The Growth and Welfare Effects of Macroeconomic Volatility

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Abstract

This paper studies the welfare properties of competitive equilibria in economies with financial frictions hit by aggregate shocks. In particular, it shows that competitive financial contracts can result in excessive borrowing ex ante and excessive volatility ex post. Even though, from a first-best perspective the equilibrium always displays under-borrowing, from a second-best point of view excessive borrowing can arise. The paper identifies two channels through which inefficient borrowing can emerge, a wage channel and an asset price channel. Inefficiency arises because private financial contracts fail to internalize their effect on the equilibrium volatility of these prices.

Keywords: Credit Market Imperfections, Constrained Efficiency, Credit Booms, Financial Fragility, Corporate Hedging, Fire Sales.

JEL: E32, E44, E61, G13, G18

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1 Introduction

Over the last two decades both industrialized and emerging economies have experienced "credit booms," periods of high investment, associated to a credit expansion, high asset prices and fast growth. In some cases these booms have been followed by a bust associated with low investment, bankruptcies, a credit contraction and an output contraction.[1] A common feature of many of these episodes is that the balance sheet of some agents in the economy was highly vulnerable to negative aggregate shocks because of the liabilities (or the assets) accumulated during the boom. Typical examples are: companies with excessive debt; commercial banks saddled with non-performing loans; financial institutions with large holdings of devalued real estate; emerging market companies with dollar-denominated debt. These balance sheet exposures can be a source of amplification and propagation during the downturn. One can define a macroeconomic notion of "financial fragility" as a measure of how sensitive agents' balance sheets are to aggregate shocks.[2] What determines the degree of financial fragility of an economy? In what cases does the equilibrium financial structure display excessive fragility, i.e. excessive sensitivity to aggregate shocks? This paper tackles these questions in a model where the equilibrium financial contracts, and thus the degree of aggregate fragility, are endogenously determined.

Policy discussions about excessive borrowing and excessive fragility tend to focus on the macroeconomic risks associated with the crisis episodes. However, the discussion is rarely framed in terms of *constrained efficiency*. If the private sector correctly perceives the risk of a negative shock, it will incorporate that risk in its optimal decisions. If agents still decide to borrow heavily during the boom, this means that the expected gains from increased investment today more than compensate for the expected costs of being financially constrained tomorrow. Therefore, to assess the need of policy interventions, one needs to understand how and under what conditions this private calculation leads to inefficient decisions at the social level. In this paper, I attack this question using a model of financial constraints based on limited commitment in financial contracts. I define a constrained efficient financial contract as the contract that would be chosen by a social planner facing the same financial constraints present in the private economy. I study in what cases the competitive equilibrium displays excessive borrowing and excessive fragility with respect to the social planner.

[1] The main stylized facts on credit booms and on boom-bust cycles are described in Gourinchas et al. (2001), Borio and Lowe (2001), Bordo and Jeanne (2002) and Tornell and Westermann (2002), Ranciere, Tornell and Westermann (2003).

[2] See Borio (2003) for a discussion of this notion of fragility and its relevance in recent episodes of financial instability.

In models with financial constraints little attention has been devoted to the inefficiencies associated with the boom side of a credit cycle. From a first-best perspective models with financial constraints always display under-borrowing and under-investment. It is almost tautological that in a model with borrowing constraints agents borrow *less* than they would in a frictionless world. In a dynamic framework, however, the borrowing decisions taken in one period affect the probability of being financially constrained in future periods. In this case, a borrower has to balance the desire to increase borrowing and investment today with the risk of being financial constrained in the future. This paper shows that, if one considers a dynamic model *and* one adopts a second-best perspective, over-borrowing is possible.

The paper studies a dynamic model with financial frictions due to limited commitment. There are three periods. In the first period the credit boom takes place: firms with limited internal resources finance investment by borrowing outside funds. Firms are controlled by entrepreneurs. An entrepreneur finances his initial investment by offering a profile of state-contingent payments in the following periods. However, these promises of repayment need to be credible. Lack of commitment on the entrepreneur's side implies that he can only sell to outsiders a fraction of future returns. In the intermediate period, an aggregate shock affects the firm's cash flow. If a negative shock hits and the entrepreneur has promised a large repayment, entrepreneurial net worth shrinks and investment has to be cut back. Entrepreneurs can invest more in the first period by increasing the funds they promise to repay in the intermediate period. However, this will jeopardize their financial position if the negative shock hits. Entrepreneurs face a trade-off between high investment ex ante and a high probability of being financially constrained ex post. Their choice of financial contracts determines, at the same time, the level of borrowing in the first period and the volatility of investment and aggregate activity in the following periods.

The low level of aggregate investment in the bad states of the world has two general equilibrium effects. It reduces the demand for labor, and thus wages, and it reduces the demand for capital, and thus the price of real assets. Both effects increase the rate of return for entrepreneurs in the intermediate period. This induces them to reduce borrowing in order to be able to increase investment in these states. Therefore, equilibrium prices give an incentive to entrepreneurs to protect their net worth against negative aggregate shocks, i.e. they provide a motive for net worth insurance. The main contribution of this paper is to show that there can be a *wedge* between the private and the social benefits of this insurance. In particular, I show that the social benefits may be larger than the private benefits, so that excessive borrowing arises. Inefficiency arises solely because of the financial constraint: it is not due to irrational pricing of financial assets, nor to the lack of state-contingent clauses in financial contracts, nor to the presence of government guarantees and moral hazard.[3] Ex post the economy is faced with a debt-overhang problem. Even though entrepreneurs correctly forecast the probability of the debt-overhang and have access to fully state-contingent contracts, they use them less than optimally and leave their balance sheets over-exposed to the aggregate shock.

The source of the inefficiencies studied here resides in two pecuniary externalities arising in the labor and in the asset market. As described above, a decrease in entrepreneurial net worth in a bad state of nature slows down capital accumulation and decreases the demand for assets and for labor. This generates a reduction in asset prices and in real wages. Both effects can generate an inefficient reallocation of resources. On one hand, the reduction in asset prices is most relevant in the immediate aftermath of the crisis (i.e. in the intermediate period), when entrepreneurs become *net sellers* of the assets in order to restore their financial balance. The asset price drop generates a reallocation of wealth *from entrepreneurs to the rest of the economy* when entrepreneurs' wealth is particularly scarce. On the other hand, the effect on wages is more relevant during the recovery phase of the crisis (i.e. in the final period), when the reduced capital accumulation affects labor demand. This effect generates a reallocation *from workers to entrepreneurs* in the period when entrepreneurs would like to pay back the resources received during the crisis. In other words, workers would be willing to transfer resources to entrepreneurs during the crisis in exchange for higher wages in the recovery phase. Since private contracts fail to internalize the effect of these two inefficient reallocations, excessive borrowing can arise.

Two sets of observations suggest that these two channels are empirically relevant. First, financial crises are accompanied by substantial declines in asset prices and these declines seem to be accompanied by fire sales, that is, sales of assets by agents in financial distress[4]. Second, financial crises seem to be associated to substantial declines in investment, output and real wages that are relatively long lasting[5].

[3] The role of government guarantees in generating over-borrowing is the topic of a large literature. See Mc Kinnon and Pill (1996), Corsetti et al. (1999), Tornell and Schneider (2004). The inefficiencies associated to public bailouts guarantees are relatively well understood and I abstract from them in the present paper.

[4] The behavior of asset prices is also widely documented in the papers in footnote 1. For evidence of fire sales during emerging market crises see Aguiar and Gopinath (2005). For evidence on the effect of fire sales on asset prices see Pulvino (1998).

[5] The behavior of output and investment is widely documented in the papers cited in footnote 1. For the effect on real wages see Calvo, Izquierdo and Talvi (2005). They look at systemic financial crises in emerging economies. They report a 10% decline in real wages in the first phase of a crisis and a 7% additional decline in the recovery phase.

The constrained efficiency analysis in this paper shows that the fire sales and the wage decline can be a symptom of aggregate welfare losses.

To simplify the analysis, I discuss the two channels using two separate models. For each model, I provide an analytical characterization of the inefficiency described and discuss in what circumstances inefficient borrowing is likely to arise.

The paper is organized as follows. In section 2 I introduce the baseline model where only the wage channel is present. In section 3 I introduce a notion of constrained efficiency and describe the associated planner problem. To clarify the role of the pecuniary externality, in 3.3 I first present a very basic case of constrained inefficiency, which arises in a simple deterministic economy. Then, I turn to a model with aggregate shocks and show an example of a competitive equilibrium where over-borrowing and excessive volatility are present. In the same section, I present a theoretical characterization of constrained efficient allocations, and discuss in what circumstances over-borrowing is more likely to arise. In section 4, I introduce the model with asset prices and show how over-borrowing and excessive volatility can arise in that framework. In section 5 I discuss some preliminary policy implications of the analysis. Section 6 concludes.

1.1 Related Literature

The paper is related to the large literature on the role of financial frictions in the amplification and propagation of macroeconomic shocks.[6] Existing papers have compared the volatility arising in models with financial constraints with a first-best benchmark in which no financial constraints are present. The main contribution of this paper is to conduct a complete welfare analysis from a second-best perspective and to analyze cases in which over-borrowing and excessive volatility arises. To stress that my results do not depend on the lack of non-state-contingent debt, I allow entrepreneurs to write fully state-contingent contracts. In this sense, the closer precedent to this paper is Krishnamurty (2000), which analyzes a model *à la* Kiyotaki and Moore (1998) allowing for state-contingent contracts. Krishnamurty (2000) used such a model to argue that, in presence of state-contingent contracts, the degree of amplification is smaller than in the case of non-state-contingent debt. Here, on the other hand, I allow for state-

[6] See Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Holmstrom and Tirole (1997), Aghion, Banerjee and Bachetta (2000), Tornell and Schneider (2003), Cooley, Marimon and Quadrini (2004) and references in Bernanke et al. (2001).

contingent contracts mostly for the sake of generality and I discuss only in passing issues related to amplification and propagation. Some interesting results on the degree of amplification arise in the asset price model of section 4 (see Remark 12).

From a methodological standpoint, the idea that the competitive equilibrium in economies with endogenous borrowing constraints can be constrained inefficient goes back, at least, to Kehoe and Levine (1993). They show that in an economy with limited enforcement the first welfare theorem holds when there is only one good, but fails to hold with more than one good.[7] The model presented here differs in many respects from that model, but the root of the inefficiency result is essentially the same. Private contracts fail to internalize their effect on equilibrium prices in some spot markets, and, in turns, these equilibrium prices affect the financial constraints.

Krishnamurty (2000) and Caballero and Krishnamurty (2001, 2003) use a notion of constrained efficiency analogous to the one adopted in this paper. Krishnamurty (2000) derives a result of constrained inefficiency, however, the direction of the inefficiency and the possibility of over-borrowing are not explored.[8] Caballero and Krishnamurty (2001, 2003) show the possibility of excessive external borrowing in a model of international lending. Even though the environment studied here is quite different I share their emphasis on general equilibrium effects and on pecuniary externalities. In their model financial frictions make transactions within some group of agents (domestic entrepreneurs) less costly than with other agents (outside investors) and the crucial pecuniary externality arises from a reallocation of wealth within the first group. The main structural difference between my model and theirs is that they concentrate on the effects that equilibrium prices have on the reallocation of wealth *across* entrepreneurs, while here I concentrate on the effect they have in reallocating wealth between entrepreneurs and outside investors.

The paper is also related to the large and growing literature on the optimal policy response to an expansion in private credit and to the associated asset price boom. Most of the recent literature has focused on the use of monetary policy when an investment boom is driven by an irrational fad, or "bubble."[9] A recent paper by Bordo and Jeanne (2003) attacks the problem from a different perspective. They argue that asset price movements, whether rational or irrational, can be disruptive if some agents

[7] In turns, this result is reminiscent of inefficiency results in economies with incomplete markets, Geanakoplos and Polemarchakis (1986).

[8] In fact, given that in his model entrepreneurs are always net buyers of physical capital and the only endogenous price is the price of capital, my conjecture is that in that model there is always under-borrowing.

[9] See Bernanke and Gertler (2001), Cecchetti et al. (2000), Blanchard (2001) and Dupor (2002).

in the economy hold highly leveraged positions. In this framework, they study optimal monetary policy. In this paper, I take a similar approach and show that in a world of imperfect capital markets, even in the absence of irrational mispricing, it is possible to have over-investment and excessive fluctuations in investment and output. In my framework, over-investment is simply due to the presence of pecuniary externalities and the effects of different policies can be studied in terms of standard welfare measures. Bordo and Jeanne (2003) take as given the objective function of the central bank and assume that output stabilization is desirable from a social point of view. Here, instead, I define welfare directly in terms of agents' preferences and show in what circumstances the social and the private value of financial stability differ. Another difference between the approach in this paper and that in Bordo and Jeanne (2003) is that they use a monetary model and emphasize the effects of investment on aggregate demand, while I use a real model and focus on the relation between investment, asset prices and capital accumulation.

This paper is also related to the corporate finance literature on hedging in presence of financial constraints. In particular, Froot, Scharfstein and Stein (1993) make the case that firms that faces costs of raising external finance and have a concave technology should hedge cash-flow shocks. In my model firms have a constant returns to scale technology. However, a similar motive for hedging cash-flow shocks arises in general equilibrium, since wages and asset prices drop when aggregate entrepreneurial wealth is low. Finally, the general equilibrium implications of asset liquidations by distressed firms, which plays a central role in section 4, was first analyzed in Schleifer and Vishny (1992).

2 The Model

There are three periods, $t = 0, 1, 2$, and two groups of agents: consumers/workers, C, and entrepreneurs, E. There is a large number of agents of each type, normalize the population of each type to 1. There are two goods, a consumption good and a capital good. Entrepreneurs accumulate capital in periods 0 and 1, the capital is used in period 2 in the production of consumption goods.

Both the entrepreneur and the consumers are risk neutral with linear preferences represented by the utility function $c_0^j + c_1^j + c_2^j$, where $j = C, E$. The consumers have a fixed endowment of consumption good e in each period. In period 2 they inelastically supply a unit of labor, that is employed in the entrepreneurial sector.

The entrepreneur has an initial endowment of consumption goods n_0 in period 0 and no capital. Entrepreneurs have access to the following technology to turn the consumption good into capital. On date 0 they have access to a risky technology: for each unit they invest at date 0 they obtain x units of capital at date 2, where x is stochastic and $E[x] > 1$. On date 1, they have access to a safe technology: for each unit they invest at date 1 they obtain 1 unit of capital at date 2. Letting i_0 and i_1 denote investment at date 0 and 1 total capital is equal to:

$$k = xi_0 + i_1.$$

Capital k is used to produce consumption goods at time 2 according to the constant returns to scale production function $f(k, l)$. In this section, for simplicity, investment is reversible, i.e. it is possible to have $i_1 < 0$. This implies that the price of capital will be constant and equal to 1 at date 1.

Assume that the endowment e is sufficiently large relative to the entrepreneurs' wealth n_0 so that equilibrium consumption c_t^C is positive in all periods and all states.[10] In this case the interest rate is equal to zero and all random income streams are valued at risk neutral prices.

The rate of return on early investment, x, is the only source of uncertainty in the economy, and all uncertainty is resolved in period 1. The return x is the same for all entrepreneurs, so there is only aggregate uncertainty. There is a discrete set S of states of the world. At date 1 the state of the world $s \in S$ is realized with probability π_s and productivity takes the value x_s. The assumption that $E[x] > 1$ means that early investment is on average more productive than late investment. However, early investment exposes the entrepreneur to the cash flow shock x and makes his wealth more volatile. Absent financial frictions the volatility of entrepreneurial wealth would only affect their consumption but would have no effect on investment and output. In presence of financial frictions, instead, investment and output will be positively correlated with entrepreneurial wealth and the choice between early and late investment will have implications for aggregate volatility.

Goods and factor markets are competitive at all dates. The real wage rate at date 2 is denoted by

[10] Sufficient conditions for this are

$$\begin{aligned} e &> \frac{1}{1-\theta} n_0, \\ e &> k^*, \end{aligned}$$

where k^* is the first best level of investment, and θ is a parameter determining the liquidation value of the firm's assets, they are both defined below.

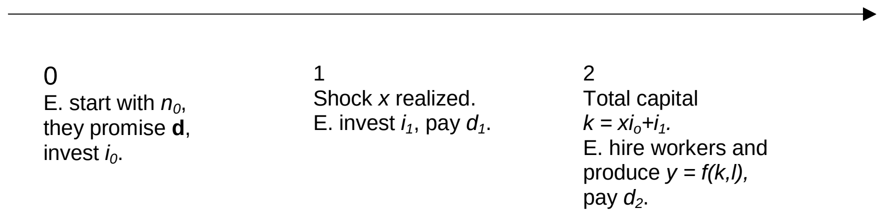

Figure 1: Timeline.

w. Because of constant returns to scale entrepreneurs profits at date 2 can be written as rk where r is the shadow rental rate on capital r defined by:

$$r \equiv \max_l \{f(1,l) - wl\}.$$

The wage rate, and hence the rate of return r, are taken as given by the individual entrepreneur.

All variables dated $t = 1, 2$, (consumption of both agents, investment, the wage and the rental rate on capital) are function of the state s. When referring to the random variable I omit the subscript s, while I use the subscript to denote a realization of the random variable.

2.1 Financial contracts with limited commitment

The financial market works as follows. At date 0 entrepreneurs offer a financial contract to consumers. A financial contract is a vector $\mathbf{d}$ that specifies state-contingent payments d_{1s} and d_{2s} from entrepreneurs to consumers in periods 1 and 2.[11] Figure 1 summarizes the timing of events for the entrepreneur.

Financial contracts are subject to default and renegotiation. At the beginning of periods 1 and 2 the entrepreneur controls the cash flow of the firm (xi_0 and rk) and can divert it for his own benefit. The entrepreneur can choose wether to use the cash flow to make the contractual payments d_{ts} or not. If he fails to make the payments d_{ts} he makes a take-it-or-leave-it offer to the outside investors regarding

[11] The payments can be positive or negative.

current and future payments. If the investors refuse the offer the firm is liquidated and the capital invested produces a flow of θ units of consumption good per unit of capital.[12] Given these assumptions a financial contract at date 0 is credible only if payments satisfy:

$$d_{1s} + d_{2s} \leq \theta i_0, \quad (1)$$

$$d_{2s} \leq \theta k_s. \quad (2)$$

If one of these inequalities is violated the entrepreneur will default and make an offer to reduce his net liabilities to θi_0 in period 1 and θk_1 in period 2. The outsiders would accept since this is the maximum they can recover upon liquidation.

The presence of default and renegotiation limits the amount of resources that entrepreneurs can credibly commit to deliver to outside investors at date 1 and 2. This makes the investment levels at dates 1 and 2 positively related to entrepreneurial wealth. There is a number of alternative models of financing constraints that generate a positive relation between entrepreneurial wealth and investment.[13] The present model of borrowing constraints is closer to Kiyotaki and Moore (1997) and emphasizes a specific form of limited commitment. The crucial difference between their model and the framework in this paper is that I allow for *state-contingent debt*. That is, I allow entrepreneurs to make their financial obligations contingent on the realization of the aggregate state.

It is useful to characterize the financial contracts in terms of the present value of the entrepreneur financial liabilities at dates 1 and 2. Define the *borrowing ratio* b_t as the ratio of total outstanding liabilities to total assets in period t:[14]

$$b_{1s} = (d_{1s} + d_{2s}) / i_0,$$

$$b_{2s} = d_{2s}/k_s,$$

[12] Assume that

$$f(k, 1) \geq \theta k,$$

for any k, so that liquidation is never optimal.

[13] For example, in Bernanke and Gertler (1986) such a relationship arises in a model with costly state verification. Holmstrom and Tirole (1996) derive it from a moral hazard problem where the entrepreneur takes an unobservable action affecting firms payoffs. Closer to my assumptions is the model of Albuquerque and Hopenhayn (2004) with limited enforcement of financial contracts.

[14] Lemma 16 in the Appendix guarantees that $i_0 > 0$ in equilibrium, so this ratio is always well defined.

with this notation the financial constraints can be written as:

$$\begin{aligned} b_{1s} &\leq \theta, \\ b_{2s} &\leq \theta. \end{aligned}$$

In this setup the capital invested offers a form of "collateral" for financial contracts. The value of an entrepreneur's outstanding liabilities cannot exceed at any point in time a fraction θ of the capital stock invested in the firm.

On the other hand, for the moment, I do not introduce any commitment problem on the consumers' side. That is, I assume that consumers can commit to make positive net present value transfers to entrepreneurs in some states of the world, $b_{ts} < 0$. The analysis in Holmstrom and Tirole (1998) studies economies where consumers have limited commitment to provide liquidity to the entrepreneurs, and a constraint of the type $b_{ts} \geq 0$ is present. In section 4 I will introduce limited commitment on the consumers' side and will study its implications.

2.2 Optimal hedging

The market for financial contracts is modeled as a Walrasian market. Entrepreneurs offer financial contracts $\mathbf{d}$ at date 0. Entrepreneurs take the price of financial contracts as given. Given that in equilibrium consumers will price financial contracts at risk neutral prices, the price of financial contract $\mathbf{d}$ is given by:

$$v(\mathbf{d}) = E[d_1 + d_2].$$

Therefore, the entrepreneur problem can be described as choosing the financial contract **d** and the investment and consumption levels that solve:

$$\begin{aligned}
&\max E\left[c_0^E + c_1^E + c_2^E\right] \\
&s.t.\ c_0^E + i_0 = n_0 + E\left[d_1 + d_2\right], \\
&\quad c_1^E + k = x i_0 - d_1, \\
&\quad c_2^E = rk - d_2, \\
&\quad d_1 + d_2 \leq \theta i_0, \\
&\quad d_2 \leq \theta k.
\end{aligned} \tag{3}$$

To characterize the optimal financial contract I study the entrepreneur's problem in two stages, using a simple recursion. Define the *entrepreneur's wealth*, or *net worth*, at date 1 as the value of the firm assets minus the total liabilities to outside investors:

$$\begin{aligned}
n_1 &\equiv x i_0 - (d_1 + d_2) \\
&= (x - b_1)\, i_0.
\end{aligned}$$

The behavior of n_1 —namely its response to the aggregate shock x— will be at the center of the analysis in this paper.

The entrepreneur problem from date 1 onwards can be written as:

$$\begin{aligned}
V(n_1, r) = \max_{c_1^E, c_2^E, k, b_2} \quad & \left[c_1^E + c_2^E\right] \\
s.t. \quad & c_1^E + k = n_1 + b_2 k, \\
& c_2^E = (r - b_2)\, k, \\
& b_2 \leq \theta.
\end{aligned}$$

It is straightforward to show that

$$V(n_1, r) = z n_1,$$

where z is stochastic and equal to

$$z_s = \frac{\partial V(n_{1s}, r_s)}{\partial n_1} = \max\left\{\frac{r_s - \theta}{1-\theta}, 1\right\}, \tag{4}$$

the variable z represents the rate of return on entrepreneurial wealth at date 1. At date 1 entrepreneurs can leverage their wealth by a factor of $1/(1-\theta)$. At date 2 they receive a net payoff of $r - \theta$ on the capital invested. Although entrepreneurs preferences are linear the presence of a financial constraint implies that the marginal utility of their wealth at date 1 is stochastic and depends on r.

From the point of view of date 0 the entrepreneur's problem can then be written as

$$\begin{aligned} \max_{c_0^E, n_1, b_1} \quad & E\left[c_0^E + V(n_1, r)\right] \\ s.t. \quad & c_0^E + i_0 = n_0 + E[b_1]\, i_0, \\ & n_1 = (x - b_1)\, i_0, \\ & b_1 \leq \theta. \end{aligned} \tag{P}$$

By choosing the state-contingent borrowing ratios b_{1s} an entrepreneur can choose different levels of investment at date 0 and different profiles of net worth n_{1s} in different states. For example, setting $b_{1s} = \theta$ in all states the entrepreneur achieves maximum leverage and can invest $i_0 = n_0/(1-\theta)$. However, given that his liabilities are fixed he obtains a volatile net worth and thus volatile investment in period 1. At the other extreme, the entrepreneur can make his debt depend on the cash-flow realizations by setting

$$b_{1s} = \theta - (\overline{x} - x_s),$$

where $\overline{x} = \max\{x_s\}$. This liability structure completely stabilizes the net worth of entrepreneurs at date 1, by making n_1 constant and equal to $(\overline{x} - \theta)\, i_0$. However, with this financial structure leverage is smaller and investment is only equal to

$$i_0 = \frac{1}{1 - \theta + \overline{x} - E[x]} n_0 < \frac{1}{1-\theta} n_0.$$

To characterize the optimal financial contract it is useful to define the expected gross rate of return on entrepreneurial wealth at date 0:

$$z_0 = E\left[z\frac{x - b_1}{1 - E[b_1]}\right]. \tag{5}$$

Entrepreneurs can leverage their wealth by a factor of $1/(1 - E[b_1])$ in period 0 and obtain a (state-contingent) net payoff of $x - b_1$ on each dollar of capital invested. This determines their wealth in period 1 that then gives a (state-contingent) return z. Notice that z_0 corresponds to the Lagrange multiplier on the budget constraint of the entrepreneur's problem P. With this notation I can characterize the optimal financial contract.

Lemma 1 *Suppose $z_s \leq z_0$ in all states s, then the optimal financial contract is characterized by the first order conditions:*

$$z_s \leq z_0, \qquad b_{1s} \leq \theta, \tag{6}$$

with at least one strict equality in each state s.

The lemma leaves aside the case $z_s > z_0$. However, if that is the case in some state s, then the optimal financial contract would entail $i_0 = 0$. Lemma 16 in the Appendix shows that this case can be ruled out in equilibrium. The two remaining cases are $z_s < z_0$ and $z_s = z_0$. When $z_s < z_0$ it is more profitable for the entrepreneur to have an extra dollar available for early investment at date 0 than to have an extra dollar in state s. In this case the entrepreneur chooses to commit all the resources available in state s in order to maximize investment at date 0. If instead $z_s = z_0$ the entrepreneur is indifferent between investment at date 0 and investment in state s and the borrowing constraint can be slack. This means that the entrepreneur is willing to leave some unused borrowing capacity in order to protect his wealth in these states of the world.

2.3 Equilibrium

A competitive equilibrium is defined by a financial contract $\mathbf{d}$ and factor prices $\{r, w\}$ such that the financial contract solves (3) and goods and factor markets clear in all periods.

The next proposition gives a characterization of the equilibrium at date 1.

Proposition 2 *There is a unique competitive equilibrium characterized by two cutoff levels x', x'' with $x' \leq x''$, such that:*

1. *If $x_s \geq x''$ then $z_s = 1$ and $b_{1s} = \theta$ (first-best investment, borrowing capacity is exhausted)*;

2. *If $x' \leq x_s < x''$ then $z_s > 1$, z_s is increasing in x_s and $b_{1s} = \theta$ (borrowing capacity is exhausted);*

3. *If $x_s < x'$ then $z_s = z_0$ and $b_{1s} = \theta - (x' - x_s) < \theta$ (borrowing capacity is slack).*

The characterization above is driven by the equilibrium relation between the level of entrepreneurial net worth and the equilibrium rate of return z. For high realizations of x_s entrepreneurial net worth is large and can finance the first-best level of investment at date 1. In these states the outside finance premium $z_s - 1$ is zero. If productivity is in an intermediate range entrepreneurs are credit constrained at date 1, capital is scarcer and the return z is higher. As long as z is smaller than z_0 entrepreneurs borrow up to the maximum and save no funds. When productivity falls below the level x' entrepreneurial capital becomes so scarce and the rate of return z so high that entrepreneurs prefer to borrow less than the maximum amount in order to protect their net worth in these states of the world. Figure 2 illustrates the equilibrium relation between the shock x and four variables: the rate of return on entrepreneurial wealth, z, the borrowing ratio, b_1, entrepreneurial wealth, n_1, and investment, k.

Everyone in this economy is risk neutral, however the concavity of f and the presence of financial constraints induces a motive for insuring the net worth of entrepreneurs. In the absence of financial frictions investment would be independent of entrepreneurial net worth and the return to entrepreneurial net worth would be constant and equal to 1. When financial frictions are present, though, investment depends on entrepreneurial net worth and the rate return on capital is negatively related to the total net worth of entrepreneurs in the economy. This generates a motive for financial stability. The economy faces a trade-off between financial stability ex post and investment ex ante. Given the limited collateral available in good states, in order to raise additional funds at date 0 entrepreneurs must promise part of the collateral available in bad states. By doing so, they reduce their net worth in bad states and this increases the return z_s.[15]

The degree of financial stability obtained in a competitive equilibrium depends on the slope of the relation between z and entrepreneurial net worth. Let n_1 be the equilibrium level of entrepreneurial net

[15] It is useful to point out that a trade-off between investment ex ante and stability ex post would arise also in a model with state contingent θ, provided that the difference $x_s - \theta_s$ is increasing in x_s.

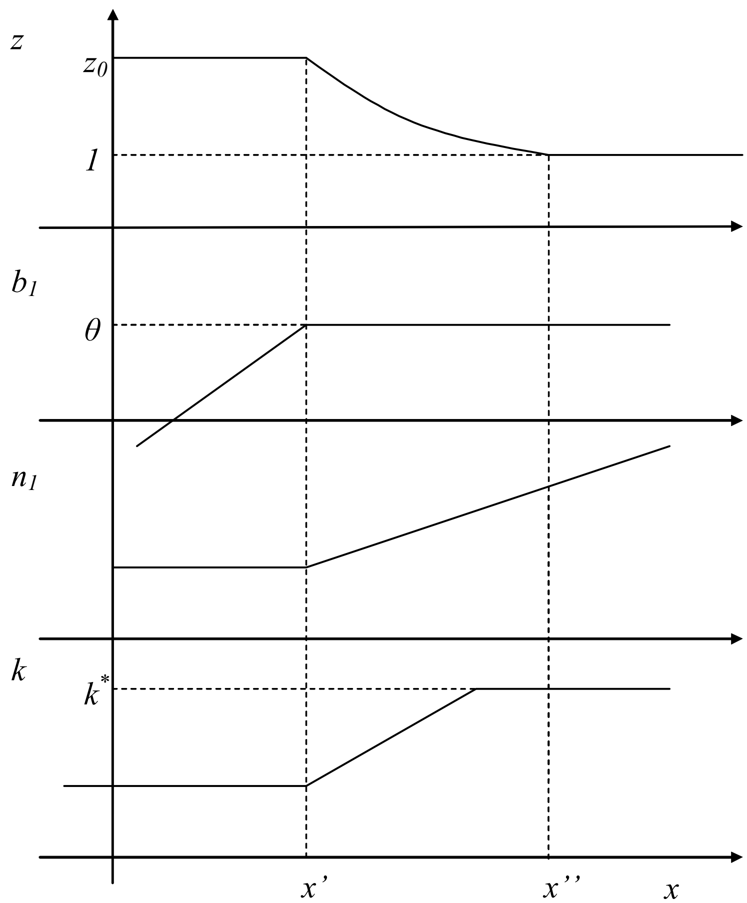

Figure 2: Equilibrium rates of return, borrowing, net worth and investment.

worth in the economy. This relation can be written as $z = h(n_1)$, where the function h is defined by

$$h(n) = \max \left\{ \frac{f_K\left(\frac{n}{1-\theta}, 1\right) - \theta}{1-\theta}, 1 \right\}. \tag{7}$$

The form of the function h depends on the production function f. I can capture the effective "risk aversion" of the entrepreneurial sector, i.e. the desire for net worth insurance in the economy defining the coefficient of absolute risk aversion

$$\sigma(n) = \left| \frac{h'(n)}{h(n)} \right|$$

Using this definition a simple comparative statics result follows.

Proposition 3 *Consider two economies that differ only for the production function, which is f in the first economy and $\hat{f}$ in the second. Suppose the associated functions h and $\hat{h}$ are such that the first economy has a higher degree of absolute risk aversion, —i.e. $\hat{\sigma}(n) > \sigma(n)$ for all n. Let b_1 and $\hat{b}_1$ be the equilibrium borrowing ratios in the two economies, then*

$$\begin{aligned} \hat{b}_{1s} &\leq b_{1s} \text{ for all } s \\ \hat{\imath}_0 &< i_0 \\ Var\left(\ln \hat{k}\right) &< Var\left(\ln k\right). \end{aligned}$$

If the production function $\hat{f}$ is associated to a steeper function $\hat{h}$, this induces a more cautious behavior on the part of entrepreneurs, and results in smaller investment ex ante and a greater degree of financial stability. A special case is the case of a linear production function, in this case z is constant, the economy is "risk neutral", in equilibrium $b_{1s} = \theta$ in all states and the economy displays maximum investment and maximum volatility.

3 Efficiency

3.1 First-best benchmark

Consider now the efficiency properties of the competitive equilibrium. As a first step, I briefly characterize the allocation arising in an economy with no financial constraints. This is the first-best benchmark.

Since $E[x] > 1$ all the endowment available at date 0 is invested[16] so

$$i_0 = e + n_0.$$

In period 1 r and k are constant across states and satisfy:

$$\begin{aligned} r_s &= r^* = 1, \\ k_s &= k^*, \end{aligned}$$

where k^* satisfies $f_K(k^*, 1) = 1$.

Absent financial frictions, investment at date 1 and output at date 2 are independent of the cash flow shock, x. The following proposition summarizes the comparison between the first-best and the competitive economy.

Proposition 4 *There is a cutoff n^* such that if $n_0 \geq n^*$ the competitive equilibrium coincides with the first-best allocation. If $n_0 < n^*$ then*

$$\begin{aligned} i_0 &< e + n_0, \\ k_s &\leq k^*, \\ Var(\ln k) &> Var(\ln k^*) = 0. \end{aligned}$$

Not surprisingly, from a first-best point of view the model can only display under-borrowing and under-investment. In the first-best economy there is no trade-off between investment and financial stability. Since entrepreneurs have unlimited access to outside funds the economy can achieve maximum investment at date 0 and maximum stability at date 1.

3.2 Constrained Efficient Financial Contracts

Let us turn to second-best analysis. Consider a planner that, at date 0, can set the financial contract **d** and make a transfer τ between consumers and entrepreneurs. The rest of the allocation is determined by competitive markets as in the previous section. In particular, the planner does not intervene in the

[16] In the first-best allocation $c_0^C = 0$, and the shadow interest rate is equal to $E[x] > 1$.

financial market at date 1 or in the factor markets at date 2. The financial contract is still subject to default and renegotiation, so the planner faces the same constraints faced by private contracts, i.e. (1) and (2). The contract that solves this social planner problem is dubbed a "constrained efficient financial contract". The point of this exercise is to understand the inefficiencies associated with the choice of the financial contracts at date 0, taking as given the contractual frictions in the economy.

By choosing the financial contract at date 0 the planner can affect investment at date 0 and the profile of net worth at date 1. In particular the financial contract **d** determines the level of entrepreneurs' wealth:

$$n_1 = xi_0 - d_1 - d_2,$$

and the equilibrium capital stock will be given by:

$$k = \min\left\{k^*, \frac{1}{1-\theta}n_1\right\}.$$

The factor prices w and r can then be written as a function of entrepreneurs' wealth n_1

$$\begin{aligned} w(n_1) &= f_L(k,1), \\ r(n_1) &= f_K(k,1). \end{aligned}$$

Consumers expected utility is given by:

$$\begin{aligned} &E\left[e - \tau + (e + d_1) + (e + w + d_2)\right] = \\ &E\left[3e - \tau - b_1 i_0 + w\right]. \end{aligned}$$

The entrepreneurs utility at date 1 can be written as $V(n_1, r)$ using the indirect utility function derived in the previous section.

Therefore, the planner problem can be written in terms of the borrowing ratio b_1 and of the transfer τ.

$$\max_{c_0^E, n_1, b_1, \tau} E \quad \left[c_0^E + V\left(n_1, r\left(n_1\right)\right)\right] \tag{P'}$$

$$\begin{aligned} s.t. \quad & c_0^E + i_0 = n_0 + \tau, \\ & n_1 = \left(x - b_1\right) i_0, \\ & b_1 \leq \theta, \\ & E\left[3e - \tau + b_1 i_0 + w\left(n_1\right)\right] \geq U^C \end{aligned}$$

where U^C is the level of consumers' utility. As in the previous section, I am assuming that a constrained efficient allocation is characterized by $c_t^C > 0$. Let $\hat{U}^C$ be the consumers utility in the competitive equilibrium. Let $\hat{b}_1$ and $\hat{\imath}_0$ be the competitive equilibrium levels of borrowing and investment. A competitive equilibrium is constrained efficient if $\hat{b}_1$ and $\hat{\tau} = \hat{b}_1 \hat{\imath}_0$ solve problem P' for $U^C = \hat{U}^C$.

Since the competitive equilibrium is characterized by problem P the analysis in this section amounts to a comparison between problems P and P'. The crucial difference between problems P and P' is that factor prices are taken as given in the former and are endogenous in the latter. The reason why factor prices have welfare consequences is that they affect the total amount that entrepreneurs can commit to pay to the rest of the economy in future periods. In private financial contracts entrepreneurs commit *directly* to make payments that are backed by the collateral θk_t. However, on top of that, by choosing a given investment policy entrepreneurs commit *indirectly* to pay the equilibrium wage level on the spot labor market in period 2. If they increase the demand for labor at date 2 by accumulating more capital they indirectly commit to pay higher wages.

A very basic example can illustrate the potential sources of inefficiency in this model. This is the case where x is non-stochastic and is equal to 1. In this case periods 0 and 1 can be collapsed into one period and the only effective choice variable for the planner is a pure transfer from workers to entrepreneurs at date 0. This simple case does not bear directly on the theme of the paper, i.e. the efficient choice of the borrowing ratios b_1. However, it provides a very useful starting point for the analysis of the general case.

3.3 A deterministic example

Consider the case where x is non-stochastic and equal to 1. Since $x = 1$ the entrepreneurs' borrowing at date 0 is irrelevant, given that the return on the private project is identical to the market interest rate. Therefore, the choice of b_1 is irrelevant and can be set to 0 without loss of generality. In this case the only choice of the planner is the transfer τ and the planner problem can be written as:

$$\begin{aligned} \max_{c_0^E, n_1, \tau} \quad & c_0^E + V(n_1, r(n_1)) \\ s.t. \quad & n_1 = n_0 + \tau - c_0^E, \\ & [3e - \tau + w(n_1)] \geq U^C. \end{aligned}$$

Essentially, the planner can only transfer resources from consumers to entrepreneurs at date 0. This increases entrepreneurs' net worth, and thus investment and wages at date 2.

Can a simple transfer like this increase the ex ante utility of *both* the entrepreneur and the consumer? The answer is positive if in the competitive equilibrium $w'(n_1) > 1$ and $dV/dn_1 > 0$. In this case the transfer is on net beneficial for the consumers and leads to a Pareto improvement. Consumers are not willing to make the transfer τ individually because it only entails a loss at the individual level. However, since it relaxes the financial constraint of the firms and induces more investment and higher wages, the transfer results in a net benefit for consumers.

Suppose the production function takes the form

$$f(k, l) = \theta k + k^\alpha l^{1-\alpha}.$$

Let us describe the utility possibility frontier, i.e. the pairs $\left(U^E(\tau), U^C(\tau)\right)$ corresponding to different levels of the transfer τ. Suppose $n_0 + \tau < n^*$. Then the welfare of entrepreneurs and consumers are given by the following pair of parametric equations:

$$\begin{aligned} U^E(\tau) &= \alpha (1-\theta)^{-\alpha} (n_0 + \tau)^\alpha, \\ U^C(\tau) &= (1-\alpha)(1-\theta)^{-\alpha} (n_0 + \tau)^\alpha - \tau. \end{aligned}$$

For $n_0 + \tau > n^*$ investment is at its first-best level and the utility possibility set is linear. The utility possibility frontier is depicted in Figure 3. The Pareto frontier corresponds to the decreasing part of it.

In this simple case there are two cutoffs,[17] n^* and n^o, such that: (1) if $n_0 > n^*$ the economy achieves first-best investment and the Pareto frontier is linear; (2) if $n_0 \in (n^o, n^*)$ the equilibrium is in the decreasing section of the utility possibility frontier and the equilibrium is constrained efficient; (3) if $n_0 < n^o$ the equilibrium is in the increasing section of the utility possibility frontier and the equilibrium is constrained inefficient. In the last case a $\tau > 0$ leads to a Pareto improvement.

Notice that the possibility of a Pareto improvement depends crucially on two things: (1) the presence of first-best inefficiency in investment ($f_K > 1$), (2) the presence of a pecuniary externality ($w'(n_1) > 0$). Since $f_K > 1$ it is possible to increase the social surplus $f(k, 1) - k$ by transferring resources to the entrepreneurs. However, this surplus will be realized in period 2 and the very problem of this economy is that entrepreneurs cannot commit to transfer resources to consumers. The pecuniary externality, i.e. the effect of k on wages, matters because it allows the economy to reallocate some of the increased surplus to the workers. When the pecuniary externality is sufficiently strong a Pareto improvement is possible, since consumers receive enough of the increased surplus through a wage increase.

As a further interpretation, notice that the inequality $w'(n_1) > 1$ holds when the following condition holds

$$\theta + f_{LK} > 1. \tag{8}$$

Entrepreneurs can make direct and indirect commitments to pay out future output. The term θ correspond to the direct commitments they make on financial markets. The term f_{LK} correspond to the indirect commitment to pay out future output in the form of wages. When the sum of the two terms is greater than one it means that, from the social point of view, the financial constraint is not locally binding: entrepreneurs can increase investment by dk and at the same time increase the pledgeable fraction of output by *more than dk*. In this way, from a social point of view, the investment dk can be fully financed with outside funds.

[17] The two cutoffs are given by

$$\begin{aligned} n^* &= \alpha^{\frac{1}{1-\alpha}} (1-\theta)^{-\frac{\alpha}{1-\alpha}}, \\ n^o &= (1-\alpha)^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} (1-\theta)^{-\frac{\alpha}{1-\alpha}}. \end{aligned}$$

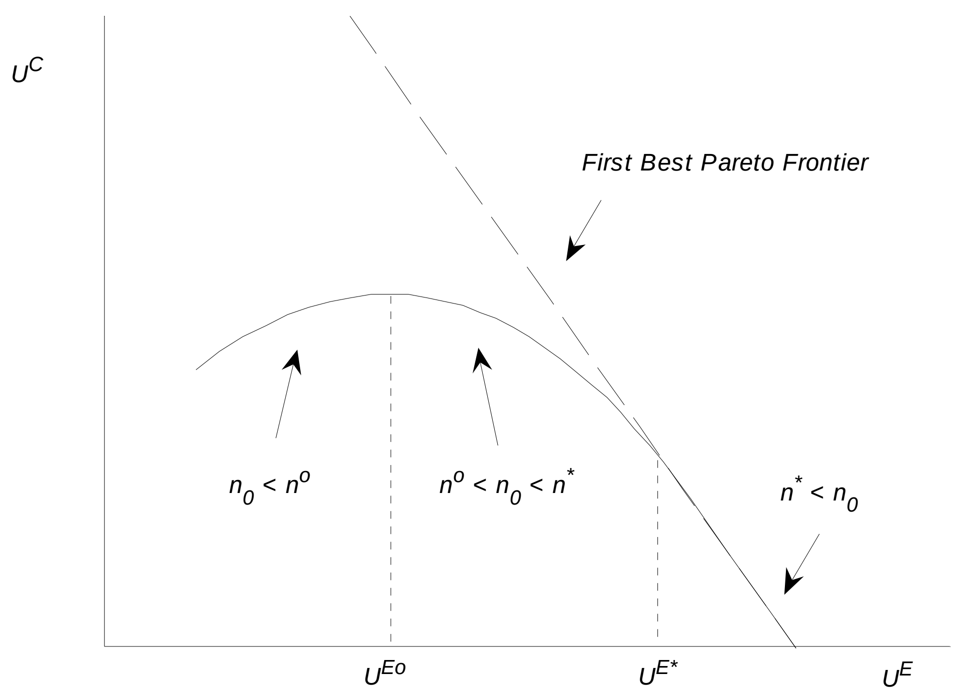

Figure 3: Constrained efficiency in the deterministic case.

3.4 Investment and financial stability: an example

Now, let us turn to the characterization of the constrained efficient allocation in the general case where x is stochastic, and describe the social planner problem in terms of a trade-off between investment and financial stability analogous to the one faced by the single entrepreneur. Now the planner can induce entrepreneurs to reallocate their wealth to the states of world where the social benefit of investment is the largest. Section 2 shows that the private sector faces market incentives to contain financial instability. In this Section I show that the market incentives may not reflect the marginal value of entrepreneurial wealth in different states of the world from a social point of view.

Let us begin with a simple example that illustrates the possibility of inefficient volatility and excess borrowing. Then, I will characterize constrained efficient financial contracts in general and derive the theoretical result that lies behind the example.

Consider the following two states example. The production function is

$$f(k,l) = \theta k + k^{\alpha} l^{1-\alpha},$$

with $\theta = 1/2$ and $\alpha = 1/2$. Let x take the two values $\{5/6, 2\}$ with equal probability. The initial wealth of entrepreneurs is $n_0 = 3/16$.

In competitive equilibrium there are maximal borrowing ratios, i.e. $b_{1,s} = \theta$ for $s = 1, 2$. The marginal utilities of entrepreneurial wealth at date 0 and at date 1, in states 1 and 2, are:

$$z_0 = 2.166 > z_1 = 2, \quad z_2 = 1,$$

that is, the return on early investment is greater than the return on late investment, irrespective of the aggregate shock. Therefore, entrepreneurs find it optimal to maximize investment at date 0 and choose maximal borrowing ratios.

Now, suppose the planner requires entrepreneurs to reduce the borrowing ratio in the low state to

$$\tilde{b}_{1,1} = 0.243 < \theta,$$

and enforces a transfer

$$\tilde{\tau} = 0.146$$

at date 0 between consumers and entrepreneurs. Table 1 illustrates the resulting allocation and compares it with the competitive equilibrium allocation. The reduction in borrowing at date 0 leads to an allocation that Pareto dominates the competitive equilibrium. In particular, $\tilde{b}$ and $\tilde{\tau}$ achieve a constrained efficient allocation that leaves the entrepreneurs as well off as under the competitive allocation. The constrained efficient Pareto frontier for this economy is plotted in Figure 4.

	Competitive equilibrium	Constrained efficient allocation
b_1	$\begin{cases} 0.5 \\ 0.5 \end{cases}$	$\begin{cases} 0.243 \\ 0.5 \end{cases}$
i_0	0.375	0.333
n_1	$\begin{cases} 0.125 \\ 0.563 \end{cases}$	$\begin{cases} 0.197 \\ 0.5 \end{cases}$
k	$\begin{cases} 0.25 \\ 1 \end{cases}$	$\begin{cases} 0.40 \\ 1 \end{cases}$
U^E	0.406	0.406
U^C	0.375	0.385

Table 1. Over-borrowing Example

By reducing the level of investment at date 0 and reducing the borrowing ratio in the low state the planner induces a more stable profile for entrepreneurs' wealth, for investment and for output. The gains associated with the increase in financial stability can be illustrated using a graph analogous to the one in Figure 3. Figure 5 plots the realized payoffs u_E and u_C for the entrepreneur and the consumers in the two states of the world at date 1. The two curves in the figure represent the combination of payoffs that are feasible at date 1, *for a given level of investment* i_0. Each curve can be derived exactly as the utility possibility frontier in Figure 3, by reallocating wealth between consumers and entrepreneurs. The outermost curve corresponds to the investment level in competitive equilibrium (0.375) while the innermost curve corresponds to the investment level at the constrained efficient allocation (0.333). Each

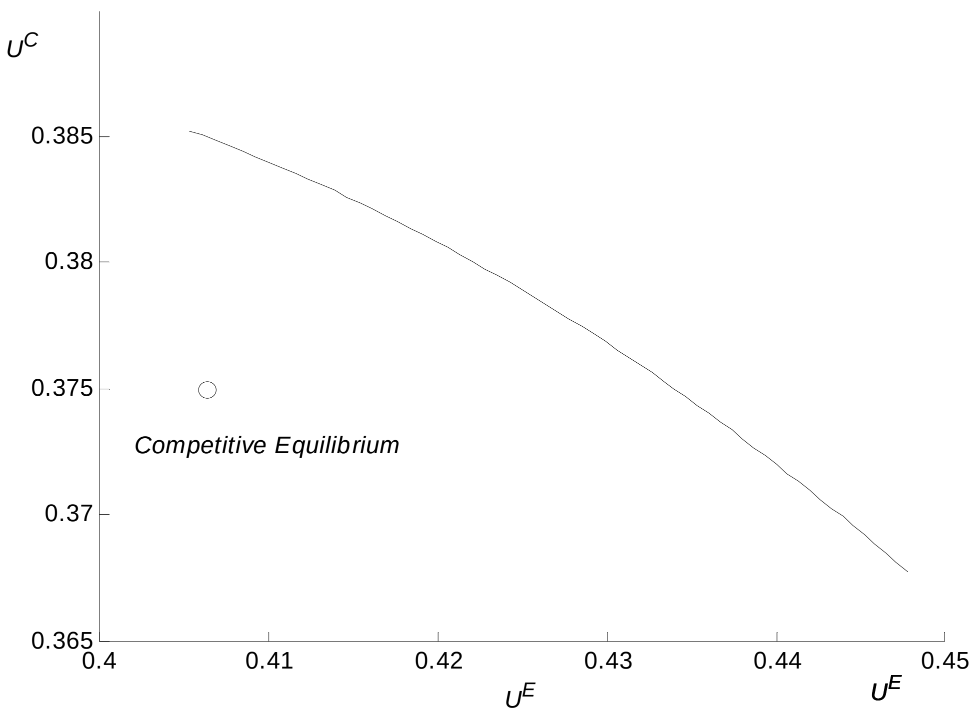

Figure 4: Constrained efficient frontier.

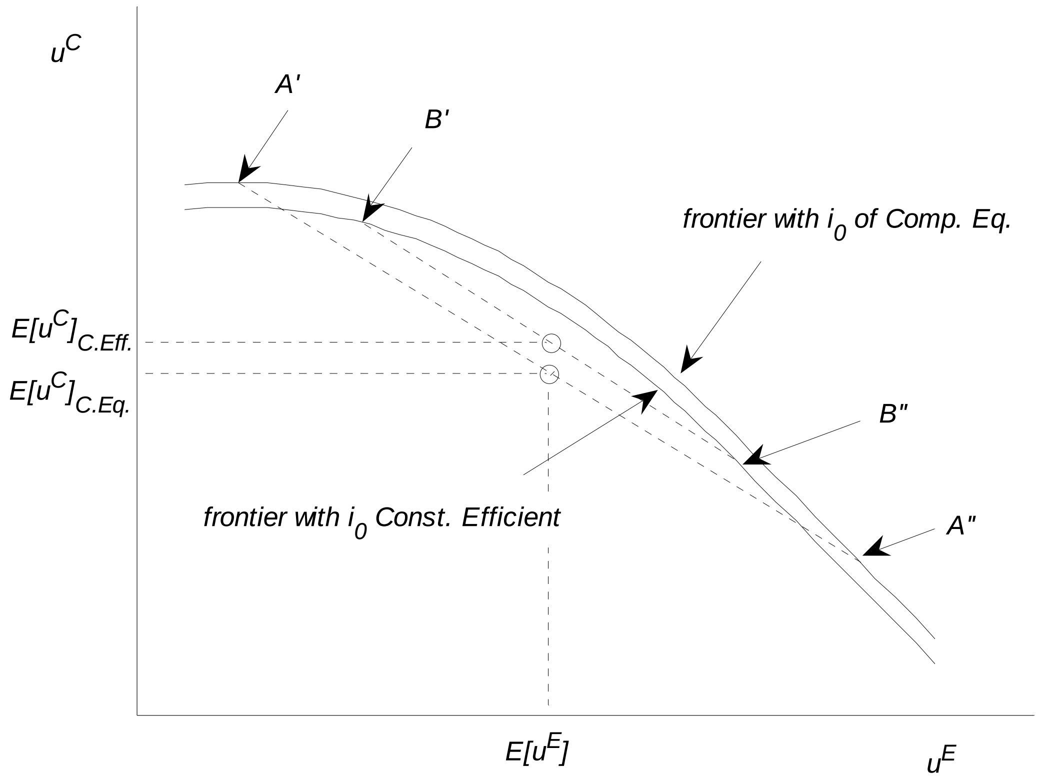

Figure 5: Utility possibility frontiers at date 1. Comparison of the competitive and efficient allocations.

allocation also determines the levels of n_1 in the two states of the world and thus the two points selected on the utility possibility frontier. In the competitive equilibrium the two points are denoted A' and A'', while in the constrained efficient allocation they are B' and B''. By taking a convex combination of the two points one can derive the ex ante expected utility in the two allocations. The picture shows that the expected utility for the entrepreneurs is the same in the two allocations, while for the consumers it is higher at the efficient allocation.

While in Figure 3 I emphasized the non-monotonicity of the relation between the payoffs of the two agents, in Figure 5 I want to emphasize the concavity of this relation. The concavity of the utility possibility frontier at date 1 means that a more stable profile for entrepreneurs' wealth is associated to higher expected payoffs for both agents. The surplus gain at low levels of n_1 is larger than the surplus loss at low levels of n_1.

Even though the competitive equilibrium entails higher investment and more resources available at date 1, it also implies a more volatile allocation of resources at date 1. Because of financial constraints the volatility of the allocation at date 1 determines a surplus loss. As the figure shows the losses due to lower investment are more than compensated by the gains in terms of increased financial stability.

3.5 Investment and financial stability: analytics

Under what circumstances are the social gains from financial stability larger than the losses associated to lower investment? The following two propositions characterize constrained efficient financial contracts and allow us to answer this question. First, I show that a constrained efficient financial contract is characterized by optimality conditions analogous to the optimality conditions for the individual entrepreneur in equilibrium. The difference between the social and the private margins account for the possibility of constrained inefficiency. Second, I investigate under what circumstances the inefficiency takes the form of excess borrowing and excess volatility at date 0.

Let $\tilde{b}_1$ be entrepreneurs liabilities in a constrained efficient financial contract and $\tilde{n}_1$ be the corresponding net worth.

Proposition 5 *Suppose a constrained efficient financial contract has $\tilde{c}_t^C > 0$ at all t. Then there is a constant $\tilde{z}_0$ and a random variable $\tilde{z}$ such that:*

$$\begin{aligned} \tilde{b}_{1s} &\leq \theta \\ \tilde{z}_s &\leq \tilde{z}_0 \end{aligned} \tag{9}$$

with at least an equality in each state. The $\tilde{z}_0$ and $\tilde{z}$ satisfy

$$\begin{array}{ll} \tilde{z}_s = z_s + \frac{\tilde{z}_0 - 1}{1-\theta} f_{LK} & \text{if } \tilde{n}_{1s} < n^*, \\ \tilde{z}_s \in \left[z_s, z_s + \frac{\tilde{z}_0 - 1}{1-\theta} f_{LK} \right] & \text{if } \tilde{n}_{1s} = n^*, \\ \tilde{z}_s = z_s & \text{if } \tilde{n}_{1s} > n^*, \end{array} \tag{10}$$

$$\tilde{z}_0 = E\left[\tilde{z} \frac{x - \tilde{b}}{1 - E\tilde{b}} \right], \tag{11}$$

where z is defined in (4).

Notice the similarity between the first order conditions (9) and the first order conditions (6) characterizing the equilibrium financial contract. The expression in (10) can be interpreted similarly to expression (4): it gives the social rate of return on entrepreneurial wealth at date 1. The difference between z and $\tilde{z}$ is that the first only captures the private return on an extra dollar of net worth n_1, while the second captures the additional effect that an extra dollar of net worth has on wages. When entrepreneurs are constrained at date 1 an extra dollar of net worth increases investment, thus it increases labor demand at date 2 and it increases wages by

$$dw = f_{KL}\frac{1}{1-\theta}.$$

An increase in entrepreneurial wealth generates a pecuniary transfer from entrepreneurs to workers, if this pecuniary transfer was incorporated in private contracts it would increase the resources available to entrepreneurs at date 0 and reduce their resources at date 2. Since the entrepreneurs' marginal value of wealth at date 0 is $\tilde{z}_0$ while their marginal value of wealth at date 2 is 1 the total welfare effect of this transfer corresponds to $(\tilde{z}_0 - 1)\, dw$. As in the deterministic case, the presence of first-best inefficiency $(\tilde{z}_0 > 1)$ means that there is a surplus gain from reallocating resources to the entrepreneurs, while the presence of a pecuniary externality allows the surplus gain to be partly allocated to consumers.

In order to discuss over-borrowing and excess volatility let us introduce a function $\tilde{h}$ analogous to the function h introduced in 2.3, that, to each level of net worth n_1, associates the corresponding social rate of return $\tilde{z}$.[18] As the slope of h captures the "risk aversion" of the private sector, the slope of $\tilde{h}$ captures the social degree of "risk aversion". The possibility of over-borrowing and excess volatility is related to the relative slopes of these two functions. In particular, what matters is an appropriate measure of "absolute risk aversion". Given that $\tilde{h}$ is non-differentiable I use the following definition of "absolute risk aversion".

[18] The function $\tilde{h}$ is, more precisely, a correspondence and is defined by

$$\tilde{h}(n) = \begin{cases} \frac{f_K - \theta + (\tilde{z}_0 - 1) f_{KL}}{1-\theta} & \text{if } n < n^*, \\ \left[\frac{f_K - \theta}{1-\theta}, \frac{f_K - \theta + (\tilde{z}_0 - 1) f_{KL}}{1-\theta}\right] & \text{if } n = n^*, \\ 1 & \text{if } n > n^*, \end{cases} .$$

where $k = \frac{n}{1-\theta}$.

Definition 6 *The function $\tilde{h}$ displays a higher degree of absolute risk aversion than h iff*

$$\frac{\tilde{h}(n')}{\tilde{h}(n)} \geq \frac{h(n')}{h(n)}$$

for any pair $n' < n$.

I can now compare the competitive allocation with the constrained efficient financial contract. Let b_1 be entrepreneurs liabilities in the competitive equilibrium. Take any constrained efficient allocation that weakly dominates the competitive allocation. Let $\tilde{b}_1$ be the value of entrepreneur liabilities in this allocation.

Proposition 7 *Suppose $\tilde{h}$ displays a higher degree of absolute risk aversion than h. Then the following inequalities hold*

$$b_{1s} \geq \tilde{b}_{1s} \text{ for all } s,$$

$$Var(\ln k) \geq Var\left(\ln \tilde{k}\right).$$

That is, either the equilibrium is constrained efficient or the equilibrium displays excess borrowing and excess volatility.

At this point the next question is: under what conditions is $\tilde{h}$ more risk averse than h? In general this depends on the production function. However, independently of the production function there is a crucial effect that tends to make $\tilde{h}$ more risk averse than h. In states in which entrepreneurial wealth n_{1s} is sufficiently high the economy achieves the first-best level of investment and $z_s = \tilde{z}_s = 1$. This happens because, at high levels of entrepreneurial wealth, the pecuniary externality captured by $w'(n_1)$ is muted and the social and private return on entrepreneurial wealth are identical. For lower levels of wealth, however, the pecuniary effect is positive and $\tilde{z}_s > z_s$. This tends to make $\tilde{h}$ more risk averse than h.

With a general production function the curvature of f can undo this effect and it is possible to construct examples in which there is too little borrowing and too little volatility in equilibrium. A sufficient condition for over-borrowing is

$$\frac{f_{KK}}{f_K - \theta} \leq \frac{f_{LKK}}{f_{LK}}$$

This condition is satisfied by the Cobb-Douglas production function used in the example above.

The proposition above gives a result in terms of borrowing ratios: in equilibrium the ratio of borrowed funds to total assets $E[b_1]$ is too large. The constrained efficient level of investment i_0, on the other hand, depends on the way in which the surplus is divided between consumers and entrepreneurs. Suppose one compares the competitive equilibrium with the constrained efficient allocation that leaves the entrepreneurs indifferent. Then, in the case of a Cobb-Douglas production function, it is possible to prove that efficient investment is lower than equilibrium investment. In particular, this applies to the example illustrated in Table 1, where, indeed the economy displays excess investment.

3.6 Volatility and shocks to long run productivity

I conclude this section with some remarks on the effects of changing the volatility of x and on the effects of shocks to long run productivity.

Remark 8 *Suppose there is no uncertainty regarding x. Then both the competitive equilibrium and the constrained efficient allocation entail $b = \tilde{b} = \theta$. If $w'(n_1) < 1$ the competitive equilibrium is constrained efficient.*

When there is no uncertainty regarding x and $x > 1$ the social return on investment at date 0 $\tilde{z}_0$ is equal to $\tilde{z}(x-\theta)/(1-\theta)$ and it is always the case that $\tilde{z}_0 > \tilde{z}$. In this case delaying investment is never beneficial neither from a private nor from a social point of view. As there is no trade-off between investment and volatility maximum investment is always optimal. The equilibrium can still be constrained inefficient, through the channel discussed in 3.3. However, excess borrowing can only arise in the presence of uncertainty. This is hardly surprising: excess borrowing arises only as a side effect of excess volatility, and volatility is absent when x is non-random.

Apart from this limit case it is not easy to obtain comparative statics results for changing levels of the volatility of x. Let us look at the relation between volatility and efficiency in the example above. To look at changes in volatility I fix $E[x]$ and vary $x_2 - x_1 = \Delta$. For each Δ I compute the mean borrowing ratio $E[b]$ at the competitive equilibrium and at the constrained efficient allocation that makes entrepreneurs indifferent. The mean borrowing ratios are plotted in Figure 6. For low levels of volatility ($\Delta > 0.42$) the gains from financial stability are low, both from the private and from the

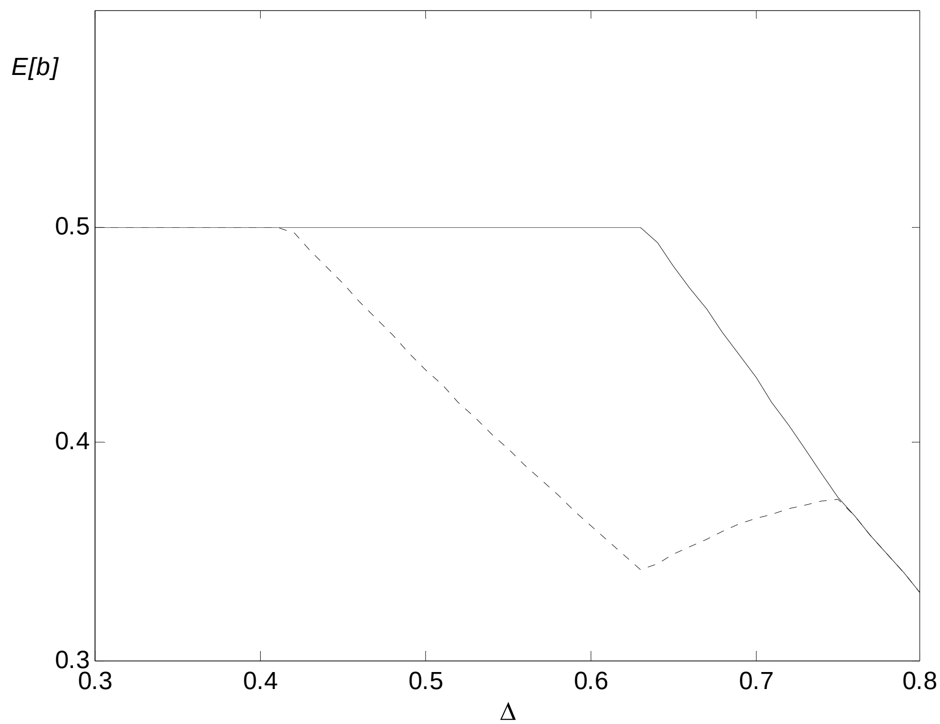

Figure 6: Mean borrowing ratios and the volatility of x.
Solid line: competitive equilibrium. Dotted line: constrained efficient allocation.

social point of view. Both the competitive equilibrium and the constrained efficient allocation display maximum leverage, with $E[b] = \theta$. On the other hand, for high levels of volatility ($\Delta > 0.75$) the gains from financial stability are large. In equilibrium entrepreneurs respond to the high volatility by reducing early investment i_0. This implies that the capital stock is low in both states, i.e. we have $k_s < k^*$ for $s = 1, 2$. When this is the case, with a Cobb-Douglas technology, the private and the social margins are perfectly aligned and the equilibrium borrowing ratios are constrained efficient.[19] For intermediate levels of volatility ($0.42 < \Delta < 0.75$), instead, the capital stock is first-best efficient in the good state, $k_2 = k^*$. In this case the effect on wages is zero in the high state, $w'(n_{1,2}) = 0$ and positive in the low state $w'(n_{1,1}) > 0$. This means that the social gains from financial stability are larger than the private gains, given that the ratio $\tilde{z}_2/\tilde{z}_1$ is larger than the ratio z_2/z_1. In this intermediate region excess borrowing arises.

[19] This follows from the fact that with a Cobb-Douglas f_{LK} and $z = \frac{f_K - \theta}{1-\theta}$ are proportional for each level of k.

In our model the shock x affects entrepreneurial wealth in the intermediate period but it does not affect the long run productivity of the project, i.e. the production function f is unaffected. In general it may be more realistic to assume that the realization of the shock x is positively correlated with long run productivity. It is easy to modify the model to allow for shocks to long run productivity. Consider a simple example where the production function takes the form

$$\theta k + a k^{\alpha} l^{1-\alpha}$$

and a is stochastic and positively correlated with x. In presence of this correlation the benefits from financial stability are weaker, both from the private and from the social point of view. The following proposition describes a case in which the correlation between x and a completely eliminates any incentive for financial stability, both at the private and at the social level.

Proposition 9 *Suppose productivity a is random and*

$$a_s = \kappa (x_s - \theta)^{1-\alpha}$$

then both the competitive equilibrium and the constrained efficient allocation entail $b_s = \tilde{b}_s = \theta$ for all s, independently of the volatility of x. If $E\left[w'\left(n_1\right)\right] < 1$ then the equilibrium is constrained efficient.

In this case the return on entrepreneurial wealth is constant across states, even though entrepreneurial wealth is volatile. Therefore, z is constant and z_0 is always larger than z_s for each state s. Here, entrepreneurs' net worth tends to be low precisely in those states in which productivity is low and the two effects offset each other. When a is stochastic the first-best level of investment $k^*\left(a\right)$ is also stochastic and is proportional to $a^{\frac{1}{1-\alpha}}$. On the other hand, in the economy with financial frictions the equilibrium level of investment is proportional to $a^{\frac{1}{1-\alpha}}$ since x is perfectly correlated with a. In terms of output volatility this means that

$$Var\left[\ln y\right] = \frac{1}{1-\alpha} Var\left[\ln a\right] = Var\left[\ln y^*\right]$$

even though $y < y^*$ in each state.

The result above can be used to argue that constrained inefficiency is more likely to arise when the shocks that affect entrepreneurs balance sheets in the short run are weakly correlated with long run productivity. This hints to the idea that a credit expansion associated to "non-fundamental" movements in asset markets is more likely to be socially costly.

4 A Model with Asset Prices and Fire Sales

This section presents a model with endogenous asset prices. The focus is on the effect of asset price movements on the entrepreneurs' balance sheet at date 1 and on the feed-back between net worth and asset prices. I define a "fire sale" as a state of the world in which entrepreneurs have to sell part of their capital stock in order to satisfy the financial constraint. During a fire sale entrepreneurs are *net sellers* of assets, so a drop in asset prices has a negative effect on entrepreneurs' balance sheet. In this section, I analyze the pecuniary externality associated to this drop in asset prices. The main point of this section is to show that the pecuniary externality arising during fire sales can determine excessive borrowing from an ex ante point of view.

To study the asset price channel separately from the wage channel studied above consider a variant of the economy described in section 2 with no labor and a linear production function. I introduce a second sector in the economy, the traditional sector, which will absorb the capital of the entrepreneurs in the intermediate period in the event that entrepreneurs need to sell some of their capital stock. The fact that investment is irreversible at the aggregate level and that the traditional sector has a concave technology implies that the price of capital will be endogenous and will depend on the amount of capital that the entrepreneurs are selling.

There are three periods,$t = 0, 1, 2$, and two goods, consumption and capital. There is a unit mass of entrepreneurs and a unit mass of consumers. Each consumer receives an endowment e of the consumption good each period. The entrepreneurs have an endowment n_0 in period 0. Both consumers and entrepreneurs have linear utility as in section 2. As in section 2 I assume that the consumers endowment e is large relative to n_0 so that the equilibrium interest rate is zero and financial contracts are valued at risk neutral prices in all periods.

Entrepreneurs invest in period 0 and 1, i_0 and i_1. Early investment, i_0, delivers i_0 units of capital and xi_0 units of the consumption good in the intermediate period. The payoff x is random and takes the values $\{x_s\}$ that depend on the aggregate state s. The support of x is $[\underline{x}, \overline{x}]$, the values of x can be negative. A negative shock x means that the entrepreneur has to face "restructuring costs". In that case the entrepreneur has two options: incur the cost xi_0 and continue or shut down the firm, sell the entire capital stock, and receive zero returns in period 2. Assume that $E[x] > 0$ so early investment is more profitable than late investment. In period 1 entrepreneurs make the additional investment i_1 and sell k^S units of capital, so they obtain $k = i_0 - k^S + i_1$. Investment is irreversible at the aggregate level. That is, while consumption goods can be transformed into capital goods, one for one, at each point in time, existing capital cannot be transformed into consumption. Thus, entrepreneurs face the constraint $i_1 \geq 0$. The production function of the entrepreneurs in period 2 is linear and equal to $f(k) = Ak$ with $A > 1$.

Each consumer owns a firm in the traditional sector. In fact, we can think of the traditional sector as a back yard technology. Firms in the traditional sector invest capital in period 1 to produce consumption goods in period 2. The production function of the traditional sector displays decreasing returns and is given by $F(k^T)$ with $F' > 0$, $F'' < 0$.

As a simplifying assumption let

$$F'(0) = 1$$

so the firms in the traditional sector will only operate when entrepreneurs are selling capital at a price smaller than one.

To ensure concavity of the second-best problem I also make the following assumption.

Assumption A. *The functions* $(F'(k) - \theta) k$ *and* $F(k) - F'(k) k$ *are concave in* k.

This assumption is satisfied, in particular, when F is Cobb-Douglas.

4.1 Financial contracts

The financial structure is analogous to the one in section 2. Entrepreneurs sell financial contracts at date 0, subject to limited commitment. If the entrepreneur defaults the liquidation value of the firm at date 1 is proportional to the value of capital invested at date 1 and is equal to $\theta q i_0$, with $\theta \leq 1$.[20] The

[20] Notice that here I am assuming that, if the entrepreneur is active during the sale of firm assets, then he is able to recover the whole value qi_0, while if the sale is forced under default only $\theta q i_0$ is recovered. This reflects the idea that the entrepreneur's human capital is also required in the transfer of used capital.

liquidation value of the firm at date 2 is θk. Therefore the financial contract $\mathbf{d}$ is credible if

$$\begin{aligned} d_{1s} + d_{2s} &\leq \theta q_s i_0, && (12) \\ d_{2s} &\leq \theta k_s. && (13) \end{aligned}$$

So far I have concentrated my attention on limited commitment on the entrepreneurs' side. Now, I also assume that there is limited commitment on the consumers' side. Specifically, consumers can default on their financial obligations and their future income cannot be seized.[21] Consumers can hold securities issued by entrepreneurs and use them as collateral for their financial obligations. However, given that the financial contract $\mathbf{d}$ describes the *net* financial obligations between the consumers and entrepreneurs I can omit the description of these financial arrangements and simply require that $\mathbf{d}$ satisfies:

$$\begin{aligned} d_{1s} + d_{2s} &\geq 0, && (14) \\ d_{2s} &\geq 0. && (15) \end{aligned}$$

Cross-holdings of securities across entrepreneurs are irrelevant in this context, given that there is only aggregate uncertainty. For a thorough discussion of the role of aggregate and idiosyncratic shocks in this type of framework see Holmstrom and Tirole (1998).

Again the financial contract can be summarized by the borrowing ratios:

$$\begin{aligned} b_{1s} &= (d_{1s} + d_{2s})/i_0, \\ b_{2s} &= d_{2s}/k_s. \end{aligned}$$

To simplify the analysis I make a further assumption. Let $\underline{q} = F'(n_0/(1-\theta))$.

Assumption B. *The economy parameters satisfy:*

$$\begin{aligned} &\underline{q} > \theta \\ &\frac{A-\theta}{1-\theta}\left(\underline{q}(1-\theta) + \underline{x}\right) > 1-\theta \end{aligned}$$

[21] For simplicity, here I assume that the output in the traditional sector $F\left(k_t^T\right)$ is a private return for the consumers, and cannot be used as collateral in financial contracts.

This assumption is sufficient to ensure that entrepreneurs never sell the entire firm in period 1, i.e. that only partial liquidation occurs in equilibrium. This assumption together with Assumption A also rules out multiple equilibria in period 1.

4.2 Optimal hedging

As a preliminary step, notice that entrepreneurs budget constraint at date 1 takes the form

$$c_1^E + i_1 = \chi_{(k>0)} x i_0 + q k^S - d_1$$

where $\chi_{(k>0)}$ is an indicator function that takes the value 1 when k is positive, and zero otherwise. One can substitute in this expression the sales of used capital

$$k^S = i_0 + i_1 - k.$$

Then, a simple arbitrage argument shows that if $q < 1$ new investment i_1 will be zero, while $q > 1$ is incompatible with equilibrium. Therefore, the budget constraint can be rewritten as

$$c_1^E + qk = \chi_{(k>0)} x i_0 + q i_0 - d_1.$$

This equation has a simple balance sheet interpretation: the current cash flow $\chi_{(k>0)} x i_0$ can be used to pay insiders and outsiders (c_1^E and d_1) or to finance capital purchases $q(k - i_0)$. If the cash flow is negative, i.e. the firm is facing restructuring costs, this has to be financed either by an injection of funds from outsiders ($d_1 < 0$) or by sales of capital.

The entrepreneurs' problem now takes the form:

$$\max E\left[c_0^E + c_1^E + c_2^E\right]$$
$$s.t.\ c_0^E + i_0 = n_0 + E\left[d_1 + d_2\right],$$
$$c_1^E + qk = \left(\chi_{(k>0)} x + q\right) i_0 - d_1,$$
$$c_2^E = Ak - d_2,$$
$$0 \leq d_1 + d_2 \leq \theta q i_0,$$
$$0 \leq d_2 \leq \theta k.$$

This problem is analogous to problem (3) in section 2.2, except that: (1) the gross return on capital is now given by $\left(\chi_{(k>0)} x + q\right)$ where the asset price q is endogenous, (2) the technology at date 2 is linear, and (3) the financial contract is subject to limited commitment on the consumers' side. Notice that, thanks to Assumption B, one can show that $\chi_{(k>0)} = 1$ in equilibrium.[22] Therefore, from now on I disregard the option to shut down the firm.

As in section 2.2 I can characterize the optimal financial contract in terms of the marginal return on entrepreneurial wealth z. The variable z is defined now as:

$$z_s = \max\left\{\frac{A-\theta}{q_s-\theta}, 1\right\}, \tag{17}$$

a dollar of entrepreneur's wealth at date 1 can be levered to give $1/(q-\theta)$ units of capital invested in the firm and these will pay a net return of $A-\theta$ to the entrepreneur on date 2.

Similarly, the rate of return on entrepreneurial wealth at date 0 is

$$z_0 = E\left[z\frac{q+x-b_1}{1-E\left[b_1\right]}\right]. \tag{18}$$

With this notation the optimal financial contract is characterized as follows.

Lemma 10 *In an optimal financial contract one of the following three sets of conditions has to hold in*

[22] See the proof of Proposition 11 in the Appendix.

each state s:

$$\begin{array}{rclll} z_s & < & z_0, & b_{1s} = \theta q_s, & or \\ z_s & = & z_0, & 0 \leq b_{1s} \leq \theta q_s, & or \\ z_s & > & z_0, & b_{1s} = 0. & \end{array} \tag{19}$$

Notice that, compared to conditions (6) in Lemma 1, conditions (19) allow for the case $z_s > z_0$. In this case, the entrepreneurial rate of return in state s is larger than the rate of return at date 0. The entrepreneur would like to receive a net transfer of resources in this state of the world, i.e. hold negative liabilities. However, due to limited commitment on the consumers' side the entrepreneur can only set its liabilities to zero in this state of the world.

4.3 Equilibrium

In a competitive equilibrium entrepreneurs choose optimally a consumption plan, $\{c_t^E\}$, an investment plan, $\{i_t\}$, and a financial contract, $\mathbf{d}$, that satisfy the budget constraint and the financial constraints (12)-(13) and (14)-(15); consumers choose $\{c_t^C\}$ and k^T; goods market, financial markets and the market for used capital clear. Before giving a characterization of the equilibrium financial contracts it is useful to describe the equilibrium in the asset market, or market for used capital, at date 1.

The asset market

Denote the price of capital at date 1 as q. Then, consumers receive the net profits

$$F(k^T) - qk^T$$

from investment in the traditional sector.

The equilibrium in the market for used capital at date 1 can be of two types, depending on the sign of investment in the entrepreneurial sector, i_1. Suppose that $i_1 > 0$. Then given that entrepreneurs can turn consumption goods into capital goods one for one, an arbitrage argument implies that $q = 1$. In this case optimality for firms in the traditional sector implies that $k^T = 0$.

If, instead, $i_1 = 0$ then market clearing in the used capital goods market requires

$$k^T = k^S = i_0 - k,$$

that is, the investment in the traditional sector must absorb the disinvestment in the entrepreneurial sector. Then, optimality for firms in the traditional sector implies that the asset price q satisfies:[23]

$$q = F'(i_0 - k).$$

Let me describe briefly the determinants of investment at date 1, for a given financial contract. Rewrite the entrepreneur budget constraint at date 1 in terms of the borrowing ratios b_1 and b_2:

$$qk = (q + x)\, i_0 - b_1 i_0 + b_2 k. \tag{20}$$

As noticed above, this constraint describes the entrepreneur balance sheet at date 1.If the entrepreneur is facing restructuring costs today, i.e. has negative revenues $x < 0$, then he can cover these costs by selling assets, $(qi_0 - qk)$, or by increasing his liabilities, $(b_2 k - b_1 i_0)$, or both. The optimal financial contract will determine whether asset sales arise in equilibrium.

Since $A > 1 > q$ the entrepreneur will always maximize total capital invested at date 1, k, and set $b_2 = \theta$. Then, one can substitute and obtain the following expression for investment at date 1:

$$(k - i_0) = \frac{x + \theta - b_1}{q - \theta} i_0. \tag{21}$$

This function defines investment by entrepreneurs as a function of the asset price and is illustrated in Figure 7. Notice that the behavior of the demand for capital is very different depending on the entrepreneurs balance sheet. If $b_1 < x + \theta$, the indebtedness is relatively low with respect to the firm cash flow and the firm borrowing capacity. In this case the firm is able to raise new capital and $k > i_0$. Then, equilibrium in the capital market is given by $q = 1$ and all capital is invested in the entrepreneurial sector. When, instead $b_1 > x + \theta$ the level of debt is so high that firms have to sell capital to pay their debts. This is a case of fire sales. In this case $k < i_0$ and the equilibrium price of capital will be $q < 1$.

[23] The assumption that e is sufficiently large than $e > i_0$, also implies $e > qi_0$ so the consumers can always acquire all the capital stock without need for borrowing.

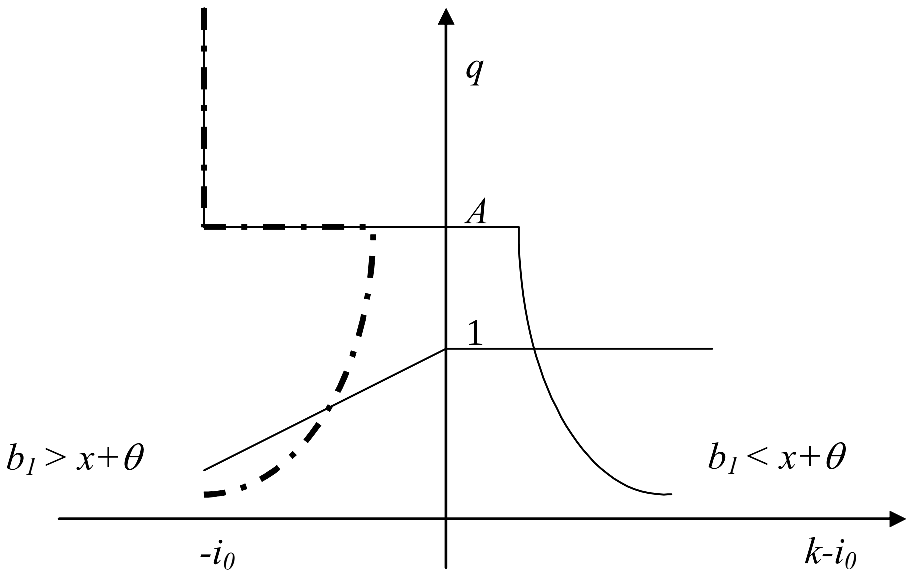

Figure 7: The asset market

In the case of fire sales the demand for capital by entrepreneurs is locally *increasing* in the price of capital q. This happens because an increase in the price of capital improves entrepreneurs balance sheets, as it allows entrepreneurs to sell a smaller amount of assets to pay their debts. The fact that entrepreneurs are *net sellers* of assets during a fire sale will turn out to be crucial for the welfare analysis.

Equilibrium contracts

As in section (2.3) it is possible to characterize the equilibrium in terms of cutoffs for the cash flow shock x.

Proposition 11 *Under assumptions A and B there is a unique competitive equilibrium characterized by a cutoff level $\hat{x} < 0$ where the financial contract has the following properties:*

1. *If $x_s \geq \hat{x}$ then $b_s = \theta q_s$;*

2. *If $\hat{x} - \theta\hat{q} \leq x_s < \hat{x}$ then $b_s = \theta\hat{q} - (\hat{x} - x)$;*

3. *If $x_s < \hat{x} - \theta\hat{q}$ then $b_{1s} = 0$;*

The value $\hat{q}$ is the equilibrium price of capital when $x = \hat{x}$.

The type of equilibrium arising in the capital market in period 1 will depend on the realization of the cash flow shock. If $x < 0$ entrepreneurs have to sell some capital in order to cover the negative cash flow shock. However, if the negative shock is not too large (i.e. for $x > \hat{x}$) the firm will still be borrowing up to its maximum capacity. When the cash flow shock is sufficiently bad (i.e. for $x < \hat{x}$) the entrepreneurs start to cut back on their borrowing, up to the point where they set their borrowing to zero. Finally, for very low values of x (i.e. for $x < \hat{x} - \theta\hat{q}$) entrepreneurs would like to receive positive transfers from consumers, and the limited commitment on the consumers side becomes the binding constraint. An immediate consequence of the characterization above is that if the borrowing capacity θ is sufficiently large, then $\underline{x} > -\theta$ and there are no fire sales in equilibrium.

In the fire sales region the asset market response tends to amplify the cash flow shocks. This effect is summarized in the following remark.

Remark 12 *(Amplification) In equilibrium the response of investment to a cash flow shock is larger in the fire sale region.*

The response of investment to a cash flow shock in the fire sales region is equal to

$$\frac{1}{1 + F''/(q-\theta)^2} \frac{1}{q-\theta}.$$

This expression is always larger than $1/(1-\theta)$, which measures the response in the case of positive investment. The first factor in this expression captures the amplification through asset markets. As more entrepreneurs in distress sell used capital the price of capital drops further increasing the required adjustment. The bigger is the price response on the used capital market (i.e. the bigger is $|F''|$) the greater the amplification of the original shock.

4.4 Constrained Efficiency

Let us now turn to constrained efficiency. As in the previous section constrained efficient allocations can be characterized in terms of a maximization problem where the planner takes into account the endogeneity of equilibrium prices. The planner maximization problem is presented in the Appendix.

The following proposition is the analogous of Proposition 5 in the current setup and characterizes a constrained efficient allocation.

Proposition 13 *Suppose a constrained efficient financial contract has* $\tilde{c}_t^C > 0$ *at all* t. *Then there is a constant* $\tilde{z}_0$ *such that in each state* s *one of the following holds:*

$$\begin{aligned} z_s &< \tilde{z}_0, \qquad \tilde{b}_{1s} = \theta q_s, \ \textit{or} \\ z_s &= \tilde{z}_0, \qquad 0 \leq \tilde{b}_{1s} \leq \theta q_s, \ \textit{or} \\ z_s &> \tilde{z}_0, \qquad \tilde{b}_{1s} = 0. \end{aligned} \tag{22}$$

The constant $\tilde{z}_0$ *satisfies:*

$$\tilde{z}_0 = z_0 - E\left[\left(z - \tilde{z}_0\right)\rho\right],$$

where z_0 *and the random variable* z *are given by (17) and (18). The random variable* ρ *is given by:*

$$\rho_s = -\frac{b_1 - x - \theta}{q - \theta + F''\left(k^T\right)k^T} F''\left(k^T\right)k^T \ \textit{if}\ k^T > 0$$

and zero otherwise.

The only difference between the competitive allocation and an efficient allocation is the cutoff $\tilde{z}_0$. In particular if $\tilde{z}_0 < z_0$ then there will be states of the world in which the planner would choose a lower level of borrowing b_{1s}, thus obtaining lower (log) volatility of investment and output. This leads to the following.

Corollary 14 *If the following condition is satisfied*

$$z_0 E\left[\rho\right] > E\left[z\rho\right] \tag{23}$$

then $\tilde{z}_0 < z_0$. *Then, as long as there is some state with* $z_s = z_0$, *the equilibrium displays excess borrowing and excess volatility.*

Let me first provide some intuition for inequality (23) and then discuss in what cases the inequality holds.

The externality associated to endogenous asset prices in this model works as follows. Suppose entrepreneurs could coordinate to reduce their investment in the initial period by $di_0 < 0$. This implies a reduced supply of used capital in fire sale states. This reduced supply results in a higher price of used capital. Let the effect in each state be dq_s. Since entrepreneurs are sellers of capital in fire sale states an increase in the asset price dq reallocates $k^T dq$ dollars from consumers to entrepreneurs. In order to compensate consumers for this reallocation the entrepreneurs can pay them $E\left[k^T dq\right]$ at date 0. The marginal cost of this transfer is $z_0 E\left[k^T dq\right]$ since z_0 is the marginal utility of funds for the entrepreneurs. On the other hand, the expected marginal benefits associated to the reallocation are $E\left[zk^T dq\right]$. The net effect of the reallocation is equal to

$$E\left[zk^T dq\right] - z_0 E\left[k^T dq\right].$$

One can show that the net effect of the reallocation $k_s^T dq_s$ is equal to $\rho_s di_0$. Therefore, if (23) holds the net effect of this reallocation is positive.

Now, suppose that the entrepreneurs reduce the investment di_0 by reducing their borrowing level b_s in a state of the world where $z_s = z_0$, that is, in a state of the world where they are exactly indifferent between early and late investment. In this case the reduction in investment di_0 has no *direct* effects on the entrepreneurs ex ante welfare. The only effect is the *indirect* effect working through the asset price adjustment described above. Therefore, if (23) holds this reduction in borrowing and investment has a positive net effect on welfare. Summing up, by committing to reduce borrowing and investment at date 0 entrepreneurs reduce the size of the fire sales in the bad states and support the price of used capital in those states. If the associated reallocation has positive net effects, the reduction in borrowing is welfare improving.

The next question is, what determines wether inequality (23) holds? First, notice that z is high in those states where there are large fire sales (low x and high k^T). Second, notice that the value of ρ also tends to be high in the states of the world with large fire sales, because in these states there tends to be a larger reallocation. In particular, notice that, in a competitive equilibrium, the cash flow shortfall $(b_1 - x - \theta)$ is larger for larger values of $-x$, and thus for bigger values of k^T. Moreover, if F is Cobb-Douglas, then the expression:

$$\frac{-F''\left(k^T\right)k^T}{q - \theta + F''\left(k^T\right)k^T}$$

is also increasing in k^T. So in the Cobb-Douglas case we know that ρ is negatively correlated with k^T

and z. This does not necessarily implies that $E[(z - z_0)\rho] < 0$, given that $E[z]$ is in general different from z_0. However, if the distribution of x is sufficiently dispersed then there will be states with $z < z_0$. In this case it is easy to construct examples where (23) holds.

4.5 Discussion

Notice that the presence of excess borrowing is essentially due to the fact that there are states of the world in which entrepreneurs have insufficient instruments to protect their wealth. In particular, absent commitment problems on the consumer side, inequality $z < z_0$ will never hold and excess borrowing will not arise. Holmstrom and Tirole (1998) show that the supply of public liquidity can alleviate the lack of commitment on the consumers' side. In this model if there is enough public liquidity, in the sense of Holmstrom and Tirole (1998), the constraint $b_1 \geq 0$ is never binding and an inefficient credit expansion would never arise.

The type of pecuniary transfers associated to the asset price channel and the wage channel are quite different: the wage channel increases wages in the *last period* when entrepreneurs are consuming and their marginal utility of wealth is relatively low (it is $1 < z_0$ in the model). In that period entrepreneurs would like to increase their ability to repay consumers, which is limited by lack of commitment. The wage increase on the labor market helps them commit, indirectly, to reallocate wealth to consumers. The asset channel instead changes asset prices in the *intermediate period*, when the entrepreneurs have just been hit by the negative shock, and their marginal utility of wealth is relatively high ($z_s > z_0$ in the model). In that period entrepreneurs would like to increase their ability to extract resources from consumers, which is limited by lack of commitment on the consumers' side. The asset price increase helps them, indirectly, to reallocate wealth in their own favor. The first externality is driven by the inability of entrepreneurs to commit to payments in the long run when investment returns are realized (period 2), the second externality is driven by the inability of consumers to commit to payments in the short run when entrepreneurs need additional liquidity (period 1).

It is useful to remark that in this framework the inefficiency is *not* due to fact that the price q affects the collateral available to entrepreneurs. The pecuniary externality in (23) matters for efficiency only when the constraint $b_1 \geq 0$ is binding. In that case the collateral constraint $b_1 \leq \theta q$ is not binding and the positive effect of the asset price on the borrowing capacity θq is irrelevant for entrepreneurs. That is, asset prices matter because they determine the asset side of entrepreneurs balance sheet at date 1, not because of their effects on entrepreneurs ability to borrow. In a fully dynamic model it is possible to introduce positive effects on the value of collateral in future periods (i.e. from period 2 onward), and so have a "collateral channel" different from the asset price channel discussed here. However, the transfers associated with this additional effect are different and require a separate analysis.

5 Remarks on Policy

The cases of over-borrowing identified in the previous sections can be addressed using a number of possible policy instrument. Here, I will discuss briefly some of the model's implications for prudential regulation and for monetary policy. I will make reference to the model presented in section 2, but the remarks apply as well to the model in section 4.

In presence of over-borrowing and excess volatility a simple policy that restores the second-best is a capital requirement. Regulatory interventions that impose minimum capitalization on financial firms are widespread in industrialized economies, and often their introduction is justified based on the idea that excessive leverage in the financial sector may bring about excessive volatility at the macroeconomic level.[24] The models presented here gives a welfare-based rationale to this idea.

Consider a *capital requirement* at date 0 of the type

$$\frac{n_0}{k_1} \geq \nu$$

this imposes a lower bound on the ratio of inside funds to total assets. The presence of this constraints effectively reduces the rate of return on investment at date 0, z_0, by increasing the shadow cost of outside funds. This tilts the trade-off in favor of financial stability, expands the set of states of the world in which entrepreneurs acquire net worth insurance and increases net worth in these states. The next proposition shows that a capital requirement can implement a constrained efficient allocation.

[24] For a discussion of the so called "macroprudential approach" to financial regulation see Borio (2003).

Proposition 15 *Suppose $\tilde{h}$ is more risk averse than h. Consider a constrained efficient allocation that dominates the competitive equilibrium. Let*

$$\tau = \left(1 - E\tilde{b}_1\right)\tilde{\imath}_0 - n_0$$
$$\nu = \left(1 - E\tilde{b}_1\right)$$

Then a capital requirement ν and a transfer τ to entrepreneurs at date 0 implement the constrained efficient allocation.

In practice, capital requirements are imposed on a specific class of firms, typically on commercial banks and financial intermediaries. To have a fully fledged theory of capital requirements would require an explicit model of financial intermediation. If financial intermediaries specialize in the provision of contingent credit lines and other forms of net worth insurance then the stability of the intermediaries wealth is instrumental in providing net worth insurance to the non-financial corporate sector. Moreover, if entrepreneurial firms which rely more on outside funding are more dependent on bank credit, capital requirements on banks can help to stabilize the balance sheet of the firms that need it most. The analysis of capital requirements in an explicit framework with intermediation remains an important topic for future research.[25]

An open question is how capital requirements should be calibrated for investments with different risky profiles. Existing capital requirements are usually based on the riskiness of the individual investment, using some measure of "value at risk". The framework of this paper could be extended to analyze models with different types of investment. In that case the assets that are more positively correlated with the aggregate economy are the ones that generate larger external costs in case of under-capitalization. The approach taken here points to optimal capital requirements that depend on macroeconomic correlations and not just on individual risk. In particular, it might be desirable that investments with higher correlation with macroeconomic conditions be subject to tighter requirements.[26]

The second type of intervention that one could consider is a *bail-out policy*, i.e. an intervention that transfers resources to distressed firms at date 1. This type of intervention has no effects in our framework.

[25] Allen and Gale (2004) study capital requirements in a model of intermediation from the point of view of constrained efficiency.

[26] See Borio (2003, p.10) for a discussion of recent policy proposals that go in this direction.

If the private incentives to protect net worth are unchanged private contracts will exactly neutralize the effect of expected government transfers. More precisely, suppose the government can induce a non-distortionary transfer of resources t_s from investors to entrepreneurs at date 1, and suppose that these transfers are compensated by an equivalent transfer $E[t]$ from entrepreneurs to investors at date 0. Then it is straightforward to show that equilibrium prices and quantities are unchanged and the equilibrium borrowing will simply be equal to $bi_0 + t$.

Let me now consider the effects of a *precautionary monetary contraction* in period 0, oriented at reducing borrowing. This is the policy that Bordo and Jeanne (2002) call a "pro-active" monetary policy. The models considered here are real models and do not allow an explicit treatment of monetary policy. However, suppose the policy maker has access to some monetary instrument at date 0, and decides to use it to curb over-borrowing. A monetary tightening at date 0 can work through two different channels. First, monetary policy can affect investment through a standard interest rate channel: an increase in the interest rate would reduce the cost of funds for entrepreneurs and reduce borrowing. Second, monetary policy can operate through a balance sheet channel:[27] an increase in interest rates at date 0 can reduce current activity, reduce current profits and thus affect the initial net worth of entrepreneurs n_0.

Both channels would induce a reduction in investment ex ante, however they achieve it in very different ways. The interest rate channel has a beneficial substitution effect because it makes outside finance more costly, it reduces the return on investment financed with outside funds, reduces the shadow value z_0 and induces entrepreneurs to increase the degree of financial stability. This effect reduce investment ex ante but it also induces a more stable profile of net worth in different states of the world. On the other hand, the balance-sheet channel tends to reduce investment by simply reducing the wealth of entrepreneurs. This effect tends to depress net worth in *all* states of the world, so it achieves a reduction in investment *without* achieving greater financial stability.

Compared to this monetary tightening, the virtue of a capital requirement is that it increases the cost of outside funds without generating a negative effect on entrepreneurs' wealth. In short: if monetary policy affects investment decisions through a balance sheet channel then it tends to reduce *both inside and outside sources of finance*. This makes it a relatively blunt tool to deal with the type of over-borrowing discussed in this paper.

[27] See Bernanke Gertler and Gilchrist (2001) for a discussion of the balance sheet channel and for an explicit monetary model that incorporates a balance sheet channel.

A very different type of monetary intervention that can affect equilibrium financial contracts is a state-contingent monetary policy at date 2. In particular an expansionary policy in bad states of the world that increases the rate of return on capital, r_2, would gives a reward to firms that have maintained a sufficient level of capitalization. This policy increases z in the bad states and encourages net worth insurance ex ante. Abstracting from the distortions associated to the monetary intervention this type of state-contingent policy would have beneficial effects on the choice of financial contracts. From this point of view, a monetary policy that tends to stabilize output ex post may also have favorable incentive effects ex ante.

Clearly, an explicit monetary model is required to evaluate the quantitative significance of the effects described and to study the desirability of different forms of intervention.

6 Concluding Remarks

The model and the examples presented in this paper show that over-borrowing and excessive volatility are possible even if entrepreneurs have access to fully state-contingent contracts. In this paper I have emphasized the forces that tend to generate over-borrowing. However, the models do not bear unambiguous qualitative predictions regarding the direction of the inefficiency. Whether the effects identified in this paper are quantitatively relevant and what is the size of the welfare loss associated to them remains an open question for future research.

The policy debate on financial supervision and regulation has been recently shifting towards a "macroprudential" approach.[28] According to that approach the regulator should be concerned most of all about the macroeconomic consequences of financial instability and the main source of instability is identified in the common exposure to macroeconomic risks across financial institutions. The present paper provides at the same time a warning and a justification for that approach. The warning is that aggregate volatility and some degree of financial fragility are unavoidable in presence of financial constraints, and that a reduction in financial fragility can only be achieved at the cost of reducing investment ex ante. The presence of highly levered positions is a symptom that the expected gains from investment are high. In short, writing the objective function of the central bank or of the regulator only in terms of output volatility, and disregarding the productive effects of capital accumulation may

[28] See Borio (2003) and references therein.

be misleading. On the other hand, the welfare analysis in this paper provides a justification for a macroprudential approach. In a framework with financial constraints private agents may underestimate the damage associated to a contraction in their wealth, after a negative aggregate shock, and therefore prudential requirements that limit the volatility of their wealth may be welfare improving.

The model has focused on the effects of financial volatility on wages and on asset prices. Other general equilibrium effects of financial volatility are potentially relevant and deserve further attention. In particular it would be useful to extend the model to allow for a "collateral channel" for asset prices as discussed at the end of section 4. Furthermore, risk premia can be endogenized. In this paper I have assumed risk neutrality to show that net worth insurance can arise solely because of financial constraints and a concave technology. In a model with risk averse investors the risk premia used at date 0 to evaluate financial contracts will depend on the future distribution of wages. A lower volatility of future wages will reduce risk premia. In turn, if entrepreneurs can raise funds at a lower premium at date 0 they can finance the same level of investment with lower borrowing at date 1, and their wealth at date 1 may be less sensitive to aggregate shocks. If these effects are large multiple, Pareto-ranked equilibria can arise.[29] I leave the analysis of models with endogenous collateral constraints and endogenous risk premia to future research.

Another limitation of the model presented is that the simple three periods structure prevents a full analysis of the dynamics. Among other things, this limits the analysis of monetary policy given that the effectiveness of monetary policy is typically associated to monetary policy inertial character and to its effect on future expectations.[30] A full dynamic extensions with an explicit role for monetary policy would allow to answer some of the questions raised in section 5.

Finally, the models studied allow for fully state-contingent contracts. This modeling choice was dictated by the concern of showing that inefficiency can arise *even when* fully state-contingent contracts are available. The results can be easily extended to the case of incomplete markets, and in particular to the case of non-state-contingent debt. The question of why private financial contracts are not fully state-contingent with respect to observable aggregate shocks, especially in presence of large crisis episodes, is a crucial question for understanding the nature of the financial frictions and their macroeconomic implications. This question remains open for future research.

[29] Examples in this direction are presented in Holmstrom and Tirole (2001, sec. 6.1) and in Lorenzoni (2005).

[30] For example in Dupor (2001) the effects of monetary policy that are quantitatively most relevant arise from the effect of monetary policy on future expected profits, rather than from the effect on current profits and on current interest rates.

Appendix

Lemma 16 *If $E[x] > 1$ then $i_0 > 0$ in equilibrium.*

Proof. Rewrite problem P as

$$\begin{aligned} \max \quad & E[zn] \\ & i_0 = n_0 + E[B] \\ & n = xi_0 - B \\ & B \leq \theta i_0 \\ & i_0 \geq 0 \end{aligned}$$

in order to allow for the case $i_0 = 0$. Then, $i_0 = 0$ is optimal if

$$\begin{aligned} E[zx] &\leq \lambda \\ z_s &\leq \lambda \end{aligned}$$

in this case we can have $z_s > E[zx] > z_{s'}$.

However, suppose we have $\max\{z_s\} > z_{s''}$. Then we have $B_{s''} = 0$ and $B_{s'} < 0$ for some $s : z_s = \max\{z_s\}$. (Otherwise $n_0 + E[B] = 0$ would be violated). This means that in equilibrium $k_{s''} = 0 < k_{s'}$ and implies $z_{s''} \geq z_{s'}$, a contradiction.

Therefore, it must be that if $i_0 = 0$ then z_s is constant across states and $n_1 = n_0$. But when this is the case we have

$$E[zx] = zE[x] > z$$

and the first f.o.c. is violated. A contradiction. ■

Proof of Proposition 2

It is useful to first prove the characterization part and then prove existence and uniqueness.

Notice that the definition of z implies $z_s \geq 1$. The chain of inequalities

$$E\left[z\frac{x-b}{1-Eb}\right] \geq \frac{Ex - Eb}{1-Eb} > 1$$

implies $z_0 > 1$. Consider the states $S_0 = \{s : z_s < z_0\}$. We want to show that if $s \in S_0$ and $x_{s'} > x_s$ then $s' \in S_0$. Notice that if $z_0 > z_s$ then $b_{1s} = \theta$, $b_{1s'}$ satisfies $b_{1s'} \leq \theta$ and we have $n_{1s'} = (x_{s'} - b_{1s'})\, i_0 > (x_s - \theta)\, i_0 = n_{1s}$ which implies $k_{s'} \geq k_s$. Concavity of f then implies $z_{s'} \leq z_s < z_0$.

Consider now the states $S_1 = \{s : z_s = 1\}$. Since $z_0 > 1$ it follows that $S_1 \subset S_0$. If $s \in S_1$ and $x_{s'} > x_s$ then $b_{1s'} = b_{1s} = \theta$ and therefore $k_{s'} \geq k_s$ and $z_{s'} = 1$.

By entrepreneurs' optimality conditions if $b_{1s} < \theta$, then $z_s = z_0$. This implies that n_{1s} is constant in S/S_0. This implies $x_s - b_{1s} = x' - \theta$. Denote the cutoff as $\hat{x}$. Therefore, the optimal financial contract b_1 can be expressed in terms of the cutoff $\hat{x}$ as

$$b_{1s} = \theta + \min\{x_s - \hat{x}, 0\}.$$

Therefore an optimal financial contract is fully characterized by $\hat{x}$.

Let $H(x) = E[yh(n_1)]$ where

$$y_s = \iota_s - \frac{x_s - \theta}{1 - \theta} \tag{24}$$

where $\iota_s = 1/\pi_s$ if $b_{1s} < \theta$, and zero otherwise and the function h is defined in the text in (7). Notice that $H(\overline{x})$ is equal to $\left(1 - E\left[\frac{x-\theta}{1-\theta}\right]\right)$ multiplied by a positive factor, so $H(\overline{x}) < 0$.

Differentiating H one obtains

$$H'(x) = E\left[y_s h'(n_{1s}) \frac{dn_{1s}}{dx}\right]$$

where

$$\begin{aligned} \frac{dn_{1s}}{dx} &= \left(1 - \pi_s \frac{x_s - b_{1s}}{1 - Eb_1}\right) k_0 \text{ if } b_{1s} < \theta \\ \frac{dn_{1s}}{dx} &= -\pi_s \frac{x_s - b_{1s}}{1 - Eb_1} k_0 \text{ if } b_{1s} > \theta \end{aligned}$$

Consider an x^* such that $H(x^*) = 0$. Then in all states with $b_{1s} < \theta$ we have $1 - \pi_s \frac{x_s - b_{1s}}{1 - Eb_1} > 0$ (otherwise $z_s \leq \pi_s \frac{x_s - b_{1s}}{1 - Eb_1} z_s < z_0$) and $y_s > 0$ (otherwise $H(x^*) > 0$). In the remaining states we have $\frac{dn_{1s}}{dx} < 0$ and $y_s < 0$. Therefore $H'(x^*) < 0$. So H is quasiconcave, either $H(\underline{x}) < 0$ and there is an equilibrium with $b_s = \theta$ for all s, or $H(x^*) = 0$ for $\underline{x} < x^* < \overline{x}$. In both cases the equilibrium is unique.

Proof of Proposition 3

From Proposition 2 we know that the equilibrium is fully characterized by the cutoff x', so we only need to prove that $\hat{x}' > x'$.

Let n_1 be the equilibrium net worth in the first economy. Let $\underline{n} = \min\{n_{1s}\}$. The condition $h(\underline{n}) = E\left[\frac{x-\theta}{1-\theta}h(n_1)\right]$ implies $\tilde{h}(\underline{n}) > E\left[\frac{x-\theta}{1-\theta}\tilde{h}(n_1)\right]$ because $\tilde{h}$ is more risk averse than h. Let $\tilde{H}$ be defined as in Proposition 2. The last inequality implies $\tilde{H}(x') > 0$ and, by the quasiconcavity of $\tilde{H}$, we have $x' < \hat{x}'$.

Proof of Proposition 5

The proposition follows from long but straightforward manipulation of the first order conditions of problem P'.

Proof of Proposition 7

First one can show that the when $\tilde{h}$ is more risk averse than h then $f_L(k,1)$ is concave. By contradiction: if $f_{LKK} > 0$ for some $k < k^*$ then the coefficient of absolute risk of h and $\tilde{h}$ are

$$\left|\frac{f_{KK} + \kappa f_{LKK}}{f_{KL} - \theta + \kappa f_{KL}}\right| < \left|\frac{f_{KK}}{f_{KL} - \theta}\right|.$$

One can show that the consumers participation constraint in program P' is binding. Then the objective of program P' can be rewritten as

$$\begin{aligned} & \left[c_0^E + V(n_1, r(n_1))\right] + E\left[3e - \tau + b_1 i_0 + w(n_1)\right] - U^C \\ = \; & c_0^E + E\left[f(k,1)\right] + 3e - \tau + b_1 i_0 - U^C \end{aligned}$$

where $k = \min\left\{k^*, \frac{1}{1-\theta}n_1\right\}$. Rewritten in this form, and provided that f_L is concave, the program is concave.

Proceeding as in the proof of 2 it is possible to show that an efficient allocation is fully characterized by a cutoff $\tilde{x}$ such that $\tilde{b}_1 = \theta + \min\{x - \tilde{x}, 0\}$.

Let me prove the proposition considering the constrained efficient allocation corresponding to $U^C = U_c^C$, the proof can be extended to any efficient allocation that weakly dominates the equilibrium.

Let n_1 be the equilibrium wealth levels and let x be defined as in (24). Since x and n_1 are monotonically related and $E[xh(n)] = 0$ it follows that $E\left[x\tilde{h}(n)\right] \geq 0$. The concavity of the program can then be used to show that the constrained efficient cutoff satisfies $\tilde{x} \geq \hat{x}$.

Proof of Proposition 11

The proof is analogous to that of Proposition 2. First, one can partition the states in the three sets $S_1 = \{s : z_s < z_0\}, S_2 = \{s : z_s = z_0\}, S_3 = \{s : z_s > z_0\}$ and show that they define two cutoffs for x. Secondly, for the states in S_2 it must be the case that q_s is constant, which requires i_1 to be constant. This pins down the values for b_s in S_2.

It remains to show that entrepreneurs never want to shut down the firm in the intermediate period and that there are no multiple equilibria in period 1. To show the first claim notices that the investment level at date 0 is bounded by $n_0/(1-\theta)$, this means that assets sales at date 1 are bounded by $n_0/(1-\theta)$ which implies that

$$q > \underline{q} = F'(n_0/(1-\theta)).$$

Then notice that the entrepreneurs prefer continuation as long as

$$\frac{A-\theta}{q-\theta}(x+q-b) > q-b \tag{25}$$

now either $x+\theta-b > 0$ or $x+\theta-b < 0$. In the first case $q = 1$ and assumption B ensures that

$$\frac{A-\theta}{1-\theta}(x+1-b) > 1-b$$

given that $\frac{A-\theta}{1-\theta} > 1$ and $b < \theta$. In the second case the expression on the left of (25) is a concave function of q, so we only need to check that the inequality holds for the extreme values $q = 1$ and $q = \underline{q}$. For both values the second inequality in assumption B is sufficient.

To show that there are no multiple equilibria notice that an equilibrium in the used capital market solves the equation:

$$\left(F'\left(k^T\right) - \theta\right) k^T = (b - x - \theta)\, i_0$$

The function on the left hand side is concave by assumption A and is zero at $k^T = 0$. Therefore the equation has only one solution in $[0, i_0]$ as long as $(F'(i_0) - \theta)\, i_0 > (b - x - \theta)\, i_0$. This inequality holds as long as

$$\underline{q}(1-\theta)+\underline{x}>0,$$

which holds if the second inequality in assumption B is satisfied.

With these two results in place existence can be proved following similar steps as for Proposition 11.

Constrained Efficiency in Section 4 and proof of Proposition 13

The planner takes as given the determination of equilibrium from date 1 on. Therefore, the planner problem can be written as in section 3.2 as:

$$\begin{aligned}
\max_{\tau,i_0,b_1,q,k^T,\tau} \quad & E\left[c_0^E+(A-\theta)(i_0+i_1)\right] \\
s.t. \quad & c_0^E+i_0=n_0+\tau, \\
& \left(F'\left(k^T\right)-\theta\right)k^T+(x+\theta-b_1)\,i_0\geq 0, \\
& E\left[3e-\tau+b_1i_0+\left(F\left(k^T\right)-F'\left(k^T\right)k^T\right)\right]\geq U^C, \\
& k^T=\max\left\{-i_1,0\right\}, \\
& 0\leq b_1\leq\theta,
\end{aligned}$$

we made assumptions to ensure that this program is concave, namely we assumed that $\left(F\left(k^T\right)-F'\left(k^T\right)k^T\right)$ and $\left(F'\left(k^T\right)-\theta\right)k^T$ are concave.

This program can be rewritten in terms of i_0, b_1, k^T as:

$$\begin{aligned}
\max_{i_0,b_1,k^T} \quad & E\left[\frac{A-\theta}{F'\left(k^T\right)-\theta}\left(x+F'\left(k^T\right)-b_1\right)i_0\right] \qquad (P'_A) \\
s.t. \quad & \left(F'\left(k^T\right)-\theta\right)k^T=\max\left\{(b_1-x-\theta)\,i_0,0\right\}, \\
& E\left[3e+n_0-i_0+b_1i_0+F(k^T)-F'\left(k^T\right)k^T\right]\geq U^C, \\
& 0\leq b_1\leq\theta.
\end{aligned}$$

from this problem one can derive the first order conditions that appear in Proposition 13.

Proof of Proposition 15 The entrepreneur problem subject to capital requirements can be written as

$$\begin{aligned} \max_{k_1,\{b\}} \quad & E\left[c_0^E + V(n_1, r)\right] \\ s.t. \quad & c_0^E + i_0 \leq n_0 + \tau + E[b]\, i_0 \\ & i_0 \leq \nu n_0 \\ & n_1 = (x - b_1)\, i_0 \\ & b_s \leq \theta \end{aligned}$$

let μ be the Lagrange multiplier on the capital requirement constraint then the first order conditions are

$$\begin{aligned} (1 - Eb)\, z_0 + \mu &= E[z(x-b)] \\ z_0 - z &\geq 0 \\ b &\leq \theta \\ (z_0 - z)(b - \theta) &= 0 \end{aligned}$$

Consider a constrained efficient allocation and let $\{z_s\}$ be the rate of return on inside funds at the constrained efficient allocation. Let $\hat{n} = \min\{n_{2s}\}$ and let $x = \frac{x-b}{1-Eb}$. The constrained efficient allocation satisfies

$$\tilde{h}(\hat{n}) = E\left[x\tilde{h}(n_1)\right]$$

This implies that

$$h(\hat{n}) > E[xh(n_1)]$$

and we can let

$$\begin{aligned} z_0^* &= h(\hat{n}) \\ \mu^* &= \frac{E[z(x-b)]}{(1-Eb)} - z_0^* \geq 0 \end{aligned}$$

The Lagrange multipliers (z_0^*, μ^*) and the constrained efficient levels of b^* satisfy the first order conditions for the entrepreneur.

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