) denotes the expected utility at any time j conditional on the entire history of endowment shocks. In what follows, we label g_t the "output gap" at time t and impose on it a simple autoregressive law of motion:

$$g_t = \rho g_{t-1} + e_t \qquad (2)$$

where e_t are iid drawings from a known uniform distribution defined over the compact symmetric support $[-e, +e]$, yielding a corresponding, proportional compact symmetric support $[-\epsilon, +\epsilon]$ for g_t, with ϵ being the interval's positive finite upper bound. $0 \leq \rho \leq 1$ measures persistence. Notice that, as all households are identical in preferences and endowment possibilities, and hence subject to the same random shocks ϵ_t, g_t stands for both the *individual* and the *aggregate* output gaps at time t.

The absence of idiosyncratic shocks to individual endowments has two key implications for our modelling strategy. Firstly, since all residents choose to self-insure against the same kind of aggregate uncertainty at the same time, there is no need to allow for a *domestic* capital market. In fact, such a market would not be operative at all, as there would be no scope for risk pooling among citizens operating in the same economy, while there would always be a domestic demand for foreign claims to finance a less variable profile of consumption relative to national income. Secondly, locating transactions in financial claims "outside the boundaries" of the economy introduces a "natural" dimension of market incompleteness: that related to asymmetric information between lenders and borrowers in uncollateralised international consumption loans. In other words, it becomes natural under such circumstances to assume that domestic households, while having unlimited access to a world capital market in one-period *lending* contracts, nevertheless face a *borrowing* limit which precludes bankruptcy with probability one. Following the lead of Schechtman and Escudero (1977), Hayashi (1982), Hubbard and Judd (1986, 1987), along with Krusell and Smith (1998), we implement such a restriction in its most extreme version, by imposing an outright non–negative wealth condition. While econometric evidence lends support to the conjecture that agents might be prevented from borrowing as much as they would like to

[1] In reality, of course, microeconomic income processes are very different from their macroeconomic aggregates, so that while individuals share in the general growth, the variance in their earnings is dominated by idiosyncratic components.

smooth consumption, our characterisation of this friction has been widely canvassed in the literature as a first approximation to reality and seems to entail no serious loss of generality.[2] As it is argued in the remainder of the paper, this form of market friction, coupled with the government's ability, under certain conditions, to access the international financial market on behalf of its citizens, lends a key role to government as a financial intermediary.

At the beginning of each period, *after observing the current shock to individual endowments*, resident household(s) must decide how much to consume and how many financial claims to purchase or sell in the international capital market. A financial claim costs one consumption good and entitles its owner to $(1+r)$ goods next period, with $n < r < \delta$. [3] Consumption and accumulation decisions are made to satisfy an infinite set of period flow budget constraints,

$$c_t\,(g_t)\;+w_t-(y^*+g_t)+s_t-w_{t-1}(1+r)\;+\;k_t\;\leq 0,\;\forall t\geq 0 \tag{3}$$

and an infinite set of contemporary borrowing constraints,

$$w_t \geq 0,\,\forall t \geq 0 \tag{4}$$

for some predetermined w_{-1}. In (3) and (4), w_{t-1} and w_t denote real outside assets carried over from the earlier period and into the next period, respectively, $y^* + g_t$ is the realised level of individual endowment at time t, s_t is a universal real tax paid by the entire population present at time t, and k_t is a non-negative real tax that is equal to zero for everybody except the native household. For simplicity, we assume a fixed exchange rate.

After observing the realisation of the endowment shock in the first period, the native household draws up its contingent plan for non-negative consumption and lending in such a way as to maximise its lifetime utility (1) subject to the infinite succession of time budget and borrowing restrictions (3) and (4), for any arbitrary fiscal plan characterised by the series $\{s_t,\, k_t;\, t \geq 0\}$. The generalised Euler condition associated with our native household's individual optimisation problem at any time t can be written in the following compact form:[4]

$$U'\left(c_t(g_t)\right)=\max\left[U'\left(y^*+g_t\;-(s_t+k_t)+(1+r)w_{t-1}\right),\;\;\beta^{-t}\int_{-\epsilon}^{\epsilon}U'\left(c_{t+1}(g_{t+1})\right)\,dG^{t+1}\right]\;,\;\forall t\geq 0 \tag{5}$$

with β standing for the ratio between the gross rate of time preference and the gross

[2]A more general setting, allowing agents to accumulate a limited amount of net financial liabilities, is studied by Clarida (1987) and Aiyagari (1994). Aiyagari, in particular, argues that present-value budget balance and a non-negativity condition on individual consumption together *imply* a borrowing restriction, although the limit imposed on the period holdings of wealth would be less tight than ours in that agents would generally be allowed to borrow some positive amount of resources.

[3]Clarida (1990) proves that, in a multi-country general equilibrium model whereby each economy's aggregate income is subject to random shocks and individuals are borrowing-constrained, the interest rate prevailing in the international financial market is constant in the stationary state and strictly less than the rate of time preference of the least impatient country. In what follows, for the reasons spelled out below, we assume $r < \delta$.

[4]See, for example, Deaton (1991).

rate of interest prevailing in the capital market, i.e. $\beta = \frac{1+\delta}{1+r}$ (with $1 < \beta < 1+r$)[5], and $U'(c(g_t))$ representing the marginal utility of consumption at some generic time t. Clearly, at the decision juncture $t = 0$, if individual borrowing from international capital markets is ruled out, consumption at time zero can be no higher than the contemporary amount of spendable resources, $y^* + g_0 - (s_0 + k_0) + (1+r)w_{-1}$, given by the sum of the after-tax income realised at zero, $y^* + g_0 - (s_0 + k_0)$, and the wealth carried over from the past compounded at the prevailing real interest rate, $(1+r)w_{-1}$. If consumption cannot fall short of spendable resources, marginal utility cannot be higher than $U'(y^* + g_0 - (s_0 + k_0) + (1+r)w_{-1})$, i.e. the marginal utility index evaluated at the quantity of current spendable resources. Hence, the borrowing constraint will bind at zero if marginal utility evaluated at $y^* + g_0 - (s_0 + k_0) + (1+r)w_{-1}$ exceeds the anticipated discounted marginal utility at any future period, whose expectation is conditional on the current realisation of income. Otherwise, the current and the expected marginal utilities are equated in the usual way.

Schechtman and Escudero (1977), Clarida (1987) and Deaton and Laroque (1992) prove that, under a number of mild conditions on U (and $\delta > r$), the optimisation problem admits a solution. However, the set of generalised Euler conditions identified by (5) generally does not yield a closed-form law of motion for consumption, not even under fairly simple functional specialisations of utility. The problem lies, of course, in the presence of borrowing restrictions. The inability to access a frictionless market for consumption loans adds a "kink" to the expectations operator used in the intertemporal marginal condition – the second term on the right-hand side of (5) – in that expectations of future marginal utilities have to be formed by taking in due account the possibility that borrowing restrictions might sooner or later become binding. This, as emphasised by Carroll (2001a and 2001b), Deaton (1991) and Aiyagari (1994), augments the degree of prudence – that is, as defined in Kimball (1990), households' tendency to frontload saving in anticipation of future hardships – as compared to a scenario contemplating the same risk preferences but no borrowing restrictions. However, there is no closed analytical form capturing the degree to which this effect impact on consumption relative to a situation in which borrowing is unlimited and the household can consume at the prescribed life-cycle optimum.

In the remainder of this study we tackle the complexity of the problem just mentioned in a twofold manner. First, we specialise the period utility to a quadratic function of consumption, with the bliss point indicated by C:

$$U(c_t) = -(C - c_t)^2 \tag{6}$$

This representation of preferences allows, in a complete-market framework that is in the absence of borrowing limits, a simple closed-form solution to the optimisation problem under uncertainty, namely that implied by the application of the certainty equivalence principle.[6] We call the solution to this linear-quadratic problem in the absence of borrowing constraints the *permanent-income optimum.* Secondly, we approximate the optimal solution to the linear-quadratic problem facing households *in the presence of borrowing restrictions* by the following

[5] The condition that $\beta < 1+r$ is needed in order to guarantee that the present value of the expected stream of dated consumption is bounded away from infinity.

[6] See, for example, Abel (1990).

rule of thumb. Households would try to consume at their permanent-income optimum, unless prevented from doing so by lack of sufficient spendable resources, that is unless $y^* + g_0 - (s_0 + k_0) + (1 + r)w_{-1}$ is less than needed to finance the life-cycle consumption level at time zero. In technical terms, households maximise their quadratic utility (6) with respect to consumption taking into account the whole succession of flow budget constraints (3), $t \geq 0$, and the *contemporary* borrowing restriction $w_1 \geq 0$, but ignoring the fact that borrowing restrictions may come to bind in the future, constraining w_2, w_3, etc. also to be above zero. This amounts to having households solve the intertemporal Euler condition in (5) with the rightmost one-period-ahead expectation operator censored over the "good" interval in which the income realisation is sufficiently favourable (i.e., the output gap $g_1$is large enough) to allow a consumption level in excess of total spendable resources. In this way, households draw up their consumption plans *as if* they were subject to a borrowing restriction in the period of decision – as apparent from the former term in the max operator of (5) – but no borrowing restriction were anticipated to bite in the future. [7]

Appendix I proposes an algorithm to find the numerical optimal solution to the "true" optimisation problem, i.e. the exact solution implied by (5), (3) and (4). We conduct stochastic simulations based on the parameterisations presented in Section 3 below. We show that, under quadratic preferences, and for C normalised to equal trend output y^*, the exact optimal decision at any arbitrary time

Our rule of thumb can be represented in the following cut-off representation:

$$c_t^* \ = Min\left[C, \ \hat{c}_t\right] \tag{7}$$

with:

$$\hat{c}_t = \begin{cases} c_t^{PI} = \ y^* + \left[\frac{1+r-\beta}{1+r-\rho}\right] g_t + (1+r-\beta)w_{t-1} \ - \left[\frac{1+r-\beta}{1+r}\right] \sum_{j=t}^{\infty} \left(\frac{1}{1+r}\right)^j \int_{-\epsilon}^{\epsilon} (s_j + k_j)\, dG^j, \\ \qquad \text{if } y^* + g_t \ + (1+r)w_{t-1} - (s_t + k_t) \geq c_t^{PI} \\ c_t^{B} = \ y^* + g_t \ + (1+r)w_{t-1} - (s_t + k_t), \\ \qquad \text{if } y^* + g_t \ + (1+r)w_{t-1} - (s_t + k_t) < c_t^{PI} \end{cases} \tag{8}$$

First, notice that, since (6) admits a bliss (or saturation) point for period consumption, at an optimum consumption – at any date t – cannot exceed C. This explains the Min operator in (7). Secondly, the renewal process formulated in (8) captures our simple rule of thumb, in that the household sets a target for period consumption equal to its permanent-income optimal choice, i.e. c_t^{PI}, which in turn is equal to the annuity value of the sum of financial wealth and the expected present discounted value of future take-home income. Appendix

[7] Appendix I proposes an algorithm to find the numerical optimal solution to the "true" optimisation problem, i.e. the exact solution implied by (5), (3) and (4).

I provides the details of the analytical expression for c_t^{PI}. If disposable resources at time t, $y^* + g_t + (1+r)w_{t-1} - (s_t + k_t)$, are large enough to allow for the financing of c_t^{PI}, it consumes c_t^{PI} and saves the difference between current resources and target consumption. Otherwise, it sets its consumption at c_t^B, equal to its entire current resources – driving its wealth to zero. In this way, at any t, there exists a unique critical value c_t^{PI}, such that, for draws of income bad enough to make spendable resources fall short of c_t^{PI}, the household decides to spend everything it has on hand and enter the next period with zero assets; conversely, for realised spendable resources exceeding c_t^{PI}, something of the cash currently on hand will be held over and carried forward into the next period, and current consumption will be lower than spendable resources at zero. As it is apparent, this consumption schedule displays excess sensitivity of consumption to even transitory earnings innovations over the liquidity-constrained region – that is, the marginal propensity to consume out of an even non-permanent income innovation is one. Notice that our assumption that households are relatively impatient and unpersuaded by the rewards of waiting (i.e. $\beta > 1$) guarantees that the current value – at time zero – of the expected marginal utility at an infinitely distant time converges to zero almost surely. As discussed in Aiyagari (1994), this condition is necessary – but not sufficient – in order to bound the household's accumulated assets away from infinity at some distant point in the future.[8] In the remainder of the paper we imagine that the native household enters period zero with zero wealth, i.e. $w_{-1} = 0$.

Before concluding on the household's decision problem, it is worth saying a few words regarding our choice of quadratic preferences.[9] Quadratic utilities are notoriously scarcely appealing as a description of consumers' behaviour towards risk, as they imply an increasing willingness to pay to avoid a given bet as wealth increases. However, quadratic utilities imply risk-averse and yet imprudent agents, i.e. agents who do not save for precautionary motives. Since our main result – that non-altruistic agents would choose to purchase insurance by means of costly debt reduction – is strongly dependent on agents' attitude towards risk, and would be induced by preferences displaying a high degree of prudence, we chose to minimise this analytical bias by partially ruling out precautionary saving[10]. Our choice of quadratic utilities, beyond serving the purpose of taming analytical complexity, also reflects this concern.

2.1 Fiscal policy and consumption

The only policy problem facing the fiscal authority in this economy is that of deciding – at time zero – over the level at which the debt-to-population ratio should be *stabilised* in the

[8] Clarida (1987) establishes that, even under $\beta > 1$, the process of wealth accumulation can be explosive if households' rate of relative risk aversion is not uniformly bounded. A prominent example of such preferences is the exponential utility function.

[9] Our utility function has been used by Hall (1978), Hayashi (1985), Abel (1990) and, in a model of fiscal adjustment, Bertola and Drazen (1993). Blanchard and Mankiw (1988) review the arguments in favour of more plausible approaches to the formalisation of preferences.

[10] As shown by Carroll and Kimball (2001), even in the quadratic utility case precautionary saving takes place.

long run.[11] This, as we shall see in the remainder of this Section, involves a key distributional choice.

At time zero the government starts out with an inherited debt – held on behalf of the native household – which carries the value b_0 in terms of consumption good. This debt is denominated in one-period risk-free real bonds issued on the international capital market. Since the size of the population at date zero is normalised to one, b_0 stands for both total and per-capita real public liabilities at the start of history. Imagine now that the government decides to stabilise debt at its historical level b_0. In this case, in order to preserve stationarity in the future evolution of $b_t = \left(\frac{1+r}{1+n}\right)(b_{t-1} - s_{t-1})$, the government has to commit to an infinite series of taxes $\{s_t;\, t \geq 0\}$ satisfying the following restriction:

$$b_0 = \sum_{t=0}^{\infty} \left(\frac{1+n}{1+r}\right)^t \int_{-\epsilon}^{\epsilon} s_t \, dG^t \tag{9}$$

According to (9) the government must identify an infinite succession of per-capita primary surpluses s_t that can guarantee systematic coverage of the interest payments due on b_t, given the expected rate of population growth n, thus keeping its real per capita burden sufficiently close, on average, to its long-term stabilisation level b_0.[12] Notice that in this case, all generations of taxpayers – the native household *and* all the successive generations of immigrants from their respective time of settlement onwards – are called upon to shoulder a uniform amount of real debt b_0: solvency and (stochastic) stationarity thus impose that this amount be backed by the present discounted value of the whole stream of taxes (or, equivalently, per-capita primary surpluses) s_t that each resident – whether native or immigrant – will have to pay over its lifetime.

Assume, in contrast, that the policy decision were to stabilise at some level $b^* < b_0$. Since the level at which stabilisation is effected, b^*, identifies the real burden of debt that the government is going to assign, upon entry, to each of the successive generations of immigrants entering the country from time one onwards, and since default is ruled out, stabilising at some $b^* < b_0$ means that the native household will have to be held liable for covering the difference between b_0 and b^*. In formulae, the native household's inherited burden of debt at zero, b_0, would be partitioned into two shares according to the following accounting condition:

$$b_0 = b^* + \bar{k}\,; \quad b^* = \sum_{t=0}^{\infty} \left(\frac{1+n}{1+r}\right)^t \int_{-\epsilon}^{\epsilon} s_t \, dG^t\,, \quad \bar{k} = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \int_{-\epsilon}^{\epsilon} k_t \, dG^t \tag{10}$$

The former fraction of the debt borne by the native dynasty at zero – what we shall call the *kernel* or *universal* debt and indicate with b^* in (10) – would be the real burden which the native taxpayer would share with each future generation of newcomers. Again, solvency and

[11] We will assume, without loss of generality, that government consumption is zero throughout, so that no resources are absorbed from the economy beyond what is needed to ensure the payment of interest on debt.

[12] Condition (9) stems from solving forward the first-order difference equation describing the law of motion of per-capita debt, i.e. $b_{t+1} = \left(\frac{1+r}{1+n}\right)(b_t - s_t)$, and imposing the usual no-Ponzi-game condition that rules out explosive paths of deficit financing.

stationarity would impose that this amount of liabilities be backed by $\sum_{t=0}^{\infty}\left(\frac{1+n}{1+r}\right)^{t}\int_{-\epsilon}^{\epsilon} s_t\, dG^t$, i.e. the expected discounted value of the *universal taxes* (or primary surpluses) to be paid by *all* residents – whether native or immigrant – at any generic time t, paying due attention to the expectation of an expanding tax base (which justifies n in the compounding factor). By construction, the second fraction of the debt burden weighing upon the native household at zero, $\bar{k}$, would not be backed by the infinite stream of universal taxes s_t, and thus it would *not* be borne by any taxpayer other than the native one. Consequently, it would have to be backed by the present discounted value $\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^{t}\int_{-\epsilon}^{\epsilon} k_t\, dG^t$ of the anticipated *excess taxes* k_t levied on the first generation alone until debt reaches its optimal value: this justifies the absence of n from the numerator of the compounding factor.

We focus on the beginning of history, time zero, when only the native dynasty is resident in the country and all policy decisions take place. Naturally, there are time consistency pitfalls to the policy exercise, in particular questions pertaining to the degree to which governments can credibly commit to a policy whose target moves over time. If the government were allowed to re-optimise at some later date $t \geq 1$ *before* the native household had fully repaid its excess debt burden $\bar{k}$, the mere anticipation of this later action by the native household at time zero would suffice to distort its policy preferences at the same point of history and the whole policy exercise outline above would thereby unravel. This is why we impose that the government be capable of firmly committing itself to a detailed set of actions and be prevented from deviating and reneging on previous promises at any future date.

After making the founding decision about who is bearing which burden of public debt at time zero, the fiscal authority is entirely confined to the implementation of the policy plan established at the start of history. This mainly involves the collection of taxes from national taxpayers and the transfer of the relative proceeds to the international holders of national bonds as interest payment and – at times of income adversity – it may encompass the channelling of credit from outside capital markets to borrowing-constrained national residents in the codified form outlined below (in condition (11)). In any case, the real public debt burden weighting upon the native household, b^* plus $\bar{k}$, will evolve according to the following laws of motion. The *kernel debt* will follow $b^*_{t+1} = \left(\frac{1+r}{1+n}\right)(b^*_t - s_t)$, with the initial condition $b^*_0 = b^*$ and $b^* = \left(\frac{1+r}{r-n}\right)s^*$, with s^* denoting – as better explained in (11) below – the primary surplus run in the absence of shocks to output, when all previous cyclical influences have abated.[13] The *excess debt,* instead, will follow $\sum_{j=t}^{T}\left(\frac{1}{1+r}\right)^{j-t} E_t k_j = (1+r)\left(\bar{k} - \sum_{i=0}^{t-1}\left(\frac{1}{1+r}\right)^{i} k_i\right)$, with $T < \infty$ under certain conditions to be specified below.

In order to cut down on the dimensionality of the policy space, and to focus the problem on the choice over $b^*$and $\bar{k}$ at time zero only, we will restrict the search for a policy optimum to the sequences of dated contingent taxes $\{s_t,\, k_t;\, t \geq 0\}$ satisfying the linear restrictions posited by the following Assumption 1:

[13]Notice that $b^* = \left(\frac{1+r}{r-n}\right)s^*$ is the solution to $b_{t+1} = \left(\frac{1+r}{1+n}\right)(b_t - s_t)$ when $s_t = s^*$ and $b_t = b^*\ \forall t$.

Assumption 1 – *We assume s_t and k_t in (9) and (10) to obey:*

$$s_t(g_t) = s^* + a\, g_t + r(b_t^* - b^*) \tag{11}$$

and:

$$k_t(g_t) = \begin{cases} y^* + g_t - c_t^*(g_t) \ - s_t(g_t), & y^* + g_t \ - s_t(g_t) \geq c_t^*(g_t) \\ 0, & y^* + g_t \ - s_t(g_t) < c_t^*(g_t) \end{cases} \tag{12}$$

Equation (11) renders, in a standard linear fashion,[14] two key features of a modern fiscal regime operating in a *stationary* stochastic context. Firstly, it captures the built-in countercyclical features of modern budgets, whereby gross revenues typically rise – and/or primary expenditures accruing to individual citizens decline – as the realised output gap widens. This income-induced fraction of taxes is represented by $a\, g_t$, where a measures the sensitivity of the individual (and aggregate) net tax liabilities to the individual (and aggregate) output gap.[15] Secondly, (11) incorporates the condition that, in a stochastic steady state, the sequence of current and future primary surpluses must ensure that the present discounted value of current and future revenues used to sustain debt satisfy the government's intertemporal budget constraint – as formulated in (9) or (10) *at each point in time* – not only at time zero – that is, for all initial debt stocks and following aggregate shocks of all possible magnitudes. (11) does so by centering the primary surplus around a long-term norm s^* univocally linked with the debt target b^* – via the identity $s^* = \left(\frac{r-n}{1+r}\right) b^*$ – and by imposing a self-correcting mechanism that ensures systematic mean reversion once the shocks hitting the economy have abated – via $r(b_t^* - b^*)$. Whenever taxes are residually determined *at each point in time* to satisfy the government solvency constraint, this contingent (closed-loop) rule can be assumed to hold, and vice versa.

Condition (12), for its part, posits an "opportunistic" scheme for the time distribution of the payments $k_t(g_t)$ due on account of $\bar{k}$. Assuming that both $\bar{k} > 0$ and $\left[\bar{k} - \sum_{i=0}^{t-1} \left(\frac{1}{1+r}\right)^i k_i\right]$ > 0 – i.e. provided that the excess debt $\bar{k}$ established at the start of history was strictly positive, and that it has not yet been repaid – at any generic t the tax due by the native taxpayer against its excess debt obligation $\bar{k}$ will be calibrated in a way to extract no more than the difference between its time-t realised disposable income – net of the universal tax $s_t(g_t)$ levied on *all* residents – and the optimal consumption target defined by (7). A scheme of this sort is *opportunistic* in that the government takes advantage of "good" states of nature – when the native household's disposable resources suffice to finance at least $c^*(g_t)$ as in the upper section of (12) – to tax income heavily, in order to temporarily lift the excess tax in periods in which the household is liquidity constrained – as in the lower section.Appendix II

[14] Buiter (1998) proposes a rule similar to ours in a model of fiscal policy and price determination.

[15] In a broader analytical context than the one presented in this paper, a condition similar to (11) would endogenously flow from the policy optimisation problem rather than being imposed as an institutional foundation of the policy process itself. The literature on the optimality of deficit rules reminiscent of (11) – whereby governments run a surplus in "good times" and a deficit in "bad times" – is too vast to be summarised here. As a prominent recent example see Chari, Christiano and Kehoe (1994).

proves that an opportunistic scheme of this nature is self-liquidating in expectations at time zero, i.e. there exists a $T < \infty$ such that $\sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^t \int_{-\epsilon}^{\epsilon} k_t(g_t)\, dG^t = \overline{k}$.

The following Lemma, which concludes this Section, reveals the impact of different distributional choices upon the native household's consumption at zero in good and bad income conditions.

Lemma 1 – *For any given b_0, if liquidity constraints do not bind, consumption is an increasing function of the level at which stabilisation occurs, b^*; conversely, under binding liquidity constraints, consumption is a negative function of b^*. The likelihood of the household finding itself borrowing constrained increases with b^*. At $t = 0$ the new rule for consumption now looks as follows:*

$$\hat{c}_0 = \begin{cases} c_0^{PI} = y^* + \left[\frac{(1-a)(1+r-\beta)}{1+r-\rho}\right] g_0 + \left(\frac{1+r-\beta}{r(1+r)}\right) [n\, b^* - r\, b_0] \; + o(g_0), & g_0 \geq \Gamma_0 \\ c_0^{B} = y^* + (1-a)\, g_0 - s^* = y^* + (1-a) g_0 - b^* \left(\frac{r-n}{1+r}\right) & g_0 \; < \Gamma_0 \end{cases} \tag{13}$$

with:

$$\Gamma_0 = \left(\frac{1+r-\rho}{(1-Aa)(\beta-\rho)(1+r)}\right) \left[b^* \left(r - n \left(\frac{\beta - 1}{r} \right) \right) - b_0 (1+r-\beta) \right] \tag{14}$$

where $o(g_0)$ is a term in the realised output gap of a second order of magnitude and $A = \left((\beta - \rho) + \frac{r(1+r)(1+r-\beta)}{n+r} \right) / (\beta - \rho)$.

Proof. Use (11), (12), (10), the initial condition for the kernel debt, $b_0^* = b^*$, and $s^* = \left(\frac{r-n}{1+r}\right) b^*$ to eliminate $\sum_{j=t}^{\infty} \left(\frac{1}{1+r}\right)^j \int_{-\epsilon}^{\epsilon} (s_j + k_j)\, dG^j$ and $s_t + k_t$ from (8). This yields a new cutoff rule for individual consumption, with both c_t^{PI} and c_t^{B} expressed as functions of b^*. In order to make the threshold for disposable resources $y^* + g_t - s_t + w_t(1+r) < c_t^{PI}$ – below which consumption switches analytical forms – a function of the realised output gap at time t, substitute the new expression for c_t^{PI} into it and collect terms in g_t. Finally, particularise at time $t = 0$ and recall that we assume $w_{-1} = 0$. **[c.q.d.]**

Notice that, under any reasonable parameterisation relevant to this paper, coefficient A can be safely approximated to unity. In what follows, we shall therefore assume $A = 1$.

Within a dynastic model with borrowing constraints, like the one we use, the result of the Lemma is hardly surprising. Imagine, for the time being, as in Weil (1989), a frictionless world, where agents are never prevented from consuming at their preferred permanent-income level $c_t^{PI}(g_0)$. Provided the consumer is moderately impatient (i.e. $1 \leq \beta < 1 + r$), and assuming a positive anticipated rate of population growth n, agents would *not* be indifferent as to the level of debt at which stabilisation occurs: they would gain from a policy that stabilises at a "high" b^*, as c_0^{PI} is a *positive* function of b^*. This anti-Modigliani-Miller feature is simply a reflection of the basic non-Ricardianness of a framework where eternal dynasties overlap without motives for mutual altruism. If the population is expected to be sustained by a *positive* stream of immigrants (i.e. $n > 0$), offloading more of b_0 onto these aliens becomes an attractive choice for the native household living in a world of complete markets. This result is similar in spirit to that recently discussed by Ireland (2001) in the same analytical context augmented by the presence of money. Likewise, Detken (1999) uses a Weil model in a similar context and discusses its non-Ricardian properties.

Now introduce frictions into the credit market and the need to switch from $c_t^{PI}(g_0)$ to $c_t^{B}(g_0)$ under sufficiently bad income conditions. When borrowing restrictions bind, as apparent from the lower section of (13), having to pay a lower tax on average, s^* – i.e. having stabilised around a lower debt ratio $b^* = \left(\frac{1+r}{r-n}\right) s^*$ – would be most valuable, as under such conditions the marginal propensity to consume out of a tax cut would be one. Therefore, when the household is prevented from consuming at its preferred level because of credit market imperfections, a policy of low debt stabilisation would appear the most advantageous. Again, the notion that the timing – and the scale – of taxation does matter when households are prevented from borrowing freely mirrors earlier results in the Ramsey-problem literature. See, for example, Barsky, Mankiw and Zeldes (1986).

Notice also two features about the threshold Γ_0. Firstly that, as $\beta \to 1^+$, Γ_0 is never positive and entirely driven by the difference between b_0 and b^*. A status-quo solution to the stabilisation choice (one in which $b^* = b_0$) would thus entail $\Gamma_0 = 0$, and it would consequently put the threshold of income under which the household feels borrowing constrained at y^*, average income. This result replicates Deaton's (1991) simulations on optimal cutoff rules under different utility specifications. Secondly, as $\beta \to 1^+$, $\Gamma_0 \leq 0$ is a negative function of b^* and a. Hence, with liquidity constraints, stabilising at a b^* lower than b_0 – although costly – can make the bad occurrence less likely and alleviate the hardship when it occurs. Likewise, boosting the sensitivity of the budget to the income gap has the intuitive advantage of pushing the gap threshold more into negative territory, thus alleviating the probability of incurring borrowing constraints.

In conclusion, (13) points to the fundamental tension existing between the policy preferences that the native household would advance under a *good* and a *bad* state, respectively. If the native household – or the government on its behalf – were to express its preferences over the split of b_0 into b^* and $\overline{k}$ after having received a "good" draw of income (with g_0 such that $\Gamma_0 \leq g_0 \leq \epsilon$), it would decide to set $\overline{k}$ to zero and to offload as much debt as possible to future generations. By contrast, if it were to express its preferences under a bad income draw, it would vote for as low a b^* as it would be compatible with the feasibility constraints facing the policy decision.

The remainder of this paper proves that, at the margin, there may exist an intermediate level of $b^* < b_0$ (associated with a strictly positive self-imposed excess burden $\overline{k}$ payable in good times according to (12)) at which the utility gain perceived under binding borrowing restrictions from a small reduction in the tax would be exactly balanced off by the anticipated utility loss of having to shoulder an excess tax $k_t(g_t)$ in good times.

2.2 Optimal debt with no deficit limits

Policy decisions over the relevant fiscal variables are taken at the beginning of period zero, on behalf of the only household resident at that time and *before* anybody (native agent or fiscal authority) can observe the realisation of the contemporary endowment shock, g_0. Thus the government ranks stabilisation levels for the debt-to-population ratio and the associated time-sequences of (possibly) contingent tax schedules $\{s_t,\ k_t \geq 0;\ t \geq 0\}$ on the basis of a criterion function provided by the lifetime indirect utility of the *native household,* determined by (6). To make the problem non-trivial, we implicitly disallow confiscatory taxes on accumulated wealth and immigrants' incomes. After the fully credible policy plan has been announced, the native household gets to decide over its preferred intertemporal allocation of consumption.

Definition 1 – *We define an* optimum *for the debt ratio and the budget sensitivity as the solution to the government problem of maximising the native household's intertemporal indirect utility* $\sum_{t=0}^{\infty}\left(\frac{1}{1+\delta}\right)^t \int_{-\epsilon}^{\epsilon} U[c_t^*(g_t)]dG^t$ *by choice of alternative stabilisation options* $\{b^*,\ \bar{k}\ \geq 0;\ b_0 = b^* + \bar{k}\}$ *and budget sensitivities* $0 \leq a^* \leq 1$*, with* $c_t^*(g_t) > 0$ *– the optimal consumption choice – given by (7) and (13).*
A local optimum *is the vector* $\Omega^* = \{b^*,\ a^*\} = \arg\max \sum_{t=0}^{\infty}\left(\frac{1}{1+\delta}\right)^t \int_{-\epsilon}^{\epsilon} U[c_t^*(g_t \mid b_0)]dG^t$*, where consumption is conditional on* b_0*.*
A global optimum *is the fixed point of the government local optimisation problem, i.e. a vector* $\Omega_G^* = \{b_G^*,\ a_G^*\} = \arg\max \sum_{t=0}^{\infty}\left(\frac{1}{1+\delta}\right)^t \int_{-\epsilon}^{\epsilon} U[c_t^*(g_t \mid b_G^*)]dG^t$*.*

Assumption 2 – *We assume that the government policy choice is made under steady state conditions, i.e. with* $g_{-1} = 0$*. Also, for ease of exposition and in order to suppress notational clutter, assume* $\beta \to 1^+$*.*[16]

The set up of our welfare optimisation problem poses two sorts of issues related to the objective of the government's benign care and to the set of attainable policy options. On the former note, it is fair to say that identifying the policy objective function with the lifetime utility of the *native household* alone amounts to raising the latter to the status of *the* representative citizen in this economy. This assumption would no doubt look arbitrary within a standard overlapping generations model. However, we believe this identification to be warranted in the present context, where the foundations of the fiscal regime for the years to come have – by construction – to be laid down at the start of history, and where all generations of not-yet resident taxpayers at time zero bear no connection to the native resident. On the latter point, concerning the admissible policy set, notice that we do *not* impose model closure conditions on the government optimisation problem, as stated in Definition 1. This condition would simply state that the ex-post difference between realised income and optimal consumption at any date explains the change in the country's net asset position. However, within this partial equilibrium model such a condition of feasibility of foreign current transactions, provided the government and the household budget constraints are satisfied, is simply an identity.

Our problem admits two kinds of solutions. The *local* solution is defined as a set Ω^* that solves the constrained optimisation exercise given the initial debt inherited from history, b_0. The local solution thus answers the following question: what is the value of $b^*$and a^* the

[16] Results for the case $\beta > 1$ are available form the authors upon request.

government would choose on behalf of the taxpayers, conditional on starting off from a given fiscal position identified by b_0? In contrast, the global solution is identified by a set Ω^*_G which represents the fixed point of the optimum problem. In other words, conditional on starting off from a fixed-point couple $\{b^*_G,\ a^*_G\}$, taxpayers would not feel any incentive to move in any direction.

The following proposition establishes a necessary and sufficient condition for an interior solution to the government optimisation problem, i.e. for $b^* < b_0$.

Proposition 1 – *The necessary and sufficient condition for an interior solution to the government optimisation problem is that $r > 2n$. If this condition is satisfied, the following applies:*

(i) If the government treats a as a given in its optimisation problem, the local optimum for b^* takes on the following generic form:

$$b^* = b_0 - \Phi\left(a, \epsilon, \rho | \, r - n\right) \tag{15}$$

with:

$$\begin{aligned} \Phi\left(a, \epsilon, \rho | \, r - n\right) &\geq 0 \\ \Phi\left(1, \epsilon, \rho | \, r - n\right) &= \Phi\left(a, 0, \rho | \, r - n\right) = \Phi\left(a, \epsilon, 1 | \, r - n\right) = \Phi\left(a, \epsilon, 1 | \, n\right) = 0 \\ \Phi_\epsilon\left(a, \epsilon, \rho | \, r - n\right) &> 0, \quad \Phi_{r-n}\left(\rho, \epsilon, a | \, r - n\right) > 0; \quad \Phi_a\left(a, \epsilon, \rho | \, r - n\right), \ \Phi_\rho\left(a, \epsilon, \rho | \, r - n\right) < 0 \end{aligned}$$

where Φ_i denotes the partial derivative of function Φ with respect to its argument i. The global optimum is given by:

$$b^*_G = \left[-\frac{1}{4} \frac{(1-\rho)(1+r)\epsilon}{(r-n)(1+r-\rho)} \right] (1-a) + \left[\frac{1}{4} \, \frac{(1+r)\ \epsilon}{(r-n)(1+r-\rho)} \ \frac{r^2\ (1+r)}{n+r} \right] a \tag{16}$$

(ii) If a is a policy choice variable, then:

$$\begin{aligned} a^* &= a^*_G = 1 \\ b^* &= b^*_G = b_0 \end{aligned} \tag{17}$$

Proof. See Appendix II.

Lemma 2 in Appendix II reduces the dynamic optimisation problem to its static counterpart, and makes use of a first-order Taylor expansion to transform a problem of utility maximisation into one in which consumption is being maximised instead. The resulting necessary conditions for a welfare optimum – under the opportunistic scheme for excess debt repayment posited in (12) – are the following:

$$\left(y^* - E_0\left[c_0(g_0,\, b^*,\, a \,|\, b^*)\right]\right) \frac{\partial E_0\left[c_0(g_0,\, b^*,\, a \,|\, b_0)\right]}{\partial b^*} = 0 \tag{18}$$

$$\left(y^* - E_0\left[c_0(g_0,\, b^*,\, a \,|\, b^*)\right]\right) \frac{\partial E_0\left[c_0(g_0,\, b^*,\, a \,|\, b_0)\right]}{\partial a} = 0 \tag{19}$$

with the latter one, taken with respect to a, disactivated when the government treats a as a given, and where $E_0\left[c_0(g_0,\, b^*,\, a)\right] = \int_{-\epsilon}^{\Gamma_\theta} c_0^B(g_0,\, b^*,\, a \mid\, k_t(g_0))\, dG^0 + \int_{\Gamma_\theta}^{\epsilon} c_t^{PI}(g_0,\, b^*,\, a \mid\, k_t(g_0)) dG^0$. The two conditions simply state that, off corners, i.e. provided that the optimisation occurs in the range where the native household's marginal utility is positive – when expected consumption still falls short of the saturation point C, which we normalised to y^*, i.e. $E_0\left[c_0(g_0,\, b^*,\, a)\right] < C = y^*$ – the optimal value of b^* (or a) is the one which sets to zero the own derivative of expected consumption – the right hand side of (18) or (19). Otherwise, past the bliss level of expected consumption, the optimum is nailed down by the left-hand side of (18) or (19), i.e. $E_0\left[c_0(g_0,\, b^*,\, a)\right] = C = y^*$. The right-side marginal conditions are the ones that pin down the *local optima* for b^* and a, respectively, whereas the expression on the left-side, when it becomes zero, identifies the *global* maximum for expected consumption, and, by implication, expected utility. Figure 1 graphically displays these dynamics of the solution in Proposition 1.

[INSERT FIGURE 1]

Proposition 1 synthesises our results under a fiscal regime that features no restrictions on the level of the deficit that the government can run period-by-period. It points to the notion that under such a regime there exist two *complementary* forms of government-provided insurance available to the native household against bad luck, i.e. low consumption when borrowing constraints happen to bite. These two competing insurance packages can be ranked in utility terms. The first, and the more efficient, package is automatic stabilisation, i.e. the promise of a subsidy a – per each unit of income shortfall – that households automatically receive from the budget when realised endowments are below normal levels. A value of a as close to one as possible can both mitigate the consequences of a bad shock once it occurs – as evident from the lower section of (13) – and make the bad state less likely to occur in the first place – as apparent from (14).

The second form of self-insurance, when the first one is unavailable or restricted, contemplates the payment of a premium $k_t(g_t)$ – collected only in good states of nature, according to (12), and with a present discounted value equal to $\bar{k} = b_0 - b^*$ – against the promise of having the non-contingent fraction of taxes s^* permanently reduced. Clearly, the former insurance package dominates the latter, as it can be activated *ex post* when income shortfalls eventually materialise.

The complementarity between automatic stabilisation and debt reduction explains both (15)-(16) and (17). When a is a fully viable policy option, the government – acting on behalf of the household – is well-advised to "purchase" full coverage in the form of automatic stabilisation ($a^* = a_G^* = 1$) – which can be activated *ex post* at no cost – and zero coverage in the alternative form of debt reduction ($b^* = b_G^* = b_0$), which requires a cumulative payment of $\bar{k}$. Conversely, when the choice of a is restricted and so that it is sufficiently smaller than unity, the native agent is generally ready to accept a policy of accelerated debt reduction whose cost falls entirely upon itself, provided the associated payments $k_t(g_t)$ earmarked against the $\bar{k}$ excess obligation are opportunistically designed, and the growth rate of population is not too large.

Figure 2 provides a graphical representation of the channels through which these two insurance schemes can cushion consumption against unfavourable income shocks. The first panel shows how an increase in a can be thought of as causing a clockwise rotation of the two relevant branches of the consumption schedule, thus lifting consumption levels attainable to the left of its kink marked by Γ_0. Γ_0 itself gets pushed into more negative territory as a consequence of the rotation, which itself improves expected utility as it reduces the perceived likelihood of a bad state. The second panel portrays the impact of a cut in b^* as a downward shift in the consumption line applying to the right of Γ_0 and an upward shift of the line applying to the left of Γ_0. The crossing point between the two lines – marking the kink in the overall consumption schedule and identified by Γ_0 – is unambiguously shifted to the left. This, again, reduces the perceived riskiness attached to income fluctuations.

Intuitively, the desirability of insurance coverage increases with uncertainty. This explains why reducing the amplitude of the interval that spans the possible shocks to the income gap in (2) makes debt reduction a less and less attractive option. Accordingly, Φ is rising in ϵ and reaches a minimum of zero as $\epsilon \to 0$. Finally, income persistence also is detrimental to prudent behaviour. If ρ is large, a good draw of income indicates that more good draws are to be expected in the future, thus making a prudential debt cut less desirable. Since, by assumption, policy decisions are taken under relatively good income conditions – the steady state – this explains why raising ρ towards unity makes the *status quo* option $b^* = b_0$ more attractive.

[INSERT FIGURE 2]

2.3 Optimal debt with a deficit limit

Most likely, unfettered fiscal regimes of the sort considered in the preceding Section are an abstraction or, at best, a very loose approximation to reality. As conjectured by Bertola and Drazen (1993), trigger points may exist that restrict the scope for fiscal policy decisions when, say, fiscal outcomes are subject to financial market scrutiny. Values of government spending – or deficit – to output considered critical by consumers or market participants may induce discrete stabilisations when they are approached. Seen from a different angle, trigger points may be institutionalised by the imposition of hard constitutional limits on government borrowing. Europe's Stability and Growth Pact is a prominent example of a quasi-constitutional arrangement restricting the capability of signatory governments to choose a deficit in excess of 3 per cent of output, even when operating at the trough of serious recessions. Section 3 builds upon this example more extensively.

To account for these various implicit and explicit constraints, in what follows we reformulate the policy problem of the preceding Section by imposing an additional constraint on the government. What will result from this re-optimisation under a deficit limit is a non-discrete and non-discretionary version of the recurrent deficit cycles of Bertola and Drazen who explicitly recognise that their model can be re-interpreted as a model of infinitesimal fiscal interventions once the trigger is reached.

Definition 2 – *We define a deficit constraint as:*

$$\left[b^*\left(\frac{r-n}{1+r}\right)\right] + a\epsilon(\omega-1) - rb^* \leq -\,d_{\max}[\,y^* + \epsilon(\omega-1)] \tag{20}$$

where d_{max} is the maximum deficit allowed by the constraint, and $0 < \omega < 1$.

The form that we give to the deficit constraint in the steady state deserves further discussion. One issue concerns how *soft* the constraint is allowed to be in our model. Normally, provided it is credible, a deficit limit must be observed in virtually *all* circumstances of nature to which a positive probability, no matter how small this might be, is attached. In our context, such a "hard deficit" regime would dictate that fiscal variables be calibrated so as to insure against a breach of the rule even under the worst possible realisation of output, no matter how unlikely that is. However, in practice – and notably in the light of the implementation arrangements put in place by the Stability and Growth Pact – credible deficit rules can still leave scope for contingent provisions tolerating breaches of the deficit limit under particularly unfavourable circumstances.[17] Consequently, we allow for a continuum of degrees to which the rule can be made stringent, depending on the parameter ω. If ω is zero, the deficit limit $d_{\max}$ may not not be violated even in the case of (per-capita) output being at its worst possible reading of $y^* - \epsilon$. At the other extreme of the spectrum of strictness, if, say, $\omega = 1$, overall deficit would be permitted to be as high as $d_{\max}$ per cent of trend output under zero-output-gap conditions, which, in turn, would imply that it could end up considerably overshooting that value in a less favourable output scenario, given its countercyclical properties. In between these two extremes, we allow for a continuum of output thresholds at which the escape (or tolerance) clause is triggered, i.e. $0 \leq \omega < 1$.

Further notice that, starting from the steady state (i.e. when $b^*_{t-1} = b^*_t = b^*$), $s^* = b^*\left(\frac{r-n}{1+r}\right) + a\,\epsilon(\omega-1)$ is the *minimum* per-capita primary surplus that can be tolerated under the deficit rule. Hence, $b^*\left(\frac{r-n}{1+r}\right) + a\,\epsilon(\omega-1) - rb^*$ is the *maximum* that the overall deficit can reach without triggering penalties or market reactions.[18] Both deficit constraints are calibrated in real per-capita output terms, by multiplying the deficit threshold $d_{\max}$ by $y^* + \epsilon(\omega-1)$, to make them applicable to a setting where the trigger for deficits is expressed in per cent of trend output.

Definition 3 – *We define a* deficit-constrained optimum *for the debt ratio and the budget sensitivity as the solution to the government problem of maximising the native household's intertemporal indirect utility $\sum_{t=0}^{\infty}\left(\frac{1}{1+\delta}\right)^t \int_{-\epsilon}^{\epsilon} U[c^*_t(g_t)]dG^t$ by choice of alternative stabilisation options $\{b^*,\ \bar{k} \geq 0;\ b_0 = b^* + \bar{k}\}$ and budget sensitivities $0 \leq a^* \leq 1$, with $c^*_t(g_t) > 0$*

[17] The Stability and Growth Pact contains a provision allowing for a deficit-to-GDP ratio in excess of 3 per cent if it results from an unusual event outside the control of the member state that has a major impact on the financial position of the general government or if it results from a "severe" economic downturn.

[18] In order to keep complexity at a minimum, we overlook the possibility that a sufficiently long string of bad draws of output may inflate the mean-reverting term $r(b^*_t - b^*)$ in (11). This simplification can make our calibration of the deficit constraint by the conditions prevailing over the steady state too lax. As a partial compensation to this bias, when simulating fiscal plans under the Stability and Growth Pact requirements, we set ω equal to zero.

– the optimal consumption choice – given by (7) and (13), and the steady state per-capita deficit $d = b^*\left(\frac{r-n}{1+r}\right) + a\epsilon(\omega - 1) - rb^*$ *restricted by: (20), with some* $0 \leq \omega < 1$. *The usual characterisations of local and global optima follow from Definition 1.*

Proposition 2 – *The* local *solution to the deficit-constrained optimum problem is yielded by the solution to the the following sets of first order conditions:*
(i) For values of $E_0\left[c_0(g_0,\, b^*,\, a)\right] < y^*$*: the deficit constraint (20), and*

$$\frac{\partial E_0 c_0(g_0,\, b^*,\, a)}{\partial b^*}\left(\frac{\partial E_0 c_0(g_0,\, b^*,\, a)}{\partial a}\right)^{-1} = \left[\left(\frac{r-n}{1+r}\right) - r\right]\frac{1}{\epsilon(\omega - 1)} \tag{21}$$

(ii) For values of $E_0\left[c_0(g_0,\, b^*,\, a)\right] \geq y^*$*: the deficit constraint (20), and*

$$E_0\left[c_0(g_0,\, b^*,\, a \mid b^*)\right] = y^* \tag{22}$$

while the global *solution* (b^*_G, a^*_G) *to the deficit-constrained optimum problem is given by the solution to (16) and (20).*

Proof. The conditions are immediate from the first order conditions to the problem in Definition 3.

Again, provided the optimisation occurs in a range along which the native household's marginal utility is still positive – which occurs when expected consumption is lower than the bliss point, i.e. $E_0\left[c_0(g_0,\, b^*,\, a)\right] < y^*$ – the operative necessary condition for an optimum is the one that exploits the equality of the marginal rate of substitution between the two forms of insurance – that "purchased" through a cut in b^* and that acquired by augmenting a – and the marginal rate of transformation of the one into the other, given the deficit condition. However, past the bliss level of expected consumption, the optimum is nailed down by the condition that expected consumption should be equal to its saturation point y^*.

The local solution to the problem set out in Proposition 2 has to be numerical. For sensible calibrations of the parameters r, n, ϵ, and ρ, – as we will see in detail in the next Section – the local solution obtained in Proposition 2 is such that $b^* < b_0$, and $a^* \leq 1$. In order to numerically evaluate the sensitivity of the obtained solution to the chosen parametric configuration we present Figure 3, that shows some graphical intuition about the form of the solution.

[INSERT FIGURE 3]

It is not difficult to see that *given* a, the solution for b^* can be obtained by solving a quadratic function of the form $f(b^* \mid a) = B_1(b^*)^2 + B_2 b^* + B_3$, where B_1, B_2 and B_3 are non-linear functions of a, r, n, ρ, ϵ, and b_0. Figure 3 plots the locus of points lying on the function $f(b^* \mid a) = 0$. The thick curve corresponds to the $f(b^* \mid a)$ associated with an economy with an initial debt slightly above 60 per cent of GDP, a real interest rate – equal to the rate of individual time preference – of 3.5 per cent, an annual rate of real income growth of 1.5 per cent, a maximum output gap ϵ of almost 4 per cent of potential and an autoregressive parameter for output gaps of 0.6.[19] The couple of points at which this curve intersects the horizontal axis – only one of which is visible in the Figure – identify the two values of debt b^* at which the necessary conditions of the policy problem are satisfied. However, in order to choose the debt ratio which, starting from around $b_0 = 60$, the government should set as the target for fiscal policy, one needs to evaluate the welfare of the taxpayer at the two pairs of roots for b^* and a. We checked that, within the subspace of parameter values that is of interest to us, the welfare is maximised at the *higher interior root* for b^*, i.e. $b^* < b_0$. Thus, referring to the archetypal case portrayed in Figure 3, the native household in that economy would be ready to accept an excess burden around one third of its normal income relatively to future generations of taxpayers – the difference between 60 per cent and the 32 per cent target indicated by the optimising b^* – in exchange for having: (i) the "core" non-contingent primary surplus s^* curtailed from the current 1.2 per cent of normal income, associated with a debt of 60 per cent, to the 0.6 per cent required to support a debt of 25 per cent in the long run; (ii) the budgetary elasticity of the primary surplus to the gap boosted to 0.6.

These are *local optima*, however. An economy with the same structural parameters but starting off from a debt burden of 100 per cent of normal income would obtain a quite different indication in terms of the appropriate target for fiscal policy – the crossing point between the thin line and the horizontal axis indicates an optimal target for debt between 65 and 70 per cent. Now, if optimal debt can change according to the initial conditions, and actually declines at progressively lower initial debt ratios, two issues arise. The first one, raised earlier in the paper, relates to the need for governments to credibly commit to a policy at time zero. The second concerns the global optimum, which once attained, constitutes a fixed point of the optimisation problem solved by the government.

As regards *global optima,* their characterisation in the deficit constraint case is relatively easy to get. The solution to the system formed by (16) and (20) yields a relatively transparent expression for b^*_G:

$$b^*_G = \left[-\frac{1}{4}\frac{(1-\rho)(1+r)\epsilon}{(r-n)(1+r-\rho)}\right](1-a_G) + \left[\frac{1}{4}\;\frac{(1+r)\;\epsilon}{(r-n)(1+r-\rho)}\;\frac{r^2\;(1+r)}{n+r}\right]a_G \tag{23}$$

and making use of the deficit constraint evaluated at a_G,

$$b^*_G = -\frac{1}{4}\frac{(1-\rho)(1+r)\left[d_{max}(y^*-\epsilon)-\epsilon\right] + \left(\frac{r^2(1+r)^2 d_{\max}(y^*-\epsilon)}{n+r}\right)}{(r-n)(1+r-\rho)+(1-\rho)(r^2+n)+\left(\frac{r^2(1+r)^2(n+r^2)}{n+r}\right)}$$

that provides the reduced form solution for b^*_G. For sensible values of the parameters the previous expression can be approximated by

$$b^*_G = -\frac{1}{4}\frac{(1-\rho)(1+r)\left[d_{max}(y^*-\epsilon)-\epsilon\right]}{(r-n)(1+r-\rho)+(1-\rho)(r^2+n)} \tag{24}$$

Hence, provided that output gaps do *not* follow a random walk (with $\rho = 1$), the sign of (24) depends on whether $[d_{max}(y^* - \epsilon) - \epsilon]$ is positive or negative. In turn, the latter

[19] This particular configuration of parameter values broadly fits the parameter configuration of Germany. See Section 3 below discussing our quantitative results.

expression is non negative if and only if:

$$\epsilon \leq \frac{d_{max}\, y^*}{1 + d_{max}} \tag{25}$$

A superficial comparison between (25) and (23) suffices to suggest the two key results of our paper. Namely, that countries expecting output fluctuations of moderate amplitude would afford targeting a less ambitious – i.e. non-negative – debt even in the *very* long run. This notwithstanding, they would be able to target a very high value for a_G. This latter conclusion can be drawn by observing that (25) is equivalent to $a_G \geq 1$ – see the term after the former equality sign in (23).

Conversely, countries subject to more pronounced cycles ($\epsilon > \frac{d_{\max}\, y^*}{1+d_{\max}}$) would have to settle for *both* less automatic stabilisation ($a_G < 1$) *and* a negative debt position in the *very* long run. As is apparent from (25), the threshold between countries belonging to the two different groupings is traced by the numerical deficit limit, d_{max}.

The next Section shall illustrate the general validity of this pattern of dependence by means of a calibrated example.

3 A case study: Europe's Stability and Growth Pact

Issues in how to best reconcile welfare-enhancing fiscal planning with deficit constraints command strong interest in contemporary Europe. The move to an explicit rule-based fiscal regime has accompanied European Monetary Union, in which fiscal positions must respect the 3 per cent maximum deficit rule embedded in the Maastricht Treaty and the Stability and Growth Pact. Moreoever, Europe's awkward demographics and advanced stage of development in terms of attainable productivity levels also make it an ideal laboratory case for setting our optimal debt algorithm into work.

In an attempt to operationalise the Pact, several studies have examined which country-specific fiscal positions can act as benchmarks guiding policy under the 3 per cent deficit ceiling. The basic premise used to identify these benchmarks is simple. Since budget deficits are counter-cyclical and thus tend to worsen during downturns, and since the maximum deficit-to-GDP ratio that countries are allowed to run without incurring penalties has to be obeyed in almost *any* state of the world – except, perhaps, the most extreme – economies that are more prone to macroeconomic fluctuations and/or whose public budget is more sensitive to cyclical conditions should aim for a lower deficit *on average*. This practice suffers from a fundamental defect. For it is true that, *given* a certain measure of the degree to which budgets respond to cyclical variations, it is possible to pin down the deficit-to-GDP ratio that they should target in the long run if they want to insure themselves against the risk of breaching the limit once at a trough of a recession. It is also true that this deficit benchmark bears with itself, by implication, the extent to which current taxpayers should accelerate debt repayment to bring their fiscal positions close to that target. Given the deficit benchmark d_{max}, it is very easy – just resorting to standard public finance arithmetic – to calculate the debt level b^* which would be consistent with sustaining a normal deficit of d^* forever.

But, on the other hand, if at that juncture debt repayment and the target for debt b^* have to be treated as the *outcome* of a policy choice, a should also be thought of as *endogenously* determined through the same policy process. If governments can decide over *both* their target debt level b^* *and* the degree to which they want their deficits to vary in response to cyclical fluctuations, a, then simple stock-flow fiscal arithmetics – linking b^* to a via the deficit constraint – do not suffice anymore to nail down a new fiscal regime in its entirety.

As shown in the preceding Section, in the presence of constraints on government borrowing, our model is capable to provide indications about both the optimal level of debt *and* the optimal degree of response of deficits to income fluctuations. The rest of the paper is devoted to apply the results stated at the end of the preceding Section to the case of the European Union (EU) under the Stability and Growth Pact. Sub-section 3.1 explains how we calibrate our model to obtain illustrative estimates for a number of Member States of the EU. Sub-section 3.2 then proceeds to the discussion of our numerical results.

3.1 Calibration

In obtaining illustrative quantitative estimates on the basis of parameters in selected EU countries[20], some parameters (such as y^*, r, n, b_0, ω, ϵ) are calibrated, while the degree of persistence of the output gap (ρ) is estimated.

Values for the initial debt ratios, b_0, are calibrated to the estimates for general government consolidated gross debt (Maastricht definition) as a percentage of gross domestic product at market prices, as contained in European Commission (2000). Interest rates, r, are set at 3.5% for all countries, in the vicinity of long-run observations for long-term real interest rates. The parameter calibration for population growth, n, is taken as fixed at 1.5%, while the real rate of time preference, β, is set to one for the baseline scenario. As pointed out earlier, the degree of tolerance with respect to the institutional deficit constraint, ω, is set to zero in the baseline scenario for all countries. The parameter y^* is normalised to 100 in order to interpret output gap realisations as deviations in percentage points.

Country-specific estimates of the persistence (ρ) and magnitude (ϵ) are extracted from output gaps as measured by the European Commission. This output gap measure is based on the percentage deviation of actual output from trend output, with the Commission measure of g_t based on an estimation of trend output using the Hodrick-Prescott Filter, which by design induces stationarity-inducing transformation to the data.[21] The calibrated degree of persistence of period-t output gap shocks for each country is obtained from a first-order autoregressive model of the form proposed in (2), estimated over the period 1960-1999. When assessed against higher-order auregressive models, this specification best characterises the data, as would be expected given the use of HP-filtered series.

Estimates of ϵ, representing the ex-post variability of output gaps, are obtained by taking the average of the maximum and minimum values of the observed output gap over the sample

[20] Greece and Portugal are excluded to avoid any bias that could be associated with a "catching up" process, in which special factors could distort a comparison with other EU countries.

[21] Note that the adopted measure of output gaps is equivalent to the per capita output gap, given that offsetting adjustment for population must be made to both actual and trend GDP in the calculation.

period used to obtain estimates of ρ. This methodology used to calibrate this parameter is similar to that adopted by the European Commission (2000) in defining its set of budgetary benchmarks, whereby the largest weighted average of the two largest negative output gaps recorded in each EU Member State between 1960 and 1997 is used in generating these benchmarks. In order to compensate for the absence of an adequate treatment of structural breaks in the Commission estimates of g_0, the maximum amplitude of maximum output gaps was censored for observed output gaps in excess of 7.5% of trend output. This results in a censoring of ϵ of output gap observations only for Finland (in 1989 and 1993).

For the sake of simplicity, the estimates of ϵ are reproduced on the basis of iid draws from a uniform distribution defined over the compact symmetric support $[-e, +e]$. This yields a uniform distribution distributed over the interval $[-\epsilon, \epsilon]$, where the variance of e_t is such that $\sigma_e^2 = \frac{e^2}{3}$. When $t = 0$, and given $g_{-1} = 0$, the implied gap g_0 is uniform over the same closed support as the shock e_0. In general, however, the presence of persistence in the output gap generates an ex-post distribution for g_t uniformly distributed over a different support, $[-\epsilon, \epsilon]$, where ϵ is calculated as follows. On the one hand, from the expression of the gap (2), $\sigma_g^2 = \frac{1}{1-\rho^2}\sigma_e^2 = \frac{1}{1-\rho^2}\frac{e^2}{3}$. Additionally, given that g_t is uniform, it has to be that, $\sigma_g^2 = \frac{\epsilon^2}{3}$. Then, equating both expressions for the variance of the gap, it is easy to see that, $\epsilon = e/\sqrt{1-\rho^2}$.

The calibration of e is made in order to replicate ϵ according to the above formula. The distribution function associated to each occurrence of the gap, g_t, is such that $dG^j = \frac{1}{2\epsilon}dg_j$. It is worth noticing that, as the time horizon increases, $t \to \infty$, the uniform distribution, by virtue of the Central Limit Theorem, would converge to a normal distribution. Even when our maximisation problem has an infinite horizon, this convergence fact is not relevant for our theoretical results, although when simulating might imply that, ex-post, a small percentage of the realisations of the gap would lie outside the theoretical limits $[-\epsilon, \epsilon]$. Nevertheless, as the sample size increases, the gap becomes the sum of an increasing number of uniformly distributed random variables, with a decreasing weight as time goes by, $g_t = \sum_{j=0}^{t} \rho^j e_{t-j}$. This way, the gap shows a zero expected value, and a variance such that, $\sigma_g^2 = \frac{e^2}{3}(1 + \rho^2 + \rho^4 + \ldots) = \frac{e^2}{3(1-\rho^2)}$. So, as the sample size increases, $T \to \infty$, the distribution of the output gap converges to a normal distribution, with mean zero and standard deviation given by $\frac{e}{\sqrt{3}\sqrt{1-\rho^2}}$.

3.2 Results

In order to quantify local and global optima for the set of our fiscal choice variables, b^* and a, we proceed as follows. We first search for the locally optimal pair of policy variables b^* and a from the first-order condition of the government, given the country-specific value of b_0, and on the basis of a schedule for the non-negative excess tax $k_t(\overline{k}, g_t)$ which would imply *no* excess duty levied on the current generations in a situation in which these should find themselves operating under binding borrowing restrictions (i.e. whenever $g_0 < \Gamma_0$). Then we identify the globally optimal pair of policy variables, b_G^* and a_G^*. In obtaining results, a sequence of uniformly-distributed independent random shocks to the period-t output gap are applied to the model in order to assess simulated convergence properties. It is found that the model converges to its steady state under all sensible parameter configurations, while breaches of the deficit limit over the transition path to the steady state remains below five per cent of total observations under the baseline configuration. The existence of a small number of breaches during the transition can be understood in that the deficit limit is formulated to be valid over the steady state, (20). Clearly, this only an approximation to the "true" limit that should be observed when total debt *fluctuates around* b^* in a random fashion, and in this sense it need not necessarily be *always equal to* b^* in all states of nature. The *actual* interest spending that the government would have to sustain at the trough of a recession along the stochastic steady state, $rb_t^*(-\epsilon)$, would be higher than the interest rb^* payable when the debt level is *exactly equal* to its zero-output-gap steady state target b^*– because $rb_t^*(-\epsilon) \geq rb^*(0) = rb^*$. The introduction of the latter concept of interest into our steady state condition relating s^* to b^*, which we then used for calculating our optimal pair $\{b^*,\ a\}$, is compensated for by being more cautious in some other respects, such as setting tolerance of breaches of the 3% limit of the Pact to zero. Table 1 summarises the baseline parameterisation and results for local and global optima.

[INSERT TABLE 1]

3.2.1 Local optima

As expected, countries experiencing large output fluctuations on average, as measured by ϵ, have a lower local target for debt. For instance, according to our exercise, the living electorate in Finland would be ready to pay a substantial sum in order to have the non-contingent primary surplus reduced in the steady state from the level needed to permanently sustain a debt ratio at the current level of around 40 per cent of GDP – to a target of a balanced primary budget consistent with a near-zero target debt ratio prescribed by the policy exercise. Voters in Denmark and Spain would be almost as ambitious as those in Finland, although their final target debt would be higher. By contrast, other countries with less volatile GDP and/or with larger initial debt burdens would content themselves with a more measured approach to fiscal stabilisation. Indeed, economies such as that of Finland and Denmark, with strong self-insurance motives, have embarked on a precipitous reduction of their debt levels in the recent past. By contrast, other member states, such as France and Austria, with less risky aggregate income profiles – and thus lesser incentives toward fiscal retrenchment according to our line of reasoning – could indeed afford a more relaxed time schedule for debt redemption.

The local targets for the sensitivity of primary surpluses to output gaps are considerably larger than the current values as estimated in the literature. Indeed, while recent studies estimate actual a to be in the vicinity of 0.2 for the area as a whole – see Wyplosz (1999) and Mélitz (2000) – according to our model these electorates would be happy to pay a tax for having the automatic subsidy accruing from the budget in bad times increased to between half and 80 per cent of realised income shortfalls. Countries with smooth output fluctuations – France, Austria and the Netherlands – would target a budgetary sensitivity higher than countries at the opposite extreme. This can be understood in that for them the cost of this form of insurance, based on an ex-post rather than ex-ante arrangement, would be lower given initial debt in terms of the fiscal adjustment if the deficit constraint is to be observed in all circumstances.

3.2.2 Global optima

The diversity in initial fiscal positions is the primary factor behind the wide range of optimal local debt targets that the same optimisation problem would indicate for different governments. A less dispersed set of fixed points for b_G^* and a_G in the *very* long run characterises the convergence trajectories of the various countries under analysis. The numerical targets for a, given in the last column of Table 1, point to the existence of a quite substantial pent-up demand for automatic stabilisation for these countries. If freed from the burden of their inherited debts, electorates would be happy to forego a great deal of consumption in good times in exchange for robust income subsidisation in bad times. When brought to roughly the same initial conditions, results indicate that electorates would choose that no less than two thirds of their envisaged income shortfalls be compensated by an endogenous correction of taxes and transfers. Accompanying this result is the readiness of electorates for footing the cost of turning a debt into an asset position, if that is what the frequency and severity of recessions require them to do in order to respect the deficit ceiling of the Stability and Growth Pact.

Notice that this finding – strong and unambiguous as it is – has been derived on the basis of a model which, if anything, was constructed to bias the final results *against* the voluntary purchase of insurance through debt repayment. Indeed, our households are risk averse – which is why they choose to self-insure anyway – but act in a totally imprudent manner in the face of the uncertainty that looms ahead. Although they dislike uncertainty, they basically behave as if there were none. More prudent households – saving *also* against the possibility of being liquidity constrained in the future – would demand larger amounts of automatic stabilisation a and would be willing to pay more in exchange for it.

The targets for b_G^* are also more tightly dispersed than their local counterparts. In the baseline scenario, these net asset positions would numerically be comprised between a quarter of GDP – under Finland's extreme output volatility – and near-zero, with five out of eleven countries clustering around a balanced net asset position and two other countries – Germany and Sweden – not far from there.

On the whole, countries expecting output fluctuations of moderate amplitude would afford targeting a less ambitious – i.e. non-negative – debt even in the very long run. Notwithstanding this, they would be able to target a very high value for a_G. Conversely, countries subject to more pronounced cycles would have to settle for both less automatic stabilisation ($a_G < 1$) and a negative debt position in the very long run. The threshold between countries belonging to the two different groupings is traced by the numerical deficit limit.

3.2.3 Robustness

In order to gauge the extent to which the results are dependent on the chosen baseline parameterisation, the sensitivity of results to perturbations to the key calibrated parameters is considered. The results of this exercise, contained in Table 2, indicate a good degree of robustness to changes in r and n, with results not significantly different from the baseline for movements in these calibrated parameters. Changes to the rate of time preference, on the other hand, tend to have a more pronounced effect. Holding all else equal, an increase in δ results in a reduction in the steady-state debt ratio with respect to the baseline accompanied by an increase in the optimal degree of budgetary stabilisation. This can be interpreted simply as a translation of the increased degree of impatience into a desire for a lower degree of core taxation, s^*, combined with a heightened degree of automatic stabilisation.

[INSERT TABLE 2]

4 Conclusions

The model presented in this paper formalises the basic trade-off faced by policymakers when households are not free to borrow against their future income and an excess tax designed to bring down the level of debt can be designed in a way not to excessively penalise the household's disposable resources in low-endowment conditions.

Adopting a quadratic utility function, we are able to derive a closed-form *local* and *global* analytical solution of optimal debt for a given degree of budgetary responsiveness to the economic cycle and optimal budgetary stabilisation. The derived expressions show how the two insurance schemes can cushion consumption against unfavourable income shocks, and how the desirability of insurance coverage in the form of increased budgetary responsiveness to cyclical fluctuations – purchased via debt reduction – increases with uncertainty.

We take this analysis one step further by including a deficit constraint, which allows us to solve *jointly* for the desired level of public debt and degree of automatic stabilisation. Illustrative numerical simulation results are derived from a calibration of the model to a subset of EU Member States, and yield the unambiguous policy indication from a *normative* standpoint that these electorates, with no exception, regard their current fiscal situation as unsatisfactory. In addition to the finding that current public liabilities are too burdensome to be considered sustainable in periods of economic distress, when the payment of taxes detracts directly from consumption one-by-one, it is found that budgets are too rigid and the operation of automatic stabilisers too feeble to be of help in smoothing consumption across states of nature. The amount of resources that our mildly risk-averse, selfish individuals would surrender to bring about more balanced, flexible and responsive public finances is potentially quite sizeable. From a *positive* standpoint, the results suggest reasons why different countries in Europe have recently embarked on policies of debt re-absorption of such different intensities. Our results suggest, again, a self-insurance perspective for looking at the speed at which distinct countries decide to redeem public debt. In this logic, electorates who face particularly erratic income profiles can develop an appetite for dramatic debt cutbacks, whereas voters who feel less pressing motives to be concerned about income riskiness might be more relaxed about fiscal adjustment.

While we would not regard these policy prescriptions as definitive before conducting further analysis on the basis of a richer model – notably including physical capital – they are nevertheless very suggestive in pointing to a univocal direction. Namely, that countries expecting output fluctuations of moderate amplitude will target a less ambitious debt reduction both in the short run – that is, looking forward from their current initial conditions – and in the *very* long run and will always afford a more responsive system of fiscal stabilisers. Conversely, countries subject to more pronounced cycles will always be willing to accept *both* less automatic stabilisation *and* a more aggressive policy of debt repayment, in the short run as well as in the very long haul.

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Appendix I

I A. Solution to the consumption problem

This sub-section A of Appendix I illustrates how the rule of thumb for consumption given in (8) can be derived under our assumption that households set a target for consumption equal to the level at which they would consume if they were not borrowing constrained, i.e. under the permanent-income hypothesis. As explained in the text, the adoption of such a rule implies that households: first, find the level of consumption that would be dictated – given the prevailing income conditions – by the permanent-income hypothesis, i.e. c_t^{PI}; and then try to set actual consumption equal to c_t^{PI} unless current spendable resources $y^* + g_t + (1+r)w_{t-1} - (s_t + k_t)$ fall short of c_t^{PI}, in which case they exhaust current resources and put $c_t = y^* + g_t + (1+r)w_{t-1} - (s_t + k_t)$. Appendix IB shows that this simple rule of thumb performs extremely well in tracking the truly optimal consumption decision implied by the exact solution to the optimisation problem under borrowing limits.

Under the rule of thumb, at a generic time t, the necessary and sufficient conditions for the maximisation of (1) subject to (3) and $w_{t+1} \geq 0$, are provided by the following set of Kuhn-Tucker relations:

$$\frac{\partial U(c_t)}{\partial c_t} - \lambda_{1,t} = 0 \tag{26}$$

$$\lambda_{1,t} + \lambda_{2,t} - \left(\frac{1+r}{1+\delta}\right) E_t \lambda_{1,t+1} = 0 \tag{27}$$

$$\lambda_{2,t} w_{t+1} = 0, \lambda_{2,t} \geq 0 \tag{28}$$

where $\lambda_{1,t}$ and $\lambda_{2,t}$ are the Lagrange multiplier associated with the time-t constraint (3) and with $w_{t+1} \geq 0$, respectively. Notice that under this rule of thumb the household does not internalise the future borrowing constraints w_{t+j}, for $j > 1$. We can differentiate two cases depending on whether the borrowing restriction binds or not at the time t of decision.

Case i. Assuming $w_{t+1} \geq 0$, then $\lambda_{2,t} = 0$, and from (26) and (27), the optimum consumption path is implied by the following, standard condition:

$$c_t = y^* \left[1 - \frac{1+r}{1+\delta}\right] + \left[\frac{1+r}{1+\delta}\right] E_t[c_{t+1}] \;\; \forall t \tag{29}$$

Using (29), w_t can be solved forward by means of (3). This gives us the expression for consumption, called c_t^{PI}, in the upper part of (8). The optimal solution for wealth is then given by the resource constraint (3).

Case ii. If given a solution for time-t consumption from *Case i.*the implied solution for wealth from the resource constraint (3) is such that $w_{t+1} < 0$. Then, taking account of the non-negativity constraint on *contemporary* net worth, $w_{t+1} \geq 0$, the set of optimality conditions given by (26), (27) and (28) imply $\lambda_{2,t} \geq 0 \Leftrightarrow w_{t+1} = 0$ and then we get that the implied solution for consumption is given by the lower part of (8).

I B. Algorithm to compute an exact solution to the consumer problem

[TO BE ADDED]

Appendix II

In this Appendix we prove the result stated in Proposition 1. To start with, we use Lemma 2 to contract the highly dimensional dynamic problem faced by the government at time zero in its static counterpart.

Lemma 2 – *Under Assumption 2, and ruling out the possibility of reschedulings at a later stage, for any given self-liquidating repayment scheme,* $k_t(g_t) \geq 0$,

$$\begin{aligned} &Arg \max_{\{b^*,\, a,\, \bar{k}\}} \sum_{t=0}^{\infty} \left(\frac{1}{1+\delta}\right)^t \int_{-\epsilon}^{\epsilon} U[c_t^*(b^*,\, a,\, g_t)]dG^t \\ &= Arg \max_{\{b^*,\, a,\, \bar{k}\}} U\left(\int_{-\epsilon}^{\epsilon} c_0^*(b^*,\, a,\, g_t)dG^0\right) = Arg \max_{\{b^*,\, a,\, \bar{k}\}} \int_{-\epsilon}^{\epsilon} c_0^*(b^*,\, a,\, g_t)dG^0 \end{aligned} \tag{30}$$

where $\int_{-\epsilon}^{\epsilon} c_0^*(b^*,\, a,\, g_t)dG^0$ *is the expected individual consumption function at date zero, conditional on the* $k_t(g_0)$*, and* $c_0^*(g_t)$ *is given by (7) and (13) and expected consumption must satisfy:* $\int_{-\epsilon}^{\epsilon} c_0^*(b^*,\, a,\, g_t)dG^0 \leq C = y^*$.

Proof. The former equality sign follows from the linear-quadratic and Markov properties of our system (see Svensson, 1999, and Ljungqvist and Sargent, 2000, Chapter 1). That the state of the system obeys a Markov process is proved by Clarida (1987), in his Corollary 2.2. The latter equality sign follows from a first-order Taylor approximation of the quadratic expected utility at time zero around expected consumption at the same time. [c. q. d.]

Proof of Proposition 1: Part (i). Under the conditions in Lemma 2, given a, the optimal solution for b^* comes from equation (18). For this optimality condition to hold, it has to be that,

$$\frac{\partial E_0(\hat{c}(b^*, a/b_0))}{\partial b^*} = 0$$

Computing the expected individual consumption function at time zero,

$$E_0(\hat{c}(b^*, a)) = \int_{-\epsilon}^{\epsilon} c_0^*(b^*,\, a,\, g_t)dG^0$$

and taking its partial derivative with respect to b^*it is possible to obtain (15) with a bit of laborious algebra, where

$$\Phi\left(a, \epsilon, \rho \middle| r-n\right) = \frac{(r-2n)\,(1-\rho)^2(1+r)(n+r)(1-a)^2\epsilon}{r^4(1+r-\rho)\left[(n+r)(1-\rho)(1-a)+(1+r)a\right]}$$

For the solution in (15) to yield a stable reduction of debt from any arbitrary b_0, it is a necessary and sufficient condition that the coefficient $\Phi\left(a, \epsilon, \rho \middle| r-n\right)$ be positive. For $a<1$ and $\rho<1$, it is clear that it is necessary and sufficient that $r>2n$. As regards the global optimum, for a stationary solution to the government problem to be such that $b_0 = b^* = b^*_G$, it is necessary and sufficient that

$$\frac{\partial E_0(\hat{c}(b^*, a/b_0 = b^* = b^*_G))}{\partial b^*} \neq 0$$

and also that,

$$\left(y^* - E_0\left[\hat{c}(b^*, a/b_0 = b^* = b^*_G)\right]\right) = 0$$

Plugging into this expression the expected individual consumption function at time zero, $E_0(\hat{c}(b^*, a)) = \int_{-\epsilon}^{\epsilon} c_0^*(b^*,\, a,\, g_t\,) dG^0$,particularised at $b_0 = b^* = b^*_G$ one can get the global optimum in the first part of Proposition 1.

As regards the proof of Part (ii) : to solve simultaneously for b^* and a it is necessary to solve (18) and (19) simultaneously. From the solution to

$$\frac{\partial E_0(\hat{c}(s^*, a/b_0))}{\partial a} = 0$$

one gets

$$b^* = b_0 - \Phi_2\left(a, \epsilon, \rho \middle| r-n\right), \;\; \Phi_2\left(a, \epsilon, \rho \middle| r-n\right) = \frac{(1-\rho)(1+r)\epsilon(1-a)}{r(1+r-\rho)} \tag{31}$$

To solve for b^* and a we have, then, two conditions given by (15) and (31). By simple substitution it is clear that the pair $a = 1$ and $b^* = b_0$ solves both equations. Also, this solution is unique, in the sense that there is no other combination of (b^*, a) that it is a solution to the government problem. If, for any a, we subtract (15) from the alternative solution for b^*, (31), and given that $r^3 \to 0$, the result is,

$$(r-2n)\left((1-\rho)\;(n+r)\;(1-\;a)^2\right) = 0$$

that, given that by hypothesis $r - 2n > 0$, it is only solved by $a \to 1$. [c. q. d.]

Figure 1. Optimal solution for *b given *a***

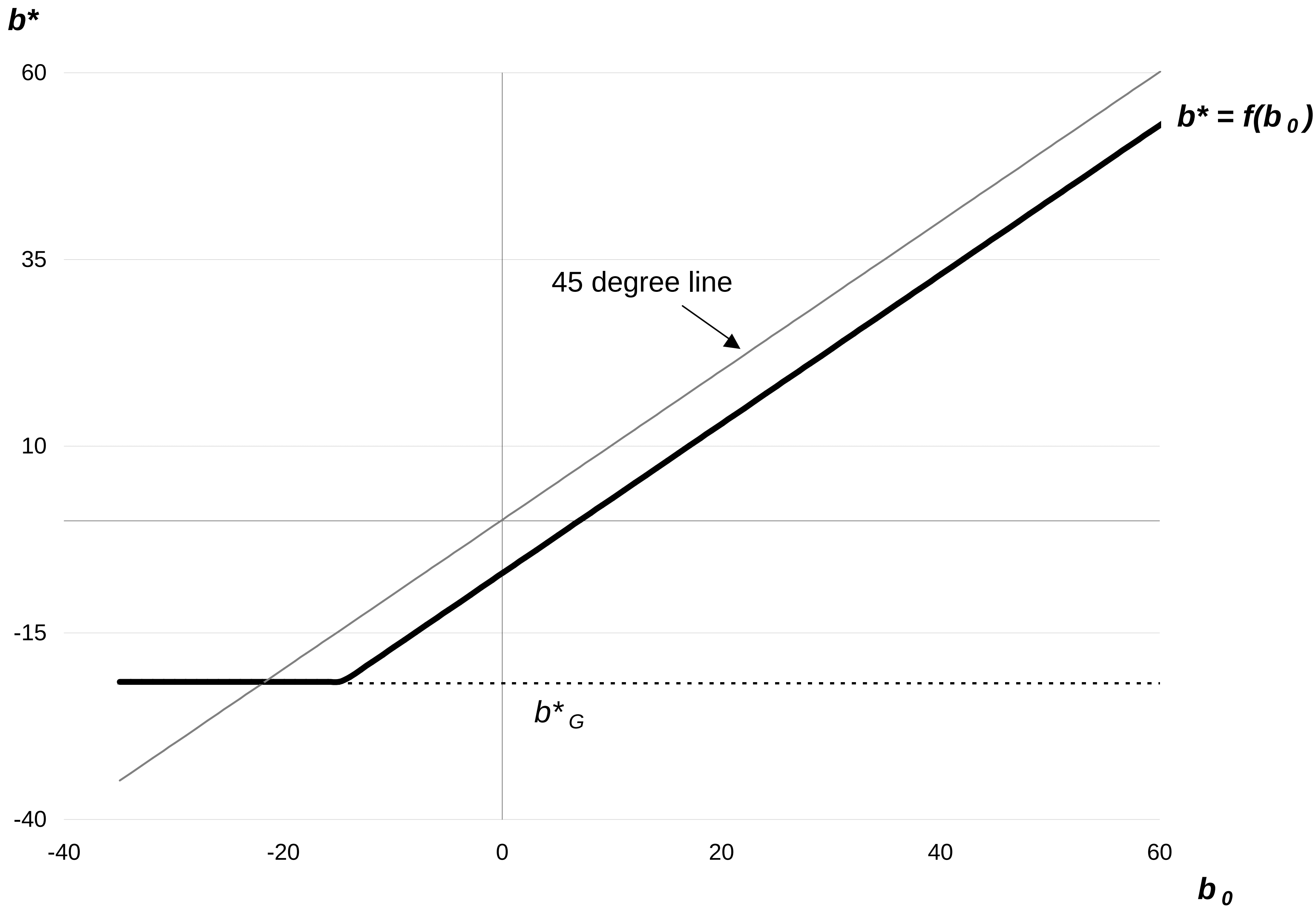

Figure 2 (Panel 1). Response of optimal consumption to an increase in *a*

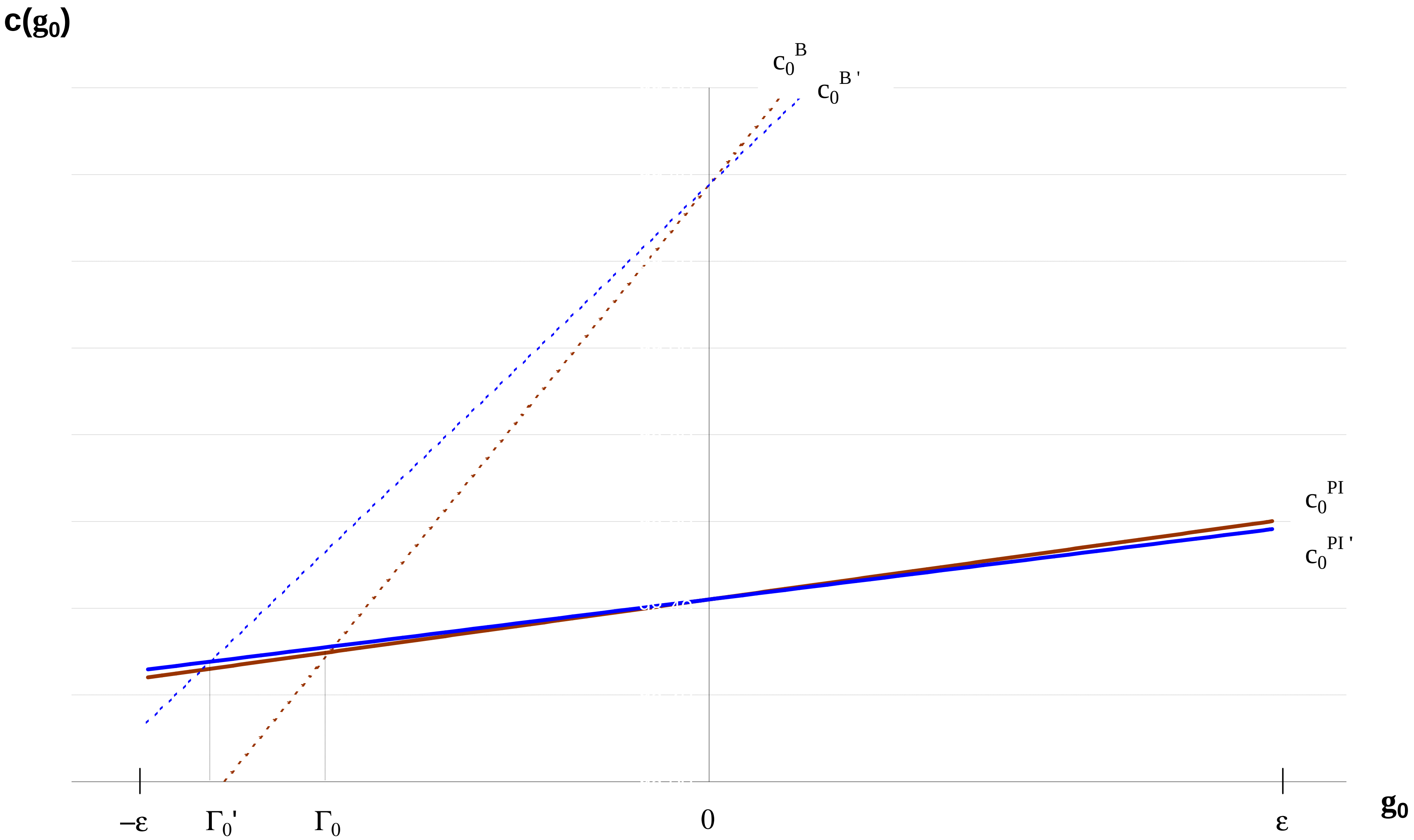

Figure 2 (Panel 2). Response of optimal consumption to a reduction in *b**

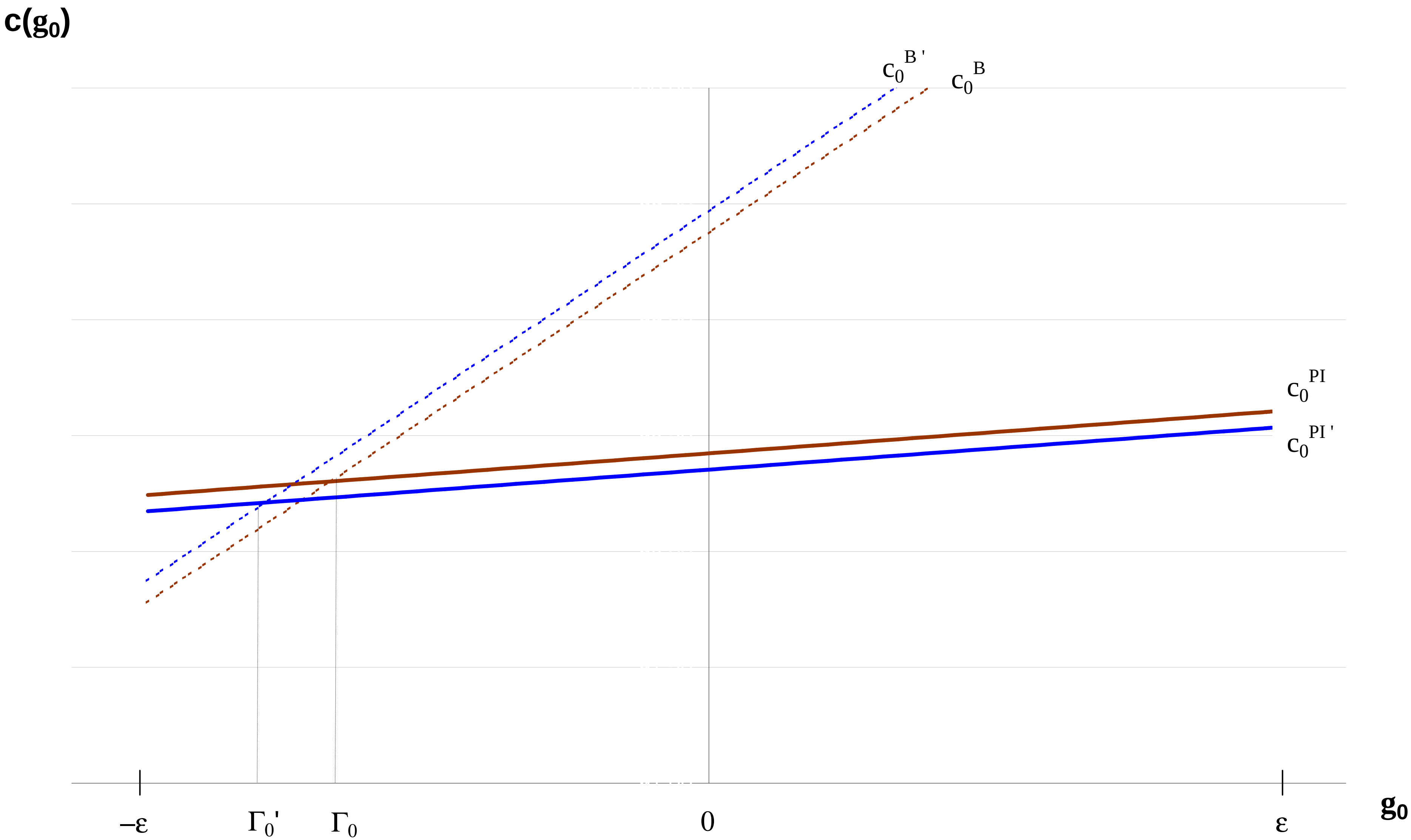

Figure 3. **Sensitivity of local** optima **to alternative parameter configurations**

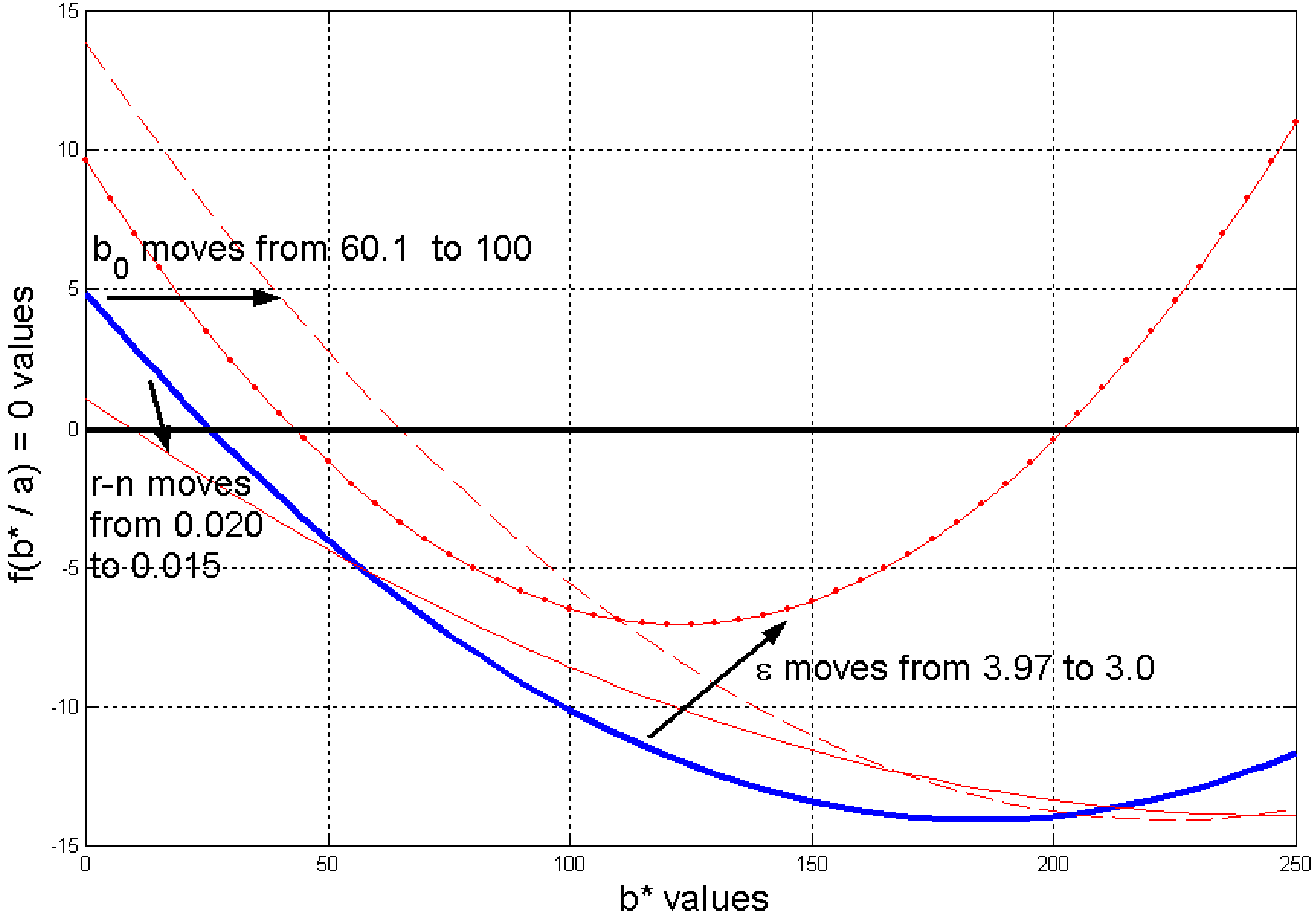

Table 1. Baseline parameterisation and results

	Baseline parameterisation*				Local solution					Global solution	
	ρ	ε	b_0	s_0	b^*_L	s^*_L	$b_0 - b^*_L$	$(b_0-b^*_L)/b_0$	a^*_L	b^*_G	a^*_G
Belgium	0.526	3.21	110.01	2.13	81.16	1.57	28.85	26.23	0.51	-1.47	0.91
Denmark	0.598	4.95	49.27	0.95	9.35	0.18	39.92	81.01	0.55	-19.00	0.64
Germany	0.576	3.97	60.66	1.17	32.31	0.62	28.35	46.74	0.60	-9.14	0.76
Spain	0.732	4.91	62.27	1.20	21.91	0.42	40.36	64.81	0.51	-17.05	0.64
France	0.681	2.59	58.17	1.12	51.18	0.99	6.99	12.02	0.82	5.50	1.00
Italy	0.456	3.20	110.82	2.14	81.55	1.58	29.27	26.42	0.51	-1.63	0.92
Netherlands	0.625	2.90	58.67	1.13	46.82	0.90	11.85	20.20	0.75	2.07	0.99
Austria	0.593	2.99	63.99	1.24	49.50	0.96	14.49	22.64	0.72	1.06	0.97
Finland	0.728	5.53	42.65	0.82	-2.19	-0.04	44.84	105.14	0.52	-23.30	0.58
Sweden	0.640	4.28	61.31	1.18	28.87	0.56	32.44	52.91	0.57	-11.82	0.72
United Kingdom	0.615	4.60	42.36	0.82	9.39	0.18	32.97	77.84	0.59	-15.29	0.68

* note that r=0.035, n=0.015, ω=0 and β=1 for all countries

Table 2. Robustness of results to changes in parameters

(changes in level with respect to baseline)

	r=0.0375				n=0.01				n=0.02				1/(1+d)=0.96			
	b^*_L	a^*_L	b^*_G	a^*_G	b^*_L	a^*_L	b^*_G	a^*_G	b^*_L	a^*_L	b^*_G	a^*_G	b^*_L	a^*_L	b^*_G	a^*_G
Belgium	0.96	-0.01	0.28	0.00	4.30	0.11	0.35	0.00	-2.75	-0.10	-0.44	0.01	-13.09	0.06	-0.48	0.00
Denmark	1.60	-0.01	2.03	-0.01	-2.87	0.02	2.77	-0.02	2.53	-0.02	-3.78	0.03	-9.68	0.03	-2.33	0.01
Germany	1.10	-0.01	1.05	0.00	0.21	0.04	1.41	-0.01	0.19	-0.04	-1.91	0.02	-9.05	0.04	-1.30	0.01
Spain	1.72	-0.01	1.95	-0.01	-1.81	0.03	2.59	-0.02	1.83	-0.03	-3.53	0.03	-11.32	0.04	-2.47	0.01
France	0.28	0.00	-0.36	-0.10	3.57	0.08	-0.54	-0.10	-2.63	-0.07	0.80	-0.10	-6.19	0.04	0.08	-0.10
Italy	0.95	-0.01	0.28	0.00	4.31	0.11	0.35	0.00	-2.76	-0.11	-0.45	0.01	-13.12	0.06	-0.45	0.00
Netherlands	0.47	0.00	-0.04	0.00	2.99	0.07	-0.10	0.00	-2.02	-0.06	0.18	0.00	-6.80	0.04	-0.79	0.00
Austria	0.55	-0.01	0.05	0.00	2.99	0.07	0.03	0.00	-1.98	-0.07	0.01	0.00	-7.55	0.04	-0.29	0.00
Finland	1.96	-0.01	2.58	-0.01	-4.25	0.01	3.45	-0.03	3.63	-0.01	-4.71	0.04	-10.33	0.03	-3.14	0.01
Sweden	1.30	-0.01	1.35	0.00	-0.47	0.03	1.81	-0.02	0.74	-0.04	-2.45	0.03	-9.79	0.04	-1.67	0.01
United Kingdom	1.09	0.00	1.68	-0.01	-2.16	0.01	2.27	-0.02	1.96	-0.02	-3.09	0.03	-8.24	0.03	-1.98	0.01