6 Imperfections in International Capital Markets

The last chapter explored models in which there are virtually no restrictions on the range of financial contracts people can sign, and where contracts are always honored. In reality, difficulties in enforcing contracts ex post limit the range of contracts agents will agree to ex ante. Without doubt, enforcement problems are a major reason why financial trading falls far short of producing the kind of efficient global equilibrium that the Arrow-Debreu model of complete asset markets portrays.

The problem of contract enforcement is particularly severe in an international setting. The sanctions that foreign creditors can impose on a country that defaults are limited and often fairly indirect. The first part of our analysis considers how such limitations may or may not reduce a country's ability to tap international capital markets for consumption insurance, and the following section looks at how they can curtail efficient investment. Among the questions we address are the "debt overhang" problem that some observers hold responsible for the Latin American recession of the 1980s and the implications of various types of financial restructuring. The third and fourth sections of the chapter assume that the binding constraint on contracts is private information rather than the limited ability of creditors to impose penalties. We first look at an environment where countries are free to misrepresent domestic economic conditions in order to increase their insurance payments from abroad. We then show how investment and international capital flows can be dampened by moral hazard problems at the firm level.

It is important to contrast the capital market imperfections studied here with the stochastic bonds-only model of Chapter 2. The earlier model simply assumed without any explicit justification that some markets are closed to trade (specifically, international markets for risky assets). Here, the nature of any limitations on asset trade is determined *endogenously* based on underlying information or enforcement problems. A central lesson of the analysis is that endogenous imperfections in international capital markets will not necessarily cause those markets to collapse completely. Instead, capital markets usually will still be able to facilitate risk sharing and intertemporal trade, but only to a limited extent.

6.1 Sovereign Risk

Perhaps the most fundamental reason why international capital markets may be less integrated than domestic capital markets is the lack of a supranational legal authority, capable of enforcing contracts across borders. In the first part of this chapter, we will study some of the implications of "sovereign risk," which, broadly interpreted, can refer to any situation in which a government defaults on loan contracts with foreigners, seizes foreign assets located within its borders, or prevents domestic residents from fully meeting obligations to foreign creditors. We have already mentioned the developing-country debt crisis of the 1980s, in which a large number of countries, especially in Latin America and Africa, renegotiated debt obligations to foreign creditors. (See Chapter 2.) Eastern Europe followed in the 1990s. This recent experience is hardly unique. Some of the same countries defaulted on their debts during the 1930s and during the 1800s. Indeed, countries have been defaulting on debts to foreign creditors periodically since the inception of international lending. It is important to understand, though, that in the vast majority of cases, sovereign default has been *partial* rather than complete. A country may stand in default for years if not decades, but it generally reaches some type of accommodation with its creditors before reentering capital markets.

Because foreign lenders have only limited powers directly to punish sovereign borrowers, especially governments, the binding constraint on debt repayments is generally a country's *willingness to pay* rather than simply its *ability to pay*. This fundamental distinction was first emphasized in a classic paper by Eaton and Gersovitz (1981). In this section, we will look at two different mechanisms by which foreign creditors can enforce repayment, at least up to a certain level. The first consists of direct punishments. Generally speaking, we think of these as being based on rights that the creditors have within their own borders, rights which allow them to impede or harass the international trade and commerce of any borrower that unilaterally defaults. (Gone are the days when gunboats would steam into third-world harbors to protect the financial claims of American or European investors.) Thus, although creditors may not be able to seize plant and equipment within a defaulting country's borders, they can often prevent it from fully enjoying its gains from trade.[1] The second motive for repayment we shall consider is reputation: a country may be willing to repay loans to foreigners in order to ensure access to international capital markets in the future. Creditors' legal rights of direct punishment can also make it difficult for a country in default to gain access to new international loans. There are many subtle issues here, and the legal framework is complex (see Box 6.1 on the legal doctrine of foreign sovereign immunity). But as we shall emphasize later, there is a fundamental level at which creditors must have some legal or political rights to enforce repayment or international capital markets would collapse.

Throughout our analysis, we will treat each sovereign borrower as a single unified entity, "the country." We will not distinguish between government and private borrowers. In many developing countries, government and government-guaranteed debt accounts for the bulk of foreign borrowing, and in this section we will generally be thinking of the government as the borrower.[2] We recognize that the costs

1. In earlier days countries might pledge specified future customs revenues to debt service. (See Box 6.1.) Such pledges, which themselves are revocable, would offend nationalistic sensibilities today.

2. Even if a domestic firm wants to repay foreign creditors, it can be prevented from doing so by a government that blocks its access to the necessary foreign exchange. Sometimes creditors have been able to pressure borrowing governments to take responsibility for private domestic debts to them. Díaz-

and benefits of default typically fall very unevenly across groups within a country, but we do not explore the implications of this issue. Instead we focus on the overall gains and losses to a country of sovereign borrowing and default.

6.1.1 Sovereign Default and Direct Creditor Sanctions

The topic of sovereign risk raises a host of interesting but difficult modeling issues. A simple starting point is to assume that a sovereign's creditors can impose direct sanctions with a current cost proportional to the sovereign's output. Broadly interpreted, we have in mind trade sanctions, including the confiscation of exports or imports in transit and the seizure of trade-related foreign assets.[3] Concern over access to short-term trade credits has often been an especially important consideration for modern borrowers contemplating default. Good relations with international financial intermediaries, who specialize in gathering and processing information on creditworthiness, have become increasingly essential to international trade in complex modern economies.

Just as we do not model the tensions across different groups within debtor countries, we will not place too much emphasis in this chapter on tensions across various creditors (see Eaton and Fernandez, 1995). In practice, cross-default clauses in loans from banks and provisions for the organization of bondholders' committees serve to coordinate the actions of lenders in the event of default.[4] We assume, however, that lenders behave competitively in making loans, so that they cannot extract monopoly rents from a borrowing country. This assumption is realistic, since a country in good standing on its debt is generally free to pay off one lending consortium with a new loan from another one. Foreign claim holders have no legal rights to apply sanctions unless a country violates its contract with them.

The present section (section 6.1) focuses on insurance aspects of international capital markets. Throughout, unless otherwise noted (as in section 6.1.3), we will assume a fundamental asymmetry between foreign providers of insurance and country recipients. In particular, we will assume that foreign insurers can credibly make commitments to a future state-contingent payment stream whereas the

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Alejandro (1985) discusses one prominent case, that of Chile in the early 1980s. In other cases, by assuming private debts, governments may have actually made default easier. This is especially the case in countries where foreign creditors might have some hope of pressing claims in domestic courts against private companies, but not against the government.

3. Generally, the net gain to creditors from sanctions will be much less than the cost to the debtor. This point is not central to the analysis of this section, but can be important in a broader bargaining context, such as the one we consider in appendix 6A. The assumption that the pain of sanctions is proportional to output is proposed by Sachs (1984) and by Cohen and Sachs (1986). It is far from innocuous, as we shall see. Nor is it obviously valid—the marginal cost of trade disruption, say, might sometimes be higher for a poorer economy.

4. For further discussion, see Bulow and Rogoff (1989a, appendix).

Box 6.1
Sovereign Immunity and Creditor Sanctions

The legal doctrine of sovereign immunity would appear to exempt the property of foreign governments from the jurisdiction of domestic courts. (In most countries, foreign diplomats generally cannot even be forced to pay parking tickets.) Historically, sovereign immunity has sometimes limited the direct sanctions creditors can apply in cases of sovereign default. Over the years, however, as a result of considerable evolution, the practical application of the doctrine has increasingly given creditors leverage to retaliate against defaulting sovereigns. In modern times, the ability of countries expressly to waive sovereign immunity in their commercial contracts has strengthened the rights of their creditors, thereby paving the way for an expansion of international lending.

The idea of sovereign immunity is an old one. Chief Justice John Marshall of the United States Supreme Court invoked it as early as 1812 in a famous decision. The American schooner *Exchange*, seized at sea in 1810 in the name of the French Emperor Napoleon, later docked in Philadelphia. When its previous owners tried to recover the *Exchange* in federal district court, their case was dismissed on the grounds that the ship was a state vessel of France employed in the pursuit of national objectives. The circuit court reversed this decision, but the Supreme Court overturned the reversal and affirmed the district court's original judgment leaving the ship in France's hands. Chief Justice Marshall argued that by welcoming a friendly nation's "public armed ship" into its port, the United States had implicitly exempted the ship from its jurisdiction, that is, extended sovereign immunity. In general, Marshall stated,

> [F]ull and absolute territorial jurisdiction being alike the attribute of every sovereign, and being incapable of conferring extra-territorial power, would not seem to contemplate foreign sovereigns nor their sovereign rights as its objects. One sovereign being in no respect amenable to another, and being bound by obligation of the highest character not to degrade the dignity of his nation, by placing himself or its sovereign rights within the jurisdiction of another, can be supposed to enter a foreign territory only under an express license, or in the confidence that the immunities belonging to his independent sovereign station, though not expressly stipulated, are reserved by implication, and will be extended to him. (Quoted in Bishop, 1971, p. 660)

Where courts showed some reluctance to help creditors in pursuing claims on sovereign debtors before World War I, national governments could be more compliant. Political pressure and even military force might be deployed on behalf of aggrieved domestic creditors (though usually when creditor interests matched their government's foreign-policy goals). Examples abound. Britain, France, and Spain intervened in Mexico on behalf of creditors during the years 1859–61. When Egypt, a province of the Ottoman Empire, repudiated its debts in 1879, Britain and France induced the Ottoman sultan, a heavy borrower himself, to turn control of Egypt's finances over to British and French functionaries. (Turkey itself put foreign creditors in charge of important revenues in 1881 in return for debt reduction and continued access to foreign loans.) Invasions by U.S. Marines gave the United States control over the Dominican Republic's customs revenues in 1905 and over Nicaragua's during 1911–12. Britain sent a battleship to Guatemala's waters in 1913 to persuade the country to continue servicing debt held by British subjects.*

Box 6.1 *(continued)*

In the postimperialist era after 1945, a middle ground has emerged between jurists' respect for sovereign rights and politicians' willingness to disregard them. Starting in 1952, the United States adopted a policy of restricted sovereign immunity, which distinguished between governmental activities sui generis (for example, diplomatic missions) and governmental activities (including commercial activities) that private persons also can conduct. The latter, but not the former, can be subject to standard domestic commercial law. This doctrine was formalized in the United States by the Foreign Sovereign Immunities Act (FSIA) of 1976, and in Britain by the State Immunity Act of 1978.

By strengthening creditors' rights, these legal changes made sovereign borrowing easier. A key feature of the FSIA is that it permits countries to waive sovereign immunity in many commercial transactions. Most developing-country government debt contracts after 1976 have contained explicit waivers of sovereign immunity, with the details of the waiver an important bargaining point. The waivers have made it more difficult for sovereigns that repudiate their debts to engage in international trade, and their existence supports the assumption that creditors can impose direct sanctions on a reneging sovereign debtor.

* For details, see Feis (1930), Lindert and Morton (1989), and the latter authors' references to the intervening literature.

country may or not be able to do so (as we shall illustrate). One can think of justifying this asymmetry in two ways. First, many of the basic models here can easily be reformulated as models of equity investment or lending with state-contingent repayments rather than pure insurance. If the foreign investors provide cash up front, their credibility is not at issue. Indeed, we generally have in mind this interpretation of the models, and we use the example of pure insurance contracts to highlight the analogies with the complete-markets models of Chapter 5. Second, interpreting the country as a developing economy, one can think of foreign insurers as having access to a stronger legal system that allows them to make financial commitments. Thus, if a British bank legally promises to make a payment to a small country, the country can generally enforce the claim in British courts. This asymmetry seems quite realistic in the developing-country context, although we will not attempt to model the broader underpinnings of the industrialized-country legal system.

6.1.1.1 The Model

Some central points about sovereign risk's impact can be made in a bare-bones model. Consider a small endowment economy inhabited by a risk-averse representative agent who lives for two periods, labeled 1 and 2, in which date 1 consumption yields no utility and the country's date 1 endowment is zero. These assumptions together imply that the country can neither save nor dissave on date 1: its only economic activity on that date is to enter into contracts with foreign insurers so as to reduce the consumption risks posed by an uncertain date 2 output level.

There is a single good on date 2, and the representative agent's lifetime utility equals the date 1 expected utility of date 2 consumption

$$U_1 = \mathrm{E}u(C_2),$$

where we now follow our usual notation that identifies individual with aggregate domestic consumption when there is a single representative agent.[5] Date 2 output is uncertain as of date 1 and is given by

$$Y_2 = \bar{Y} + \epsilon,$$

where $\mathrm{E}\{Y_2\} = \bar{Y}$ and the mean-zero shock ϵ can take any of N values $\underline{\epsilon} = \epsilon_1 < \epsilon_2 < \ldots < \epsilon_N = \bar{\epsilon}$, $\bar{Y} + \underline{\epsilon} > 0$. The shock ϵ is the only source of potential consumption uncertainty for the small country. The term $\pi(\epsilon_i)$ denotes the probability that $\epsilon = \epsilon_i$, and $\sum_{i=1}^{N} \pi(\epsilon_i) = 1$.

On date 1, the country contracts with foreign insurers to pay them the shock-contingent amount $P(\epsilon)$ on date 2. (The value that ϵ takes on date 2 is observed by everyone.) A negative value of $P(\epsilon)$ means that the insurers make a payment to the country in state ϵ, a positive value that the country pays an insurance "premium." Insurers compete against each other in offering contracts, and they are risk-neutral. (One could equivalently assume that insurers are risk-averse but that the country's output shock ϵ can be completely diversified away in international capital markets.) Because insurers put no money down on date 1, they are willing to sign any contract under which the sovereign can credibly promise to make payments $P(\epsilon)$ satisfying the zero-expected-profit condition:

$$\sum_{i=1}^{N} \pi(\epsilon_i) P(\epsilon_i) = 0. \tag{1}$$

Of course, the sovereign's credibility is only ever an issue when $P(\epsilon) > 0$![6]

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5. Lifetime utility could, alternatively, be of the discounted form $U_1 = \beta \mathrm{E}u(C_2)$, but the multiplicative constant β is inconsequential for the analysis and therefore can be omitted. The conditional expectations operator will be denoted throughout by $\mathrm{E}\{\cdot\}$ rather than $\mathrm{E}_1\{\cdot\}$ when there can be no confusion surrounding the information on which expectations are conditioned.

6. It will sometimes prove convenient to interpret $\pi(\epsilon)$ as the probability density function for a continuously distributed ϵ, in which case eq. (1) becomes

$$\int_{\underline{\epsilon}}^{\bar{\epsilon}} \pi(\epsilon) P(\epsilon) \mathrm{d}\epsilon = 0.$$

Condition (1) would not hold if a single insurer or a small collusive group could extract monopoly rents from the country.

6.1.1.2 A Benchmark Case: Full Insurance

Let's initially assume away any risk of default, so as to obtain a benchmark case against which we can later compare our main results. Thus the country can commit itself to any schedule $P(\epsilon)$ of date 2 payments such that $P(\epsilon) \leq Y_2$. In this case we get an equilibrium familiar from the discussion of complete markets in Chapter 5. The payments schedule $P(\epsilon) = \epsilon$ satisfies (1)—because the shock ϵ has mean zero—and results in a date 2 consumption level that is *independent* of ϵ,

$$C_2(\epsilon) = Y_2 - P(\epsilon) = Y_2 - \epsilon = \bar{Y}.$$

Stabilizing consumption across all states of nature is the best the country can do; so, in equilibrium, the country will diversify away its output risk completely.[7] We will refer to the contract with payments schedule $P(\epsilon) = \epsilon$ as the *full insurance* contract.

The full insurance contract solves the problem of maximizing expected utility given the availability of binding Arrow-Debreu contracts at the actuarially fair prices for consumption contingent on the state ϵ. Alternatively, one can think of the country as selling its uncertain endowment forward to the outside world at the risk-neutral equity price (measured in date 2 consumption units)

$$\sum_{i=1}^{N} \pi(\epsilon_i) Y_2 = \sum_{i=1}^{N} \pi(\epsilon_i) \bar{Y} + \sum_{i=1}^{N} \pi(\epsilon_i) \epsilon_i = \bar{Y}.$$

This forward sale guarantees the consumption level $\bar{Y}$ on date 2. On any interpretation, the country receives $-\epsilon$ from insurers when $\epsilon < 0$, but must hand over to insurers any $\epsilon > 0$.

This last part of the full insurance contract is troublesome. We have assumed away the possibility that the insurers themselves fail to make scheduled payments when $P(\epsilon) < 0$. (Exercise 1 shows how to relax this assumption.) But when $P(\epsilon) > 0$, a sovereign that maximizes its citizens' welfare will choose not to pay ex post unless it perceives some cost to default. If the sanctions foreign creditors can impose in the event of default cost the country only a fraction $\eta \in (0, 1)$ of its output, there is no guarantee that the country will always honor its end of the full insurance contract. Indeed, the country would prefer to default and pay nothing if $P(\epsilon) = Y_2 - \bar{Y} > \eta Y_2$. Thus, unless repudiation is ruled out by sufficiently strong sanctions, the full insurance contract would never be offered in the first place.[8]

7. It is straightforward to check that this allocation describes the solution to maximizing $\mathrm{E}u(C_2) = \sum_{i=1}^{N} \pi(\epsilon_i) u[C_2(\epsilon_i)]$ subject to eq. (1) and $C_2(\epsilon_i) = Y_2 - P(\epsilon_i)$.

8. Assuming that the country repays in cases of indifference, a default occurs whenever $\eta Y_2 < Y_2 - \bar{Y} = \epsilon$, that is, whenever $\epsilon > \eta \bar{Y}/(1 - \eta)$.

6.1.1.3 Optimal Incentive-Compatible Contracts

What type of contracts would we see instead? Since the foreign insurers themselves never default, these contracts will have three features. First, the contract can never call on the sovereign to make a payment to foreign creditors in excess of the sanction cost. Thus the payments schedule $P(\epsilon)$ satisfies (for every $i = 1, \ldots, N$) the *incentive-compatibility constraint*,

$$P(\epsilon_i) \leq \eta(\bar{Y} + \epsilon_i). \tag{2}$$

Second, competition among the risk-neutral insurers must result in an equilibrium that yields them expected profits of zero. Third, competition will ensure that the contract is optimal for the sovereign, subject to eqs. (2) and (1)—otherwise, the sovereign would offer to pay insurers slightly positive expected profits for a contract slightly more favorable to itself.

Together, these three features imply that the optimal incentive-compatible insurance contract solves the problem:

$$\max_{C_2(\epsilon), P(\epsilon)} \sum_{i=1}^{N} \pi(\epsilon_i) u[C_2(\epsilon_i)]$$

subject to the incentive-compatibility constraint (2), the zero-profit condition (1), and the N budget constraints

$$C_2(\epsilon_i) = \bar{Y} + \epsilon_i - P(\epsilon_i). \tag{3}$$

To solve, we substitute eq. (3) into the maximand and set up the Lagrangian

$$\begin{aligned}\mathcal{L} = &\sum_{i=1}^{N} \pi(\epsilon_i) u[\bar{Y} + \epsilon_i - P(\epsilon_i)] - \sum_{i=1}^{N} \lambda(\epsilon_i)[P(\epsilon_i) - \eta(\bar{Y} + \epsilon_i)] \\ &+ \mu \sum_{i=1}^{N} \pi(\epsilon_i) P(\epsilon_i),\end{aligned}$$

as directed by the Kuhn-Tucker theorem for problems with inequality constraints (see Supplement A to Chapter 2). Differentiate the Lagrangian with respect to $P(\epsilon_i)$, for each ϵ_i. Along with eqs. (1) and (2), necessary conditions for an optimal $P(\epsilon)$ schedule are (for all ϵ, dropping the i subscripts)

$$\pi(\epsilon) u'[C_2(\epsilon)] + \lambda(\epsilon) = \mu \pi(\epsilon), \tag{4}$$

$$\lambda(\epsilon)[\eta(\bar{Y} + \epsilon) - P(\epsilon)] = 0, \tag{5}$$

for nonnegative multipliers $\lambda(\epsilon)$. The first of these conditions, eq. (4), shows how positive multipliers on the incentive constraint, $\lambda(\epsilon) > 0$, may induce unequal consumption across different realizations of ϵ. The second, eq. (5), is the complementary slackness condition, which implies that $\lambda(\epsilon) = 0$ for ϵ values at which eq. (2) holds as a *strict* inequality.

How does the optimal incentive-compatible contract look? For simplicity, let us assume that the distribution of ϵ is *continuous*. A plausible guess is that incentive inequality (2) will not hold as an equality for the lowest values of ϵ: these are states in which insurers make net payments to the country, or where the country's payments to insurers are strictly smaller than the costs of punishment.[9] Across these states $\lambda(\epsilon) = 0$ according to eq. (5), so eq. (4) reduces to $u'[C_2(\epsilon)] = \mu$, implying that consumption is constant irrespective of ϵ. From eq. (3), it follows that across states where $\lambda(\epsilon) = 0$, $P(\epsilon) = P_0 + \epsilon$ for some constant P_0. This repayment function makes $C_2(\epsilon)$ equal to $\bar{Y} + \epsilon - P(\epsilon) = \bar{Y} - P_0$, which is independent of ϵ. We will know P_0's value only at the end of our calculation of the optimal repayment schedule. The reason is that the level of consumption the country can assure itself in the "bad" (low ϵ) states of nature depends on how much it can credibly promise to repay creditors in the good states.

Since the last paragraph's analysis shows that P_0 satisfies $u'(\bar{Y} - P_0) = \mu$, eqs. (3) and (4) tell us that in states of nature such that the incentive constraint (2) holds with equality, it must be true that

$$\begin{aligned} u'(\bar{Y} - P_0) - u'[C_2(\epsilon)] &= u'(\bar{Y} - P_0) - u'[\bar{Y} + \epsilon - P(\epsilon)] \\ &= u'(\bar{Y} - P_0) - u'[(1-\eta)(\bar{Y} + \epsilon)] \\ &= \frac{\lambda(\epsilon)}{\pi(\epsilon)} \geq 0. \end{aligned} \tag{6}$$

Notice that the left-hand side of the last equality falls as ϵ falls. Consider the critical value of ϵ, denoted by e, such that $u'(\bar{Y} - P_0) - u'[(1-\eta)(\bar{Y} + e)] = 0$, and, therefore, $\lambda(e) = 0$.[10] For ϵ above e, eq. (6) shows that $\lambda(\epsilon)$ is strictly positive, so that, by eq. (5), $P(\epsilon) = \eta(\bar{Y} + \epsilon)$. For ϵ below e, the country is not constrained by eq. (2): since Kuhn-Tucker forbids a negative $\lambda(\epsilon)$, $\lambda(\epsilon) = 0$ and $P(\epsilon) = P_0 + \epsilon$ in this region. Our definition of e therefore implies that

$$\bar{Y} - P_0 = (1-\eta)(\bar{Y} + e), \tag{7}$$

which can be rewritten as

$$P_0 + e = \eta(\bar{Y} + e). \tag{8}$$

Equation (8) implies that $P_0 = \eta\bar{Y} - (1-\eta)e$, which shows that the repayment schedule is

9. Note that $\epsilon - \eta(\bar{Y} + \epsilon) = (1-\eta)\epsilon - \eta\bar{Y}$, the difference between the full insurance payment and the cost of default, is an increasing function of ϵ.

10. We assume that e lies in the interior of $[\underline{\epsilon}, \bar{\epsilon}]$. If $e = \bar{\epsilon}$, the certain component of the country's output $\bar{Y}$ is large enough, given η, to make full insurance feasible.

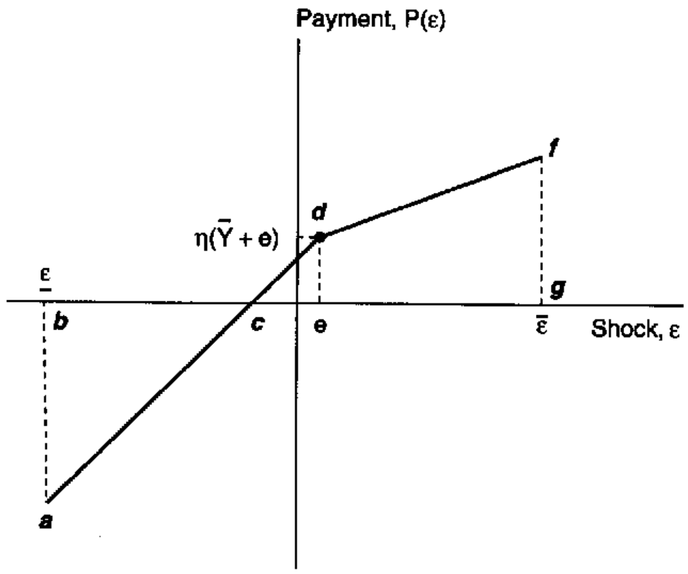

Figure 6.1
The optimal incentive-compatible contract

$$P(\epsilon)=\begin{cases}\eta\bar{Y}-(1-\eta)e+\epsilon=\eta(\bar{Y}+e)+(\epsilon-e), & \epsilon\in[\underline{\epsilon},e),\\ \eta(\bar{Y}+\epsilon)=\eta(\bar{Y}+e)+\eta(\epsilon-e), & \epsilon\in[e,\bar{\epsilon}].\end{cases} \tag{9}$$

Thus at $\epsilon=e$, as elsewhere on $[\underline{\epsilon},\bar{\epsilon}]$, the repayment schedule $P(\epsilon)$ is continuous, as shown in Figure 6.1. Note that $P(\epsilon)$ rises dollar for dollar in states of nature where $\epsilon<e$. As ϵ rises above e, $P(\epsilon)$ rises only at rate η since the incentive constraint is binding.

To complete the derivation of the optimal repayment schedule, we have only to tie down e [and hence, by eq. (8), P_0] through the zero-profit condition (1). In Figure 6.1, we assume that ϵ is uniformly distributed over $[\underline{\epsilon},\bar{\epsilon}]$ (implying $\underline{\epsilon}=-\bar{\epsilon}$, since $\mathrm{E}\epsilon=0$). The condition that the optimal incentive-compatible contract must yield insurers zero expected profits is represented by the equality of the areas of triangle ***abc*** and quadrilateral ***cdfg***.

6.1.1.4 An Example

The assumption that ϵ is continuous and uniformly distributed allows explicit computation of the optimal repayment schedule, that is, of the parameter e. Since this exercise serves to make our discussion more concrete, we describe it in detail. All that we need do is ensure that e makes the contract in eq. (9) consistent with zero expected profits. When ϵ is uniformly distributed over $[-\bar{\epsilon},\bar{\epsilon}]$, its probability density function is $\pi(\epsilon)=1/2\bar{\epsilon}$, and so eq. (1) can be written

$$\int_{-\bar{\epsilon}}^{e}[\eta(\bar{Y}+e)+(\epsilon-e)]\frac{\mathrm{d}\epsilon}{2\bar{\epsilon}}+\int_{e}^{\bar{\epsilon}}[\eta(\bar{Y}+e)+\eta(\epsilon-e)]\frac{\mathrm{d}\epsilon}{2\bar{\epsilon}}=0.$$

By evaluating these two integrals, we find (after some algebra) that the foregoing equation in e reduces to the quadratic equation

$$e^2 + 2\bar{\epsilon}e + \left(\bar{\epsilon}^2 - \frac{4\eta\bar{\epsilon}\bar{Y}}{1-\eta}\right) = 0.$$

The quadratic has two roots, one of which is less than $-\bar{\epsilon}$ and is disregarded. The economically relevant solution is

$$e = -\bar{\epsilon} + 2\sqrt{\frac{\eta\bar{\epsilon}\bar{Y}}{1-\eta}}. \tag{10}$$

You can verify that $e < \bar{\epsilon}$, giving a range over which the incentive-compatibility constraint actually does bind, provided $\bar{\epsilon} > \eta(\bar{Y} + \bar{\epsilon})$. The last inequality means the country would rather default at $\epsilon = \bar{\epsilon}$ than make the full-insurance payment $\bar{\epsilon}$ to creditors. It is simply the condition that sanctions are not severe enough to support full insurance.

6.1.1.5 Discussion

With this example under our belts, it is easier to grasp the intuition behind the optimal incentive-compatible contract in Figure 6.1. For sufficiently low realizations of ϵ, there is no enforcement problem. As a result, the country can smooth consumption across these states. For higher values of ϵ, though, the temptation to default would be too great under full insurance. So the optimal contract calls on the country to transfer only a fraction η of any unexpected output increase to creditors, which is the most they can extract through the threat of sanctions. This provision has two effects. First, limitations on how much the country can promise to repay in good states of nature reduce the level of consumption its insurers can afford to guarantee it in bad states of nature. Second, the country is limited in how much it can smooth consumption across the good states. Figure 6.2 shows the constrained consumption locus compared with the full insurance locus $C_2(\epsilon) = \bar{Y}$.

Consider the first contract feature described in the preceding paragraph: given the contract's asymmetric treatment of low and high ϵ values, insurers can earn zero expected profits only if the contract guarantees them higher net payments than the full insurance contract over a range of the lowest ϵ values. This observation implies that $(1-\eta)(\bar{Y} + e) < \bar{Y}$ (as shown in Figure 6.2), which is equivalent, by eq. (7), to $P_0 > 0$ (as in Figure 6.1).[11] The optimal contract therefore requires the country to make positive transfers to insurers even for some negative values of ϵ. Interestingly, this prediction of the model matches the observation that economies with temporarily low outputs often have made positive transfers to creditors.

11. The form of the constrained consumption locus in Figure 6.2 implies that the country in effect exchanges its risky output Y_2 for the asset with date 2 payoff $(1-\eta)Y_2$ and a put option.

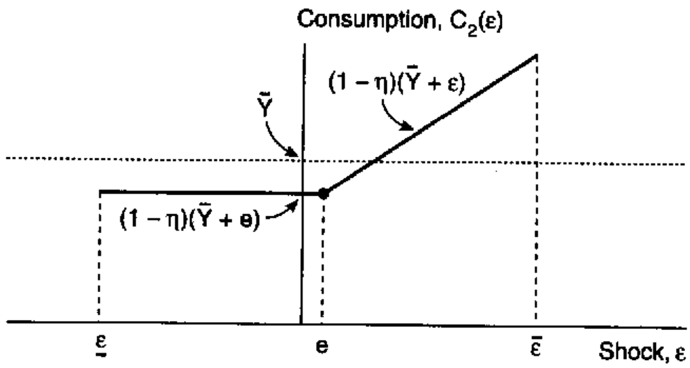

Figure 6.2
Optimal incentive-compatible consumption

Equation (10) illustrates the effects of higher sanctions, η. These raise e, allowing consumption stabilization over a higher range of shocks. Notice that e could well be negative (just take η low enough); as $\eta \to 0$, so that sanctions become powerless, $e \to \underline{\epsilon} = -\bar{\epsilon}$ and contracting becomes altogether infeasible in this model.

Because insurers earn zero expected profits, the optimal contract under default risk still sets the country's expected consumption to equal $\bar{Y}$. However, the contract's failure to equalize consumption across states of nature leaves the country worse off than it would be were full insurance possible. Perhaps surprisingly, it is in the country's interest for sanctions to be as dire as possible! As η rises, consumption can be stabilized across more states of nature, to the country's benefit. The sanctions are never exercised in equilibrium anyway, so their only role here is the positive one of enhancing the credibility of the country's promise to repay. Only if there were some contingencies that could bring the sanctions into play might higher potential punishments be a mixed blessing.

We have assumed that creditors are precommitted to imposing their maximal sanctions ηY_2 in the event of any default. How would the analysis change if creditors might somehow be bargained into settling for less than they are owed? Appendix 6A discusses a model of this type. The main impact on the preceding analysis is quantitative. The country will still obtain partial insurance, but only through contracts inferior to those it could get were creditors truly committed to applying maximal sanctions after any infraction.

The reader may find the pure risk-sharing contracts we have considered rather unrealistic. After all, most international capital-account transactions take the form of noncontingent money loans, equity purchases, or direct foreign investment. All we have done in our analysis, though, is to separate out two features that these more standard contracts typically combine: a riskless intertemporal loan and a pure risk-sharing contract. For example, if a home firm were to sell equity to a foreign investor, it would be receiving money up front in return for a share of a risky future profit stream. Funds obtained by issuing bonds or by borrowing from foreign banks are technically noncontingent, but the long history of sovereign lending shows that the payments may be rescheduled, renegotiated, or even changed unilaterally when the borrower's economy falters. Lenders as well as borrowers almost certainly anticipate such possibilities, so that interest rates on loans contain a premium to compensate for states of nature in which scheduled payments are not made in full. Thus *implicit* lending contracts involve risk sharing even if the *explicit* contracts do not.

"Stripping out" the pure risk-sharing component of a foreign investment from its lending component makes the analysis simpler and cleaner, and this advantage will become increasingly apparent as we move to explicitly dynamic models. In interpreting the results, however, it is important to bear in mind that in reality the two components typically come as a package. With pure risk-sharing contracts, the danger of the country's "defaulting" appears only in the good states of nature because in bad states the country receives resources from abroad rather than having to pay. If sovereign lending takes the form of equity arrangements, this still makes sense. If, however, one reinterprets the analysis as a model of loans, then the binding constraint becomes the country's willingness to meet its obligations in bad states of nature (where the lender's leverage to enforce repayment is lowest). Though the bond or bank-loan interpretation would seem to give very different results, in fact, it does not, as we illustrate in end-of-chapter exercise 2. By either interpretation, the implicit contract calls for the country to make relatively larger net payments when output is high and relatively smaller payments when output is low.[12]

This hyperrational interpretation of sovereign borrowing may seem strained given the experience of the developing-country debt crisis in the 1980s. Many borrowers that paid relatively modest interest rate premiums prior to 1982 fell into serious debt-servicing difficulties thereafter, and world secondary market prices for their government-guaranteed debt plummeted. In some cases (for example, Bolivia and Peru), discounts relative to face value exceeded 90 percent. Some have argued that lenders could not possibly have foreseen even the possibility that the debt crisis would be so severe.[13] Of course, many sovereign debtors in western Europe and Asia also seemed potentially risky in the 1970s, but loans to these countries

12. Bulow and Rogoff (1989a) argue that many contingencies, even though observable by both parties in the event of default, may be difficult to write contracts on. Therefore, lenders and borrowers write noncontingent loans, fully anticipating that they may have to be renegotiated. See also H. Grossman and Van Huyck (1988).

13. Bulow and Rogoff (1988a) argue that banks in industrial countries made loans recognizing that their governments, out of concern for the stability of world trade and the world financial system, could be gamed into making side payments to avoid a creditor-debtor showdown. See Dooley (1995) for a retrospective on the debt crisis of the 1980s.

generally paid off handsomely. One cannot evaluate overall investor returns just on the basis of the countries that ran into difficulties.

6.1.1.6 The Role of Saving

The last "two-period" model made the unrealistic assumption that there is no consumption or saving in the first period. What happens if the small country maximizes

$$U_1 = u(C_1) + \beta \mathrm{E} u(C_2), \qquad \beta < 1,$$

receives the endowment $Y_1 = \bar{Y}$ in the first period, and starts out with neither foreign assets nor debt? We again assume that $Y_2 = \bar{Y} + \epsilon$ and that risk-neutral insurers compete on date 1 to offer the country zero-expected-profit contracts for date 2, but we also allow the country to borrow or lend at a given world interest rate $r > 0$, where $\beta(1+r) = 1$.

To see how things change, we have to be very precise about what happens in the event of default. First, we assume that if the country defaults on its contracts with insurers, it forfeits any repayments on savings it may have invested abroad, up to the amount in default.[14] This provision amounts to assuming that aggrieved creditors can seize a defaulting sovereign's foreign assets as compensation. Second, we assume that default on an amount that exceeds the sovereign's own foreign claims triggers sanctions that cost the country a fraction η of its output.[15]

We reserve a detailed analysis of this model for appendix 6B, but its main predictions are easily grasped. Absent default risk there is no saving and the country fully eliminates its second-period consumption risk, as in section 6.1.1.2. With default risk, however, the country recognizes that its own saving effectively gives creditors collateral to seize in case of default. Thus, by saving, the country expands its access to insurance. (Indeed, through this mechanism the country can get partial insurance even when $\eta = 0$, something that wasn't possible in the last model.) But insurance is incomplete, and the repayment schedule still has slope $\eta < 1$ once ϵ reaches a cutoff analogous to e in Figure 6.1. In the extended model, the country distorts its intertemporal consumption profile, consuming less than it otherwise would on date 1, in order to reduce its date 2 consumption variability.

14. It will be in the interest of the country to put its first-period savings into assets that can be seized, since in this way it can expand its insurance opportunities.

15. Technically, in states of nature where the cost of maximal sanctions exceeds the shortfall in repayment, sanctions could be imposed at the minimal level required to ensure repayment. Indeed, this is the natural outcome predicted by bargaining models such as the one considered in appendix 6A. In the absence of private information, default does not take place in equilibrium.

6.1.1.7 Observability and Loan Contracts

One way in which models such as the previous one can be misleading is the tacit assumption that creditors (insurers) can fully observe all the contracts the country engages in. If they cannot, insurers may have no way to be sure that incentive-compatibility constraints like eq. (2) actually are being respected. Their doubts would seriously limit the sovereign's ability to enter into any agreements at all. Problems of observability raise fascinating and important questions, but we shall continue to place them aside until we discuss the consequences of hidden borrower actions in sections 6.3 and 6.4.[16]

6.1.2 Reputation for Repayment

The preceding analysis assumed that a sovereign in default faces sanctions proportional to its income. One of the most severe punishments a defaulting country can face, however, would seem to be a long-term cutoff from foreign capital markets. History furnishes many examples of countries that were largely shut out of private world capital markets for long periods after defaults, for example, much of Latin America for roughly four decades starting in the early 1930s. Certainly, the idea that a country with a bad "reputation" loses access to further credit is intuitively appealing—as anyone who has gone through a thorough credit check can attest. Thus much of the literature on sovereign debt focuses on the question: How much net uncollateralized lending can be supported by the threat of a capital-market embargo? As we shall see, the answer depends in sometimes subtle ways on a detailed specification of the economic environment.[17]

6.1.2.1 A Reputational Model with Insurance

To isolate the role of reputation, we now deprive creditors of any ability to interfere actively with a defaulting debtor's trade or to seize its output. Instead the *only* cost of default is a loss of reputation that brings immediate and permanent exclusion from the world capital market, including the abrogation of current creditor financial obligations to the debtor. We will assume for now, as before, that creditors as a group can precommit to carry out this threat if the country does not make promised payments. (All the results below are easily modified when defaulters suffer only temporary exclusion from capital markets, although less severe penalties naturally can support only more limited sovereign borrowing.) We remind the reader of our continuing assumption that creditors never repudiate their own commitments to the sovereign unless the sovereign defaults first.

16. For a model that explicitly considers hidden actions by a sovereign in a model with default risk, see Atkeson (1991).

17. Surveys of sovereign borrowing that focus on this issue include Eaton and Fernandez (1995) and Kletzer (1994).

A small country has stochastic output $Y_s = \bar{Y} + \epsilon_s$ for dates $s \geq t$. Importantly, the mean-zero shock ϵ_s is i.i.d. As before, it takes values $\epsilon_1, \ldots, \epsilon_N \in [\underline{\epsilon}, \bar{\epsilon}]$, and $\pi(\epsilon_i)$ is the probability that $\epsilon = \epsilon_i$. On date t the country's infinitely-lived representative resident maximizes

$$U_t = \mathrm{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \right\} \tag{11}$$

subject to the constraints that[18]

$$B_{s+1} = (1+r)B_s + \bar{Y} + \epsilon_s - C_s - P_s(\epsilon_s), \tag{12}$$

where B denotes national holdings of noncontingent claims on foreigners, $B_t = 0$, and, for every date s, insurance payments $P_s(\epsilon_i)$ satisfy

$$\sum_{i=1}^{N} \pi(\epsilon_i) P_s(\epsilon_i) = 0. \tag{13}$$

The world interest rate r satisfies $\beta(1+r) = 1$. It is easy to verify that full insurance contracts, which set $P_s(\epsilon) = \epsilon$, will be equilibrium contracts if the country can precommit to meet its obligations to creditors. (In particular, the full insurance contracts are time independent.) Under full insurance, consumption is $C_s = \bar{Y}$ in every period, and B remains steady at 0.

If the country cannot precommit to pay, is the threat of being cut off from world capital markets enough to support full insurance? We answer the question by comparing the country's short-run gain from default to its long-run loss from financial autarky.

Suppose that on date t a country contemplates default on the full insurance contract. Its short-run gain is the extra utility on date t from avoiding repayment:

$$\mathrm{Gain}(\epsilon_t) = u(\bar{Y} + \epsilon_t) - u(\bar{Y}). \tag{14}$$

The punishment for default (even partial default) is that the country loses access to world markets forever after, and is consigned to consuming its random endowment rather than $\bar{Y}$. The date t cost associated with default therefore is

$$\mathrm{Cost} = \sum_{s=t+1}^{\infty} \beta^{s-t} u(\bar{Y}) - \sum_{s=t+1}^{\infty} \beta^{s-t} \mathrm{E}_t u(\bar{Y} + \epsilon_s).$$

By the economy's stationarity, we can drop all time subscripts and write Cost as the time-invariant quantity:

18. As is implicit in the following constraint, we consider only contracts making the country's payment a function of the current shock (rather than of current and past shocks). In the applications that follow, this assumption does not restrict the generality of the conclusions.

$$\text{Cost} = \frac{\beta}{1-\beta}[u(\bar{Y}) - \mathrm{E}u(\bar{Y}+\epsilon)]. \tag{15}$$

Because $u(C)$ is strictly concave, $u(\bar{Y}) > \mathrm{E}u(\bar{Y}+\epsilon)$, so there is a positive penalty for defaulting.[19] That cost does not depend on how small or large an infraction the country has committed. Because of this knife-edge property, a punishment such as the one reflected in eq. (15) is called a *trigger strategy*. Notice also that the cost in eq. (15) becomes unboundedly large as $\beta \to 1$.

The gain from defaulting, eq. (14), is highest when ϵ_t assumes its maximum possible value, $\bar{\epsilon}$. As a result, the full insurance contract is sustainable in all states of nature (and on all dates) only if

$$\text{Gain}(\bar{\epsilon}) \leq \text{Cost},$$

that is, when

$$u(\bar{Y}+\bar{\epsilon}) - u(\bar{Y}) \leq \frac{\beta}{1-\beta}[u(\bar{Y}) - \mathrm{E}u(\bar{Y}+\epsilon)]. \tag{16}$$

If this last inequality holds (as it will if β is close enough to 1), then the country has a strong enough interest in maintaining its reputation for repayment that it will always honor the full insurance contract, even when the temptation to renege is highest.

Note that the reputational equilibrium we have just described would collapse if the country had a *finite* horizon. Let T be the model's last period. Then a debtor has nothing whatsoever to lose by defaulting completely on date T, and will do so if it owes money. Potential creditors understand this fact, and thus will not enter into unsecured contracts on date $T-1$. But then the threat of a future cutoff carries no weight on date $T-2$: since it will happen in any case, debtors will certainly default beforehand, on date $T-2$. By backward induction, you can see that on no date will creditors ever be paid a penny of what they are owed. Thus they won't lend in the first place. Reputational considerations can never support repayment in this model if there is a known finite date beyond which access to international capital markets offers no further gains. However, one should not think of reputational arguments as narrowly applying to infinite-horizon models. Equilibria in which reputation

19. A second-order Taylor approximation around $\epsilon = 0$ gives

$$u(\bar{Y}+\epsilon) \approx u(\bar{Y}) + u'(\bar{Y})\epsilon + \tfrac{1}{2}u''(\bar{Y})\epsilon^2,$$

implying that $\mathrm{E}u(\bar{Y}+\epsilon) \approx u(\bar{Y}) + (1/2)u''(\bar{Y})\mathrm{E}\epsilon^2$, and thus that

$$\text{Cost} \approx \frac{-\beta}{2(1-\beta)}u''(\bar{Y})\mathrm{Var}(\epsilon) > 0,$$

where $\mathrm{Var}(\epsilon) = \mathrm{E}\epsilon^2$ is the variance of ϵ.

is important can occur in finite-horizon models where the borrowing country has private information, for example, about its direct costs of default. Our focus on the preceding infinite-horizon trigger-strategy equilibrium is in part due to its relative analytical tractability.

Application: How Costly Is Exclusion from World Insurance Markets?

Even when the world capital market allows a country fully to insure its output, is the fear of future permanent exclusion from that market likely to suffice to deter default? To answer the question, we calculate empirical measures of the long-term cost of a capital-market embargo, using data from a selection of developing countries.

In our calculations we return to the framework used in Chapter 5 (pp. 329–332) to discuss the gains from international risk sharing. By analogy with that application, the stochastic process generating a country's GDP is assumed to be

$$Y_s = (1+g)^{s-t}\bar{Y}\exp[\epsilon_s - \tfrac{1}{2}\mathrm{Var}(\epsilon)],$$

where $\bar{Y}$ is the trend level of output on the initial date t and where the shock ϵ_s is i.i.d. and distributed normally with mean zero and constant variance $\mathrm{Var}(\epsilon)$.

The representative resident's period utility function takes the isoelastic form

$$u(C) = \frac{C^{1-\rho}}{1-\rho},$$

where $\rho > 0$ is the coefficient of relative risk aversion (here equal to the inverse of the intertemporal substitution elasticity, see Chapter 5). To take a polar but tractable case, we assume that under a full insurance contract the country would completely diversify its output risk in world capital markets before date t. (The country's GDP risk is purely idiosyncratic.) In this case consumption, C_s, equals mean output, $(1+g)^{s-t}\bar{Y}$, on every date $s \geq t$.[20] Accordingly, the compo-

20. Recall that if X is a normally distributed random variable with mean μ_X and variance σ_X^2, then $\exp X$ is lognormal with mean

$$\mathrm{E}\{\exp X\} = \exp\left(\mu_X + \tfrac{1}{2}\sigma_X^2\right).$$

Thus, in the case at hand,

$$\begin{aligned}\mathrm{E}_t Y_s &= (1+g)^{s-t}\bar{Y}\exp\left[-\tfrac{1}{2}\mathrm{Var}(\epsilon)\right]\mathrm{E}_t\exp(\epsilon_s)\\ &= (1+g)^{s-t}\bar{Y}\exp\left[-\tfrac{1}{2}\mathrm{Var}(\epsilon)\right]\exp\left[\tfrac{1}{2}\mathrm{Var}(\epsilon)\right]\\ &= (1+g)^{s-t}\bar{Y}.\end{aligned}$$

nent of the representative national resident's lifetime date t utility accruing after date t is

$$\beta \bar{U}_{t+1} = \frac{1}{1-\rho} \sum_{s=t+1}^{\infty} \beta^{s-t}(1+g)^{(1-\rho)(s-t)}\bar{Y}^{1-\rho} = \frac{\bar{Y}^{1-\rho}}{1-\rho} \times \frac{\beta(1+g)^{1-\rho}}{1-\beta(1+g)^{1-\rho}}$$

[assuming $\beta(1+g)^{1-\rho} < 1$]. In autarky, however, the country must consume its random endowment instead of mean output. On date t expected utility accruing from date $t+1$ onward therefore is

$$\begin{aligned}
\beta \mathrm{E}_t U^{\mathrm{A}}_{t+1} &= \frac{1}{1-\rho} \sum_{s=t+1}^{\infty} \beta^{s-t}\mathrm{E}_t Y_s^{1-\rho} \\
&= \frac{\bar{Y}^{1-\rho}}{1-\rho} \sum_{s=t+1}^{\infty} \beta^{s-t}(1+g)^{(1-\rho)(s-t)}\mathrm{E}_t \exp\left\{(1-\rho)\left[\epsilon_s - \tfrac{1}{2}\mathrm{Var}(\epsilon)\right]\right\} \\
&= \frac{\bar{Y}^{1-\rho}}{1-\rho} \sum_{s=t+1}^{\infty} \beta^{s-t}(1+g)^{(1-\rho)(s-t)} \exp\left\{\tfrac{1}{2}[(1-\rho)^2 - (1-\rho)]\mathrm{Var}(\epsilon)\right\} \\
&= \frac{\bar{Y}^{1-\rho}}{1-\rho} \times \frac{\beta(1+g)^{1-\rho}}{1-\beta(1+g)^{1-\rho}} \exp\left[-\tfrac{1}{2}\rho(1-\rho)\mathrm{Var}(\epsilon)\right] < \beta \bar{U}_{t+1}.
\end{aligned}$$

Exclusion from world capital markets will support full insurance in the present case if and only if inequality (16) holds for all output realizations Y_t, on all dates t. Since the left-hand side of inequality (16) is strictly increasing in date t output, an equivalent condition is

$$\lim_{Y_t \to \infty} [u(Y_t) - u(\bar{Y})] \le \beta(\bar{U}_{t+1} - \mathrm{E}_t U^{\mathrm{A}}_{t+1}).$$

Invoking the isoelastic form of the period utility function and dividing the preceding inequality through by $\bar{Y}^{1-\rho}$, we see that full insurance will be feasible if and only if the following time-invariant inequality holds:

$$\begin{aligned}
&\frac{\lim_{\epsilon_t \to \infty} \exp\{(1-\rho)[\epsilon_t - \frac{1}{2}\mathrm{Var}(\epsilon)]\} - 1}{1-\rho} \\
&\quad \le \frac{\beta(1+g)^{1-\rho}}{(1-\rho)[1-\beta(1+g)^{1-\rho}]} \left\{1 - \exp\left[-\tfrac{1}{2}\rho(1-\rho)\mathrm{Var}(\epsilon)\right]\right\}.
\end{aligned}$$

Notice first that when $\rho \le 1$, this inequality never holds: because marginal period utility falls off relatively gently as consumption rises, there is always some finite output realization high enough that inequality (16) is violated. Thus, concern for reputation can support full insurance in the present model only when the risk aversion coefficient ρ exceeds 1.[21]

When $\rho > 1$, $\lim_{\epsilon_t \to \infty} \exp(1-\rho)[\epsilon_t - \frac{1}{2}\mathrm{Var}(\epsilon)] = 0$ and the last inequality therefore reduces to

$$1 \leq \beta(1+g)^{1-\rho} \exp\left[\tfrac{1}{2}\rho(\rho-1)\mathrm{Var}(\epsilon)\right].$$

Intuitively, higher values of β and $\mathrm{Var}(\epsilon)$ make it more likely that a full-insurance equilibrium is sustainable. A higher trend growth rate, g, makes the full-insurance equilibrium less likely by making future output uncertainty progressively less costly in utility terms.

For eight developing countries, Table 6.1 presents estimates of g and $\mathrm{Var}(\epsilon)^{1/2}$, the mean and standard deviation in the growth rate of real per capita GDP. The table assumes that $\rho = 4$ and $\beta = 0.95$. Also reported are two measures of the cost of capital-market exclusion. The column labeled Cost/Y shows the total cost of reputation loss as a ratio to *current* mean output. That ratio can be measured by the solution κ to the equation

$$u[(1+\kappa)\bar{Y}] - u(\bar{Y}) = \beta(\bar{U}_{t+1} - \mathrm{E}_t U^{\mathrm{A}}_{t+1}),$$

which is, in the present example,

$$\kappa = \left[\frac{1-\beta(1+g)^{1-\rho}\exp\left[\frac{1}{2}\rho(\rho-1)\mathrm{Var}(\epsilon)\right]}{1-\beta(1+g)^{1-\rho}}\right]^{-\frac{1}{\rho-1}} - 1.$$

Notice that for $\rho > 1$, $\mathrm{Cost}/Y \to \infty$ as $\beta(1+g)^{1-\rho}\exp\left[\frac{1}{2}\rho(\rho-1)\mathrm{Var}(\epsilon)\right] \to 1$ from below, so Cost/Y is undefined (effectively infinite) for countries such that full insurance is sustainable by reputation. The column Cost per Year reports the permanent fractional increase in GDP equivalent to access to full insurance. This number is the same as the "cost of consumption variability" τ calculated on p. 330, and it is therefore given by $\tau = \{\exp[\frac{1}{2}(1-\rho)\rho\mathrm{Var}(\epsilon)]\}^{1/(1-\rho)} - 1$.

For the preference parameters underlying Table 6.1, only Venezuela, with the lowest per capita growth rate in the group, would never default on a full insurance contract if the penalty were future exclusion from the world capital market. For the other countries, the total cost of exclusion in terms of current output, κ, is a finite number equivalent to anywhere from 4 (for Colombia) to 53 (for Lesotho) percent of one year's GDP. Thus, positive output shocks of the same size would be enough to induce default on a full-insurance contract, implying that the lenders would never offer the contract in the first place. Remember however, that we have unrealistically assumed the possibility of un-

21. This peculiar feature of the model stems from the assumption that output shocks are potentially unbounded from above. We assumed a lognormal distribution for output, however, purely to facilitate the exact calculations in the text. For more reasonable probability distributions making output bounded on every date, we wouldn't necessarily be able to rule out full-insurance equilibria when $\rho \leq 1$.

Table 6.1
Output Processes and Cost of Capital-Market Exclusion, 1950–92

Country	g	$\mathrm{Var}(\epsilon)^{1/2}$	Cost/Y (κ)	Cost per Year (τ)
Argentina	0.015	0.099	0.36	0.020
Brazil	0.040	0.117	0.24	0.028
Colombia	0.023	0.050	0.04	0.005
Lesotho	0.053	0.160	0.53	0.052
Mexico	0.030	0.088	0.13	0.016
Philippines	0.023	0.100	0.24	0.020
Thailand	0.043	0.081	0.08	0.013
Venezuela	0.011	0.118	Undefined	0.028

Source: Penn World Table, version 5.6. The calculations assume $\beta = 0.95$ and $\rho = 4$.

bounded positive output shocks. With a more realistic bounded distribution, positive output shocks as much as 53 percent of GDP would be zero-probability events, so a country with Lesotho's high output-growth variability around trend (16 percent per year) might well be deterred from default by its fear of reputation loss. The final column of Table 6.1, showing the cost of consumption variability τ as an annuitized flow, reports estimates substantially larger than those applicable to most industrialized countries (recall Chapter 5). Since τ is not a present value, it does not depend on the economy's growth rate or discount rate.

The trend-stationary stochastic process used to capture output variability underestimates the cost of exclusion from the world capital market if output shocks are persistent, and especially if there is a unit root in output.[22] For several reasons, however, Table 6.1 is more likely to convey an exaggerated picture of the deterrent power of reputation loss. First, countries usually cannot eliminate *all* output risk through financial contracts. Second, the results are quite sensitive to the assumed taste parameters. (Were β equal to 0.85 rather than 0.95—imagine that a somewhat myopic government, one facing some probability of losing office, makes the default decision—Argentina would reckon the cost of reputation loss as equivalent to only 11 percent of current GDP, not 36 percent.) Third, the possibilities of investment or disinvestment at home, absent in the preceding model, create self-insurance possibilities that reduce the gains from external risk sharing. Finally, it could occur in reality that a country can still lend in international markets, even when it can no longer borrow. As discussed in section 6.1.2.4 below, this possibility, along with domestic investment possibilities, can facilitate self-insurance and thereby reduce the cost of losing one's reputation as a good borrower. ∎

22. See Obstfeld (1994b, 1995).

6.1.2.2 The Feasibility of Partial Insurance

What if eq. (16) does not hold? Can the country still obtain partial insurance, as in our two-period analyses? The answer, first illustrated by Eaton and Gersovitz (1981), is yes. It simplifies presentation of the main points to begin by adopting a setup analogous to the one in section 6.1.1.1. In that spirit, we assume that the country can neither save nor dissave, and can sign only one-period contracts to share the following period's output risk with competitive, risk-neutral foreign insurers.[23] (We discuss how to relax the somewhat artificial no-saving assumption in the next section.) Thus the country maximizes the function (11) subject to

$$C_s(\epsilon_s) = \bar{Y} + \epsilon_s - P_s(\epsilon_s) \tag{17}$$

(which is the same constraint as in section 6.1.1.1, for every period s), the zero-profit condition for foreign insurers, eq. (13), and an incentive-compatibility constraint that guarantees payment for all $P_s(\epsilon_s) > 0$. Our setup precludes the accumulation of any collateral to secure risk-sharing contracts, as in section 6.1.1.6, and ensures that expected consumption always equals $\bar{Y}$. This loss of generality is harmless for present purposes, as it is only when debts are at least partially unsecured that there can be a meaningful default.

The form of the incentive compatibility constraint can be derived by modifying our analysis of the full insurance case. A major simplifying factor is the time-independent or stationary nature of the country's problem (recall there is no saving or dissaving and ϵ is i.i.d.). Stationarity implies that the optimal incentive-compatible contract covering any date s (given that no default has occurred) will be time-independent, that is, $P_s(\epsilon_s) = P(\epsilon_s)$. If the country defaults on this contract on date t after observing ϵ_t, its short-term gain is

$$\text{Gain}(\epsilon_t) = u(\bar{Y} + \epsilon_t) - u[\bar{Y} + \epsilon_t - P(\epsilon_t)].$$

The cost of future exclusion from the world capital market (given that an optimal incentive-compatible one-period insurance contract otherwise would have been signed in every future period) is the time-independent quantity

$$\text{Cost} = \frac{\beta}{1-\beta}\left\{\mathrm{E}u[\bar{Y} + \epsilon - P(\epsilon)] - \mathrm{E}u(\bar{Y} + \epsilon)\right\}.$$

Thus the incentive-compatibility constraint, $\text{Gain}(\epsilon_t) \leq \text{Cost}$, has the form

$$u(\bar{Y} + \epsilon_t) - u[\bar{Y} + \epsilon_t - P(\epsilon_t)] \leq \frac{\beta}{1-\beta}\left\{\mathrm{E}u[\bar{Y} + \epsilon - P(\epsilon)] - \mathrm{E}u(\bar{Y} + \epsilon)\right\}$$

$$= \frac{\beta}{1-\beta}\sum_{j=1}^{N}\pi(\epsilon_j)\left\{u[\bar{Y} + \epsilon_j - P(\epsilon_j)] - u(\bar{Y} + \epsilon_j)\right\}. \tag{18}$$

23. Grossman and Van Huyck (1988) base their analysis on a similar assumption.

Consider the country's position on any arbitrary date. Since the country is signing a contract covering next-period consumption only, and since its problem is stationary, the best it can do is to choose the schedule $P(\epsilon)$ to maximize

$$\sum_{i=1}^{N} \pi(\epsilon_i) u[\bar{Y} + \epsilon_i - P(\epsilon_i)]$$

subject to constraints (18) (one for each state $i = 1, \dots N$) and eq. (1). The Lagrangian (which does not depend on the date) therefore is

$$\begin{aligned}
\mathcal{L} = {} & \sum_{i=1}^{N} \pi(\epsilon_i) u[\bar{Y} + \epsilon_i - P(\epsilon_i)] \\
& - \sum_{i=1}^{N} \lambda(\epsilon_i) \Bigg(u(\bar{Y} + \epsilon_i) - u[\bar{Y} + \epsilon_i - P(\epsilon_i)] \\
& - \frac{\beta}{1-\beta} \sum_{j=1}^{N} \pi(\epsilon_j) \left\{ u[\bar{Y} + \epsilon_j - P(\epsilon_j)] - u(\bar{Y} + \epsilon_j) \right\} \Bigg) \\
& + \mu \sum_{i=1}^{N} \pi(\epsilon_i) P(\epsilon_i).
\end{aligned}$$

The associated Kuhn-Tucker necessary conditions (which must hold for all ϵ) are

$$\left[\pi(\epsilon) + \lambda(\epsilon) + \frac{\beta \pi(\epsilon)}{1-\beta} \sum_{j=1}^{N} \lambda(\epsilon_j) \right] u'[C(\epsilon)] = \mu \pi(\epsilon) \tag{19}$$

and the complementary slackness condition

$$\begin{aligned}
\lambda(\epsilon) \Bigg(& \frac{\beta}{1-\beta} \sum_{j=1}^{N} \pi(\epsilon_j) \left\{ u[\bar{Y} + \epsilon_j - P(\epsilon_j)] - u(\bar{Y} + \epsilon_j) \right\} \\
& - u(\bar{Y} + \epsilon) + u[\bar{Y} + \epsilon - P(\epsilon)] \Bigg) = 0,
\end{aligned} \tag{20}$$

for nonnegative $\lambda(\epsilon)$.[24]

Equations (19) and (20) look more forbidding than their analogs in the sanctions model of section 6.1.1, eqs. (4) and (5), but their implications are pretty much the same. For relatively low values of ϵ, incentive-compatibility constraint (18) doesn't

24. In taking the partial derivatives leading to eq. (19), recall that we are seeking the optimal $P(\epsilon)$ schedule, which requires that we maximize for every $P(\epsilon_i)$, $i = 1, \dots, N$. To compute a specific partial $\partial\mathcal{L}/\partial P(\epsilon_i)$, for example, $\partial\mathcal{L}/\partial P(\epsilon_2)$, simply write out $\mathcal{L}$ term by term and differentiate with respect to $P(\epsilon_2)$. You should end up with eq. (19) for $\epsilon = \epsilon_2$. Notice that in eq. (19), the time subscript attached to $C(\epsilon)$ can be suppressed thanks to the problem's stationarity.

bind and $\lambda(\epsilon)=0$. For these states, eq. (19) implies that

$$u'[C(\epsilon)] = \frac{\mu}{1+\frac{\beta}{1-\beta}\sum_{j=1}^{N}\lambda(\epsilon_j)}. \tag{21}$$

Because the right-hand side of eq. (21) is the same for *all* ϵ, consumption is again stabilized in the face of the worst downside risks. As before, this fact means that $P(\epsilon)=P_0+\epsilon$ for some constant P_0, and therefore that $C(\epsilon)=\bar{Y}-P_0$ as long as $\lambda(\epsilon)=0$.

When $\lambda(\epsilon)>0$, constraint (18) holds as an equality and fully determines the functional dependence of $P(\epsilon)$ upon ϵ. Implicit differentiation of the equality constraint corresponding to eq. (18) gives the slope

$$\frac{\mathrm{d}P(\epsilon)}{\mathrm{d}\epsilon} = \frac{u'[\bar{Y}+\epsilon-P(\epsilon)]-u'(\bar{Y}+\epsilon)}{u'[\bar{Y}+\epsilon-P(\epsilon)]}.$$

Because constraint (18) never binds unless $P(\epsilon)$ is positive, the strict concavity of $u(C)$ implies that $0<\mathrm{d}P(\epsilon)/\mathrm{d}\epsilon<1$. By eq. (17), $C(\epsilon)$ must therefore increase with ϵ when eq. (18) binds in order to deter debt repudiation.

Now we tie together the two portions of $P(\epsilon)$—over the range of relatively low ϵ where $\lambda(\epsilon)=0$ and over the range of higher ϵ where $\lambda(\epsilon)>0$. (We took an analogous step in the model of section 6.1.1.) Equation (21) holds for ϵ such that $\lambda(\epsilon)=0$, and for such ϵ, $C(\epsilon)=\bar{Y}-P_0$, as we saw a moment ago. Thus, eq. (21) implies that

$$\mu = \left[1+\frac{\beta}{1-\beta}\sum_{j=1}^{N}\lambda(\epsilon_j)\right]u'(\bar{Y}-P_0).$$

Using this expression to eliminate μ from eq. (19), we get

$$\left[1+\frac{\beta}{1-\beta}\sum_{j=1}^{N}\lambda(\epsilon_j)\right]\left\{u'(\bar{Y}-P_0)-u'[C(\epsilon)]\right\} = \frac{\lambda(\epsilon)u'[C(\epsilon)]}{\pi(\epsilon)},$$

which holds for all ϵ.

Assume for simplicity that ϵ has a continuous distribution function. We have seen that $C(\epsilon)$ falls as ϵ falls over the range of ϵ with $\lambda(\epsilon)>0$. Thus the left-hand side of the preceding equation also falls as ϵ falls until ϵ reaches $e\in(\underline{\epsilon},\bar{\epsilon})$, where $u'(\bar{Y}-P_0)=u'[C(e)]$ and $\lambda(e)=0$.[25] Since $C(e)$ therefore equals $\bar{Y}-P_0$, the consumption schedule is continuous at $\epsilon=e$, where constraint (18) switches from nonbinding to binding as ϵ rises. Because consumption thus is continuous

25. As in section 6.1.1, cases with $e=\bar{\epsilon}$ imply that full insurance is feasible.

over states, the two portions of $P(\epsilon)$ must coincide at $\epsilon = e$, where the incentive-compatibility constraint first starts to bite.

A picture similar to Figure 6.1 illustrates the optimal incentive-compatible contract, but now the constrained arm of $P(\epsilon)$ will not in general be linear.

6.1.2.3 The Fully Dynamic Case

What happens when full insurance is not initially possible, but the country can save or dissave according to eq. (12)? Equivalently, what if we relax constraint (13) and require instead only that foreign lenders offer contracts with expected present values of zero? This case has been analyzed formally by Worrall (1990); here we offer an intuitive sketch.

As in the two-period model of section 6.1.1.6, a partially binding incentive-compatibility constraint gives the country an additional motive for saving: by accumulating a positive foreign asset position that creditors can seize in the event of default, the country provides a hostage that modifies its own incentive to withhold payment. This, in turn, allows the country fully to insure its income over more states of nature.

In a dynamic setting, the country continues to accumulate foreign assets as long as the incentive-compatibility constraint binds in any state of nature. But the marginal return to those extra assets falls as the country's foreign wealth increases, so mean consumption rises over time. The country stops saving once it owns just enough foreign wealth that default no longer pays even in the highest state of nature, $\bar{\epsilon}$. This occurs when the full insurance contract is completely collateralized, that is, when the foreign asset stock reaches the value $\bar{B}$ at which $(1+r)\bar{B} = \bar{\epsilon}$. At this point the country can credibly promise always to fulfill the full insurance contract, and consumption thus occupies the steady state $\bar{C} = \bar{Y} + r\bar{B}$ thereafter.

The country's long-run consumption is higher than in the full insurance case, but to earn its collateral it has had to distort the flat first-best intertemporal consumption profile it would have preferred. While saving allows the country fully to insure its output in the long run, it is still worse off than if it had been able to commit to full repayment at the outset.

6.1.2.4 The Significance of Reputation

Concern over maintaining a reputation for creditworthiness can support some uncollateralized international lending between sovereign nations. But one should not conclude that reputation alone, absent *any* legal rights for creditors at home or abroad other than the right not to lend in the future, can support a significant level of sovereign lending. Indeed, the deterrent effect of reputation loss depends critically on our implicit assumptions regarding creditor rights and incentives.

The preceding models assumed that defaulting countries are simply cut off from world capital markets. While it is plausible that potential lenders would shun a country with a past record of nonrepayment, it is much less plausible that foreign banks would worry about a reputation for repayment when *accepting* the country's deposits, or that foreign firms would worry about it when selling the country shares. Of course, if the country tried to place deposits, for example, in the same banks it had borrowed from, the banks might, with legal justification, confiscate the country's funds. But what about other banks, possibly even banks in other countries? Throughout this section we have relied on the assumption that lenders throughout the world will present a united front, either in imposing direct trade sanctions or in enforcing a total capital-market embargo on an offending sovereign borrower. For this to be a reasonable assumption, even as an approximation, creditors must have rights at home and abroad that go far beyond the right simply to stop lending.

Why does a debtor's ability to accumulate assets make any difference? Perhaps surprisingly, the threat that a transgression will be punished by loss of future borrowing possibilities does not deter a country from default when its lending opportunities are not simultaneously curtailed. To see how a candidate reputational equilibrium can unravel when creditors cannot touch a sovereign's foreign assets, let us revisit the simple model of section 6.1.2.1. There, the threat of financial market autarky could be sufficient to support complete insurance, provided the borrower did not discount the future too much.

Suppose we now relax the implicit assumption that creditors can seize assets held abroad. In fact, we assume that after defaulting a country is free to hold any type of asset and write any type of *fully collateralized* insurance contract. (A fully collateralized insurance contract is one where the country posts a large enough bond to cover any possible payment it might be called upon to make.)

With this option, will the country still have an incentive to honor its reputation contract? As before, it is sufficient to consider its incentive to default in the most favorable state of nature, $\bar{\epsilon}$, in which the reputation contract calls upon the country to make the maximum payment, $P(\bar{\epsilon})$. Let us imagine now that state $\bar{\epsilon}$ occurs, but that instead of paying $P(\bar{\epsilon})$ to creditors, the country defaults on its reputation contract. Rather, it takes the money it would have paid to its creditors and invests it abroad in a riskless bond paying the world interest rate r. At the same time, the country writes an explicit insurance contract with a new group of foreign insurers, providing it with the *exact same* payout function $P(\epsilon)$, as in its original (possibly implicit) reputation contract. Crucially, the new insurers do not need to rely on the country's (now defunct) reputation because it can put up its bond as collateral. Under this scheme, the country must come out ahead. Its new insurance contract fully duplicates its old insurance contract. At the same time, the country can consume the interest on its bond in each future period while still maintaining the necessary amount of collateral. (As an alternative to writing a new insurance contract, the country could invest in a portfolio of foreign stocks and bonds having a return that covaries negatively with its output.)

Why does it matter if the reputation contract fails in state $\bar{\epsilon}$? If one node on the equilibrium tree fails, the whole reputation contract cannot be an equilibrium, since foreign insurers must be able to break even on average. Might there not be another reputation contract, providing perhaps a bit less insurance, that still works? The answer is no. For any reputation contract, there must always be some state of nature in which the country's payment is higher than (or at least as high as) the payment in any other state of nature. The country will always default in that state. Therefore, *no* level of reputation-based insurance is possible!

The foregoing argument assumed a stationary endowment economy, but it is in fact quite general and requires virtually no assumptions on the production or utility functions (see Bulow and Rogoff, 1989b). The main nuance in extending the result to more general environments is that in a growing economy, the largest possible reputation payment may also be growing over time. The proof involves noting that the world market value of a claim to all the expected future payments by a country can never exceed the world market value of a claim to its entire future net output.

The no-reputation result we have just derived is quite remarkable but, as Bulow and Rogoff note, there are some important qualifications. For example, the country may not be able to construct an asset portfolio that exactly mimics its reputation contract, and this consideration may sustain a limited amount of reputation insurance. Countries that default on debt may lose reputation in other areas (e.g., trade agreements).[26] A limited amount of reputation lending may also be possible if creditors cannot perfectly observe a country's actions or preferences. The overall conclusion from this analysis, however, is that if countries with poor credit histories can safely lend abroad, the threat of reputation loss becomes much weaker as a lever to deter default.[27]

So far in this section we have ignored the possibility that creditors (insurers), rather than being unfailingly honest themselves, may break their financial promises. The more general question is whether creditors' threats and promises are credible. To think about answers, we need a framework in which borrowers and lenders are treated symmetrically.

6.1.3 A General-Equilibrium Model of Reputation

We turn to a setup in which no country can effectively commit itself to pay uncollateralized debts. Thus the positions of all participants in the world capital market are symmetrical. In this context, there do exist equilibria in which the cost of losing reputation is sufficient to support international contracts.

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26. See also Cole and P. Kehoe (1995, 1996).

27. Remember that throughout this chapter, we have presented a very simplistic notion of default. Real world default is complex and generally involves bargaining between debtors and creditors of the nature sketched in appendix 6A.

Consider a world composed of a very large number of small countries j, all of which share the utility function

$$U_t^j = \mathrm{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u(C_s^j) \right\}.$$

Country j's endowment is

$$Y_t^j = \bar{Y} + \epsilon_t^j + \omega_t,$$

where ω_t is a mean-zero global shock common to all countries, and ϵ_t^j is a mean-zero idiosyncratic country shock such that

$$\sum_j \epsilon_t^j = 0. \tag{22}$$

Shocks are assumed to be i.i.d. and bounded within $[\underline{\epsilon}, \bar{\epsilon}]$ and $[\underline{\omega}, \bar{\omega}]$, respectively, so that every country's output is always positive.

Given assumption (22), the efficient (Arrow-Debreu) allocation for this economy sets

$$C_t^j = \bar{Y} + \omega_t, \qquad \forall j, t.$$

To attain this first-best (full insurance) allocation through the market, countries sell off their positive idiosyncratic shocks and insure themselves against negative realizations, all at actuarially fair prices. Can this equilibrium be supported if countries have no direct sanctions to punish a sovereign that breaches its insurance contract? The answer is yes, provided countries follow the right kind of trigger strategy in response to a default.

Specifically, suppose that any country j that defaults on its contract is completely and permanently cut off from world markets. This exclusion requires (a) that country j lose its reputation for repayment, so that everyone believes it will always default in the future if given the opportunity; *and* (b) that all other countries lose their own reputations for repaying country j. [Part (b) is needed to prevent country j from purchasing bonded insurance contracts in favorable states of nature, thereby eliminating its dependence on foreign insurers by analogy with the example in section 6.1.2.4.]

Under these assumptions about expectations, no country will lend to a defaulting country j. Nor will the defaulter itself lend abroad, because after defaulting, country j believes that any potential insurer i will default at the first opportunity. (Hence country j would confirm country i's beliefs by again defaulting were country i nevertheless to sign a contract with j. Similarly, country i, believing country j will never make promised payments, would perceive no loss from seizing any assets j foolishly entrusted to i.) Thus the punishments on a defaulter are self-enforcing. In terminology from game theory, the equilibrium is subgame perfect because the threats that support it are credible.

The short-term gain to country j from repudiating the first-best insurance contract on date t is

$$\text{Gain}(\epsilon_t^j, \omega_t) = u(\bar{Y} + \epsilon_t^j + \omega_t) - u(\bar{Y} + \omega_t),$$

while the expected future cost is

$$\begin{aligned}\text{Cost} &= \mathrm{E}_t \sum_{s=t+1}^{\infty} \beta^{s-t}[u(\bar{Y} + \omega_s) - u(\bar{Y} + \epsilon_s^j + \omega_s)] \\ &= \frac{\beta}{1-\beta}[\mathrm{E}u(\bar{Y} + \omega) - \mathrm{E}u(\bar{Y} + \epsilon^j + \omega)].\end{aligned}$$

(We can drop time subscripts in the final expression thanks to the problem's stationarity.) The foregoing formulas for gain and cost are analogous to the ones we derived in eqs. (14) and (15), except for the presence of the global shock ω. Note especially that the world shock causes the gain from default to fluctuate over time. The temptation is greatest when the world is in an extreme recession ($\omega = \underline{\omega}$) and country j in a relative boom ($\epsilon^j = \bar{\epsilon}$). As we have noted, the cost of default is constant (because of the i.i.d. shocks). Thus the first-best allocation can be supported by reputation if

$$\text{Gain}(\bar{\epsilon}, \underline{\omega}) \leq \text{Cost}.$$

This condition can always be met if β is close enough to 1 (that is, if countries place high enough weight on continued capital-market access). If not, partial insurance may still be possible, as in the small-country case.

The model shows that reputation *may* support international lending, not that it will. There is a vast multiplicity of trigger-strategy equilibria supporting different degrees of international risk sharing, including none. We have not provided any argument to show why countries should coordinate on the particular expectations assumed.

An obvious shortcoming of the permanent exclusion scheme we have examined is that, after a transgression by one party, countries willingly forgo potential gains from trade forever. Might they not find it mutually advantageous to reopen asset trade at a later date? In the parlance of game theory, the equilibrium on which our example focuses is subgame perfect but not obviously *renegotiation-proof*: it is conceivable that after a default on a first-best insurance contract, all players would wish to interrupt the defaulter's punishment and proceed with insurance restrictive enough to deter default in the future. Here we note only that for high enough discount factors β, renegotiation-proof equilibria that support the first-best allocation can be constructed.[28]

28. See Kletzer (1994) for a detailed discussion of renegotiation- and coalition-proof equilibria in debt models.

Application: How Have Prior Defaults Affected Countries' Borrowing Terms?

A basic tenet of reputational models of sovereign borrowing is that default reduces a sovereign's future gains from the international capital market. What is the historical record?

Many sovereign borrowers of the 1920s defaulted in the 1930s and weren't able to return to world capital markets until the 1970s. It would be misleading, however, to view these exclusions as independent cases in which individual defaulters were shut out of an otherwise well-functioning world financial system. In reality, the defaults were a symptom of a much larger contraction of world capital markets and trade, in which even some countries that continuously met their foreign obligations suffered denial of new loans. The situation could be modeled using the last subsection's general-equilibrium default model, modified to allow even honest borrowers who did not default to lose reputation.[29]

Exclusion from capital markets is virtually never permanent. As documented by Lindert and Morton (1989), fewer than a third of borrowers with some default history over 1820–1929 fully repaid foreign debts in the 1930s. Seventy percent of those with payments problems over 1940–79 fell into arrears or rescheduled on concessionary terms in the first half of the 1980s (a period of generalized debt crisis that we will discuss further in section 6.2.3). Even Mexico, Turkey, and the Soviet Union, all of which lost access to foreign credits in the 1920s after new revolutionary governments repudiated *ancien régime* debts in the 1910s, eventually regained private market access in the 1970s (only to experience renewed debt problems in the 1980s).

Elements of an explanation are suggested by the fact that many defaulting borrowers eventually settled with creditors. In many of the defaults that took place over the first part of the twentieth century, the terms of the final settlements tended to be generous enough so that, on average, British and U.S. investors ended up earning rates of return slightly above what they could have earned on U.S. or British government debt (see Eichengreen, 1991, for a survey of estimates).[30] Thus creditors may have viewed many defaults as "excusable" and been willing to accept, at least ex post, the implicit state contingency of their prior loans. Alterna-

29. Why, contrary to such a model, did some debtors continue to repay after the world capital market dried up? The answer may be that creditors had additional sanctions to deploy in these cases. Argentina, which had borrowed extensively from Britain, had an important export surplus with that country and feared commercial retaliation. It therefore continued to service debt through the 1930s even after most other Latin American countries defaulted (see Díaz-Alejandro, 1983).

30. Loans to prerevolutionary Mexico, Turkey, and Russia were not settled quickly and yielded low rates of return after the fact, a circumstance that may help explain why the successor governments were kept from borrowing in the 1920s.

tively, settlement of old debts may have represented a renegotiation process by which defaulting debtors restored their standing in world capital markets.

Experience also suggests that lenders face considerable uncertainty about borrower characteristics and preferences. As a result, changes in a borrowing country's political regime or economic prospects can have a big impact on its capital-market access, despite past sins. Peru's development of guano exports aided the country in settling prior foreign claims and reentering world capital markets in 1849 (see Fishlow, 1985). More recently, radical economic liberalization and macroeconomic stabilization in Argentina, Chile, and Mexico returned those countries to world capital markets around 1990 after the debt crisis of the 1980s (although investors in Mexico were soon burned in a financial crisis sparked by the country's 1994 currency devaluation).

Econometric studies indicate that lenders typically base their country risk assessments on past debt-servicing behavior as well as on newer information. After controlling for current economic and political determinants of default risk, Özler (1993) finds that among countries with borrowing histories, those with earlier debt problems faced higher commercial-bank interest rates in the 1970s. Lenders apparently do take default histories into account, at least to some extent. ■

6.2 Sovereign Risk and Investment

Because sovereign debt problems have been most acute for low- and middle-income countries, concerns about their economic effects have centered more on possible harm to investment and growth than on limited risk sharing. These areas of concern are not unrelated, of course. But a number of interactions between sovereigns' borrowing and investment decisions are most easily understood in a setting without uncertainty. Indeed, several main points can be made most simply in a framework based on the two-period model of Chapter 1.

While a certainty setting serves well to illustrate some basic concepts, such as the importance of borrower commitments, it is inadequate for a realistic account of other issues, notably the pricing of sovereign debt in world secondary markets. Uncertainty therefore reappears in the latter part of this section when we discuss the interaction between investment and the market value of sovereign debt. One of the robust conclusions that will emerge is that international capital flows do not necessarily equalize countries' marginal rates of return on investment when creditors fear sovereign default.

6.2.1 The Role of Investment under Direct Sanctions

The small country is inhabited by a representative agent with utility function

$$U_1 = u(C_1) + \beta u(C_2).$$

On date 1 the country receives the endowment Y_1, but no capital is inherited from the past ($K_1 = 0$). Date 2 output depends on date 1 investment, $I_1 = K_2 - K_1 = K_2$, according to the production function

$$Y_2 = F(K_2).$$

As usual, $F'(K) > 0$ and $F''(K) < 0$.

In deference to the conventions of the vast literature on international debt, we depart from our usual notation for a country's foreign *assets*, B, and instead throughout the remainder of this chapter refer to D, its foreign *debt*. (Clearly, $D = -B$.) Using this notation, let D_2 be the country's borrowing from foreign lenders on date 1, and $\mathfrak{R}$ the amount of loan repayment the country makes on date 2.

The first-period finance constraint is

$$K_2 = Y_1 + D_2 - C_1,$$

whereas that for the second period is

$$C_2 = F(K_2) + K_2 - \mathfrak{R},$$

assuming that capital does not depreciate and can be "eaten" at the end of the second period. We do not presume that the sum of interest and principal, $(1+r)D_2$, is repaid in full. Thus we interpret $\mathfrak{R}$ broadly, as the lesser of the face value owed to creditors and the sanctions they impose in the event of default, which here (as in section 6.1.1) take the form of a proportional reduction in the country's date 2 resources. Specifically, we assume creditor sanctions reduce the country's date 2 resources by the fraction η in case of default, so that

$$\mathfrak{R} = \min\{(1+r)D_2, \eta[F(K_2) + K_2]\}, \tag{23}$$

with full repayment in case of a tie.

If the country could commit to repay in full we would be in the world of Chapter 1, in which investment continues up to the point at which

$$F'(K_2) = r$$

and consumption obeys the Euler equation

$$u'(C_1) = (1+r)\beta u'(C_2).$$

Suppose, however, that the country cannot commit to repay, so that its repayment never exceeds the cost of sanctions

$$\mathfrak{R} \leq \eta[F(K_2) + K_2].$$

There are two cases to consider, which differ in allowing the country to commit to an investment strategy before receiving any loans. Investment is significant for lenders because by raising date 2 output, it raises the power of their sanctions to deter default. (The assumption that the cost of sanctions is a fixed *fraction* of output, rather than a constant amount, is crucial in giving investment this strategic role.)

6.2.1.1 Discretion over Investment: Calculating the Debt Ceiling

Perhaps more realistic is the case in which the country is free to choose any investment strategy it wants after borrowing. Here potential creditors must ask themselves, "If we lend D_2 today, will the country choose to invest enough to make $\eta[F(K_2) + K_2] \geq (1 + r)D_2$?" If not, lenders won't be repaid in full. Their task, therefore, is to figure out how much they can safely lend. We denote by $\bar{D}$ the most they can lend without triggering default. The first part of the present problem is to calculate this credit limit.

This problem turns out to be surprisingly tricky, though quite instructive. The basic issue is that lenders must calculate their returns under each of two scenarios, depending on whether the borrower chooses investment with the intent of repaying or chooses it intending to default. We find that the equilibrium debt level has a knife-edge quality, such that a small increase in debt could lead to very large decreases in both investment and payments to creditors. (On a first pass, the reader may choose to skip to section 6.2.1.2, where we treat $\bar{D}$ as given and look at the implications. However, skipping the intermediate step of calculating $\bar{D}$, though conventional in the literature, obscures some fundamental issues.)

To calculate $\bar{D}$, let's put ourselves in the sovereign's position after lenders have given it money. Given date 1 borrowing of D_2, it is free to choose C_1 and K_2 and then set repayments according to eq. (23). Substituting the relevant finance constraints into U_1, we formulate the country's problem as

$$\max_{K_2} u(Y_1 + D_2 - K_2) + \beta u\left[F(K_2) + K_2 - \min\{(1+r)D_2, \eta[F(K_2) + K_2]\}\right]. \tag{24}$$

Its solution tells us whether the sovereign defaults, and $\bar{D}$ is the largest value of D_2 such that full repayment is the sovereign's preferred action.

The simplest way to see what is going on is through a diagram. Figure 6.3 graphs the country's production and consumption possibilities over C_1 and C_2, both for a *given* debt D_2, in analogy to the PPFs for GDP and GNP that we saw in Chapter 1.

In Figure 6.3, the **GDP** PPF is indicated by the broken line. It intersects the horizontal axis at $Y_1 + D_2$, a sum equal to the total resources the country has available for consumption or investment on date 1. **GDP** plots date 2 resources, $F(K_2) + K_2$, against $K_2 = I_1$, where K_2 is measured from right to left starting at $Y_1 + D_2$. There are two other transformation loci in the figure. The one labeled $\mathbf{GNP^D}$ (the D stands for "default") plots $(1 - \eta)[F(K_2) + K_2]$, the output the country can consume on date 2 after it defaults and suffers sanctions, against K_2. The one labeled $\mathbf{GNP^N}$ (the N stands for "nondefault") plots $F(K_2) + K_2 - (1+r)D_2$, the output the economy can consume on date 2 if it repays in full. $\mathbf{GNP^N}$ is simply **GDP** shifted vertically downward by the distance $(1 + r)D_2$. According to eq. (23), the outer

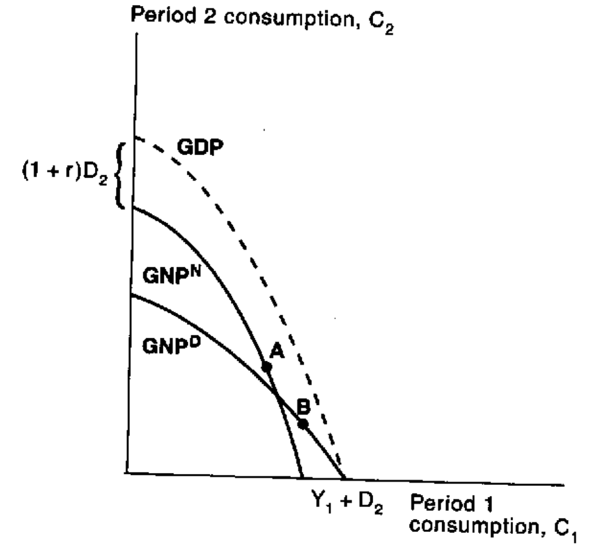


Figure 6.3
Post borrowing consumption possibilities

envelope of **GNP**^{D} and **GNP**^{N}, the locus of maximal consumption possibilities, is what constrains utility. Unless contracted debt repayments are very big, there will be some investment levels high enough that repayment is optimal.

But what investment level will the sovereign choose, given D_2? The optimal value of K_2 is given by the tangency of the consumption-possibilities locus with the highest consumption indifference curve. The unusual feature of the present problem is that the **GNP**^{D}-**GNP**^{N} outer envelope is nonconcave, meaning that the sovereign's optimal investment decision may *not* be uniquely determined as a function of D_2! The kink, it is important to note, occurs precisely where $\eta[F(K_2) + K_2] = (1 + r)D_2$, at the intersection of **GNP**^{D} and **GNP**^{N}. It is possible in Figure 6.3 that two different investment levels, such as those at points **A** and **B**, yield equal utility. This seemingly peculiar feature of the problem is the key to solving for $\bar{D}$.

Since we have a badly behaved (nonconvex) problem on our hands, it is prudent to work out thoroughly a simple example that conveys the intuition behind more general cases. The utility function we assume is $U_1 = \log C_1 + \beta \log C_2$, and the production function, $Y_2 = \alpha K_2$, where $\alpha > r$.[31] A critical inequality assumption is needed to make what follows interesting:

31. This case would not make sense, of course, absent default risk: without that risk, all the world's savings would flow into the country's capital stock until r was driven up to α. Think of the present example as one in which the marginal domestic product of capital is approximately constant over the small scale on which the country can invest.

$$1 + r > \eta(1 + \alpha). \tag{25}$$

This inequality—which holds for any empirically plausible values of r, η, and α—ensures that a higher debt makes default more attractive even when all additional borrowing is invested.[32]

We see how the sovereign's investment and repayment decisions depend on D_2 by solving two maximization problems, one of which assumes full repayment and the other default. The utility maxima for these problems, U^{N} and U^{D}, respectively, are compared to see whether the sovereign actually defaults.

To find U^{N}, solve the problem of maximizing U_1 subject to $K_2 = Y_1 + D_2 - C_1$ and $C_2 = (1+\alpha)K_2 - (1+r)D_2$, which, when combined, imply the intertemporal constraint

$$C_1 + \frac{C_2}{1+\alpha} = Y_1 + \frac{(\alpha - r)}{1+\alpha} D_2. \tag{26}$$

(This equation describes $\mathbf{GNP}^{\mathrm{N}}$.) Optimal consumption levels are

$$C_1 = \frac{1}{1+\beta}\left[Y_1 + \frac{(\alpha - r)}{1+\alpha}D_2\right], \qquad C_2 = \frac{(1+\alpha)\beta}{1+\beta}\left[Y_1 + \frac{(\alpha - r)}{1+\alpha}D_2\right], \tag{27}$$

implying a maximized lifetime utility of

$$U^{\mathrm{N}} = (1+\beta)\log\left\{\frac{1}{1+\beta}\left[Y_1 + \frac{(\alpha - r)}{1+\alpha}D_2\right]\right\} + \beta\log\left[(1+\alpha)\beta\right].$$

To find U^{D}, maximize lifetime utility U_1 subject to $K_2 = Y_1 + D_2 - C_1$ and $C_2 = (1-\eta)(1+\alpha)K_2$, which, when combined, imply the equation for $\mathbf{GNP}^{\mathrm{D}}$,

$$C_1 + \frac{C_2}{(1-\eta)(1+\alpha)} = Y_1 + D_2. \tag{28}$$

Optimal consumptions in this case are

$$C_1 = \frac{1}{1+\beta}(Y_1 + D_2), \qquad C_2 = \frac{(1-\eta)(1+\alpha)\beta}{1+\beta}(Y_1 + D_2), \tag{29}$$

so that

$$U^{\mathrm{D}} = (1+\beta)\log\left[\frac{1}{1+\beta}(Y_1 + D_2)\right] + \beta\log[(1-\eta)(1+\alpha)\beta].$$

Now calculate the utility difference between default and nondefault as a function of the debt-output ratio at the end of period 1, D_2/Y_1:

32. Even with $r = 0.05$, $\alpha = 1$, and $\eta = 0.5$, so that α is absurdly large relative to r and creditors are endowed with overwhelming retaliatory power, eq. (25) is still satisfied.

$$U^{\mathrm{D}} - U^{\mathrm{N}} = (1+\beta)\log\left[\frac{1+\dfrac{D_2}{Y_1}}{1+\dfrac{(\alpha-r)}{1+\alpha}\left(\dfrac{D_2}{Y_1}\right)}\right] + \beta\log(1-\eta).$$

For D_2 close to zero, this difference is close to $\beta\log(1-\eta) < 0$; but it rises as D_2/Y_1 rises. Thus a higher debt incurred on date 1 makes default a relatively more attractive strategy on date 2. The point at which the sovereign is indifferent between default and full repayment (but, in a tie, repays) occurs when $U^{\mathrm{D}} - U^{\mathrm{N}} = 0$, or, exponentiating this equality, when

$$1 = \left[\frac{1+\dfrac{D_2}{Y_1}}{1+\dfrac{(\alpha-r)}{1+\alpha}\left(\dfrac{D_2}{Y_1}\right)}\right]^{1+\beta} (1-\eta)^{\beta}.$$

Solving for D_2/Y_1, we find that the limit beyond which lenders will not extend credit is

$$\bar{D} = \left[\frac{\left(\dfrac{1}{1-\eta}\right)^{\beta/(1+\beta)} - 1}{1 - \dfrac{(\alpha-r)}{(1+\alpha)}\left(\dfrac{1}{1-\eta}\right)^{\beta/(1+\beta)}}\right] Y_1, \tag{30}$$

a positive number in view of inequality (25).[33] They will not extend credit beyond this point because they do not wish to forfeit full repayment. As you can see, making the force of sanctions greater (raising η) increases the borrowing limit, as does greater patience (higher β) and more productive domestic capital (higher α). A higher world interest rate r, by making default more attractive, lowers $\bar{D}$.

A better understanding of the debt limit comes from looking directly at the investment incentives of higher debt. Provided the sovereign is not going to default, its preferred investment level is given by eq. (27) as

$$K_2 = Y_1 + D_2 - C_1 = \frac{\beta}{1+\beta}(Y_1 + D_2) + \frac{(1+r)D_2}{(1+\beta)(1+\alpha)}.$$

Once debt is high enough that default is the preferred option, eq. (29) shows that investment is lower, at only

33. Inequality (25) holds if and only if

$$\frac{1+r}{1+\alpha} > \eta \Leftrightarrow 1 - \frac{1+r}{1+\alpha} < 1-\eta \Leftrightarrow \frac{\alpha-r}{1+\alpha} < 1-\eta.$$

Because $\beta/(1+\beta) < 1$, however, $1-\eta < (1-\eta)^{\beta/(1+\beta)}$.

$$K_2 = \frac{\beta}{1+\beta}(Y_1 + D_2).$$

Thus, in deciding investment when default is planned, the sovereign treats the initial debt as "owned" resources that need not be repaid.

Were lenders to allow the country's borrowing to rise beyond $\bar{D}$, the point at which it is indifferent between default and repayment, investment would crash discontinuously as the sovereign moved to reduce its vulnerability to the anticipated creditor sanctions. Figures 6.4a and 6.4b convey the discontinuity graphically.[34] In Figure 6.4a, debt is initially at a level where full repayment is optimal, the utility maximum is at point **A**, and the associated investment level is denoted K^{A}.[35] An increase in D_2 causes both **GNP**$^{\mathbf{D}}$ and **GNP**$^{\mathbf{N}}$ to shift upward, but the flatter **GNP**$^{\mathbf{D}}$ schedule takes the relatively larger vertical upward shift.[36] The differentially shifting curves move the economy to a position at which $U^{\mathrm{N}} = U^{\mathrm{D}}$ (the same indifference curve has tangencies at **B** and **B**′), and investment is determined at K^{B} rather than $K^{\mathrm{B}'}$ only because we've assumed repayment in case of ties. Thus the debt level associated with this second equilibrium must be the debt ceiling $\bar{D}$. A further small increase in D_2, as in Figure 6.4b, moves the optimum to point **C**, where default is preferred, and causes a sharp investment decline from K^{B} to K^{C}. (Notwithstanding these discontinuous shifts in action, higher borrowing raises the sovereign's utility level continuously.)

One further point is noteworthy: the kink in the solid **GNP**$^{\mathbf{D}}$-**GNP**$^{\mathbf{N}}$ outer envelope in Figure 6.4b has the property that if $\bar{K}$ is investment at that point, the cost of sanctions equals the gain from default, that is, $\eta[F(\bar{K}) + \bar{K}] = (1+r)\bar{D}$. Thus, at point **B**, we find the surprising result that $\eta[F(K^{\mathrm{B}}) + K^{\mathrm{B}}]$ is *strictly* greater than $(1+r)\bar{D}$: although the country really is on the verge of default, creditor sanctions appear superficially more than sufficient to discourage it. Nonetheless, an

34. As we shall discuss later, the discontinuity could be removed by sufficient uncertainty over period 2 investment productivity.

35. To avoid cluttering the diagram, we do not actually show K^{A}, which corresponds to the distance between the horizontal-axis intercept of **GNP**$^{\mathbf{D}}$ and the point on the horizontal axis vertically below point **A**. Similarly, the investment levels associated with other labeled points in Figure 6.4 are not shown explicitly but can be inferred. The investment level marked $\bar{K}$ will be brought in momentarily.

36. The equation of **GNP**$^{\mathbf{N}}$ follows from eq. (26) as

$$C_2 = -(1+\alpha)C_1 + (1+\alpha)Y_1 + (\alpha - r)D_2.$$

That of **GNP**$^{\mathbf{D}}$ follows from eq. (28) as

$$C_2 = -(1-\eta)(1+\alpha)C_1 + (1-\eta)(1+\alpha)(Y_1 + D_2).$$

Thus higher D_2 shifts the vertical intercept of **GNP**$^{\mathbf{N}}$ upward by $(\alpha - r)\Delta D$ and shifts that of **GNP**$^{\mathbf{D}}$ upward by $(1-\eta)(1+\alpha)\Delta D$. As we saw in footnote 33, however, inequality (25) implies that $\alpha - r < (1-\eta)(1+\alpha)$.

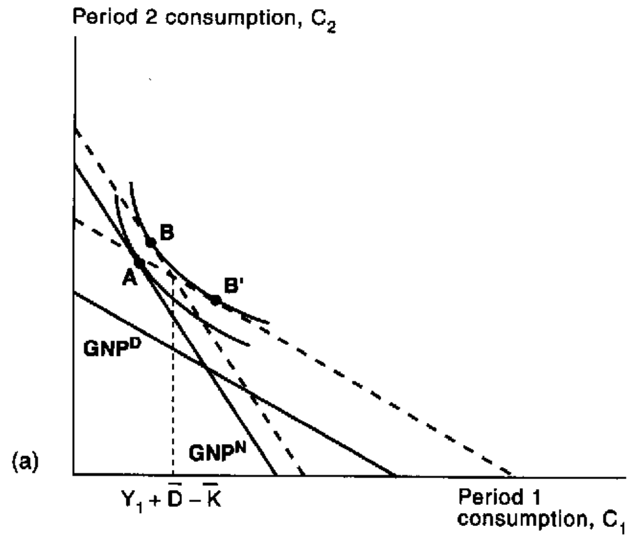


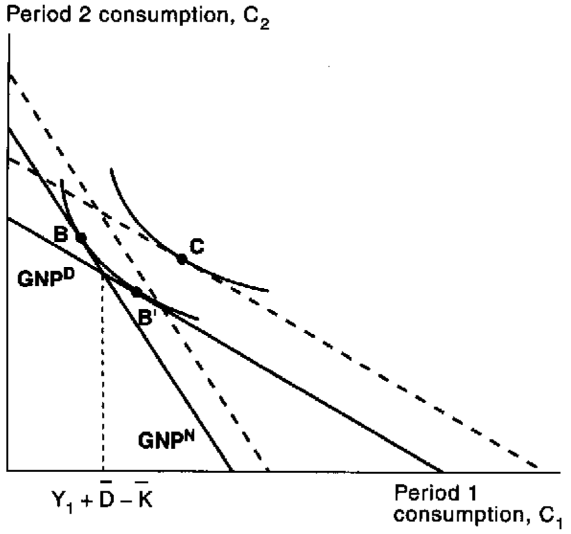
Figure 6.4
Investment effects of growing debt: (a) full repayment is optimal; (b) default is preferred

extra penny of foreign borrowing causes a default. The reason, of course, is the catastrophic investment decline that the extra penny of borrowing sets off.[37]

The discontinuity in investment with respect to debt can, as we have noted, be removed if there is sufficient uncertainty over second-period investment productivity. (Sections 6.2.3 through 6.2.5 will rely on such models.) With enough uncertainty, the country doesn't know for sure whether it will default on date 2: given date 1 borrowing D_2, the ex post repayment decision depends not only on today's investment choice K_2, but also on the realized value of domestic productivity. Even when the country would be sure of repaying under certainty, there is a chance output will turn out so low that default is preferred. Thus, other things being equal, the country reduces the prospective force of creditor sanctions by investing somewhat less than it would under certainty. Conversely, a country that would be sure to default under certainty will invest somewhat more under uncertainty to cover the possibility of

37. As the similar aspect of Figure 6.3 makes clear, this "bang-bang" behavior can occur even when production functions aren't linear.

We urge you to approach the relevant published literature on the foregoing problem with caution, as much of it is incorrect. The usual treatment argues that the sovereign's investment K_2 is a function $K(D_2)$ of debt, with $\bar{D}$ determined so as to equate the output cost of default to the gain from nonrepayment: $\eta\{F[K(\bar{D})] + K(\bar{D})\} = (1+r)\bar{D}$. You can now see the flaws in this line of argument. First, investment is not necessarily a well-defined function of debt under certainty. Second, because the condition $\eta[F(K) + K] = (1+r)D$ occurs at the kink in the GNP^D-GNP^N outer envelope, it generally *cannot* characterize any kind of optimum for the sovereign, let alone an optimum where it is indifferent between default and nondefault. (The preceding statement assumes standard preferences with strictly convex indifference curves. An exception would be the case of Leontief preferences over C_1 and C_2.)



(b)

Figure 6.4 *(continued)*

unexpectedly high ex post productivity values. In general, a small increase in D_2 is likely to have only a small negative effect on the probability of full repayment, and thus dictate only a small optimal change in current investment. Future production uncertainty therefore can make investment a single-valued continuous function of first-period borrowing. The investment effect of uncertain future productivity is illustrated in Figure 6.5, which compares the preceding model's investment response to debt under certainty (solid line) with investment under uncertainty (broken line).

6.2.1.2 Optimal Investment and Consumption Given the Debt Limit

The sovereign takes the upper borrowing limit $\bar{D}$, which we have just calculated, as a given constraint. As our previous discussion has shown, creditors set $\bar{D}$ so that for any $D_2 \leq \bar{D}$, $\min\{(1+r)D_2, \eta[F(K_2)+K_2]\} = (1+r)D_2$ when the sovereign chooses K_2 optimally after the loan has been extended. So the maximand in eq. (24) simplifies to

$$U_1 = u(Y_1 + D_2 - K_2) + \beta u[F(K_2) + K_2 - (1+r)D_2], \tag{31}$$

which the sovereign maximizes over K_2 and D_2 subject to the constraint

$$D_2 \leq \bar{D}. \tag{32}$$

If λ is the nonnegative Kuhn-Tucker multiplier on this inequality constraint and the Lagrangian is

$$\mathcal{L} = u(Y_1 + D_2 - K_2) + \beta u[F(K_2) + K_2 - (1+r)D_2] - \lambda(D_2 - \bar{D}),$$

necessary conditions for an optimum are

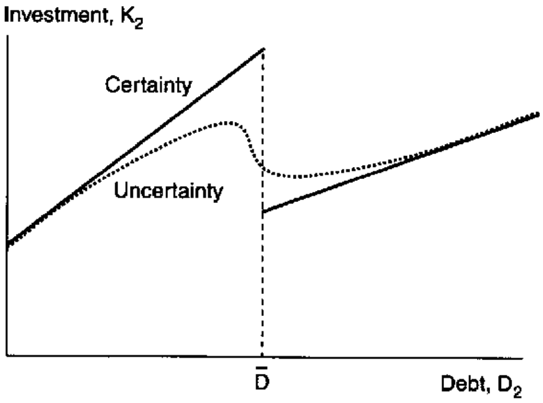


Figure 6.5
Uncertainty and the investment response to debt

$$u'(C_1) = (1+r)\beta u'(C_2) + \lambda,$$

$$u'(C_1) = [1 + F'(K_2)]\beta u'(C_2),$$

$$\lambda(\bar{D} - D_2) = 0.$$

When $\lambda = 0$, these conditions reduce to those governing the model of Chapter 1 with no default risk. This situation might occur if the sanctions η are very powerful, or if the country needs to borrow only a little to attain the Chapter 1 optimum. But when constraint (32) binds so that λ is positive, the domestic interest rate $F'(K_2)$ exceeds the world rate r, and if $\beta(1+r) = 1$, consumption is tilted upward, that is, $u'(C_1) > u'(C_2)$. The tilt reflects a domestic "shadow" rate of interest above the world rate r. Despite consumption's upward tilt, the country's inability to push investment all the way to the efficient point can result in second-period consumption being below its unconstrained level.

6.2.1.3 Precommitment in Investment

An alternative setup assumes the country can commit to an investment strategy *before* creditors lend it any money. One can think of this as a case of partial commitment: the country can commit to an investment strategy but not to repaying loans. For example, the government could prepay some of the cost of a major investment project or subscribe to an International Monetary Fund program that placed credible limits on government consumption.

If the country actually can choose K before lenders extend credit, the latter are always willing to lend any amount up to $\eta[F(K) + K]$. The borrower's problem therefore is to maximize U_1 as given by eq. (31) subject to

$$(1+r)D_2 \leq \eta[F(K_2) + K_2]. \tag{33}$$

The associated Kuhn-Tucker Lagrangian is

$$\mathcal{L} = u(Y_1 + D_2 - K_2) + \beta u[F(K_2) + K_2 - (1+r)D_2]$$
$$- \lambda \{(1+r)D_2 - \eta[F(K_2) + K_2]\}.$$

Notice the difference between the country's problem here and the one it faced with an inflexible upper bound $\bar{D}$ for D_2. Here, the country can always borrow more by committing to invest more. Before, such promises were empty, since lenders knew exactly how much the borrower would wish to invest once the loan had been disbursed.

Differentiating $\mathcal{L}$ with respect to D_2 and K_2 and invoking complementary slackness, we have

$$u'(C_1) = (1+r)[\beta u'(C_2) + \lambda], \tag{34}$$

$$u'(C_1) = [\beta u'(C_2) + \lambda\eta][1 + F'(K_2)], \tag{35}$$

$$\lambda \{\eta[F(K_2) + K_2] - (1+r)D_2\} = 0,$$

where the multiplier λ is nonnegative. Condition (34) shows that if the inequality constraint is binding (and, consequently, $\lambda > 0$), consumption will have an upward tilt when $\beta(1+r) = 1$, as in the discretionary investment model. Condition (35) shows that, contrary to the latter model, $1 + F'(K_2) < u'(C_1)/\beta u'(C_2)$, that is, the marginal gross return to investment is below the marginal rate of substitution of future for present consumption. This policy is optimal because the country expands its borrowing possibilities by $\eta[1 + F'(K_2)]$ for every additional date 1 output unit it invests. However, $F'(K_2)$ must exceed r in order for λ to be strictly positive.

Although the ability to commit investment in advance does not do the country as much good as being able to commit to repay, the ability to tie its hands even in a limited way helps it. The country must benefit, since it can always commit to the investment level that would arise under complete discretion.

6.2.1.4 Dynamic Inconsistency in Policy

The two contrasting models we have just sketched are useful vehicles for a first look at the general problem of *dynamic inconsistency* in economic policymaking. A future policy that the government finds optimal today, taking account of its influence over the actions of others, may no longer be optimal once those actions have been taken. Policymaking is subject to dynamic inconsistency when the optimal policy rule for a *given* date changes as time passes. Unlike the dynamic inconsistency problem in intertemporal consumer choice (Chapter 2), policy choice can be dynamically inconsistent with unchanging policymaker preferences: at bottom, the phenomenon is due to constraints on policy that change over time as an initially optimal plan is implemented.

The preceding two subsections illustrate the problem nicely. The policy the sovereign finds optimal when investment influences lenders' decisions (section 6.2.1.3) is different from the one it finds optimal after the loans have been made (section 6.2.1.1).

To demonstrate the point in detail, let us return to the specific case underlying Figure 6.4, in which $u(C) = \log C$ and $F(K) = \alpha K$, with $\alpha > r$. In the last subsection we derived in general terms the optimal *precommitment* investment level, call it K^{P}. This is simply the investment level the government finds it optimal to promise when it is constrained by eq. (33). Combining eqs. (34) and (35) by solving for λ, we find that

$$\frac{u'(C_1)}{\beta u'(C_2)} = \frac{C_2}{\beta C_1} = \frac{(1+r)(1+\alpha)(1-\eta)}{1+r-\eta(1+\alpha)} > 1+\alpha.$$

[Be sure to verify the asserted inequality using eq. (25).] In Figure 6.6, the precommittment consumption ratio C_2/C_1 lies along the ray **OC**.[38] The maximum loan D^{P} lenders are willing to make under precommitted investment is linked to K^{P} by the repayment constraint (33) (which we assume binds),

$$\eta(1+\alpha)K^{\mathrm{P}} = (1+r)D^{\mathrm{P}}.$$

Figure 6.6 shows the equilibrium that results, with consumption at point **P**, if the government's investment commitment is carried out.[39] What if the government cannot be held to its commitment? In that case, once credit D^{P} has been extended, eq. (33) no longer is relevant for the country. This result gives rise to dynamic inconsistency: the country can do better for itself after it has borrowed if it is not actually forced to follow its initial plan. In Figure 6.6, it chooses the lower investment level K^{D}, defaults, and consumes at point **D**, which is on a higher indifference curve than **P**.

Rational lenders understand the dynamic inconsistency of the optimal plan the government adopts before loans are made. Unless the government can somehow precommit its future actions, lenders therefore won't consider the investment level specified in the plan to be *credible*. Instead of believing that the government will implement it once loans have been made, lenders will do the calculation described in section 6.2.1.1 and offer no more than the amount $\bar{D}$ in eq. (30). The government

38. The slope of **OC** is

$$\frac{\beta(1+r)(1+\alpha)(1-\eta)}{1+r-\eta(1+\alpha)}.$$

39. How does the constraint $\eta(1+\alpha)K = (1+r)D$ prevent the government from raising without limit borrowing, investment, and (since $\alpha > r$) date 2 consumption? Because $1+r > \eta(1+\alpha)$ [inequality (25) again], $D/K < 1$ along the repayment constraint. Thus the country must cut current consumption every time it raises investment. The increasing marginal utility of current consumption as D and K rise in proportion thus places a limit on borrowing in the precommitment optimum.

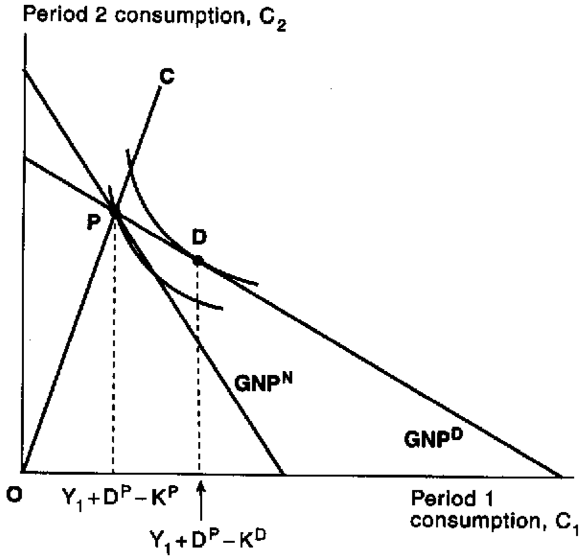


Figure 6.6
Dynamic inconsistency in investment plans

thus will have no choice but to optimize taking $\bar{D}$ as given, as in the equilibrium of section 6.2.1.1.[40]

6.2.2 Reputation and Investment

In the preceding two-period model, countries are able to borrow only if creditors can impose direct sanctions. As in the consumption insurance case of section 6.1.2, one can dispense with direct sanctions and rely on reputation arguments if the horizon is infinite. There is an important sense, however, in which it is the consumption-smoothing rather than the investment motive that underpins reputation-for-repayment models of sovereign debt. Even in the two-period case, a country with enough first-period output could self-finance the efficient investment level with no utility loss if it did not care about smoothing consumption across periods. In fact, the analysis of section 6.1.2.4, which suggested that the scope for *purely* reputation-based lending is limited, applies with even greater force to the investment case.

As a simple example of a more general problem, think of a borrowing country in a deterministic environment: it has the production function $Y = AF(K)$, where both A and the world interest rate r are constant. Once the country reaches the

40. The seminal references on dynamically inconsistent policy problems are Kydland and Prescott (1977) and Calvo (1978). A lucid survey is Persson and Tabellini (1990). Of course, a very basic example of a dynamically inconsistent policy is at the heart of the sovereign debt problem: a country promises to repay lenders but, once loans have been made, would rather not!

steady-state capital stock $\bar{K}$ at which $AF'(\bar{K}) = r$, it no longer needs the world capital market. Fear that it will lose reputation therefore will not deter repudiation of its foreign debt, which makes the country better off in every subsequent period. Lender anticipation of this eventual default leaves the country unable to borrow even when its capital stock is far below $\bar{K}$. If lenders cannot deploy direct sanctions, there will be no sovereign borrowing.[41]

6.2.3 Debt Overhang

During the 1980s many developing countries, notably in Latin America, found it hard to pay their foreign creditors. The booming growth these countries had experienced in the 1970s—growth aided in large part by low world real interest rates and ready foreign credit—came to a screeching halt as both the intertemporal terms of trade (the real interest rate) and the intratemporal terms of trade dramatically and simultaneously worsened.[42] These developments led in many cases to severe debt-servicing problems.

Many have argued that the causality between debt problems and the growth slowdown was bidirectional. That is, the huge foreign debt borrowers had run up by the early 1980s itself made a direct contribution to slower growth. The channel for this effect is that a legacy of foreign debt effectively generates a tax on investment. The following example illustrates the claim.

On date 1, the first of two periods, a country has an inherited debt of face value D that will come due on date 2. (We are not going to be concerned here with how the debt was acquired.) The country's income is Y_1 in period 1 and $AF(K_2)$ in period 2, where the productivity shock A now is a random variable with mean $\mathrm{E}(A) = 1$, distributed over $[\underline{A}, \bar{A}]$ with probability density function $\pi(A)$. It is convenient here to assume that capital depreciates by 100 percent in use. Thus, the only capital available for date 2 production is the amount the economy invested on date 1, that is, $K_2 = I_1$. Similarly, because K_2 dissipates entirely in production, $AF(K_2)$ equals the economy's total resources available for consumption or debt repayment on date 2.

To focus squarely on the problem's investment aspect, we assume the country is risk-neutral with expected utility function

$$U_1 = C_1 + \mathrm{E}(C_2)$$

(which we have simplified further by setting the subjective discount factor, β, to 1). Purely as a notational simplification, the world interest rate, r, is set to 0. This is a direct sanctions model, in which creditors penalize the country in the amount

41. If capital depreciates rapidly enough, the incentive to repudiate may be altered. Thomas and Worrall (1994) examine such a case under the assumption that capital must be provided by a foreign direct investor, for example, a multinational firm.

42. See Bulow and Rogoff (1990).

$\eta AF(K_2)$ (a random quantity from the perspective of date 1) should it default. We will assume in that case that creditors actually *gain* the same fraction η of total debtor output $AF(K_2)$. (It is simple to modify the model so that seizure of debtor goods or curtailment of its trade involves deadweight costs that drive a wedge between what the country pays and what its creditors gain.)

Eliminating consumption levels by using the constraints

$$C_1 = Y_1 - K_2, \quad C_2 = AF(K_2) - \min[\eta AF(K_2), D],$$

we write the country's utility as a function of its investment choice, K_2:

$$U_1 = U(K_2) = Y_1 - K_2 + \mathrm{E}\{AF(K_2) - \min[\eta AF(K_2), D]\}.$$

The assumption $\mathrm{E}\{A\} = 1$, which implies $\mathrm{E}\{AF(K_2)\} = F(K_2)$, converts the country's maximization problem to

$$\max_{K_2} U(K_2) = Y_1 - K_2 + F(K_2) - V(D, K_2), \tag{36}$$

where $V(D, K_2)$ is the payment creditors actually expect to receive on date 2. (This sum is the debt's *market* value.) Since the borrower will default for A realizations such that $\eta AF(K_2) < D$, that is, when $A < D/\eta F(K_2)$, we see that

$$V(D, K_2) = \eta F(K_2) \int_{\underline{A}}^{\frac{D}{\eta F(K_2)}} A\pi(A)\mathrm{d}A + D \int_{\frac{D}{\eta F(K_2)}}^{\overline{A}} \pi(A)\mathrm{d}A. \tag{37}$$

The first of the two summands on the right-hand side of eq. (37) captures payments in default states, the second, payments in nondefault states. In default states, creditors cannot collect in full but in effect can levy a "tax" equal to η percent of output. Only when A is sufficiently high are creditors fully repaid (in which event the sum they are paid is independent of output). Importantly, the probability of default is *not* exogenous: it depends on how much the country invests.

How does an increase in its inherited debt affect the country's optimal investment choice? Substituting eq. (37) into eq. (36), differentiating with respect to K_2, and equating the resulting derivative to zero, we get the first-order condition

$$F'(K_2)\left[1 - \eta \int_{\underline{A}}^{\frac{D}{\eta F(K_2)}} A\pi(A)\mathrm{d}A\right] = 1. \tag{38}$$

This condition[43] states that the debtor will invest up to a point where the expected marginal product of investment, net of expected additional penalty payments to creditors, equals the current consumption cost of investing (that is, 1). We denote the optimal investment choice by $K(D)$, and assume sufficient uncertainty that it

43. Contrary to first appearances, we have not forgotten to differentiate the integration limits in eq. (37) in deriving eq. (38). The derivative of $U(K_2)$ with respect to K_2 is

is uniquely defined. At first glance, it might appear that raising D can move investment either way: a rise in D raises the effective creditor tax on investment, and while a fall in K_2 raises $F'(K_2)$, it also widens the range of A realizations over which losses to creditors rise with total output. One can show, however, that $K'(D) < 0$ using the fact that the second-order condition for the country's maximization problem is met at an interior optimum. Thus an inherited liability to foreigners may indeed have a negative, *debt overhang effect* on the debtor's investment.[44,45]

6.2.4 The Debt Laffer Curve

Krugman (1989) and Sachs (1989) have argued that a severe enough debt overhang may enable creditors as a group to *raise* expected debt repayments $V(D, K_2)$ simply by forgiving (that is, canceling) a portion of what they are owed. Let us differentiate eq. (37) with respect to D, taking account of the dependence of K_2 on D. The total derivative is

$$\frac{\mathrm{d}V[D, K(D)]}{\mathrm{d}D} = \int_{\frac{D}{\eta F(K_2)}}^{\bar{A}} \pi(A)\mathrm{d}A + \left[\eta F'(K_2) \int_{\underline{A}}^{\frac{D}{\eta F(K_2)}} A\pi(A)\mathrm{d}A\right] K'(D). \quad (39)$$

$$U'(K_2) = -1 + F'(K_2)\left[1 - \eta \int_{\underline{A}}^{\frac{D}{\eta F(K_2)}} A\pi(A)\mathrm{d}A\right]$$
$$+ \left\{\eta F(K_2)\frac{D}{\eta F(K_2)} - D\right\} \pi \left[\frac{D}{\eta F(K_2)}\right] \frac{DF'(K_2)}{\eta F(K_2)^2}.$$

But the last term, which comes from differentiating the integration limits in $U(K_2)$, is 0. (Because the integration limits are chosen optimally, this is another example of the envelope theorem.) Equation (38) shows that a high enough value of $\lim_{K\to 0} F'(K)$ guarantees that K_2 will be chosen strictly positive.

44. Differentiating $U(K_2)$ twice with respect to K_2, we see that the second-order condition for a maximum is

$$U''(K_2) = F''(K_2)\left[1 - \eta \int_{\underline{A}}^{\frac{D}{\eta F(K_2)}} A\pi(A)\mathrm{d}A\right] + \frac{D^2 F'(K_2)^2 \pi \left[\frac{D}{\eta F(K_2)}\right]}{\eta F(K_2)^3} < 0.$$

[This inequality need not hold globally (for all K_2), but it must hold at the optimal (interior) investment level.] Implicit differentiation of eq. (38) gives

$$\frac{\mathrm{d}K_2}{\mathrm{d}D} = K'(D) = \frac{DF'(K_2)\pi \left[\frac{D}{\eta F(K_2)}\right]}{U''(K_2)\eta F(K_2)^2},$$

which is negative if $U''(K_2) < 0$. Interestingly, eq. (38) also implies that the sign of $\mathrm{d}K_2/\mathrm{d}\eta$ is ambiguous. A higher η raises the creditor "tax" on investment in default states, but also lowers the probability that a default state occurs. Second-order conditions do not rule out the possibility that $\mathrm{d}K_2/\mathrm{d}\eta > 0$; see Bulow, Rogoff, and Zhu (1994).

45. With concave utility, it is no longer true that higher debt necessarily reduces investment. (This effect can be seen in Figure 6.5.) Imagine a small country that is excluded from new international borrowing on date 1 but that nonetheless becomes liable then to pay a very small transfer to foreign creditors on date 2. For the usual consumption-smoothing reasons, the country will cut current as well as future consumption, and invest more on date 1. (Helpman, 1989, emphasizes this point.) Notice that this example is predicated on the upward tilt in the stream of expected transfer obligations to foreigners.

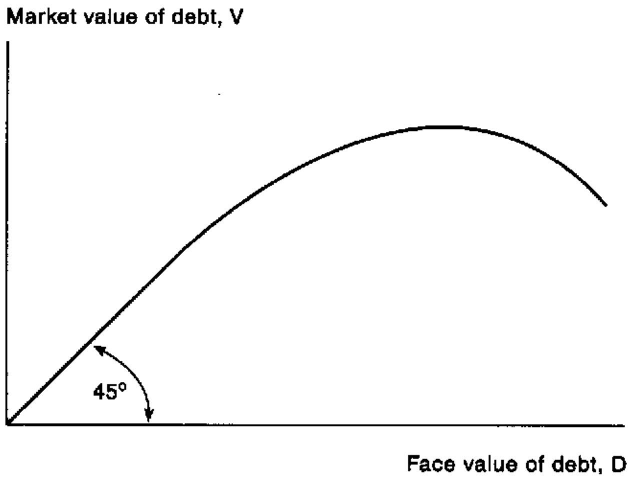


Figure 6.7
The debt Laffer curve

The first term on the right-hand side is the probability of full repayment and clearly is nonnegative. Conditional on the country repaying in full, creditors do better if the face value of its obligations is higher. The second term is negative, however, because a higher face value of debt depresses investment and thus makes default more probable.[46] In principle the second term can dominate the first for D sufficiently large. Thus, if we graph $V[D, K(D)]$ against D (as in Figure 6.7), V may be declining with D for large D, as shown. Krugman (1989) has dubbed Figure 6.7 the *debt Laffer curve*, by analogy with the usual tax Laffer curve showing how the revenue from a tax first rises and then falls as the tax is progressively raised from zero.

Because a rise in D both depresses investment and raises the chances of default, V rises less than in proportion to D (except for D small enough that full repayment is assured). The Laffer curve therefore is concave, as drawn.

If a country has so much debt that it is on the wrong side of the Laffer curve, creditors can make themselves better off as a group by unilaterally writing down the debt's face value. This result occurs because $V(D, K_2)$, the payment they expect to receive, rises. (The debtor naturally is better off as well.)

If this free lunch is readily available, why is voluntary debt forgiveness rarely observed in practice? Sachs (1989) argues that it may be difficult to *coordinate* debt forgiveness among a large group of creditors: each has an incentive to hold out for full repayment on its own claims and watch their value rise when others forgive.

46. It has been argued, more generally, that external debt discourages governments from needed but harsh economic reform efforts, since most of the short-term benefits would accrue to creditors (in the form of higher secondary-market prices for sovereign debt). This is another possible factor behind the debt Laffer curve's eventual negative slope.

The free-rider problem can be solved if a very large buyer purchases most of a country's debt and forgives some of it. The buyer would thereby internalize the externalities that prevent numerous small holders from coordinating on forgiveness. The problem with this idea, however, is the same free-rider problem that prevents coordination on forgiveness: why should any of the existing small debt holders sell, except at the higher postforgiveness price? The result is that the large buyer will not realize profits and therefore won't undertake the deal.[47]

Some observers have concluded that the inability of private debtors to negotiate deals for debt forgiveness is prima facie evidence that there is scope for Pareto-improving intervention by some public entity such as a multilateral lending agency. This is debatable. While there is some evidence that debt indeed impedes investment, the effect generally seems to be fairly weak [see, for example, Bulow and Rogoff (1990), Warner (1992), or Cohen (1993)]. And even if large debt levels do act as a tax on investment, this fact does not prove that any countries have actually been on the wrong side of the debt Laffer curve. Cohen's (1990) evidence, for example, suggests that the far side of the debt Laffer curve was not relevant for highly indebted countries even during the peak of the 1980s developing-country debt crisis. Of course, one might still argue that even if public intervention is not literally Pareto improving, the costs (to private creditors and to industrialized-country taxpayers) are still relatively small compared to the potential benefits for highly indebted developing countries. This remains an important and unresolved question.

6.2.5 Debt Buybacks

As Figure 6.8 illustrates, secondary-market prices for developing-country debt fell to deep discounts during the 1980s. These discounts inspired proposals that countries buy back their own debt on the open market at seemingly bargain-basement prices. Despite some legal obstacles, many countries did carry out such debt buybacks. It may seem obvious that a country benefits if it can effectively cancel a dollar of its debt by paying much less than one dollar. But a closer look using the model we have developed shows that the problem is harder than it appears at first glance. In truth, when buybacks are not accompanied by negotiated creditor concessions, they are likely to harm a highly indebted country while helping its creditors.

Let us write the market price of the country's debt on date 1, p, as the ratio of total expected repayments to total face value outstanding:

$$p = \frac{V(D, K_2)}{D}.$$

47. A similar free-rider problem can discourage even socially productive corporate takeover attempts, as shown in a classic paper by S. Grossman and Hart (1980).

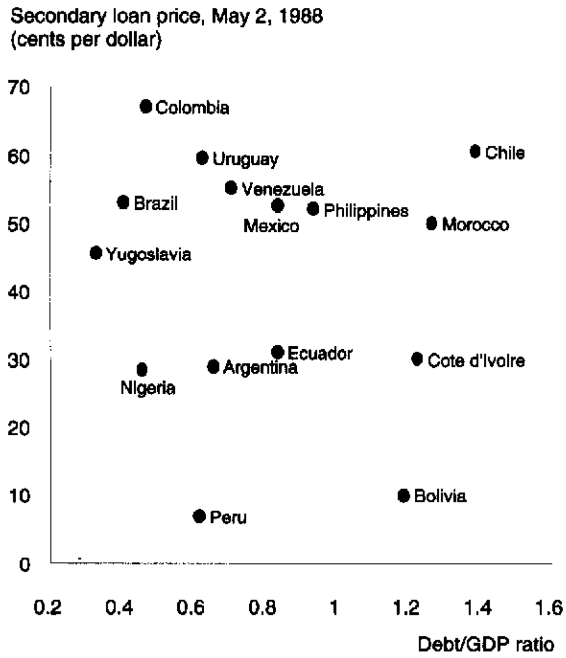


Figure 6.8
Market price of debt for 15 highly indebted developing countries

We assume that buybacks occur before investment and that buybacks are publicized before they are executed. On the assumption of rational expectations, debt owners understand that the function $K(D)$ defined in section 6.2.3 determines how the country's investment decision will be altered by the reduction in its debt's face value. This point is important because the country will have to pay the higher *post*buyback price for every unit of debt repurchased. No rational seller who knew that the price was about to jump up to a new equilibrium would sell at a lower price.

Suppose the country uses some of its first-period endowment Y_1 to buy back an amount Q of its debt on date 1 at a market price p, where p is the postbuyback price and incorporates rational expectations of the buyback's investment effect. Based on eq. (36) the country's expected utility after the buyback is

$$\begin{aligned}U_1 &= Y_1 - pQ - K_2 + F(K_2) - V(D-Q, K_2)\\ &= Y_1 - \frac{V[D-Q, K(D-Q)]}{D-Q}Q - K(D-Q) + F[K(D-Q)]\\ &\quad - V[D-Q, K(D-Q)],\end{aligned}$$

where the second line reflects the optimal dependence of investment on debt implicit in the function $K(D)$. To assess the effects of a small buyback, observe that

$$\left.\frac{\mathrm{d}U_1}{\mathrm{d}Q}\right|_{Q=0} = -\left\{F'[K(D)] - 1\right\} K'(D) - \left\{\frac{V[D, K(D)]}{D} - \frac{\mathrm{d}V[D, K(D)]}{\mathrm{d}D}\right\}. \tag{40}$$

The foregoing derivative can be split into two terms.[48] The first of these, $-\{F'[K(D)] - 1\}K'(D)$, is an unambiguous gain for the country. By eq. (38), the debt-overhang investment effect makes $F'[K(D)] > 1$; because the buyback reduces debt and spurs investment [remember that $K'(D) < 0$], it moves the economy closer to a first-best investment allocation.

However, the second term in eq. (40),

$$-\left\{\frac{V[D, K(D)]}{D} - \frac{\mathrm{d}V[D, K(D)]}{\mathrm{d}D}\right\},$$

represents a net loss for the country. This term is the difference between what the country pays to repurchase its discounted debt, which is the debt's *average* price, and the reduction in total expected future debt payments, which one can think of as the debt's *marginal* price.[49] By eq. (39), the debt's marginal price is the slope of the debt Laffer curve in Figure 6.7, and the curve's concavity implies that marginal price is below average price. The buyback is costly because the country is paying average price for marginal debt units that have a below-average effect on what the country expects to repay.[50] Notice that this loss to the country is a pure gain to creditors, who are paid the debt's average price on each unit they sell and lose only the reduction in expected country repayments, equal to the debt's marginal price.

Contrary to appearances, therefore, the buyback's effect on debtor welfare need not be positive (although creditors always gain). Only if the buyback provides an investment stimulus strong enough to overcome the effect of the gap between average and marginal debt prices will the debtor gain. But is this outcome even possible? Remember that the debt Laffer curve's bowed shape is related to the strength of the investment effect: it is precisely when the investment effect is strong

48. For arbitrary $Q > 0$ the derivative is

$$\frac{\mathrm{d}U_1}{\mathrm{d}Q} = \left[\frac{(D-Q)(V_D + V_K K') - V}{(D-Q)^2}\right] Q - \frac{V}{D-Q} + K' - F'K' + V_D + V_K K'.$$

Equation (40) is obtained by evaluating at $Q = 0$ and noting that the total derivative $\mathrm{d}V/\mathrm{d}D$ equals $V_D + V_K K'$. The proof that even large buybacks are also detrimental to debtors is fairly straightforward for the proportional seizure technology assumed here. It can, in fact, be generalized to allow for more general (possibly nonlinear) seizure technologies, under fairly mild restrictions. See Bulow and Rogoff (1991).

49. The distinction is discussed in greater detail by Bulow and Rogoff (1988b).

50. Having a little more debt outstanding raises the country's payments only in nondefault states. However, the totality of debt yields payoffs in default as well as nondefault states.

that the gap between average and marginal debt price is also high. So there is no presumption, after all, that a strong investment effect makes the debtor more likely to come out ahead.

We settle the question by using eq. (38) to eliminate $F'[K(D)] - 1$, eq. (37) to eliminate $V[D, K(D)]$, and eq. (39) to eliminate $\mathrm{d}V[D, K(D)]/\mathrm{d}D$ from eq. (40). The result is the surprisingly simple expression

$$\left.\frac{\mathrm{d}U_1}{\mathrm{d}Q}\right|_{Q=0} = -\frac{\eta F(K_2)}{D} \int_{\underline{A}}^{\frac{D}{\eta F(K_2)}} A\pi(A)\mathrm{d}A < 0.$$

The country's investment gains go entirely into increased expected payments to creditors; on balance the country therefore must *lose* when it repurchases its discounted debt.

There is a more intuitive way to see why investment stimulus cannot make a buyback helpful in this model. Because the country is continuously optimizing its investment, investment changes can have only second-order welfare effects for the country. Thus the envelope theorem implies that the change in debtor utility is approximately the same as in the case of unchanged investment.[51]

Application: Debt Buybacks in Practice

During the late 1980s and early 1990s, heavily indebted countries throughout the world, but especially in Latin America, engaged heavily in various forms of debt buybacks. The case of Bolivia provides a much-discussed example.

Like many other countries in the developing world, Bolivia accumulated large foreign debts during the years after the 1973 oil-price shock. In the early 1980s, however, facing plummeting terms of trade for its commodity exports, a sharp rise in world real interest rates, and worldwide recession, Bolivia allowed its debt to fall into arrears. By September 1986, when discussions of a buyback first began, Bolivian debt traded on world secondary markets at a mere 6 cents on the dollar. Using money largely contributed by foreign donors (including the Netherlands, Spain, and Brazil), Bolivia spent \$34 million in March 1988 to repurchase debt with a face value of \$308 million—nearly half of the country's \$670 million privately held debt. After the buyback, the country's remaining debt was priced at 11 cents on the dollar, a fact many contemporary observers interpreted as evidence that the buyback had sharply improved Bolivia's economic prospects.

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51. For further analysis, including cases of buybacks beneficial to the debtor, see Bulow and Rogoff (1991) and the following application. Alternative discussions include Detragiache (1994) and Diwan and Rodrik (1992).

Table 6.2
Bolivia's March 1988 Debt Buyback

	Prebuyback	Postbuyback
Face value of debt, D	\$670 million	\$362 million
Price, p (fraction of a dollar)	0.06	0.11
Total market value, $p \times D$	\$40.2 million	\$39.8 million

Table 6.2 suggests a very different interpretation, however. Bolivia spent \$34 million = \$308 million $\times$ 0.11 on debt reduction, the product of the face value of repurchased debt and the debt's postbuyback secondary-market price. As a result, the total *market* value of Bolivia's debt (expected repayments to creditors in our model) fell only \$40.2 million $-$ \$39.8 million = \$400, 000. Bolivia thus recouped less than 1.2 percent of the money it spent. Why did the country gain so little? Our model suggests the answer.

With an average debt price of only 6 cents on the dollar, the marginal value of Bolivian debt was nearly zero. The repurchase nearly doubled p mainly because the face value of debt outstanding, D, fell by just under a half without significantly affecting the country's expected future trade balance surpluses. It is possible, of course, that other factors contributed to the rise in Bolivia's debt price, but related evidence strongly suggests that this was not the case; see Bulow and Rogoff (1988b). For example, over the same period where Bolivian debt rose sharply in price, secondary-market prices for the debts of all other heavily indebted countries fell by a weighted average of 30 percent.

Bolivia did not bear the cost of the buyback, but the donors who contributed the bulk of the funds used presumably had no intention that the main beneficiaries of their largesse be American, British, and Japanese banks. Of course, one can equate creditors' gain with Bolivia's loss only when the transaction does not produce pure efficiency gains. If debt reduction ameliorates debt overhang, for example, then a buyback might no longer be a zero-sum game. Our theoretical analysis has shown that efficiency considerations are unlikely to reverse the overall conclusion that the cost of a straight buyback exceeds the benefit. At a more pragmatic level, the efficiency gains from a buyback come mainly from a higher probability of full repayment, and since Bolivian debt traded at only 11 cents after the repurchase, it is hard to imagine that any efficiency gains were large.[52]

52. Table 6.2 ignores official debts (such as money owed to the IMF and the World Bank), which nominally are senior to private debts. If some of the funds used to pay off private creditors might have ended up instead being used to repay official debts, then the benefit to Bolivia of the buyback is higher. Bulow and Rogoff (1988b) and Bulow, Rogoff, and Bevilaqua (1992) argue that in practice private debt is, if anything, de facto senior to official debt, so that the calculations in Table 6.2 would not be affected by incorporating official debt into the analysis.

The basic problem illustrated here extends to many other popular buyback schemes. These include debt-for-equity swaps, in which shares in debtor-country firms are used to repurchase debt, and "debt-for-nature" swaps, in which contributions from "green" organizations finance the buyback in return for the country's promise to preserve endangered natural habitats. Not all debt-reduction schemes are necessarily inefficient for the debtor, however. As part of an overall debt reduction deal with creditors, countries can sometimes negotiate repurchase prices much closer to the marginal rather than average value of debt. The key to such plans is usually an agreement by all creditors that those who hold on to their debt must make concessions (say, agree to a lower interest rate). Such concessions push down the postbuyback price of the debt and, therefore, lower the price at which creditors are willing to sell. Creditors as a group may agree to such buybacks if their best alternative option is the status quo. Mexico's early 1990 debt reduction under the "Brady plan" (named for former U.S. Treasury secretary Nicholas Brady) is a good example of a negotiated repurchase. Subsequent calculations generally suggest that the leakage of donated funds to creditors was much smaller than it would have been under a straight buyback.[53]

Postscript: In May 1993, Bolivia conducted another large buyback, although this time at a negotiated rather than market-determined price. Total principal extinguished was $170 million, at a price of 16 cents on the dollar. This buyback followed a concessional refinancing in March 1993 that covered roughly $500 million including arrears, in which Bolivia was granted a 67 percent reduction in its stock of officially held debt. As of this writing, the country has not yet been able to return to private capital markets. ■

*6.3 Risk Sharing with Hidden Information

In the risk-sharing models we have analyzed so far, we have assumed that the events upon which payments are conditioned (explicitly or implicitly) are observable by the country, its creditors, and all potential lenders. This section briefly considers how risk sharing is compromised when key contingencies upon which agents would like to contract are private information, directly observed by only one of the parties. Here the focus is not on willful default, as in the case of sovereign debt; rather, it is on the adverse incentives caused by informational asymmetries.

53. For analyses of Mexico's 1990 Brady plan debt reduction, see Bulow and Rogoff (1991) and van Wijnbergen (1991).

In the case we examine, a country's output cannot be observed perfectly, perhaps because of incomplete, inaccurate, or falsified data on its economy.[54] The country would like to share output risk with other countries through Arrow-Debreu contracts, making insurance payments when output is above the world average and receiving them when it is below. But if other countries cannot check the country's reports about its own output, the country has an irresistible incentive to misrepresent its output as being lower than it really is, so as to receive insurance payments to which it is not entitled.

Potential trading partners understand this incentive, of course, and the nature of the contracts they will sign therefore is limited. Is there any scope at all for trade in state-contingent assets in this situation? Perhaps surprisingly there may be, although the conditions supporting trade are fragile.

6.3.1 The Model

The world economy produces and consumes on two dates and consists of a continuum of very small countries, indexed by [0, 1], that receive exogenous output endowments each period. On the first date, date 1, half the countries receive low output, $\underline{Y}$, and half receive high output, $\overline{Y}$, where average output is denoted by

$$Y = \frac{\underline{Y} + \overline{Y}}{2}.$$

On the second date, date 2, everyone receives the same output, equal to average date 1 output, Y. So only date 1 output is risky. The 50 percent of countries that receive low date 1 output are chosen randomly and independently—for example, through a simultaneous toss of fair coins for every country in [0, 1].

Let us imagine that, *prior* to date 1, countries can sign contracts to diversify the risk of their date 1 output.[55] Each country knows it will have low (high) output with probability $\frac{1}{2}$ and wishes to maximize expected utility

$$\mathrm{E}U_1 = \mathrm{E}\{\log(C_1) + \log(C_2)\},$$

where the discount factor β has again been set to 1 for simplicity. It is easy to see the Arrow-Debreu equilibrium for this simple world economy. Since there is no *world* output risk on date 1, actuarially fair full consumption insurance will be available. Each country agrees to deliver the amount $\overline{Y} - Y$ to insurers on date 1 in the event its output is high, and to receive $Y - \underline{Y}$ if its output is low.

54. While we assume unobservability, the results for the two-period model of this section really only require that there is no way for a third party (say, a court of law) to *verify* the private information upon which countries would like to write contracts. In a multiperiod setting, this would not necessarily be the case.

55. You can think of this prior date as date 0 and view the model as describing a three-period economy in which only date 1 endowments are uncertain and in which consumption occurs only on dates 1 and 2.

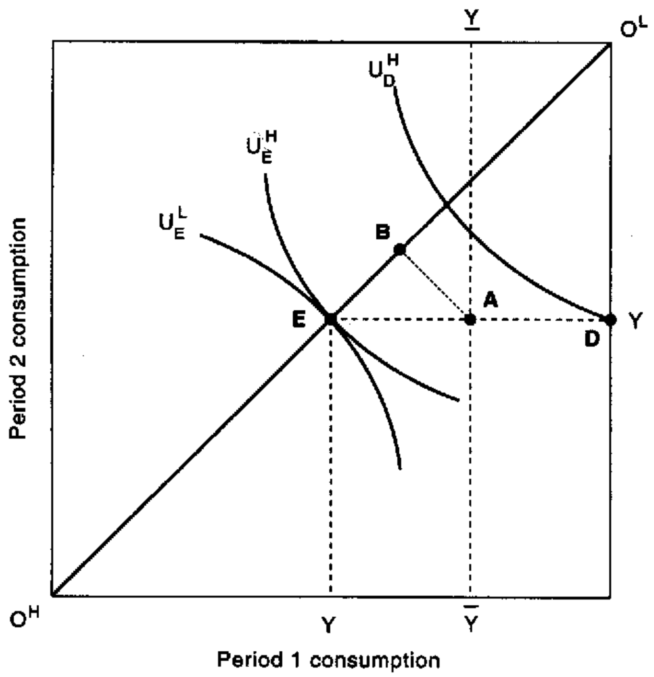

Figure 6.9
Efficient and borrowing solutions

In this way each country can be sure of consuming $C_1 = Y$ with certainty: all consumption risk can be diversified. Since everyone's consumption is also constant over time and $\beta = 1$, the equilibrium world interest rate r is zero.[56] Indeed the constancy over time of total world output implies that the world interest rate is zero in *any* equilibrium allowing riskless borrowing and lending, as you can see by aggregating individual countries' consumption Euler equations. We shall refer to this fact again.

Figure 6.9 is an Edgeworth box diagram showing intertemporal endowments and consumption allocations for high date 1 output (type H) and low date 1 output (type L) countries. (We measure type H quantities starting from the southwest corner of the box and type L quantities starting from the northeast corner. Each side's length equals $2Y$, the sum of the two types' incomes.) Point **A** is the autarky point and **E** represents the Arrow-Debreu equilibrium, where both types achieve equal consumption and utility levels, U_E^H and U_E^L ex post.

56. Let $p(\underline{Y}^j)$ be the price of a unit of date 1 output contingent on country j's output being low, and $p(\overline{Y}^j)$ the price contingent on country j's output being high. Since $p(\underline{Y}^j) = p(\overline{Y}^j) = \frac{1}{2}$ (there is no global uncertainty), it is optimal for the country to choose equal consumption levels across possible date 1 states.

If outputs cannot be observed, then simple Arrow-Debreu contracts will not be traded and markets will not be able to reach point **E**. Consider the situation of a type H country on date 1 given the efficient Arrow-Debreu contingent consumption trades just described. Since its output cannot be observed, it will claim it is a low-output type, hoping to receive $Y - \underline{Y}$ rather than paying $\overline{Y} - Y$. This ploy, if successful, would place the type H country's consumption at point **D** and raise its utility to U_{D}^{H}. But potential insurers, aware of the incentive to claim falsely that output is low, will refuse to promise insurance in the first place.

International debt contracts are noncontingent here and thus do not require revelation of hidden information. If we assume these are enforceable, the world economy will reach consumption point **B** in Figure 6.9 through borrowing and lending. Point **B** is on the diagonal contract curve, along which both types' ex post marginal rates of intertemporal substitution are equal. Why does trade in riskless bonds produce the ex post efficient allocation described by **B**?

At **B** a type H country consumes

$$C_1^H = C_2^H = \tfrac{1}{2}(\overline{Y} + Y)$$

by lending $\frac{1}{2}(\overline{Y} - Y)$ to foreigners on date 1 and receiving a repayment of the same amount on date 2. Similarly, a type L country consumes

$$C_1^L = C_2^L = \tfrac{1}{2}(\underline{Y} + Y)$$

by borrowing exactly what a type H country wishes to lend on date t,

$$\tfrac{1}{2}(\overline{Y} - Y) = \tfrac{1}{2}(Y - \underline{Y}),$$

and repaying that amount on date 2. It is simple to check that $r = 0$ is indeed the equilibrium interest rate.

Trade in bonds thus brings the world economy to the diagonal contract curve in Figure 6.9. The intertemporal marginal rates of substitution of country types H and L are equalized at **B**, so the bond-intermediated allocation welfare-dominates **A** ex post. Using the concavity of utility, you can easily show that allocation **B** also guarantees higher *expected* utility to a country that does not know ex ante what type it will be. Because the two types end up with different lifetime consumption levels at **B**, however, consumption uncertainty is not eliminated as in the Arrow-Debreu allocation. Trade in bonds cannot replicate the allocation that trade in Arrow-Debreu securities achieves.

6.3.2 Incentive-Compatible Risk Sharing

An interesting question is whether there exist feasible state-contingent contracts that bring the world economy to allocations better than point **B**. The answer is a qualified yes. These contracts are structured in such a way that, even though date 1 endowments are unobservable, no country can gain through deception.

The basic idea is simple.[57] We have seen that a type H certainly lies when covered by full Arrow-Debreu insurance: lying brings a positive payment on date 1 without any cost on date 2, moving consumption from point **A** to point **D** in Figure 6.9. Suppose, however, that contracts could be structured so that any country reporting low output on date 1 were penalized by having to make a payment on date 2. (We assume away problems of enforcing that payment.) If the gain from reporting low output were made small enough and the subsequent penalty large enough, a type H country might be deterred from pretending to be of type L.

Formally, a feasible incentive-compatible contract satisfies market-clearing and incentive-compatibility or "truth-telling" constraints that deter each type of country from posing as the other. Suppose that a country reporting high output on date 1 makes payments P_1 on date 1 and P_2 on date 2 while a country reporting low output on date 1 pays $-P_1$ and $-P_2$ on dates 1 and 2, respectively. (Payments can be negative.) Clearly such a contract satisfies resource constraints and is incentive-compatible if it induces both types to report honestly. Expected utility is the average utility of types H and L,

$$\mathrm{E}U_1 = \tfrac{1}{2}[\log(\overline{Y} - P_1) + \log(Y - P_2)] + \tfrac{1}{2}[\log(\underline{Y} + P_1) + \log(Y + P_2)].$$

The constraint that a type H does not gain from posing as a type L is

$$\log(\overline{Y} - P_1) + \log(Y - P_2) \geq \log(\overline{Y} + P_1) + \log(Y + P_2), \tag{41}$$

and the constraint that a type L does not gain from posing as a type H is

$$\log(\underline{Y} + P_1) + \log(Y + P_2) \geq \log(\underline{Y} - P_1) + \log(Y - P_2). \tag{42}$$

Notice that both constraints cannot simultaneously hold with equality: this would imply $\overline{Y} = \underline{Y}$, a contradiction.

It is easy to give examples of incentive-compatible contracts that produce allocations Pareto-superior to **B**. Consider, for example, setting

$$P_1 = \frac{\overline{Y}}{\overline{Y} + Y}(\overline{Y} - Y), \qquad P_2 = \frac{-Y}{\overline{Y} + Y}(\overline{Y} - Y), \tag{43}$$

so that a country of type H makes a positive payment in period 1 but receives a transfer in period 2. As the reader can confirm, the preceding payment schedule implies that in both periods, a type H consumes

57. The initial reference is Green (1987). Related papers include Thomas and Worrall (1990), Green and Oh (1991), and Taub (1990). Atkeson and Lucas (1992) and Lucas (1992) consider insurance for unobservable preference shocks, as well as implications for the distribution of wealth.

$$\left(\frac{\overline{Y}}{\overline{Y}+Y}\right)2Y,$$

while a type L consumes

$$\left(\frac{Y}{\overline{Y}+Y}\right)2Y.$$

This allocation is denoted by point **C** in Figure 6.10. Because $P_1 > (\overline{Y} - Y)/2$ and $-P_2 < (\overline{Y} - Y)/2$, the proposed payment scheme involves a net present-value transfer to type L. Thus point **C** is closer to **E** than **B** is and yields higher expected utility than **B**.[58] You can verify algebraically that eq. (41) holds with equality at **C** whereas eq. (42) holds as a strict inequality: the operative constraint here is to prevent the high-income country from posing as poor, not the reverse. In Figure 6.10, a false claim of poverty would place type H at point **C**′, which is on the same utility contour U_{C}^{H} as the truthful allocation **C**. (When indifferent, agents tell the truth.)

Allocation **C** is not the *best* that can be done through a truth-telling mechanism. The optimal incentive-compatible allocation lies to the northwest of **C** in Figure 6.10. Since the derivation of the optimal contract is not particularly edifying, we leave it as an exercise. We note, however, that the optimal incentive-compatible contract does *not* lie on the contract curve.[59]

This last observation suggests it may be hard to make the preceding ideas work in a market setting. Our discussion has implicitly assumed that the incentive-compatible contract is the only financial commitment agents can make. But what happens if countries can borrow and lend freely after they announce their type and receive the endowment specified in the contract? Return to the contract that led to allocation **C**. If a type H announces it is poor, receiving the endowment **C**′ in Figure 6.10 thanks to the contract, it can then smooth its consumption by lending until it reaches point **C**″ on utility contour $U_{\mathrm{C}''}^{H} > U_{\mathrm{C}}^{H}$. Thus type H countries will no longer tell the truth.

The contract penalizes lying through the "punishment" of an uneven intertemporal consumption path, but that punishment is empty if transgressors can always turn to the international bond market to smooth out their consumption. In this case, the best the market can do is indeed the borrowing-lending allocation, point **B**. The

58. The closer we get to **E** along the contract curve, the smaller is the ex post consumption difference between the two types on both dates. This unambiguous reduction in consumption variability implies a higher expected utility level ex ante. (We know **C** is on the contract curve in Figure 6.10 because $\overline{Y} - P_1 = Y - P_2$ and $\underline{Y} + P_1 = Y + P_2$.)

59. It may seem restrictive to limit the search for an optimal contract to ones in which each country truthfully reveals its type. The revelation principle of Myerson (1979) and Harris and Townsend (1981) assures us that we cannot do better by allowing for nontruthful revelation.

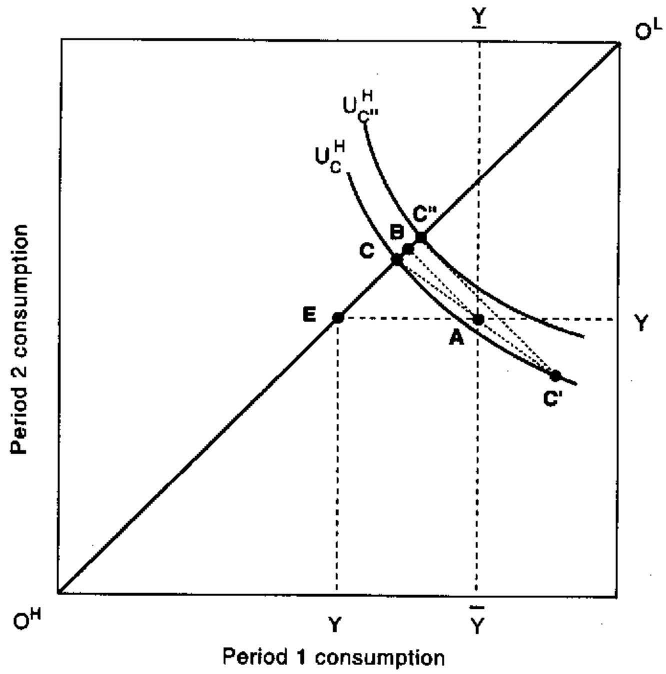

Figure 6.10
Incentive-compatible risk sharing

model thus provides some justification for the prevalence of bond-intermediated lending that we assumed in the stochastic models of Chapter 2.

Limited risk-sharing through incentive contracts is possible only if insurers can monitor and control countries' *other* financial trades, perhaps using some sort of credit-rationing scheme. But as we saw in section 6.1.2.4, these things are extremely hard to do. Indeed, the point here is quite analogous to the one we made in connection with reputation in the sovereign-debt context. The power of an explicit or implicit penalty to support financial trades is crucially dependent on the inability of a country to enter into financial agreements with others that would undo the penalty's effects.

6.4 Moral Hazard in International Lending

In the last section we saw how international risk sharing is restricted when the output risks that countries wish to diversify are not directly observable. Even when final outputs are observable and verifiable, and even if the terms of international contracts are enforceable by a supranational legal body, hidden information about borrower investment or work effort may still limit the scope for intertemporal trade and risk sharing. We illustrate these points using a model in which a firm's foreign creditors cannot observe how it allocates borrowing between investment and disguised consumption.[60]

6.4.1 Moral Hazard in Investment

A small country facing the world interest rate r is populated by a large number of two-period-lived entrepreneurs who invest on date 1 and consume only on date 2. To abstract, for now, from consumption insurance aspects, we assume the representative individual has the linear utility function

$$U_1 = U(C_1, C_2) = C_2.$$

On date 1 an individual entrepreneur receives an exogenous endowment Y_1 that can be converted into date 2 income either by investing abroad at the riskless rate of return r or by investing at home in a risky "family firm." Home investment at level I yields a random output Y_2 distributed as follows:

$$Y_2 = \begin{cases} Z & \text{with probability } \pi(I) \\ 0 & \text{with probability } 1 - \pi(I). \end{cases}$$

We assume that $\pi'(I) > 0$, $\pi''(I) < 0$, $\pi(0) = 0$, and $\pi'(0)Z > 1 + r$.[61] As more traditional production functions imply, higher investment raises expected output at a decreasing marginal rate. Firms' outputs are mutually independent.

The efficient (full information) investment level $\bar{I}$ occurs at the maximum of expected profits

$$-I + \frac{\pi(I)Z}{1+r},$$

that is, where the expected marginal product of investment equals the gross world interest rate:

$$\pi'(\bar{I})Z = 1 + r. \tag{44}$$

We shall assume that

$$\bar{I} > Y_1,$$

so that domestic entrepreneurs cannot finance their optimal investment levels without foreign loans.

In our earlier models of intertemporal trade there was no need to look separately at gross borrowing and lending levels. That is not the case here. Let L denote *gross*

60. The model is based on Gertler and Rogoff (1990), which relaxes some of the simplifying assumptions we will make here.

61. The last assumption implies that a positive investment level is efficient under symmetric information.

foreign lending by the small country on date 1 and D its *gross* foreign borrowing. A domestic individual's date 1 finance constraint is

$$I + L = Y_1 + D, \tag{45}$$

where

$$L \geq 0, \qquad D \geq 0.$$

We are allowing for gross foreign lending to capture the idea that domestic entrepreneurs may covertly invest borrowed funds abroad rather than at home, a round-tripping strategy reminiscent of "capital flight." (Similar results obtain if we introduce the possibility of secret first-period consumption.)

Foreign lenders are risk-neutral and competitive, so they will earn the expected return r on any loan to an entrepreneur. If borrowers could *commit* to investing $\bar{I}$, they would borrow $D = \bar{I} - Y_1$ and choose $L = 0$ (since the expected return to domestic investment is higher than r until $\bar{I}$ is reached).[62] However, no repayment would be possible in the bad (zero-output) state of nature. Thus promised payments on securities issued against family-firm output have to be of the state-contingent form $P(Y_2)$, with $P(0) = 0$ and $P(Z)$ determined by the lenders' zero-profit condition, in this case

$$\pi(\bar{I})P(Z) = (1 + r)(\bar{I} - Y_1).$$

This is the first-best borrowing contract.

Under asymmetric information, however, the borrower might be unable to commit credibly to an investment of $\bar{I}$. Suppose the information structure is as follows: lenders directly observe the borrower's first-period endowment Y_1, gross borrowing D, and second-period output Y_2, but not first-period investment I or the gross foreign assets L that the borrower may secretly accumulate on date 1.[63] Moreover, the borrower doesn't choose I and L until after lenders set the amount and terms of the loan, D and $P(Y_2)$. The timing assumption alone wouldn't be a problem if repayments could be indexed to I, but as lenders can't observe investment, the best they can do is index to Y_2. This *is* problematic because if an entrepreneur's investment goes sour, creditors have no way to prove if he has failed to act in good faith.

Let's look at the borrower's problem in this private information setting. The borrower maximizes

62. In this model, there would be no point in borrowing more than $\bar{I} - Y_1$ at rate r, investing $\bar{I}$, and sending the balance of what had been borrowed back abroad to earn r. Thus we can safely assume $L = 0$ under full information.

63. The assumption $L \geq 0$ rules out secret borrowing from abroad. An alternative model would have lenders unable to observe final output, in the spirit of the last section's model. Greenwood and Williamson (1989) consider a model of that type in which there is *costly state verification*; that is, lenders can observe the output realization at some cost.

$$\begin{aligned}\mathrm{E}C_2 &= \pi(I)[Z - P(Z)] - [1 - \pi(I)]P(0) + (1+r)L \\ &= \pi(I)[Z - P(Z)] - [1 - \pi(I)]P(0) + (1+r)(Y_1 + D - I),\end{aligned} \tag{46}$$

where eq. (45) was used to eliminate L. For the borrower the contract terms $[P(Z), P(0), D]$ are given. The first-order condition for a maximum is

$$\pi'(I)\{Z - [P(Z) - P(0)]\} = 1 + r. \tag{47}$$

Now recall the features of the investment contract under commitment, which sets $P(Z) = (1+r)(\bar{I} - Y_1)/\pi(\bar{I})$ and $P(0) = 0$. Comparing eq. (47) with eq. (44), we see that the borrower will actually choose an investment level $I < \bar{I}$ if he can take up the first-best contract despite freedom to choose investment ex post. Because the lender agrees to share in the risk of a bad outcome whenever $P(Z) > P(0)$, the borrower has less incentive to invest in a good outcome; he would rather secretly lend some money abroad and earn the sure return r, which, when $I = \bar{I}$, exceeds the net return given by the left-hand side of eq. (47) [see eq. (44)]. This moral hazard problem, which is reminiscent of the sovereign debt overhang problem we studied earlier, implies that if lenders offer the full information first-best contract, they will earn an expected rate of return strictly below r. Understanding the borrower's incentives, they therefore will not offer that contract.

There is, however, an optimal incentive-compatible contract, one that earns lenders r *given* the borrower's proclivity to underinvest in a successful outcome. The contract provisions $[P(Z), P(0), D]$ maximize eq. (46) subject to the lender's zero-profit condition,

$$\pi(I)P(Z) + [1 - \pi(I)]P(0) = (1+r)D, \tag{48}$$

given that investment is determined by eq. (47), which we interpret as an incentive-compatibility constraint. We shall also assume that $P(0) = 0$. The only way for the borrower to pay a positive amount when $Y_2 = 0$ is by drawing on his own assets abroad; but since these are unobservable by lenders, the borrower could always feign bankruptcy.[64] Thus the incentive-compatibility constraint (47) reduces to

$$P(Z) = Z - \frac{1+r}{\pi'(I)},$$

which is graphed as the downward-sloping curve **IC** in Figure 6.11. The curve has a negative slope because a reduction in the amount $P(Z)$ that lenders appropriate in the event of success stimulates the borrower to invest more. Notice that **IC** intersects the horizontal axis at $\bar{I}$, because only when the repayment in the good

64. Alternatively, the borrower could ensure bankruptcy in the bad state by raising C_1 if we relaxed the assumption that first-period consumption yields no utility.

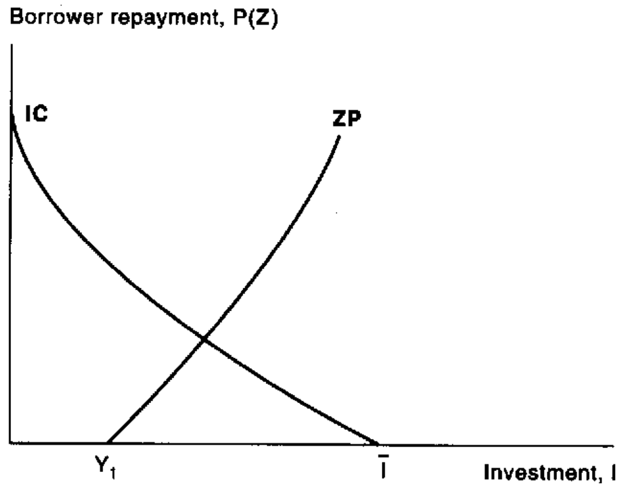

Figure 6.11
Small-country equilibrium

state equals that in the bad state, $P(Z) = P(0) = 0$, is investment not distorted [consult eq. (47)].

It is straightforward to show that the optimal contract induces the borrower to choose $L = 0$. If the contract led to $L > 0$ instead, $P(Z)$ would have to be higher for lenders to break even, which would reduce the incentive to invest.[65]

65. The Lagrangian for the borrower's problem is

$$\begin{aligned}\mathcal{L} &= \pi(I)[Z - P(Z)] + (1+r)(Y_1 + D - I) \\ &\quad + \psi[\pi(I)P(Z) - (1+r)D] + \mu\{\pi'(I)[Z - P(Z)] - (1+r)\} \\ &\quad - \lambda(I - Y_1 - D).\end{aligned}$$

Here, ψ is the Lagrange multiplier on the zero-profit constraint, μ that on the incentive-compatibility constraint $\pi'(I)[Z - P(Z)] - (1+r) = 0$, and λ that on the constraint $L = Y_1 + D - I \geq 0$. The Kuhn-Tucker necessary conditions can be written [after using the incentive-compatibility condition to eliminate $Z - P(Z)$] as

$$\psi\pi'(I)P(Z) + \mu[(1+r)\pi''(I)/\pi'(I)] - \lambda = 0, \quad \text{(i)}$$

$$(1+r)(1-\psi) + \lambda = 0, \quad \text{(ii)}$$

$$(\psi - 1)\pi(I) - \mu\pi'(I) = 0, \quad \text{(iii)}$$

$$\lambda(Y_1 + D - I) = 0, \quad \text{(iv)}$$

where (i)–(iii) are the first-order conditions for I, D, and $P(Z)$, and (iv) is a complementary slackness condition. We wish to show that $\lambda > 0$, which, by (iv), implies $L = 0$.

Proof: We reason by contradiction. If a positive amount D is borrowed, then, given $P(0) = 0$, $P(Z)$ must be positive for eq. (48) to hold. (If $D = 0$, all output would be invested domestically, leaving $L = 0$.) Suppose $\lambda = 0$. Then (ii) implies $\psi = 1$. Because $P(Z)$ and $\pi'(I)$ are strictly positive, (i), $\psi = 1$, and $\pi''(I) < 0$ imply $\mu > 0$. But this means that (iii) can't be satisfied unless $\pi'(I) = 0$, which is ruled out. We see that the assumption $\lambda = 0$ leads to a contradiction.

(Notice that even though the borrower *chooses* to place no covert funds abroad when offered the optimal incentive-compatible contract, it is precisely his option of doing so that constrains borrowing.) Using eq. (45) to eliminate D from eq. (48) and remembering that $P(0) = 0$ and $L = 0$, we rewrite the zero-profit condition for lenders as

$$P(Z) = \frac{(1+r)(I - Y_1)}{\pi(I)}. \tag{49}$$

This equation defines the upward-sloping **ZP** locus in Figure 6.11. Since Y_1 is fixed, a rise in I implies a rise in borrowing which means $P(Z)$ must go up.[66] This second locus intersects the horizontal axis at Y_1.

Figure 6.11 shows that equilibrium investment is strictly below $\bar{I}$. It also allows us to do some comparative statics exercises. A rise in first-period income Y_1, for example, shifts **ZP** to the right: any given $P(Z)$ is consistent with a higher I. Thus a rise in Y_1 lowers $P(Z)$ and raises investment. Investment clearly rises by less than Y_1, however, and because $L = Y_1 + D - I = 0$, capital inflows D decline. A rise in Z shifts **IC** upward, raising $P(Z)$, investment, and borrowing. A rise in the world interest rate shifts both curves leftward, lowering investment.

Because the number of firms is large and their date 2 output realizations independent, per capita aggregate output on date 2 equals $\pi(I)Z$, and, like investment, is lower than under full information. But countries with higher date 1 wealth, Y_1, enjoy higher investment and higher date 2 output.

One interesting implication of the model is that even when international capital markets are perfectly integrated with *riskless* rates of return equal across countries, expected marginal products of capital exceed riskless interest rates and depend on country characteristics. The expected (gross) marginal product of capital is

$$\pi'(I)Z > 1 + r.$$

Since I is an increasing function of the initial endowment Y_1, the model predicts that initially richer countries will have higher investment and lower gaps between the expected marginal products of capital and the risk-free rate, other things being equal.

We have worked so far with a representative entrepreneur, but an interesting question concerns the effect of inequality in initial endowments on aggregate in-

66. The slope of **ZP** is

$$\left.\frac{\mathrm{d}P(Z)}{\mathrm{d}I}\right|_{\mathbf{ZP}} = (1+r)\frac{\pi(I) - (I - Y_1)\pi'(I)}{\pi(I)^2}.$$

But since $\pi(I)$ is strictly concave with $\pi(0) = 0$, $\pi(I)/I > \pi'(I)$ for any I. Furthermore, because $I > Y_1$, $\pi(I)/(I - Y_1) > \pi(I)/I > \pi'(I)$, implying $\pi(I) > (I - Y_1)\pi'(I)$. Thus the numerator in the slope is positive. Investing an extra dollar of borrowing cannot raise the probability of a good outcome enough to warrant a lower repayment in the event of a good outcome.

vestment and date 2 output. Under plausible conditions capital-market imperfections make I a strictly concave function of Y_1, in which case greater wealth inequality within a country lowers average per capita investment and, with it, date 2 output.

6.4.2 A Two-Country Model

A general-equilibrium version of the model confirms that richer countries will tend to have lower expected marginal returns to capital. It also yields some surprising predictions concerning the possible direction of international capital flows between rich and poor countries.

Two countries, Home and Foreign, have equal populations. A fraction s of each population comprises savers, the rest being entrepreneurs. Savers do not have access to investment projects and can save only by acquiring the securities entrepreneurs issue. By diversifying across a large number of independent firms, savers can assure themselves a riskless (gross) return of $1 + r$, which will now be endogenously determined. Home savers and Home entrepreneurs both have a date 1 endowment y_1; both types of Foreign agent have an endowment y_1^*. (We switch to lower-case quantity variables here because populations within each country are heterogeneous. As a result, per capita quantity variables are no longer interchangeable with aggregates.) Preferences and technologies are the same as in the small-country case and identical across countries, with productivity outcomes statistically independent between as well as within countries. We assume that Home is the richer country, so that $y_1 > y_1^*$. (What really matters in determining the global allocation of investment is that Home entrepreneurs have higher wealth than Foreign entrepreneurs.)

As before, everyone has the utility function $U(c_1, c_2) = c_2$, so all of the world's first-period output is invested in equilibrium.

Absent informational asymmetries, investment levels in Home and Foreign would be governed by

$$\pi'(I)Z = 1 + r, \qquad \pi'(I^*)Z = 1 + r,$$

where $Z = Z^*$ because both countries' technologies are the same. Under full information we therefore would have $\bar{I} = \bar{I}^*$, with the world interest rate equal to the common expected marginal product of capital,

$$\pi'(\bar{I})Z = \pi'(\bar{I}^*)Z = \pi'\left[\frac{y_1 + y_1^*}{2(1-s)}\right] Z.$$

(Please note that in this section we interpret I and I^* as *per entrepreneur* investment in each country; similarly for Z and Z^*. Since only $1 - s$ percent of all agents are entrepreneurs, one must divide world per capita income by $1 - s$ to convert to investment funds per entrepreneur.)

Let us assume that in equilibrium both $y_1 < \bar{I}$ and $y_1^* < \bar{I}^*$, so that neither Home nor Foreign entrepreneurs can finance the first-best equilibrium investment levels without drawing on the resources of savers. Under asymmetric information, the loan contracts entrepreneurs in the two countries are offered therefore will have to satisfy the incentive-compatibility constraints

$$P(Z) = Z - \frac{1+r}{\pi'(I)}, \qquad P(Z)^* = Z - \frac{1+r}{\pi'(I^*)}, \tag{50}$$

and the zero-profit conditions

$$P(Z) = \frac{(1+r)(I - y_1)}{\pi(I)}, \qquad P(Z)^* = \frac{(1+r)(I^* - y_1^*)}{\pi(I^*)}. \tag{51}$$

If we knew $1 + r$, we could use these conditions, as before, to calculate repayments in case of investment success, investment levels, and borrowing for each country.

To calculate investment levels and the world interest rate, substitute for $P(Z)$ in eq. (50) using eq. (51); then solve the result for $1 + r$ to get the Home interest rate equation,

$$1 + r = \frac{\pi'(I)Z}{1 + \frac{\pi'(I)(I-y_1)}{\pi(I)}} \equiv \rho(I, y_1). \tag{52}$$

(There is a parallel definition for Foreign.) Notice that $\partial\rho/\partial I < 0$ and $\partial\rho/\partial y_1 > 0$.[67] The locus of investment pairs along which Home and Foreign face a common risk-free interest rate is given by

$$\rho(I, y_1) = \rho(I^*, y_1^*), \tag{53}$$

and it has a positive slope, as the corresponding $\rho\rho$ locus in Figure 6.12 shows. Curve **IS** graphs the equality of world saving and investment,

$$\frac{y_1 + y_1^*}{1 - s} = I + I^*.$$

Curve **IS** has slope -1. A key observation is that $\rho\rho$ cannot intersect **IS** at point **A**, the first-best allocation given the identical Home and Foreign technologies. Because $y_1 > y_1^*$, eq. (52) shows that $\rho(I, y_1) > \rho(I^*, y_1^*)$ at **A**, creating an incentive

67. To see the negative relationship between I and r, recall that in Figure 6.11, a rise in r shifts both curves to the left and lowers I. If you prefer a calculus proof, compute

$$\frac{\partial\rho}{\partial I} = \frac{\pi''(I)\pi(I)^2 Z + \pi'(I)^2[\pi'(I)(I - y_1) - \pi(I)]Z}{[\pi(I) + \pi'(I)(I - y_1)]^2}.$$

In the numerator of this fraction, $\pi''(I) < 0$ by the strict concavity of $\pi(I)$. As shown in footnote 66 $\pi'(I)(I - y_1) - \pi(I) < 0$. So $\partial\rho/\partial I < 0$, as claimed.

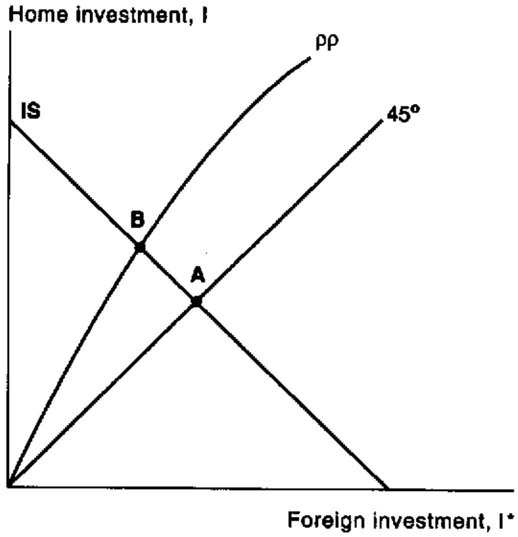

Figure 6.12
Two-country world equilibrium

for world savings to flow from Foreign to the richer country, Home. Thus $\rho\rho$ and **IS** must intersect at a point like **B**, with investment higher in the richer country (just as the partial-equilibrium model suggested). Only with an equal distribution of initial wealth among entrepreneurs worldwide would the world economy attain efficient investment. With unequal entrepreneurial wealth, expected world output therefore is lower than under full information.

Home's higher income implies that it saves more, but, as we have seen, it also invests more. Thus it is by no means clear that richer Home lends to poorer Foreign. Instead, a seemingly perverse flow of savings from Foreign to Home can occur. The model therefore suggests an explanation of the phenomenon that capital sometimes flows from low-income to high-income countries.

It is easy to show that if Foreign's government has a debt to Home, and taxes Foreign firms (on either date) to service it, Foreign investment is depressed. (As Foreign's debt rises, $\rho\rho$ in Figure 6.12 shifts upward.) This effect, a variant of the debt overhang effect, has implications for the transfer problem analyzed in Chapter 4. When one country transfers income to another, credit-market imperfections may magnify the direct costs. (See end-of-chapter exercise 4.)

Given initial wealth distributions, however, the equilibrium is constrained Pareto optimal. Despite the higher rate of return on marginal investment in the poor country, there is no way for a world planner to engineer a Pareto-improving allocation unless the planner has access to more information than do lenders. (See end-of-chapter exercise 5.)

6.4.3 Implications for Consumption Insurance

Under risk aversion, informational asymmetries may lead to suboptimal insurance, with repercussions on investment. To see this relationship we modify the small-country model of section 6.4.1 so that the representative entrepreneur's utility function is strictly concave in date 2 consumption,

$$U_1 = U(C_1, C_2) = u(C_2).$$

To focus squarely on insurance considerations, we make the entrepreneur's date 1 endowment more than sufficient to finance the first-best investment level: $Y_1 > \bar{I}$, where, as before, $\pi'(\bar{I})Z = 1 + r$. The entrepreneur's sole reason for using capital markets is to reduce consumption variability.

Now the entrepreneur's date 2 consumption is

$$C_2 = \begin{cases} Z - P(Z) + (1+r)(Y_1 - I) & \text{with probability } \pi(I) \\ -P(0) + (1+r)(Y_1 - I) & \text{with probability } 1 - \pi(I). \end{cases}$$

The first-best insurance contract would be

$$P(Z) = [1 - \pi(\bar{I})]Z, \qquad P(0) = -\pi(\bar{I})Z,$$

which stabilizes consumption at its expected value of $\pi(\bar{I})Z + (1+r)(Y_1 - \bar{I})$ and satisfies the zero-profit condition for insurers,

$$\pi(\bar{I})P(Z) + [1 - \pi(\bar{I})]P(0) = 0.$$

Under asymmetric information, however, the entrepreneur has no reason to invest anything at all once insurers have guaranteed his date 2 consumption. The precise form of the optimal incentive-compatible contract is messy, but it is analogous to those analyzed earlier. The contract involves a trade-off between efficient production and efficient risk sharing, one that leaves domestic consumption subject to domestic production uncertainty and investment below its first-best level.

6.4.4 Discussion

The possibility of moral hazard is clearly an important reason why the complete-markets model of Chapter 5 squares so poorly with the data. We have explored moral hazard in the context of physical capital investment, but it arises in many other contexts, for example, investment in human capital. A graduate student who could buy full insurance on his future lifetime income would face a diminished incentive to study hard!

The preceding models also illustrate how moral hazard in *government* investment may interfere with international (or interregional) insurance markets. Suppose one interprets investment in the last model as tax-financed public investment in infrastructure, schools, and so on. If a government can commit to the first-best investment level, it may be able to obtain full insurance. But if commitment is impossible, full insurance would give voters an ex post incentive to elect a government that invests (and taxes) at levels below the ex ante optimum. This is another example of dynamic inconsistency in government policy.[68]

More generally, asymmetric information has broad ramifications for the functioning of credit markets, domestically as well as in an international context, although informational distortions are likely to be even more severe in the latter setting. Transactions may be limited not only by moral hazard, as in the models just examined, but also by adverse selection problems—the tendency for "bad" borrowers (those with a low likelihood of repayment) to drive out "good" borrowers when lenders cannot observe borrower quality. If sufficiently severe, adverse selection may lead to a collapse of the market, as shown in a pioneering paper by Akerlof (1970). Gertler (1988) provides a good survey of the roles of moral hazard and adverse selection problems in models of financial intermediation.

One theme of the moral hazard model is that a rise in initial borrower wealth can mitigate the dampening effects on investment—recall how an increase in the borrower's initial endowment, Y_1, shifted the locus **ZP** outward in Figure 6.11. In an economy where the value of wealth is endogenous and depends on expectations about future economic conditions, a collapse in economic confidence can reduce borrower wealth, depressing investment and consumption and inducing self-fulfilling cycles of bust and boom. Kiyotaki and Moore (1995) present a theoretical model of credit cycles.

Mishkin (1978), Bernanke (1983), and others have argued that a general credit collapse linked to declining asset values helped deepen the Great Depression of the 1930s. A body of more recent evidence points to similar effects of borrower net worth on economic activity, as the following application illustrates.

Application: Financing Constraints and Investment

If there are no impediments to borrowing at the firm level—as, for example, in the q investment model of Chapter 2—then it should not matter whether a firm finances additions to physical capital out of retained earnings or out of borrowed funds. The logic is closely analogous to that for the small country model of Chapters 1 and 2, where the efficient level of national investment is independent of national savings. In the neoclassical investment model, firm-level savings (or, more generally, firm financial structure) should be irrelevant for investment allocations.

When informational problems constrain firm borrowing, however, the firm's current financial condition can have a critical effect on its investment. Firms with high current cash flow (high current income net of wages, taxes, and interest payments)

68. Persson and Tabellini (1996) discuss a different model of moral hazard in government investment.

have the means to self-finance a greater proportion of their investment. One strong empirical implication of the class of models we have just studied is that firms with high cash flow actually should invest more. A substantial body of research suggests that this is indeed the case.

In an early and influential study, Fazzari, Hubbard, and Peterson (1988) showed that cash flow can help explain firm-level investment empirically, even after one controls for a firm's q ratio and other factors suggested by standard neoclassical models of investment. They study a large panel data set consisting of publicly-traded United States manufacturing firms for the years 1970–84. Reasoning that borrowing constraints are likely to be more severe for rapidly growing firms than for mature firms, and that mature firms tend to pay the highest dividends, they divided their sample into three groups. Class I firms consist of those with dividend-earnings ratios less than 0.1 for at least 10 years, Class II firms consist of those with dividend-earnings ratios greater than 0.1 but less than 0.2, and Class III consists of all other firms. They then ran panel regressions of the general form

$$\left(\frac{I}{K}\right)_t^i = a_0 + a_1 q_t^i + a_2 \left(\frac{cashflow}{K}\right)_t^i + \epsilon_t^i,$$

where q^i is a measure of Tobin's q for firm i (see Chapter 2) and *cashflow* is a measure of firm i's cash flow.[69] Theoretically, once one controls for q, the cash flow variable should not have any explanatory power absent borrowing constraints. However, Fazzari, Hubbard, and Peterson found that cash flow is consistently significant in their regressions, and it is significant for all three firm groups. Interestingly, the coefficient is much larger for Class I firms (those most likely to be constrained) than for the other two classes, with the Class III firms having the lowest coefficient. The authors emphasize that this last result is their most important. It is possible that all three groups face a differential cost between external and external finance, but it is also possible that the cash flow variable is simply proxying for other factors. If the Class II and III firms are viewed as control groups, one can (loosely) think of the differential between the cash flow coefficients for these firms versus the Class I firms as measuring the importance of cash flow.

Similar results have been found by other researchers using different time periods and different methods of classifying firms.[70] Smaller firms, for example, might be expected to have less access to equity financing and therefore be more reliant on bank loans and other forms of intermediated credit. Gertler and Gilchrist (1993)

69. Recall that q measures the ratio of the shadow value of a unit of capital in place to the cost (not including installation cost) of new investment. In practice, q is sometimes measured by the ratio of the market value of a firm to the book value of its assets, but this measure is very crude because it can be very sensitive to accounting conventions. Also, technically, the right variable to include in the regression is *marginal* q rather than *average* q. In Chapter 2 we demonstrated conditions under which marginal and average q are equal, but these conditions might not always be met in practice.

70. For a recent survey of the evidence, see Bernanke and Gertler (1995).

find that small U.S. firms are indeed more sensitive to general financial conditions and argue that the differential cost of internal versus external financing is a plausible explanation. Further confirming evidence comes from studies based on countries outside the United States; see, for example, Devereux and Schiantarelli (1990) on the United Kingdom and Hoshi, Kashyap, and Scharfstein (1991) on Japan. (The latter study tests for an internal-external financing differential by classifying firms according to whether or not they belong to an industrial group.)

Unquestionably the biggest problem plaguing this literature is the difficulty in measuring Tobin's q. If q is badly measured, it is hard to be sure that the cash flow variable is capturing the effects of credit constraints rather than say, expected future earnings. (Expectations would be fully embodied in q if that variable were correctly measured.) One interesting approach to dealing with this problem has been suggested by Gilchrist and Himmelberg (1995). They measure q by using vector autoregressions to forecast a firm's expected future earnings, including cash flow as one of the predictive variables. Then, in their second-stage regressions, they include only the part of cash flow that is orthogonal to q. Cash flow remains a consistently important variable in their investment regressions, even after controlling for its predictive power for future earnings.

None of the tests described is foolproof, in the sense that one can construct models with perfect asset markets that generate the same empirical regularities. Given the uniformity of the empirical results and the lack of convincing positive documentation for alternative explanations, however, it is hard to deny some role to asset market imperfections in limiting both flows of outside funds to firms and investment. ■

Appendix 6A Recontracting Sovereign Debt Repayments

In the models of sections 6.1.1 and 6.2.1, we assumed that if creditors could threaten a country with ηY in default costs (where Y is output), they could force it to make up to ηY in debt payments. For two reasons, this magnitude probably overstates creditors' power to enforce repayment. First, imposing sanctions is costly for creditors and does not necessarily yield them any direct benefits—other than the satisfaction of revenge! Second, a country may be able temporarily to avoid sanctions or seizure and buy time for negotiation with creditors by delaying and rerouting goods shipments.

If either of these channels is important (or if both are), creditors will be unable to make credible take-it-or-leave-it offers to debtors, such as "Pay in full or we will annihilate a fraction η of your output." Instead, actual debt repayments will be the outcome of a bargaining process. Consider, for example, the following very stylized infinite-horizon model of a small endowment economy, which is specialized in producing an exportable but consumes only an importable.[71]

The sovereign maximizes the intertemporal utility function

71. The model is based on Bulow and Rogoff (1989a).

$$U_t = \sum_{s=0}^{\infty} \frac{hC_{t+hs}}{(1+\delta h)^s}, \tag{54}$$

where C is consumption of the import good. The length of a period here is h; we leave this as a parameter so as to be able to consider the limit of continuous bargaining ($h \to 0$). Because the length of period is not fixed, we have written the discount factor as $1/(1+\delta h)$ instead of β as we usually do, and we interpret δ as the subjective *rate* of time preference. We assume that $\delta > r$, where $1 + rh$ is the fixed exogenous gross world rate of interest on importable goods. Finally, notice the assumption of a linear period utility function; this feature simplifies our analysis of bargaining considerably.

Each period, the country is endowed with Yh units of its export good, each of which is worth P units of the imported good on the world market. (P is constant.) While the country does not consume the export good, it cannot be forced to export it in any given period. Instead, the country may avail itself of a storage technology whereby S_t stored in period t yield $(1-\theta h)S_t$ units after a period,

$$S_{t+h} = (1-\theta h)S_t,$$

where $\theta > 0$ and $1 > \theta h$.[72] Storage is inefficient, but may nevertheless be relevant in situations where the country is trying to renegotiate its international debts. Think of a debtor country as producing bananas that may be seized by creditors once they are shipped abroad, but cannot be seized while they are still in the country. Thanks to the storage technology, the country can credibly threaten creditors with delayed payment if they are unwilling to reschedule or simply write down debts.

Because period utility is linear in eq. (54), there is no consumption-smoothing motive for borrowing here. Instead, the country's sole motive for borrowing is that its subjective discount rate exceeds the world interest rate ($\delta > r$). Thus the country will do all its consuming on the initial date, date t, by immediately borrowing the entire present discounted value of its future income and spending the rest of eternity repaying PYh per period. In this case the initial amount borrowed (measured in terms of imports) would be

$$D = \frac{PYh}{rh} = \frac{PY}{r}. \tag{55}$$

Let us suppose that the country actually did borrow and consume this much in the initial period. Could it actually be forced to repay PYh to lenders in each ensuing period? In general, the answer is no, even if lenders can seize 100 percent of any international shipments ($\eta = 1$ in terms of the text's model, with creditors obtaining an equal benefit). If the country has absolutely nothing to gain by shipping its goods, it will put them in storage and bargain with its creditors for repayments below the sum PYh that it owes.

Exactly how much of a reduction in its contracted payment PYh the country can get depends on the nature of the bargaining process, but the country clearly should be able to get something. After all, creditors are impatient: their discount rate is r and the goods they would otherwise seize are deteriorating at rate θ in storage. Thus creditors have something to gain by making an immediate concession that induces the sovereign to ship its output and pay at least partially what it owes.

72. Without affecting the results we could allow for a storage technology yielding a nonnegative return ($\theta \le 0$), provided $-\theta < r$.

One simple model of how the country's income is divided each period draws on A. Rubinstein (1982). (Here, though, bargaining is over a flow rather than a stock.) It predicts that a key factor governing actual repayments is the relative impatience of the two countries. For the creditors, the effective discount rate (in the continuous-time limit) is $r + \theta$. (Any delay in reaching an agreement costs the creditors both because they must wait to relend any repayments they receive and because the sum being bargained over shrinks in storage.) For the debtor country, the effective discount rate is $\delta + \theta$.

Imagine first that the country simply cannot sell any of its output until it has reached an agreement with creditors (that is, $\eta = 1$). Absent private information, a Rubinstein-type model predicts that in the continuous-time limit ($h \to 0$), the two parties will reach an agreement *immediately*, with creditors receiving at most a fraction

$$\frac{\delta + \theta}{(\delta + \theta) + (r + \theta)} = \frac{\delta + \theta}{\delta + r + 2\theta} \tag{56}$$

of the country's output PY and the country receiving at least

$$\frac{r + \theta}{(\delta + \theta) + (r + \theta)} = \frac{r + \theta}{\delta + r + 2\theta}. \tag{57}$$

Of course, creditors will be repaid in full if debtors initially borrowed a fraction of PY no greater than expression (56), but only then. Anticipating that the country will try to bargain over repayments, creditors therefore will never make an initial loan bigger than

$$D = \left[\frac{\delta + \theta}{(\delta + \theta) + (r + \theta)}\right]\frac{PY}{r},$$

which is strictly below the amount in (55).[73]

More realistic assumptions might allow the country to consume the good it produces, or allow creditors to seize only a fraction $\eta < 1$ of exported goods. Either of these possibilities can create an "outside option" for the country that may influence the outcome of bargaining. For example, if the fraction of shipments creditors can seize, η, is less than the share in

73. The Rubinstein (1982) solution assumes an alternating offers framework in which the debtor and its creditors take turns making offers each period (so that the exogenous component of bargaining power is equal). When it is the country's turn to make an offer, its best strategy is to offer creditors a share just large enough so that they would rather reach an agreement immediately than wait a period to make a counteroffer. The reverse holds when it is the creditors' turn to make an offer. Thus, on its turn, the debtor will offer creditors

$$x_t = \frac{1 - \theta h}{1 + rh} x'_{t+h},$$

where x_t is the share of output and goods in storage the country offers the creditors on date t and x'_{t+h} is the share the creditors will offer themselves if they wait a period. When it is the creditors' turn, they offer the country

$$1 - x'_t = \frac{1 - \theta h}{1 + \delta h}(1 - x_{t+h}).$$

The creditors' equilibrium share, eq. (56), is found by solving for the stationary state of these two difference equations (that is, remove time subscripts and solve for x and x'), and then taking the limit as $h \to 0$. A recent exposition is contained in Mas-Colell, Whinston, and Green (1995).

eq. (56), the sovereign will be able to borrow only η percent of the present discounted value of his output, and the bargaining factors highlighted in eq. (56) may no longer be relevant. If $\eta > (\delta + \theta)/(\delta + r + 2\theta)$, however, improvements in creditors' power to seize a country's goods abroad may do little to enhance their bargaining position or, accordingly, the debtor's ability to borrow. Also, the present model endows neither creditors nor borrowers with any type of private information. Kletzer (1989) shows that with private information, debtors and creditors may reach agreement only after some delay, so that bargaining results in inefficiencies. It is also easy to make the model stochastic. In a stochastic setting, Bulow and Rogoff (1989a) reinterpret the bargaining model as an account of debt-rescheduling agreements. If shocks are observable to the two parties in the rescheduling agreement but difficult to verify in a court of law, the optimal loan contract may involve a high face value of debt, with both parties anticipating the likelihood of debt rescheduling later.

At one level, the main results derived earlier in this chapter are easily modified to incorporate the possibility of bargaining. We can then reinterpret the parameter η as the outcome of a bargaining process rather than simply an exogenous seizure-technology parameter. However, a bargaining perspective raises other important issues that are somewhat obscured by the more mechanical repayment model of the text. Perhaps the most important is the possibility that creditor-country governments might be drawn into the bargaining process and gamed into making side payments. Think of the sovereign's foreign creditors as private agents representing only a small fraction of creditor-country taxpayers. By interfering in trade with the debtor country, private creditors inflict damage on their compatriots as well as on debtor-country citizens. Therefore, creditor-country governments may be willing to make side payments to "facilitate" rescheduling agreements. This view assumes that creditor-country governments will not simply abrogate international loan contracts and deprive creditors of their legal right of retribution. The creditor country may be reluctant to do so if it is concerned that such abrogation will undermine the reputation of its constitution and its legal system.

Bulow and Rogoff (1988a) develop a model of three-way bargaining among debtor-country governments, creditor-country governments, and private creditors. They show that expected future government side payments may increase the borrowing limits of small debtor countries. They also show that from an ex ante perspective, debtors facing competitive lenders capture the entire surplus from anticipated side payments. In practice, side payments can take many forms, ranging from trade concessions to subsidized loans channeled through multilateral lenders or bilateral export-import banks.

Appendix 6B Risk Sharing with Default Risk and Saving

This appendix derives the results summarized in section 6.1.1.6. Recall the assumptions made there, that a small country maximizes

$$U_1 = u(C_1) + \beta \mathrm{E}\{u(C_2)\}, \qquad \beta < 1,$$

receives an endowment $Y_1 = \bar{Y}$ in the first period, and begins that period without foreign assets or debt. In the second period the (stochastic) endowment is

$$Y_2 = \bar{Y} + \epsilon.$$

The mean-zero shock ϵ can take any of the values $\epsilon_1, \ldots, \epsilon_N$ in the closed interval $[\underline{\epsilon}, \bar{\epsilon}]$, where $\bar{Y} + \underline{\epsilon} > 0$. Risk-neutral insurers compete on date 1 to offer the country zero-expected-profit contracts covering uncertain date 2 output, and on date 1 the country can borrow or lend at a given world interest rate $r > 0$, where $\beta(1+r) = 1$.

The full insurance allocation is essentially the same as in the model without date 1 consumption and saving. If the country could somehow credibly forswear default, it would be able to obtain a riskless date 2 consumption level of $\bar{Y}$ by committing to the insurance-payment schedule $P(\epsilon) = \epsilon$. Given that $C_2 = \bar{Y}$ is feasible (for all ϵ) and that $\beta(1+r) = 1$, the optimal choice for C_1 is $\bar{Y}$, making optimal date 1 saving zero.

With default risk, however, the full-insurance contract is not incentive compatible. The optimal incentive-compatible contract maximizes U_1 subject to the intertemporal budget constraints (which must hold for each ϵ)

$$C_2(\epsilon) = \bar{Y} + \epsilon - P(\epsilon) + (1+r)(\bar{Y} - C_1),$$

the zero-profit condition (1), which requires that $\mathrm{E}\{P(\epsilon)\} = 0$, and the incentive compatibility constraints, which, in the present context, are (for each ϵ)

$$P(\epsilon) \leq \eta(\bar{Y} + \epsilon) + (1+r)(\bar{Y} - C_1) \tag{58}$$

instead of eq. (2). These incentive constraints reflect the assumption of section 6.1.1.6 that a sovereign defaulting on payments to insurers forfeits any interest and principal on its date 1 foreign investment. (Foreign creditors can compensate themselves by seizing the sovereign's own foreign assets if it defaults.)[74]

The Lagrangian for this problem is

$$\mathcal{L} = u(C_1) + \sum_{i=1}^{N} \pi(\epsilon_i)\beta u[\bar{Y} + \epsilon_i - P(\epsilon_i) + (1+r)(\bar{Y} - C_1)]$$

$$- \sum_{i=1}^{N} \lambda(\epsilon_i)[P(\epsilon_i) - \eta(\bar{Y} + \epsilon_i) - (1+r)(\bar{Y} - C_1)] + \mu \sum_{i=1}^{N} \pi(\epsilon_i)P(\epsilon_i).$$

The Kuhn-Tucker necessary conditions for C_1 and $P(\epsilon)$ are

$$u'(C_1) = \beta(1+r) \sum_{i=1}^{N} \pi(\epsilon_i)u'[C_2(\epsilon_i)] + (1+r) \sum_{i=1}^{N} \lambda(\epsilon_i), \tag{59}$$

$$\pi(\epsilon)\beta u'[C_2(\epsilon)] + \lambda(\epsilon) = \mu\pi(\epsilon), \tag{60}$$

and the complementary slackness condition is

$$\lambda(\epsilon)[\eta(\bar{Y} + \epsilon) + (1+r)(\bar{Y} - C_1) - P(\epsilon)] = 0, \tag{61}$$

where $\lambda(\epsilon) \geq 0$, for all ϵ.

The first of these conditions differs from the standard Euler equation in that higher date 1 consumption lowers the amount of insurance available next period by raising the benefit of default relative to its cost. This effect tends to encourage saving. The second and third conditions are analogous to eqs. (4) and (5).

To proceed, sum both sides of eq. (60) over $i = 1, \ldots, N$ and infer from eq. (59) that

74. In case $C_1 > \bar{Y}$, the constraint says that the total sum owed to foreign creditors in state ϵ, $P(\epsilon) + (1+r)(C_1 - \bar{Y})$, must be no greater than the cost of their sanctions in that state.

$$u'(C_1) = \beta(1+r)\sum_{i=1}^{N}\pi(\epsilon_i)u'[C_2(\epsilon_i)] + (1+r)\sum_{i=1}^{N}\lambda(\epsilon_i) = (1+r)\mu.$$

Let's simplify the problem as before by dividing $[\underline{\epsilon}, \bar{\epsilon}]$ into the disjoint intervals $[\underline{\epsilon}, e)$, on which constraint (58) does *not* bind [and where $\lambda(\epsilon) = 0$ by eq. (61)], and $[e, \bar{\epsilon}]$, on which constraint (58) holds with equality. Combining the preceding equation $u'(C_1) = (1+r)\mu$ with eq. (60) for $\epsilon \in [\underline{\epsilon}, e)$ yields

$$u'(C_1) = (1+r)\beta u'[C_2(\epsilon)] = u'[C_2(\epsilon)] \tag{62}$$

[recall $\beta(1+r) = 1$], which implies that the country equates consumption across dates for those states in which sanctions more than suffice to compel compliance with loan contracts. For ϵ such that $\lambda(\epsilon) > 0$, however, constraint (58) holds as an equality. But in these cases we can solve for $P(\epsilon)$ and $C_2(\epsilon)$ from constraint (58) and the intertemporal budget constraint:

$$P(\epsilon) = \eta(\bar{Y} + \epsilon) - (1+r)(C_1 - \bar{Y}), \qquad C_2(\epsilon) = (1-\eta)(\bar{Y} + \epsilon). \tag{63}$$

The implications thus are similar to those of the simpler case worked out in the text. Where the repayment constraint is binding, a small unexpected drop in output increases net payments to the country by only a fraction η of the output decline. On the other hand, where the constraint is not binding, net payments to creditors rise one-for-one with ϵ, which is the only way $C_2(\epsilon)$ can be maintained at C_1 ex post in that region, as eq. (62) requires.

It is straightforward to solve for the shape of the optimal $P(\epsilon)$ schedule. Observe that at the critical value $\epsilon = e$ when $\lambda(\epsilon)$ first becomes zero, the last two eqs. (62) and (63) both hold, implying that

$$C_1 = (1-\eta)(\bar{Y} + e). \tag{64}$$

This equation leads to part of the solution for $P(\epsilon)$. For $\epsilon \in [\underline{\epsilon}, e)$,

$$C_2(\epsilon) = \bar{Y} + \epsilon - P(\epsilon) + (1+r)(\bar{Y} - C_1) = C_1$$

by eq. (62), so substituting eq. (64) for C_1, one can infer the equation for $P(\epsilon)$ on the unconstrained region $[\underline{\epsilon}, e)$. Similarly, eq. (64) and the first equation in (63) give an equation describing $P(\epsilon)$ over the constrained region $[e, \bar{\epsilon}]$. The results are

$$P(\epsilon) = \begin{cases} \epsilon + (2+r)[\eta\bar{Y} - (1-\eta)e], & \epsilon \in [\underline{\epsilon}, e) \\ \eta\epsilon + (2+r)\left[\eta\bar{Y} - \dfrac{1+r}{2+r}(1-\eta)e\right], & \epsilon \in [e, \bar{\epsilon}]. \end{cases} \tag{65}$$

A figure similar to Figure 6.1 shows the implied $P(\epsilon)$ schedule.

Solution is straightforward when ϵ is uniformly distributed. We leave the general case as an exercise, and restrict ourselves to solving the interesting special case $\eta = 0$. Under these assumptions $\underline{\epsilon} = -\bar{\epsilon}$, and eq. (65) becomes

$$P(\epsilon) = \begin{cases} \epsilon - (2+r)e, & \epsilon \in [-\bar{\epsilon}, e) \\ -(1+r)e, & \epsilon \in [e, \bar{\epsilon}]. \end{cases} \tag{66}$$

This implies that the zero-profit condition (1) is

$$\int_{-\bar{\epsilon}}^{e}[\epsilon - (2+r)e]\frac{d\epsilon}{2\bar{\epsilon}} - \int_{e}^{\bar{\epsilon}}(1+r)e\frac{d\epsilon}{2\bar{\epsilon}} = 0,$$

which reduces to the quadratic

$$e^2 + [2(3+2r)\bar{\epsilon}]e + \bar{\epsilon}^2 = 0.$$

The relevant root of this equation is

$$e = -\bar{\epsilon}[3 + 2r - \sqrt{(3+2r)^2 - 1}] \in (-\bar{\epsilon}, 0).$$

What explains the availability of partial insurance ($e > -\bar{\epsilon}$) despite the total inefficacy of sanctions ($\eta = 0$)? By eq. (64), the country's date 1 saving when $\eta = 0$ is $\bar{Y} - (\bar{Y} + e) = -e > 0$. Thus creditors are in a position to confiscate $-(1+r)e$ on date 2 should the country renege on its contract. For realizations of $\epsilon \in [-\bar{\epsilon}, e)$, eq. (66) calls for the country to pay insurers an amount $P(\epsilon) = \epsilon - (2+r)e$ strictly below the amount of country principal and interest that insurers could seize.[75] For $\epsilon \in [-\bar{\epsilon}, e)$, date 2 consumption therefore is stabilized at

$$\begin{aligned} C_2(\epsilon) &= \bar{Y} + \epsilon - P(\epsilon) - (1+r)e \\ &= \bar{Y} + \epsilon - [\epsilon - (2+r)e] - (1+r)e \\ &= \bar{Y} + e. \end{aligned}$$

In contrast, for $\epsilon \in [e, \bar{\epsilon}]$, eq. (66) has the country pay out an amount exactly equal to its own claims on foreigners. In this constrained region of $[-\bar{\epsilon}, \bar{\epsilon}]$, the country thus is restricted to the autarky consumption level $C_2(\epsilon) = \bar{Y} + \epsilon$.

The result is that by saving $-e$ on date 1, the country can credibly promise to comply with a zero-profit contract that insures it against shocks $\epsilon < e$.

Exercises

1. *Two-sided default risk.* Consider the following one-period, two-country version of the model in section 6.1.1, in which Home and Foreign agents have identical utility functions $u(C)$ [$u(C^*)$]. Home's endowment is given by $Y = \bar{Y} + \epsilon$, while Foreign's is given by $Y^* = \bar{Y} - \epsilon$, where ϵ is zero-mean random shock that is symmetrically distributed around 0 on the interval $[-\bar{\epsilon}, \bar{\epsilon}]$. Home and Foreign agents write insurance contracts prior to the realization of the relative output shock, which specify a payment by Home to Foreign of $P(\epsilon)$ [$= -P^*(\epsilon)$]. Obviously, in the absence of default risk, $P(\epsilon) = \epsilon$, and $C = C^* = \bar{Y}$: there is perfect risk-sharing. Assume, however, that due to enforcement limitations, any equilibrium contract must obey the incentive compatibility constraints:

   $$P(\epsilon) \le \eta Y, \qquad P^*(\epsilon) \le \eta Y^*.$$

   The questions below refer to the efficient symmetric incentive-compatible contract. (You may answer using a graph.)

   (a) Show that there is a range $[-e, e]$ such that $C = C^*$ for $\epsilon \in [-e, e]$. Solve for e. (This is not hard.)

   (b) Characterize $C(\epsilon)$ and $C^*(\epsilon)$ for ϵ outside the interval $[-e, e]$.

75. The inequality $\epsilon - (2+r)e < -(1+r)e$ is equivalent to $\epsilon < e$.

2. *Indexed debt contracts in lieu of insurance.* Again reconsider the small-country model of section 6.1.1, where consumption takes place only in period 2. Now, instead of being able to write insurance contracts, the country is only able to borrow in the form of equity or output-indexed debt contracts. In particular, it borrows D in period 1 and makes *nonnegative* payments $P(\epsilon) \geq 0$ in period 2 subject to the zero-profit condition

$$\sum_{i=1}^{N} \pi(\epsilon_i)P(\epsilon_i) = (1+r)D$$

and the incentive compatibility constraint

$$P(\epsilon_i) \leq \eta[\bar{Y} + (1+r)D + \epsilon_i], \qquad \forall i.$$

We justify this last constraint as follows. As in the text, the country receives a second-period endowment of $\bar{Y}$ and has no first-period endowment (nor inherited capital stock). In addition, it has access to a linear local production technology such that $F(K) = (1+r)K$. Thus the country can invest borrowed funds locally and still earn the world market rate of return. Note that K will equal D, given our assumptions. Observe also that in this formulation, creditors pay cash up front so their credibility is never at issue.

(a) Treating D as given, characterize the optimal incentive-compatible $P(\epsilon)$ schedule. [Hint: $C(\underline{\epsilon}) = \bar{Y} + \underline{\epsilon} + (1+r)D$.] Draw a diagram illustrating your answer.

(b) If the country has access only to equity contracts, is it equally well off as in the text's case of pure insurance contracts? (Consider how large η must be to achieve full insurance in each of the two cases.)

3. *A problem on reputational equilibrium.* This problem places a number of restrictions on contracts and investment which you should take as given for now; we will allow you to critique them at the end. Suppose that the infinitely lived representative agent in a small country has utility function given by

$$U_t = \mathrm{E}_t\left\{\sum_{s=t}^{\infty} \beta^{s-t}u(C_s)\right\},$$

where $\beta(1+r) = 1$. The country cannot lend abroad or obtain pure insurance contracts. It can borrow but exclusively in the form of one-period bonds that must pay risk-neutral foreign lenders the expected return $1+r$. Repayments $P(\epsilon_t)$ may, however, be indexed to ϵ_t (explicitly or implicitly). Consumption each period is given by

$$C_t = F(D_t) + \epsilon_t - P(\epsilon_t),$$

where $P(\epsilon_t) \geq 0$ and $\epsilon_t \in [\underline{\epsilon}, \bar{\epsilon}]$ is a positive ($\epsilon > 0$), serially uncorrelated shock such that $\mathrm{E}_{t-1}\{\epsilon_t\} = e > 0$. The term $F(D_t)$ comes from the assumption that only fresh foreign capital may be used for investment; capital depreciates by 100 percent in production. The production function satisfies $F'(D) > 0$ and $F''(D) < 0$ for $D < \bar{D}$, and $F'(D) = 1 + r$ for $D \geq \bar{D}$, where $\bar{D}$ is a constant.

(a) Assume that the country can commit to any feasible repayment schedule. Characterize the optimal contract. Under what conditions can this contract be enforced as a trigger-strategy equilibrium where the only penalty to default is that the country is excluded from all future borrowing?

(b) Briefly: How reasonable is the assumption here that the country cannot use its own income to finance investment, even though its income can be used to make debt repayments to foreigners? Recall the discussion of the text in section 6.2.2.

(c) Briefly: How important is the assumption that the country is prohibited from lending money abroad? Recall the discussion of section 6.1.2.4.

4. *Collateralizable second-period endowment.* Take the small-country model with moral hazard in investment of section 6.4.1. Assume now that in addition to receiving first-period endowment Y_1, each entrepreneur receives *exogenous* second-period endowment income E_2. This income is in addition to any income received from the investment project or from secret lending abroad. E_2 is fully observable and can be used either for second-period consumption or to help pay off loans.

(a) How does the introduction of collateralizable future income change the analysis? You need not make your answer self-contained; you can just show how the **IC** and **ZP** curves are modified, and why.

(b) Suppose now that there is only first-period endowment, but that in the second period, the government must pay off a per capita debt D^G. It finances the debt by placing a second-period tax of τ on successful entrepreneurs. (Obviously, unsuccessful entrepreneurs cannot be made to pay any tax in the second period, as they have no observable income.) Show how the overhang of government debt reduces investment.

5. *Fiscal policy with moral hazard.* Consider again the model of section 6.4.1 (with only a first-period endowment), but now assume that for every entrepreneur, there is a "saver." Savers have the same utility function and initial endowment Y_1 as entrepreneurs, but do not have access to any investment project. They can either lend their money to local entrepreneurs or lend abroad at rate $1 + r$. Note that the presence of local savers does not change the determination of equilibrium investment, since they do not affect the world interest rate. Assume that Y_1 is sufficiently small so that in market equilibrium, investment is below its full-information efficient level.

Can you think of any way a home social planner can make some agents better off without making any others worse off? Assume that the social planner faces the same information constraints as other agents; that is, the planner is not able directly to observe an individual entrepreneur's choice of I, only final project output Y_2 (which equals either Z if successful or 0 if not). Consider a scheme whereby the planner makes each saver pay a first-period tax of τ_1, transferring the income to entrepreneurs. Then, in the second period, the planner places a tax τ_2 on successful entrepreneurs, transferring the money back to savers.

6. *Debt overhang and debt forgiveness.* Consider a small open economy that inherits a very large (effectively infinite) debt, D, which is scheduled to be paid off in the second period. The representative agent in the country has the utility function

$$U_1 = \log C_1 + \beta \log C_2.$$

First-period endowment income is Y_1. Capital depreciates by 100 percent in production and second-period output is $Y_2 = I^\alpha$, where $I = K_2$ is date 1 investment. In the second period, creditors will be able to force the country to pay ηY_2 in debt repayments. (Assume that the debt is so large that the country cannot fully repay its debt even if it invests all its resources.)

(a) Solve for the country's optimal choice of investment, I^{D}, and the implied level or repayments to creditors, $\eta(I^{\mathrm{D}})^{\alpha}$.

(b) Now assume that entering period 1, creditors decide partly to forgive the country's debt, writing down the face value to $\eta(I^{\mathrm{D}})^{\alpha}$, the amount they expect to be repaid if they do nothing. Does this cost the creditors anything? Can the debtor country benefit?

(c) Assume that creditors have no interest in the welfare of the country and care only about maximizing debt repayments. How far should they write down the country's debt, if at all? You may find it convenient to answer this part with a graph.