Optimal Fiscal and Monetary Policy: Equivalence Results*

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first draft: January, 2001
this version: March, 2002

Abstract

In this paper, we analyze the implications of price setting restrictions for the conduct of cyclical fiscal and monetary policy. We consider an environment with monopolistic competitive firms, a shopping time technology, prices set one period in advance, and government expenditures that must be financed with distortionary taxes. We show that the sets of (frontier) implementable allocations are the same indepedendently of the degree of price stickiness. Furthermore, the sets of policies that decentralize each allocation are also the same except in the extreme cases of flexible and sticky prices, where the sets are larger but still include that same set of policies. In this sense we establish

*We thank Andy Neumeyer, Sergio Rebelo, and participants at seminars at The University of Chicago, University of Pennsylvania, U. Di Tella, at the 2001 Winter Camp in International Economics, NBER workshop on "Nominal Rigidities at the F.R.B. of San Francisco, 2001 SED Meetings in Stockholm, and the 2002 AEA Meetings in Atlanta . Errors are our own. Correia and Teles gratefully acknowledge financial support of FCT.

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an irrelevance or equivalence of environments. We also describe the minimal set of instruments, in the different environments, and thus discuss equivalence and neutrality of fiscal and monetary instruments. If the government cannot issue state contingent debt, it is still possible to implement the common set of allocations with high volatility of consumption taxes and labor income taxes.

1 Introduction

In this paper, we analyze the implications of price setting restrictions for the conduct of cyclical fiscal and monetary policy. We consider as the benchmark a stochastic dynamic general equilibrium economy with a transactions technology and monopolistic competitive firms that set prices contemporaneously. This economy is compared to the same economy with prices set in advance and to an economy with both flexible and sticky firms. We characterize the sets of fiscal and monetary policies and equilibrium allocations that finance exogenous government expenditures, i.e., the sets of implementable allocations and policies.

We show that the sets of (frontier) implementable allocations are the same independently of the degree of price rigidity. Furthermore the sets of policies that decentralize each allocation are also the same except in the extreme case of flexible and sticky prices, where the sets are larger but still contain that common set of policies. In this sense we establish an irrelevance, or equivalence, of environments. This equivalence would be lost if we were to consider idiosyncratic shocks. The sets of allocations would, then, depend on the degree of price rigidity. However the sets of policies would not. In this stricter sense there is still an irrelevance of environments.

The sets of policies, common across environments, that decentralize each allocation are characterized by the following principles: Money supply policy is conducted so that there are no surprises in prices, as under sticky prices; fiscal policy is conducted so that there are no surprises in mark ups, as under flexible prices. In each extreme environment there are other policies that decentralize each allocation. In particular, under sticky prices, fiscal policy is not uniquely pinned down. In this sense there is short run neutrality of taxes which is analogous to the neutrality of money under flexible prices.

In the mixed economy with both flexible and sticky firms, fiscal and monetary policies are pinned down, according to those principles. When the common policy is followed, in this mixed environment the same set of allocations can be achieved as under flexible and sticky prices. This set constitutes a frontier of allocations, so that it is uniquely optimal to conduct fiscal and monetary policies so that there are no surprises in prices and mark ups. Thus, fiscal policy should be conducted as if prices were flexible.

We further characterize the set of frontier allocations by showing that, independently of the degree of price stickiness, that set is characterized by the Friedman rule, of zero nominal interest rates, and by full taxation of profits.

We describe the minimal set of instruments, in the different environments, and thus discuss equivalence and neutrality of fiscal and monetary instruments. In particular, we show that, if the government cannot issue state contingent debt, it is still possible to implement the common set of allocations with high volatility of consumption taxes and labor income taxes.

Under flexible prices, there are many money supply policies that decentralize the same allocation. Either this neutrality or the equivalence of the consumption and labor income tax is lost if there is no state contingent debt. This means that either the price level or the variability of the consumption tax are pinned down when public debt is not state contingent. The principle is the one of a fiscal theory of the price level gross of consumption taxes.

The neutrality of taxes under sticky prices is also lost when public debt is not state contingent. In fact real state contingent debt can be simulated with variability of consumption taxes. Both the neutrality of money and the neutrality of taxes are not present in the mixed economy. Fiscal and monetary policy must be conducted in a single way so that there are no surprises in prices or mark ups. When public debt is not state contingent, the variability of the consumption tax and labor income tax are also pinned down.

We also consider the relevance of the restriction on taxes that they may not be state contingent. This restriction is not binding when the optimal policy is to set constant distortions across states, unless public nominal debt cannot be state contingent. In line with previous literature, as in Lucas and Stokey (1983), Chari, Christiano and Kehoe (1991), or Zhu (1992), we analyze the conditions under which it is optimal to set constant distortions across states. When the restriction is binding, policies and allocations will depend on the degree of price rigidity. The equivalence of environments is lost.

This paper is related to Adão, Correia and Teles (2000). They analyze a similar environment to the one in this paper, but restrict policy instruments to monetary policy and lump sum taxes. Because of the non negativity of the nominal interest rate it is not possible to eliminate the mark up distortion arising from monopolistic competition. The two other restrictions, prices set one period in advance and the cash in advance restriction can be eliminated. They show that the Friedman rule is optimal, but that in general it is not optimal to replicate the flexible price allocation. Under flexible prices the Friedman rule is optimal so that proportionate distortions are constant across states. The flexible prices allocation is optimal whenever, under sticky prices, it is optimal to set constant distortions across states. Adão, Correia and Teles (2000) identify the conditions under which it is optimal to do so. They conclude that when there are shocks to government expenditures it is not optimal, in general, to smooth proportionate distortions, and therefore replicate flexible prices.

Another closely related paper is Adao, Correia and Teles (1999), where the point is made that the policies that decentralize the optimal allocation are independent of the degree of price or portfolio rigidity. For this reason they argue that the strength of the monetary transmission mechanism is not relevant for policy.

This paper builds on the recent literature on cyclical monetary policy with nominal rigidities, such as Ireland (1996), Carlstrom and Fuerst (1998b), Carlstrom and Fuerst (1998a), Goodfriend and King (1997), King and Wolman (1998), Khan, King and Wolman (2000), Rotemberg and Woodford (1999). It also builds on the literature on the Ramsey fiscal and monetary policies, such as Lucas and Stokey (1983), Chari, Christiano and Kehoe (1996), Correia and Teles (1996, 1999), Siu (2001) and Schmitt-Grohe and Uribe (2001b). Goodfriend and King (2000) belongs to both literatures.

2 The model

The environment is a standard real business cycles model with labor only to which we add restrictions on transactions and the setting of prices. The agents are identical households, a continuum of firms indexed by $i \in [0, 1]$, and a government. Each firm produces a distinct, perishable consumption good, indexed by i. The production uses labor, according to a linear technology. We impose that transactions must be made according to a shopping time technology as in Kimbrough (1983) and De Fiore and Teles (1998). A fraction of firms are restricted to set prices one period in advance. Exogenous government purchases must be financed with distortionary taxes.

The state of the economy is represented by the random variable $\sigma_t \in \Sigma$, that follows a Markov process. There are government purchases shocks, $G_t = G(\sigma_t)$, and productivity shocks, $s_t = s(\sigma_t)$.

Households The households start period t with nominal wealth $\mathbb{W}_t$, and decide to buy money balances, M_t. They also buy B^h_{t+1} units of money in nominal bonds that pay $R_{t+1}B^h_{t+1}$ units of money one period later; and Z^h_{t+1} units of state contingent nominal securities, that cost z_{t+1} in units of currency today, and each of them pays one unit of money at the beginning of period $t+1$ in a particular state. They can also buy $A(i)_{t+1}$ units of stocks of firm i, that cost $a(i)_t$ in units of currency. At the end of the period, the households receive the labor income, $W_t N^w_t$, and the profits from the firms $\Pi_t = \int_0^1 \Pi_t(i)di$.

The two sources of income, labor income and dividends, are taxed at the rates τ^w_t and τ^d_t. There are also consumption taxes, τ^c_t.

We assume a transactions technology, as in De Fiore and Teles (1998), that relates consumption, money balances and shopping time N^s_t

$$N^s_t \geq l(C_t, \frac{M_t}{(1+\tau^c_t)P_t}) \tag{1}$$

where P_t is the aggregate price level,

$$P_t = \left[\int P_t(i)^{1-\theta}di\right]^{\frac{1}{1-\theta}}.$$

In order to guarantee that consumption expenditures are non-negative, τ^c_t is restricted to be $\tau^c_t \geq -1$. The transactions function l is homogenous of degree $k \geq 0$. It can thus be written as $l(C_t, m_t) = L\left(\frac{m_t}{C_t}\right)C^k_t$, where $m_t = \frac{M_t}{(1+\tau^c_t)P_t}$. We restrict $L' < 0$, and $L'' > 0$. We define the point of full liquidity as $\frac{m}{C}^*$ where $L'\left(\frac{m}{C}^*\right) = 0$. Furthermore we assume that at that point time used for transactions is zero, $L\left(\frac{m}{C}^*\right) = 0$. This implies that $l_C = 0$ at that point.

The budget constraints of the households can be written as

$$M_t + B^h_t + E_t Z^h_{t+1} z_{t+1} + \int_0^1 A(i)_t a(i)_t di \leq \mathbb{W}_t \tag{2}$$

$$\mathbb{W}_{t+1} = M_t + R_t B_t^h + Z_{t+1}^h - (1+\tau_t^c)\int_0^1 P_t(i)c_t(i)di + \tag{3}$$
$$+(1-\tau_t^w)W_t N_t^w + \int_0^1 A(i)_t\left[a(i)_{t+1} + (1-\tau_t^d)\Pi(i)_t\right]di$$

Initial nominal wealth, $\mathbb{W}_0^- = \mathbb{W}_0 - \int_0^1 a(i)_0 di$, is given. Given the total available time, normalized to one, we can write labor, N_t^w, as

$$N_t^w = 1 - h_t - l(C_t, \frac{M_t}{(1+\tau_t^c)P_t}) \tag{4}$$

The preferences of the households are given by

$$U = E_0\left\{\sum_{t=0}^{\infty}\beta^t u(C_t, h_t)\right\} \tag{5}$$

over leisure, h_t, and the composite consumption good,

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}}, \theta > 1. \tag{6}$$

The first order conditions of the households problem include the following marginal conditions:

$$\frac{c_t(i)}{C_t} = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} \tag{7}$$

$$\frac{u_C(t) - u_h(t)l_C(t)}{u_h(t)} = \frac{(1+\tau_t^c)P_t}{(1-\tau_t^w)W_t} \tag{8}$$

$$-l_m(t) = \frac{(1+\tau_t^c)P_t}{(1-\tau_t^w)W_t}(R_t - 1) \tag{9}$$

$$\frac{u_h(t)}{(1-\tau_t^w)W_t} = E_t\left[R_{t+1}\frac{\beta u_h(t+1)}{(1-\tau_{t+1}^w)W_{t+1}}\right] \tag{10}$$

$$z_{t+1} = \beta\frac{u_h(t+1)}{u_h(t)}\frac{R_{t+1}(1-\tau_t^w)W_t}{R_t(1-\tau_{t+1}^w)W_{t+1}} \tag{11}$$

$$a(i)_t = \beta E_t\left[z_{t+1}\left[a(i)_{t+1} + (1-\tau_t^d)\Pi(i)_t\right]\right] = 0 \tag{12}$$

From these conditions we get $E_t z_{t+1} = \frac{1}{R_{t+1}}$.

Government The government must finance a given path of government purchases $(G_t)_{t=0}^{\infty}$, such that

$$G_t = \left[\int_0^1 g_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > o \tag{13}$$

Given the prices on each good, $P_t(i)$, the government minimizes expenditure on government purchases by deciding according to

$$\frac{g_t(i)}{G_t} = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} \tag{14}$$

A government policy consists of a sequence of a government expenditures, money supplies, taxes and debt supplies indexed by dates and states, $\left(G_t, M_t, R_t, \tau_t^c, \tau_t^w, \tau_t^d, B_t^g, Z_{t+1}^g\right)_{t=0}^{\infty}$. The following restrictions apply to the tax rates: $R_t \geq 1$, $\tau_t^c \geq -1$, $\tau_t^w \leq 1$, $\tau_t^d \leq 1$. If $R_t < 1$, it would be possible to make infinite profits issuing debt and holding money. If $\tau_t^c < -1$, the consumers would be able to purchase an infinite amount of consumption. If $\tau_t^w > 1$, labor supply would be zero. Finally, if $\tau_t^d > 1$, it would be optimal to minimize profits.

Firms In this economy there are $0 \leq \alpha \leq 1$ firms that set prices one period in advance. The remaining firms set prices contemporaneously. Each firm i has the production technology

$$y_t(i) \leq s_t n_t^w(i) \tag{15}$$

where $y_t(i)$ is the production of good i and s_t is an aggregate technology shock. $y_t(i)$ can be used for private and public consumption, $y_t(i) = c_t(i) + g_t(i)$.

The problem of the firm is to choose the price in order to maximize profits that can be used for consumption in period $t+1$ taking the demand function,

$$\frac{y_t(i)}{Y_t} = \left(\frac{P_t(i)}{P_t} \right)^{-\theta}, \tag{16}$$

as given, where $Y_t = C_t + G_t$, and satisfying the technology constraint, (15). The demand is obtained from (7) and from (14).

The firms that can choose prices every period, at each date t maximize the nominal value of profits $E_t\left[z_{t+1}(1-\tau_t^d)\Pi_t(i)\right] = \frac{(1-\tau_t^d)}{R_t}\Pi_t(i)$ where

$$\Pi_t(i) = P_t(i)y_t(i) - W_t n_t^w(i)$$

Since the consumption tax is restricted to be $\tau_t^c \geq -1$, the maximization of the nominal value of profits also maximizes the value in units of consumption. Since $\tau_t^d \leq 1$, the maximization of $\frac{(1-\tau_t^d)}{R_t}\Pi_t(i)$ is equivalent to the maximization of $\Pi_t(i)$, where $P_t(i)$ satisfies the demand function (16). The firms choose a single price according to

$$P_t(i) = P_t^f = \frac{\theta}{(\theta-1)}\frac{W_t}{s_t}. \tag{17}$$

The price is set equal to a constant mark-up over marginal cost.

We consider now the problem of the firms that set the prices one period in advance, and can only sell output in period t at the previously chosen price. As of time t, the firms are constrained in terms of the price at which they can sell, but are not constrained in terms of the quantity. Thus, at time t, and given a previously chosen price, they do choose quantities to maximize profits. These firms sell the output on demand in period t as long as the value of profits is non negative, $E_t\left[z_{t+1}(1-\tau_t^d)\Pi_t(i)\right] = \frac{(1-\tau_t^d)}{R_t}\Pi_t(i) \geq 0$. If this value was negative the firm would choose to produce zero. We make assumptions on the mark up $\frac{\theta}{(\theta-1)}$ and the magnitude of shocks to guarantee that the profits when the firms sell the output on demand are non negative in every state.

When firm i sets prices one period in advance, it solves the problem of choosing at $t-1$ the price $P_t(i)$ that maximizes the value of profits, i.e.

$$E_{t-1}\left[z_t z_{t+1}(1-\tau_t^d)\left(P_t(i)y_t(i) - W_t n_t^w(i)\right)\right] \tag{18}$$

The firm's problem is to maximize (18) subject to (15) and (16).

The firms will choose the price according to the following first order condition

$$E_{t-1}\left[z_t z_{t+1}(1-\tau_t^d)y_t(i)\left(1-\frac{\theta}{(\theta-1)}\frac{W_t}{s_t P_t(i)}\right)\right] = 0 \tag{19}$$

Using (11), this can be written as

$$E_{t-1}\left[(1-\tau_t^d)\frac{u_h(t+1)R_{t+1}}{(1-\tau_{t+1}^w)W_{t+1}}y_t(i)\left(1-\frac{\theta}{(\theta-1)}\frac{W_t}{s_t P_t(i)}\right)\right] = 0 \tag{20}$$

Therefore, the price chosen by the firms that set prices in advance is

$$P_t(i) = P_t^s = \frac{\theta}{(\theta - 1)} E_{t-1}\left[\upsilon_t \frac{W_t}{s_t} \right] \tag{21}$$

where,

$$\upsilon_t = \frac{(1-\tau_t^d)\frac{u_h(t+1)R_{t+1}}{(1-\tau_{t+1}^w)W_{t+1}} y_t^s}{E_{t-1}\left[(1-\tau_t^d)\frac{u_h(t+1)R_{t+1}}{(1-\tau_{t+1}^w)W_{t+1}} y_t^s \right]}.$$

Firms charge a mark-up over the expected value of a weighted marginal cost, where the weights depend on period $t+1$'s marginal utility of consumption, the price levels, the nominal interest rates and the velocity shocks as well as on period t's output of the sticky firms.

Market clearing The market clearing conditions are

$$c_t^s + g_t^s = y_t^s \tag{22}$$

$$c_t^f + g_t^f = y_t^f \tag{23}$$

$$\alpha y_t^s + (1-\alpha) y_t^f = s_t (1 - h_t - l(C_t, m_t))$$

$$B_t^h = B_t^g$$

$$Z_{t+1}^h = Z_{t+1}^g, \text{ for all possible states at } t+1$$

$$A(i)_t = 1$$

3 Implementable allocations

In this section we show that the sets of implementable allocations are the same in the two extreme environments, under flexible and sticky prices. Fiscal and monetary policies affect the economy very differently in the two environments. However it is still possible to decentralize the same set of allocations. The nominal rigidity gives rise to an additional distortion. Still it does not restrict the set of implementable allocations. Even if it neutralizes the effect of fiscal instruments, as we will discuss in the next section, it generates the monetary non-neutrality that can be used to achieve the same set of allocations as under flexible prices. In this sense the nominal rigidity is irrelevant for the choice of the optimal allocation. In the intermediate case, where the economy is composed of both firms that set the prices contemporaneously and firms that set prices one period in advance, the set of implementable allocations is larger, because it includes allocations where relative prices are distorted. However we will show that there is a frontier of allocations that coincides with the implementable set in the two extreme environments.

Adao, Correia and Teles (2002) discuss the issue of whether to one monetary policy can correspond multiple allocations both in flexible and sticky price environments. They show that when the policy is defined in the way we do it here, that is, monetary policy is defined by a path of interest rates and a path of money supplies, it is the case that for each policy there is a single allocation. This means that for each specific allocation that is feasible and implementable there exists one policy that implements only that allocation.

The set of implementable allocations must include the intertemporal budget constraint of the households

$$\begin{aligned} & E_0 \sum_{t=0}^{\infty} Q_{t+1} \left[(1+\tau_t^c) P_t C_t + M_t (\frac{Q_t}{Q_{t+1}} - 1) \right] \\ = \; & E_0 \sum_{t=0}^{\infty} Q_{t+1} \left[(1-\tau_t^w) W_t \left(1 - h_t - l(C_t, m_t)\right) - (1-\tau_t^d) \Pi_t \right] + \mathbb{W}_0^- \end{aligned}$$

where $Q_t = \prod_{j=0}^{t} z_j$, $Q_0 = 1$. The profits, Π_t, are

$$\Pi_t = \alpha y_t^S \left(P_t^S - \frac{W_t}{s_t} \right) + (1-\alpha)\, y_t^F \left(P_t^F - \frac{W_t}{s_t} \right)$$

The intertemporal prices $Q_{t+1} = \frac{\beta^{t+1} R_{t+1} (1-\tau_0^w) W_0}{R_0 (1-\tau_{t+1}^w) W_{t+1}} \frac{u_h(t+1)}{u_h(0)}$ can be replaced to obtain:

$$\begin{aligned} & E_0 \sum_{t=0}^{\infty} \beta^t u_h(t) \left\{ \frac{(1+\tau_t^c)}{(1-\tau_t^w) w_t} C_t + \frac{(1+\tau_t^c)}{(1-\tau_t^w) w_t} (R_t - 1)\, m_t \right\} \\ = \; & E_0 \sum_{t=0}^{\infty} \beta^t u_h(t) \left\{ (1 - h_t - l(C_t, m_t)) + \frac{(1-\tau_t^d)}{(1-\tau_t^w)} \frac{\Pi_t}{W_t} \right\} + \frac{R_0 u_h(0)}{(1-\tau_0^w)} \frac{\mathbb{W}_0^-}{W_0} \end{aligned}$$

where $w_t = \frac{W_t}{P_t}$ is

$$w_t = \left[\alpha \left(w_t^s\right)^{(\theta-1)} + (1-\alpha)\left(w_t^f\right)^{(\theta-1)}\right]^{\frac{1}{\theta-1}}$$

where $w_t^j = \frac{W_t}{P_t^j}$, $j = f, s$. Let $\mathbb{W}_0^- = 0$. Then, the intratemporal conditions, (8) and (9), can be used to replace τ_t^c and R_t, to obtain

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u_C(t) C_t - u_h(t)\left(1 - h_t - (1-k) l(t)\right) - u_h(t) \frac{\left(1-\tau_t^d\right)}{\left(1-\tau_t^w\right)} \frac{\Pi_t}{W_t} \right\} = 0 \tag{24}$$

We also use the fact that the transactions technology is homogeneous of degree k.

From (17) and (19), and using the law of iterated expectations as well as (10), the price setting conditions for the flexible and sticky firms are, respectively,

$$w_t^f = \frac{\theta - 1}{\theta} s_t \tag{25}$$

$$E_{t-1}\left[\frac{(1-\tau_t^d) u_h(t)}{(1-\tau_t^w) w_t^s}\left(1 - \frac{\theta}{\theta-1}\frac{w_t^s}{s_t}\right) y_t^s\right] = 0 \tag{26}$$

Thus, we have, for $t \geq 1$,

$$E_0\left[\frac{(1-\tau_t^d) u_h(t)}{(1-\tau_t^w) w_t^s} y_t^s\right] = E_0\left[\frac{\theta}{\theta-1}\frac{(1-\tau_t^d) u_h(t)}{(1-\tau_t^w)}\frac{y_t^s}{s_t}\right] \tag{27}$$

Real profits, on (24), are

$$\frac{\Pi_t}{W_t} = \alpha y_t^S \left(\frac{1}{w_t^s} - \frac{1}{s_t}\right) + (1-\alpha) y_t^f \left(\frac{1}{w_t^f} - \frac{1}{s_t}\right) \tag{28}$$

From (25) and (27), we have

$$E_0 \sum_{t=0}^{\infty} \beta^t u_h(t) \frac{\left(1-\tau_t^d\right)}{\left(1-\tau_t^w\right)} \frac{\Pi_t}{W_t} = u_h(0) \frac{\left(1-\tau_0^d\right)}{\left(1-\tau_0^w\right)} \left(\alpha y_0^s \left(\frac{1}{w_0^s} - \frac{1}{s_0}\right) + (1-\alpha)\frac{y_0^f}{s_0}\frac{1}{\theta-1}\right) \tag{29}$$

$$E_0 \sum_{t=1}^{\infty} \beta^t u_h(t) \frac{\left(1-\tau_t^d\right)}{\left(1-\tau_t^w\right)} \left(1 - h_t - l(t)\right) \frac{1}{\theta - 1}$$

The degree of price stickiness α does not restrict the value of profits, and therefore the government, or households, equilibrium budget constraints, as stated in the following lemma.

Lemma 1 *The restriction on allocations* $\{C_t, h_t, m_t\}$ *from implementability condition (24) is the same for any* $0 \leq \alpha \leq 1$.

Proof From (29), the value of profits from $t = 1$ on is the same for any α. At time 0, any value of non negative profits can be achieved with any one of the three instruments w_0^s, τ_0^d or τ_0^w, for $0 < \alpha < 1$. When $\alpha = 1$, the instruments are w_0, τ_0^d or τ_0^w, and when $\alpha = 0$ these are τ_0^d or τ_0^w. All these instruments are equivalent instruments in what concerns their effect on profits.■

From (6), (7), (13), (14), (22) and (23), we can write the resources constraints as

$$\left[\alpha\left(\frac{w_t^f}{w_t^s}\right)^{1-\theta}+1-\alpha\right]^{\frac{\theta}{1-\theta}}(C_t+G_t)=y_t^f \tag{30}$$

$$\left[\alpha+(1-\alpha)\left(\frac{w_t^f}{w_t^s}\right)^{\theta-1}\right]^{\frac{\theta}{1-\theta}}(C_t+G_t)=y_t^s \tag{31}$$

$$\alpha y_t^s+(1-\alpha)\,y_t^f=s_t\,(1-h_t-l(C_t,m_t)) \tag{32}$$

Let $D(\frac{w_t^f}{w_t^s}) \equiv \left\{\alpha\left[\alpha+(1-\alpha)\left(\frac{w_t^f}{w_t^s}\right)^{\theta-1}\right]^{\frac{-\theta}{\theta-1}}+(1-\alpha)\left[\alpha\left(\frac{w_t^s}{w_t^f}\right)^{\theta-1}+(1-\alpha)\right]^{\frac{-\theta}{\theta-1}}\right\}^{-1}$.
Then, from (30), (31) and (32), we have

$$C_t+G_t=D(\frac{w_t^f}{w_t^s})s_t\,(1-h_t-l(C_t,m_t)) \tag{33}$$

The implementability conditions for the allocations $\{C_t, h_t, m_t\}$ and $\left\{y_0^s, y_0^f\right\}$, real wages $\left\{w_t^f, w_t^s\right\}$, and the taxes $\{\tau_t^d, \tau_t^w\}$ are (24) and (29), the resources constraints, (33), and for period $t = 0$, (30), (31) and (32), as well as the price setting conditions, (25), (26). The restrictions on taxes and profits also apply.

In the following proposition we state that the sets of implementable allocations are the same in the extreme cases of flexible and sticky prices.

Proposition 2 *The sets of implementable allocations coincide in the cases where* $\alpha = 0$ *and* $\alpha = 1$.

Proof Under flexible prices, when $\alpha = 0$, the set of implementable allocations $\{C_t, h_t, m_t\}$ and taxes $\left\{\tau_t^d, \tau_t^w\right\}$ are given by

$$\begin{aligned} 0 &= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u_C(t) C_t - u_h(t) \left(1 - h_t - (1-k) l(t)\right) \right\} - \\ & \quad E_0 \sum_{t=0}^{\infty} \beta^t u_h(t) \frac{(1-\tau_t^d)}{(1-\tau_t^w)} \left(1 - h_t - l(t)\right) \frac{1}{\theta - 1}, \end{aligned} \tag{34}$$

together with the resources constraints,

$$C_t + G_t = s_t \left(1 - h_t - l(C_t, m_t)\right) \tag{35}$$

When $\alpha = 1$, the set of implementable allocations $\{C_t, h_t, m_t\}$, real wages $\{w_t\}$, and taxes $\left\{\tau_t^d, \tau_t^w\right\}$, is given by

$$\begin{aligned} 0 &= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u_C(t) C_t - u_h(t) \left(1 - h_t - (1-k) l(t)\right) \right\} - \\ & \quad u_h(0) \frac{(1-\tau_0^d)}{(1-\tau_0^w)} \left(1 - h_0 - l(0)\right) \left(\frac{s_0}{w_0} - 1 \right) - \\ & \quad E_0 \sum_{t=1}^{\infty} \beta^t u_h(t) \frac{(1-\tau_t^d)}{(1-\tau_t^w)} \left(1 - h_t - l(t)\right) \frac{1}{\theta - 1}, \end{aligned}$$

and the resources constraints, (35). Under sticky prices there is an additional instrument at time 0, which is the surprise effect of monetary policy on w_0. However, as was stated in lemma 1, the effect on profits of w_0 can be achieved under flexible prices with τ_0^d or τ_0^w.■

In the mixed economy with both flexible and sticky firms, where $0 < \alpha < 1$, the set of implementable allocations is a larger set. However, there is a frontier of allocations that coincides with the sets under flexible and sticky prices. The other allocations, such that relative prices are distorted, are interior allocations that would not be chosen for any government preferences that depend on aggregate consumption and leisure. We state this in the following lemma and proposition.

Lemma 3 *When $0 < \alpha < 1$, there is a frontier of allocations with $w_t^s = w_t^f = \frac{\theta-1}{\theta} s_t$, $t \geq 0$.*

Proof As stated in lemma 1, the implementability condition (24) defines a set of allocations that is independent of α. By setting $w_t^s = w_t^f = \frac{\theta-1}{\theta} s_t$, $t \geq 0$, the set of implementable allocations will be given by (24) together with the resources constraints, (35). If the policy was such that $w_t^s \neq w_t^f$, then the resources constraint would be satisfied with inequality,

$$C_t + G_t = D(\frac{w_t^f}{w_t^s}) s_t \left(1 - h_t - l(C_t, m_t)\right)$$

since $D(\frac{w_t^f}{w_t^s}) < 1$ iff $w_t^s \neq w_t^f$.■

Thus, allocations are not dependent on the value of alfa, i.e., on the strength of the monetary transmission mechanism. We state this result in the following proposition:

Proposition 4 *The set of (frontier) implementable allocations is independent of the degree of price stickiness,* $0 \leq \alpha \leq 1$.

4 Decentralization

In this section we discuss how the allocations in the common set of implementable allocations can be decentralized with the policy instruments described above, in the different environments. This will clarify how the policy instruments affect the economy depending on the price setting restrictions.

In each environment, for each allocation we can recover the taxes, the debt and monetary policies and support that allocation. We can use the following equilibrium condition, that is independent of α, to recover the nominal interest rate,

$$-l_m(t) = \frac{u_C(t) - u_h(t) l_C(t)}{u_h(t)} \left(R_t - 1\right), \, t \geq 0 \tag{36}$$

The taxes on profits will not be unique, and they can be determined by the single equation, (29).

We have shown that the planner´s choice will always be characterized by $w_t^s = w_t^f$. We can characterize additional properties of the frontier. This frontier is the set of implementable allocations from which any planner whose preferences are on aggregate consumption and leisure will chose. We will restrict further the set defined by (34) and (35).

For any degree of price stickiness, α, the choices of τ_t^d and m_t are independent of government preferences. Because profits cannot be negative, it is optimal to tax profits completely, $\tau_t^d = 1$. Since at $\tau_t^d = 1$, the production decisions are indeterminate, we consider the case of τ_t^d arbitrarily close to one. The government can decentralize the same allocation as in the perfect competition case.[1]

The choice of $\{C_t, h_t, m_t\}_{t=0}^{\infty}$, is subject to

$$E_0 \sum_{t=0}^{\infty} \beta^t \{u_C(t)C_t - u_h(t)(1 - h_t - (1-k)l(t))\} = 0$$

and (35).

Whatever are the preferences of the planner, and including the ones of the benevolent planner, the first order conditions of the planner problem for real money m_t are

$$-l_m(t)\left[\lambda_t s_t - \varphi(k-1)u_h(t)\right] = 0$$

The optimal solution is thus always characterized by

$$-l_m(t) = 0$$

which is decentralized with the Friedman rule

$$R_t = 1$$

This is the result of De Fiore and Teles (1998), for the deterministic case, that extends the result of Correia and Teles (1996) when instruments include consumption taxes. The result is in contrast with Mulligan and Sala-i-Martin (1996) and Schmitt-Grohé and Uribe (2000). Mulligan and Sala-i-Martin (1996) use the specification of the transactions technology proposed by Kimbrough (1986) that was not restricted to exhibit unitary elasticity of money with respect to the price level gross of consumption taxes. Schmitt-Grohe and Uribe (2000) don't allow for complete taxation of profits and/or for consumption taxes. The inflation tax is replacing the consumption tax, with an efficiency loss.

[1]Full taxation of profits could also be achieved with an arbitrary large consumption tax and an arbitrary low labor income tax.

Because the choice of real money balances when $R_t = 1$ is indeterminate, we consider the limiting case, when R_t approaches one.

In characterizing the set of implementable allocations, we have considered that the government can issue state contingent debt, and collect state contingent consumption taxes, labor income taxes and profit taxes. Not all these instruments are necessary, to decentralize the sets of allocations identified above. In particular, there are ways of simulating state contingent real debt, when the government issues risk free nominal debt, even in a world where price variability is costly. This is related to the issue addressed by Chari, Christiano and Kehoe (1991), and recently by Siu (2000) and Schmitt-Grohe and Uribe (2001).

Chari, Christiano and Kehoe (1991) show that in the absence of state contingent debt the price level can react to shocks so as to simulate state contingent real debt. The high volatility of prices associated with such a policy lead Siu (2000) and Schmitt-Grohe and Uribe (2001) to analyze optimal policies with sticky prices. They find that the price volatility is substantially reduced once costly price adjustments are considered. The planner that has to compare the costs of changing prices with the gains of simulating state contingent, decides in favor of very low costs of changing prices. It turns out that this result is due only to the fact that both authors do not consider consumption taxes. With consumption taxes it is possible to simulate state contingent debt, still keeping the prices from changing in response to contemporaneous shocks. With flexible prices, state contingent debt can be simulated with price volatility or consumption tax volatility.

When there is a share of sticky firms, the set of (frontier) implementable allocations is such that prices are predetermined. Thus each of the allocations in the set of implementable allocations described above, will be achieved with high volatility of both consumption and labor income taxes, with the objective of simulating state contingent real debt.

Under flexible prices, the three instruments are equivalent: state contingent debt, unanticipated money supply policy and consumption taxes. Chari, Christiano and Kehoe (1991) needed only one instrument to simulate state contingent debt and they chose to use unanticipated money supply policy. With sticky prices, the three instruments are no longer equivalent. Both instruments are necessary to decentralize the optimal allocation: unanticipated money supply policy and the consumption taxes. Therefore the optimal policy pins down consumption taxes. As with prices in Chari, Christiano and Kehoe (1991), the taxes on consumption will be highly volatile to replicate state contingent real debt.

4.0.1 Flexible prices

For each allocation we can recover the taxes, the debt and monetary policies that support that allocation using the following equilibrium conditions,

$$\frac{(u_C(t) - u_h(t) l_C(t))}{u_h(t)} s_t = \frac{(1+\tau_t^c)}{(1+\tau_t^w)} \frac{\theta}{\theta - 1}, \ t \geq 0 \tag{37}$$

$$(1+\tau_t^c) P_t m_t = M_t, \ t \geq 0 \tag{38}$$

$$\frac{u_C(t-1) - u_h(t-1) l_C(t-1)}{(1+\tau_{t-1}^c) P_{t-1}} = E_{t-1}\left[R_t \frac{\beta\left(u_C(t) - u_h(t) l_C(t)\right)}{(1+\tau_t^C) P_t}\right], \ t \geq 1 \tag{39}$$

$$\begin{aligned} & E_t \sum_{s=0}^{\infty} \beta^s \left\{u_C(t+s) C_{t+s} - u_h(t+s)\left(1 - h_{t+s} - (1-k) l(t+s)\right)\right\} \\ = \ & R_t \left(u_C(t) - u_h(t) l_C(t)\right) \frac{\mathbb{W}_t^-}{(1+\tau_t^c) P_t}, \ t \geq 0 \end{aligned} \tag{40}$$

where

$$\begin{aligned} \mathbb{W}_t^- = \ & M_{t-1} + R_{t-1} B_{t-1}^h + Z_t^h - (1+\tau_t^c) P_t C_t + \\ & \frac{u_h(t)}{u_C(t) - u_h(t) l_C(t)} (1+\tau_t^c) P_t \left(1 - h_t - l(t)\right) \end{aligned} \tag{41}$$

If there are no consumption taxes, $\tau_t^c = 0$, and if public debt is not state contingent, then $\{\tau_t^w, M_t, B_t^g\}$ and $\{P_t\}$ are uniquely pinned down. From (37), τ_t^w is determined in each date and state. Let Φ_t be the number of states in period t. $\Phi_0 = 1$. Conditions (38) and (40), for $t = 0$, determine M_0 and P_0, since $\mathbb{W}_0^-$ is given. At $t = 1$, there are $2\Phi_1 + 1$ equations, (38), (39) and (40), to determine, $2\Phi_1 + 1$ variables, M_1, P_1, and $\mathbb{W}_1^-$ or B_0^g. Using this argument we are able to determine the policy variables for any t. In this case the price level variability simulates real state contingent debt. Alternatively, without labor income taxes and state contingent debt, can determine $\{\tau_t^c, M_t, B_t^g\}$ and $\{P_t\}$. Also here the price level variability simulates real state contingent debt. The two minimal sets of instruments are equivalent sets, in the sense that they are alternative policies that decentralize the same allocation.

When the set of instruments is the labor income tax, τ_t^w, money supply, M_t, and state contingent debt Z_t^g, in this case there is a unique tax policy $\{\tau_t^w\}$, and multiple paths for money supply, $\{M_t\}$, corresponding to multiple paths for the debt $\{Z_t^g\}$ and the price level $\{P_t\}$. The same is true if instead of the labor income tax, a consumption tax is used. These are all equivalent policies that exhibit a (fiscal) neutrality of money under flexible prices. If both taxes are considered and debt is not state contingent, there are still multiple paths for the money supply, $\{M_t\}$, and the price level gross of consumption taxes $\{(1+\tau_t^c)P_t\}$. Real state contingent debt can be simulated with the variability of both the price level and the consumption tax.

There is one money supply and tax policy such that the prices are predetermined, as under sticky prices. The price is determined by

$$\begin{aligned} & \frac{u_C(t-1) - u_h(t-1)l_C(t-1)}{(1+\tau_{t-1}^c)P_{t-1}} \\ = \; & \frac{1}{P_t} E_{t-1}\left[R_t \frac{\beta\left(u_C(t) - u_h(t)l_C(t)\right)}{(1+\tau_t^c)}\right], \end{aligned}$$

4.0.2 Sticky prices

Under sticky prices there is only one price to determine in each period, but there are also less equilibrium conditions that can be used to recover the policy variables. When all the firms set prices one period in advance, the relevant equilibrium conditions are as under flexible prices (38), (39), (40), except that conditions (37) are replaced by

$$E_{t-1}\left[(1-\tau_t^d)u_h(t)\left(1-h_t-l(t)\right)\left(\frac{\frac{(u_C(t)-u_h(t)l_C(t))}{u_h(t)}s_t}{\frac{(1+\tau_t^c)}{(1+\tau_t^w)}\frac{\theta}{\theta-1}} - 1\right)\right] = 0, \; t \geq 1 \tag{42}$$

that hold from period $t=1$ on. The initial price level P_0 is given.

If there are no labor income taxes and if public debt is not state contingent, then $\{\tau_t^c, M_t, B_t^g\}$ and $\{P_t\}$ are uniquely determined. At $t=0$, τ_0^c is determined using (40) for $t=0$, according to a fiscal theory of the price level gross of consumption taxes. M_0 is determined using (38). At $t=1$, there are $2\Phi_1+2$ equations, (42), (38), (39) and (40), to determine, $2\Phi_1+2$ variables, $\{\tau_1^c, M_1\}$, P_1, and $\mathbb{W}_1^-$ or B_0^g. Given $\{\tau_1^c\}$ in the different states, (39) can be used to determine the price level P_1. The intratemporal condition (42) determined an average value for the consumption tax. Together with (40) can be used to determine $\{\tau_1^c\}$ and $\mathbb{W}_1^-$ or B_0^g. The money supply can be recovered using (38). Since the price level does not depend on the state, in order to replicate real state contingent debt, the consumption tax will vary with the state.

If public debt was state dependent, the consumption taxes would only be pinned down on average. In that case there would be multiple paths for $\{\tau_t^c, M_t, Z_t^g\}$ or $\{\tau_t^w, M_t, Z_t^g\}$ or yet $\left\{\frac{(1+\tau_t^c)}{(1+\tau_t^w)}, M_t, Z_t^g\right\}$, associated with the same real allocation. This means that there is a short run neutrality of taxes, under sticky prices, analogous to the neutrality of money under flexible prices.

One of the tax policies is such that:

$$\frac{\frac{(u_C(t)-u_h(t)l_C(t))}{u_h(t)}s_t}{(1+\tau_t^c)\frac{\theta}{\theta-1}} = 1,$$

as under flexible prices. Otherwise, this condition will only be satisfied on average, according to (42). The real wage will be different from the one under flexible prices and will compensate the deviations in the tax policy from this path, satisfying (8).

If the policy instruments were labor income taxes, money supply, and non state contingent public debt, it would not be possible to decentralize the same set of allocations.

Thus, in each extreme environment, under flexible and under sticky prices, there are many policies that decentralize the same allocation. Each of these allocations can be decentralized with a common policy to both environments. Use fiscal and monetary policy so that

$$\frac{\frac{(u_C(t)-u_h(t)l_C(t))}{u_h(t)}s_t}{\frac{(1+\tau_t^c)}{(1+\tau_t^w)}\frac{\theta}{\theta-1}} = 1$$

and

$$\begin{aligned} & \frac{u_C(t-1)-u_h(t-1)l_C(t-1)}{(1+\tau_{t-1}^c)P_{t-1}} \\ = \; & \frac{1}{P_t}E_{t-1}\left[R_t\frac{\beta\left(u_C(t)-u_h(t)l_C(t)\right)}{(1+\tau_t^c)}\right] \end{aligned}$$

Fiscal policy is conducted so that there are no surprises in the mark ups; monetary policy is conducted so that there are no surprises in prices.

4.0.3 The economy with flexible and sticky firms

We have seen that the allocations on the frontier are such that there are no relative price distortions. Thus it must be the case that the real wage is given by

$$w_t = \frac{\theta - 1}{\theta} s_t, \tag{43}$$

as under flexible prices. The price level is constant across states. For each allocation we can recover the taxes, the debt and monetary policies that support that allocation using the equilibrium conditions, as under flexible prices, (37), (38), (39), (40).

If the policy instruments are consumption taxes, money supply and state contingent debt, then these are uniquely pinned down by (37), (38), (39), (40). (37) determines $\{\tau_t^c\}$ for any state and date $t \geq 0$. From (39), we obtain $\{P_t\}$ for any $t \geq 1$. P_0 is given. $\{M_t\}$ are determined using (38) and $\{Z_t^g\}$ or $\left\{\mathbb{W}_t^-\right\}$ are determined using (40). If instead of consumption taxes, labor income taxes are used, the policies are also uniquely determined.

If the policy instruments are consumption taxes, labor income taxes and non state contingent public debt, the policies will not be uniquely pinned down. Since P_0 and $\mathbb{W}_0^-$ are given, (40) for $t = 0$ can be used to determine τ_0^c, (37) for $t = 0$ determines τ_0^w, and (38) determines M_0. At $t = 1$, there are $3\Phi_1 + 1$ equations, (38), (39) and (40), to determine, $3\Phi_1 + 2$ variables, $\{\tau_1^c, \tau_1^w, M_1\}$, P_1, and $\mathbb{W}_1^-$ or B_0^g. The argument applies for any t.

In this case the price level variability simulates real state contingent debt. There is one degree of indeterminacy. The average value of consumption taxes, τ_1^c, is not determined. Thus neither is the average labor income tax, τ_1^w, and the price level P_1. In this case state contingent real debt is simulated with the variability of the consumption tax.

The minimal set of instruments are $\{\tau_t^w, M_t, Z_t^g\}$ or $\{\tau_t^c, M_t, Z_t^g\}$ and $\{\tau_t^c, \tau_t^w, M_t, B_t^g\}$, even if in the last case the policy variables are not determined uniquely.

In this environment, with both flexible and sticky firms, there is neither fiscal nor monetary neutrality. Policy has to satisfy the following two principles: Fiscal policy should be conducted as if all prices were flexible; and monetary policy should be conducted as to replicate the flexible prices equilibrium. In this sense it is optimal to eliminate gaps, i.e. deviations to an allocation under flexible prices, once this allocation is the best implementable when prices are flexible.

The policies are not dependent on the value of alfa, that is on the strength of the monetary transmission mechanism. We can state this result in the following corollary:

Corollary 5 *(Adao, Correia and Teles, 1999) The optimal policies do not depend on the alfas.*

4.1 Restrictions on tax instruments

We have seen above that the restriction on the public debt that it may not be state dependent is not relevant since it can be replaced by the use of other fiscal instruments. In this section we inquire of the relevance of the restriction that taxes may not be state contingent. We first state the conditions under which it is optimal for a Ramsey planner to set constant distortions across states. Under those conditions the constraint is not binding.

When it is the case that optimal (second best) distortions are state dependent, then in the extreme case of $\alpha = 1$, it is still possible to use state dependent monetary policy to achieve the optimal solution. The optimal allocations will be characterized by gaps, i.e. deviations from the flexible price allocation. In general the equivalence of environments in terms of allocations and policies is lost when the restricting is imposed that taxes are not state dependent. Adao, Correia and Teles (2001) is an example of this principle. Since they only consider monetary policy there is a natural restriction on policy, that the nominal interest rate cannot be negative. For this reason the set of allocations under flexible prices is smaller that the set of allocations under sticky prices. In that paper, conditions are provided under which the optimal solution belongs to both sets. They claim that in general it doesn't.

When $0 < \alpha < 1$, and taxes cannot be state dependent, if the optimal solution is characterized by varying distortions across states, then the second best cannot be achieved. The third best will be characterized by deviations from the flexible price allocation and policies.

In the following subsection we provide the conditions under which it is optimal to set constant distortions across states.

4.1.1 Ramsey distortions.

In this section we describe the conditions under which it is optimal to smooth proportionate distortions. At the Friedman rule, we assume that the time used for transactions is zero. From homogeneity, $l_C(t) = 0$.

Then allocations are characterized by:

$$\frac{u_C(C_t, h_t)s_t}{u_h(C_t, h_t)} = \frac{\theta}{(\theta - 1)} \frac{(1 + \tau_t^c)}{(1 - \tau_t^n)}$$

$$C_t + G_t = s_t (1 - h_t)$$

From the FOC of Ramsey problem

$$\frac{u_C(t)}{u_h(t)} s_t = \frac{1 + \varphi \left(1 + \frac{u_{hC}(t)C_t}{u_{ht}} - \frac{u_{hh}(t)(1-h_t)}{u_{ht}}\right)}{1 + \varphi \left(1 + \frac{u_{CC}(t)C_t}{u_C(t)} - \frac{u_{Ch}(t)(1-h_t)}{u_C(t)}\right)}$$

Under what conditions on preferences is it optimal to set constant taxes?

i) Additively separable and constant elasticities

$$u = \frac{C_t^{1-\sigma}}{1 - \sigma} - \alpha N_t^{\psi}, \ \sigma \geq 0, \ \psi \geq 1,$$

$$\frac{u_{Ct}}{u_{ht}} s_t = \frac{1 + \varphi \left(1 + \frac{u_{Ch}}{u_h} C_t - \frac{u_{hh}}{u_h} (1 - h_t)\right)}{1 + \varphi \left(1 + \frac{u_{CC}}{u_C} C_t - \frac{u_{Ch}}{u_h} C_t \frac{u_h}{u_C s_t} \frac{s_t(1-h_t)}{C_t}\right)}$$

$$\frac{u_{Ct}}{u_{ht}} s_t = \frac{1 + \varphi\psi}{1 + \varphi (1 - \sigma)}$$

ii) Balanced growth

$$u = \frac{\left(C_t \mathcal{F}(h_t)\right)^{1-\sigma} - 1}{1 - \sigma}, \mathcal{F}' > 0, \ \sigma \geq 0.$$

which includes the isoelastic

$$u = \frac{(C_t {h_t}^{\psi})^{1-\sigma} - 1}{1 - \sigma}, \ \sigma \geq 0,$$

$$\frac{u_{Ct}}{u_{ht}} s_t = \frac{1 + \varphi \left(1 + \frac{u_{Ch}}{u_h} C_t - \frac{u_{hh}}{u_h} (1 - h_t)\right)}{1 + \varphi \left(1 + \frac{u_{CC}}{u_C} C_t - \frac{u_{Ch}}{u_h} C_t \frac{u_h}{u_C s_t} \frac{s_t(1-h_t)}{C_t}\right)}$$

$$\frac{u_{Ct}}{u_{ht}}s_t = \frac{1+\varphi\left(1+1-\sigma-\psi\left(1-\sigma\right)\frac{1-h_t}{h_t}\right)}{1+\varphi\left(1-\sigma-\left(1-\sigma\right)\frac{u_h}{u_C s_t}\frac{1}{1-\frac{G_t}{s_t(1-h_t)}}\right)}$$

$$\frac{u_{Ct}}{u_{ht}}s_t = \frac{1+\varphi\left(1+1-\sigma-\psi\left(1-\sigma\right)\frac{1-h_t}{h_t}\right)}{1+\varphi\left(1-\sigma-\left(1-\sigma\right)\frac{u_h}{u_C s_t}\frac{1}{1-\frac{G_t}{s_t(1-h_t)}}\right)}$$

In equilibrium

$$\frac{u_{Ct}}{u_{ht}}s_t = \frac{h_t}{\psi C_t}s_t$$

$$C_t + G_t = s_t\left(1-h_t\right)$$

If

$$G_t = s_t G \text{ or } G_t = g y_t$$

then, $\frac{u_{Ct}}{u_{ht}}s_t$ will depend only on h_t.

Therefore it is optimal to set constant taxes when preferences are separable and isoelastic in consumption and labor or preferences are consistent with balanced growth and government expenditures shocks are perfectly correlated with the productivity shock.

4.2 Conclusions:

In this paper we analyze the implications of nominal rigidities for the conduct of fiscal and monetary policy in response to shocks. We find that the sets of implementable allocations are the same under flexible and sticky prices.

Each allocation can be decentralized with a common policy to both environments. Under flexible prices money supply policy is conducted so that there are no surprises in prices. Under sticky prices, fiscal policy is conducted so that there are no surprises in mark ups. In each environment there are other policies that decentralize the same allocation. In particular, under sticky prices, fiscal policy is not uniquely pinned down. This means that there is short run neutrality of taxes, which is analogous to the neutrality of money under flexible prices. Thus both under flexible prices and under sticky prices there are neutral and non neutral fiscal and monetary instruments. The roles are reversed across environments.

In the mixed economy with both flexible and sticky firms, fiscal and monetary policies are pinned down. It is possible to determine a frontier where fiscal and monetary policies are conducted so that there are no surprises in prices and mark ups. The frontier is independent of the share of sticky firms and the policies that decentralize those allocations are also independent of the share of sticky firms. This second point was made by Adao, Correia and Teles (1999), that argue that the monetary transmission mechanism is not relevant for policy.

Because the frontier allocations are independent of the degree of rigidity, it is possible to use the results on optimal taxation under flexible prices as in Lucas and Stokey (1983), Chari, Christiano and Kehoe (1991), Zhu (1992). In this paper we analyze the conditions for optimal smoothing of distortions. Since policies do not depend on the degree of stickiness, exogeneity of the degree of stickiness is not a problem.

We also show in this paper that the frontier is characterized by the Friedman rule. Furthermore, we address the issue of which is the minimal set of instruments. Without state contingent debt, high volatility of consumption and income taxes can simulate state contingent debt.

The results in this paper hold under staggered price setting (or Rotemberg, 1982) and cash-in-advance.

References

[1] Adão, Bernardino, Isabel Correia and Pedro Teles, 1999, The Monetary Transmission Mechanism: Is it Relevant for Monetary Policy?, mimeo, Banco de Portugal.

[2] Adão, Bernardino, Isabel Correia and Pedro Teles, 2001, Gaps and Triangles, Federal Reserve Bank of Chicago wp-01-13.

[3] Carlstrom C. T. and Timothy S. Fuerst, 1998a, Price Level and Interest Rate Targeting in a model with Sticky Prices, mimeo, Federal Reserve Bank of Cleveland.

[4] Carlstrom C. T. and Timothy S. Fuerst, 1998b, A Note on the Role of Countercyclical Monetary Policy, *Journal of Political Economy,* vol 106, no. 4.

[5] Cooley, Thomas F. and Gary D. Hansen, 1989, The Inflation Tax in a Real Business Cycle Model, *American Economic Review*, 79, no. 4, 733-748.

[6] De Fiore, Fiorella and Pedro Teles, 1998, The Optimal Mix of Taxes on Money Consumption and Income, wp, Banco de Portugal

[7] Gali, Jordi and Tommaso Monacelli, Optimal Monetary Policy and Exchange Rate Volatility in a Small Open Economy, mimeo, U. Pompeu Fabra.

[8] Goodfriend, M., and R. G. King, 1997, The New Neoclassical Synthesis and the Role of Monetary Policy, NBER Macroannual.

[9] Greenwood, J., Hercowitz, Z., and G. Huffman, 1988, Investment, Capacity Utilization and the Real Business Cycle, American Economic Review 78, 402-417.

[10] Ireland, P., 1996, The Role of Countercyclical Monetary Policy, *Journal of Political economy*, vol. 104, n. 4.

[11] R. E. Lucas, Jr. and N. L. Stokey, 1983, Optimal Fiscal and Monetary Theory in an Economy without Capital, *Journal of Monetary Economics* 12, 55-93.

[12] King, Robert G. and Alexander L. Wolman, 1998, What Should the Monetary Authority do When Prices are Sticky?, mimeo, University of Virginia.

[13] Schmitt-Grohé, Stephanie and Martin Uribe, 2000, Optimal Fiscal and Monetary Policy under Imperfect Competition, mimeo, U. of Pennsylvania.

[14] Zhu, X. 1992, Optimal Fiscal Policy in a Stochastic Growth Model, Journal of Economic Theory, 58, 250-289.