A Theory of Growth and Volatility at the Aggregate and Firm level*

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Abstract

This paper presents an endogenous growth model that explains the evolution of the first and second moments of productivity growth at the aggregate and firm level during the post-war period. Growth is driven by the development of both (i) idiosyncratic R&D innovations and (ii) general innovations that can be freely adopted by many firms. Firm-level volatility is affected primarily by the Schumpeterian dynamics associated with the development of R&D innovations. On the other hand, the variance of aggregate productivity growth is determined mainly by the arrival rate of general innovations. Ceteris paribus, the share of resources spent on development of general innovations increases with the stability of the market share of the industry leader. As market shares become less persistent, the model predicts an endogenous shift in the allocation of resources from the development of general innovations to the development of R&D innovations. This results in an increase in R&D and in firm-level volatility,and in a decline in aggregate volatility. The effect on productivity growth is ambiguous.

On the empirical side, this paper documents an upward trend in the instability of market shares.It shows that firm volatility is positively associated with R&D spending, and that R&D

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is negatively associated with the correlation of growth between sectors leading to a decline in aggregate volatility.

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1 Motivation

The literature on endogenous growth has made substantial progress in the past 15 years. In spite of these advances, however, there remains much to be learnt about the determinants of long-run productivity growth. In our opinion, the existing literature suffers from two main limitations.

First, state-of-the-art models (Aghion and Howitt [1998, Ch. 12], Dinopoulos and Thompson [1998], Jones [1995], Kortum [1997], Peretto [1998], Segerstrom [1998] and Young [1998]) predict a positive relationship between the growth rate of productivity and the share of Research & Development (R&D) in GDP. However, this prediction does not seem to be true of data for the United States (US) during the post-war period. Figure 1 illustrates the smoothed growth rate of productivity as well as the evolution of the share of private R&D in GDP as measured by the NSF. No clear relationship seems to exist between the two variables.[1] In fact, Comin [2004] shows that the R&D expenditures as defined by the NSF can account for only a small fraction of productivity growth in the US during the post-war period. This finding suggests that there are other (probably) purposeful investments that lead to important improvements in productivity, investments that are not embodied in new products and, as a result, are not included in the NSF's definition of R&D.[2]

Second, in addition to having trouble explaining the first moments of growth processes, the existing theories have left the second moments out of their scope, making the implicit assumption that their determinants were orthogonal to the determinants of the first moments.[3] This presumption, however, is debatable in light of the interesting dynamics of volatility during the post-war period. Two strands of the literature have characterized the evolution of volatility at the aggregate and firm level. McConnell and Perez-Quiros [1999] and Stock and Watson [2003] have shown that the volatility of aggregate variables such as output, hours worked and labor productivity growth has declined during the post-war period. At the firm level, however, these same variables have become more volatile (Comin and Mulani [2003], Chenney et al. [2003] and this paper). Perhaps most importantly, these diverging trends are also true of the evolution of productivity growth. Figure 2

[1] Examination of TFP growth or output growth results in similar conclusions.

[2] These alternative innovations should not be confused with what the NSF classifies as process innovation. Process innovation applies to the development of new industrial processes such as those that lead to the production of steel or chemical products. In our context, this is the same as standard R&D that leads to a new product or an improved version of an existing product.

[3] There exists literature that has attempted to explore the effects of exogenous increases in aggregate volatility on growth (Ramey and Ramey [1995], Barlevy [2003]). A key difference between that literature and this paper is that here volatility (both aggregate and firm-level) is endogenous to analysis.

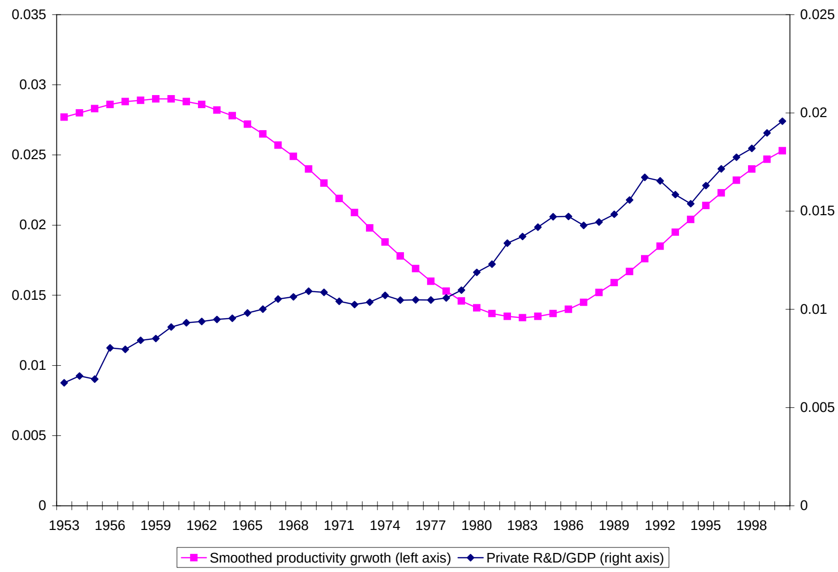

Figure 1: Evolution of (Smoothed) Productivity Growth and Private R&D share in GDP.

illustrates the evolution of productivity growth at the aggregate and firm level. The left axis plots the standard deviation of a 10 year-centered rolling window of annual productivity growth. The right axis plots evolution of the same variable averaged for firms in the COMPUSTAT data base.[4] The opposite trends are evident.[5]

Standard macroeconomic models are not equipped to explain these diverging trends in volatility. Existing representative-agent models cannot account for the divergence since they predict that the second moments of the aggregate and individual variables are identical. In principle, models with firm heterogeneity such as Bertola and Caballero (1990) can accommodate different trends in aggregate and firm-level volatility by assuming different trends in the variance of the exogenous aggregate and firm-specific shocks. However, Comin and Philippon [2005] provide evidence that these diverging trends are not just a coincidence. The goal should then be to build a model where in response to a shock, the firm-level and aggregate second moments respond in opposite ways. This is not the case in current models of firm heterogeneity because the interactions between firms embedded in these models are not adequate: most of them are partial equilibrium models and treat

[4] Comin and Mulani [2003] shows that the upward trend in firm-level volatility is not the result of a compositional bias in the COMPUSTAT sample. See also Comin and Philippon [2005] for a more detailed discussion.

[5] As mentioned earlier, Comin and Mulani [2003] and Chenney et al. [2003] have documented upward trends in the volatility of growth rate of firm-level sales and employment. This paper is, to the best of our knowledge, the first to document the upward trend in the firm-level volatility of the growth rate of sales per worker.

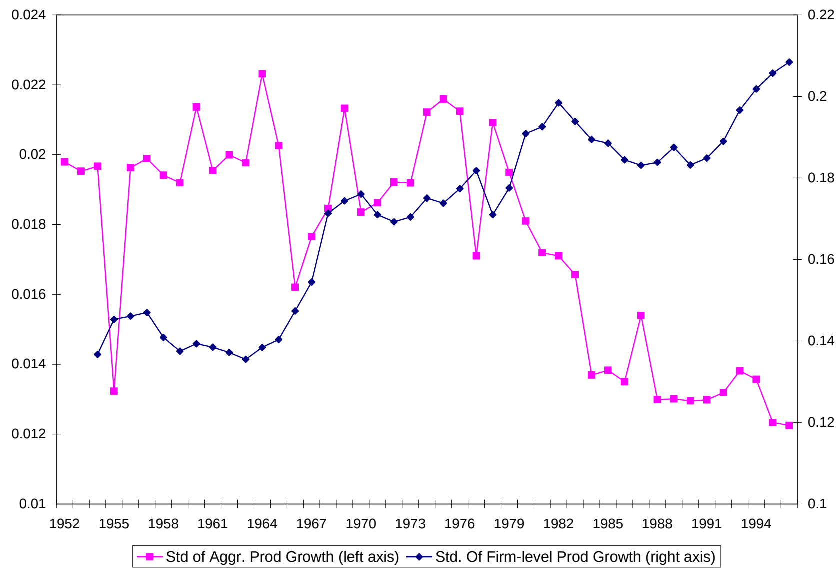

Figure 2: Evolution of the Aggregate and Firm-level Volatility of productivity

firms as independent entities. Even though more recent versions of these models have incorporated general equilibrium interactions, they seem insufficient to generate the co-movement patterns that drive the diverging trends in volatility.

Addressing the above concerns, this paper builds an endogenous growth model that attempts to enhance our understanding of the determinants of productivity growth and that has implications for firm and aggregate volatility that are consistent with the evidence.

To this end, this paper takes on a fresh route that, we believe, is more promising. It builds on the quality-ladder models of Aghion and Howitt [1992] and Grossman and Helpman [1991, Ch. 4]. In this context, standard R&D investments lead firms to develop new products that replace the current leading products. Such improvements in productivity lead to substantial firm-level volatility since incumbents incur losses while entrants enjoy capital gains. However, these innovations are to a large extent sector specific and hence, their effects on volatility at the aggregate level are relatively minor.

To explain the movements in aggregate growth and volatility, it is necessary to consider a second type of innovation - general innovations.[6] General innovations have two properties. First, they are

[6]Most of the innovations that fit our notion of general innovations are explicitly excluded by the NSF from their definition of R&D. See Comin [2004] for more on the NSF definition of R&D.

applicable to several firms and sectors.[7] Second, a firm that develops a general innovation (for the most part) cannot appropriate the benefits enjoyed by other firms when adopting it. This is the case because general innovations -such as the mass production system, mass customization, flow manufacturing and other organizational innovations, improved process controls, product development, testing practices and pre-production planning, new personnel and accounting practices, development of financial innovations and credit instruments such as the credit card, general programming languages such as Basic, Hypertext or Fortran, the use of electricity as the source of energy in a plant,...- are hard to patent and relatively easy to reverse-engineer.[8]

These two properties have interesting implications. First, general innovations have a large effect on aggregate growth because they affect many firms symmetrically. Furthermore, since general innovations do not arrive smoothly over time, investments in the development of general innovations may lead to substantial volatility in aggregate productivity growth. Second, the inability of innovators to appropriate the social value of general innovations means that their incentives to develop them depend on their firm's productivity gain from implementing the innovation. These productivity gains are larger for more valuable firms. As a result, general innovations are typically conducted by (large) leading firms.

In equilibrium, there is a negative relationship between the resources spent on R&D and on the development of general innovations. Since (1) R&D leads to turnover in market leaders and to a decline in the value of leading firms and (2) the private return to a general innovation increases in the value of the firm, a force that leads the economy to invest more in R&D may induce a decline in the rate of development of general innovations.

This trade off between R&D and general innovations accounts for the trends observed in growth and volatility at the aggregate level. Productivity growth increases with the development of both R&D and general innovations. But since an increase in the R&D intensity leads to a decline in the arrival rate of general innovations, the relationship between R&D and productivity growth is ambiguous. Second, aggregate volatility is primarily affected by the rate of arrival of general

[7] This feature links our general innovations to the General Purpose Technologies (GPT's) of Bresnahan and Trajtenberg [1995] and Helpman and Trajtenberg [1998]. Unlike GPT's, general innovations do not have to be revolutionary. An interesting difference from a modelling point of view is that the development of GPT's and their components is usually modelled as either being exogenous or supported by patents. Our general innovations are developed endogenously and are not patentable.

[8] Hellwig and Irmen [2001] and Boldrin and Levine [2000] have also drawn attention to the importance of innovations that are not patentable.

innovations because these generate simultaneous fluctuations in several sectors of the economy. Hence, a decline in investments in general innovations, leads to a decline in aggregate volatility. Finally, firm-level volatility is primarily driven by market turnover. An increase in R&D intensity leads to turnover in the market leader and an increase in firm-level volatility.

In addition to developing a new model of growth and volatility, this paper also provides empirical evidence of the forces and mechanisms emphasized by the model. First, it documents the significant increase in private R&D intensity and the market turnover rate in the US during the postwar period. Second, it shows that turnover and firm volatility have increased to a greater extent in sectors that have experienced higher increases in R&D intensity. Finally, this paper establishes that R&D is negatively associated with aggregate volatility by showing that sectors that experienced higher increases in R&D, also experienced greater declines in the correlation between their own growth and the rest of the economy.[9]

In addition to providing a qualitative explanation of the evidence, the mechanisms emphasized by the model are quantitatively important. A calibration of the model shows that it can account for (1) the lack of a relationship between R&D and productivity growth, (2) 75 percent of the increase in firm volatility and, (3) 50 percent of the decline in aggregate volatility.

The rest of the paper is organized as follows. Section 2 formalizes these intuitions with a model and undertakes the comparative statics exercises. Section 3 discusses and evaluates predictions of the model in both quantitative and qualitative terms. Section 4 concludes.

2 Model

The following describes an endogenous technological change model that delivers endogenous growth, and endogenous volatility at the aggregate and firm-level. For purposes of simplicity, we present the basic mechanisms in a one-sector context. Section 2.4 extends this basic model to a multisector environment in order to highlight implications for the co-movement of growth across sectors, implications that are essential to understand the evolution of volatility at the aggregate level.

[9] Comin and Philippon [2005] decompose the variance of aggregate growth into 2 components - (a) a sectoral variance component and (b) a correlation-between-sectors component and show that the decline in aggregate variance is due to the decline in the correlation of sectoral growth.

2.1 Set up

Preferences

The representative consumer enjoys a utility flow that is linear on the units of final output consumed (c_t). The present discounted value of utility can then be represented as

$$U = \int_0^{\infty} c_t e^{-rt} dt, \tag{1}$$

where r denotes the instantaneous discount rate. Consumers supply, inelastically, a mass of L units of labor.

Aggregate output

Final output (y) is produced by combining two goods denoted as leading (y_l) and standard (y_s) as specified in the following production function:

$$y = y_l^{\beta} y_s^{1-\beta} \tag{2}$$

This Cobb-Douglas aggregation of y_l and y_s simplifies the analysis later on since it implies that the nominal sales of each good is proportional to aggregate output regardless of the good's quality. Formally, the demands for y_l and y_s are given by:

$$\begin{aligned} \beta y &= p_l y_l \\ (1-\beta) y &= p_s y_s, \end{aligned}$$

where p_l and p_s denote, respectively, the prices of the leading and standard goods received by their producers. The price of aggregate output is normalized to 1.

Both leading and standard goods are produced competitively. The leading good is produced with labor (L_l) and the leading intermediate good (x_l) according to the function:

$$y_l = q_l x_l^{\alpha} L_l^{1-\alpha}, \tag{3}$$

where q_l denotes the quality of the leading intermediate good.

The production of standard output entails the use of labor (L_s) and m different standard intermediate goods (x_{si}) as follows:

$$y_s = q_s \left(m^{\sigma-1} \sum_{i=1}^{m} x_{si}^{\sigma} \right)^{\frac{\alpha}{\sigma}} L_s^{1-\alpha}, \tag{4}$$

where q_s is the efficiency of the standard intermediate goods and $\sigma \in (0,1)$ is the elasticity of substitution between the different standard intermediate goods.

The production of a unit of an intermediate good requires a_x units of labor. a_x increases with the complexity of the economy measured by q_l and declines with the efficiency of the production process denoted by h. Specifically, we assume that a_x takes the following form: $a_x = q_l^{\psi_q}/h^{\psi_h}$, where $\psi_q, \psi_h > 0$.

The final-good producers demand the following quantities of intermediate goods and labor:

$$\begin{aligned} x_l &= L_l \left(\alpha \frac{p_l q_l}{p_l^x} \right)^{\frac{1}{1-\alpha}} &(5) \\ L_l &= (1-\alpha) p_l y_l / w \\ x_{si} &= \frac{L_s}{m} \left(\frac{\alpha p_s q_s}{p_s^x} \right)^{\frac{1}{1-\alpha}} \left(\frac{p_s^x}{p_{si}^x} \right)^{\frac{1}{1-\sigma}} &(6) \\ L_s &= (1-\alpha) p_s y_s / w, \end{aligned}$$

where w is the wage rate, p_l^x and p_{si}^x denote the prices of the leading and the i^{th} standard intermediate goods, and $p_s^x = \left(m^{-1} \sum_{i=1}^m (p_{si}^x)^{\frac{-\sigma}{1-\sigma}} \right)^{\frac{-(1-\sigma)}{\sigma}}$.

Intermediate goods

There are a fixed number of $m+1$ intermediate goods in the economy, each manufactured by one and only one producer at any moment in time. Only the producer of the leading intermediate good can manufacture a good with the highest quality (q_l). Other intermediate good producers, whose goods are dominated by the leader's, can only manufacture a (differentiated) standard intermediate good with a fixed quality q_s.

Intermediate good producers can make two types of investments. First, they can try and develop an intermediate good with higher quality than q_l. In particular, after spending n_{si}^q units of aggregate output, they face a probability $\lambda_i^q = \lambda_0^q n_{si}^q / y$ of developing a new leading good with quality $\delta_q q_l$ ($\delta_q > 1$).[10,11] In this formulation, λ_0^q measures the probability of succeeding in the development of a superior intermediate good per fraction of GDP spent on R&D.[12]

[10] Griliches [1984] finds evidence in favor of the linearity of the R&D production technology using firm-level data.

[11] This formulation has several interesting features. First, the lower demand elasticity of the leading intermediate good is instrumental to generate cross-sectional variation in sales per worker. Second, by not having to carry around the distribution of intermediate goods qualities, we can make substantial progress in solving the model analytically. Third, the absence of entry and exit simplifies the computation of the second moments.

[12] Note that we can think off λ_0^q as being equal to $\frac{\bar{\lambda}_0^q}{1-\tau_s}$, where $\bar{\lambda}_0^q$ is a term that has to do with the technology to conduct R&D and τ_s is a government subsidy to R&D financed by lump sum taxes. In other words, in this setting, changes in the technology to conduct R&D are isomorphic to changes in the R&D subsidies. These alternative interpretations will be relevant when interpreting the comparative statics conducted below.

Second, intermediate goods producers can also invest in improving the production process of their intermediate good (i.e. reducing the cost of production, a_x). Specifically, each intermediate goods firm can invest n^h units of aggregate output and face a probability $\lambda^h = \max\left\{0, -f_0^h + \lambda_0^h \left(n^h/y\right)^{\rho_h}\right\}$, with $0 < \rho_h < 1$, of successfully increasing h up to $\delta_h h$, with $\delta_h > 1$. Note that the technology to develop such general innovations exhibits (local) increasing returns to scale at $n^h = 0$.

These two types of innovations differ in their appropriability. Firms that invent a new product or improve the quality of an existing product can patent the innovation and extract a substantial fraction of the surplus enjoyed by other firms from such an innovation. On the other hand, firms that develop a general innovation such as an improvement in the management practices cannot appropriate the benefits experienced by other firms when they adopt the same practices. This is the case because such general innovations are easy to reverse engineer and because many of them are not embodied in a good and therefore are hard to patent. These characteristics are reflected in the assumption that general innovations are immediately (and costlessly) adopted by all producers.

A second difference between the two types of innovations is their applicability. The impact of new or improved goods is often restricted to a small number of sectors. General innovations, such as the improvements in management or the organization of production mentioned above, can be applied to many different economic activities. As a result, a general innovation developed in a particular sector can be applied in several other sectors. This characteristic is important to explain the large effect of general innovations on aggregate volatility.

Given the demand functions (5) and (6), the profits of the intermediate goods producers are:

$$\begin{aligned} \pi_l &= (p_l^x - a_x w)x_l - \left(\left(\frac{\lambda_l^h + f_0^h}{\lambda_0^h}\right)^{\frac{1}{\rho_h}} + \frac{\lambda_l^q}{\lambda_0^q}\right) y \\ \pi_{si} &= (p_{si}^x - a_x w)x_{si} - \left(\left(\frac{\lambda_{si}^h + f_0^h}{\lambda_0^h}\right)^{\frac{1}{\rho_h}} + \frac{\lambda_{si}^q}{\lambda_0^q}\right) y, \end{aligned}$$

where λ_l^x and λ_{si}^x denote the hazard rates for innovations of type x faced by the leader and the i^{th} standard intermediate goods firms, for $x = h,\ q$.

Intermediate goods producers sell their goods monopolistically. Optimal pricing of the intermediate goods implies that $p_l^x = a_x/\alpha$, and that $p_{si}^x = a_x/\sigma$.

Profits of the intermediate goods producers evaluated at the optimal pricing rule are denoted as $\bar{\pi}_l$ and $\bar{\pi}_s$. Let V^l and V^{si} denote the values of a intermediate goods producer of the leading and i^{th} standard good. In a minor abuse of notation, let $\overrightarrow{\lambda_{-i}^z}$ denote the vector that contains the hazard rates for innovations on z (for $z = q, h$) for all the intermediate goods producers other than i (for

$i = l, si$). Finally, G^z denotes the law of motion for $\overrightarrow{\lambda^z_{-i}}$. Using this notation, V^l and V^{si} are defined by the following Bellman equations:

$$
\begin{aligned}
V^l\left(q_l, h; \overrightarrow{\lambda^q_{-l}}, \overrightarrow{\lambda^h_{-l}}\right) &= \max_{\lambda^h_l, \lambda^q_l} \bar{\pi}_l + (1+rdt)^{-1}[\lambda^q_l V^l(q_l\delta_q, h; \overrightarrow{\lambda^{q\prime}_{-l}}, \overrightarrow{\lambda^{h\prime}_{-l}}) + \sum_{i=1}^{m} \lambda^h_{si} V^{si}(q_l\delta_q, h; \overrightarrow{\lambda^{q\prime}_{-l}}, \overrightarrow{\lambda^{h\prime}_{-l}}) \\
&\quad (\lambda^h_l + \sum_{i=1}^{m} \lambda^h_{si}) V^l(q_l, h\delta_h; \overrightarrow{\lambda^{q\prime}_{-l}}, \overrightarrow{\lambda^{h\prime}_{-l}}) + \\
&\quad (1 - \lambda^q_l - \sum_{i=1}^{m} \lambda^q_{si} - \lambda^h_l - \sum_{i=1}^{m} \lambda^h_{si}) V^l(q_l, h; \overrightarrow{\lambda^{q\prime}_{-l}}, \overrightarrow{\lambda^{h\prime}_{-l}})] \\
&\quad s.t. \\
\overrightarrow{\lambda^{q\prime}_{-l}} &= G^q(q_l, h; \overrightarrow{\lambda^q_{-l}}, \overrightarrow{\lambda^h_{-l}});\ \ \overrightarrow{\lambda^{h\prime}_{-l}} = G^h(q_l, h; \overrightarrow{\lambda^q_{-l}}, \overrightarrow{\lambda^h_{-l}})
\end{aligned}
$$

$$
\begin{aligned}
V^{si}\left(q_l, h; \overrightarrow{\lambda^q_{-si}}, \overrightarrow{\lambda^h_{-si}}\right) &= \max_{\lambda^h_{si}, \lambda^q_{si}} \bar{\pi}_s + (1+rdt)^{-1}[\lambda^q_l V^s(q_l\delta_q, h; \overrightarrow{\lambda^{q\prime}_{-si}}, \overrightarrow{\lambda^{h\prime}_{-si}}) + (\sum_{i'\neq i} \lambda^q_{si'}) V^{si}(q_l\delta_q, h; \overrightarrow{\lambda^{q\prime}_{-si}}, \overrightarrow{\lambda^{h\prime}_{-si}}) \\
&\quad +\lambda^q_{si} V^l(q_l\delta_q, h; \overrightarrow{\lambda^{q\prime}_{-si}}, \overrightarrow{\lambda^{h\prime}_{-si}}) + (\lambda^h_l + \sum_{i=1}^{m} \lambda^h_{si}) V^{si}(q_l, h\delta_h; \overrightarrow{\lambda^{q\prime}_{-si}}, \overrightarrow{\lambda^{h\prime}_{-si}}) + \\
&\quad (1 - (\lambda^q_l + \sum_{i=1}^{m} \lambda^q_{si}) - (\lambda^h_l + \sum_{i=1}^{m} \lambda^h_{si})) V^{si}(q_l, h; \overrightarrow{\lambda^{q\prime}_{-si}}, \overrightarrow{\lambda^{h\prime}_{-si}})] \\
&\quad s.t. \\
\overrightarrow{\lambda^{q\prime}_{-si}} &= G^q(q_l, h; \overrightarrow{\lambda^q_{-si}}, \overrightarrow{\lambda^h_{-si}});\ \ \overrightarrow{\lambda^{h\prime}_{-si}} = G^h(q_l, h; \overrightarrow{\lambda^q_{-si}}, \overrightarrow{\lambda^h_{-si}})
\end{aligned}
$$

The functional equations are self-explanatory. They simply capture the capital gains enjoyed and losses suffered by each type of firm when an innovation, R&D-driven or general, arrives. The only noteworthy element is that firms take as exogenous the hazard rates generated by the innovation activities of other firms.

2.2 Optimal Investments and Stationary Symmetric Equilibrium

Optimal Investments

Producers of standard intermediate goods have the option of introducing a good of higher quality. Optimal investment in developing this superior intermediate good induces followers to equalize the marginal cost of the R&D investment to its expected marginal benefit:

$$\overbrace{y}^{\text{Marginal Cost}} = \overbrace{\lambda_0^q (V^l(q\delta_q, h) - V^{si}(q,h))}^{\text{Expected Mg. Benefit from Embodied Innovations}}$$

Current leaders, in principle, can also increase the quality of their intermediate good. They face the same marginal cost as followers, but the expected marginal benefit is now equal to $\lambda_0^q(V^l(q\delta_q, h) - V^l(q,h))$. This implies that, if in equilibrium $V^l > V^{si}$, only followers will invest in increasing the quality of the leading intermediate good, as is the case in standard quality-ladder models.

Leaders have incentives to come out with general innovations that reduce the marginal cost of producing intermediate goods for all producers. In an interior solution, the optimal investment in general innovations by the leader results in the following equality:

$$\overbrace{\frac{y}{\rho_h}\left(\frac{\lambda^h + f_0^h}{\lambda_0^h}\right)^{\frac{1-\rho_h}{\rho_h}}}^{\text{Marginal Cost}} = \overbrace{\lambda_0^h (V^l\,(h\delta_h, q) - V^l\,(h,q))}^{\text{Expected Mg. Benefit from general Innovations}}$$

Followers, in principle, can also come out with general improvements in productivity. In equilibrium, however, since the private value of these innovations is proportional to the value of the firm, their incentive to undertake these innovations is lower. In what follows, we simplify the analysis by assuming that followers do not find it profitable to indulge in investments that lead to general innovations.[13]

Stationary Symmetric Equilibrium

The analysis presented here is restricted to the Stationary Symmetric Equilibrium (SSE) of this economy.[14] The SSE is characterized by (a) the optimality conditions derived thus far, (b) the equilibrium in the market for labor and (c) the fact that optimal investments in the development of innovations lead to constant hazard rates over time and over intermediate goods producers for any given category (i.e. leader vs. standard, R&D vs. general).

[13] This assumption is a result of the model if $\frac{y}{\rho_h}\left(\frac{f_0^h}{\lambda_0^h}\right)^{\frac{1-\rho_h}{\rho_h}} > \lambda_0^h(V^{si}(q, \delta_h h) - V^{si}(q,h))$. This is the case if f_0^h is sufficiently large or if λ_0^h is lower for followers than for the leader.

[14] The comparative statics results that we obtain below, however, also hold outside the steady state. In particular, it is easy to normalize the value functions so that they have no state variable. This means that the transition to the new steady state after a change in the parameters is instantaneous and that the steasy state comparative statics hold generally. We have proved this formally for the comparative statics results that we use in section 3 to interpret the evolution of US growth.

Labor market clearing implies that

$$L = L_l + L_s + L_l^x + \sum_{i=1}^{m} L_{si}^x \tag{7}$$

Using the demands of final output producers, the demands of the producers of leading and standard goods, and the linear technology in the production of intermediate goods, it is straightforward to solve for the allocation of labor and profit flows. The latter are given by the following expressions:

$$\begin{aligned}
\bar{\pi}_l &= y((1-\alpha)\alpha\beta - (\frac{\lambda^h + f_0^h}{\lambda_0^h})^{\frac{1}{\rho_h}}) \\
\bar{\pi}_s &= y(\frac{(1-\sigma)\alpha(1-\beta)}{m} - \frac{\lambda^q}{\lambda_0^q m})
\end{aligned}$$

Imposing symmetry on the investments of standard intermediate goods producers, using the result that only standard intermediate good producers invest in R&D and the assumption that only the leader undertakes general innovations, V^l and V^s are solved for in terms of the aggregate hazard rates for embodied and general innovations, λ^q and λ^h.

$$\begin{aligned}
V^s &= \frac{1}{\Omega_s}\left[\bar{\pi}^s + \frac{\lambda^q}{m}\frac{\triangle q \pi^l}{r + \lambda^q - \lambda^h(\triangle h - 1)}\right] \\
V^l &= \frac{1}{\Omega_l}\left[\bar{\pi}^l + \frac{\lambda^q \triangle q \pi^s}{r - \lambda^h(\triangle h - 1) - (\frac{m-1}{m}\triangle q - 1)\lambda^q}\right],
\end{aligned}$$

where

$$\begin{aligned}
\triangle q &\equiv \delta_q^{\frac{\beta - \psi_q}{1-\alpha}} \\
\triangle h &\equiv \delta_h^{\frac{\psi_h}{1-\alpha}} \\
\Omega_s &\equiv r - \lambda^h(\triangle h - 1) - (\frac{m-1}{m}\triangle q - 1)\lambda^q - \frac{(\lambda^q \triangle q)^2}{m(r + \lambda^q - \lambda^h(\triangle h - 1))} \\
\Omega_l &\equiv r - \lambda^h(\triangle h - 1) + \lambda^q - \frac{(\lambda^q \triangle q)^2}{m(r - (\frac{m-1}{m}\triangle q - 1)\lambda^q - \lambda^h(\triangle h - 1))}
\end{aligned}$$

The equilibrium conditions that determine λ^q and λ^h can be expressed as:

$$1 = \lambda_0^q \left(\triangle q v^l - v^s\right) \tag{8}$$

$$c'(\lambda^h) = \lambda_0^h(\triangle h - 1)v^l, \tag{9}$$

where $v^l \equiv V^l/y$, $v^s \equiv V^s/y$, and $c'(\lambda^h)$ is defined as $\frac{1}{\rho_h}\left(\frac{\lambda^h+f_0^h}{\lambda_0^h}\right)^{\frac{1-\rho_h}{\rho_h}}$.

$\boldsymbol{\sigma} = 1$ case

For purposes of simplicity, we focus our analysis on the special case where the standard intermediate goods are not differentiated (i.e. $\sigma = 1$). This simplification preserves most of the interesting insights of the model. The only new insight that arises when $\sigma < 1$ is discussed in footnote 15.

If $\sigma = 1$, the value of standard intermediate good firms, v^s, is zero. This follows from the fact that when standard intermediate goods are not differentiated, the producers make zero operating profits i.e. they incur loses equal to the cost of undertaking R&D (i.e. $\pi_{si} = -\frac{\lambda^q}{\lambda_0^q m}$). In equilibrium, these static loses precisely compensate the expected capital gains from becoming market leaders, making the net value of a standard intermediate good producer zero.

To see this formally, note that the equilibrium condition that determines the R&D intensity can be rewritten as:

$$0 = \frac{\lambda^q}{m}\left(\frac{-1}{\lambda_0^q} + \left(\triangle q v^l - v^s\right)\right) \tag{10}$$

Using the expression for π_{si} and taking advantage of the linearity of the R&D technology, expression (10) can be rewritten as:

$$0 = \pi_{si} + \frac{\lambda^q}{m}\left(\triangle q v^l - v^s\right)$$

In the symmetric equilibrium, the value of a standard intermediate good producer can be expressed as:

$$rv^s = \pi_{si} + \frac{\lambda^q}{m}(\triangle q v^l - v^s) + \lambda^q(\frac{m-1}{m})(\triangle q - 1)v^s + \lambda^h(\triangle h - 1)v^s. \tag{11}$$

Plugging expression (10) into (11) yields:

$$rv^s = \lambda^q(\frac{m-1}{m})(\triangle q - 1)v^s + \lambda^h(\triangle h - 1)v^s.$$

But note that $v^s = 0$ is the only solution to this equation.

Similarly, it is straightforward to show that when $\sigma = 1$, v^l is given by the following expression:

$$v^l = \frac{((1-\alpha)\alpha\beta - (\frac{\lambda^h+f_0^h}{\lambda_0^h})^{\frac{1}{\rho_h}})}{r + \lambda^q - \lambda^h(\triangle h - 1)} \tag{12}$$

The SSE is characterized by equation (12) and by conditions (Lq) and (Lh) that ensure the optimal investment rates in R&D-driven and general innovations:

$$1 = \lambda_0^q \triangle q v^l \tag{Lq}$$

$$c'(\lambda^h) = \lambda_0^h(\triangle h - 1)v^l. \tag{Lh}$$

Isolating v^l from condition (Lq), it follows that, in the SSE, $v^l = (\lambda_0^q \triangle q)^{-1}$. Plugging this back into condition (Lh), it follows that:

$$c'(\lambda^h) = \frac{\lambda_0^h(\triangle h - 1)}{\lambda_0^q \triangle q}.$$

Plugging in the functional form specified for $c(.)$, we obtain the following expression for the rate of arrival of general innovations:

$$\lambda^h = \lambda_0^h \left(\frac{\rho^h \lambda_0^h(\triangle h - 1)}{\lambda_0^q \triangle q} \right)^{\frac{\rho^h}{1-\rho^h}} - f_0^h \tag{13}$$

Proposition 1 characterizes the effects of the various parameters on the arrival rate of general innovations (λ^h).

Proposition 1 *If σ is equal to 1, λ^h increases with λ_0^h, ρ^h and $\triangle h$, declines with λ_0^q, $\triangle q$ and f_0^h and is unaffected by α, β and r.*

Investments in general innovations are optimal when the leading firm equalizes the marginal cost and the marginal expected benefit from them. The rate of general innovations in the SSE is increasing in λ_0^h and ρ^h since both these parameters reduce the marginal cost of developing general innovations. Similarly, λ^h declines with the fixed cost of undertaking general innovations, f_0^h, since, *ceteris paribus*, it raises the marginal cost of general innovations. λ^h also increases with $\triangle h$ since it increases the marginal benefit from these innovations.

The rest of the comparative static exercises described in proposition 1 operate through the value of leading firms, v^l. In the SSE, v^l is equal to $(\lambda_0^q \triangle q)^{-1}$. This is the case since, in equilibrium, standard intermediate goods firms undertake R&D investments until the expected marginal cost of developing an embodied innovation ($1/\lambda_0^q$) equals the capital gain experienced when developing a superior leading product ($\triangle q v^l$). As a result, the rate of arrival of R&D-driven innovations (λ^q) adjusts until v^l equals $(\lambda_0^q \triangle q)^{-1}$. This is why changes in the demand elasticity (α), the market share of leading intermediate goods (β) or the discount rate (r) have no effect on v^l or on λ^h.[15]

Changes in the efficiency of R&D investments (λ_0^q) or on the quality of improvements associated with them ($\triangle q$), however, do have an effect on λ^h. Increases in λ_0^q or $\triangle q$ make the development of

[15] This result contrasts with standard models of R&D, where there is only one form of innovation. In these models, it is typically the case that when the market size increases, more resources are allocated in equilibrium to the innovation activity.

an embodied innovation more attractive and lead to increases in the rate of arrival of R&D-driven innovations. This mechanism reduces the value of leading firms and hence the marginal value of general innovations for leading firms. As a result, λ^h is decreasing in λ_0^q and $\triangle q$ in the SSE.

Plugging back the SSE expression for λ^h (13) into equation (Lq) allows us to solve for the rate of arrival of embodied innovations in the SSE:

$$\lambda^q = \lambda_0^q \triangle q(1-\alpha)\alpha\beta + (1-\rho^h)\left(\lambda_0^h(\triangle h - 1)\left(\frac{\rho^h}{\lambda_0^q \triangle q}\right)^{\rho^h}\right)^{\frac{1}{1-\rho^h}} - r - f_0^h(\triangle h - 1).$$

Proposition 2 characterizes the effects of the various parameters on λ^q.

Proposition 2 *If σ is equal to 1, λ^q increases with λ_0^q, $\triangle q$, λ_0^h and β,declines with r, f_0^h and α for $\alpha > 1/2$. The effects of ρ^h and $\triangle h$ on λ^q are ambiguous.*

The comparative statics exercises undertaken here are also intuitive. r, f_0^h and α - when α is greater than 1/2 - reduce λ^q since they reduce v^l and hence the capital gain enjoyed by a standard intermediate good producer who succeeds in developing the new leading intermediate good. r operates through the effective discount rate while f_0^h and α reduce the profit flows of the leading firm.

β and λ_0^h have a positive effect on the rate of arrival of R&D-driven innovations since they raise v^l. The leader's market share, β, increases the leader's profits, while the efficiency of development of general innovations, λ_0^h, reduces the cost of sustaining a given development intensity.

The effects of λ_0^q and $\triangle q$ on λ^q are similar. Both higher λ_0^q and $\triangle q$ raise the expected capital gain from investing in R&D innovations, for a given v^l. Since the marginal cost is constant, the value of the leader must decline to equalize the marginal benefit and the marginal cost of these innovations in equilibrium. The higher arrival rate of embodied innovations, λ^q brings about the decline in v^l.[16]

In general, the effects of ρ^h and $\triangle h$ on λ^q are ambiguous. While the conditions that determine the sign of these effects are straightforward to derive, they are not particularly enlightening for purposes of this discussion.

Combining propositions 1 and 2, it follows that, if $\sigma = 1$, increases in λ_0^q or $\triangle q$ lead to increases in λ^q and declines in λ^h. In other words, λ_0^q and $\triangle q$ cause the rate of R&D-driven and general

[16] In principle, one might think that v^l could also adjust by varying λ^h. However, because of the envelope theorem, $\partial v^l / \partial \lambda^h = 0$ at the optimum.

innovations to move in opposite directions. Clearly, since all the functions are continuous, this result also holds when standard intermediate goods are not too differentiated (i.e. σ is in the neighborhood of 1).[17] This negative co-movement between R&D-driven and general innovations is the key theoretical result to understand the post-war dynamics of growth and volatility at the aggregate and firm level.

2.3 Moments

Now that we have solved for the SSE of the economy, we can express the statistics in terms of the hazard rates, λ^q and λ^h. The expected growth rate of both aggregate output ($E\gamma_y$) and labor productivity ($E\gamma_{y/L}$) is equal to

$$E\gamma_y = E\gamma_{y/L} = \ln(\triangle q)\lambda^q + \ln(\triangle h)\lambda^h.$$

Growth is the result of both embodied and general innovations. But, we have just established a negative co-movement between R&D and general innovations in response to changes in the efficiency of R&D (λ_0^q). Therefore, the effect of an increase in λ_0^q on expected growth is ambiguous.[18] In particular, an increase in λ_0^q reduces expected productivity growth if and only if the relative productivity gain associated with an embodied innovation is relatively small: $\ln(\triangle q)/\ln(\triangle h) \leq -\frac{\partial\lambda^h}{\partial\lambda_0^q}/\frac{\partial\lambda^q}{\partial\lambda_0^q}$.

Equation (14) provides the formula for the variance of the aggregate growth rate of the economy. Since R&D-driven and general innovations follow independent Poisson processes, the variance of the growth rate of aggregate output (and productivity) is linear in the hazard rates, where the coefficients of λ^q and λ^h are the squared contribution to productivity growth from each type of

[17] Moving away from the $\sigma = 1$ case yields one additional insight. When σ is smaller than 1, the market value of a follower (V^s) is positive and varies with λ^q, λ^h and β. In this case, an increase in the leading good's market share, β, not only increases v^l but may also reduce v^s. Therefore, an increase in β widens the gap between the values of the leader and a follower more than when $\sigma = 1$, enhancing the response of λ^q to an increase in the leader's market share. This larger response may break the insensitivity of λ^h to changes in β that we observe when $\sigma = 1$. In particular, when $\triangle q$ is close to 1, the positive direct effect of β on v^l is more than compensated by the indirect effect of β on v^l through the increase in λ^q. As a result, an increase in β leads to an increase in λ^q and to a decline in λ^h. This prediction gives us an additional channel to understand the observed dynamics of growth and volatility that surely reinforces the interpretation provided in the main text.

[18] In what follows, we focus on the comparative statics of an increase in λ_0^q. However, all the results hold for some parameter range if the driving force is an increase in the leader's market share, β.

innovation.

$$var(\gamma_y) = var(\gamma_{y/l}) = (\ln(\triangle q))^2 \lambda^q + (\ln(\triangle h))^2 \lambda^h \tag{14}$$

The effect of λ_0^q on the variance of aggregate productivity growth in the one sector version of the model is also ambiguous. In particular, aggregate volatility declines with an increase in λ_0^q if and only if $(\ln(\triangle q)/\ln(\triangle h))^2 \leq -\frac{\partial\lambda^h}{\partial\lambda_0^q}/\frac{\partial\lambda^q}{\partial\lambda_0^q}$.[19]

At the firm level, general innovations symmetrically affect the expected growth rate of sales for all firms. As a result, general innovations affect the expected growth rate of firm sales - denoted by $E\gamma_{salesi}-$ only through the effect they have on $E\gamma_y$.

$$E\gamma_{salesi} = \begin{cases} E\gamma_y - \lambda^q \ln(\beta m/((1-\beta))) \text{ for } i = l \\ E\gamma_y + \lambda^q/m \ln(\beta m/((1-\beta))) \text{ for } i = s \end{cases}$$

Embodied innovations also affect the expected growth rate of sales through the effect on $E\gamma_y$. However, the arrival of an innovation embodied in a new intermediate good generates market turnover leading to a reduction in the sales of the leaders as well as an expected increase in the sales of the followers. This results in the prediction that at the firm level, there is a positive relationship between R&D intensity and expected growth.

These same considerations help us understand the determinants of the expected growth rate of sales per worker. Here, market turnover affects the firm' sales per worker since market leaders charge higher markups than producers of standard intermediate goods. The possibility of a change in the market position creates an expected gain (loss) in the sales per worker for standard (leading) intermediate goods producers, as is clear in the next expression:

$$E\gamma_{salesi/L_i} = \begin{cases} \gamma_y - \lambda^q \ln(\sigma/\alpha) \text{ for } i = l \\ \gamma_y + \lambda^q/m \ln(\sigma/\alpha) \text{ for } i = s \end{cases}.$$

The firm-level volatility of the growth rates of sales and sales per worker depends on the variance of the aggregate growth rate of the economy and on the risk of turnover in the market leader. Expressions (15) and (16) present the average variances of the growth rate of sales and sales per worker.

[19] Note that, if the productivity gain from an embodied innovation is smaller than the gain from a GPT-style innovation (i.e. $\ln(\triangle q)/\ln(\triangle h) < 1$), the fact that the expected growth rate of the economy does not increase following an increase in λ_0^q is sufficient to generate a decline in aggregate volatility.

$$
\begin{aligned}
var(\gamma_{sales_i}) &= var(\gamma_y) + \lambda^q \left(\frac{1+\beta(m-1)}{m}\right)\left(\ln(\frac{\beta m}{(1-\beta)})\right)^2 &(15)\\
var(\gamma_{sales_i/L})) &= var(\gamma_y) + \lambda^q \left(\frac{1+\beta(m-1)}{m}\right)(\ln(\sigma/\alpha))^2 &(16)
\end{aligned}
$$

Since the variance of aggregate output in the US data is approximately two orders of magnitude smaller than the variance of firm-level volatility, quantitatively, the important contribution comes from the turnover rate measured by λ^q. An increase in λ_0^q increases investments in R&D-driven productivity that raise the turnover rate (λ^q) and firm-level volatility.[20]

2.4 Multisector Economy

The analysis thus far has been conducted on a one-sector economy. This simplification is especially restrictive because of the difference in the applicability of R&D-driven and general innovations. In this section,[21] we extend the model to a multisector economy to explore the role of the covariance of sectoral growth in aggregate variance. As we shall see, this channel eliminates the ambiguity in the evolution of aggregate volatility in response to an increase in λ_0^q.

Consider a multisector economy where aggregate output (y) is produced by combining N sectoral outputs (y_i) according to the following technology:

$$
y = \prod_{n=1}^{N} y_n^{\frac{1}{N}}.
$$

In this new context, the model economy developed thus far can be interpreted as corresponding to a sector. In particular, sectoral output (y_n) is produced combining the leading and standard sectoral goods according to technology (2). Each of these, in turn, is produced by combining labor and the corresponding (sector-specific) intermediate goods as specified in the production functions (3) and (4). The leading firm in sector n faces a probability λ_n^h of developing a general innovation.

[20] When the increase is in β rather than in λ_0^q, there are two additional effects on firm-level volatility. The first of these effects is due to the weighting scheme and leads to a positive co-movement between β and firm-level volatility. The second effect applies only to the firm-level volatility of sales. Specifically, an increase in β enhances the stake in the competition for leadership and therefore increases the variance of the firm-level growth rate of sales. Since this force does not operate in the case of the growth rate of sales per worker, an increase in β should result in a larger increase in the firm-level volatility of the growth rate of sales than of the growth rate of sales per worker. This prediction is borne by the data.

[21] A more detailed description of this multisector model is conducted in Appendix 1.

Followers in the sector produce an embodied innovation that leads to a change in sectoral leadership with probability λ_n^q. For the time being, we capture the wider applicability of general innovations by assuming that general innovations developed in one sector can be freely used by the producers of all the sectors to reduce their marginal cost of production. R&D-driven innovations, though, are sector specific.

Let $(\lambda_s^q, \lambda_s^h)$ denote the stationary symmetric equilibrium levels of $(\lambda_n^q, \lambda_n^h)$. The conditions that determine the levels of $(\lambda_s^q, \lambda_s^h)$ are still given by expressions (8) and (9). The only difference is that now the value of the firms that produce different intermediate goods must be modified to reflect the increase in aggregate demand on account of embodied innovations in other sectors. It is easy, though, to characterize situations where this effect is not sufficient to dominate the negative effect that a higher turnover (i.e. λ^q) has on the value of firms producing the leading intermediate goods.[22] This is the main prediction that is borrowed from the previous analysis for the derivations that follow.

For any given sector n, the growth rate of the sector's output (or productivity) (γ_{y_s}) is:

$$\gamma_{y_s} = \gamma_{y/l_s} = \#q_n^s * \triangle q + \#h * \triangle h,$$

where $\#q_n^s$ is the number of new embodied innovations developed in the sector during the period, and $\#h$ is the number of new general innovations developed in the economy.

The growth rate of the economy (γ_y) is simply the average of the sectoral growth rates:

$$\gamma_y = \gamma_{y/l} = \frac{\sum_{n=1}^N \gamma_{y_n}}{N} = \frac{\sum_{n=1}^N \#q_n^s}{N} * \triangle q + \#h * \triangle h.$$

Since new technologies arrive with a Poisson rate, it is straightforward to compute the instantaneous expectation and variance of the growth rate of productivity at the sector and aggregate level:

$$\begin{aligned}
E\gamma_{y_s} &= \lambda_s^q \ln(\triangle q) + \lambda^h \ln(\triangle h) \\
E\gamma_y &= \lambda_s^q \ln(\triangle q) + \lambda^h \ln(\triangle h) \\
V\gamma_{y_s} &= \lambda_s^q (\ln(\triangle q))^2 + \lambda^h (\ln(\triangle h))^2 \\
V\gamma_y &= \frac{\lambda_s^q}{N} (\ln(\triangle q))^2 + \lambda^h (\ln(\triangle h))^2
\end{aligned}$$

[22] In this multisector model, $v^l = ((1-\alpha)\alpha\beta - (\frac{\lambda^h + f_0^h}{\lambda_0^h})^{\frac{1}{\rho_h}})/(r + \lambda^q(1 - (\triangle q - 1)(N-1))/N - \lambda^h(\triangle h - 1))$. λ^h decreases unambiguously with λ_0^q. λ_s^q increases with λ_0^q if the following condition holds: $\frac{\left[(1-\alpha)\alpha\beta - \left(\frac{\rho_h \lambda_0^h (\triangle h - 1)}{\lambda_0^q \triangle q}\right)^{1/(1-\rho_h)} \left(\frac{N-\rho_h}{1-\rho_h}\right)\right]}{(1-(\triangle q - 1)(N-1))} > 0.$

where $\lambda^h = N * \lambda^h_s$.

These expressions show that aggregation does not have any effect on the expected growth rate of productivity since the same rates of arrival are expected at the aggregate and sector levels. However, this is no longer the case for the second moments. R&D-driven innovations are sector specific and are averaged away at the aggregate level. Hence, R&D-driven innovations have a larger effect on the volatility of the sectoral growth rate than on the volatility of the aggregate growth. General innovations, on the other hand, are adopted across the economy. Thus, their impacts are the same at the aggregate and sectoral level.

Another way to illustrate this point is by undertaking a variance-covariance decomposition of the variance of aggregate growth. Recall that $\gamma_y \equiv \frac{\sum_{n=1}^N \gamma_{y_n}}{N}$. Hence,

$$\begin{aligned} V(\gamma_y) &= E\left(\frac{\sum_{n=1}^N \gamma_{y_n}}{N} - E\gamma_y\right)^2 \\ &= E\left(\frac{\sum_{n=1}^N \gamma_{y_n} - E\gamma_{y_s}}{N}\right)\left(\frac{\sum_{n'=1}^N \gamma_{yn'} - E\gamma_{y_s}}{N}\right) \\ &= \frac{V(\gamma_{y_s})}{N} + E\frac{\sum_{n=1}^N \sum_{n'\neq n}\left(\gamma_{y_n} - E\gamma_{y_s}\right)\left(\gamma_{yn'} - E\gamma_{ys}\right)}{N^2} \\ &= \frac{V(\gamma_{y_s})}{N} + \frac{N(N-1)}{N^2} Cov(\gamma_{y_n}, \gamma_{y_{n'}}), \end{aligned} \tag{17}$$

where $cov(\gamma_{y_n}, \gamma_{y_{n'}})$ denotes the covariance between the growth rates of two generic sectors n and n'.

In expression (17), as the number of sectors (N) increases, the importance of the sectoral variance in the aggregate variance declines and aggregate volatility depends more on the covariance of growth across sectors. Sectoral variance $V(\gamma_{y_s})$ depends on the arrival rate of embodied innovations developed in the sector (λ^q_s) and on the arrival of general innovations developed in the economy (λ^h). The sectoral covariance, instead, is equal to $(\ln(\triangle h))^2\ \lambda^h$ and depends solely on the hazard rate for general innovations. Therefore, as the number of sectors increases, the variance of aggregate growth increasingly depends of the intensity of general innovations while the arrival rate of R&D-driven innovations becomes less relevant.

This observation has important implications for the comparative statics associated with λ^q_0. Propositions 1 and 2 showed that an increase in λ^q_0 leads to an increase in λ^q_s and a decline in λ^h. Since sectoral volatility depends positively on both of these, λ^q_0 has an ambiguous effects on the variance of sectoral growth. However, since the sectoral covariance depends only on the frequency of arrival of general innovations, an increase in λ^q_0 unambiguously leads to a decline in the covariance of sectoral growth. Furthermore, since the covariance component dominates sectoral variance in economies with a relatively large number of sectors, the variance of aggregate growth declines unambiguously when λ_0^q increases.

The covariance of sectoral growth can trivially be decomposed in to the product of standard deviations and correlation:

$$cov(\gamma_{y_s}, \gamma_{y_s'}) = \sqrt{V(\gamma_{y_s})V(\gamma_{y_{s'}})} * corr(\gamma_{y_s}, \gamma_{y_{s'}})$$

Since when looking at actual data, the variance of growth in a sector typically depends on other factors such as the sector size and age, it is useful to examine the implications of the model for the correlation of growth across sectors.

$$corr(\gamma_{y_s}, \gamma_{y_{s'}}) = \frac{(\triangle h)^2 \lambda^h}{(\triangle q)^2 \lambda_s^q + (\triangle h)^2 \lambda^h}. \tag{18}$$

Expression (18) corresponds to the correlation between the growth rates in any two sectors. The correlation of sectoral growth is decreasing in the rate of R&D innovations and increasing in the rate of general innovations.

2.4.1 Imperfect diffusion of general innovations

We have just shown the importance of the covariance of growth across sectors for understanding the evolution of aggregate volatility. It is also illuminating to explore the model' predictions for the cross-section variation in the covariance of sectoral growth. In this section, we do that by relaxing the assumption that general innovations are applicable to all the sectors in the economy. Specifically, we replace that by two assumptions: (i) the intermediate good producers of a given sector can freely adopt all the general innovations developed in its sector and (ii) the random variable, that determines whether a general innovation is suitable to be adopted in a sector other than the one where it was developed, follows a Bernoulli distribution that is independent across sectors and innovations.

Let ξ denote the probability that a general innovation is adopted in a sector other than the one where it was developed. The previous assumptions imply that the arrival rate of general innovations in sector n is equal to $\lambda_{sn}^h + \xi(N-1)\overline{\lambda}_{s(-n)}^h$, where λ_{sn}^h denotes the rate of development of general innovations in sector n and $\overline{\lambda}_{s(-n)}^h$ denotes the average rate of development of general innovations in the sectors other than n. The covariance between the growth in two different sectors depends on how frequently they adopt the same general innovations. Clearly, the probability of such a coincidence is higher for the technologies developed in either of the sector. For the technologies developed in other sectors, the probability that a given innovation is suitable for adoption in two random sectors is ξ^2. Taking these considerations into account, it is easy to see that the covariance between the growth in sectors n and n' is:

$$cov(\gamma_{y_n}, \gamma_{y_{n'}}) = \xi(\lambda_n^h + \lambda_{n'}^h) + \xi^2(N-2)\overline{\lambda}_{-(n,n')}^h,$$

where $\overline{\lambda}_{-(n,n')}^h$ denotes the average rate of development of general innovations in the sectors other than n and n'. Averaging over all the sectors n', the average covariance of the growth of sector n with the growth rate in the other sectors is

$$cov_n = \xi(\lambda_n^h + \overline{\lambda}_{(-n)}^h) + \xi^2(N-2)\overline{\lambda}_{(-n)}^h.^{23} \tag{19}$$

To explore the cross-section variation in this covariance, suppose that the efficiency of the investments in the development of embodied innovations (λ_0^q) varies across sectors. From proposition 1, we know that in sectors with higher values of λ_0^q, leading firms will have fewer incentives to develop general innovations. As a result, we should observe a negative correlation between λ_0^q and λ_n^h in the cross-section. Moreover, from expression (19), there should also be a negative cross-sectoral relationship between λ_0^q and the average covariance of a sector.

2.5 Predictions

In summary, the following are the predictions arising from the model that can be tested using available data:

- Expected growth:
  - (E1) At the firm level, there is a positive relationship between firms' R&D expenses and productivity growth.
  - (E2) At the sector and aggregate level, the relationship between R&D expenses and productivity growth is ambiguous.
- Turnover (T): There is a positive relationship between the turnover rate and the R&D intensity in a sector.

[23] As in the previous subsection, it is useful to have in mind that the comparative statics for the correlation of sectoral growth are qualitatively identical to those for the covariance.

- Volatility:
  - (V1) R&D has a positive effect on average firm-level volatility of the growth rate of sales and sales per worker.
  - (V2) At the sector level, the relationship between average R&D intensity and volatility is ambiguous.
  - (V3) At the aggregate level, an increase in λ_0^q leads to a decline in the volatility of the growth rate of output, labor and labor productivity.
- Co-movement:
  - (C1) An increase in λ_0^q leads to a decline in the covariance and correlation between the growth rate of productivity across sectors.
  - (C2) Since λ_0^q has a positive effect on firm-level volatility (and on R&D), the model predicts a negative relationship between the average firm-level volatility in a sector (or the R&D to sales ratio in a sector) and the correlation between the sector's growth and the growth rate in the other sectors of the economy.

3 Evidence and Discussion

This section addresses three questions. Is there any evidence that λ_0^q has increased in the US during the postwar period? Is there any indication that the evolution of growth, volatility and co-movement of productivity growth at the aggregate and firm level are associated with the mechanisms described in the model? Finally, in a quantitative sense, are the mechanisms presented above able to generate the dynamics of volatility and growth observed in the US economy?

3.1 Driving forces

As we shall see below, market turnover has increased significantly since the 1950's. This upward trend may be due in part to various exogenous factors that lead to an increase in λ_0^q, the probability of developing an R&D innovation per unit of output invested in R&D by the private sector.

One such exogenous factor that increases λ_0^q is R&D subsidies: Over the past two decades, tax credits for R&D have become widespread and increasingly generous. Currently, over half (29) of all

US states offer an R&D tax credit.[24] Secondly, though harder to quantify, the fact that the US has become an "information society" is also a likely source of the increase in λ_0^q : it is now easier for potential innovators to obtain access to frontier knowledge and therefore more likely that they will develop a product or service that dominates those produced by the current market leaders. Finally, oftentimes workers learn to produce a product or deliver a service to the point that they find it advantageous to create their own company and compete with the market leader. These business dynamics require that workers obtain a holistic understanding of the process of production. The gradual disappearance of Taylorism and workers with better analytic abilities due to the spread of college education have meant that workers now acquire such a holistic understanding. As a result, these business dynamics have probably become a more important source of turnover.[25]

Figure 3 plots a measure of the inverse of the turnover rate for the sample of firms in the COMPUSTAT data base. Specifically, for each 2 digit sector and year, firms are ranked by the level of sales per worker. After creating a vector of percentiles for every year in the postwar period, persistence in rankings is measured by computing the correlation between the vectors of rankings in two years, five and ten years apart (i.e. 1950 with 1955, and 1950 with 1960, respectively). Repeating the same exercise for all the years in the postwar period results in a time series for the turnover in market leadership.[26]

Both of these statistics indicate that there has been an increase in market turnover. In the early 50's, the correlation of rankings was 0.9 for the 5-year-apart measure and 0.8 for the 10-year-apart measure. These correlations have declined in a fairly monotonic manner reaching 0.71 and 0.66, respectively, at the end of the sample in 2002. Comin and Philippon [2005] document similar trends for rankings based on market capitalization and profit rate.

[24] There is a literature that has shown that these R&D tax credits have lead to a substantial increase in the share of private R&D in GDP (for example, Hall [1993], Mamuneas and Nadiri [1996], or Bloom, Griffith, and Van Reenen [2002]).

[25] One example that illustrates this view is Mountain Hardwear, an outdoors gear company founded in 1993 by workers that left North Face and Sierra Designs. They justify their success as follows: "we decided to take a fresh approach to making great gear. Figuring that if we made innovative, technologically advanced tents, outdoor clothing and sleeping bags, consumers would buy them. We were right. [...] But it wasn't just about making great gear. From all those years of working in the outdoor industry, we knew what we liked about the business, and we also knew what we wanted to change."

[26] This measure of turnover is unlikely to be affected by entry into the COMPUSTAT sample. This is the case because when there are more firms in sample, it is more likely that a firm is taken over by some other firm, but the decline in the percentile associated with this decline in the ranking will be smaller if there were fewer firms in sample.

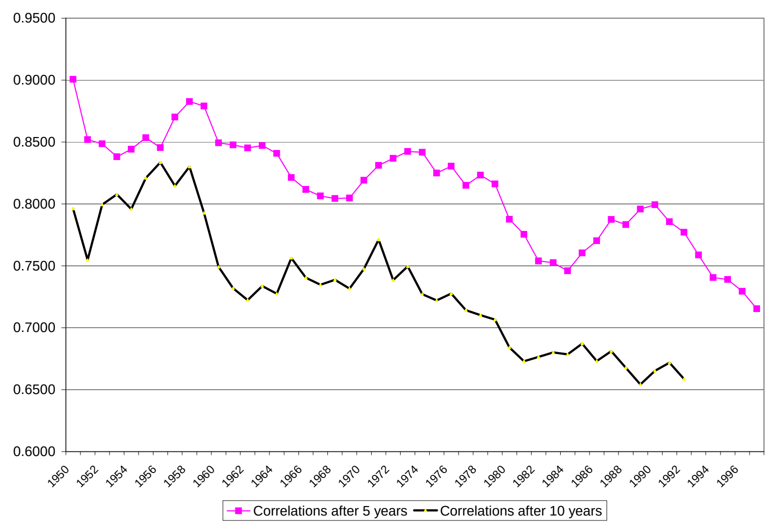

Figure 3: Correlations of firm percentiles by sales per worker.

This increase in turnover, however, is not only the result of exogenous factors but also the result of endogenous choices made by firms. An important path to market leadership is technological superiority.[27] Baumol [2002] emphasizes the role of research and development of superior goods as a competitive mechanism far more important than competition in prices. Figure 1 has showed that one measure of these efforts in developing superior products (the NSF measures of non-federally financed R&D over GDP) has almost tripled since the early 50's. Similarly, the firms in other G-7 countries were conducting much less R&D than the US in 1970 but had matched R&D intensity in the US by the turn of the century.

3.2 Evaluation of model predictions

The model's predictions are consistent with the facts described in the introduction. It predicts the lack of a clear relationship between R&D intensity and productivity growth at the aggregate level, the upward trend in firm-level volatility and the downward trend in aggregate volatility. Next, we provide evidence of other trends predicted by the model and that these facts are driven by the mechanisms of the model.

Expected growth

[27] R&D however is by no means the only investment that leads firms to leadership. In the case of Mountain Hardware, the founders had acquired product-specific knowledge from working for the previous market leaders, North Face and Sierra Designs, that gave them a competitive edge.

Firms that invest more in R&D are more likely to experience the improvements in productivity associated to becoming the new market leader. This is why the model predicts a positive relationship between R&D spending and expected growth in sales per worker at the firm level (prediction E1). Griliches [1980, 1986] and Griliches and Mairesse [1984] have examined panels of firm-level data covering the post-war period and observed that there is a strong, significant relationship between R&D intensity and productivity or TFP growth at the firm level, even after including firm-level fixed effects.

As we move to the sector or aggregate levels, the model's predictions about the relationship between R&D and productivity growth become ambiguous (E2). This ambiguity follows from the negative relationship that exists between R&D and the development of general innovations. If, in addition, general innovations do not diffuse perfectly across sectors, sectors that develop fewer general innovations are also the sectors that implement fewer innovations. As a result, the model predicts that in sectors with more R&D investments, the contribution to growth from general innovations will be lower and the resulting relationship between R&D and growth will be ambiguous.

There are many studies that have estimated the relationship between R&D intensity and TFP growth using sector level data.[28] These studies, typically find a significant positive relationship when examining the cross-section. However, once sector level fixed effects are introduced as regressors, the coefficient of the R&D intensity becomes insignificant (Jones and Williams [1998]). This may be the case for two reasons. First, noise in the data may make it difficult to identify the relationship between R&D and TFP growth in the time series.[29] Second, it may be the case that after allowing for a sector-specific average growth rate, R&D has no effect on sectoral TFP growth. In any case, the fact that it is possible to identify the effect of R&D on TFP growth in the time dimension using firm-level data but not using sector-level data seems to indicate that the relationship becomes less clear as we move to more aggregate levels. As discussed above, one reason for the disappearance of this relationship may be the negative effect that R&D has on the development of general technologies in the sector.[30]

[28] See Jones and Williams [1998] for references.

[29] However, this attenuation bias should be stronger when using firm-level data which is probably noisier than sector-level data. Yet, Griliches and Mairesse [1984] and others have no problem identifying a strong and significant effect of R&D on productivity growth at the firm level.

[30] Abdih and Joutz [2005] reach similar conclusions using a different methodology. They estimate cointegration relationships between R&D labor, patent applications (i.e. the R&D output), the stock of patents and TFP. Theis results provide evidence of the ambiguous effect of R&D on TFP because they show that, though there is a very strong positive relationship between R&D labor and patent applications, there is no statistical relationship between

This econometric exercises provide indirect evidence on the joint hypothesis that general innovations are an important source of productivity growth and that R&D dampens the development of general innovations. We are not the first ones to highlight the importance of general innovations. Mokyr [2002] has emphasized the role of some general innovations for growth. In particular, he claims that "much of the productivity increase in the twentieth century was the result of the perfection of production techniques and process innovation. [...] These led to a continuous transformation in organizational methods, most obviously in mass production in manufacturing techniques but eventually in services and agriculture as welll"

Unfortunately, direct measures on the intensity of investment in general innovations are not available. This makes it difficult to directly test the negative effect of R&D on the development of general innovations. A very distant substitute is to create a (very incomplete) list of general innovations and show that most of them were conducted either before WWII or between the 50's and early 60's when firm turnover was low. Table 1 contains our list of general innovations most of which were developed before 1970. A detailed description of each technology and why they qualify as general is relegated to appendix 2.

Turnover, Volatility and R&D

The model predicts that an increase in λ_0^q leads to more R&D. Furthermore, the increase in R&D leads to higher turnover and firm volatility (predictions T and V1). The process of developing and marketing innovations until they take over the current market leader requires time. [31] As a result, the interaction between R&D and turnover/volatility should generate an interesting lead-lag relationship. In particular, past R&D should lead to current volatility, and current volatility should lead to subsequent R&D. To investigate this relationship, we build a panel of annual R&D intensities, turnover rates and average firm volatility in 35 2-digit sectors that cover the US economy between 1950 and 1996. For each sector, we compute the ratio of R&D expenses to total sales, the median standard deviation of a 10 year rolling window of sales per worker growth and the persistence in the rankings of sales per worker as in figure 3. Then, we estimate the regressions:

$$\begin{aligned}
\lambda_{it}^q &= \alpha_{0i} + \alpha_1 t + \beta(j) * (R\&D/Sales)_{it-j} + \epsilon_{it} \\
\sigma_{it} &= \alpha_{0i} + \alpha_1 t + \beta(j) * (R\&D/Sales)_{it-j} + \epsilon_{it}
\end{aligned}$$

In this specification, we introduce both a sector-level fixed effect and a time trend, to reduce

patents and TFP.

[31] Shakerman and Pakes [1994] suggests developing innovations and marketing them can take between 2 and 4 years.

the possibility of spurious correlations between R&D and volatility. Figures 4a and 5a report the estimates of $\beta(j)$ for various lags (j) of R&D. Figures 4b and 5b report the associated $p-values$ of these estimates (on an inverse scale) after computing Newey-West standard errors. The lead-lag relationship between R&D and volatility/turnover is evident from these figures. As we suspected, current volatility has a significant impact on future R&D that peaks at approximately $t+3$. In addition, there is an evident effect of past R&D on current volatility/turnover that peaks at $t-5/t-4$. This effect is always positive, statistically significant and at least as large as the contemporaneous correlation between R&D and firm volatility. Also noteworthy is the symmetrical nature of the lead-lag relationships between R&D and volatility and between R&D and turnover.

These estimates are checked for robustness using two variations. First, the time trend is replaced by year dummies. In addition, other measures of volatility such as mean sales growth, median sales growth or mean sales per worker growth, are used. The use of these variations continues to result in the same bimodal cross-correlogram between R&D and firm-volatility.

Of course, there is always the possibility that the estimated relationship between R&D and volatility is driven by some third variable omitted from the regression. However, we consider it unlikely that the cross-correlogram between R&D and volatility is the result of omitted variable bias. For this to be the case, the omitted variable should be significantly correlated with current volatility, with R&D at t-4, t-5, t and the leads of R&D from t+1 to t+4 but not correlated with R&D at t-3, t-2 and t-1. Furthermore, all of these correlations should have the same sign. Our model, instead, satisfies all these restrictions very naturally.

As we move to higher levels of aggregation, the trend in volatility as well as its relationship with R&D changes. At the sector level, the model predicts an ambiguous relationship between average R&D intensity and volatility (V2). In the model, aggregate volatility depends by-and-large on the arrival rate of general innovations. As turnover increases, the market leaders' private value of developing general innovations declines and so does aggregate volatility (prediction V3). This is how the model accounts for the observed downward trend in aggregate volatility.

Co-movement

Since general innovations are widely applicable in the economy, a decline in the intensity of their development should lead to a decline in the correlation of growth across sector (prediction C1).

To explore the evolution of the correlation of growth across sectors, we proceed as follows. First, $corr([\gamma_{s,\tau}]_{t-4}^{t+5}, [\gamma_{j,\tau}]_{t-4}^{t+5})$ is defined as the correlation between the annual growth rate in sectors s and j during the 10 year period centered at t. Then, for every sector s, the average correlation with the rest of the sectors is computed as follows:

$$corr^{\text{sec}}_{s,t} = \sum_{j \neq s} \frac{\omega^{\text{sec}}_j}{\sum_{h \neq s} \omega^{\text{sec}}_h} corr([\gamma_{s,\tau}]^{t+5}_{t-4}, [\gamma_{j,\tau}]^{t+5}_{t-4}) \ , \tag{20}$$

where ω^{sec}_j denotes the average share of sector $j's$ sale in the total sales of the economy. Finally, aggregate correlation is defined as a weighted average of the sectoral correlations:

$$corr^a_t = \sum_s \omega^{\text{sec}}_s corr^{\text{sec}}_{s,t}.$$

Figures 7 and 8 show a clear downward trend in the average correlation ($corr^a_t$) of productivity and TFP growth across sectors during the post-war period.[32] Comin and Philippon [2005] show that the decline in the correlation of sectoral growth is entirely driven by the decline in the covariance of growth across sectors (as opposed to an increase in the variance of sectoral growth).

To gain insight into the importance of R&D in the decline of the co-movement of growth across sectors, we exploit the cross-sectional implications of the model. Recall that once we recognize that general innovations are more likely to diffuse in the sector where they are developed, it follows that sectors with higher turnover should develop and adopt less general innovations, and should have a lower correlation, on average, with the other sectors (prediction C2). To test this prediction, we estimate the following specification:

$$corr^{\text{sec}}_{s,t} = \alpha_s + \beta t + \gamma RD_{s,t} + \epsilon_{st}, \tag{21}$$

where $corr^{\text{sec}}_{s,t}$ is defined in expression (20) and $RD_{s,t}$ denotes the R&D intensity in sector s at time t. The first and third columns in Table 2 report the estimate of γ when $corr^{\text{sec}}_{s,t}$ is measured by the correlations of productivity and TFP growth, respectively. In both cases, R&D is associated with a significant decline in correlation. Specifically, the estimates of γ are -3.3 for productivity and $-2.5$for TFP growth, with $p - values$ of 2 percent. This implies that the increase in R&D is associated with a decline of between 5 and 6.6 percentage points of the 10 and 25 percentage point decline observed in the sectoral correlation of TFP or productivity growth. These estimates are robust to replacing the time trend with year dummies.

Columns 2 and 4 of Table 2 replace the R&D intensity as explanatory variable with the firm-level volatility in the sector. Consistent with the model, higher firm-level volatility in a sector is also

[32]See Comin and Philippon [2005] for more on this.

associated with lower correlation of the sectoral growth with other sectors.[33]

In principle, the estimated effect of R&D on sectoral correlation can be driven by omitted variable bias. For example, one may think that R&D intensity may be related to how sensitive sectors are to aggregate shocks. However, to the extent that this sensitivity has not changed much over time, this effect should be captured by the sector fixed effect.

Another explanation for the decline in aggregate volatility is proposed by Themar and Thoenig [2004]. Building on Arrow [1971], they claim that financial innovation can lead to more risk taking by firms but to less aggregate credit crunches. It follows from their analysis that sectors that benefit more from financial innovation are going to become less correlated with the rest of the economy because they will be less exposed to credit crunches and to binding collateral constraints (Bernanke, Gertler and Gilchrist [1996]). Lower exposure to financial stress will lead to lower aggregate volatility. Comin and Philippon [2005] empirically explore this hypothesis by including in regression (21) two additional controls that proxy for the degree of financial dependence in the sector: the amount of debt and equity issued in the sector over the total sales in the sector. Contrary to what we have observed with R&D, both measures of financial market dependence are positively associated to the correlation of sectoral growth (albeit this relationship is statistical insignificant). Therefore, the improvement of financial markets does not seem to be a major force in the decline in aggregate volatility. More importantly for our purposes is that the negative effect of R&D on the correlation of sectoral growth is not driven by the omission of measures of external financial dependence.

The fact that the existing theories proposed to explain the decline in aggregate volatility do not seem to be driving the negative relationship between R&D and the correlation of sectoral growth reinforces the view that, as suggested by our model, this relationship is causal.[34]

3.3 Calibration

After providing econometric evidence in favor of the specific mechanisms of the model, we undertake two calibration exercises to assess the ability of the mechanisms to generate the observed evolutions of aggregate growth, and aggregate and firm volatility.

[33] See Comin and Philippon [2005] for more comprehensive tests of this hypothesis.

[34] Philippon [2003] argues that an increase in competition in the goods market leads firms to adjust their prices faster, which reduces the impact of aggregate demand shocks. While intuitively appealing, Philippon [2003]'s is a within sector explanation with no implication about the evolution of sectoral co-movement.

Firm-level volatility

Recall that the variance of the growth rates of sales and sales per worker at the firm level are given by the following expressions:

$$
\begin{aligned}
var(\gamma_{sales_i}) &= var(\gamma_y) + \lambda^q \left(\frac{1+\beta(m-1)}{m}\right)\left(\ln(\frac{\beta m}{(1-\beta)})\right)^2 && (22)\\
var(\gamma_{sales_i/L})) &= var(\gamma_y) + \lambda^q \left(\frac{1+\beta(m-1)}{m}\right)(\ln(\sigma/\alpha))^2 && (23)
\end{aligned}
$$

The first term in both expressions represents the variance of aggregate output. In the US, this term is approximately two orders of magnitude smaller than the variance of firm-level volatility and hence irrelevant to understand the evolution of firm-level volatility. The quantitatively relevant effect of an increase in λ_0^q comes from the second term. The turnover rate, λ^q, increases both due to the exogenous increase in λ_0^q and the endogenous increase in R&D intensity. In the postwar period, R&D has increased by a factor of three in the US (figure 1). If to this we add the exogenous increase in λ_0^q, the linearity of the production function for new R&D-driven innovations implies that λ^q has increased by at least a factor of 3 in the postwar period.

Independent estimates of λ^q can be computed from the evolution of the persistence of the rankings in sales per worker in figure 3.[35] These calculations indicate that in the mid 50's, λ^q was approximately 2 percent while in the mid 90's, it was 3 times higher. Comin and Philippon [2005] conduct similar exercises using other measures of market leadership such as profit rates or market value. Specifically, they compute the probability that a firm currently ranked in the top 20^{th} percentile of profits rate or market value in its sector is not in the top 20th percentile in 5 years. These exercises imply that the turnover rate has increased by a factor of 5-6 during the postwar period.

Once we have these estimates, it is very simple to understand the power of the model to induce a very significant increase in firm volatility. In expressions (22) and (23), the first term is quantitatively irrelevant. The second term depends on fixed parameters and on λ^q. Our estimates indicate that λ^q has increased by at least a factor of 3. Firm variance has increased by a factor of approximately 4 in the postwar period. Therefore, the model can account for, at least, 75 percent of the increase in firm volatility.

Aggregate volatility and productivity growth

[35] See Appendix 3 for the formal derivation.

One way to assess the model's ability to generate the observed evolution of aggregate growth and volatility would be to calibrate all the parameters of the technology to develop general innovations and use them in the model along with the evolution of R&D-style innovations to pin down the evolution of λ^h. This route, however, is unfeasible because we have no independent information to calibrate f_0^h, ρ_h and λ_0^h.

Alternatively, we can use data on the correlation of sectoral growth to pin down the evolution of λ^h and explore what this implies, in the lens of the model, for the evolution of growth and aggregate volatility in the 50's and more recently.

Specifically, we use the following 6-step procedure:[36]

(i) Calibrate the initial turnover rate (λ_{1950}^q) to match the initial correlation of rankings in figure 3. As shown in Appendix 3, this yields an estimate of λ_{1950}^q of 2 percent.

(ii) Using the value of λ_{1950}^q and the initial correlation and variance of sectoral growth (0.5 and 0.0005 respectively), pin down the values for $\lambda_{1950}^h * (\ln(\triangle h))^2$, $\lambda_{1950}^q * (\ln(\triangle q))^2$ and $\ln(\triangle q)$.

(iii) Using the average initial growth rate of productivity (0.025), calibrate $\ln(\triangle h)$ and λ_{1950}^h.

(iv) Calibrate the final turnover rate, λ_{2000}^q, to 2.5 times the initial turnover rate (i.e. 5 percent).

(v) Using the final correlation of sectoral growth (0.25) and the calibrated value of $\ln(\triangle h)$, compute the final rate of arrival of general innovations (λ_{2000}^h).

(vi) With this information and the number of sectors (35), compute the final expected growth rate of productivity ($E\gamma_{y2000}$) and the initial and final variance of aggregate productivity growth ($V\gamma_{y1950}, V\gamma_{y2000}$).

Table 3 shows the actual as well as the model's predictions for the final expected growth rate of labor productivity, the initial and final standard deviations of aggregate productivity growth.

Moment	Data	Model
$E\gamma_{y2000}$	0.02	0.017
$Std(\gamma_{y1950})$	0.02	0.016
$Std(\gamma_{y2000})$	0.012	0.012

Table 3

This simple calibration shows that the model can easily explain the lack of relation between R&D and productivity growth at the aggregate and that it can account for an important fraction of the decline in aggregate volatility. In this exercise it accounts for 50 percent of the decline in the

[36] A more detailed explanation of this calibration is contained in Appendix 3.

standard deviation. While the model does not generate as much volatility to account for the initial level of aggregate volatility, it does match the final level of volatility.

4 Conclusion

Before undertaking any rigorous policy recommendation, we must reach a thorough understanding of the forces that drive growth in the US. This paper has presented a new growth theory for US that overcomes two hurdles we believe any candidate theory should pass. First, it explains the relationship between R&D and productivity growth at the firm-level as well as the lack of a relationship between the two at the sector and aggregate level. Second, it explains the evolution of the second moments of productivity growth at the firm and aggregate level. In particular, it explains the diverging trends in firm and aggregate volatility and the fact that the decline in aggregate volatility is in a large fraction due to a decline in the correlation of sectoral growth.

In addition to being consistent with these facts, this paper has also provided evidence on the importance of the mechanisms emphasized by the model. In particular, it has showed that firm volatility and market turnover are associated with past R&D and that current market turnover is associated with subsequent R&D. Perhaps most importantly, it has showed that sectors that have experienced higher increases in R&D have also experienced greater declines in the correlation of their growth with the rest of the economy. This indicates that there is a strong connection between aggregate and firm volatility. Furthermore, it supports the view that this connection operates mainly through the effect of R&D on the decline in the co-movement of growth across sectors.

Finally, our model suggests that sectoral co-movement is driven by the development of general innovations, and the decline of their importance in growth is at the root of the observed dynamics for the first and second moments of aggregate productivity growth. Let's hope that the current lack of light (data) around general innovations does not keep us from searching for the key (growth) in the right place, like the old economist'joke.

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Fig 4a: Effect of R&D on volatility at various lags

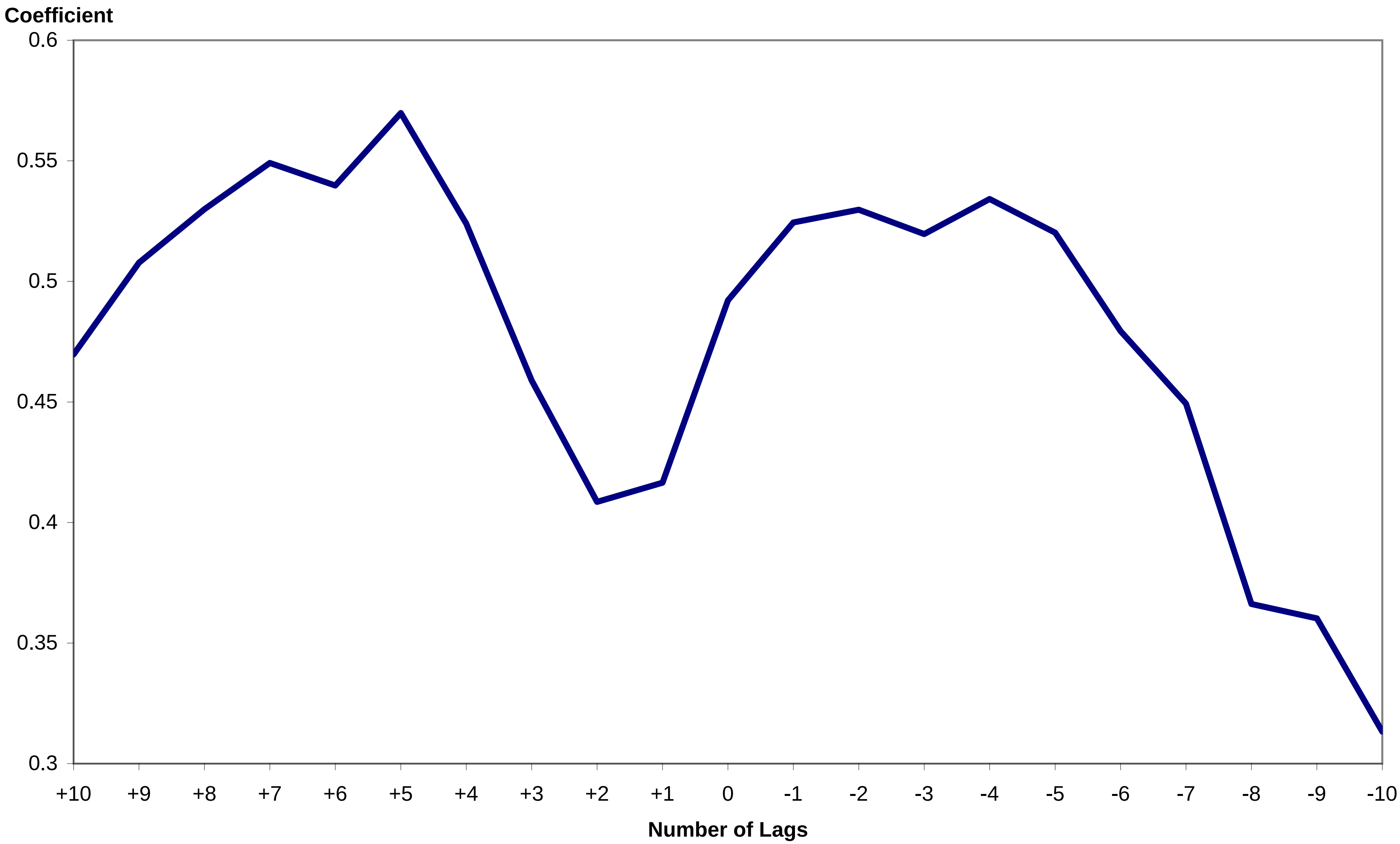

Fig 4b: P-values of Effect of R&D on volatility at various lags

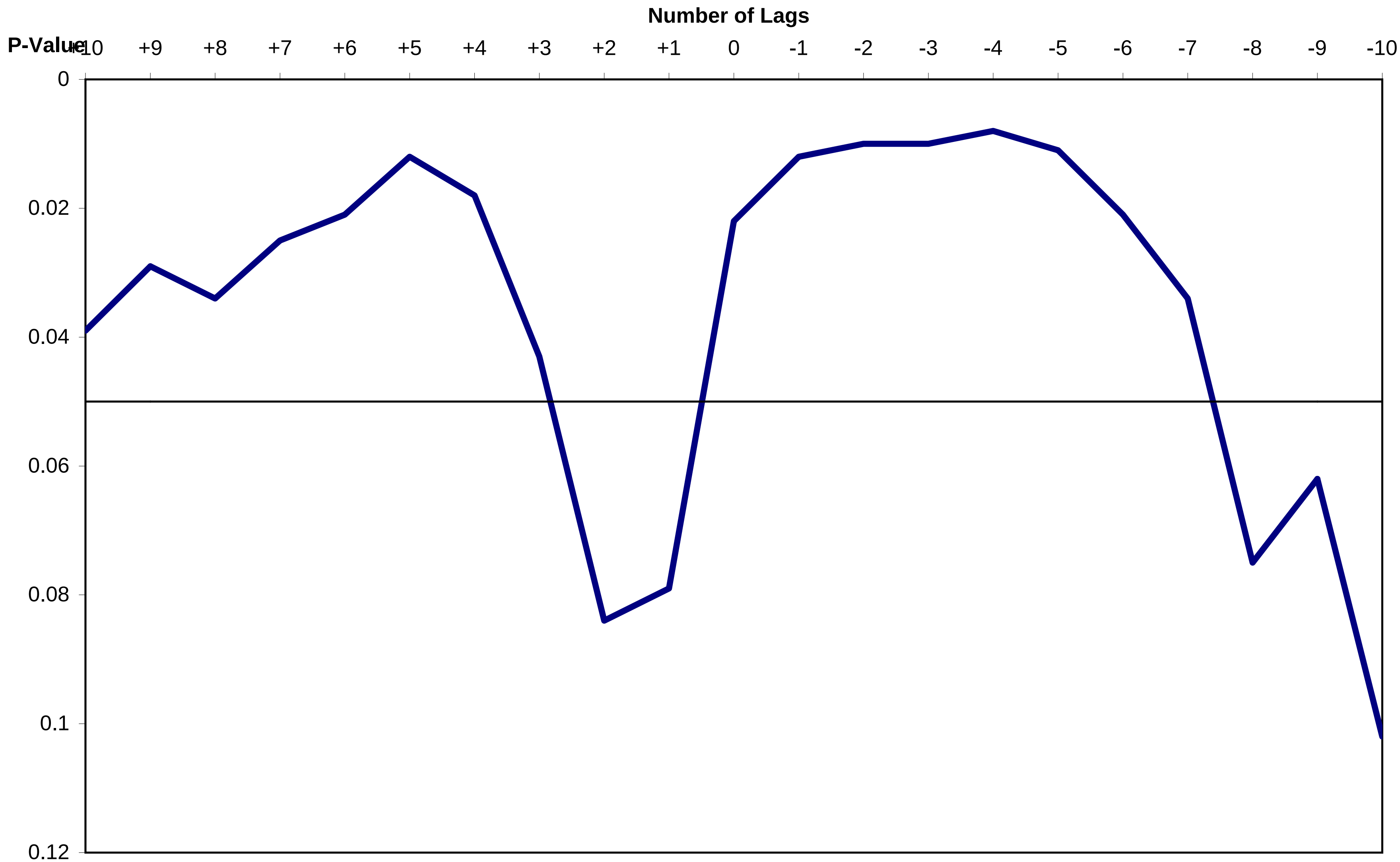

Fig. 5a: Effect of R&D on turnover at various lags

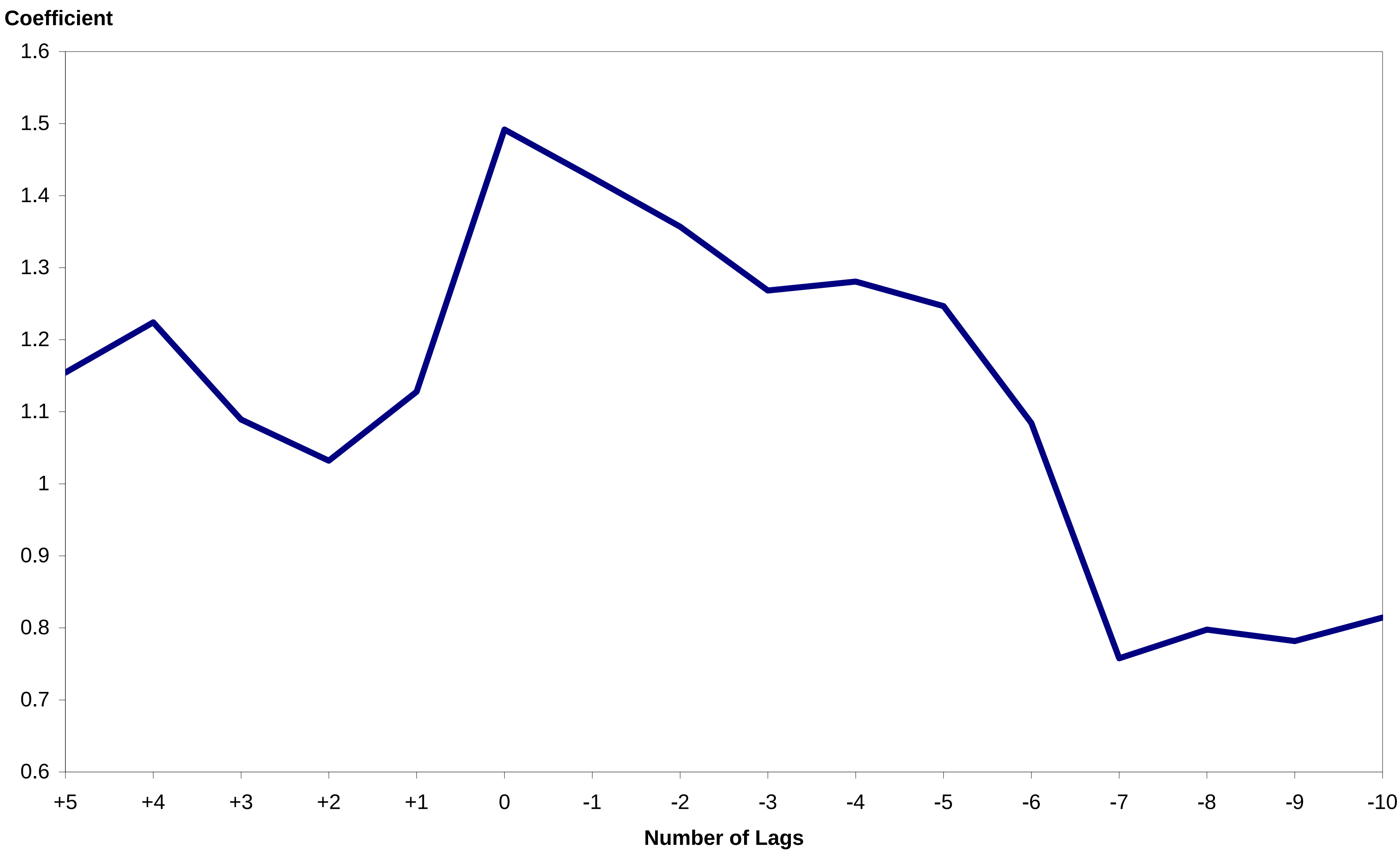

Fig 5b: P-value of Effect of R&D on turnover at various lags

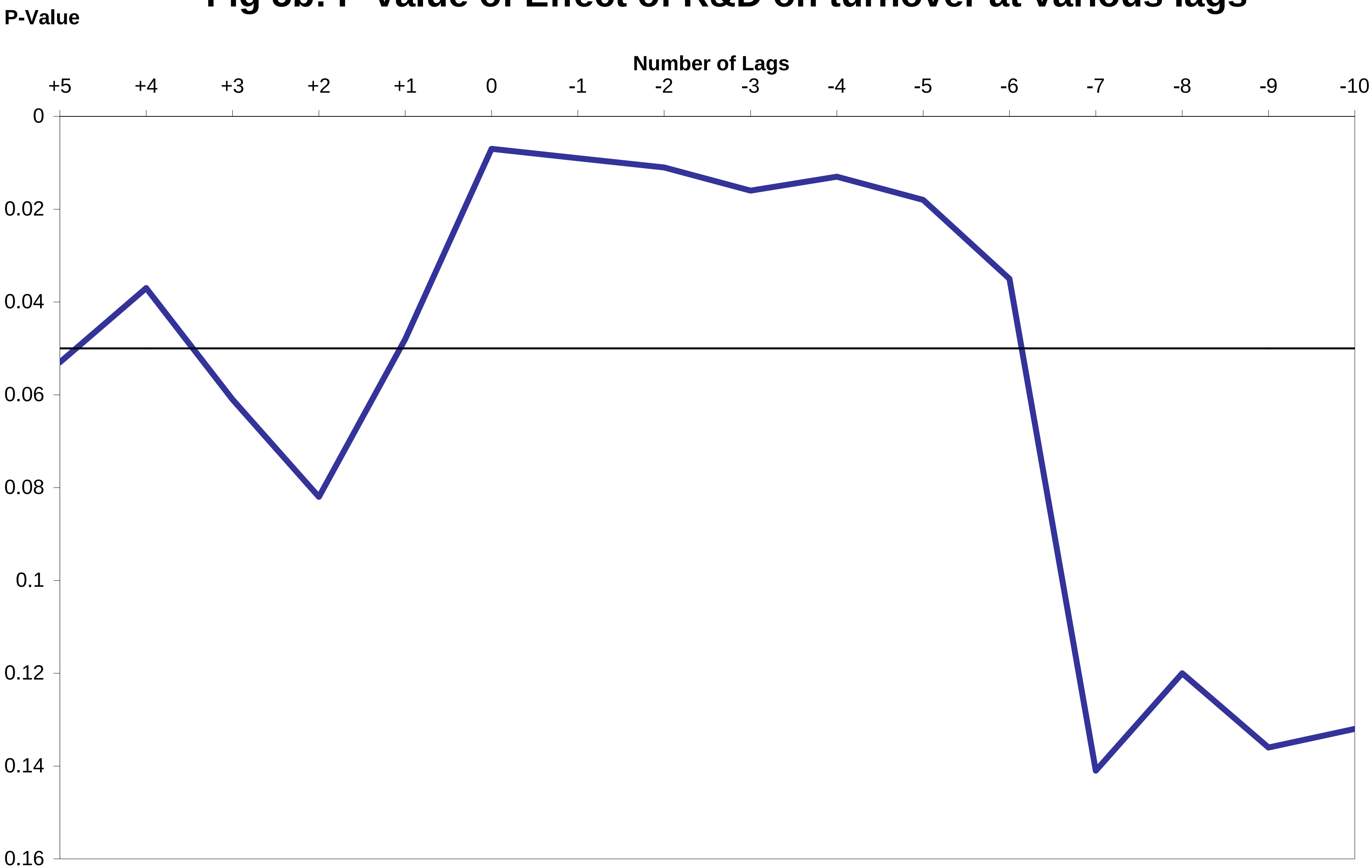

Fig 6: Evolution of sectoral Correlation of Productivity Growth

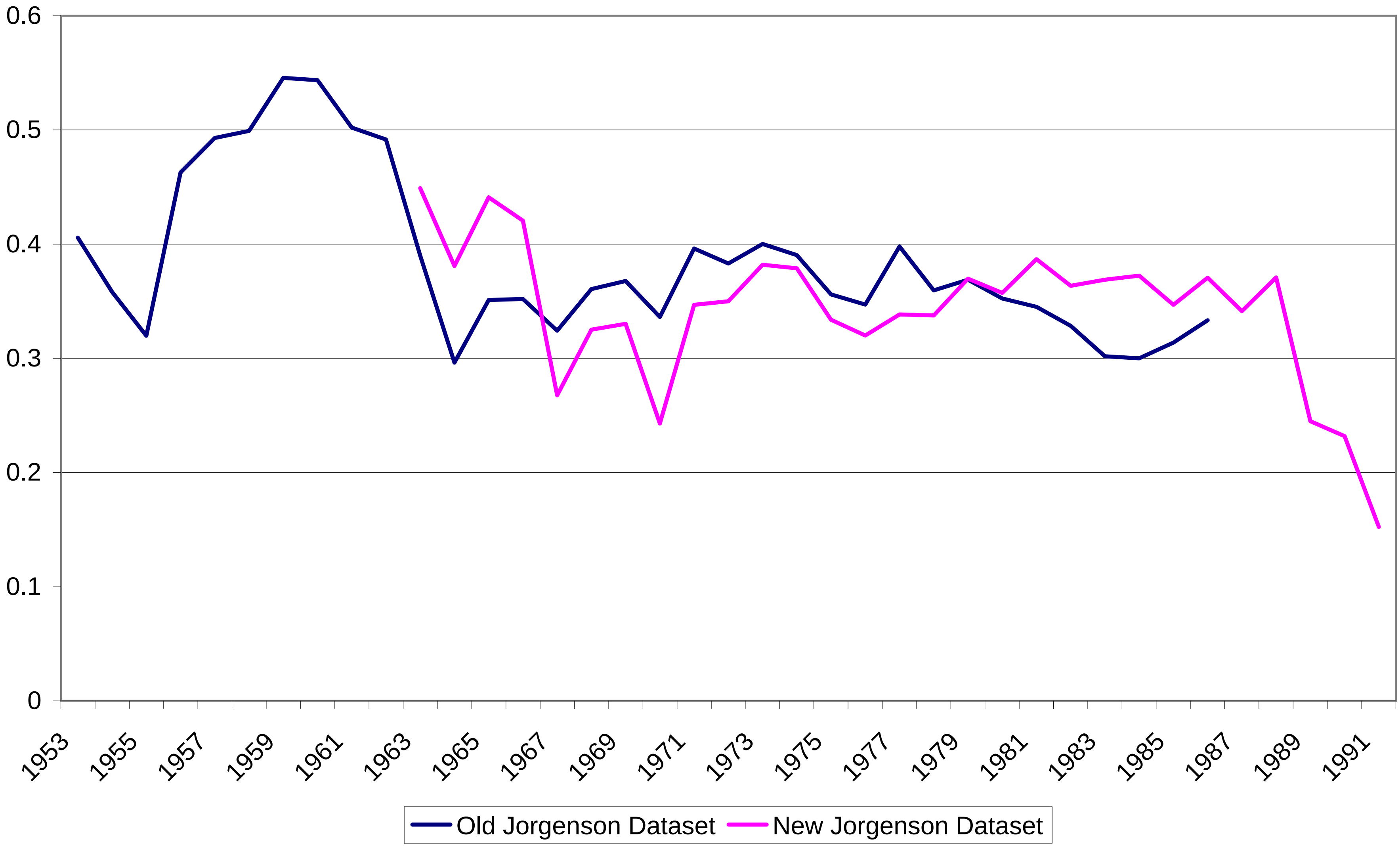

Figure 7: Correlation of Sectoral TFP Growth

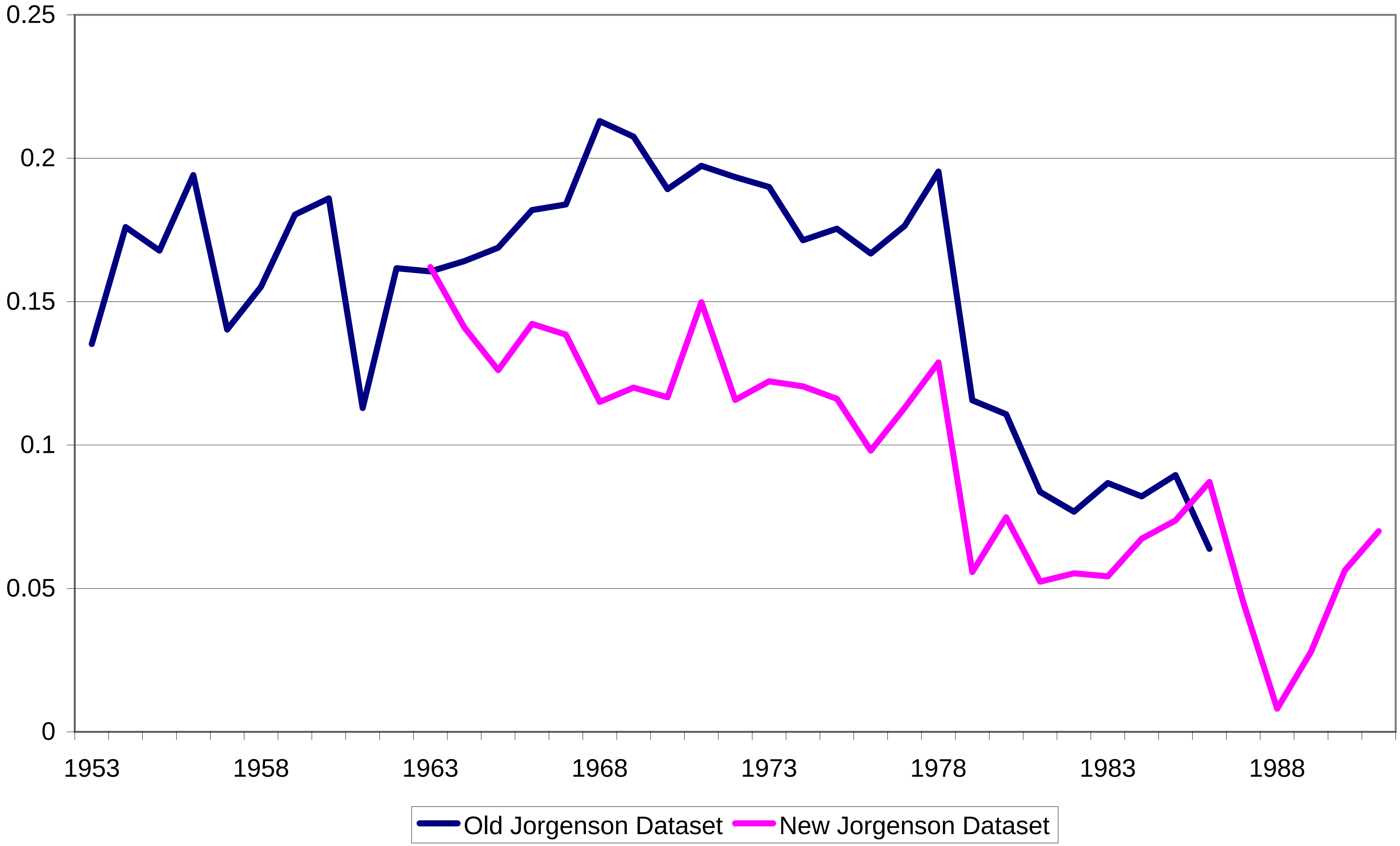

Table 1
Examples of General Innovations

Innovation	Date	Importance
Management and Production Design		
Mass Production	1900	Fixed costs spread out over larger volumes meant lower costs.
Ford assembly line	1913	Shorter assembly time resulted in lower production costs.
Scientific Management	1911	Used a scientific approach to production processes to improve productivity.
McKinsey Management Consulting	1923	Introduced a streamlined approach to consulting services.
Human Resource Management		
Hawthorne Studies	1924-1933	Addresses aspects of the employee's environment that were most important to productivity.
Industrial Psychology	1940's-50's	Emphasized contextual variables for purposes of training and positive organization change.
Survey Feedback	1940's	Highlighted the importance of sharing feedback with employees.
Sensitivity Training	1946	Focused on the importance of open discussion in small groups.
Trade		
Mall	1922	Started the modern-day one-stop shop for all consumers.
Department store	1877	First one to use wholesale purchases to bring down retail prices.
Marketing		
Coupons	1895	Effective promotion/marketing tool.
Mail order catalog	1872	Enabled businesses to target consumers that did not access to retail outlets.
Credit/Banking		
Credit card	1950	Helped businesses undertake credit transactions in a more extensive and systematic manner.
Magnetic Ink Character Recognition	1950's	Allowed computerized tracking and accounting of check transactions.
Electronic Recording Method of Accounting	1950's	Helped computerize the banking industry.
Networks/Computer Science		
Hypertext	1945	Basis of the eventual World Wide Web.
Fortran	1957	High-level programming language that made for improved scientific, engineering and mathematical applications.
Arpanet	1969	Enabled the exchange of information over large geographic distances.
Chemical Engineering	1920's	Improved the design and control of similar operations at plants in several different industries.

Table 2: R&D, Firm-level Volatility and Co-movement

	Dependent variable			
	Correlation in productivity growth		Correlation in TFP growth	
R&D	-3.28		-2.49	
	(1.42)		(1.09)	
Firm-level volatility		-0.264		-0.22
		(0.126)		(.08)
N	1011	1011	1011	1011

Notes:
Newey-West Standard Errors are reflected in parentheses.
Firm volatility is measured by the sectoral average of the firm-level volatility of the growth rate of sales
All regressions include sector and year dummies.

Appendix I: Multisector Extension

In this appendix we provide more details on the extension of the basic model to a multisector setting presented in section 2.4.

Preferences are still given by (1). Aggregate output (y) results from combining sectoral outputs (y_n) with the following Cobb-Douglas aggregator.

$$y = \prod_{n=1}^{N} y_n^{\frac{1}{N}}.$$

The n^{th} sector's output is produced by combining the sectoral leading and standard outputs as follows:

$$y_n = y_{ln}^{\beta} y_{sn}^{1-\beta} \tag{24}$$

Leading and standard sectoral outputs are produced with the following technologies:

$$y_{ln} = q_{ln} x_{ln}^{\alpha} L_{ln}^{1-\alpha} \tag{25}$$

$$y_{sn} = q_{sn} \left(m^{\sigma-1} \sum_{i=1}^{m} x_{sni}^{\sigma} \right)^{\frac{\alpha}{\sigma}} L_{sn}^{1-\alpha}, \tag{26}$$

where, for each sector n, q_{ln} denotes the quality of the leading intermediate good, $x_{\ln}$ is the number of units of leading intermediate good, q_{sn} is the (fixed) quality of standard intermediate good, x_{sni} is the number of units of the i^{th} standard intermediate good, and $L_{\ln}$ and L_{sn} are the amounts of labor employed in the production of the leading and standard sectoral outputs.

Aggregate, sectoral, leading and standard outputs are produced competitively.

As in the one sector model, the production of a unit of an intermediate good requires a_{xn} units of labor. a_{xn} increases with the complexity of the sector measured by $q_{\ln}$ and declines with the efficiency of the production process denoted by h. Specifically, we assume that a_{xn} takes the following form: $a_{xn} = q_{\ln}^{\psi_q} / h^{\psi_h}$, where $\psi_q, \psi_h > 0$.

Intermediate goods are produced non-competitively.

Optimal demands at each stage of production yields the following relationships:

$$
\begin{aligned}
p_n y_n &= \frac{y}{N} \\
p_{\ln} y_{\ln} &= \beta p_n y_n \\
p_{sn} y_{sn} &= (1-\beta) p_n y_n \\
x_{\ln} &= L_{\ln} \left(\alpha \frac{p_{\ln} q_{\ln}}{p^x_{\ln}} \right)^{\frac{1}{1-\alpha}} \\
L_{\ln} &= (1-\alpha) p_{\ln} y_{\ln} / w \\
x_{sni} &= \frac{L_{sn}}{m} \left(\frac{\alpha p_{sn} q_{sn}}{p^x_{sn}} \right)^{\frac{1}{1-\alpha}} \left(\frac{p^x_{sn}}{p^x_{sni}} \right)^{\frac{1}{1-\sigma}} \\
L_{sn} &= (1-\alpha) p_{sm} y_{sm} / w
\end{aligned}
$$

where w is the wage rate, the following are the n^{th} sector prices of output (p_n), leading output ($p_{\ln}$), standard output (p_{sn}), leading intermediate good ($p^x_{\ln}$), the i^{th} standard intermediate good (p^x_{sni}), and the price index of the standard intermediate goods, $p^x_s = \left(m^{-1} \sum_{i=1}^m (p^x_{si})^{\frac{-\sigma}{1-\sigma}} \right)^{\frac{-(1-\sigma)}{\sigma}}$.

Though it is not critical in any respect, it is convenient to scale the cost of undertaking innovations by the sectoral output. The cost of facing a probability λ^h of developing a general innovation in sector n is $(\frac{\lambda^h + f^h_0}{\lambda^h_0})^{\frac{1}{\rho_h}} p_n y_n$, and the cost of developing a new leading product with probability λ^q_s/m is $\frac{\lambda^q_s}{\lambda^q_0 m} p_n y_n$. The pricing decisions of the intermediate good producers are the same as in the one sector model. The resulting profit flow for leading and standard intermediate good producers in the symmetric equilibrium are:

$$
\begin{aligned}
\bar{\pi}_l &= \frac{y}{N}((1-\alpha)\alpha\beta - (\frac{\lambda^h_s + f^h_0}{\lambda^h_0})^{\frac{1}{\rho_h}}) \\
\bar{\pi}_s &= \frac{y}{N}(\frac{(1-\sigma)\alpha(1-\beta)}{m} - \frac{\lambda^q_s}{\lambda^q_0 m})
\end{aligned}
$$

Optimal investment in the development of general and R&D innovations implies:

$$
\begin{aligned}
1 &= \lambda^q_0 \triangle q v^l && \text{(Lq)} \\
c'(\lambda^h/N) &= \lambda^h_0 (\triangle h - 1) v^l, && \text{(Lh)}
\end{aligned}
$$

where

$$
v^l = \frac{((1-\alpha)\alpha\beta - (\frac{\lambda^h/N + f^h_0}{\lambda^h_0})^{\frac{1}{\rho_h}})}{r + \lambda^q(1 - (\triangle q - 1)(N-1))/N - \lambda^h(\triangle h - 1)}.
$$

The description of the symmetric equilibrium concludes with the new labor market clearing condition:

$$L = \sum_{n=1}^{N} \left[L_{\ln} + L_{sn} + L_{\ln}^{x} + \sum_{i=1}^{m} L_{sni}^{x} \right]$$

Appendix II: Discussion of General Technologies

The following is a list of general technologies, general inventions that have resulted in pervasive improvements in productivity across sectors and over time.

Production Design

Mass production of cars and Ford's assembly line

Mass production first originated in the automobile industry in the United States in 1901. American car manufacturer Ransome Eli Olds (1864-1950) invented the basic concept of the assembly line and mass produced the first automobile, the Curved Dash Oldsmobile. Henry Ford (1863-1947) invented an improved version of the assembly line by installing the first conveyor belt-based assembly line in his car factory in Ford's Highland Park, Michigan plant, around 1913-14. The assembly line reduced production costs for cars by reducing assembly time. After installing the moving assembly lines in his factory in 1913, Ford became the world's biggest car manufacturer. By 1927, 15 million Model Ts had been manufactured.

The philosophy of mass production was simple: the factory, itself thought of as a machine, would monitor the progress of its human elements and immediately signal where a unit was not accomplishing its job satisfactorily by the buildup of work at that station. The pace of work could be increased. Unskilled workers could be substituted for skilled labor. The task of management was made much simpler: the assembly line forced the pace of the slower workers and made it obvious where bottlenecks were occurring. Fixed overhead costs were spread out over larger and larger volumes of production, thus lower and lower prices became possible.

This strategy that characterized mass production–invest heavily in fixed capital, try to produce the maximum output at low prices, and use the productive expertise gained to forge technological leadership and lowest cost positions–was to become the defining characteristic of American industry throughout the twentieth century.

Scientific Management

Scientific management is the study of relationships between workers and machines and was first introduced by Frederick Taylor, regarded as the Father of Scientific Management, in 1911. He published Principles of Scientific Management, in which he proposed work methods designed to increase worker productivity. Taylor realized that organization productivity could be increased by enhancing the efficiency of production processes. This involved breaking down each task to its smallest unit and to figure out the one best way to do each job. Emphasis was laid on the ensuring the worker indulged in only those motions essential to the task. Taylor attempted to make a science for each element of work and restrict behavioral alternatives facing worker. Taylor looked at interaction of human characteristics, social environment, task, and physical environment, capacity, speed, durability, and cost. The overall goal was to remove human variability.

The results were profound. Productivity under Taylorism went up dramatically. In a famous experiment on the output of a worker loading pig iron to a rail car, Taylor increased the worker's output from 12 to 47 tons per day. New departments arose such as industrial engineering, personnel, and quality control. There was also growth in middle management as there evolved a separation of planning from operations. Rational rules replaced trial and error; management became formalized and efficiency increased. This model in its pure form was a dramatic improvement over the previous model of organization which was a feudal model based on fixed status and position by birth, not merit and unquestioned authority.

Management Consulting

McKinsey and Co. was one of the first management consulting firms established in 1923 in Chicago. While the consulting industry had originated before then, the innovative approach Mckinsey introduced in its consulting services was an important general technology. The McKinsey way of consulting can be decomposed in the following x steps. First the consultant gathers as much factual information about the client's organization as possible. Second, after a thorough analysis of the facts, an initial hypothesis is determined, to be tested with the client. Finally, a set of recommendations are presented to the client. These recommendations are limited to what can be realistically done given the resources of the client, the consulting firm and the amount of time required. Further, the recommendations are proposed along with milestones to be achieved as intermediate steps towards the ultimate target.

Human Resources Management

The Hawthorne Studies

Beginning in 1924 and continuing until 1933, the Western Electric Company sponsored a series of experiments for studying worker productivity and morale at its Hawthorne Works near Chicago. The researchers, from the Harvard Business School were led by Fritz Roethlisberger, T.N. Whitehead, Elton Mayo, and George Homans, and by W. J. Dickson of Western Electric. The intent of these studies was to determine the effect of working conditions on productivity.

There were four major phases to the Hawthorne Studies. The first were the illumination experiments. Lighting was changed in a variety of ways for a test group consisting of women and the results were compared with a control Group. In a surprising result, better lighting alone did not have any relationship with productivity of the women. Productivity improved under both conditions of improved lighting and worse lighting. The increase in productivity was attributed to the increased attention received by the women.

The second set of studies, the relay assembly group experiments, studied the importance of shorter working periods, incentive pay, personal health and supervision. Special work areas, consultation of the women before changes affecting them led to permanent increases in productivity of 30% over a 2 year period as well as improvements in morale. However, productivity continued to increase even when changes were made in the negative direction. The conclusion was there was no relationship between working conditions and productivity. The subjects themselves identified the primary reasons for their increase in productivity:

1. More freedom on the job.
2. Having no boss.
3. Setting their own work pace.
4. Having a smaller group: Their pay was based on their performance as group. By having a small group they had more control over the output and therefore over their pay than in the regular 30 (or more) people groups.

The studies showed the importance of worker attitude and provided information about factors other than physical working conditions that contribute to positive worker attitude.

In an attempt to investigate attitude more thoroughly, the third set of studies was launched in 1928. This consisted on an extensive employee interviewing program of 21,000 interviews. The interviews allowed researchers to learn about employees attitude towards supervision, worker relations and perceived status. A major outcome of this study was to teach supervisor how to handle employee complaints.

To investigate on the job worker relationships more extensively, researchers conducted what has been known as the bank-wiring experiments which involved fourteen male employees whose job consisted on wiring and soldering banks of equipment for central connecting services. These experiments were similar to the relay assembly group experiments, except that there was no change of supervision. Again, in the relay and bank wiring phases, productivity increased and was attributed to group dynamics. In particular, researchers found that a group norm regarding the rate of productivity significantly affects individual performance, and that informal authority from influential group members often overrode formal authority from the supervisor.

Industrial Psychology

Industrial psychology, which involved the testing of morale and efficiency at businesses, industrial and military organizations, came into its own during Second World War. Edwin A. Fleishman (1953) undertook what was a typical project of its time at the International Harvester Company. Fleishman (1953) studied the relationship of training programs on the leadership of supervisors and the consideration they extended to their subordinates (their sensitivity to and consideration of subordinates' needs and feelings). The results of the study focused his attention on the need low levels of consideration supervisors had towards their subordinates and the training program was created to address the same. While supervisors showed an initial response to the training program by being more considerate towards their subordinates, in due course, they reverted back to their original behavior. The reversal of the behavior was attributed to the culture or climate of the department the subjects came from. The climates had as much an effect on the trainee as did the training. In what came to be known as a critical point in organizational change, the study highlighted the difference between focusing on the individual and focusing on contextual variables (such as group norms and organizational culture). These lessons and others since then have been applied in an extensive manner across industries as constant attempts to improve productivity.

Survey Feedback

The organizational survey feedback method first showed up in the late 1940's. Questionnaires were being used to systematically assess employee morale and attitudes in organizations. Floyd Mann's study in 1957, guided by Rensis Likert, went a long way in developing what we now know as the Survey Feedback method. The method involved data collection by questionnaire to determine employee's perceptions of the management of the organization. The second aspect of the method was reporting the results back to the employees who answered the questionnaire. Once the results of the survey had been conveyed, managers, using the help of the subordinates, would chart out a plan to undertake positive changes in areas of concern as reflected in the survey results. The study emphasized that the effectiveness of the method relied on what the manager did with the information from the survey. Positive changes occurred when the manager discussed the results with his subordinates

Sensitivity Training

Sensitivity training refers to small group discussions where the primary, almost exclusive source of learning is the behavior of the group members themselves. Participants receive feedback from one another regarding their behavior in the group. Sensitivity training, also known as T-groups, became the earliest tool of what came to be known as organizational development. Kurt Lewin discovered the concept when undertaking a training workshop in Connecticut in 1946. He was asked to conduct a workshop that would help improve community leadership in general and interracial relationships in particular. Lewin brough in trainers and researchers and along with the participants engaged in lectures, role play and general group discussions. In the evenings, the trainers and researchers would evaluate the events of the day. The workshop acquired its significance however when participants happened to observe and participate in the evaluations as well. Participants began to object to the interpretation of their behavior on several occasions. The observation by the participants resulted in the three-way discussion among the researchers, trainers and participants. The participants in turn became more sensitive to their own behavior in terms of how they were being perceived by others and the impact their behavior was having on others. Carl Rogers labeled this mode of learning as "perhaps the most significant social invention of the century".

Credit/banking

Credit card

The credit card industry actually began in the United States in the 1930s when oil companies and hotel chains began issuing credit cards to customers for purchases made at their own gas stations and hotels. Frank McNamara, founder of Diners' Club, invented the first universal credit card for use in restaurants in 1950. He issued his card to 200 customers who could use it at 27 restaurants in New York.

The bank credit card was also introduced in the 1950s. While store or book credit allowed irregular repayment and installment loans required regular repayment, the credit cards of the early 1950s combined both types of credit. In 1951, Franklin National Bank released the first revolving charge card. The revolving line of credit was an attraction for early customers. Using the revolving card a customer could borrow money, repay it, borrow again, repay some, borrow again, and all without having to be approved for each new line of credit as long as the borrower remained under their credit limit. But credit card customers wanted to use their cards outside of their local area. The organizations that are now called Visa and MasterCard sprang up to create interchange, a nation-wide system designed to settle credit card transactions between banks, merchants and customers.

Today, with help from Visa and MasterCard, financial institutions are marketing credit cards to people all over the world. Credit cards have allowed consumers to carry debt, something that previously required a bank loan – a much more intensive process than a credit-card approval. Credit cards have been the primary instrument that fueled international consumerism and high consumer debt, each of which has spurred multiple trickle-down industries.

Credit Reporting

In Manhattan during the 1830s, Lewis Tappan handled the credits in his brother's wholesale silk business and developed extensive credit records in their line of business. Tappan recognized that this aspect of their wholesale business could be extended to other suppliers who needed information. By separating out the credit-information activity and serving many suppliers, Tappan realized what economists call "scale economies" and helped to found the business of credit reporting in the United States. Tappan contracted with agents and correspondents throughout the country to "gossip" about the solvency, prospects, and character of local businesses. He established an information hub that could rapidly service new inquiries and add new information. Tappan's agency later became known as R. G. Dun & Co., and merged in 1933 with The Bradstreet Company to form Dun & Bradstreet, which now dominates the field of commercial credit reporting.

Prior to World War II, few retailers sold on credit, and those that did confined their credit business to well-known customers. Creditors kept their own accounts and engaged in information exchanges with each other, sharing lists of names known to be poor credit risks. The first bureaus were non-profit cooperatives, owned by the merchants who participated. A national association called Associated Credit Bureaus was organized in 1937.

Sensing a need for standards, Fico Credit-scoring was invented in 1956 by two Stanford mathematicians, Bill Fair and Earl Isaac. They correctly guessed that lenders didn't really want credit "history"; they wanted point-accurate prediction. By crunching the right numbers, Bill and Earl could come up with a figure between 300 and 900, which would precisely forecast an individual's future creditworthiness.

Today there are three main credit reporting systems – Equifax, Experian and Trans Union. Each of these has a large number of smaller, affiliated credit bureaus. As of 1998, ACB had 591 member credit bureaus around the U.S., selling 600 million credit reports annually.

ERMA and MRCI

During the 1950s, researchers at the Stanford Research Institute invented "ERMA", the Electronic Recording Method of Accounting computer processing system. ERMA began as a project for the Bank of America in an effort to computerize the banking industry. ERMA computerized the manual processing of checks and account management and automatically updated and posted checking accounts. Stanford Research Institute also invented MICR (magnetic ink character recognition) as part of ERMA. MICR allowed computers to read special numbers at the bottom of checks that allowed computerized tracking and accounting of check transactions.

ERMA was first demonstrated to the public in 1955, and first tested on real banking accounts in the fall of 1956. Production models (ERMA Mark II) of the ERMA computer were built by General Electric. Thirty-two units were delivered to the Bank of America in 1959 for full-time use as the bank's accounting computer and check handling system. ERMA computers were used into the 1970s.

Electronic money

Today's Fedwire funds transfer service traces its roots back to 1918 when Federal Reserve Banks first moved currency via telegraph. However, the widespread use of electronic currency didn't begin until the automated clearinghouse (ACH) was set up by the US Federal Reserve in 1972 to provide the US Treasury and commercial banks with an electronic alternative to check processing. Similar systems emerged in Europe around the same time, so electronic currency has been widely used throughout the world on an institutional level for more than two decades.

Payments made today in nearly all of the deposit currencies in the world's banking systems are handled electronically through a series of interbank computer networks. One of the largest of these networks is CHIPS (Clearing House Interbank Payments System), which is owned and operated by the New York Clearing House. It is used for large-value funds transfers. In 1994, CHIPS and Fedwire combined handled 117.5 million transactions for a total value of US$506.6 trillion.

Although banks have been able to move currency electronically for decades, only recently has the average consumer had the capability to use electronic transfers in any meaningful way. The increasing power and decreasing cost of computers — coupled with advancements in communication technology that make global interaction available at vastly reduced costs — have together made the digital transfer of funds a reality for millions of individuals around the world. As a result, we are now witnessing the early stages of development of the digital economy.

<u>Codes and communications</u>

Hypertext

Ted Nelson coined the term Hypertext in 1965. In simple terms, hypertext is text which links to other texts. It formed the basis of what eventually became the World Wide Web and the internet. Current day internet as we know it is based on the easy navigation of content across web pages using hypertext links.

Arpanet

Advanced Research Projects Agency (ARPA), a branch of the military that developed top secret systems and weapons during the Cold War, created the ARPAnet. It was created to meet the need for large powerful computers in the country that were networked with each other to overcome geographic differences.

Four computers were the first connected in the original ARPAnet. They were located in the respective computer research labs of UCLA, Stanford Research Institute, UC Santa Barbara, and the University of Utah. As the network expanded, different models of computers were connected, creating compatibility problems. The solution rested in a better set of protocols called Transmission Control Protocol/Internet Protocol (TCP/IP) designed in 1982.

To send a message on the network, a computer broke down its data into IP (Internet Protocol) packets, like individually addressed digital envelopes. TCP (Transmission Control Protocol) ensured the packets were delivered from client to server and reassembled in the right order.

Several other innovations occurred under ARPAnet - email (or electronic mail), the ability to send simple messages to another person across the network (1971); telnet, a remote connection service for controlling a computer (1972); and file transfer protocol (FTP), which allowed information to be sent from one computer to another in bulk (1973).

Fortran

At IBM in 1954, John Backus and a group started to design the FORmula TRANslator System, or FORTRAN0. Computers were slow and unreliable and all programming was done in machine or assembly code. Work was completed in 1957 and the authors claimed that the resulting code would be as efficient as handcrafted machine code. For many years, FORTRAN dominated programming, and was the common tongue for computer programmers.

<u>Trade</u>

The Mall

A shopping center, shopping mall, or shopping plaza, is the modern adaptation of the historical marketplace. The mall is a collection of independent retail stores, services, and a parking area, which is conceived, constructed, and maintained by a separate management firm as a unit. They may also contain restaurants, banks, theaters, professional offices, service stations etc.

The first shopping mall was the Country Club Plaza, founded by the J.C. Nichols Company and opened near Kansas City, Mo., in 1922. The first enclosed mall called Southdale opened in Edina, Minnesota (near Minneapolis) in 1956. In the 1980s, giant megamalls were developed. The West Edmonton Mall in Alberta, Canada, opened in 1981 - with more than 800 stores and a hotel, amusement park, miniature-golf course, church, "water park" for sunbathing and surfing, a zoo and a 438-foot-long lake.

Department Stores

In 1877, John Wanamaker opened "The Grand Depot" a six story round department store in Philadelphia. He is credited with developing one of the first true department stores in the country, and with creating the first White Sale, modern price tags, and the first in-store restaurant. He also pioneered the use of money-back guarantees and newspaper ads to advertise his retail goods. Along with the retail giants of the day including, Marshall Field in Chicago, Alexander T. Steward in New York, Wanamker was one of the first to discover the vast power of buying wholesale and how it could cut costs to reduce retail prices.

Marketing

Coupons

A Philadelphia pharmacist named Asa Candler invented the coupon in 1895. Candler bought the Coca-Cola Company from the original inventor Dr. John Pemberton, an Atlanta pharmacist. Candler placed coupons in newspaper for a free Coke from any fountain - to help promote the new soft drink.

Today coupons are an integral part of promotion campaigns for every business. Cut-out coupons are included in newspapers as an advertising tool. They may be embedded in the product so as to encourage repeat purchases. Over the years, coupons have been adopted as marketing tool across industries to help businesses build a brand image and target their customers in a more efficient manner.

Mail Order Catalog

Aaron Montgomery Ward invented the idea of a mail order catalog. As a traveling salesman, he realized that his rural customers could be better served by mail-order, a revolutionary idea at the time. The first catalog consisted of a single sheet of paper with a price list, 8 by 12 inches, showing the merchandise for sale with ordering instructions. The catalogs gradually expanded, became bigger and more heavily illustrated, often referred to as “dream books” by rural families.

Today, mail-order catalogs are an integral part of any business. They have helped retail business across sectors to tap into the market of consumers who are unwilling or unable to access the retail outlets. Serving as an effective marketing medium, mail order catalogs have opened up new segments of consumers previously unavailable to these businesses.

Chemical Engineering

Chemical engineering emerged early in the twentieth century as a separate body of knowledge that could guide the design as well as the operation of chemical process plants, including plants producing well-established products, such as ammonia, in addition to new ones. It has an extensive effect on economic activity across sectors since it provides essential guidance to the design of a very wide range of plants. Arthur D. Little in 1915 set the stone for the field of knowledge that came to be known as Chemical Engineering. He introduced the concept of the ‘unit operations’ which referred to activities such as mixing, heating, filtering, verizing among others that featured in any chemical process. Chemical engineering research was directed towards the improvement of such processes and the selection and development of the equipment in which they were carried out. The concept of unit operations brought focus on the fact that there were a limited number of similar operations common to many industries. The discipline went on to accumulate a set of methodological tools that provided the basis for a wide range of activities connected with the design of chemical process plants.

Such advances in the field of chemical engineering have had a substantial impact across several sectors. Few would argue with tremendous impact they have had on the petrochemical industry. However, even in its stages of infancy, Chemical Engineering research was applied to the paper and pulp industry and contributed to the at the time new sulfite process of converting wood pulp into paper.

Appendix III: Discussion of Calibration

In this appendix we discuss in more detail the calibration conducted in section 3.3 to explore the model predictions for aggregate volatility and growth. In particular, we explain each of the 6 steps.

(i) and (iv) Calibrate the turnover rates (λ^q_{1950} and λ^q_{2000}) to match the initial correlation of rankings in figure 3.

We proceed in two steps. First, we use the model to compute the productivity percentiles of the leader and the followers in a sector. Second, we use the model to compute the expected correlation of the percentiles over time as a function of λ^q.

At any given moment in time, the market leader has a higher productivity than the m followers. These in turn have the same level of sales per worker. The percentile of the leader $p_l = 1/(2(m+1))$, while the percentile of the followers is $p_f = (m+2)/(2(m+1))$. Let's denote by $\overrightarrow{p_t}$ the $(m+1)$ x 1 vector that contains the percentile of each firm at year t. The mean and variance of $\overrightarrow{p_t}$ are constant and given by $\mu_p = 0.5$ and $Var_p = m/(2(m+1))^2$, respectively.

The correlation of percentiles between years t and $t+1$ is given by the following expression:

$$\begin{aligned} Corr(\overrightarrow{p_t}, \overrightarrow{p_{t+1}}) &= \frac{Cov(\overrightarrow{p_t}, \overrightarrow{p_{t+1}})}{Var_P} \\ &= \frac{E\left[\sum_{i=1}^{m+1}(p_{it}-\mu_p)(p_{it+1}-\mu_p)/(m+1)\right]}{Var_p}, \end{aligned} \tag{27}$$

where E denotes the expectation of $\overrightarrow{p_{t+1}}$ conditional on $\overrightarrow{p_t}$.

With probability $1-\lambda^q$, no firm will take over the market leader and $\overrightarrow{p_{t+1}}$ will be the same as $\overrightarrow{p_t}$. In that event, $\sum_{i=1}^{m+1}(p_{it}-\mu_p)(p_{it+1}-\mu_p)/(m+1) = \sum_{i=1}^{m+1}(p_{it}-\mu_p)^2/(m+1) = Var_p$. With probability λ^q, one firm will take over the market leader and they will swap their percentiles at year $t+1$. For the market leader, $(p_{it}-\mu_p) = -m/(2(m+1))$, while for the followers, $(p_{it}-\mu_p) = 1/(2(m+1))$. Hence,

$$\begin{aligned} Cov(\overrightarrow{p_t}, \overrightarrow{p_{t+1}}) &= (1-\lambda^q)Var_p + \lambda^q\left[\frac{m-1}{m+1}\frac{1}{(2(m+1))^2} - \frac{2}{m+1}\frac{m}{(2(m+1))^2}\right] \\ &= (1-\lambda^q)Var_p - \frac{2\lambda^q Var_p}{m(m+1)} \\ &\simeq (1-\lambda^q)Var_p, \end{aligned}$$

where the last approximation holds when m is sufficiently large. Substituting into (27) it follows that

$$Corr(\overrightarrow{p_t}, \overrightarrow{p_{t+1}}) \simeq (1-\lambda^q)$$

It follows also that for λ^q small,

$$Corr(\overrightarrow{p_t}, \overrightarrow{p_{t+5}}) \simeq (1 - 5\lambda^q).$$

Therefore since in 1950 $Corr(\overrightarrow{p_t}, \overrightarrow{p_{t+5}}) \simeq 0.9$, we calibrate λ^q_{1950} to 0.02. Similarly, since in 2000 $Corr(\overrightarrow{p_t}, \overrightarrow{p_{t+5}}) \in (0.7, 0.75)$ we calibrate λ^q_{2000} to (0.05, 0.06).

(ii) Using the value of λ^q_{1950} and the initial correlation and variance of sectoral growth, pin down the values for $\lambda^h_{1950} * (\ln(\triangle h))^2$, $\lambda^q_{1950} * (\ln(\triangle q))^2$ and $\ln(\triangle q)$.

In the multisector version of the model, we have seen that the variance of sectoral growth and the correlation of sectoral growth are given by the following expressions:

$$V\gamma_{y_s} = \lambda^q_s(\ln(\triangle q))^2 + \lambda^h(\ln(\triangle h))^2 \tag{28}$$

$$corr(\gamma_{y_s}, \gamma_{y_{s'}}) = \frac{(\triangle h)^2\lambda^h}{(\triangle q)^2\lambda^q_s + (\triangle h)^2\lambda^h} \tag{29}$$

It follows that:

$$\lambda^q_s(\ln(\triangle q))^2 = V\gamma_{y_s}/(1 + \Phi),$$

where

$$\Phi \equiv \frac{corr(\gamma_{y_s}, \gamma_{y_{s'}})}{1 - corr(\gamma_{y_s}, \gamma_{y_{s'}})}.$$

It also follows from (28) and (29) that $\lambda^h(\ln(\triangle h))^2 = \Phi V\gamma_{y_s}/(1 + \Phi)$ and (trivially) $\ln(\triangle q) = \sqrt{\lambda^q_s(\ln(\triangle q))^2/\lambda^q_s}$.

We calibrate $corr(\gamma_{y_s}, \gamma_{y_{s'}})_{1950}$ to 0.5 (figure 6) and $V_{\gamma_{y_s}}(1950)$ to 0.0005 both computed using the Jorgenson and Stiroh 35-KLEM dataset. That pins down $\lambda^q_{s1950}(\ln(\triangle q))^2$, $\lambda^h_{1950}(\ln(\triangle h))^2$ and $\ln(\triangle q)$, which is assumed to be constant.

(iii) **Using the average initial growth rate of productivity growth calibrate $\ln(\triangle h)$ and λ^h_{1950}.**

The expected growth rate of the economy is given by the following expression:

$$E\gamma_y = \lambda^q_s \ln(\triangle q) + \lambda^h \ln(\triangle h) \tag{30}$$

From this expression it follows that:

$$\ln(\triangle h) = \frac{\lambda^h(\ln(\triangle h))^2}{E\gamma_y - \lambda^q_s \ln(\triangle q)}. \tag{31}$$

Further, once $\ln(\triangle h)$ is known, $\lambda^h = \lambda^h(\ln(\triangle h))^2/\ (\ln(\triangle h))^2$. We use BLS data reported in figure 1 to calibrate $E\gamma_{y1950}$ to 0.025 and then use expression (31) to pin down $\ln(\triangle h)$ and λ^h_{1950}.

(v) Using the final correlation of sectoral growth and the calibrated value of $\ln(\triangle h)$ compute the final rate of arrival of general innovations (λ^h_{2000}).

From expression (29) it follows that

$$\lambda^h = \Phi\lambda^q(\ln(\triangle q))^2/(\ln(\triangle h))^2.$$

Substituting in Φ_{2000} , which we set to 0.25 based on figure 6, λ^q_{2000} which we have set to 0.05 based on the discussion above and the calibrated values of $\ln(\triangle q)$ and $\ln(\triangle h)$ we can pin down λ^h_{2000}.

(vi) With this information and the number of sectors (35), compute the final expected growth rate of productivity ($E\gamma_{y2000}$), the initial and final variance of aggregate productivity growth ($V\gamma_{y1950}, V\gamma_{y2000}$).

This follows by evaluating the following two expressions at λ^q_{s1950}, λ^h_{1950}, λ^q_{s2000}, λ^h_{2000}.

$$\begin{aligned} E\gamma_y &= \lambda^q_s \ln(\triangle q) + \lambda^h \ln(\triangle h) \\ V\gamma_y &= \frac{\lambda^q_s}{N}(\ln(\triangle q))^2 + \lambda^h(\ln(\triangle h))^2 \end{aligned}$$