Identifying the Responses of Hours to a Technology Shock: a Two–Step Structural VAR Approach

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Abstract

The response of hours worked to a technology shock is an important and a controversial issue. The estimated response is generally sensitive to the specification of hours in SVARs. This paper introduces a simple two step approach in order to consistently estimate technology shocks from a SVAR model and the response of hours that follow this shock. The first step considers a SVAR model with a set of a relevant stationary variable, but excluding hours. Given a consistent estimate of technology shocks in the first step, the response of hours to this shock is estimated in a second step. When applied on US data, the two step approach predicts a short–run decrease of hours after a technology improvement, whereas a delayed and hump–shaped positive response. This result is robust to different sample periods, measures of hours and output and to the variables included in the VAR at the first step.

Keywords: SVARs, long–run restriction, technology shocks, hours worked

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Introduction

The response of hours to a technology shock is the subject of many controversies in quantitative macroeconomics. The contributions of Galí (1999), Basu, Fernald and Kimball (2004) and Francis and Ramey (2005a) show that the short–run response of hours worked to a technology shock is significantly negative in the US economy. Galí (1999) and Francis and Ramey (2005a) obtain this result using a Structural Vector Autoregression (SVAR) of labor productivity growth and hours in first difference (DSVAR) with long–run restrictions (see Blanchard and Quah, 1989). Basu, Fernald and Kimball (2004) use a direct measure of aggregate technology change, controlling for imperfect competition, varying utilization of factors and aggregation effects, and find that hours fall significantly on impact after a technology improvement. Moreover, Galí (1999) and (2004) shows that the level of hours significantly decreases in the short run in all G7 countries and the euro area as a whole, with the exception of Japan. These results are in contradiction with Christiano, Eichenbaum and Vigfusson (2004). Christiano, Eichenbaum and Vigfusson use a SVAR with a level specification of hours (LSVAR) and find a positive and hump–shaped response of hours after a technology shock. Moreover, they show that the LSVAR specification encompasses the DSVAR specification.

The specification of hours in level or in difference appears the core issue of the controversies. Galí (1999), Galí and Rabanal (2004) and Christiano, Eichenbaum and Vigfusson (2004) perform various unit root tests, but it becomes hard to obtain clear–cut evidence in favor of level or difference specification. Unfortunately, recent contributions proceeding with simulation experiments point out that the specification of hours in SVARs using long–run restrictions can alter significantly the estimated effect of a technology shock on hours. For example, Chari, Kehoe and McGrattan (2005) simulate a business cycle model estimated by Maximum Likelihood on US data with multiple shocks. They show that the DSVAR approach leads to a negative response of hours under a RBC model in which hours respond positively. As pointed by Christiano, Eichenbaum and Vigfusson (2004), the DSVAR may induce strong distortions if hours worked are stationary in level.

In this paper, we propose a simple method that allows to consistently estimate technology shocks and thus the responses of hours to a technology improvement. We argue that hours must be excluded from SVARs to consistently estimate technology shocks. The proposed approach consists in the following two steps. In a first step, a SVAR model with long–run restriction that includes well-chosen covariance stationary variables allow to properly identify the technology shock series. Among these variables, the consumption to output ratio seems to be a promising candidate. In the second step, the impulse response functions of hours at different horizons are obtained by a simple OLS regression of hours on the estimated technology shocks for different lags. In this latter step, we consider hours worked in level and in difference in the regression models. Our method then combines a SVAR approach in the line of Blanchard and Quah (1989), Galí (1999) and Christiano, Eichenbaum and Vigfusson (2004) and the regression equation used by Basu, Fernald and Kimball (2004) in their growth accounting exercice.

Applications to US data show that hours decrease in the short–run after a positive technology shock but display a positive hump–shaped response. Our result is in the line of previous empirical findings which obtain that hours fall significantly on impact (see Basu, Fernald and Kimball, 2004, Francis and Ramey, 2005b) and display a positive hump pattern during the subsequent periods (see Vigfusson, 2004). This result is robust to the sample period considered, measures of hours and output, bivariate VARs, relevant larger VARs and breaks in labor productivity. The level and difference specifications of hours provide similar impulse response functions in all our experiments.

The paper is organized as follows. In a first section, we present our two step approach. Section 2 presents the empirical results. The last section concludes.

1 The Two Step Approach

The goal of our approach is to accurately identify technology shocks in a first step using adequate covariance–stationary variables in the VAR model. A large part of the performance of the two step approach depends on the time series properties of these variables, which can be interpreted as instruments allowing to estimate with more precision the true technology shocks. Christiano, Eichenbaum and Vigfusson (2004) and Gospodinov (2006) show that the introduction of a nonstationary or a nearly nonstationary variable in a VAR model leads to a weak instrument problem. As shown by these authors, this problem precludes a consistent estimates of the impulse response functions under long-run identification scheme. The objective of the first step is then to include a set of variables in the SVAR model to identify more properly the true technology shocks series. Among these variables, a promising candidate is the consumption to output ratio.[1] Cochrane (1994) argues that the consumption to output ratio contains useful information to disentangle the permanent to the transitory component. Moreover, as shown in the empirical section, the unit root can be rejected for this ratio at a conventional level and the empirical autocorrelation function indicate a less persistent process than the one of hours. So we decide to introduce this ratio as instrument to identify the technology shocks. Their impact on the variable of interest (hours worked) is evaluated in the second step. To do so, hours are projected in level and in difference on the identified technology shocks series. We show that this estimator of the impulse responses function is consistent for both specifications of the hours worked. In the applications, we also consider in the first step larger SVARs that have been used in the relevant literature (see for example, Christiano, Eichenbaum and Vigfusson, 2004) to check the robustness of our two step strategy. In what follows, we present the two step approach.

[1] Another promising candidate in our sample is the log of the investment to output ratio.

Step 1: *Identification of technology shocks*
We consider a VAR model which includes productivity growth $\Delta(y_t - h_t)$ and consumption to output ratio $c_t - y_t$ (in logs). We start by specifying a VAR(p) model in these two variables:

$$\boldsymbol{X}_t = \sum_{i=1}^{p} \boldsymbol{B}_i \boldsymbol{X}_{t-i} + \boldsymbol{\varepsilon}_t \tag{1}$$

where $\boldsymbol{X}_t = (\Delta(y_t - h_t), c_t - y_t)'$ and $\boldsymbol{\varepsilon}_t = (\boldsymbol{\varepsilon}_{1,t}, \boldsymbol{\varepsilon}_{2,t})'$ with $\mathrm{E}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t') = \boldsymbol{\Sigma}$. Under usual conditions, this VAR(p) model admits a VMA(∞) representation $\boldsymbol{X}_t = \boldsymbol{C}(L)\boldsymbol{\varepsilon}_t$ where $\boldsymbol{C}(L) = (\boldsymbol{I}_2 - \sum_{i=1}^{p} \boldsymbol{B}_i L^i)^{-1}$. The structural VMA($\infty$) representation is given by

$$\boldsymbol{X}_t = \boldsymbol{A}(L)\boldsymbol{\eta}_t$$

where $\boldsymbol{\eta}_t = (\boldsymbol{\eta}_t^T, \boldsymbol{\eta}_t^{NT})'$. $\boldsymbol{\eta}_t^T$ is period t technology shock, whereas $\boldsymbol{\eta}_t^{NT}$ is period t non–technology shock. By normalization, these two orthogonal shocks have zero mean and unit variance. The identifying restriction implies that the non–technology shock has no long–run effect on labor productivity. This means that the upper triangular element of $\boldsymbol{A}(L)$ in the long run must be zero, *i.e.* $\boldsymbol{A}_{12}(1) = 0$. In order to uncover this restriction from the estimated VAR(p) model, the matrix $\boldsymbol{A}(1)$ is obtained as the Choleski decomposition of $\boldsymbol{C}^{-1}(1)\boldsymbol{\Sigma}\boldsymbol{C}^{-1}(1)'$. The structural shocks are then directly deduced up to a sign restriction by $\boldsymbol{\eta}_t = (C(1)A(1))^{-1}\boldsymbol{\varepsilon}_t$.

Step 2: *Estimation of the responses of hours to a technology shock*
Suppose the following infinite moving average representation for hours worked as a linear function of a technology and a non–technology shocks:

$$h_t = \boldsymbol{a}_{21}(L)\boldsymbol{\eta}_t^T + \boldsymbol{a}_{22}(L)\boldsymbol{\eta}_t^{NT}. \tag{2}$$

where the individual $a_{21,k}$ measures the impact of the technology shock at lag k. The identifying restriction of Step 1 implies that non–technology shocks are orthogonal to technology shocks by construction, *i.e.* $\mathrm{E}(\boldsymbol{\eta}_{t-i}^T, \boldsymbol{\eta}_{t-j}^{NT}) = 0 \ \forall i, j$ and that the technology and non–technology shocks are serially uncorrelated which implies $\mathrm{E}(\boldsymbol{\eta}_t^T, \boldsymbol{\eta}_{t-i}^T) = 0$ and $\mathrm{E}(\boldsymbol{\eta}_t^T, \boldsymbol{\eta}_{t-i}^T) = 0 \ \forall i \neq 0$. Let $\hat{\boldsymbol{\eta}}_t^T$ denotes the estimated technology shocks obtained from the SVAR model in the first step. According to the debate on the right specification of hours worked, we examine two specifications to measure the effect of a technology shock. In the first specification, hours are projected in level on the identified technology shocks while in the second specification, hours are projected in difference.

Let us now examine in more details both specifications. In the first one, the logs of hours worked is regressed on the current and past values of the identified technology shocks $\hat{\boldsymbol{\eta}}_t^T$ in the first-step:

$$h_t = \sum_{i=0}^{q} \boldsymbol{\theta}_i \hat{\boldsymbol{\eta}}_{t-i}^T + \boldsymbol{\nu}_t \tag{3}$$

where $q < +\infty$. $\boldsymbol{\nu}_t$ is a composite error term that accounts for non–technology shocks and the remainder technology shocks. A standard OLS regression provides the estimates of the population responses of hours to the present and lagged values of the technology shocks, namely: $\hat{a}_{21,k} = \widehat{\boldsymbol{\theta}}_k$. According to the debate on the appropriate specification of hours, this variable is also regressed in first difference on the current and past values of the identified technology shocks. The response of hours worked to a technology shock is now estimated from the regression:

$$\Delta h_t = \sum_{i=0}^{q} \tilde{\boldsymbol{\theta}}_i \hat{\boldsymbol{\eta}}_{t-i}^T + \tilde{\boldsymbol{\nu}}_t. \tag{4}$$

As hours are specified in first difference, the estimated response at horizon k is obtained from the cumulated OLS estimates: $\hat{a}_{21,k} = \sum_{i=0}^{k} \widehat{\tilde{\boldsymbol{\theta}}}_i$ In the following proposition, we show that the OLS estimators of the effect of technology shocks are consistent estimators of the true ones for both specifications.

Proposition 1 *Assume the infinite moving average representation (2) for hours worked and consider the estimation of the finite VAR in the first step as defined in (1) and the two projections (3) and (4) in the second step. The OLS estimators $\hat{a}_{21,k}$ converge in probability to $a_{21,k}$ for the level and first–difference specifications, $\forall k$.*

In Proposition 1, consistency is derived under the assumption that hours follow a stationary process. While a unit process could provide a good statistical approximation in small sample, hours worked per capita are bounded and therefore the stochastic process of this variable cannot have asymptotically a unit root. By definition, the consistency property of an estimator is an asymptotically concept so only the asymptotic behavior of hours worked is of interest. Consequently, the consistency of the OLS estimators for both specifications is derived only under the assumption that hours worked per person is a stationary process. Proposition 1 implies that the specification of hours (level or first difference) does not asymptotically matter for the estimation of the effect of a technology improvement on this variable. However, the small sample behavior of the OLS estimators associated to the two specifications can differ. To shed light on the behavior on this estimator in the case where the hours worked are well approximated by a unit root process in small sample, we derive the asymptotic distribution of the OLS estimators for the level specification by considering that the hours are characterized by an nearly nonstationary process. We show in the Appendix that the estimators for this specification do not converge in probability to the true values and lead to high sampling uncertainty. This derivation allows us to propose a simple corrected estimator which is consistent whatever the process follows by the hours worked.

2 Empirical Results

We now apply the two step methodology to US data. We consider different measures of hours and output, bivariate VARs and larger VAR specifications, different sample periods and breaks in labor productivity.

We first present results based on a simple bivariate VAR in the first step. This VAR model includes the growth rate of non farm business labor productivity and the log of ratio of nominal consumption expenditures to nominal GDP.[2] In the second step, the log level h_t and the growth rate of hours Δh_t are projected on the estimated technology shocks. Hours worked in the non farm business sector are converted to per capita terms using a measure of the civilian population over the age of 16. The period is 1948Q1-2003Q4 and we will therefore refer to this as the long sample. For comparison purpose, we estimate SVAR models with the growth rate of labor productivity and the log level of hours (LSVAR) and the growth rate of hours (DSVAR). In each of the SVAR models, we identify technology shocks as the only shocks that can affect the long-run level of labor productivity.[3] In the second step, we include the current and twelve past values of the identified technology shocks in the first step, *i.e.* $q = 13$ in (3) and (4). In order to assess the dynamic properties of hours worked and consumption to output ratio (in logs), we compute their autocorrelation functions. The autocorrelation functions of hours worked always exceed those of the consumption to output ratio and decay at a slower rate. Additionally, we perform Augmented Dickey Fuller (ADF) test of unit root. For each variable, we regress the growth rate on a constant, lagged level and four lags of the first difference. The ADF test statistic is equal to -2.74 for hours and -2.93 for the consumption to output ratio. This hypothesis cannot be rejected at the 5 percent level for hours, whereas it is rejected at the 5 percent level for the consumption to output ratio. These findings suggests that this latter variable is less persistent than hours.

The estimated Impulse Response Functions (IRFs) of hours after a technological improvement are reported in Figure 1. The upper left panel shows the well known conflicting results about the effect of a technology shock on hours worked between LSVAR and DSVAR specifications. The LSVAR displays a positive hump–shaped response whereas DSAVR implies a decrease in hours. To the contrary, our two step approach delivers almost the same picture whatever hours are specified in level or first difference (see the upper right panel of Figure 1). In the very short run, the IRFs of hours are identical and when the horizon increases the positive response is a bit more pronounced when hours are taken in level rather than in first difference. On impact, hours worked decrease, but after five periods the response becomes persistently positive and hump–shaped. The bottom panel of Figure 1 reports also the centered 95 percent confidence interval.[4] The confidence interval is larger when we consider hours in level. When hours are projected in first difference, the response on impact is significantly different from zero. However, the positive hump–shaped response is less precisely estimated. Our results are in the line of previous empirical papers which obtain that hours fall significantly on impact (see Basu,

[2]Labor productivity is measured as the non farm business output divided by non far business hours worked. Consumption is measured as consumption on nondurables and services and government expenditures.

[3]The lag length for each VAR model is obtained using the Hannan–Quinn criterion and we apply a LM test to check for serial correlation. The number of lags is 3 or 4 depending on the data and the sample.

[4]These confidence intervals are computed by standard bootstrap methods, using 1000 draws from the sample residuals. Our simulations account for the *generated regressors* problem, since technology shocks used in the second step have been estimated in the first step of our procedure.

Fernald and Kimball, 2004, Francis and Ramey, 2005b), but display a hump–shaped positive response during the subsequent periods (see Vigfusson, 2004). Figure 2 reports the IRFs obtained with the corrected estimator together with the ones associated with the level and first difference specifications. As expected, the corrected IRFs are close to the ones obtained with hours in difference.

We now check the robustness of our first results to the measure of output and hours with the long sample. The alternative measure of productivity and hours is based on business sector data. Figure 3 shows that the IRFs are similar to those reported in Figure 1. Hours decreases in the short run but increase after five quarters. However, with business sector data, the responses differ according to h_t and Δh_t, which is not the case with non farm business sector data. This difference can be explained by the time series properties of business hours worked compared to non farm business hours. To measure the persistence of these series, we test the null hypothesis of a unit root for these two measures of hours using the Augmented Dickey Fuller (ADF) tests. The ADF test statistic is equal to -1.95 for business hours and -2.74 for non farm business hours. The null hypothesis of a unit root cannot be rejected at the 10 percent level for business hours, whereas it is rejected at the 10 percent level for the non farm business hours. As shown in the Appendix, the nearly nonstationary behavior of business hours drives probably the difference between both specifications. Again here, the IRFs provided by the corrected estimator are close to the IRFs obtained with the specification in difference.[5]

We maintain the bivariate SVAR in the first step but we replace the log of the consumption to output ratio by the log of the ratio of nominal investment expenditures to nominal GDP.[6] We argue that this ratio is another promising candidate in the SVAR model, since it displays lower serial correlation than hours. In addition, we perform ADF test of unit root including four lags. The ADF test statistic is equal to -3.50 for the investment to output ratio. The null hypothesis of unit root is rejected at the 1 percent level. We consider again non farm business data and the long sample.[7] Figure 4 displays the IRFs. The replacement of consumption to output ratio by the investment to output ratio does not modify the previous findings and the response of hours has the same pattern.

We now examine the robustness of the two step strategy using a larger VAR system in the first step. We maintain non farm business data four output and hours and the long sample. The SVAR model in the first step includes labor productivity growth, consumption to output ratio, investment to output ratio and the rate of inflation.[8] Results are reported in Panel (a) of Figure 5. The IRFs are very similar to those of Figure 1. Note here that the impulse responses are very close for both specifications. Using this larger system, we redo the exercise with a shorter sample. Since a bunch of business cycle literature is concerned with post–1959 data, we follow Christiano, Eichenbaum and Vigfusson (2004) and we therefore consider a second sample

[5]The results (not reported) can be obtained from the authors upon request.

[6]Investment is measured as expenditures on consumer durables and private investment.

[7]We obtain similar results with business sector output and hours.

[8]The measure of inflation is obtained using the growth rate of the GDP deflator.

period given by 1959Q1–2003Q4. Panel (b) of Figure 5 reports the estimated responses. We obtain again the same shape for the IRFs, especially when hours are projected in difference. The negative responses in the short–run differ slightly according to the specification of hours, but the two IRFs become positive and very close after five periods. The slight difference in the two IRFs can be explained by the more persistent properties of the hours series for this shorter sample. Indeed, the ADF test statistic is equal to -2.47 for the short sample compared to -2.74 for the long sample. As in the previous experiments, the IRFs obtained with the corrected estimator (not reported here) mimic very well the ones estimated when hours are specified in difference. Interestingly, the response of hours is precisely estimated when hours are taken in difference, since the positive hump–shaped response is significantly different from zero at a conventional level. We also add the federal fund rate in the larger system and consider the short sample 1959Q1–2003Q4. The results are reported in Figure 6. The negative response of hours is more pronounced in the short run compared to the previous cases, but we still find a persistent increase in the subsequent periods.

Finally, we investigate the sensitivity of our results to structural breaks in labor productivity. We consider this issue in the context of the latter experiment. Fernald (2005) shows that once we allow for trend breaks in labor productivity, the response of hours to a technology shock in LSVAR becomes persistently negative. The breaking dates identified by Fernald are 1973Q1 and 1997Q2. We first regress labor productivity growth on a constant, a pre–1973Q1 dummy variable and a pre–1997Q1 dummy variable. We then use the residuals of this regression as a new measure of labor productivity growth in the first step. The responses of hours are reported in Figure 7. The response appears unaffected as the negative response on impact is around -0.2 (see Figure 6 for a comparison). Moreover, the hump–shaped and delayed–positive response is maintained for both specifications and significant for the specification in difference.

3 Concluding Remarks

This paper proposes a simple two step approach to consistently estimate technology shocks and the responses of hours worked after a technology improvement. In a first step, a SVAR model with labor productivity growth and the log of consumption to output ratio (or a set of relevant covariance–stationary variables) allows us to estimate technology shocks. In a second step, the response of hours is obtained by a simple regression of hours on the estimated technology shocks. The two step approach, when applied on US data, predicts a short–run decrease of hours after a technology improvement, as well as a delayed and hump–shaped positive response. This findings appears robust to different sample periods, measures of hours and output and to the variables included in the VAR at the first step. The proposed approach is devoted to the estimation of the responses of hours worked. However, this empirical strategy can be easily used to evaluate the effect of a technology shock on other persistent aggregate variables.

References

Basu, S., Fernald, J. and M. Kimball (2004) “Are Technology Improvements Contractionary?”, *forthcoming American Economic Review.*

Blanchard, O.J. and D. Quah (1989) “The Dynamic Effects of Aggregate Demand and Supply Disturbances” , *American Economic Review*, 79(4), pp. 655-673.

Chari, V., Kehoe, P. and E. Mc Grattan (2005) “A Critique of Structural VARs Using Real Business Cycle Theory”, Federal Reserve Bank of Minneapolis, Research Department Staff Report 364.

Christiano, L., Eichenbaum, M. and R. Vigfusson (2004) “What Happens after a Technology Shock”, NBER Working Paper Number 9819, *revised version 2004.*

Cochrane, J. (1994) “Permanent and Transitory Components of GDP and Stock Returns”, *Quarterly Journal of Economics*, 109(1), pp. 241–266.

Fernald, J. (2005) “Trend Breaks, Long Run Restrictions, and the Contractionary Effects of Technology Improvements”, Working Paper 2005–21, Federal Reserve Bank of San Francisco.

Francis, N. and V. Ramey (2005a) “Is the Technology–Driven Real Business Cycle Hypothesis Dead? Shocks and Aggregate Fluctuations Revisited”, *Journal of Monetary Economics*, 52, pp. 1379–1399..

Francis, N. and V. Ramey (2005b) “Measures of Per Capita Hours and their Implications for the Technology-Hours Debate”, *mimeo* UCSD.

Galí, J. (1999) “Technology, Employment and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?”, *American Economic Review*, 89(1), pp. 249–271.

Galí, J. (2004) “On the Role of Technology Shocks as a Source of Business Cycles: Some New Evidence”, *Journal of European Economic Association*, 2(2–3), pp. 372–380.

Galí, J., and P. Rabanal (2004) “Technology Shocks and Aggregate Fluctuations; How Well does the RBC Model Fit Postwar U.S. Data?”, *NBER Macroeconomics Annual*, pp. 225–288.

Gospodinov, N. (2006) “Inference in Nearly Nonstationary SVAR Models with Long–Run Identifying Restriction”, *mimeo* Concordia University.

Lewis, R. and G.C. Reinsel (1985) “Prediction of Multivariate Time Series by Autoregressive Model Fitting”, *Journal of Multivariate Analysis*, 16(3), pp. 393–411.

Vigfusson R. (2004) “The Delayed Response to a Technology Shock. A Flexible Price Explanation”, Board of Governors of the Federal Reserve System, International Finance Discussion Paper 810, july.

Appendix

Proof of Proposition 1

The consistency of the second step estimators depends on the consistency of the VAR coefficients in the first step. For a sufficient number of lags and covariance stationary variables, the estimated autoregressive coefficients are consistent and asymptotically normal (see Lewis and Reinsel, 1985). The consistency of the estimated VAR coefficients ensures the consistency of the estimated technology shocks.

Consider the case where hours are a stationary variable. For the regression in the second step with hours in level, the convergence in probability is established by standard arguments. First, the estimator $\widehat{a}_{21,k}$ is centered to the true value by direct straightforward implications of the orthogonality of the permanent and the transitory shocks and by the fact that those shocks are serially uncorrelated. Second, it is easy to show that the variance of the OLS estimator converges to zero. The convergence in probability follows. For the specification with hours worked in difference, the infinite moving average representation can be rewritten as

$$\Delta h_t = \boldsymbol{\theta}_{21}(L)\boldsymbol{\eta}_t^T + \boldsymbol{\theta}_{22}(L)\boldsymbol{\eta}_t^{NT}.$$

The structural moving average coefficients can thus be retrieved by the following cumulative sum $a_{21,k} = \sum_{j=0}^{k} \theta_{21,k-j}$. The consistency of $\hat{\theta}_{21,k}$ is guaranteed by the same arguments expressed above. This completes the proof of Proposition 1.

Now, to shed light on the small sample behavior of the OLS estimator when the hours worked are characterized by a near unit root, we assume for the ease of exposition the following local-to-unity parametrization

$$h_t = \left(1 + \frac{c}{T}\right) h_{t-1} + \psi^T \boldsymbol{\eta}_t^T + \psi^{NT} \boldsymbol{\eta}_t^{NT}. \tag{5}$$

where the constant $c < 0$ and T denotes the number of observations. The simple framework allows for a highly persistent effect of a technology shock on hours as well as a highly persistent effect of a non-technology shock in small sample. When $T \to \infty$, those structural shocks have a permanent effect. Now examine the second step linear projection of hours in level on the contemporaneous technology shocks:

$$h_t = \boldsymbol{\theta}_{21}(0)\hat{\boldsymbol{\eta}}_t^T + \boldsymbol{\nu}_t.$$

The OLS estimator yields from (5)

$$\hat{\boldsymbol{\theta}}_{21}(0) = \boldsymbol{\theta}_{21}(0) + \frac{\frac{1}{T}\sum_{t=2}^{T}\left(1 + \frac{c}{T}\right) h_{t-1}\hat{\boldsymbol{\eta}}_t^T}{\frac{1}{T}\sum_{t=1}^{T} \hat{\boldsymbol{\eta}}_t^{T\ 2}}. \tag{6}$$

where $\boldsymbol{\theta}_{21}(0) = \psi^T$. We can show that the asymptotic distribution of the OLS estimator is given by:

$$\hat{\boldsymbol{\theta}}_{21}(0) - \boldsymbol{\theta}_{21}(0) \xrightarrow{L} \psi^T \int_0^1 J_c(r)dW(r) + \psi^{NT} \int_0^1 \widetilde{J}_c(r)dW(r),$$

where $W(r)$ and $\widetilde{W}(r)$ are two independent Brownian motions, $J_c(s) = \exp(cs)\int_0^s \exp(-cr)dW(r)$ and $\widetilde{J}_c(s) = \exp(cs)\int_0^s \exp(-cr)d\widetilde{W}(r)$ are Ornstein-Uhlenbeck processes.[9] $\hat{\boldsymbol{\theta}}_{21}(0)$ has then a non trivial asymptotic distribution. The asymptotic distribution implies that the OLS estimator does not converge in probability to the true impulse responses and this estimator is characterized by a large sampling uncertainty resulting from the fact that the two terms are random variables whatever the number of observations.

The asymptotic distribution suggests a simple parametric correction. Introducing h_{t-1} as regressor in the second step projection allows to correct for the second expression in the RHS term in equation (6). It is now easy to show the convergence in probability and the asymptotic normality of this corrected estimator.

[9] See Gospodinov (2006) for a similar derivation in the context of standard SVARs.

Figure 1: IRFs of Hours to a Technological Improvement

NFB Sector data and Sample Period 1948Q1–2003Q4

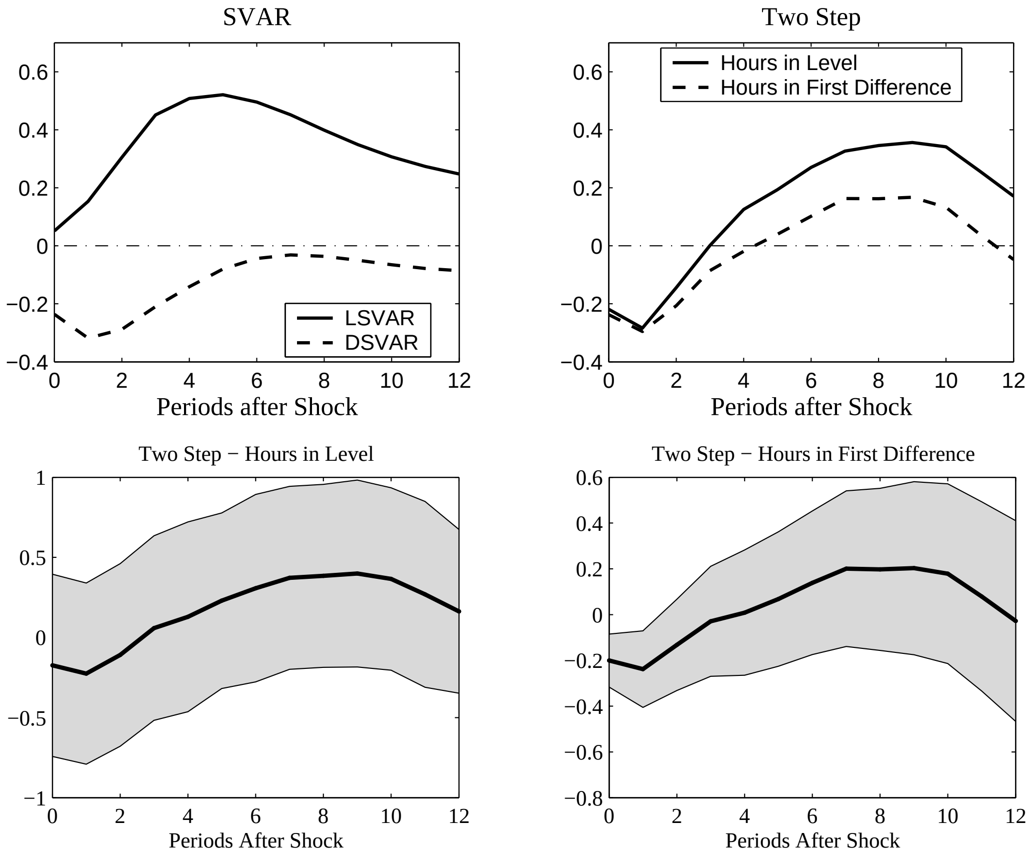


Figure 2: Corrected IRFs of Hours

NFB Sector data and Sample Period 1948Q1–2003Q4

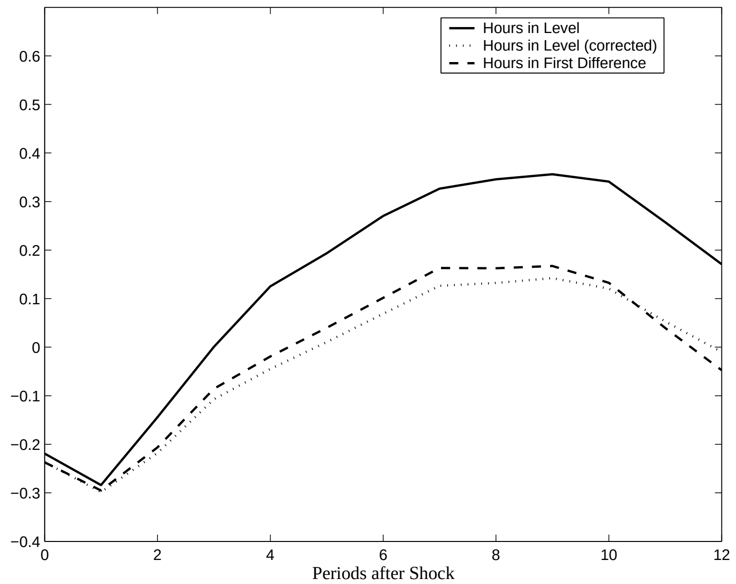


Figure 3: IRFs of Hours with Business Sector Data

Sample Period 1948Q1–2003Q4

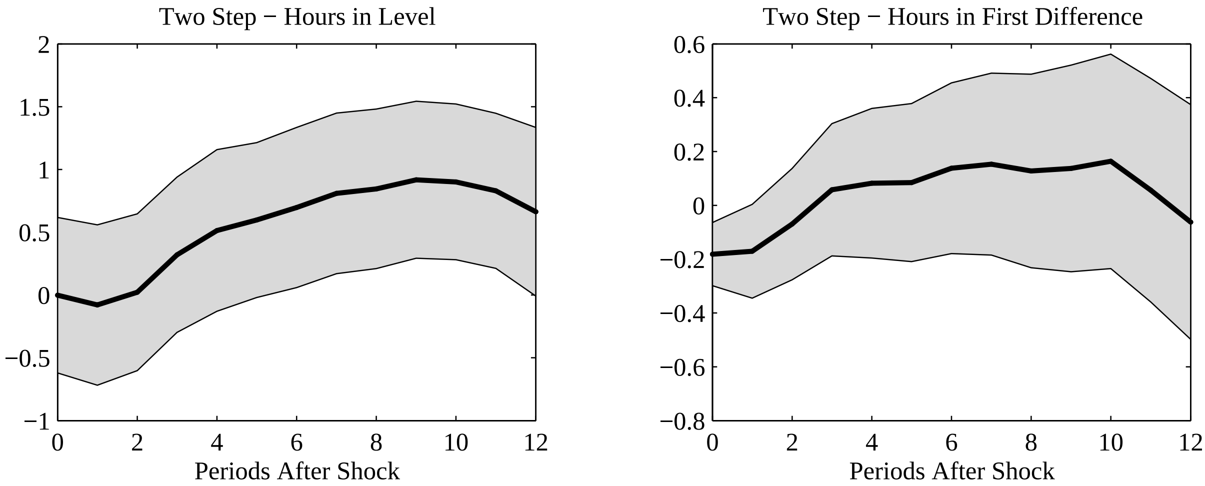


Figure 4: IRFs of Hours using Investment to Output Ratio

NFB Sector data and Sample Period 1948Q1–2003Q4

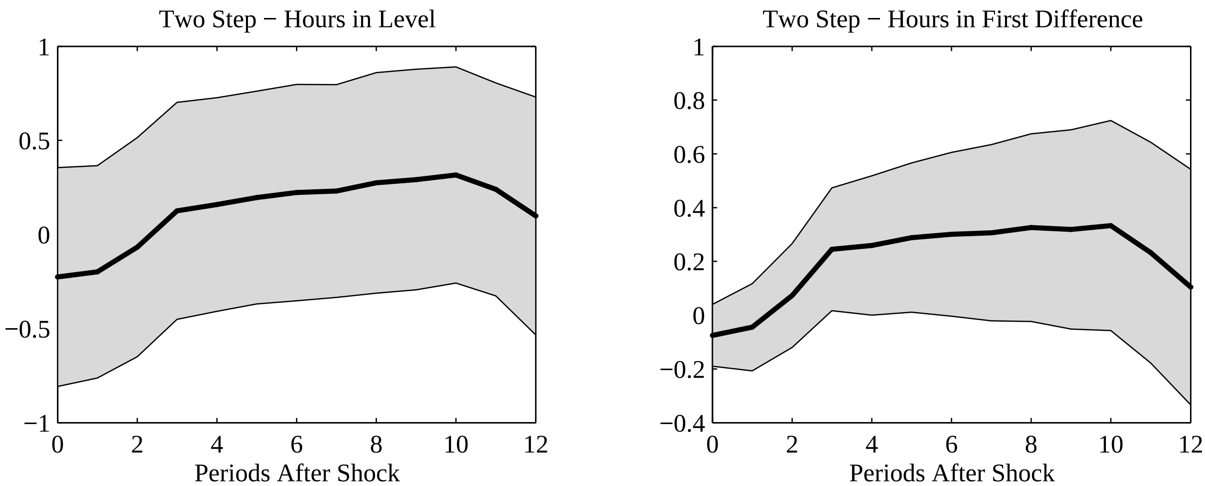
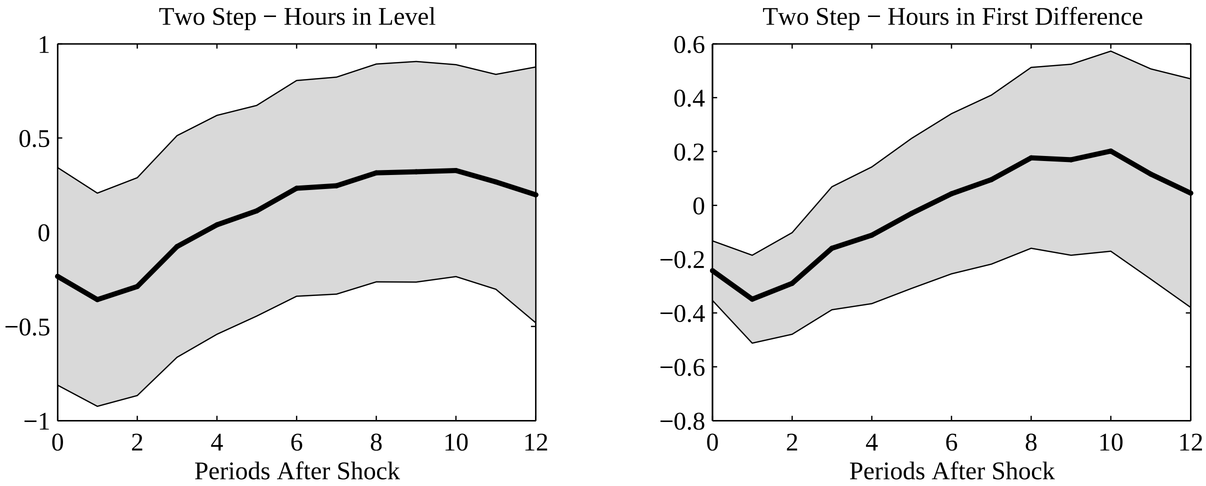


Figure 5: IRFs of Hours with a Four Variable System

Panel (a). NFB Sector data and Sample Period 1948Q1–2003Q4



Panel (b). NFB Sector data and Sample Period 1959Q1–2003Q4

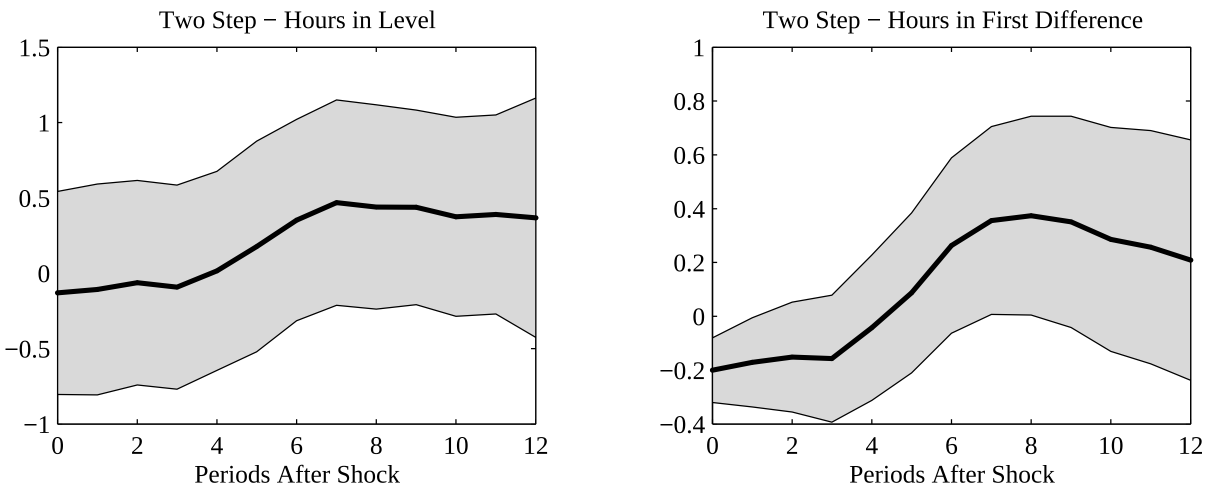


Figure 6: IRFs of Hours with a Five Variable System

NFB Sector data and Sample Period 1959Q1–2003Q4

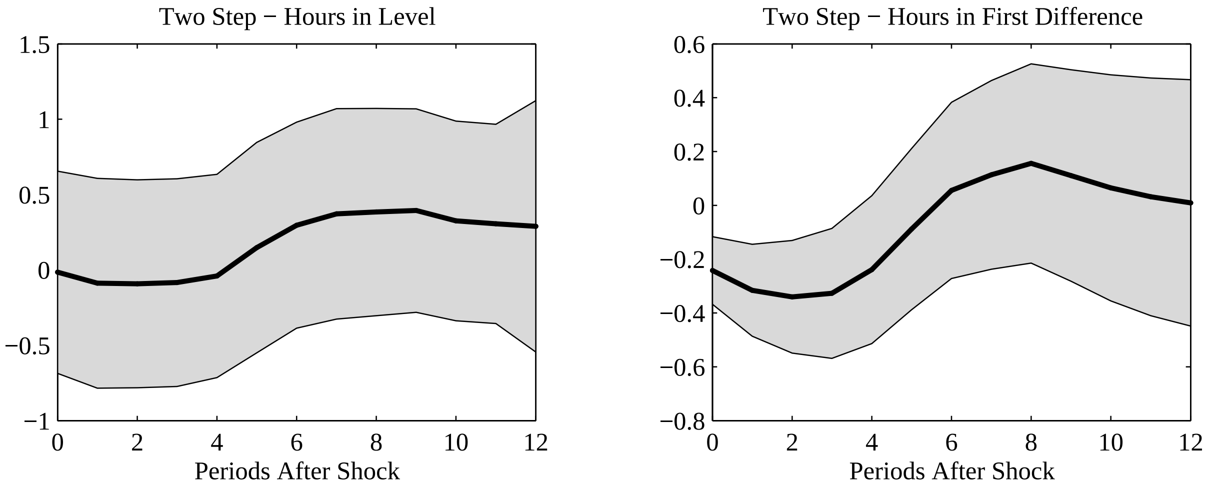


Figure 7: IRFs of Hours with Breaks in Labor Productivity

Five Variable System, NFB Sector data and Sample Period 1959Q1–2003Q4