Nominal Debt as a Burden on Monetary Policy

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Abstract

In this paper we study the role played by nominal debt on the optimal sequential choice of monetary policy. Specifically, we study a cash-in-advance economy where the consumption good must be purchased with cash carried over from the previous period as in Svensson (1985). We choose a specification of preferences that imply that if the debt were indexed, the optimal monetary policy would be time consistent. Using this same specification of preferences, first we show that when the debt is not indexed the full commitment optimal policy is time inconsistent. Then we study the recursive opitmal policy that obtains in this same case, and we show that the optimal inflation tax is non-stationary and that and that it converges to the inflation tax that obtains when there is no government debt. An implication of these results is that, when debt is not indexed, the time path followed by the optimal nominal interest is decreasing: initially the nominal interest rate is higher than the one that prevails when debt is indexed, and in the limit, since debt is being depleted, it is lower.

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1 Introduction

An economy with large stock of nominally denominated government debt can benefit from inflation surprises that reduce the need for distortionary taxation in the future. This implies that the optimal monetary policy under full commitment (the Ramsey policy) can be time inconsistent. By this we mean that if a government with the ability to commit were to reoptimize at a later date, it would choose to deviate from the policy announced originally. The purpose of this paper is to study the effects of nominal debt on the optimal sequential choice of monetary policy. To this purpose, we identify the mechanisms at work in a simple general equilibrium monetary model. Our benchmark model economy is a cash-in-advance economy with indexed debt. We characterize the optimal monetary policy in this economy and we compare it with the optimal policies that obtain when the debt is nominal, (i) under full commitment, and (ii) when the government is unable to commit to its announced policy. In this case we restrict our attention to the Markov perfect equilbrium.

The economies that we analyze are based on Nicolini (1998).[2] The structure of the optimal taxation problems that we solve is the following: We assume that the government has to finance both a given constant flow of expenditures and the interest payments on its debt with revenues levied using only seigniorage. To solve these optimal taxation problems, the government chooses the paths on signorage that maximize the household's utility subject to the implementability constraints.[3] Naturally, these optimal paths of seignorage must also satisfy the government budget.

To allow for inflation to be costly, we assume that the consumption good must be purchased with cash carried over from the previous period as in Svensson (1985). This timing of the of the cash-in-advance constraint implies that if the government decided to surprise the household with an unexpected increase in inflation in any given period, the household's consumption would be smaller than planned because its cash balances would be insufficient to purchase the intended amount of consumption. When considering whether or not to carry out such a surprise inflation, the government compares the reduction in the household's current utility that results from this lower level of consumption with the incresase in the household's future utility that results from the reduction in future seignorage.

[2]Other papers that use a similar structure and address similar issues are Marimon, Nicolini and Teles (1994 and 1997) , Chari and Kehoe (1999) and Giovannetti, Marimon and Teles (1998).

[3]The implementability constraints guarantee that the allocations that arise from the optimal taxation problems can be implemented as competitive equilibria with distortionary taxes.

Nicolini (1998) shows that, when the utility function is logarithmic in consumption and linear in leisure and the government debt is indexed, the optimal monetary policy in an economy similar to the one that we have just described is to abstain from the inflation surprises. This implies that in this model economy the solution to the Ramsey problem is time consistent.

In this paper we focus on the optimal policy that obtains when the stock of government debt is nominal. In order to have a benchmark against which to evaluate the role played by nominal debt, first we study an economy that is identical to the one discussed in Nicolini (1998). In this model economy the optimal monetary policy is to keep the price level constant over time and the Ramsey problem is time consistent. Moreover, since the solution to this problem is stationary, there is a unique interest rate that balances the government budget.[4]

Next we assume that the stock of government debt is nominal and we show that, in this case, the optimal monetary policy under full commitment is time inconsistent for the same specification of preferences. Specifically, the fact that the stock of outstanding debt is nominal creates an incentive for the government to surprise the household with an unexpected increase in the first period inflation. This implies that the government will attempt to tax the household more in the initial period than in any other period. Therefore, nominal debt creates a distorsion that prevents the full commitment Ramsey policy from being implemented. It is in this sense that nominal debt becomes a burden on monetary policy.

Since the full commitment Ramsey policy is time inconsistent and, consequently, it cannot be implemented, we also study the optimal policy that obtains in the absence of commitment. In this case, we restrict our attention to the Markov perfect equilibrium. We call this equilibrium recursive as in Cole and Kehoe (1996) and Obstfeld (1997). Two interesting features of the optimal policy that obtains under this recursive equilibrium are that the optimal inflation tax is non-stationary and that it converges to the inflation tax that obtains when there is no government debt. This result arises because in the recursive equilibrium it is optimal for the government to deplete the stock of nominal government debt to zero. An implication of these results is that in this economy the optimal nominal interest is initially higher than the one that prevails when debt is indexed and that in the limit it is lower. This decreasing path for the nominal interest rate is another indication that nominal debt is indeed a burden for monetary policy.

[4]Albanesi, Chari and Kehoe (2000) also consider an environment where the time inconsistency of monetary policy does not produce the standard Barro and Gordon (1983) result of an inflation bias. However, they study a stabilization game instead of the seignorage game that we analyze here.

The discussion about the constraints imposed by fiscal policy and government debt on monetary policy has received a lot of attention both from theoretical and from applied economists. On the theoretical camp, this issue has been studied mostly independently of the commitment issue. Some examples of these studies are the unpleasant monetarist arithmetic of Sargent and Wallace (1981), and the fiscal theory of the price level of Sims (1994) and Woodford (1996). In these approaches policies are taken to be exogenous. This is not the case both in our analysis, and in recent related literature such as Chari and Kehoe (1999), Rankin (2000) and Obstfeld (1997).

Obstfeld (1997) discusses the time consistency of optimal monetary policy when government debt is indexed. In his benchmark model economy inflation surprises are not costly and the Markov perfect equilibrium is non-monetary. This leads him to impose an *ad hoc* cost of unanticipated inflations. This new feature of his model creates the incentive for the government to deviate from the original policy path and to surprise the economy with a finite inflation. The time inconsistency of this optimal taxation problem leads him to study the Markov perfect equilibrium of this economy. In this case, the Ramsey government accumulates real assets until they earn enough interest to finance all future government expenditures. In the limit, the taxation problem disappears and so does the time inconsistency problem. Our analysis differs from Obstfeld's in that we consider nominal debt and that in our model economy the cost of unanticipated inflation arises from the timing of the cash-in-advance constraint rather than being imposed ah hoc.

On the applied camp, several recent policy proposals seem to take for granted that fiscal discipline is a prerequisite for price stability. The balanced budget proposal in the U.S. and the Stability and Growth Pact in the European Union are examples of such proposals. More specifically, applied economists have been concerned with the kind of constraints that should be imposed on the fiscal policies of member states in a monetary union. These restrictions aim at limiting the size of government debt and are mostly justified as a mechanism to reduce the amount of seignorage. Our analysis suggests that nominal debt is indeed a burden on monetary policy, but that this burden arises not so much from the need for extra seignorage but from the absence of a commitment technology that precludes the full commitment Ramsey policy from being implemented.

The rest of the paper is organized as follows: in Section 2 we describre the model economy, we define and characterize its competitive equilibrium, and we compute its stationary competitive equilibrium with zero debt; in Section 3 we discuss the optimal policy that obtains with indexed debt; in Section 4 we discuss the full commitment optimal policy with nominal debt; in Section 5 we discuss the recursive optimal policy with nominal debt; in Section 6 we compare our results; and, finally, in Section 7 we offer some concluding comments.

2 The model economy

The economy is made up of a government sector and a private sector that we describe below.

2.1 The government sector

We assume that the government in this economy issues currency, M, and nominal debt, B, to finance an exogenous and constant level of public consumption, g. We abstract from all other sources of public revenues. In each period $t \geq 0$ the government budget constraint is the following:

$$M_{t+1} + B_{t+1} \leq M_t + B_t(1 + i_t) + p_t g \tag{1}$$

where i_t is the nominal interest rate paid by each unit of currency lent to the government at time $t - 1$, and p_t is the price of one unit of the date t composite good.

A government policy is therefore a specification of $\{M_t, B_t\}$ for $t \geq 0$ and a value for g. Throughout this paper we assume that the government is benevolent in the sense that it chooses the policy that maximizes the utility of the households subject to (1) and to the implementability constraints that we discuss below.

2.2 The private sector

We assume that the economy is inhabited by a continuum of identical infinitely-lived households whose preferences over infinite sequences of consumption and time devoted to uses other than the market can be represented by the following utility function:

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) - \alpha n_t] \tag{2}$$

where $c_t > 0$ denotes consumption at time t, n_t denotes labor at time t, and $0 < \beta < 1$ is the time discount factor. We assume that function u is increasing, twice differentiable, strictly concave and that it satisfies the usual Inada conditions. More specifically, we assume that $u(c) = \log(c)$ for reasons that will become clear below.

Since we are not interested in the productive side of this economy we assume that each period labor can be transformed into either the private consumption good or the public consumption good on a one-to-one basis. Consequently, for each period $t \geq 0$ the resource constraint of the economy is the following:

$$c_t + g \leq n_t \tag{3}$$

and the competitive equilibrium real wage can be trivially shown to be $w_t = 1$ for all $t \geq 0$.

We assume that consumption in period t must be purchased using the currency carried over from period $t-1$ as in Svensson (1985). Notice that this timing of the cash-in-advance constraint implies that, when solving its maximization problem, the representative household takes both M_0 and B_0 as given.[5] Specifically, the cash-in-advance constraint faced by the representative household for every $t \geq 0$ is the following:

$$p_t c_t \leq M_t \tag{4}$$

Finally, all these assumptions imply that each period $t \geq 0$ the representative household faces the following budget constraint:

$$M_{t+1} + B_{t+1} \leq M_t - p_t c_t + B_t(1 + i_t) + p_t n_t \tag{5}$$

where M_{t+1} and B_{t+1} denote, respectively, the nominal money balances and the nominal government debt that the household carries over from period t to period $t+1$. We also assume that the representative household faces a no-Ponzi games condition.

2.3 The competitive equilibrium

2.3.1 Definition

A competitive equilibrium for this economy is an allocation $\{M_{t+1}, B_{t+1}, c_t, n_t\}_{t=0}^{\infty}$, a price vector $\{p_t, i_t\}_{t=0}^{\infty}$ and a constant level of public consumption g such that:

[5] In the Lucas and Stokey (1983) timing both M_0 and B_0 can be chosen by the household.

(i) when households take M_0, B_0 and the price vector as given, the allocation maximizes (2), subject to (4), (5) and the no-Ponzi games condition;

(ii) the allocation, the price vector, and g satisfy the government budget constraint (1); and

(iii) prices are such that the allocation and g satisfy the resource constraint (3).

2.3.2 Characterization

Given our assumptions on the utility function u, it is straightforward to show that the competitive equilibrium allocation of this economy satisfies the resource constraint (3) and the household budget constraint (5) with equality, and that the first order conditions of the Lagrangean of the household's problem are both necessary and sufficient to characterize the solution to the household's problem. Furthermore, it is also straightforward to show that, whenever $i_{t+1} > 0$, the cash-in-advance constraint (4) is binding, and the competitive equilibrium allocation of this economy is completely characterized by the following conditions that must hold for every $t \geq 0$:

$$\frac{u'(c_{t+1})}{\alpha} = 1 + i_{t+1} \tag{6}$$

$$1/\beta = (1 + i_{t+1})\frac{p_t}{p_{t+1}} \tag{7}$$

$$c_{t+1} = \frac{M_{t+1}}{p_{t+1}} \tag{8}$$

the government budget constraint (1) and the resource constraint (3).

2.4 The stationary competitive equilibrium with zero debt

For reasons that will become clear below, it is convenient to compute the stationary competitive equilibrium when there is no government debt, that is when $B_t = 0$ for every $t \geq 0$.[6]

[6] As we shall see below, when $B_0 = 0$ the solution to the optimal policy problem is time consistent.

Let $m_t = c_t/p_t$ and $(1 + \pi_t) = p_t/p_{t-1}$. Then, if we impose the steady state conditions on expressions (1), (3), (6), (7) and (8) used to characterize the competitive equilibrium in the previous section, we obtain that $\pi m = g$; $c + g = n$; $1/\alpha c = 1 + i$; $1/\beta = (1 + i)/(1 + \pi)$; and $c = m$.

Solving the system formed by these five equations we obtain that $c = m = (\beta/\alpha) - g$; $n = \beta/\alpha$; $1+i = 1/(\beta - \alpha g)$; and $1+\pi = \beta/(\beta - \alpha g)$. Moreover, this solution implies that the steady-state utility level is $\overline{u} = \log[(\beta/\alpha) - g] - \beta$.

3 Optimal policy with indexed debt

In this section we discuss the optimal policy that obtains when the government debt is indexed. This optimal policy problem is the benchmark against which we compare the optimal policy that obtains when the government debt is in nominal, which is the main focus of this article. We use superscript I to identify the optimal policy problem with indexed debt.

As we shall see below, the assumptions that the utility funtion is logarithmic in consumption and linear in labor play crucial roles in our analysis. The assumption that the utility function is logarithmic in consumption implies that the optimal taxation problem in the model economy with indexed debt is time consistent. This is because the log utility implies the elasticity of the consumption of the good at time $t = 0$ with respect to the price level and the intertemporal elasticity of substitution coincide. When debt is indexed, this implies that it is optimal to equate the taxes on the consumption good in every period. On the other hand, when debt is nominal the logarithmic utility implies that it is always optimal to shift the tax burden from the future to the present and, hence, to tax the currency brought over from the previous period by surprising the household with an unexpected increase in inflation. This implies that the solution to the optimal taxation problem with nominal debt is time inconsistent. Some of this issues are discussed in Nicolini (1998). The assumption that the utility funtion is linear in labor implies that the type of fiscal time inconsistency identified by Lucas and Stokey (1983) does not occur in our economy.

Let $q_t \equiv 1/p_t$ be the price of money and $b_t \equiv B_t q_{t-1}$be the real debt. The budget constraints of the government, (1), can be written in units of consumption as

$$q_t M_{t+1} + b_{t+1} \leq q_t M_t + b_t \frac{q_t}{q_{t-1}}(1 + i_t) + g, \; t \geq 0 \tag{9}$$

The fact that debt is indexed means that i_t adjusts to q_t, given q_{t-1}, so that

$$\frac{q_t}{q_{t-1}}(1+i_t) = \beta^{-1}, t \geq 0 \tag{10}$$

Manipulating the first order conditions (6), (7) and (8), we obtain

$$\frac{1}{\alpha q_t M_{t+1}} = \beta^{-1}, t \geq 0 \tag{11}$$

Condition (9) together with (10), and (11) give the implementability conditions that are the restrictions of the problem that we now define

Definition 1 *A Ramsey solution is a policy such that:*

The government chooses prices q_t as a function of the two state variables, nominal money and real debt by solving:

$$V^I(M_t, b_t) \quad = \quad \max_{(q_t, b_{t+1}, M_{t+1})} \left\{ \ln(q_t M_t) - \alpha(q_t M_t + g) + \beta V^I(M_{t+1}, b_{t+1}) \right\}$$

subject to the implementability conditions:[7]

$$\frac{\beta}{\alpha} + b_{t+1} - q_t M_t - \beta^{-1} b_t - g = 0 \tag{12}$$

and

$$M_{t+1} = \frac{\beta}{\alpha}\frac{1}{q_t} \tag{13}$$

Since the Ramsey problem with indexed debt is time consistent, in the sense that the continuation of the Ramsey policy is Ramsey, we can solve the Ramsey problem recursively. Hence, here the government chooses prices that maximize welfare, subject

[7]The literature refers to these conditions as implementability constraints because they are constraints on the set of allocations that can be implemented as a competitive equilibrium with distorting taxes. Constraint (12) is the consumer budget constraint with prices substituted from the first order conditions.

to the competitive equilibrium constraints that include an implementability constraint and the feasibility constraint.

The marginal condition for q_t is:

$$\frac{1}{q_t} - \alpha M_t + \beta V^I_{M_{t+1}}(-\frac{\beta}{\alpha}\frac{1}{q_t^2}) + \beta V^I_{b_{t+1}} M_t = 0 \tag{14}$$

and for b_{t+1}:

$$\beta V^I_{b_{t+1}} + \psi^I_t = 0 \tag{15}$$

where V_j denotes $\frac{\partial V}{\partial j}$, $j = M, b$ and ψ^I_t is the multiplier of the implementability condition (12).

Using the envelope theorem, we get

$$V^I_{b_t} = -\psi^I_t \beta^{-1} \tag{16}$$

$$V^I_{M_t} = \frac{1}{M_t} - \alpha q_t - \psi^I_t q_t \tag{17}$$

Using (15) and (16), gives:

$$\psi^I_t = \psi^I \tag{18}$$

i.e. the multipliers are constant when debt is indexed.

Dividing through (14) by M_t, rearranging the terms using (13), dividing through by α, and using:

$$\frac{1}{\alpha q_t M_t} = 1 + i^I_t$$

we get the expression:

$$1 + i^I_t = 1 + \beta(1 + i^I_t)\left[1 - \left(1 + \frac{\psi^I_{t+1}}{\alpha}\right)\frac{1}{1 + i^I_{t+1}}\right] + \frac{\psi^I_t}{\alpha} \tag{19}$$

It is convenient to define: $z_t^I \equiv \left(1+\frac{\psi_t^I}{\alpha}\right)\frac{1}{1+i_t^I}$, where $\psi_t^I=\psi^I$, to obtain a difference equation

$$z_t^I=(1-\beta)+\beta z_{t+1}^I$$

The solution to this difference equation is

$$z_t^I=1.$$

Notice that, otherwise, if $z_t^I>1,$ it would become arbitrarily large, meaning that the interest rate would become negative, or else, if $z_t^I<1$, it would become negative, also meaning that the interest rate was negative.

We can now write:

$$i_t^I=\frac{\psi^I}{\alpha} \tag{20}$$

i.e., the nominal interest rate is equal to the multiplier of the implementability constraint, accounted for at the marginal value of labor. By using the present value budget constraint, which is stationary, we can easily determine also the value of the nominal interest rate and therefore the multiplier.

$$1+i^I=\frac{1}{\beta-\alpha g-\alpha b\frac{1-\beta}{\beta}}$$

where b is the stationary value of the real debt.

We have solved a very simple problem: the government has in every period only one tax (i.e. the inflation tax). The solution of the maximization problem is a constant interest rate which is optimal, because the government wants to smooth distortions. What happens here is that the government internalizes the fact that it cannot default on the debt, by surprising the agents with inflation. As we shall see, whether the government can internalize or not the fact that, in equilibrium, it cannot surprise the private agents is the main difference with the other equilibrium concepts, and this shows in the equilibrium values of the multipliers of the implementability constraints.

GG: Note that when $b_0=0$ the solution coincide with that of no debt; furthermore the full commitment solution coincides with the no commitment solution; this is a special case due to the logarithmic specification of the utility function. From now on we focus on nominal debt.

4 Full commitment optimal policy with nominal debt

In this section we analyze the Ramsey (R) solutions to the government maximization problem in the case of nominal positive debt. We solve the problem from date $t = 0$ on.

If the government was able to reoptimize at time $t = 0$, and was able to commit to its policies from there on, then it would want to choose a policy that would differ from the ones obtained before. However, if this had been anticipated it would have to be the case that the ex-ante interest rate equals the ex-post rate. This is the exercise performed by Chari and Kehoe (1999).

GG: It is important to be clear about the initial conditions of our problem. Given B_0, liabilities at time $t = 0$ can be described by ...... We can consider that the government takes these liabilities as given when it chooses q_0. The RE condition is ....

Definition 2 *The Ramsey solution is a sequence of quantities and prices such that the government maximizes:*

$$Max \sum_{t=0}^{\infty} \beta^t \left[\ln(q_t M_t) - \alpha(q_t M_t + g)\right]$$

subject to the implementability conditions

$$\frac{\beta}{\alpha} + b_{t+2} - q_{t+1} M_{t+1} - \beta^{-1} b_{t+1} - g = 0,\ t \geq 0 \tag{21a}$$

$$\frac{\beta}{\alpha} + b_1 - q_0 M_0 - q_0 b_0 \frac{\beta^{-1}}{\overline{q}_0} - g = 0 \tag{22}$$

$$M_{t+1} = \frac{\beta}{\alpha} \frac{1}{q_t},\ t \geq 0 \tag{23}$$

Let $\beta^{t+1}\psi^R_{t+1}$, $t \geq 0$, ψ^R_0, and λ^R_t, $t \geq 0$, be the multipliers respectively of the constraints (21a), (22) and (23). The marginal conditions for $q_0, q_{t+1,} M_{t+1}, b_{t+1}$,and the envelope theorem allows us to get:

$$\psi^R_t = \psi^R,\ \text{all } t \tag{24}$$

i.e. constant multipliers as in the case of real debt (even though with different values), and

$$\frac{1}{q_{t+1}M_{t+1}} = \alpha + \frac{\beta}{q_{t+1}M_{t+1}}\left[\frac{1}{q_{t+1}M_{t+2}} - \alpha\frac{q_{t+2}}{q_t} - \psi^R\frac{q_{t+2}}{q_{t+1}}\right]\frac{\beta}{\alpha} + \psi^R,\ t \geq 0$$

$$\frac{1}{q_0M_0} = \alpha + \frac{\beta}{q_0M_0}\left[\frac{1}{q_0M_1} - \alpha\frac{q_1}{q_0} - \psi\frac{q_1}{q_0}\right]\frac{\beta}{\alpha} + \psi^R\left(\frac{M_0 + b_0\frac{\beta^{-1}}{q_0}}{M_0}\right) \tag{25}$$

Rearranging the expressions, we obtain:

$$1 = \frac{1}{1+i_{t+1}^R} + \beta\left[1 - \left(1 + \frac{\psi^R}{\alpha}\right)\frac{1}{1+i_{t+2}^R}\right] + \frac{\psi^R}{\alpha},\ t \geq 0 \tag{26}$$

and again defining

$$z_t^R \equiv \left(1 + \frac{\psi^R}{\alpha}\right)\frac{1}{1+i_t^R}$$

we can rearrange terms to obtain:

$$z_{t+1}^R = (1-\beta) + \beta z_{t+2}^R,\ t \geq 0$$

The solution to this difference equation is

$$z_{t+1}^R = 1,\ t \geq 0$$

so that from , $t \geq 0$

$$i_{t+1}^R = i^R = \frac{\psi^R}{\alpha} \tag{27}$$

which shows that the interest rate is again equal to the multiplier, calculated at the marginal value of labor, though with a different value for the multiplier, so that the interest rate is also different. This is not valid for some $t = 0$. In fact, the solution allows for a *surprise* inflation at $t = 0$, in the sense that agents' expectations are taken as given at time 0, so that:

$$1 + i_0^R = 1 + \beta(1 + i_0^R)\left[1 - \left(1 + \frac{\psi^R}{\alpha}\right)\frac{1}{1 + i^R}\right] + \frac{\psi^R}{\alpha}\left[1 + \frac{B_0(1 + \overline{i_0^R})}{M_0}\right] \tag{28}$$

where i_0^R is the ex-post interest rate, corresponding to

$$1 + i_0^R = \frac{u'(c_0)}{\alpha}$$

while $\overline{i_0^R}$ is the ex-ante interest rate. Naturally, if this was anticipated the two interest rates will have to be equal.

Since

$$\left(1 + \frac{\psi^R}{\alpha}\right)\frac{1}{1 + i^R} = 1$$

(28) becomes:

$$1 + i_0^R = 1 + \frac{\psi^R}{\alpha}\left[1 + \frac{B_0(1 + \overline{i_0})}{M_0}\right] \tag{29}$$

If this solution was anticipated then $\overline{i_0^R} = i_0^R$, in which case we would obtain that the nominal interest rate would be:

$$i^R = \frac{\psi^R}{\alpha} = \frac{i_0^R}{1 + \frac{B_0}{M_0}(1 + i_0^R)} \tag{30}$$

The Ramsey solution is characterized by an interest rate in period 0 that is higher than the one in the solution to the maximization problem with real debt, and a lower rate onwards. The reason is that the government aims at taking advantage of the lump-sum, or apparent lump-sum if anticipated. The government, in fact, has the perception, at some time 0, that it can surprise the agents and reduce the real value of the nominal liabilities, even if in equilibrium that will not happen. If this is anticipated by the private agents, the government cannot effectively deplete the debt.

The feature of the Ramsey problem with nominal debt is that the government behaves as if it could surprise the agents at some $t = t_0$.

5 Recursive optimal policy with nominal debt

In this section we characterize the markov perfect equilibria with nominal debt. We name this equilibrium recursive (RE). The relevant state variables are money and nominal debt at time t. Using (9) and (11), we get

$$\frac{\beta}{\alpha} + b_{t+1} \leq q_t M_t + b_t \frac{q_t}{q_{t-1}}(1 + i_t) + g,\ t \geq 0 \tag{31}$$

Since debt is nominal, and is not indexed, i_t is given at each period t. One way to say this is to write

$$\frac{\overline{q_t}}{q_{t-1}}(1 + i_t) = \beta^{-1}, t \geq 0 \tag{32}$$

where $\overline{q_t}$ is the expectation of the price of money, formed in period $t-1$. We can write the government budget constraint, using (32), as

$$\frac{\beta}{\alpha} + b_{t+1} \leq q_t M_t + b_t \frac{q_t}{\overline{q_t}} \beta^{-1} + g,\ t \geq 0 \tag{33}$$

The expectation $\overline{q_t}$ is a function of the state variables M_t and b_t, $Q(M_t, b_t)$. The maximization problem for the government is the following

$$V(M_t, b_t, Q(M_t, b_t)) =$$

$$\max_{(q_t, b_{t+1}, M_{t+1})} \{\log(q_t M_t) - \alpha(q_t M_t + g) + \beta V(M_{t+1}, b_{t+1}, Q(M_{t+1}, b_{t+1}))\}$$

subject to the implementability constraints:

$$\frac{\beta}{\alpha} + b_{t+1} - q_t M_t - b_t \beta^{-1} \frac{q_t}{Q(M_t, b_t)} - g = 0 \tag{34}$$

$$M_{t+1} = \frac{\beta}{\alpha} \frac{1}{q_t} \tag{35}$$

We define a markov perfect equilibrium, that we call recursive, as follows.

Definition 3 *The recursive monetary equilibrium is a decision rule for the inverse of the price level, $q(M_t, b_t)$, a value function, $W(M_t, b_t)$, and a price function $Q(M_t, b_t)$ such that*

(i) Given $Q(M_t, b_t)$ and $W(M_t, b_t)$, $q(M_t, b_t)$ solves

$$\max_{(q_t, b_{t+1}, M_{t+1})} \{\log(q_t M_t) - \alpha(q_t M_t + g) + \beta W(M_{t+1}, b_{t+1})\}$$

subject to the implementability constraints:

$$b_{t+1} = q_t M_t + b_t \beta^{-1} \frac{q_t}{Q(M_t, b_t)} - \left(\frac{\beta}{\alpha} - g\right) \tag{36}$$

$$M_{t+1} = \frac{\beta}{\alpha} \frac{1}{q_t} \tag{37}$$

(ii) Given $q(M_t, b_t)$ and $Q(M_t, b_t)$, $W(M_t, b_t)$ satisfies

$$W(M_t, b_t) = \log(q(M_t, b_t) M_t) - \alpha(q(M_t, b_t) M_t + g) + \beta W(M_{t+1}, b_{t+1})$$

such that

$$b_{t+1} = q(M_t, b_t) M_t + b_t \beta^{-1} \frac{q(M_t, b_t)}{Q(M_t, b_t)} - \left(\frac{\beta}{\alpha} - g\right) \tag{38}$$

$$M_{t+1} = \frac{\beta}{\alpha} \frac{1}{q(M_t, b_t)} \tag{39}$$

(iii) $q(M_t, b_t) = Q(M_t, b_t)$

Let ψ_t^{RE} be the lagrange multiplier on the first (budget) implementability constraint. Since $W(M_t, b_t) \equiv V(M_t, b_t, Q(M_t, b_t))$,, we have

$$W_{b_t} = V_{b_t} + V_Q(t) \frac{\partial Q}{\partial b_t}$$

$$W_{M_t} = V_{M_t} + V_Q(t)\frac{\partial Q}{\partial M_t}$$

From the first order conditions for q_t, we get:

$$\frac{1}{q_t} - \alpha M_t + \beta W_{M_{t+1}}\left(-\frac{\beta}{\alpha}\frac{1}{q_t^2}\right) + \beta W_{b_{t+1}}\left[M_t + b_t\beta^{-1}\frac{1}{Q(M_t,b_t)}\right] = 0 \tag{40}$$

and for b_{t+1}:

$$\beta W_{b_{t+1}} + \psi_t^{RE} = 0 \tag{41}$$

Using the envelope theorem, we get the values of the marginal derivatives with respect to Q_t, b_t and M_t respectively:

$$V_Q(t) = \psi_t^{RE} b_t \beta^{-1}\frac{q_t}{Q(M_t,b_t)^2} \tag{42}$$

$$V_{b_t} = -\psi_t^{RE}\beta^{-1}\frac{q_t}{Q(M_t,b_t)} \tag{43}$$

$$V_{M_t} = \frac{1}{M_t} - \alpha q_t - \psi_t^{RE} q_t \tag{44}$$

Using (41) and (42)-(43), as well as the fact that in a REE $q_t = Q(M_t, b_t)$, we get:

$$-\psi_{t+1}^{RE} + \psi_{t+1}^{RE}\frac{b_{t+1}}{q_{t+1}}\frac{\partial Q}{\partial b_{t+1}} + \psi_t^{RE} = 0$$

that is to say, if we denote the elasticity of Q with respect to the debt b by ϵ_b,we obtain the following difference equation for the multipliers and the elasticity:

$$\psi_t^{RE} = \left(1 - \epsilon_{b_{t+1}}\right)\psi_{t+1}^{RE} \tag{45}$$

Notice that if $\epsilon_{b_{t+1}} < 0$, and $\epsilon_{b_{t+1}} \to 0$ as the debt approaches zero, then the multipliers decline and converge to a constant. Furthermore, since

$$W_{b_t} = V_{b_t} + V_Q(t)\frac{\partial Q}{\partial b_t} = V_{b_t} - V_{b_t}\frac{b_t}{Q(M_t,b_t)}\frac{\partial Q}{\partial b_t} = (1 - \epsilon_{b_t})\, V_{b_t}$$

if the elasticity converges to zero, then $W_{b_t} \to V_{b_t}$. This is clear since the problem with nominal debt is equivalent to the problem with real debt, when debt is zero.

Rearranging (40), using (41)and (42)-(44) as well as $q_t = Q(M_t, b_t)$, we get:

$$\begin{aligned} \frac{1}{q_t} &= \alpha M_t + \frac{1}{q_t}\left[\beta\left(\frac{1}{q_t M_{t+1}} - \alpha\frac{q_{t+1}}{q_t} - \psi_{t+1}^{RE}\frac{q_{t+1}}{q_t}\right) + \psi_{t+1}^{RE}\frac{b_{t+1}}{q_t M_{t+1}}\epsilon_{M_{t+1}}\right]\frac{\beta}{\alpha} + \\ &+\psi_t^{RE}\left[M_t + b_t\beta^{-1}\frac{1}{q_t}\right] \end{aligned} \tag{46}$$

Dividing through by M_t, using (37), and $\frac{1}{\alpha q_t M_t} = 1 + i_t^{RE}$and $1 + i_{t+1}^{RE} = \beta^{-1}\frac{q_t}{q_{t+1}}$, we obtain

$$1 = \left(1 + \frac{\psi_t^{RE}}{\alpha}\right)\frac{1}{1+i_t^{RE}} + \beta\left[1 - \left(1 + \frac{\psi_{t+1}^{RE}}{\alpha}\right)\frac{1}{1+i_{t+1}^{RE}}\right] + \psi_{t+1}^{RE} b_{t+1}\epsilon_{M_{t+1}} + \frac{\psi_t^{RE}}{\alpha}\frac{B_t}{M_t} \tag{47}$$

We define, as before,

$$z_t^{RE} \equiv \left(1 + \frac{\psi_t^{RE}}{\alpha}\right)\frac{1}{1+i_t^{RE}}$$

and rearrange terms to obtain a difference equation in terms of z_t^{RE} :

$$z_t^{RE} = (1-\beta) + \beta z_{t+1}^{RE} - \psi_{t+1}^{RE} b_{t+1}\epsilon_{M_{t+1}} - \frac{\psi_t^{RE}}{\alpha}\frac{B_t}{M_t} \tag{48}$$

Proposition 4 *There is a solution of the problem in Definition* 2 *that is characterized by* $\epsilon_{M_t} = -1$, *so that in response to a marginal increase in the outstanding money balances, the sequence of equilibrium price levels increases in the same proportion. The solution is also characterized by* $\epsilon_{b_{t+1}} \leq 0$. *The multipliers,* ψ_t^{RE}, *decrease asymptotically, as debt is decreased to zero.*

Proof. Let M_t be multiplied by λ. It is straightforward to show that there is a solution of the problem such that the sequence of $\{q_t\}_{t=0}^{\infty}$ is divided by λ, and $\{M_{t+1}\}_{t=0}^{\infty}$ is multiplied by λ. This solution satisfies the difference equation (48), that can be written as

$$z_t^{RE} = (1-\beta) + \beta z_{t+1}^{RE} - \psi_{t+1}^{RE} b_{t+1} \epsilon_{M_{t+1}} - \frac{\psi_t^{RE}}{\alpha} \frac{b_t}{q_t M_t},\ t \geq 0 \tag{49}$$

where $z_t^{RE} \equiv \left(1 + \frac{\psi_t^{RE}}{\alpha}\right) \frac{1}{1+i_t^{RE}} = \left(1 + \frac{\psi_t^{RE}}{\alpha}\right) \alpha q_t M_t$, and the constraints of the problem

$$\frac{\beta}{\alpha} + b_{t+1} - q_t M_t - b_t \beta^{-1} - g = 0,\ t \geq 0$$

and $M_{t+1} = \frac{\beta}{\alpha}\frac{1}{q_t}$, $t \geq 0$, . All the other variables are kept unchanged.

Suppose now that $\epsilon_{b_{t+1}} > 0$. Then, when there is a proportionate increase in b_{t+1}, q_{t+1} also increases, and therefore b_{t+2} will go up at a rate greater than $\beta^{-1}-1$. From (37), M_{t+2} will also increase, but since $\epsilon_{M_t} = -1$, q_{t+2} will adjust in order to keep $q_{t+2}M_{t+2}$. By assumption q_{t+2} goes up so that b_{t+T} increases at a rate greater than $\beta^{-1} - 1$, which violates the government budget constraint.

As long as $b_{t+1} \neq 0$, then $\epsilon_{b_{t+1}} \neq 0$. Suppose $\epsilon_{b_{t+1}} = 0$. Then when b_{t+1} goes up, q_{t+1} is constant. Then M_{t+2} does not change, and the debt is growing at the rate β^{-1}, violating the budget. This is obvious because all that is being said is that it is not possible to finance a higher debt without ever changing the price levels.

Since $\epsilon_{M_t} = -1$, we have

$$Q(M_t, b_t) = \frac{C(b_t)}{M_t}$$

and

$$\frac{\partial C}{\partial b_t}\frac{b_t}{C(b_t)} = \frac{\partial Q}{\partial b_t}\frac{b_t}{Q(M_t, b_t)} \equiv \epsilon_{b_t}$$

We can write the recursive problem with real debt as the single state variable

$$Z(b_t, C(b_t)) =$$

$$\max_{(c_t, b_{t+1})} \left\{ \log(c_t) - \alpha(c_t + g) + \beta Z(b_{t+1}, C(b_{t+1})) \right\}$$

subject to the implementability constraint:

$$b_{t+1} = c_t + b_t \beta^{-1} \frac{c_t}{C(b_t)} + g - \frac{\beta}{\alpha} \tag{50}$$

In equilibrium, $c_t = C(b_t)$.

Money balances evolve according to

$$M_{t+1} = \frac{\beta}{\alpha} \frac{M_t}{c_t} \tag{51}$$

starting from some initial M_0.

Let $ZW(b_t) \equiv Z(b_t, C(b_t))$, and so

$$ZW_{b_t} = Z_{b_t} + Z_C(t) \frac{\partial C}{\partial b_t}$$

From the first order conditions for c_t, we get:

$$\frac{1}{c_t} - \alpha - \psi_t^{RE} \left[1 + b_t \beta^{-1} \frac{1}{C(b_t)} \right] = 0 \tag{52}$$

and for b_{t+1}:

$$\beta ZW_{b_{t+1}} + \psi_t^{RE} = 0 \tag{53}$$

and so

$$\frac{1}{c_t} - \alpha + \beta ZW_{b_{t+1}} \left[1 + b_t \beta^{-1} \frac{1}{C(b_t)} \right] = 0 \tag{54}$$

Using the envelope theorem, we get the values of the marginal derivatives with respect to C_t, and b_t, respectively:

$$Z_C(t) = \psi_t^{RE} b_t \beta^{-1} \frac{c_t}{C(b_t)^2} \tag{55}$$

$$Z_{b_t} = -\psi_t^{RE}\beta^{-1}\frac{c_t}{C(b_t)} \tag{56}$$

In equilibrium

$$Z_{b_t} = -\psi_t^{RE}\beta^{-1} = ZW_{b_{t+1}} \tag{57}$$

Using (53) and (55)-(56), as well as the fact that in a REE $c_t = C(b_t)$, we get:

$$\psi_t^{RE} = -\psi_{t+1}^{RE} + \psi_{t+1}^{RE}\frac{b_{t+1}}{C(b_{t+1})}\frac{\partial C}{\partial b_{t+1}}$$

that is to say, if we denote the elasticity of C with respect to the debt b by ϵ_b,we obtain the following difference equation for the multipliers and the elasticity:

$$\psi_t^{RE} = \left(1 - \epsilon_{b_{t+1}}\right)\psi_{t+1}^{RE} \tag{58}$$

$Z(b)$ is negative, since $c < 1$ and the function is logarithmic; $Z_b < 0$ and $Z_{bb} > 0$. Then, ψ_t^{RE} decreases when debt is reduced. We have shown that, for strictly positive debt, $\epsilon_{b_{t+1}} < 0$, and $\epsilon_{b_{t+1}} \to 0$ as the debt approaches zero. Notice that, if $b_{t+1} = 0$, $\epsilon_{b_{t+1}} = \frac{\partial Q}{\partial b_{t+1}}\frac{b_{t+1}}{Q(M_{t+1},b_{t+1})} = 0$. Thus, from (58), the multipliers will decline over time if debt is strictly positive. This implies that debt will decline, approaching zero, as the multipliers approach a constant. The interest rate converges to the value of the multipliers evaluated at the marginal productivity of labor. To summarize, the recursive equilibrium is such that $\psi_t^{RE} \searrow \underline{\psi}^{RE}$, $b_t = B_t q_t \searrow 0$ and $(1 + i_t) \searrow (1 + \frac{\underline{\psi}^{RE}}{\alpha}) = \frac{1}{\beta - \alpha g}$.

Notice that these results imply that, in the case of a recursive equilibrium, z^{RE} converges to 1, which is the solution with indexed debt.

In the next section we compare the solution in this recursive equilibrium with the solution obtained when commitment is feasible. Furthermore, we also compute a myopic solution, where the government does not take full account of the effects of its actions. We compare this to the recursive solution.

6 Comparing the results

In this section we describe the numerical solutions under the different regimes and equilibrium concepts, summarize our analytical and quantitative results, and make some policy comparisons. Recall that we have used the following notation: superscript I = policy with Indexed Debt, M = Myopic policy with Nominal Debt, R = Ramsey, RE = markov perfect or REcursive policy.

The calibration is $\beta = .98$; $\alpha = 4.45$; $g = .00822$. For $b = 0$, $n = .22$. Figs $1-3$ depict the numerical solutions under the recursive equilibrium. In Fig. 1, we have the function $b_{t+1} = B(b_t)$. This function is an increasing function from the origin with a slope less than one, so that starting from some initial b_0, b_t converges to zero. This is shown in Fig. 2 where we depict the time path of debt, starting from an initial value close to the value of output. In Fig 3, we have the time path of the nominal interest rate that starts at a value close to $i = .3$, and goes down to the value that finances government expenditures and zero debt, $i_t^{RE} = \frac{1}{\beta - \alpha g} - 1$.

The computation of the Ramsey solution is straightforward. We have

$$i^R = \frac{i_0^R}{1 + b_0 \alpha \left(1 + i_0^R\right)^2}$$

$$b = \frac{\beta}{1-\beta}\left[\frac{\beta}{\alpha} - \frac{1}{\alpha\left(1 + i^R\right)} - g\right]$$

$$\frac{\beta}{\alpha} + b - \frac{1}{\alpha\left(1 + i_0^R\right)} - b_0\beta^{-1} - g = 0$$

$$\psi^R = \alpha i^R$$

$$z^R = 1$$

$$z_0^R = \left(1 + \frac{\psi_0^R}{\alpha}\right)\frac{1}{1 + i_0^R} = \frac{1 + i^R}{1 + i_0^R}$$

In the solution with indexed debt we have:

$$c = \frac{\beta}{\alpha} - g - b\frac{1-\beta}{\beta}$$

$$1 + i^I = \frac{1}{\alpha c}$$

$$\frac{\psi^I}{\alpha} = i^M$$

$$z^I = \left(1 + \frac{\psi^I}{\alpha}\right) \frac{1}{1+i^I} = 1$$

We start by comparing the zs: In Fig. 4, we have the numerical solutions for the zs. In the solution with indexed debt (I), we have $z^I = 1$. In the myopic solution with nominal debt, the value of z is still constant but $z_t^M < 1$. As far as the Ramsey case is concerned, we know that the value of z is lower than the indexed case at $t = 0$, but coincides with the indexed case afterwards. In the recursive solution, z_t^{RE} increases and converges to one.

Fig. 5 depicts the solutions for the interest rate in the different regimes. In the case of indexed debt and in the case of the myopic policy with rational expectations imposed from $t = -1$, since both problems are stationary, the nominal interest rate is the same in both cases and is the one that allows to finance the stationary debt and government expenditures. We have that:

$$1 + i^I = \frac{1}{\beta - \alpha g - \alpha b \frac{1-\beta}{\beta}}$$

When the debt is zero we have $1 + i = \frac{1}{\beta - \alpha g}$, which is still constant but higher.

We also know from the above analysis that:

$$i^R < i^I$$

from period $t = 1$ onwards, but at $t = 0$,

$$i_0^R > i^I$$

Hence, the Ramsey path for interest rate starts at a level higher than in the indexed case, and then it decreases at $t = 1$, being constant from there on, and lower than the indexed case.

In the case of the recursive policy the equilibrium interest rate is not constant. The interest rate when the *RE*cursive policy is followed starts at a higher value than in the indexed case

$$i_0^{RE} > i^I$$

Over time, the interest rate decreases to a value lower that in all the other cases. It converges to:

$$1 + i_\infty^{RE} = \frac{1}{\beta - \alpha g} \tag{60}$$

as the debt is depleted. It converges to the level of the interest rate when debt is zero.

Finally, when we compare the multipliers ψ, we have:

$$\frac{\psi^I}{\alpha} = i^I$$

$$\begin{aligned} \frac{\psi^R}{\alpha} &= \frac{i_0^R}{1 + \frac{B_0}{M_0}(1 + i_0^R)} \\ \frac{\psi^R}{\alpha} &= i^R \end{aligned}$$

Hence:

$$\psi^R < \psi^I$$

The multiplier ψ^R is lower when debt is non indexed, because the stationary interest rate is lower and because, although the initial interest rate (i_0^R) is higher, the multiplier is discounted by the *apparent lump sum effect.*

From Fig. 6, we have

$$\psi_0^{RE} > \psi^I$$

In the markov perfect policy, the multiplier converges to:

$$\psi_{\infty}^{RE} = \alpha \left(\frac{1}{\beta - \alpha g} - 1 \right)$$

so that

$$\psi_{\infty}^{RE} < \psi^{I}$$

If we imposed that the Ramsey solution had to be stationary, then we would be back to the same solution as in the indexed case, but with different multipliers. These differing multipliers suggest the solution that would be obtained if the environment was modified to allow for a trade-off between the inflation tax and an alternative tax.

7 Concluding comments

This paper emphasizes the different roles that nominal and real debt have in affecting the sequential decision of optimal monetary policy in a general equilibrium monetary model, where the costs of an unanticipated inflation are accounted for by the specification of the timing of the cash in advance constraint, as in Nicolini (1998).

We concentrate on a regime with nominal debt but compare different regimes and equilibrium concepts. In an economy with no inherited debt, the interest rate is constant and the optimal monetary policy is time consistent. When debt is real (indexed) and the utility function is logarithmic, the solution is stationary and the same optimal monetary policy is implemented with and without full commitment. Time inconsistency arises when we consider nominal debt. The full commitment policy is stationary only from period one onwards, but the interest rate is higher in period zero, because the government is tempted to inflate away its nominal liabilities.

GG: This results in time inconstency. With nominal debt an increase in the price level not only taxes money holders but also debt holders. That is, it results in a direct cut on spending (liabilities), someting that cannot be achieved with real debt. In other words, aside from redistributional implications, nominal debt increases the temptation to use monetary policy to solve fiscal problems.

With nominal debt, and without full commitment, the markov perfect equilibrium policy has the feature that debt is progressively depleted, converging asymptotically to zero, so that, in the long run, the optimal monetary policy coincides with that of an economy without inherited debt.

GG: Hence, we have seen that the debt reduction is a component of optimal monetary policy and that this policy tends to price stability (even in our model, without alternative sources of revenues, the inflation rate is positive).

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Figure 1: The optimal stocks of indexed debt and of nominal debt under the different equilibrium concepts

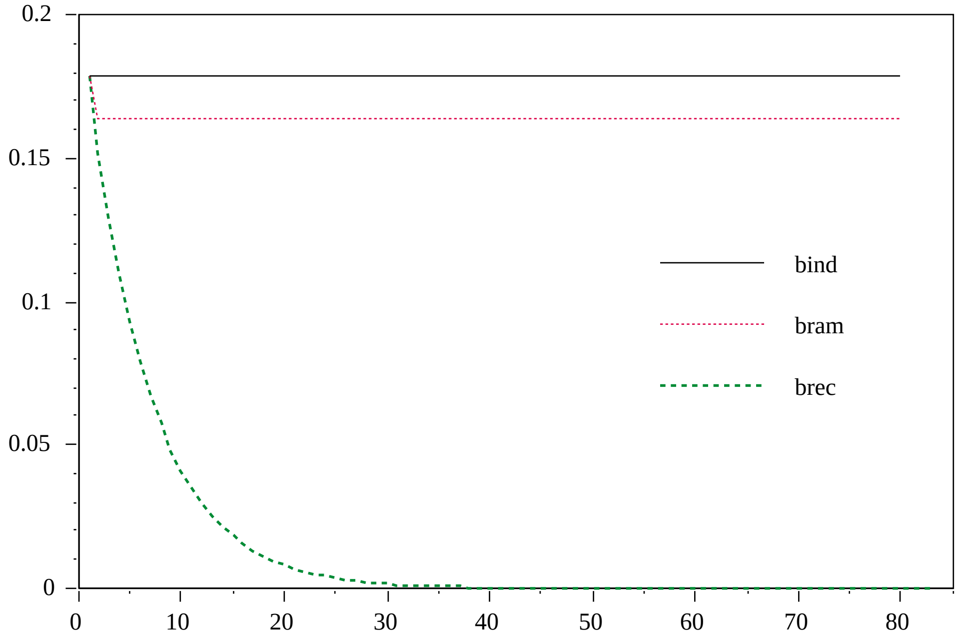

Figure 2: The optimal paths of nominal interest rates with indexed debt and with nominal debt under the different equilibrium concepts