Policy Interaction, Learning and the Fiscal Theory of Prices

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Abstract

We investigate both the rational explosive inflation paths studied by (McCallum 2001), and the classification of fiscal and monetary policies proposed by (Leeper 1991), for stability under learning of the rational expectations equilibria (REE). Our first result is that the fiscalist REE in the model of (McCallum 2001) is not locally stable under learning. In contrast, in the setting of (Leeper 1991), different possibilities can arise. We find, in particular, that there are parameter domains for which the fiscal theory solution, in which fiscal variables affect the price level, can be stable outcomes under learning. However, for another parameter domain the monetarist solution is instead the stable equilibrium. Policy formulation should systematically take account of the local stability properties, under learning, of the different REE.

JEL classification: E52, E31, D84.

1 Introduction

Interactions between fiscal and monetary policy in the determination of the price level have been the object of a great deal of new research in recent years. In particular, the fiscal theory of the price level asserts that fiscal policy can have an important influence on the price level, and an extreme specific case of the fiscalist theory asserts that, in certain specific circumstances, fiscal variables can fully determine the price level independently of monetary variables. For a long list of references see footnote 3 in (Woodford 2001).

Clearly, this extreme result is the polar opposite of the monetarist contention that the price level and the inflation rate depend primarily on monetary variables. It is thus not surprising that the fiscalist approach has aroused a great deal debate and controversy. These debates consider different aspects of the theory. One point of debate concerns the extreme specific case for which (McCallum 2001) has argued that the fiscalist equilibrium is an implausible "bubble equilibrium" with an explosive price level and, as we will see below, an explosive inflation rate.[1]

The influence of fiscal variables on the price level is, however, not limited to extreme cases in which the system is non-stationary. In a local analysis around a unique steady state (Leeper 1991) made an important distinction between "active" and "passive" policies (the precise definitions will be given below). In a standard model he showed that two combinations, either (i) active monetary and passive fiscal policy or (ii) active fiscal and passive monetary policy yield determinacy i.e. a unique stationary rational expectations equilibrium (REE). In case (i) the usual monetarist view that inflation depends only on monetary policy is confirmed. However, case (ii) is fiscalist in the sense that fiscal policy has an effect on the inflation rate. (Leeper 1991) also showed that the steady state is indeterminate when both policies are passive, while the economy is explosive when both policies are active.

As already noted, the fiscal theory of the price level is subject to debate and thus the existing literature is not very conclusive about its significance. Indeed, equilibrium analysis can shed only limited light on the issues and further criteria on the plausibility of different REE are likely to be useful in assessing the possible outcomes suggested by the fiscal theory. The learning approach to macroeconomics, which has been developed in recent years[2], provides a criterion to select "reasonable" outcomes when multiple REE exist. (The approach is also useful with in cases with unique REE as a notion of "robustness".) In this paper we re-examine some central results of the fiscal theory of the price level from a learning viewpoint. Generally speaking, this view asserts that the REE of interest are those that are stable outcomes of

[1] Another point of controversy evolves around the nature of intertemporal budget constraint of the government, compare e.g. on one hand (Buiter 1998), (Buiter 1999) and on the other Section 2 of (Woodford 2001).

[2] See (Evans and Honkapohja 2001)for a recent treatise. Surveys of the literature are provided e.g. in (Evans and Honkapohja 1999), (Marimon 1997) and (Sargent 1993).

a learning process in which agents might temporarily deviate from rational expectations, respond to these mistakes and eventually come to have correct forecast functions.

We investigate both the rational explosive inflation paths studied by (McCallum 2001), and the classification of fiscal and monetary policies proposed by (Leeper 1991), for stability under learning of the rational expectations equilibria (REE). Our first result is that the fiscalist REE in the model of (McCallum 2001) is not locally stable under learning. In contrast, in the setting of (Leeper 1991), different possibilities can arise. We find, in particular, that there are parameter domains for which the fiscal theory solution, in which fiscal variables affect the price level, can be stable outcomes under learning. However, the detailed results can in some cases be quite complicated. This implies that policy formulation should systematically take account of the local stability properties, under learning, of the different REE.

2 The Model

We consider a stochastic optimizing model that is close to (Leeper 1991) and (McCallum 2001). For the basic model, the notation and the monetary and fiscal policy rules we follow Leeper, but we use McCallum's more general class of utility functions and also his timing in which utility depends on beginning of period money balances.[3]

Households are assumed to maximize

$$\max E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[(1-\sigma_1)^{-1} c_s^{1-\sigma_1} + A(1-\sigma_2)^{-1} (m_{s-1}\pi_s^{-1})^{1-\sigma_2} \right] \right\}.$$

Here c_s denotes consumption in period s and $m_s = M_s/P_s$, where M_s is the money supply and P_s is the price level at s. Note that real money balances enter utility as $m_{s-1}\pi_s^{-1} = (M_{s-1}/P_{s-1})(P_{s-1}/P_s) = M_{s-1}/P_s$. The household's budget constraint is

$$c_s + m_s + b_s + \tau_s = y + m_{s-1}\pi_s^{-1} + R_{s-1}\pi_s^{-1} b_{s-1}, \qquad (1)$$

[3] The question whether beginning- or end-of-period real balances leads to subtle differences in the model and can have major implications, compare (Carlstrom and Fuerst 2001).

where $b_s = B_s/P_s$, $\pi_s = P_s/P_{s-1}$ is the gross inflation rate and τ_s is real lump-sum taxes. Note that B_s is the end of period s nominal stock of bonds. R_{s-1} is the gross nominal interest rate on bonds, set at time $s-1$ but paid in the beginning of period s. The household has a constant endowment y of consumer goods each period.

We assume that there is a constant flow of government purchases $g \geq 0$. As shown in the appendix, household optimality and market clearing conditions imply the Fisher equation

$$R_t^{-1} = \beta E_t \pi_{t+1}^{-1} \tag{2}$$

and the equation for money demand, in period t,

$$A\beta m_t^{-\sigma_2} E_t \pi_{t+1}^{\sigma_2 - 1} = (y-g)^{-\sigma_1}(1 - \beta E_t \pi_{t+1}^{-1}). \tag{3}$$

In addition, the equilibrium must satisfy the transversality conditions

$$\lim_{t\to\infty} \beta^t m_{t+1} = 0 \text{ and } \lim_{t\to\infty} \beta^t b_{t+1} = 0. \tag{4}$$

The specification of the model is completed by giving the government budget constraint and policy rules. The government budget constraint, written in real terms, is

$$b_t + m_t + \tau_t = g + m_{t-1}\pi_t^{-1} + R_{t-1}\pi_t^{-1} b_{t-1}. \tag{5}$$

For fiscal policy we use Leeper's tax rate rule

$$\tau_t = \gamma_0 + \gamma b_{t-1} + \psi_t. \tag{6}$$

Monetary policy is given either by Leeper's interest rate rule

$$R_t = \alpha_0 + \alpha \pi_t + \theta_t, \tag{7}$$

or by a simple fixed money supply rule

$$M_t = M + \theta_t, \tag{8}$$

as in (Sims 1999) or (McCallum 2001). Here ψ_t and θ_t are exogenous random shocks, which for simplicity are to be *iid* with mean zero. (We will later briefly take up the case where the shock are $VAR(1)$.)

In the terminology of (Leeper 1991), fiscal policy is “active” if $\left|\beta^{-1}-\gamma\right| > 1$ and “passive” if $\left|\beta^{-1}-\gamma\right| < 1$, while monetary policy is active if under (7) $|\alpha\beta| > 1$ and passive if $|\alpha\beta| < 1$. As noted by (Sims 1999), it is also natural to refer to monetary policy as active if the policy rule (8) is followed in place of (7). We want to consider the rational expectations solutions under different policy regimes and then to analyze their stability under learning. Leeper emphasized the cases of AM/PF (active monetary/passive fiscal policy) and AF/PM (active fiscal/passive monetary policy) in which, as discussed below, there is a unique stationary solution. We will be particularly interested in these cases, but will also consider explosive regimes of the model and regimes with indeterminacy, i.e. with multiple stationary solutions.

3 Bubbles and the Fiscal Theory of Prices

We begin our analysis with consideration of a prominent case of the fiscal theory of prices in which the price level path is entirely determined by fiscal policy and does not depend on monetary policy, e.g. see (Sims 1999) or (McCallum 2001). In this section we use a nonstochastic version of the model in which $\psi_t \equiv 0$ and $\theta_t \equiv 0$. Monetary policy is given by (8) and fiscal policy is given by (6) with $\gamma = 0$. Thus policy reduces to

$$\tau_t = \tau \text{ and } M_t = M,$$

which is a special case in which both monetary and fiscal policy are active.

With a nonstochastic model it is natural to assume point expectations, so that (3) becomes

$$m_t = (A\beta)^{1/\sigma_2}(y-g)^{\sigma_1/\sigma_2}[(1-\beta/\pi^e_{t+1})(\pi^e_{t+1})^{1-\sigma_2}]^{-1/\sigma_2}.$$

With constant nominal money stock we can write

$$P_t = M(A\beta)^{-1/\sigma_2}(y-g)^{-\sigma_1/\sigma_2}(\pi^e_{t+1})^{(1-\sigma_2)/\sigma_2}[1-\beta(\pi^e_{t+1})^{-1}]^{1/\sigma_2}$$

or

$$P_t = \hat{D}(\pi^e_{t+1})^{(1-\sigma_2)/\sigma_2}[1-\beta(\pi^e_{t+1})^{-1}]^{1/\sigma_2}, \qquad (9)$$

where $\hat{D} \equiv M(A\beta)^{-1/\sigma_2}(y-g)^{-\sigma_1/\sigma_2}$.

Consider first the perfect foresight solutions. Under perfect foresight we have $R_t^{-1} = \beta\pi_{t+1}^{-1}$. With a constant money supply the bond equation (5) reduces to

$$b_t = g - \tau_t + \beta^{-1} b_{t-1}.$$

With $\tau_t = \tau$ this equation is explosive and will violate the transversality conditions unless $b_1 = B_1/P_1 = (\tau - g)/(\beta^{-1} - 1)$. With B_1 given by an initial condition this equation uniquely determines, under perfect foresight, the initial price level P_1. Under perfect foresight the price equation (9) becomes

$$P_t = \hat{D}(P_{t+1}/P_t)^{(1-\sigma_2)/\sigma_2}[1 - \beta(P_{t+1}/P_t)^{-1}]^{1/\sigma_2}. \tag{10}$$

This equation has a steady state at $\hat{P} = \hat{D}(1-\beta)^{1/\sigma_2,}$ but is explosive and will diverge unless B_1 happens to be such that $P_1 = \hat{P}$. However, for $0 < \sigma_2 < 1$ and initial $P_1 > \hat{P}$ we obtain an explosive price path $P_t \to \infty$ that is consistent with the transversality conditions and the equilibrium equations. In this "fiscalist" equilibrium, the initial price level $P_1 = B_1(\beta^{-1} - 1)/(\tau - g)$ is determined by fiscal variables and P_t follows an explosive price path despite a constant money stock.

McCallum argues that this solution is less plausible than an alternative 'bubble-free' monetarist solution $P_t = \hat{P}$ and $b_{t+1} = 0$ for all $t = 1, 2, 3, \ldots$, in which (with our timing) the level of real taxes τ_t adjusts to satisfy $\tau_1 = g + \beta^{-1} b_1$ and $\tau_t = g$ for $t = 2, 3, \ldots$. One way to interpret McCallum's view, as he acknowledges, is as an argument that fiscal policy must be 'Ricardian' for all feasible sequences (not just for equilibrium sequences).[4]

3.1 Fiscalist Case Under Learning

We now take a different tack, which nonetheless comes to the same conclusion as (McCallum 2001), i.e. that the fiscalist solution is not plausible in the case under scrutiny. We suppose that the government can indeed commit to $\tau_t = \tau$ for all $t = 1, 2, 3, \ldots$, so that the only equilibrium perfect foresight price path is the explosive path given above. However, we drop the perfect foresight assumption and ask if the price path is learnable under a natural adaptive learning rule.

[4]For a related argument see (Buiter 1999).

We first note that it follows from (10) that $P_t \to \infty$ implies that $\pi_{t+1} \to \infty$ along the perfect foresight path.[5] It follows that the perfect foresight price path in this case is approximately given by

$$P_{t+1} = \bar{D} P_t^{1/(1-\sigma_2)}, \text{ where } \bar{D} = \hat{D}^{-\sigma_2/(1-\sigma_2)}.$$

From (9) we also have that the approximate temporary equilibrium for large π^e_{t+1} is given by

$$P_t = \hat{D}(\pi^e_{t+1})^{(1-\sigma_2)/\sigma_2} \tag{11}$$

Thus asymptotically, on bubble paths, prices just depend on expected inflation independently of the rest of the system.

We now give an argument that these paths are not stable under learning. We use the finding in the literature on adaptive learning, see (Evans and Honkapohja 2001), that stability under adaptive learning is generally determined by "expectational stability" (E-stability) conditions. Suppose households base their forecasts on a Perceived Law of Motion (PLM) of the form

$$P_t = D P_{t-1}^{\phi}. \tag{12}$$

(We could restrict attention to $\phi = \sigma_2/(1-\sigma_2)$ but it is also easy to treat both D and ϕ as PLM parameters). Then

$$P^e_t = D P^{\phi}_{t-1} \text{ and } P^e_{t+1} = D\left(P^e_t\right)^{\phi} = D^{1+\phi} P^{\phi^2}_{t-1}$$

so that

$$\pi^e_{t+1} = P^e_{t+1}/P^e_t = D^{\phi} P^{\phi(\phi-1)}_{t-1}. \tag{13}$$

We are here treating the information set at the time expectations are formed as including P_{t-1} but not P_t. (However, including current P_t in the information set would not make the price bubble paths stable).

Inserting into (11) gives the Actual Law of Motion (ALM) that is generated by the specified PLM:

$$P_t = \hat{D}(D^{\phi} P^{\phi(\phi-1)}_{t-1})^{(1-\sigma_2)/\sigma_2} = \hat{D} D^{\phi(1-\sigma_2)/\sigma_2} P^{\phi(1-\sigma_2)/\sigma_2(\phi-1)}_{t-1}.$$

[5] If instead we had $P_t \to \infty$ and $P_{t+1}/P_t \to \hat{\pi}$ where $0 < \hat{\pi} < \infty$, the right-hand side of (10) would tend to a finite value. This is a contradiction. (If $\hat{\pi} = 0$, there would be deflation i.e. $P_{t+1} < P_t$ for sufficiently large t, which would violate the assumption $P_t \to \infty$.)

This equation defines a mapping from the PLM parameters (D, ϕ) to the implied ALM parameters, given by

$$T(D, \phi) = (\hat{D}D^{\phi(1-\sigma_2)/\sigma_2}, \phi(\phi - 1)(1 - \sigma_2)/\sigma_2).$$

E-stability is defined in terms of the stability of the differential equation

$$d/d\tau(D, \phi) = T(D, \phi) - (D, \phi)$$

at the equilibrium of interest.

The bubble fixed point is given by $\bar{\phi} = (1 - \sigma_2)^{-1}$ and $\bar{D}$. The roots of DT are $(2\phi - 1)(1 - \sigma_2)/\sigma_2$ and $\phi((1 - \sigma_2)/\sigma_2)\hat{D}D^{\phi(1-\sigma_2)/\sigma_2 - 1}$. At the bubble solution these roots are $1 + 1/\sigma_2$ and $1/\sigma_2$. Since both roots are larger than one it follows that the bubble solution is not E-stable. Note that if we impose $\phi = (1 - \sigma_2)^{-1}$ and just examine E-stability of $\bar{D}$ we obtain the root $T'(\bar{D}) = 1/\sigma_2 > 1$ so that the bubble is E-unstable.[6]

We remark that our stability analysis has been conducted using natural but simple rules for decision-making and learning. In particular the temporary equilibrium equations specify that household demand for real balances, and thus the temporary equilibrium price level, depend only on the expected rate of inflation over the coming period. More elaborate decision (and learning) rules can be imagined in which households choose their money demands based on a forecast of the whole future price path.[7] However, our decision rule is natural because it ensures that the household attempts each period to meet its first-order condition for maximizing utility, given by the usual Euler equation. Our instability results indicate a lack of robustness of the perfect foresight price path, to small deviations, under simple learning rules of a type that are known to yield stability in other contexts, and contrasts with cases below in which these learning rules converge.

[6] There is also a fixed point of T at $\phi = 0$ and $Q = \hat{D}$. The roots at this "monetary steady state" to the approximate model are $-(1 - \sigma_2)/\sigma_2$ and 0, and so this solution is E-stable. However, at the monetary steady state the approximation based on large π^e_{t+1} is unsatisfactory and a more refined analysis based on the entire system would be needed. In the succeding sections we focus on stationary solutions (for monetary policies following interest rate rules).

[7] For example, (Woodford 2001) considers an analysis along these lines, drawing on the calculation equilibrium approach of (Evans and Ramey 1998).

3.2 Monetarist Solution under Learning

We now consider learning stability of the monetarist solution suggested by (McCallum 2001), which arises when money supply is constant and the government pays off the debt immediately, never resorting to bond finance thereafter. Clearly, this is an extreme form of "Ricardian" policies.[8] In consequence, there are no bonds in the economy and the only equation of interest is (9).

We now analyze learning following the procedure above. The solution of interest is the steady state

$$\bar{P} = \hat{D}(1-\beta)^{1/\sigma_2} \text{ with } \pi_t = 1.$$

We now log-linearize (9), which yields the approximation

$$\ln P_t = \ln \bar{P} + \left(\frac{1-\sigma_2}{\sigma_2} + \frac{\beta}{\sigma_2}(1-\beta)^{-1} \right) \ln(\pi^e_{t+1})$$

or

$$P_t = \bar{P}(\pi^e_{t+1})^L, \tag{14}$$

where $L = \frac{1-\sigma_2}{\sigma_2} + \frac{\beta}{\sigma_2}(1-\beta)^{-1}$.

Again we consider PLMs of the form (12), so that inflation expectations are given by (13). Inserting these into (14) leads to the ALM

$$P_t = \bar{P} D^{L\phi} P_{t-1}^{L\phi(\phi-1)}.$$

The mapping from the PLM to the ALM is thus

$$T(D, \phi) = (\bar{P} D^{L\phi}, L\phi(\phi-1)).$$

The monetarist steady state is the fixed point $D = \bar{P}$, $\phi = 0$. Applying the definition of E-stability as before, it is easily verified that the monetarist solution is stable under learning.

[8] Under the perfect foresight monetarist solution there is no seignorage since $\pi_t = 1$ and $\tau_t = g$ for all t. Under learning lump sum taxes adjust each period to offset seignorage.

3.3 Discussion

The results of this section therefore cast doubt upon the plausibility of the fiscal theory of the price level for the special case of constant money and taxes. If the government follows Non-Ricardian policies and the money supply is held fixed, the only REE is the explosive bubble path, but the equilibrium is not stable under learning. The economy under the specified learning rule may indeed follow some explosive path for a period of time, but this path will not converge to the fiscalist solution.

However, there are other policy regimes in which the fiscal theory of the price level has been proposed as the relevant solution. In particular, (Leeper 1991) studied situation in which the inflation rate is affected by government tax and bond variables but with finite steady state inflation. We now turn to an analysis of learning under policy rules (6) and (7) based on a linearization around the steady state. We will be particularly interested in the policy regimes in which the interaction of monetary and fiscal policy rules leads to a unique stationary solution under rational expectations, but we will also consider other policy regimes.

4 Linearized Model with Stochastic Shocks

We thus return to monetary policy following an interest rate rule, with the system specified by (3) and (5) and the policy rules given by (6) and (7). This system is nonlinear, but in a neighborhood of the steady state, we can analyze its linearization. In the Appendix it is shown that the linearized system takes the form

$$\pi_t = (\alpha\beta)^{-1} E_t^* \pi_{t+1} - \alpha^{-1}\theta_t \quad (15)$$

$$0 = b_t + \varphi_1 \pi_t + \varphi_2 \pi_{t-1} - (\beta^{-1} - \gamma) b_{t-1} + \psi_t + \varphi_3 \theta_t + \varphi_4 \theta_{t-1}, \quad (16)$$

where $E_t^* \pi_{t+1}$ denotes inflation expectations formed at t. The notation $E_t^* \pi_{t+1}$ is used to emphasize that the reduced form (15)-(16) applies whether or not expectations are rational. The coefficients $\varphi_1, \ldots, \varphi_4$ are given in the Appendix.[9] From now on we make the assumptions $\alpha \neq 0$, $\alpha\beta \neq 1$, $\gamma\beta \neq 1$ and $\beta^{-1} - \gamma \neq 1$.

[9] These reduced form equations are identical to the reduced form given by Leeper, but with coefficients that differ slightly due to differences in timing and the more general utility function used here. See (Leeper 1991), p. 136.

In the Appendix it is shown that the regular case, in which there is a unique stationary RE solution, arises when either $|\alpha\beta| > 1$ and $\left|\beta^{-1} - \gamma\right| < 1$, i.e. active monetary policy and passive fiscal policy (AM/PF), or $|\alpha\beta| < 1$ and $\left|\beta^{-1} - \gamma\right| > 1$, i.e. active fiscal policy and passive monetary policy (AF/PM). Either condition $|\alpha\beta| < 1$ or $\left|\beta^{-1} - \gamma\right| < 1$ leads to a linear restriction of the form

$$\pi_t = K_1 b_t + K_2 \theta_t \tag{17}$$

when non-explosiveness of the solution is imposed. This equation together with (16) defines the unique stationary solution in the regular case.

In the AM/PF regime we obtain $K_1 = 0$ and $K_2 = -\alpha^{-1}$, so that

$$\pi_t = -\alpha^{-1}\theta_t.$$

We will refer to this solution as the "monetarist solution", since π_t is independent of both b_{t-1} and the tax shock ψ_t. In the AF/PM. In the AF/PM regime we obtain the expression

$$\pi_t = \frac{\alpha\beta\varphi_1 + \varphi_2}{\beta^{-1} - \gamma - \alpha\beta} b_t + K_2\theta_t. \tag{18}$$

Substituting for π_{t-1} in (16) it can be seen that inflation now depends on b_{t-1} and ψ_t. We therefore refer to this REE as the "fiscalist solution."

Besides the regular cases, there are two other regimes possible, depending on policy parameters. If $|\alpha\beta| < 1$ and $\left|\beta^{-1} - \gamma\right| < 1$, so that both policies are passive, the model is "irregular" or "indeterminate," with multiple stationary solutions. If $|\alpha\beta| > 1$ and $\left|\beta^{-1} - \gamma\right| > 1$, so that both policies are active, the model is said to be "explosive," and there are no stationary solutions. As will be seen both monetarist and fiscalist solutions always exist, but need not be stationary. The different regimes are shown if Figure 1.

Clearly, the solutions can also be written in a vector autoregressive form, and this is more convenient for the analysis of learning which we now undertake. Again we will focus on E-stability conditions. Since we are now examining stationary solutions to a linearized multivariate model, the results of Chapter 10 of (Evans and Honkapohja 2001) show that E-stability conditions govern the convergence of least squares and related real-time learning schemes.

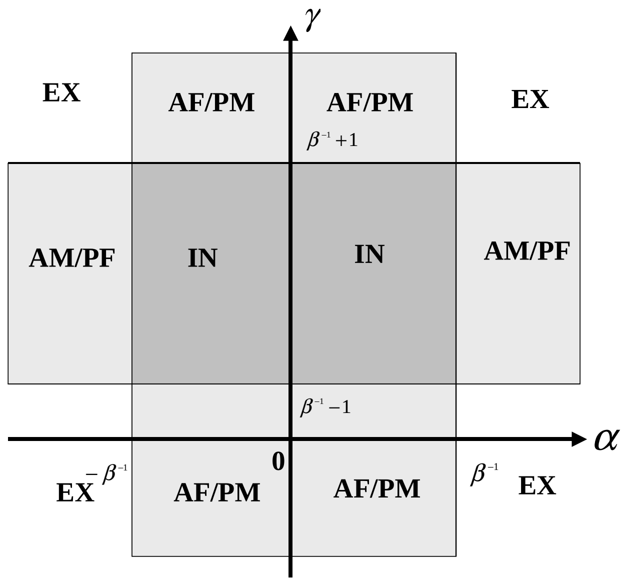

Figure 1:

4.1 REE as Fixed Points

Introducing the notation $y_t = (\pi_t, b_t)'$, the linearized model (15)-(16) can be written in the vector form

$$y_t = ME_t^* y_{t+1} + Ny_{t-1} + Pv_t + Rv_{t-1}, \tag{19}$$

where

$$M = \begin{pmatrix} (\alpha\beta)^{-1} & 0 \\ -\varphi_1(\alpha\beta)^{-1} & 0 \end{pmatrix}, \; N = \begin{pmatrix} 0 & 0 \\ -\varphi_2 & \beta^{-1} - \gamma \end{pmatrix},$$

$$P = \begin{pmatrix} -\alpha^{-1} & 0 \\ \varphi_1\alpha^{-1} - \varphi_3 & -1 \end{pmatrix}, \; R = \begin{pmatrix} 0 & 0 \\ -\varphi_4 & 0 \end{pmatrix}, \; v_t = \begin{pmatrix} \theta_t \\ \psi_t \end{pmatrix}.$$

We consider PLMs of the form

$$y_t = A + By_{t-1} + Cv_t + Dv_{t-1}. \tag{20}$$

These PLMs exclude exogenous sunspot variables by assumption (we will briefly consider such solutions below). Computing the expectation[10]

$$\begin{aligned} E_t^* y_{t+1} &= A + B(A + By_{t-1} + Cv_t + Dv_{t-1}) + Dv_t \\ &= (I+B)A + B^2 y_{t-1} + (BC + D)v_t + BDv_{t-1} \end{aligned}$$

and inserting into (19) we obtain the implied ALM

$$\begin{aligned} y_t &= M(I+B)A + (MB^2 + N)y_{t-1} \\ &\quad + (M(BC+D)+P)v_t + (MBD + R)v_{t-1}. \end{aligned}$$

Thus the mapping from the PLM to the ALM is

$$\begin{aligned} A &\longrightarrow M(I+B)A \\ B &\longrightarrow MB^2 + N \\ C &\longrightarrow MBC + MD + P \\ D &\longrightarrow MBD + R \end{aligned}$$

and the fixed points of this mapping correspond to REE of the form (20).

The second component of the mapping can have more than one solution. Given any solution for B the first component gives the unique solution for $A = 0$, provided $I - M(I+B)$ is nonsingular. Similarly, for given B the third and fourth components of the mapping are linear equations for C and D. Because of the form of M and N we have the result:

Proposition 1 *There are three types of REE taking the form (20) as listed below.*

I. $B = N$, $C = P$ and $D = R$ with $A = 0$. This is the "monetarist solution."

II. $B = \chi^{-1} \begin{pmatrix} -(\beta\gamma + \alpha\beta^2 - 1)\varphi_2 & -\beta^{-1}(\beta\gamma - 1)(\beta\gamma + \alpha\beta^2 - 1) \\ \beta\varphi_2(\alpha\beta\varphi_1 + \varphi_2) & (\beta\gamma - 1)(\alpha\beta\varphi_1 + \varphi_2) \end{pmatrix}$, where $\chi = (\beta\gamma - 1)\varphi_1 - \beta\varphi_2$, $A = 0$ and C and D also uniquely determined by the fixed point.[11] It can be verified that this is a way

[10] We make the commonly employed assumption that the agents observe the current values of the exogenous variables, but only the lagged values of the endogenous variables when they compute forecasts from the PLM.

[11] Explicit formulas for C and D are available on request. This assumes $\chi \neq 0$.

of representing the "fiscalist solution." Although this may appear to be a complicated representation, it can be verified that the eigenvalues of B are 0 and $\alpha\beta$. The zero eigenvalue corresponds to the static linear relationship (18) between π_t and b_t, which can be used to obtain alternative representations of the REE.

III. $B = \begin{pmatrix} \alpha\beta & 0 \\ -(\varphi_1\alpha\beta + \varphi_2) & \beta^{-1} - \gamma \end{pmatrix}$, $A = 0$. For C and D the solution is not unique. For D there is a two-dimensional continuum and, given a value for D, the equation for C also yields a two-dimensional continuum. We call this class of solutions the non-fundamental solutions, because of the indeterminacy in the C and D coefficients. We remark that this solution set can be expanded to allow for dependence on an exogenous sunspot variable.

In the case of AM/PF policy, the monetarist solution is stationary, while the fiscalist solution and non-fundamental solutions are explosive. In contrast, in the case of AF/PM policy, the fiscalist solution is stationary while the monetarist solution and non-fundamental solutions are explosive. In the case of PM/PF policy all the REE are stationary. We now turn to an examination of whether these solutions are stable under learning.

4.2 Stability under Learning

Let $\xi = (A, B, C, D)$ denote the parameters of the PLM and let $T(\xi)$ denote the corresponding values of the ALM given by the above mapping. The three types of RE solutions above correspond to fixed points of this map. Local stability under Least Squares learning is determined by E-stability conditions, defined as the conditions for local asymptotic stability, under the notional time differential equation

$$d\xi/d\tau = T(\xi) - \xi, \tag{21}$$

of the RE solution of interest.

For the formal analysis we note that the B component is this differential equation is nonlinear, but local stability is determined by its linearization at the fixed point of interest. The B, C and D components are matrix-valued and need to be vectorized. Moreover, it is seen that the B component of (21) is an independent subsystem, the A and D subsystems, respectively, depend on B and the C subsystem depends on both B and D. The stability conditions for (21) can be given in terms of the following matrices[12]

$$\begin{aligned} DT_A &= M(I+\bar{B}), \\ DT_B &= \bar{B}' \otimes M + I \otimes M\bar{B}, \\ DT_C &= I \otimes M\bar{B}, \\ DT_D &= I \otimes M\bar{B}, \end{aligned}$$

where $\bar{B}$ denotes the value of B at the REE of interest.

The E-stability condition for REE of type I and II is that the real parts of all eigenvalues of all four matrices DT_i, $i = A, B, C, D$, are less than one. For the class of non-fundamental solutions III the matrices DT_C and DT_D will have some eigenvalues equal to one, due to the continuum of solutions. A necessary condition for E-stability is that the non-zero eigenvalues of the four matrices have real parts less than one.

We now provide the explicit E-stability results for the three types of REE, after which we discuss the economic implications.

I. The Monetarist Solution: The eigenvalues of DT_A are 0 and $(\alpha\beta)^{-1}$. The non-zero eigenvalue of DT_B is $\frac{\beta^{-1}-\gamma}{\alpha\beta}$. All eigenvalues of DT_C and DT_D are zero. This yields the E-stability conditions

$$(\alpha\beta)^{-1} < 1 \text{ and } \frac{\beta^{-1}-\gamma}{\alpha\beta} < 1.$$

II. The Fiscalist Solution: The non-zero eigenvalues of DT_i, $i = A, B, C, D$, are $1 + \frac{\gamma+1-\beta^{-1}}{\alpha\beta}$, $1 + \frac{\gamma-\beta^{-1}}{\alpha\beta}$ and $2 + \frac{\gamma-\beta^{-1}}{\alpha\beta}$. This yields the E-stability conditions

$$\frac{\beta^{-1}-\gamma}{\alpha\beta} > 1 \text{ and } \frac{\gamma+1-\beta^{-1}}{\alpha\beta} < 0.$$

Although the matrix $\bar{B}$ depends on φ_1 and φ_2 the eigenvalues of DT_i, $i = A, B, C, D$, are in fact independent of φ_1 and φ_2, as can be verified using e.g. Mathematica (routines available on request).

III. The Non-Fundamental solutions are not E-stable, since DT_B has an eigenvalue equal to 2.

[12] For details on the technique, see Chapter 10 of (Evans and Honkapohja 2001).

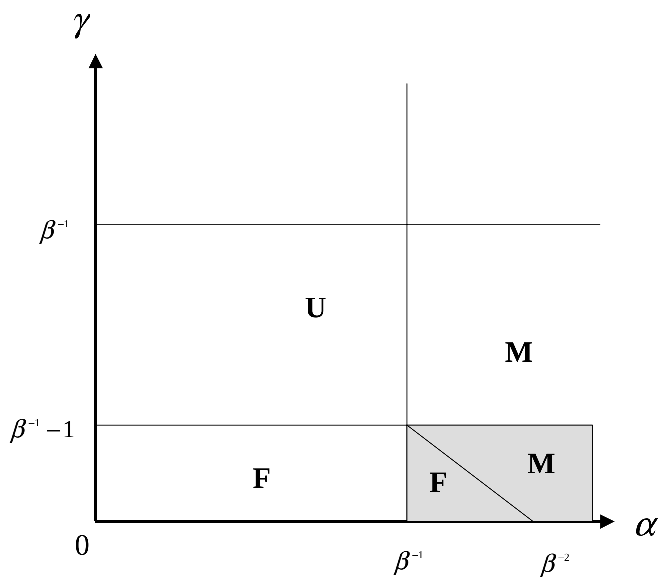

Figure 2:

4.3 Economic Implications

Looking at the economic model, it is evident that the most natural policy rules entail the parameter restrictions $\alpha > 0$ and $\gamma \geq 0$. $\alpha > 0$ means that the nominal interest rate responds positively to current inflation and $\gamma > 0$ means that the lump-sum tax responds positively to beginning-of-period debt b_{t-1}. In the case $\gamma = 0$ taxes are set independently of the debt level. Realistic values of γ would also appear to be below β^{-1}, since $\gamma > \beta^{-1}$ implies that, at the non-stochastic steady state, any shock to debt levels would lead to a tax increase that would more than pay off the debt, including interest, within one period. We therefore focus on the region of the policy parameter space, followed by a brief discussion of the other cases.

Figure 2 shows the results on learning stability for the monetarist and fiscalist solutions in this part of the parameter space. In the figure **M** indicates that the monetarist solution is stable under learning. **F** indicates that the fiscalist solution is stable and **U** indicates that neither solution is stable under learning. In none of the areas are both solutions simultaneously stable under learning. In the shaded region $\alpha > \beta^{-1}$ and $0 \leq \gamma < \beta^{-1} - 1$ the solutions are not stationary.

Within the parameter region described by Figure 2, the AM/PF regime arises with $\alpha > \beta^{-1}$ and $\beta^{-1} - 1 < \gamma < \beta^{-1}$. In this regime the monetarist equilibrium is the unique stationary solution and it is also stable under learning. In the AF/PM regime, given by $0 < \alpha < \beta^{-1}$ and $0 \leq \gamma < \beta^{-1} - 1$, the fiscalist REE is the unique stationary solution and is stable under learning.

The indeterminacy region with policy combination PM/PF is given by $0 < \alpha < \beta^{-1}$ and $\beta^{-1} - 1 < \gamma < \beta^{-1}$. Here while both solutions are stationary, they fail to be stable under learning.

The shaded explosive region with policy combination AM/AF is also divided into two cases with either the fiscalist or the monetarist solution being stable under learning. We emphasize that our results are local i.e. they are valid only in a neighborhood of the steady state. Our results for the shaded thus give only a limited amount of information because the solutions diverge from the steady state. A full analysis of learning would require examination of the nonlinear model. However, the results for this region do suggest an incipient tendency for the economy under learning to follow the indicated explosive equilibrium.

Note that active monetary policy requires $\alpha > \beta^{-1}$. This is a somewhat stronger condition than given by a usual formulation of the "Taylor principle". If instead $1 < \alpha < \beta^{-1}$ the monetarist solution becomes unstable (with either the economy becoming unstable or tending to the fiscalist solution).

4.4 Further Comments

We make a few observations about learning stability in the other regions of the policy parameters not covered by Figure 2. Throughout the AM/PF region the monetarist equilibrium is stable under learning. This solution is also stable in part of the left IN region. The fiscalist solution is stable in the top-left and bottom-right AF/PM regions of Figure 1 and it is also stable in a part of the left IN region of Figure 1. There is no stable equilibrium in the top-right AF/PM region even though this is a regular case in which the fiscalist REE is the unique stationary solution. Finally and most surprisingly, in the bottom-left AF/PM region the explosive monetarist equilibrium is stable while the stationary fiscalist solution is unstable under learning.

For convenience we have assumed that the exogenous shocks are white noise. Assume instead that they follow a jointly stationary first order vector autoregression. As we note in the appendix, this imposes additional requirements for learning stability of equilibria. In some cases the stability regions for model parameters are unchanged. However, one can also find cases in which the additional requirements tighten the domain of stability for the parameters.

5 Conclusions

We have considered the local stability under learning of the rational expectations solutions in a simple stochastic optimizing monetary model in which the interaction between monetary and fiscal policy is central. Our first finding was that in the case of constant money supply and constant taxes, the equilibrium explosive price paths dictated by the fiscal theory of the price level are not locally stable under learning. In contrast, if fiscal policy is Ricardian, then monetarist equilibrium is stable under learning. These particular results appear to cast doubt on the plausibility of the fiscal theory.

However, we then examined an alternative setting in which interest rates are set as a linear function of inflation and taxes are set as a linear function of real debt. The usual monetarist solution is locally stable under learning in the active monetary/passive fiscal policy regime in which it is the unique stationary solution. On the other hand, the fiscalist solution, in which inflation depends on the debt level and on tax shocks, is stable under learning for a subregion of the active fiscal/passive monetary regime, in which the fiscalist solution is the unique stationary solution.

There are also regions of plausible policy parameter values in which the economy is irregular, with multiple stationary solutions. For one part of this region the fiscalist equilibrium is in fact stable under learning, while the monetarist solution as well as the non-fundamental solution class are not learnable.

In total our results provide some support for the fiscalist solution. Whether the fiscalist solution emerges under learning depends on the precise specification of the fiscal and monetary policies. Careful consideration of the interaction of these policies is required to understand the qualitative characteristics of inflation and debt dynamics.

A Appendix

A.1 Household Optimality Conditions

Define the variables $W_{t+1} = m_t + b_t$ and $x_{t+1} = m_t$. Following (Chow 1996), Section 2.3, introduce the Lagrange multipliers λ_t for budget constraint and μ_t for the equation $x_{t+1} = m_t$ and write the Lagrangian

$$\begin{aligned} L &= E_0 \sum_{t=0}^{\infty} \{ \beta^t \left[(1-\sigma_1)^{-1} c_t^{1-\sigma_1} + A(1-\sigma_2)^{-1} (x_t \pi_t^{-1})^{1-\sigma_2} \right] + \\ & \quad \beta^{t+1} \lambda_{t+1} [W_{t+1} - y + c_t + \tau_t - x_t \pi_t^{-1} - R_{t-1} \pi_t^{-1} (W_t - x_t)] \\ & \quad + \beta^{t+1} \mu_{t+1} (x_{t+1} - m_t) \}. \end{aligned}$$

Here W_t, x_t are the state and c_t, m_t the control variables.

The first order conditions are

$$c_t^{-\sigma_1} - \beta E_t \lambda_{t+1} = 0, \tag{22}$$

$$E_t \mu_{t+1} = 0, \tag{23}$$

$$\lambda_t = \beta (R_{t-1} \pi_t^{-1}) E_t \lambda_{t+1}, \tag{24}$$

$$\mu_t = A \pi_t^{-1} (x_t \pi_t^{-1})^{-\sigma_2} + \beta (\pi_t^{-1} - R_{t-1} \pi_t^{-1}) E_t \lambda_{t+1}. \tag{25}$$

In addition, the household's optimal choices must satisfy the transversality conditions (4).

To derive the Fisher equation (2) we first advance (24) for one period and take conditional expectations $E_t(.)$, after which we apply the market clearing condition

$$c_t = y - g$$

and (22).

To derive (3) we substitute (24) into (25) and use (23) yielding

$$A x_{t+1}^{-\sigma_2} E_t \pi_{t+1}^{\sigma_2 - 1} + (R_t^{-1} - 1) E_t \lambda_{t+1} = 0.$$

Then use (2), the market clearing condition and (22).

A.2 Linearization

We first give the linearization of the model. Rearranging (3) we can write money demand as

$$m_t = (A\beta)^{1/\sigma_2}(y-g)^{\sigma_1/\sigma_2}[(1-\beta E_t\pi_{t+1}^{-1})(E_t\pi_{t+1}^{\sigma_2-1})^{-1}]^{-1/\sigma_2}. \tag{26}$$

(26) is of the general form

$$m_t = F[E_t(f(\pi_{t+1}), E_t g(\pi_{t+1})],$$

where

$$\begin{aligned} F(x,y) &= \hat{C}(1-\beta x)^{-1/\sigma_2}y^{1/\sigma_2}, \text{ with} \\ x &= f(z) = z^{-1} \text{ and } y = g(z) = z^{\sigma_2-1}. \end{aligned}$$

Here $\hat{C} = (A\beta)^{1/\sigma_2}(y-g)^{\sigma_1/\sigma_2}$.

Carrying out the differentiation we have

$$\begin{aligned} F_1(x,y) &= \hat{C}y^{1/\sigma_2}(-1/\sigma_2)(1-\beta x)^{-1/\sigma_2-1}(-\beta), \\ F_2(x,y) &= \hat{C}(1-\beta x)^{-1/\sigma_2}(1/\sigma_2)y^{1/\sigma_2-1} \\ f'(z) &= -z^{-2}, g'(z) = (\sigma_2-1)z^{\sigma_2-2}. \end{aligned}$$

Thus, using the chain rule

$$dm = (F_1 f' + F_2 g')dz$$

at the nonstochastic steady state π, we have the linearization

$$\tilde{m}_t = \left[\left(\frac{-\hat{C}\beta}{\sigma_2}\right)(\pi-\beta)^{-(1+\sigma_2)/\sigma_2} + \left(\frac{\sigma_2-1}{\sigma_2}\right)\hat{C}(\pi-\beta)^{-1/\sigma_2}\pi^{\sigma_2-2}\right] E_t\tilde{\pi}_{t+1}$$

or

$$\tilde{m}_t \equiv C E_t\tilde{\pi}_{t+1}. \tag{27}$$

Here $\tilde{m}_t$ and $E_t\tilde{\pi}_{t+1}$ denote the deviations from the nonstochastic steady state.

We also need to linearize the Fisher relation (2) at the nonstochastic steady state π, R. We have

$$0 = -\beta R \pi^{-2} E_t \tilde{\pi}_{t+1} + \beta \pi^{-1} \tilde{R}_t,$$

where $\tilde{R}_t$ is the deviation from the nonstochastic steady state. Since the Fisher equation also holds at the nonstochastic steady state, i.e. $\beta R \pi^{-1} = 1$, we get

$$E_t \tilde{\pi}_{t+1} = \beta \tilde{R}_t$$

which can be substituted into (27) to yield

$$\tilde{m}_t \equiv C\beta \tilde{R}_t.$$

This last expression can be used in the linearized government budget constraint.

Finally, we linearize the budget constraint, taking note that m_t is a function of R_t. We get

$$\begin{aligned} 0 &= \tilde{b}_t + \frac{\partial m}{\partial R}\tilde{R}_t + \gamma \tilde{b}_{t-1} + \psi_t - \pi^{-1}\frac{\partial m}{\partial R}\tilde{R}_{t-1} + \frac{m}{\pi^2}\tilde{\pi}_t - \\ & R\pi^{-1}\tilde{b}_{t-1} - \pi^{-1} b \tilde{R}_{t-1} + Rb\pi^{-2}\tilde{\pi}_t, \end{aligned}$$

where π, b, R are the non-stochastic steady state values and

$$\frac{\partial m}{\partial R} = C\beta$$

is the derivative of the money demand function at the non-stochastic steady state. Note that $R\pi^{-1} = \beta^{-1}$ by (2). The next step is the observation that

$$\tilde{R}_t = \alpha \tilde{\pi}_t + \theta_t$$

as a result of centering. This yields the final linearization (28) below.

Collecting everything together we have the two Leeper-type equations

$$E_t \tilde{\pi}_{t+1} = \alpha \beta \tilde{\pi}_t + \beta \theta_t$$

and

$$\begin{aligned} 0 &= \tilde{b}_t + \tilde{\pi}_t \left(C\beta\alpha + \frac{m}{\pi^2} + Rb\pi^{-2} \right) + \tilde{\pi}_{t-1}\left(-\pi^{-1}C\beta\alpha - \pi^{-1}b\alpha\right) \quad (28) \\ & + \tilde{b}_{t-1}(\gamma - \beta^{-1}) + C\beta\theta_t + \psi_t + \theta_{t-1}\left(-\pi^{-1}C\beta - \frac{b}{\pi}\right). \end{aligned}$$

Equation (28) implicitly specifies the coefficients $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ of equation (16). Here α, β, γ are just the original model parameters,

$$C = \left(\frac{-\hat{C}\beta}{\sigma_2}\right)(\pi - \beta)^{-(1+\sigma_2)/\sigma_2} + \left(\frac{\sigma_2 - 1}{\sigma_2}\right)\hat{C}(\pi - \beta)^{-1/\sigma_2}\pi^{\sigma_2 - 2},$$

in which again σ_2 is a parameter in the original model, and $\hat{C} = (A\beta)^{1/\sigma_2}(y - g)^{\sigma_1/\sigma_2}$, where π, m, b, R are the non-stochastic steady state values. The latter are given by equations

$$\beta R = \pi$$

$$b + m + \gamma_0 + \gamma b = g + m\pi^{-1} + R\pi^{-1}b$$

$$R = \alpha_0 + \alpha\pi$$

and

$$m = A^{1/\sigma_2}(y - g)^{\sigma_1/\sigma_2}\beta R(R-1)^{-1/\sigma_2}.$$

A.3 Regularity Conditions

For either specification the system under RE can be rewritten as

$$\begin{aligned} &\begin{pmatrix} 1 & 0 \\ -\varphi_2 & \beta^{-1} - \gamma \end{pmatrix}\begin{pmatrix} \pi_t \\ b_t \end{pmatrix} \\ = &\begin{pmatrix} (\alpha\beta)^{-1} & 0 \\ \varphi_1 & 1 \end{pmatrix}\begin{pmatrix} \pi_{t+1} \\ b_{t+1} \end{pmatrix} + \begin{pmatrix} (\alpha\beta)^{-1} \\ 0 \end{pmatrix}\eta_{t+1} + \\ &\begin{pmatrix} 0 \\ \varphi_3 \end{pmatrix}\theta_{t+1} + \begin{pmatrix} -\alpha^{-1} \\ \varphi_4 \end{pmatrix}\theta_t + \begin{pmatrix} 0 \\ 1 \end{pmatrix}\psi_{t+1}, \end{aligned}$$

or

$$\begin{pmatrix} \pi_t \\ b_t \end{pmatrix} = J\begin{pmatrix} \pi_{t+1} \\ b_{t+1} \end{pmatrix} + F_1\eta_{t+1} + F_2\theta_{t+1} + F_3\theta_t + F_4\psi_{t+1},$$

where

$$J = \begin{pmatrix} (\alpha\beta)^{-1} & 0 \\ (\beta^{-1}-\gamma)^{-1}(\varphi_1+\varphi_2(\alpha\beta)^{-1}) & (\beta^{-1}-\gamma)^{-1} \end{pmatrix}$$

and where $\eta_{t+1} = \pi_{t+1} - E_t\pi_{t+1}$.

The eigenvalues of J are $(\alpha\beta)^{-1}$ and $(\beta^{-1}-\gamma)^{-1}$. If either root is less than one, imposing non-explosiveness gives a linear restriction between π_t, b_t and θ_t. This is obtained as follows.[13] Diagonalize J as $J = Q\Lambda Q^{-1}$, where $\Lambda = \text{diag}((\beta^{-1}-\gamma)^{-1}, (\alpha\beta)^{-1})$. Let $(x_t, z_t)' = Q^{-1}(\pi_t, b_t)$. If $|(\alpha\beta)^{-1}| < 1$ then non-explosiveness of the solution requires that $x_t + C_1\theta_t = 0$ where C_1 depends on Q^{-1} and F_3. It can be shown that $x_t = \pi_t$ yielding the static linear relationship satisfied by the monetarist solution. If $\left|(\beta^{-1}-\gamma)^{-1}\right| < 1$, then non-explosiveness requires that $z_t + C_2\theta_t = 0$. Rewriting z_t as a linear function of π_t and b_t gives the static linear relationship satisfied by the fiscalist solution.

Finally, we remark that in Section 4.1 the fiscalist solution II can be shown to satisfy the fiscalist static relationship whether or not the model is regular. Since the matrix B is singular, one row is proportional to the other row and it can be verified that the proportionality factor is $\frac{\alpha\beta\varphi_1+\varphi_2}{\beta^{-1}-\gamma-\alpha\beta}$, which is the same as the coefficient in (18).

A.4 Serially Correlated Shocks

Suppose that the shocks $v_t = (\theta_t, \psi_t)'$ follow a $VAR(1)$ process, i.e.

$$v_t = Fv_{t-1} + e_t,$$

where e_t is white noise and the eigenvalues of F are inside the unit circle. In this case the mapping from the PLM to the ALM is unchanged for the A, B and D components. For C the mapping becomes

$$C \longrightarrow MBC + MCF + MD + P$$

and the E-stability condition for C is

$$DT_C = I \otimes M\bar{B} + F' \otimes \bar{C}.$$

[13] For the technique see the Appendix of Chapter 10 of (Evans and Honkapohja 2001).

As an illustration restrict attention to the monetarist solution in the case $\alpha > 0$ and $F = (f_{ij})$ is diagonal. It can be verified that for $f_{11}, f_{22} \geq 0$ the E-stability conditions remain unchanged. On the other hand, when f_{11} and f_{22} have different signs, the conditions can be tighter. For example, setting $\beta = 0.95$, $\alpha = 1.2$, $f_{11} = 0.99$ and $f_{22} = -0.8$ yields an unstable root for DT_C.

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