Optimal Monetary Policy in a Currency Area*

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Abstract

This paper investigates how monetary policy should be conducted in a two-region, general equilibrium model with monopolistic competition and price stickiness. This framework delivers a simple welfare criterion based on the utility of the consumers that has the usual trade-off between stabilizing inflation and output. If the two regions share the same degree of nominal rigidity, the terms of trade are completely insulated from monetary policy and the optimal outcome is obtained by targeting a weighted average of the regional inflation rates. These weights coincide with the economic sizes of the region. If the degrees of rigidity are different, the optimal plan implies a high degree of inertia in the inflation rate. But an inflation targeting policy in which higher weight is given to the region with higher degree of nominal rigidity is nearly optimal.

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"What is the appropriate domain of a currency area? It might seem at first that the question is purely academic since it hardly appears within the realm of political feasibility that national currencies would ever be abandoned in favor of any other arrangement"[1]

With the creation of the European Central Bank, what seemed to be a pure academic speculation has become a reality. Following Mundell's seminal work, several contributions have emphasized the conditions under which a currency area is optimal. However, the monetary aspects of a currency area have been neglected mainly because, as suggested by the above quotation, the abandonment of national currencies was considered politically infeasible.

The primary purpose of this paper is to investigate the optimal conduct of monetary policy in a currency area characterized by asymmetric shocks across regions. Whether monetary policy should stabilize an aggregate measure of inflation or output or whether it should take into account the dispersion of inflation or output across regions is an unsolved question. This issue has received an increasing interest in the current policy debate on the conduct of monetary policy within the Euro-area.[2] This paper contributes to the debate in two ways: first, a stylized model that helps to understand how currency areas work and second, a micro-founded welfare criterion that allows normative analysis. Our main conclusion is that monetary policy should follow a particular inflation targeting policy in which higher weight is given to the inflation rate in the region with higher degree of nominal rigidity.

This work presents a two-region model, where each region is specialized in the production of a bundle of differentiated goods and where labor is immobile across regions.[3] Money is not neutral because there are rigidities in prices. Monopolistic competition rationalizes the existence of price stickiness. A two-region model represents the minimum requirement in order to study the important role of relative prices. When different regions experience asymmetric shocks, movements in the terms of trade are important in explaining the transmission mechanism of monetary policy.

[1] Robert Mundell (1961), p 657.

[2] Among others, Bean (1999) and Dornbusch et al. (1998) present an overview of the issues concerning the implementation of monetary policy in the EMU. Peersman and Smets (1998), Rudebush and Svensson (1999) and Svensson (1999a) evaluate alternative monetary policy rules in the EMU by using closed-economy models. Weerapana (1998) studies the performance of monetary policy rules in a large open economy.

[3] Appendix E presents an extension to a K-region area.

The normative results are rooted in the analysis of the existing distortions. In our framework there are three sources of inefficiency: *i*) the monopolistic distortion that induces an inefficient level of output; *ii*) inflation in each region that creates an inefficient dispersion of prices and *iii*) price stickiness that may create a non-efficient path of the terms of trade in response to asymmetric disturbances. By using a deadweight loss evaluation, as in monetary models by Rotemberg and Woodford (1997) and King and Wolman (1998), it is possible to build a welfare criterion that accounts for the exact magnitude of these distortions. In this context the optimal policy is the one that provide the most efficient allocation of resources. Abstracting from the inefficiencies induced by monopolistic competition, monetary policymakers would be expected to stabilize prices within each region, thus avoiding the dispersion of output across resources produced using the same technology, and would be expected to induce the right changes in relative prices across regions, thus allocating resources efficiently following asymmetric shocks. However, this combined outcome is not feasible. The optimal plan implies a high degree of inertia in the inflation rates. This feasible first-best can be approximated by an inflation targeting policy in which higher weight is given to the region with higher degree of nominal rigidity. This principle is natural, given that the regions with stickier prices create more distortions in the whole area.

The idea that monetary policy should help in creating an environment in which resources are allocated efficiently is well grounded in the monetary policy agenda. The Bulletin of January 1999, ECB (1999), explicitly states that one of the main arguments for price stability is that *"price stability improves the transparency of the relative price mechanism thereby avoiding distortions and helping to ensure that the market will allocate real resources efficiently both across uses and across times. A more efficient allocation will raise the productive potential of the economy."*

As it happens, the architects of the European Monetary Union have specified a quantitative target in terms of a weighted average of the harmonized index of consumer prices of the countries belonging to the union (HICP-targeting): the weights coincide with each country's share of total consumption.

In this work, we show that the HICP-targeting is optimal only when the regions share the same degree of nominal rigidity. For example, consider two regions of equal GDP size such as France and Germany. HICP-targeting implies that each country has a weight equal to one half. Instead, if price contracts in Germany last 20% longer than in France, then the weight given to

German inflation rate should be increased by 20%. Moreover, the deadweight losses can be substantially reduced by shifting from the HICP-targeting to our proposed policy even for a small difference in the degree of nominal rigidity across regions. As with the HICP-targeting policy, our policy is transparent and easy to implement and to monitor.

In a currency area characterized by labor immobility, relative price stickiness, and decentralized fiscal policy, the impossibility of achieving the efficient outcome is similar to the Mundellian theory on the optimum area except with a new micro-founded perspective. Indeed, the interpretation instrument-toward-distortions emphasizes the lack of instruments. We show that, in an open economy, the exchange rate provides the flexibility needed in order to achieve the efficient outcome.

The work is organized as follows. Section 1 presents the structure of the model. In Section 2, the log-linear approximation to the equilibrium conditions is presented. Section 3 analyzes the positive consequences of the equilibrium. Section 4 offers the welfare analysis and determines the optimal policy. Section 5 compares the outcomes of a certain class of policies. Section 6 analyzes the optimal monetary policy in a cooperative decentralized setting. Finally, section 7 outlines some possible extensions in this research program.

1 Structure of the model

We develop a two-country optimizing model with sticky prices, incorporating elements from both the recent closed-economy literature on the effects of monetary policy and the recent open-economy literature on exchange rate determination.[4] In this section, we describe the main features of our framework, focusing on the principal elements of departure from the previous treatments.[5]

A currency area is a group of regions that share the same currency. One currency means there is one central bank that is entitled to issue money and to conduct monetary policy within that area. A different institution conducts fiscal policy. But whereas there is a common central bank, different fiscal authorities can be assigned to different regions.

[4] Goodfriend and King (1997) summarize developments in the literature on monetary policy in closed economy, while Lane (1998) surveys recent work on optimizing sticky-price models in the open-economy context.

[5] A detailed exposition of the model is in Appendix A.

The simplest form of a currency area that is of interest for our analysis is a two-region area with a single central bank and two fiscal authorities. Each fiscal authority has sovereignty over only one region. The two regions are labeled, H and F. The whole area is populated by a continuum of agents on the interval $[0, 1]$. The population on the segment $[0, n)$ belongs to the region H, while the segment $[n, 1]$ belongs to F. There is no possibility of migration across regions. A generic agent, which belongs to the area, is both producer and consumer: a producer of a single differentiated product and a consumer of all the goods produced in both regions.

Each agent derives utility from consuming an index of consumption goods and from the liquidity services of holding money, while derives disutility from producing the differentiated product. The whole area is subjected to three region-specific sources of fluctuations: demand, supply and liquidity-preference shocks. Households maximize the expected discounted value of the utility flow.

We assume that wealth can be accumulated by holding money or bonds. Within a region households are allowed to trade among themselves in a set of bonds, that span all the states of nature, thus consumption, within that region, is insured. However, we assume that markets are incomplete across regions: households can trade only in a nominal non-contingent bond denominated in the common currency.

Our stochastic model is not solvable in a closed form solution and an approximation around a steady state is needed. In an open-economy representative agent model with incomplete markets, if the real interest rate differs from the value implied by the rate of time in the consumer preferences, assets are accumulated in one region and decumulated in the other. It can be the case that following a shock that affects the real interest rate, the new steady state level of consumption differs from the initial steady state. The implied non stationary of consumptions and assets impairs the significance of any approximation. In the open-economy business cycle literature, this problem has been overcome in two ways; here we list them without further discussion: i) following Uzawa (1968), Mendoza (1991) uses an endogenous rate of time preference; ii) Cardia (1991) assumes a finite probability of death, so that the subjective discount rate becomes a function of financial wealth.[6]

In this work, without imposing complete markets, we obtain stationary

[6] Ghironi (1999) introduces in a deterministic model an overlapping generation structure with increasing population.

consumption by assuming a unitary elasticity of substitution between the consumption of the bundles of goods produced in the two regions. In this case, relative prices automatically stabilize the output risks and there is perfect insurance of consumption across regions. The idea that the structure of preference can result in a case in which the gains form international portfolio diversification are irrelevant has been exploited by Cole and Obstfeld (1991).[7] Their simulations show that for industrial economies this assumption may not be so inaccurate.

Money matters because agents derive utility from its liquidity services. If real money balances and consumption are separable in utility and prices are flexible, money is neutral. In order to give a role to monetary policy, as it is common in the literature, we introduce both nominal rigidity and a market structure characterized by monopolistic competition. The latter assumption rationalizes the existence of price stickiness, allowing producers not to violate any participation constraint. Nominal rigidity are introduced using a model *a la* Calvo (1983), thus allowing fluctuations around the equilibrium for a longer period of time.[8] In each period a seller faces a fixed probability $1-\alpha$ of adjusting its price, irrespective on how long it has been since the seller had changed its price. In this event the price is chosen to maximize the expected discounted profits under the circumstance that the decision on the price is still maintained. We have that $1/(1-\alpha)$ represents the average duration of contracts within a region. In this model, not only are regions affected by different shocks, but they are also characterized by different degrees of nominal rigidity. In a context in which shocks are asymmetric, the degrees of nominal rigidity are crucial in explaining the transmission mechanism of monetary policy. In the analysis that follows, most of the results are driven by different assumptions on the degrees of rigidity across regions.

Our analysis is focused more on normative issues: in fact, we characterize the optimal plan under different assumptions on the degrees of nominal rigidity. In this work we do not discuss issues of implementation, i.e. how monetary policy should set its instrument in order to achieve or mimic the optimal plan. However, whenever it is needed, we identify the instrument

[7] Cole and Obstfeld (1991) analyze a model with flexible prices, while Corsetti and Pesenti (1998), in a perfect foresight model, allow prices to be sticky for at most a finite period of time.

[8] Yun (1996), in a closed-economy model, and Kollmann (1996), in a open-economy model, introduce Calvo's type of price-setting into dynamic general equilibrium monetary models.

of monetary policy in terms of the one-period risk-free nominal interest rate on the nominal bond denominated in the common currency. This is consistent with the evidence of several empirical works, as Clarida, Galí and Gertler (1997), Smets (1995) and Taylor (1993), in which the transmission mechanism of monetary policy is rooted in the transmission across the term structure of an impulse given to the short-term interest rate.

In terms of our equilibrium conditions, the assumption that the instrument of monetary policy coincides with the interest rate means that the money market equilibrium condition can be neglected, provided we are not interested in characterizing the path of real money balances or that of money supply in the whole area.

In the next section we present the log-linear approximation of the structural equations of the model.

2 Equilibrium fluctuations

The equilibrium involving small fluctuations around the steady state is approximated by a solution to a log-linear approximation to the equilibrium conditions. In this section, we first focus on the fluctuations around the steady state in the case in which prices are flexible, then we will analyze the case in which prices are sticky according to Calvo's model. Given a variable X_t we denote with $\widetilde{X_t}$ the deviation of the logarithmic of that variable from its steady state in the case prices were flexible; while with $\widehat{X_t}$ we denote the deviation of the same variable under sticky prices. Other simplifying notation is useful. Given a generic variable X, a union variable X^W is defined as the weighted average of the region's variables with weights n and $1-n$

$$X^W \equiv nX^H + (1-n)X^F,$$

while a relative variable X^R is defined as

$$X^R \equiv X^F - X^H.$$

2.1 Flexible Prices

With flexible prices, prices are set as a mark-up over marginal costs, monetary policy is neutral and real variables are affected only by "real" disturbances as follows

$$\begin{aligned}
\widetilde{C}_t^W &= \frac{\eta}{\rho+\eta}(\overline{Y}_t^W - g_t^W),\\
\widetilde{Y}_t^W &= \frac{\eta}{\rho+\eta}\overline{Y}_t^W + \frac{\rho}{\rho+\eta}g_t^W,\\
\widetilde{T}_t &= \frac{\eta}{1+\eta}(g_t^R - \overline{Y}_t^R),
\end{aligned}$$

where C^W, Y^W, T are respectively union consumption, union output and the terms of trade. The latter is defined as the ratio of the price of goods produced in region F to that produced in region H. Moreover $\overline{Y}_t^i$ and g_t^i are respectively supply and demand shocks specific to region i, while η and ρ are the inverse of respectively the elasticity of labor supply and the intertemporal elasticity of substitution of consumption. Union consumption and output depend only on union supply and demand shocks. With flexible prices, the marginal utility of consumption is proportional to the marginal disutility of producing goods. While a positive supply shock, independent of the region of origin, increases in the same proportion both union consumption and output, a positive demand shock increases union output but has a crowding out effect on consumption. This is because following a demand shock agents have to increase their effort, inducing an increase in the disutility of labor supply. A lower level of consumption, by increasing its marginal utility, partially offsets the increasing disutility of supplying more output.

The terms of trade are affected only by relative disturbances. In fact its crucial role is that of balancing the burden of exerting output across regions. Risk sharing in consumption implies that the marginal disutilities of labor supply are equated between the two regions. Whenever there are asymmetric disturbances that induce the households in a region to work more, changes in the terms of trade optimally shift part of the burden to the household in the other region. A larger demand shock in region H than in region F appreciates its terms of trade, while a larger supply shock leads to a depreciation. Another characteristic of the flexible-price equilibrium is the complete insulation of the terms of trade from monetary policy, as in *ad hoc* models such as Obstfeld (1985) and Clarida and Galí (1994).

In an equilibrium in which the union inflation rate is zero, the implied path of the nominal interest rate $\widetilde{R}_t$ is

$$\widetilde{R}_t = \frac{\rho\eta}{\rho+\eta}\mathrm{E}_t[(\overline{Y}_{t+1}^W - \overline{Y}_t^W) - (g_{t+1}^W - g_t^W)].$$

This natural interest rate is only a function of union disturbances. It is worth noting that given a demand shock, no matter if the shock is originating in region H and F, the interest rate will respond in the same direction and, once we normalize the shock for the size of the region, with the same magnitude. A similar argument applies following a supply shock.

2.2 Sticky Prices

Here we discuss how the log-linear approximation of the equilibrium will behave under the hypothesis of sticky prices. The log-linear version of the Euler equation and of region H and F aggregate outputs are

$$E_t\widehat{C}^W_{t+1} = \widehat{C}^W_t + \rho^{-1}(\widehat{R}_t - E_t\pi^W_{t+1}), \tag{1}$$

$$\widehat{Y}_{H,t} = (1-n)\widehat{T}_t + \widehat{C}^W_t + g^H_t, \qquad \widehat{Y}_{F,t} = -n\widehat{T}_t + \widehat{C}^W_t + g^F_t, \tag{2}$$

where π is the inflation rate.[9] In (1) the expected growth of consumption depends positively on the real return. In (2), $\widehat{Y}_{H,t}$ and $\widehat{Y}_{F,t}$ are output respectively in region H and F. Combining (1) and (2) we obtain the intertemporal IS equation

$$\widehat{Y}^W_t = g^W_t - \rho^{-1}\mathrm{E}_t\sum_{j=0}^{\infty}(\widehat{R}_{t+j} - \pi^W_{t+j+1}),$$

which is well known in the closed-economy literature (see Kerr and King (1996) and Woodford (1996)). However in our context it is union output that depends not only upon short real interest rates, but also upon long real rates. Expectations of future monetary policy as well as expectations of the implied path of the union inflation rate affect the current equilibrium of union output. Further insights can be retrieved by analyzing the behavior of the sticky-price equilibrium compared with the flexible-price equilibrium. By defining the consumption gap as $c^W \equiv \widehat{C}^W - \widetilde{C}^W$ and the union output gap as $y^W = \widehat{Y}^W - \widetilde{Y}^W$, we have

$$c^W_t = y^W_t = -\rho^{-1}\mathrm{E}_t\sum_{j=0}^{\infty}[(\widehat{R}_{t+j} - \pi^W_{t+j+1}) - \widetilde{R}_{t+j}];$$

where the consumption and the output gap are explained by the expected deviations of current and future real rates with respect to the natural real

[9] Equation (1) represents a log-linear approximation of equation (A.8) in Appendix A, while equations (2) are derived from (A.16).

rate. Expectations of a continuous contraction of the real rate above the natural rate lead to a negative output gap.

The supply block of the model contains the aggregate supply equations of regions H and F as

$$\pi_t^H = (1-n)k_T^H(\widehat{T}_t - \widetilde{T}_t) + k_C^H(\widehat{C}_t^W - \widetilde{C}_t^W) + \beta E_t \pi_{t+1}^H, \tag{3}$$

$$\pi_t^F = -nk_T^F(\widehat{T}_t - \widetilde{T}_t) + k_C^F(\widehat{C}_t^W - \widetilde{C}_t^W) + \beta E_t \pi_{t+1}^F, \tag{4}$$

where the region-specific inflation rates depend on the expectations of future price-setting behavior as well as on the deviations of the terms of trade and consumption from their natural rates.[10] The bigger the region, the more relative prices influence the inflation rates. Focusing on the AS equation in region H, an increase in the terms of trade shifts the AS equation and increases inflation of region H through two channels. The first is the expenditure-switching effect: an increase in the price of goods produced in region F relative to goods produced in H boasts the demand of goods produced in region H, pushing up inflation in this region. The second is the reduction in the marginal utility of nominal income: the optimal response is to increase prices in order to offset the fall in revenues. We can rearrange equation (3) and (4) in a form that is familiar with the "New-Keynesian Phillips curve" literature

$$\pi_t^H = (1-n)(k_T^H - k_C^H)\cdot(\widehat{T}_t - \widetilde{T}_t) + k_C^H\cdot(\widehat{Y}_{H,t} - \widetilde{Y}_{H,t}) + \beta E_t \pi_{t+1}^H, \tag{5}$$

$$\pi_t^F = -n(k_T^F - k_C^F)\cdot(\widehat{T}_t - \widetilde{T}_t) + k_C^F\cdot(\widehat{Y}_{F,t} - \widetilde{Y}_{F,t}) + \beta E_t \pi_{t+1}^F, \tag{6}$$

in which we have emphasized the dependence on the region-specific output gap. But, in contrast with the closed-economy formulation, the terms of trade gap still matters.

The latter link has interesting implications. The "New-Keynesian Phillips curve" has attracted considerable attention from researchers. A big criticism of the closed-economy formulation is the absence of a trade-off between stabilizing inflation and output gap. Inflation can be reduced at no cost in terms of output gap. Non rationality in the formation of expectations, Roberts (1998),

[10] We have that β is the intertemporal discount factor in the consumer preferences; σ is the elasticity of substitution in consumption between goods produced in the same region. We have defined $k_C^i \equiv [(1-\alpha^i\beta)(1-\alpha^i)/\alpha^i]\cdot[(\rho+\eta)/(1+\sigma\eta)]$ and $k_T^i = k_C^i\cdot[(1+\eta)/(\rho+\eta)]$ for $i = H$ or F. Here $\pi_t^H = \ln P_{H,t}/P_{H,t-1}$, $\pi_t^F = \ln P_{F,t}/P_{F,t-1}$, where $P_{H,t}$ and $P_{F,t}$ are prices of the bundles of goods produced respectively in region H and F. A full derivation is in Appendix B.

and partially backward-looking price-setters, Galí and Gertler (1998), are the extensions that have been used in order to respond to this criticism. Without relaxing any assumption, our open-economy formulation gives a different perspective. Focusing on the AS equation of region H, we have

$$\pi_t^H = \mathrm{E}_t \sum_{k=0}^{\infty} \beta^k [(1-n)(k_T^H - k_C^H) \cdot (\widehat{T}_{t+k} - \widetilde{T}_{t+k}) + k_C^H \cdot (\widehat{Y}_{H,t+k} - \widetilde{Y}_{H,t+k})],$$

where a zero inflation rate requires both the terms of trade gap and the output gap to be zero. As we will see in the next sections, it is an exception when these two gaps can be closed simultaneously.[11] Furthermore in our specification it is inherent an implicit inertia in the inflation rate. In fact the definition of the terms of trade implies

$$\frac{T_t}{T_{t-1}} = \frac{P_{F,t}}{P_{H,t}} \frac{P_{H,t-1}}{P_{F,t-1}},$$

and in the log-linear form

$$\widehat{T}_t = \widehat{T}_{t-1} + \pi_t^F - \pi_t^H. \tag{7}$$

It follows that the terms of trade is a state variable. If monetary policy is not able to eliminate the link between the inflation rate and the terms of trade, inflation itself will be a function of its past values. Thus our specification points toward a simultaneous estimation of the two AS equations under the restriction imposed by the terms of trade identity, in order to better describe the inflation dynamics.

How important are terms of trade effects? The answer depends on the degree of openness of the regions within the union. As for the European Union as a whole, the ratio of exports of goods to union GDP, is of the order of 14%. While each country, considered separately, has an export ratio including the intra-union trade, ranging from 19% to 62%. Thus intra-union openness is high and terms of trade effects are important.

Equations (1), (2), (3), (4) and (7) combined with the log-linear version of the interest rate rule characterize completely our log-linear equilibrium dynamics.

[11] Erceg, Levin and Henderson (1999) offer a similar conclusion in a closed-economy model in which both prices and wages are sticky. In their case the stickiness of real wage creates the trade-off between output and inflation.

3 Positive Analysis

In this section we focus on some positive implication of the equilibrium under price stickiness which is described by the log-linear approximation of the previous section.

3.1 Equal Degrees of Nominal Rigidity Across Regions

First we restrict the degrees of nominal rigidity to be the same across regions, i.e. $\alpha^H = \alpha^F$. This is not a realistic scenario but it can provide a good benchmark for comparing more general frameworks. From this assumption it follows that $k_j^H = k_j^F$, for $j = C, T$.

In this case, the AS equation related to the union inflation rate is

$$\pi_t^W = k_C(y_t^W) + \beta E_t \pi_{t+1}^W, \tag{8}$$

and it has the same interpretation as a closed-economy AS equation, in which all the variables are substituted with their union correspondents. There is no trade-off in stabilizing the union inflation rate and the union output gap. Moreover using the Euler equation we have

$$\mathrm{E}_t(y_{t+1}^W) = y_t^W + \rho^{-1}(\widehat{R}_t - \widetilde{R}_t - E_t \pi_{t+1}^W). \tag{9}$$

We can 'close' this two equations with a particular interest rate rule in which the interest rate is forced only to react to a weighted average of regional variables or a weighted average of regional shocks. The weights should coincide with the economic size of the regions. As an example, a classical Taylor's rule belongs to this class

$$\widehat{R}_t = \mu \pi_t^W + \phi y_t^W,$$

in which the interest rate reacts to the union inflation rate and the output gap; μ and ϕ are policy parameters. Given this particular class of rules, equations (8), (9) combined with the rule are sufficient to determine the equilibrium path of π_t^W, y_t^W and $\widehat{R}_t$, under certain restrictions on the parameters of the rule. We have also the following proposition.

Proposition 1 *If $\alpha^H = \alpha^F$ and if the interest rate reacts only to a weighted average of variables and shocks, then if the equilibrium is determinate, asymmetric shocks do not create asymmetric responses of each of the following variables π^W, y^W and $\widehat{R}$.*

Proof. It follows directly from the previous discussion. ■

It is misleading at this stage to interpret Proposition 1 as pointing towards a direct relation between a currency union and a particular 'closed' economy in which variables are substituted with their respective union average. In fact the rule identified is highly special and if it may seem a sensible class of rules to which we can focus the attention, we should have solid arguments to prefer this class instead of others. Moreover from a positive point of view inflation rates in each region are variables of interest and the determination of relative prices is crucial for their determination. We have then

$$\pi_t^R = -k_T(\widehat{T}_t - \widetilde{T}_t) + \beta \mathrm{E}_t \pi_{t+1}^R, \tag{10}$$

where the pressure on relative inflation is given by the deviations of the terms of trade from their natural rate. Noting that $\pi_t^R = \widehat{T}_t - \widehat{T}_{t-1}$ we obtain

$$\mathrm{E}_t \widehat{T}_{t+1} - \frac{1+\beta+k_T}{\beta}\widehat{T}_t + \frac{1}{\beta}\widehat{T}_{t-1} = -\frac{k_T}{\beta}\widetilde{T}_t. \tag{11}$$

Proposition 2 *If $\alpha^H = \alpha^F$, (i) there exists a unique stable solution for the equilibrium terms of trade, (ii) the terms of trade are completely insulated from monetary policy, (iii) the terms of trade cannot be at their efficient level $\widetilde{T}_t$ at all dates t unless $\widetilde{T}_t = 0$ at all dates t.*

Proof. In Appendix D. ■

The result of insulation of the terms of trade is highly special. It holds in the flexible-prices case but also in this special case with sticky prices. When sellers set their prices, they consider as given the price indexes as well as the aggregate consumption index. Regional price indexes are determined in equilibrium, by the price-setting decisions, while consumption is influenced by monetary policy. In this special case regional price indexes react with the same magnitude to movement of the aggregate consumption, thus neutralizing monetary policy. In fact the degree of influence of aggregate consumption on sellers who are changing their price is independent of the region of residence of the sellers. Moreover when $\alpha^H = \alpha^F$ the same fraction of sellers is changing prices in each region. It follows that monetary policy cannot induce asymmetries in the inflation rates.

However here we stress that, even though monetary policy is not influential, distortions given by the stickiness of prices remain and the path of the terms of trade does not match changes in the natural level. The degree of price stickiness matters for the degree of inertia in the terms of trade. If prices adjust less frequently, the response of relative inflation to changes in the terms of trade from its natural rate decreases and the inertia in the terms of trade increases.

3.2 Different Degrees of Nominal Rigidity Across Regions

In the previous paragraph we have analyzed the case in which the duration of price contracts is equalized between regions. Here we relax this assumption in order to discuss the robustness of the above conclusions. Taking a weighted average of the AS equations we obtain

$$\pi_t^W = \theta(\widehat{T}_t - \widetilde{T}_t) + \kappa(\widehat{Y}_t^W - \widetilde{Y}_t^W) + \beta \mathrm{E}_t \pi_{t+1}^W, \tag{12}$$

where not only the deviations of union output from its natural level, but also the deviation of the terms of trade from their natural rate, generate pressures on union inflation rate.[12] It is no longer true that the closed-economy version of the New-Keynesian Phillips curve can be adapted to a currency area. Unlike the previous section, relative prices are important and a trade-off between stabilizing union inflation and union output may exist.

In this general case, relative inflation rate is

$$\pi_t^R = -\psi(\widehat{T}_t - \widetilde{T}_t) + \omega(\widehat{Y}_t^W - \widetilde{Y}_t^W) + \beta \mathrm{E}_t \pi_{t+1}^R, \tag{13}$$

where again unlike the previous section, monetary policy has an influence on the terms of trade, through the influence on union consumption or union output. In general monetary policy rules can stabilize or destabilize the terms of trade and more importantly the inertia of the terms of trade can affect the dynamics of union inflation and consumption.

In this work, we are not directly interested in evaluating only quantitative relations among variables following different rules, but we focus more on normative issues. As a result of this section, some interesting questions

[12] We have defined

$$\theta \equiv n(1-n)(k_T^H - k_T^F), \qquad \kappa \equiv n k_C^H + (1-n) k_C^F,$$
$$\psi \equiv n k_T^F + (1-n) k_T^H, \qquad \omega \equiv k_C^F - k_C^H.$$

arise. Should monetary policy be conducted in a way to stabilize the union inflation rate or should it take into consideration the distribution of inflation rates across regions? Should monetary policy stabilize union output or should it stabilize region-specific output? In order to proceed with these normative issues, a welfare criterion should be specified.

4 Normative Analysis

The objectives of the monetary policymakers in a specific country or area are a blending of different factors: structure of the economy, preferences either of the society or of the political authority, internal preferences of the central bank, independence of the central bank from the political authorities. In some cases it is the legislator that assigns the central bank specific objectives.[13]

In the European Monetary Union, the European System of Central Bank has the primary objective of maintaining price stability. Monetary policy may sustain the economic growth of the regions, but this should be done without any prejudice to price stability. As outlined by Svensson (1999b), defining price stability boils down to defining the monetary-policy loss function. But the architects of European monetary policy have been more explicit by stating that *"price stability shall be defined as a year-on-year increase in the Harmonized Index of Consumer Prices (HICP) for the euro area of below 2%"*.[14] By giving a quantitative definition of price stability, they have implicitly defined the monetary-policy loss function in terms of the HICP. In this work we argue that it is not always the case that the optimal monetary policy is achieved with the stabilization of the HICP (in our model we have $\pi^{HICP} = \pi^W$). Even if it is an appealing and simple target, it does not properly take into account all the costs that society incurs because prices do not allocate resources efficiently.

Following Rotemberg and Woodford (1997, 1998), King and Wolman (1998) and Woodford (1999a, 1999b), a standard public finance approach

[13] See Blinder (1998) for a view on central banking.

[14] The Harmonised Index of Consumer Prices inside the euro-area is a weighted average of the single-country HICP. Each country has a weight equal to the share of its total private consumption relative to the euro-area private consumption. The private consumption of each country is evaluated in the common currency and then related to the euro-area consumption. In our model the HICP is π^W.

is used in order to evaluate the magnitude of the distortions existing in the economy. As taxation creates distortions in prices and quantities, thereby causing a deadweight loss, the distortions that allow monetary policy to exist create a misallocation of quantities and prices within each region and across regions. This approach is very attractive because in the closed-economy framework it delivers a welfare criterion only in terms of squares of inflation and output gap, the latter taken in deviations from a desired level. It also justifies price stability as the optimal conduct of monetary policy, see Woodford (1999a).

In our framework, a natural welfare criterion that allows an evaluation of the deadweight loss is the discounted sum of the utility flows of the households belonging to the whole union. The average utility flows, disregarding liquidity effects, is defined at each date t as

$$w_t \equiv U(C_t) - \int_0^1 V(y_t(j), z_t^i)dj,$$

where it has been implicitly assumed that each region has a weight equal to its economic size.[15] This particular choice of weights has a convenient implication: the optimal equilibrium allocation between leisure and consumption in a central planned economy implies the efficient condition, i.e. the equality between the marginal utility of consumption and leisure. The welfare criterion of the whole union is then defined as

$$W = \mathrm{E}_0 \left\{ \sum_{t=0}^{+\infty} \beta^t w_t \right\}.$$

Following Rotemberg and Woodford (1997,1998) and Woodford (1999a), we compute a second-order Taylor series expansion of W around the deterministic steady state where all the shocks are zero. Our second-order approximation delivers an intuitive representation of the welfare function:

$$W = -\Omega \mathrm{E}_0 \left\{ \sum_{t=0}^{+\infty} \beta^t L_t \right\}, \qquad (14)$$

with

$$L_t = \Lambda \cdot [y_t^W - \overline{y}^W]^2 + n(1-n)\Gamma \cdot [\widehat{T}_t - \widetilde{T}_t]^2 + \gamma \cdot (\pi_t^H)^2 + (1-\gamma) \cdot (\pi_t^F)^2 + \text{t.i.p} + o(\|\xi\|^3);$$

[15] We have that U and V are elements of the utility function of the consumer which is described in Appendix A, where $y(j)$ is the output produced by agent j, while z^i is a region-specific shock.

we have that Ω, Λ, Γ, γ are functions of the structural parameters of the model while t.i.p. denotes parameters that are independent of the policy and $o(\|\xi\|^3)$ includes terms that are of order higher than the second in the bound $\|\xi\|$ on the amplitude of the shocks considered in the approximation.[16] Furthermore $\overline{y}^W > 0$ arises because the steady state union output, around which we are linearizing, is inefficiently low due to the monopolistic distortions.[17]

Interesting comparisons can be addressed with reference to the closed-economy case of Rotemberg and Woodford (1997,1998), Aoki (1998), Woodford (1999a,1999b).[18] Our welfare criterion can be interpreted as a generalization of their framework. If one region becomes big in size, i.e. $n \to 1$ or $n \to 0$, the welfare criterion becomes the closed-economy case of Rotemberg and Woodford, where the variability of the terms of trade is no longer important. If the stickiness vanishes in one region, i.e. if $\alpha^H \to 0$ or $\alpha^F \to 0$, the welfare criterion boils down to Aoki's welfare criterion, where both the variability of the terms of trade and the variability of the inflation rate (only in the sticky-price sector) matter.

Our general framework sheds some light on which variables are significant in order to compute the welfare function when interregional relative prices are important. What matters is the squared deviation of the union output gap from the desired level of union output. A positive output gap is desirable because of the inefficiency associated with the monopolistic distortions. Changes in the terms of trade explain divergences of the production across regions which are tolerated from the welfare point of view only if they match changes in the natural rate of the terms of trade. Here the role of relative prices emerges in allocating the resources across regions. Monetary policy should induce changes in relative prices as if prices were flexible, because in this case resources are allocated optimally when the economy experiences asymmetric shocks. The inflation rate enters in a very specific way: it is an average of the squares of the region-specific inflation rates. Inflation is costly because it induces an inefficient variability of relative prices and of output

[16] Details are in Appendix C.

[17] A similar systematic inefficiency in the steady state level of the term of trade would have arisen if we were allowing an asymmetric deterministic steady-state with $\tau^H \neq \tau^F$. This case does not add further insights to the analysis that follows.

[18] Rotemberg and Woodford (1997,1998) and Woodford (1999a,1999b) are example of closed-economy model with one sector. Aoki (1998) presents a closed-economy model with two sectors, one with flexible prices and the other with sticky prices.

within each region.

Moreover our micro-founded welfare criterion implies a determined weight, in terms of the structural parameters of the model, to be given to the squares of the inflation rates. If the degrees of nominal rigidity are equalized across regions, the relevant variable is the exact weighted average of the squares of the region-specific inflation rates; while in the case regions have different degrees of price rigidity, the inflation rate in the region with higher degree of nominal rigidity will have a higher weight in the welfare function. In what follows, we first analyze the welfare implication in the case the degree of rigidity is the same, than we will focus on the case in which one region is characterized by flexible prices, finally the general case will be considered.

4.1 Equal Degrees of Nominal Rigidities Across Regions

We begin by analyzing the case in which the duration of the price contracts is identical in both regions. The loss function simplifies to

$$L_t = \Lambda[y_t^W - \overline{y}^W]^2 + n(1-n)\Gamma[\widehat{T}_t - \widetilde{T}_t]^2 + (\pi_t^{HICP})^2 + n(1-n)(\pi_t^R)^2 + \text{t.i.p.} + o(\|\xi\|^3).$$

First we focus on the optimal policy in the case in which the monopolistic distortions are perfectly neutralized by an appropriate subsidy.[19] We have then $\overline{y}^W = 0$. In this case the efficient outcome coincides with the allocation that would arise if prices were perfectly flexible in both regions.

Proposition 3 *If $\overline{y}^W = 0$ and $\alpha^H = \alpha^F$, the optimal policy is to set $\pi_t^{HICP} = 0$ at all dates t.*

Proof. From proposition 2 we know that in this case the terms of trade and relative inflation are insulated from monetary policy. The part of the loss function that can be affected by monetary policy includes only the squares of the union output gap and of the HICP inflation rate. Given that there is no trade-off between stabilizing both these two variables, the optimal policy is to target to zero the HICP inflation rate. In this equilibrium the implied path of the interest rate will follow the natural rate. ■

[19] In our context this is possible by choosing $\tau^H = \tau^F = (1-\sigma)^{-1}$, where τ^H and τ^F are distorting taxes on the production, respectively in region H and F.

This policy maximizes the welfare function but it does not reach the efficient outcome consistently with Proposition 2. Monetary policy cannot correct the inefficiencies induced by the sluggish adjustment of relative prices. It is no longer true, as it is the case in the Goodfriend and King (1997) analysis, that by stabilizing a *core* measure of inflation — in our case a weighted average of the inflation rates — monetary policy can reach efficiency, simply because the terms of trade are out of its control.

However the policy prescription, that monetary authority should stabilize the HICP, is valid, being the optimal policy in this constrained equilibrium. Moreover even in the case in which the monopolistic distortions are not completely eliminated, i.e. $\overline{y}^W > 0$, the optimal policy invokes a long-run stabilization of the HICP.

Proposition 4 *If $\overline{y}^W > 0$ and $\alpha^H = \alpha^F$, the optimal policy is to commit to a deterministic positive path of π^{HICP} that asymptotically converges to zero.*

Proof. See Woodford (1999a) for a parallel closed-economy proof. ■

In this case $\pi_t^{HICP} = \pi_0^{HICP}\lambda^t$ where π_0^{HICP} and λ are positive, with λ less than one. The idea that the inflation rate converges to zero is consistent with the results of King and Wolman (1998), in which with Taylor-style overlapping price contracts the optimal steady-state policy is that of completely stabilizing prices, even in the presence of small distortions due to the monopolistic inefficiencies. Even though there is a long-run trade-off between union output and inflation, monetary policy does not exploit this trade-off, except at time 0 when the commitment is started. However, in the case monetary policy intends to commit to a pattern of behavior at a date far in the past, then any incentive to inflate will disappear and the optimal policy will invoke stabilization of the HICP index, as in the previous proposition. This definition of commitment, from a "timeless" perspective, seems more appropriate, as emphasized by Woodford (1999c).

It is worth stressing that even in this case efficiency is not obtained because of the combination of the inefficiency induced by the stickiness of relative prices and the existence of monopolistic distortions.

Here we move to another special case in which one region is characterized by completely flexible prices.

4.2 Flexible Prices in One Region

In this subsection we focus on the special case in which one region, F, has flexible prices while the other region, H, has sticky prices. This is an extreme case but it offers significant insights in order to understand the more general case. Again we start by assuming that the monopolistic distortions are completely offset by distorting subsidies. The only distortion remaining is the one associated with the stickiness and staggered nature of prices in region H.

Proposition 5 *If* $\overline{y}^W = 0$ *and if production in one region is characterized by flexible prices, then it is optimal to stabilize the inflation rate in the region with sticky prices. Under the optimal policy, the paths of consumption, output and terms of trade are consistent with the efficient outcome.*

Proof. In Appendix D. ■

The conclusions of this proposition are consistent with the findings of Aoki (1998) and Erceg et al. (1999). Two significant features of proposition 5 are that: *i*) monetary authority should target only the inflation rate of the region in which prices are sticky, while targeting HICP is sub-optimal, *ii*) efficiency is obtained.

In this case, there is only one distortion and one instrument, and monetary policy has the right instrument to cope with the existing distortion. This arises because inflation itself can create dispersion of output and prices within the region. In fact changes in relative prices within a region are sources of inefficiencies given that the differentiated goods are produced according to the same technology. By committing to a zero inflation rate in the region in which prices are sticky, monetary policy can avoid the dispersion of resources within that region. Moreover the terms of trade are no longer a source of distortions, given that prices in region F are flexible and they can adjust to induce an efficient path of the terms of trade.

Proposition 6 *If* $\overline{y}^W > 0$ *and if production in one region is characterized by flexible prices, it is optimal to commit to a deterministic positive path of the inflation in the sticky-price region. This path asymptotically converges to zero.*

Proof. In Appendix D. ■

We have shown that even in this case, the optimal policy implies a deterministic positive path for the inflation rate in the sticky-price region, while again targeting HICP is sub-optimal. In contrast to proposition 6, efficiency is not obtained, because the number of distortions — in this case the monopolistic distortions and the dispersion of relative prices within one region — exceed the number of instruments, only the interest rate.

A general summary can be drawn from the special cases of these last two subsections. It is useful to define the class of 'inflation targeting' policies as the policies in which monetary authority aims at stabilizing a weighted average of the region-specific inflation rates as

$$\delta\pi^H + (1-\delta)\pi^F = 0$$

where $0 \leq \delta \leq 1$. Then if the regions have identical degrees of nominal rigidities, it is optimal to set δ equal to the economic size of region H, i.e. $\delta = n$, while if region F has flexible prices, then it is optimal to give all the weight to the inflation in the region with sticky prices, i.e. $\delta = 1$, on the contrary if prices in region H are flexible, it should be $\delta = 0$.

4.3 General Case

In this subsection we analyze the general case in which both regions are characterized by different degrees of nominal rigidity. As a reminder, there are three main distortions in this context: *i*) the monopolistic distortion that induces an inefficient level of output; *ii*) inflation in each region that creates an inefficient dispersion of prices; *iii*) stickiness of prices in both regions that may create a non efficient path of the terms of trade in response to asymmetric disturbances.

First we focus on the case in which the monopolistic distortions are completely offset. In the efficient outcome inflation rates, output gap and terms of trade gap should be zero.

Proposition 7 *If* $\overline{y}^W = 0$ *and both regions have nominal rigidities, the efficient outcome is not feasible.*

Proof. In Appendix D. ■

Corollary 8 *In a currency area with pervasive nominal rigidities and monopolistic distortions, the efficient outcome is not feasible.*

The non feasibility of the efficient outcome is explained by the lack of instruments. Monetary policy has only one instrument and it cannot cope with all the distortions. As an example consider the case in which monetary policy manages to stabilize both region-specific inflation rates. Prices do not move, and this eliminates the inefficient dispersion of output within regions, but relative prices cannot adjust to optimally react to asymmetric disturbances. Reducing the distortions in both regions is inconsistent with the efficient allocation of resources across regions. The same result of inefficiency exists also in a closed-economy model with both sticky prices and wages, as discussed by Erceg et al. (1999).

What should monetary policy do? Before characterizing the optimal policy under full commitment, we analyze the optimal rule in the case in which monetary policy can commit to the class of the 'inflation-targeting' policies. This family is of particular interest because it contains both the HICP-targeting policy and the optimal policies of the special cases outlined in the previous paragraph.

Proposition 9 *If $\overline{y}^W = 0$ and prices are sticky in both regions and if monetary authority can commit only to the class of the 'inflation targeting' policies, then it is optimal to give higher weight to the region with higher degree of nominal rigidity.*

In understanding this result, it is important to note that the output gap is itself minimized by a policy in this class in which higher weight is given to the stickier-price region. The same argument applies to the terms of trade gap and to the weighted average of the squares of inflation. Figure 1 shows how the optimal choice of the weight δ varies across the overall possible degrees of nominal rigidities α^H and α^F.[20] Consistently when the degrees of rigidity are the same, δ is set equal to the economic size n, while when $\alpha^H > \alpha^F$ we have $\delta > n$.

Further insights on this class of policies can be drawn by noting that

$$\delta\pi_t^H + (1-\delta)\pi_t^F = \mathrm{E}_t \sum_{j=0}^{\infty} \beta^j [k_C^W y_t^W + k_T^W (\widehat{T}_t - \widetilde{T}_t)] = 0,$$

where k_C^W and k_T^W are combinations of the parameters of the model. In the equilibrium implied by this class of policies the output gap and the terms of

[20] The 'calibration' of the parameters is explained in the next section, where the economic size n is set equal to 0.5.

trade gap are proportional at each date t. Moreover it is possible to show that the equilibrium paths of y_t^W, c_t, π_t^W and $\widehat{T}_t$ can be described by particular linear combinations of the variables $\widehat{T}_{t-1}$ and $\widetilde{T}_t$. If $\widetilde{T}_t$ is a Markovian process, then the latter two variables represents the smallest set of state variables that contains all the information needed to forecast the future evolution of the economy. It can be shown that in the class of policies described by all the linear combinations of these two state variables, the optimal 'inflation targeting' policy is not optimal.[21] As emphasized by Woodford (1999b), with forward-looking variables, it is not the case that the analysis of the optimal plan can be restricted to the class of policies that implies an equilibrium path in which only the state variables are relevant. Optimal control behaves differently in the presence of forward-looking variables. Here we characterize the optimal plan in the case in which the monopolistic distortions are eliminated.

Proposition 10 *If* $\overline{y}^W = 0$, $\alpha^H \neq \alpha^F$ *and if* $\widetilde{T}_t$ *follows a Markovian process, then the optimal plan involves evolution of the endogenous variables according to ARMA processes*

$$\begin{aligned} G(L)\widehat{T}_t &= V(L)\widetilde{T}_t, \\ H(L)c_t^W &= R(L)\widetilde{T}_t, \\ D(L)\widehat{\pi}_t^W &= N(L)\widetilde{T}_t, \end{aligned}$$

where L is the lag operator, and where the polynomials on the LHS are of third order while the polynomials on the RHS are of second order.

Proof. In Appendix D. ■

The above proposition shows that it is not the case that the optimal plan is only a function of the current state of the economy, but also of past states. The inertia behavior of the optimal plan has two components: one is due to the intrinsic inertia of relative prices that also affects other non-optimal plans, the other component is due to the gain in credibility that monetary policy can achieve if also "in the short tun the bank regards itself as constrained to fulfill previous (explicit or implicit) commitments" (Woodford (1999b), pg.7). Only by continuing to respond to past shocks can monetary policy establish the right commitment that pushes expectations towards the optimal allocation.

[21] To this class belongs also the discretionary equilibrium, in which monetary authority re-optimizes at each date, taking as given the past values of the state variables.

Proposition 11 *If* $\overline{y}^W > 0$, $\alpha \neq \alpha^*$ *and if* $\widetilde{T}_t$ *follows a Markovian process, the optimal plan implies the same responses to the shocks as in Proposition 10.*

Proof. In Appendix D ■

The solution of Proposition 11 differs from Proposition 10 only by the existence of the incentive to increase union output gap towards a desired level. This incentive exists only at the date in which the commitment is established. This component is deterministic and it does not affect the response of the variables to the terms of trade shock. Moreover it approaches zero asymptotically, as time goes to infinity.

5 Some Numerical Comparisons

In this section we compare the welfare outcomes that can be achieved under the different policies outlined in the previous section. We focus on the case in which $\overline{y}^W = 0$. We consider three policies: 1) the HICP targeting policy, i.e. $\pi_t^{HICP} = 0$; 2) the optimal 'inflation targeting' policy, i.e. $\delta\pi_t^H + (1-\delta)\pi_t^F = 0$, with δ chosen optimally; 3) the optimal policy. It is worth noting that each class of lower number is nested in the class with higher numbers, e.g. 1) is in 2). There is also a natural rank among these policies, the welfare is increased monotonically in passing from lower numbers to higher numbers. In order to obtain the exact magnitude of these comparisons, we need to 'calibrate' the model according to values of the parameters implied by the European economies belonging to the EMU.

The parameter β is equal to the gross real rate of return. We set β equal to 0.99, implying a real rate of return equal to 1.01 on average and on a quarterly basis. On an annual basis the real interest rate is calibrated approximately at 4 percentage points, which is the average of the real rate over the past 20 years in Germany. The parameter σ, the degree of monopolistic competition, is set such that in the steady state the mark-up of prices over marginal costs is around 15%, which is a reasonable parameter for the European economies. We thus set σ equal to 7.66.

In order to calibrate the elasticity of labor supply we suppose as in Rotemberg and Woodford (1997) that output is produced according to a production function $f(N)$ where N are hours worked, while $g(N)$ is the disutility of working for the household. We have that $v(Y) = g(f^{-1}(Y))$. We can then write the elasticity of labor supply η as

$$\eta = \epsilon_{wy} - \rho - \frac{f''f}{(f')^2}$$

where ϵ_{wy} is the elasticity of the average real wage with respect to variation in production. Rotemberg and Woodford (1997) in calibrating the US economy choose a value for ϵ_{wy} of 0.3. Following Calmfors (1998) and according to works of Layard et al. (1991) and Ball (1994), this elasticity is higher in Europe, especially under unfavorable supply shocks. Moreover several arguments point toward an increase in the flexibility of wages in Europe as a consequence of the monetary union. We choose an elasticity of 0.5. The parameter $-f''f/(f')^2$ is approximated by the share of labor in total income, 0.33. The risk aversion coefficient ρ is set equal to 1/6, as in the Rotemberg and Woodford 's estimates. Thus we have that η is equal to 0.67.

In our analysis it is crucial to calibrate the parameters indicating the degrees of nominal rigidity in the two regions, α^H and α^F, as well as the economic size of each region, n and $(1-n)$. We do not have data on the average length of individual price adjustment for Europe. For the US economy a sensible value seems 3 quarters, according to the survey of Blinder et al. (1998). In a survey on UK companies, Hall et al. (1997) find that average contracts last 2 quarters. Unfortunately there are no equivalent studies for the European economies belonging to EMU.

In our context given that the duration of a price contract in each region is $1/(1-\alpha^i)$, we define the average duration of contracts in the union as the weighted average of the duration of contracts in the regions $AD \equiv (1-\alpha^H)^{-n} \cdot (1-\alpha^F)^{n-1}$, while the relative duration is defined as $RD \equiv (1-\alpha^F) \cdot (1-\alpha^H)^{-1}$. We let AD assume the values: 2, 3, 4 , 6, 8 quarters, i.e. the average duration of contracts in Europe is assumed to be between 6 months and 2 years. Given a value for the average duration, we let the relative duration vary across the spectrum of values of α. As a clarification of the definition of average and relative duration, if the average duration is 3 quarters and the relative duration is 2, then contracts last 4 quarters in region H and 2 quarters in region F. By allowing AD to assume realistic values and RD to vary across the spectrum of the possible values, we overcome the lack of data on the degree of price rigidity.

In dividing the countries belonging to the EMU in two subgroups, we choose as criterion the heterogeneity in the degrees of nominal rigidity across

Europe: as an indicator we consider the percentage increase in wages in response to a one percentage point fall in the unemployment rate (Table 2). We label as H region the countries of which the response is less than 1%, namely Belgium, Finland, Germany, Ireland, Netherlands and Spain. The other countries belong to region F. In this case n is equal to 0.523 according to Table 1 and we approximate it to 0.5. The existence of asymmetric disturbances between region H and F is crucial in our analysis and it could have been used as a criterion to partition the countries. However the distinction between 'core' and 'periphery' is controversial and it depends on the nature of the shocks, supply or demand, monetary or non monetary. However according to studies, as Bayoumi and Eichengreen (1996) and Chamie et al. (1994), a partition in two region of approximately equal size seems a good compromise.

Finally we assume that the shock $\widetilde{T}_t$ follows a Markovian process, a first order autoregressive process of the kind $\widetilde{T}_t = \phi \widetilde{T}_{t-1} + \varepsilon_t$ where ε_t is a white-noise process. For our analysis, we do not need to estimate the variance of this process, because comparisons of welfare are independent of this variance. We set it equal to one while the coefficient of persistence is arbitrarily set equal to 0.6.

Our welfare criterion is the unconditional expectation of the welfare function W

$$\mathrm{E}[W] = -\Omega \mathrm{E}\left\{ \mathrm{E}_0 \{ \sum_{t=0}^{\infty} \beta^t L_t \} \right\},$$

as in Woodford (1999b), where we have normalized $U_C \overline{C}$ to 1. The expectation E_0 is taken at time zero based upon all the information available at that date. It is then conditional upon the initial condition $\widehat{T}_{-1} = 0$ and the initial condition on the state $\widetilde{T}_{-1}$. While the expectation E is taken by integrating over the stationary distribution of $\widetilde{T}_{-1}$.

Table 3 compares the HICP-targeting policy and the optimal 'inflation targeting' policy at certain average and relative durations. Given a fixed value for the average duration, we consider differences in the relative duration of contracts between the two regions of the 0%, 20%, 50% and 100%. Column 3 and 4 present the optimal weights in the class of the 'inflation targeting' policies. When the price-contracts has the same average length in both regions, the optimal choice coincides with the economic size of the region, in this case $\delta = n = 0.5$. Even for small differences in the durations of contracts within the union, the optimal weights change by a considerable amount. If contracts in region H last the 20% more than contracts in region F, the weight to be given to the inflation rate in the region with stickier price, region H, should be increased by the 20%, passing from 0.5 to approximately 0.6. While if contracts in region H last two times more than contracts in region F, then the weight to be given to the inflation rate in region H should be increased by the 60%, passing from 0.5 to 0.8. The last column presents the percentage reduction in the deadweight loss that society can obtain by using the optimal 'inflation targeting' policy instead of the HICP-targeting. This percentage reduction is computed as

$$DR \equiv \frac{\mathrm{E}[W_1] - \mathrm{E}[W_2]}{\mathrm{E}[W_1] - \mathrm{E}[W_3]} \times 100,$$

where $\mathrm{E}[W_1]$, $\mathrm{E}[W_2]$, $\mathrm{E}[W_3]$ are the welfare criteria associated respectively with the HICP-targeting policy, the optimal 'inflation targeting' policy and the optimal plan. The reduction in the deadweight loss is always above the 95 %.

Given a fixed average duration, Figure 2 to 6 plot the comparisons among the three classes of policies for all the values of relative duration. The dotted line corresponds to the HICP-targeting policy, the dashed line to the optimal 'inflation targeting' policy, the solid line to the optimal policy. Only when the relative duration is equal to 1, consistently with our theoretical propositions, are the three policies equivalent. For other values, the HICP-targeting policy is sub-optimal, while the optimal 'inflation targeting' policy can well approximate the feasible first-best. Given the complexity of the optimal plan, the 'inflation targeting' represents a transparent and feasible policy to implement. Figure 7 to 11 plot the deadweight loss reduction, DR. The welfare gain with respect to the HICP-targeting is remarkable. In contrast to the HICP-targeting, the optimal 'inflation targeting' induces a nearly optimal adjustment in relative prices.

It can be argued that monetary union, as a regime shift, will induce a convergence in the degree of nominal rigidities across regions, justifying then the HICP targeting. This convergence might be true among industries in the same sector belonging to different regions, but not among industries in different sectors. Our model should be interpreted along these lines.

6 Obtaining Efficiency with Multiple Currencies

We have shown that, independent of the existence of monopolistic distortions, it is never possible to obtain the efficient outcome under the hypothesis that both regions have sticky prices. It is no longer the case, as suggested by Goodfriend and King (1997) and shown through an explicit welfare analysis by Aoki (1998), that by targeting a *core* measure of inflation the efficient equilibrium can be obtained.

How can we obtain efficiency in the general framework? The answer can be understood as a problem of assignment between instruments and distortions. In models like that considered here, monetary policy plays an important role, because of the combined interaction between monopolistic competition and sticky prices. In our framework we have three distortions: two are in common with the closed-economy literature, respectively the existence of monopolistic competition and the stickiness of prices in a staggered setting; the third is due to the inertia of relative prices (the terms of trade), as a consequence of both the stickiness and the staggered way of setting prices in the two regions. In a closed economy model, if taxation is able to eliminate the monopolistic distortion, monetary policy can set its instrument to offset the remaining distortions which arise because prices are set at different dates: the efficient outcome is obtained and there is no trade-off between variability of output and variability of inflation.

Instead in a currency area the inefficient outcome is a consequence of the lack of an instrument because of the additional distortion caused by the stickiness of relative prices. In an open economy framework, this problem has a solution: the exchange rate. Here we consider the model as it is in a companion paper where the exchange rate is still an endogenous variable. The structural equation of world and relative inflation are exactly the same as in the section above

$$\pi_t^R = -\psi[\widehat{T}_t - \widetilde{T}_t] + \omega[\widehat{C}_t - \widetilde{C}_t] + \beta \mathrm{E}_t \pi_{t+1}^R, \qquad (15)$$

$$\pi_t^W = \theta[\widehat{T}_t - \widetilde{T}_t] + \kappa[\widehat{C}_t - \widetilde{C}_t] + \beta \mathrm{E}_t \pi_{t+1}^W, \qquad (16)$$

while the state equation of the term of trade becomes

$$\widehat{T}_t = \widehat{T}_{t-1} + \pi_t^R + \Delta S_t.$$

where S is the nominal exchange rate. In the efficient equilibrium where $\widehat{T}_t = \widetilde{T}_t$ and $\widehat{C}_t = \widetilde{C}_t$, it follows that $\Delta S_t = \widetilde{T}_t - \widetilde{T}_{t-1}$. From the uncovered interest parity and from the Euler equation we have

$$\begin{aligned} \mathrm{E}_t \Delta S_{t+1} &= \widehat{R}_t - \widehat{R}_t^*, & (17) \\ \mathrm{E}_t \widehat{C}_{t+1} &= \widehat{C}_t + \rho^{-1} n(\widehat{R}_t - \mathrm{E}_t \pi_{t+1}^H) + \rho^{-1}(1-n)(\widehat{R}_t^* - \mathrm{E}_t \pi_{t+1}^F). & (18) \end{aligned}$$

It follows that in the efficient equilibrium the path of interest rates should satisfy

$$\begin{aligned} \mathrm{E}_t \widetilde{T}_{t+1} - \widetilde{T}_t &= \widehat{R}_t - \widehat{R}_t^*, \\ \mathrm{E}_t \widetilde{C}_{t+1} &= \widetilde{C}_t + \rho^{-1}[n\widehat{R}_t + (1-n)\widehat{R}_t^*]. \end{aligned}$$

The exchange rate provides the instrument needed to obtain the efficient outcome. Thus decentralized monetary policies can obtain the first best if conducted in a coordinated way. This points to a cost of monetary union, similar to the one emphasized by Mundellian theory. Of course, a complete normative evaluation of monetary union would have to also consider the costs associated with foreign exchange transactions (not modeled here), and the way in which these may be increased by volatility of the exchange rate. It would also have to consider possible problems with the implementation of the optimal exchange rate policy (described above) by independent national central banks. It might be in the individual interest of one country to deviate from the jointly optimal policy, a problem that is avoided in the case of a common central bank. Furthermore, even if both national central banks commit to policies consistent with the optimal equilibrium, there may be problem of indeterminacy of the equilibrium exchange rate. This problem as well is eliminated by the adoption of a common currency. These issues require further study before any general conclusions about the desirability of a monetary union can be offered.

7 Conclusion

In this work we have analyzed a currency union in a stochastic general equilibrium model. Our simple framework has provided interesting insights on the transmission mechanism within a currency area. By using the 'taxation approach', we have been able to evaluate the exact magnitude of the distortions existing in our model. A welfare criterion has been retrieved in terms of the inflation rates, the output gap and relative prices. According to this measure, we have shown that the optimal plan implies a high degree of inertia in the inflation rate. However, a particular inflation targeting policy can well approximate the feasible first-best. In this inflation targeting policy higher weight should be given to the region with higher degree of nominal rigidity.

In Appendix E the extension to a K-region economy is presented. The main results still hold. In this direction our model proposes further insights into the debate on the various measures of *core* inflation, see Bryan and Cecchetti (1994). A general equilibrium approach to *core* inflation has been introduced by Aoki (1998). In this context it is natural to define *core* inflation as the inflation rate that monetary policy should stabilize in order to maximize the welfare. Our prescription is then that monetary policy should stabilize a weighted average of the sectorial inflation rates, but with higher weight to be given to the sector with higher degree of nominal rigidity. This is consistent with the idea of Bryan and Cecchetti (1994) that *core* inflation should be inferred from the expectations-based price setters. However, even changes in prices, smaller than the average, should be excluded from the relevant inflation index, if associated with sector characterized by low degree of nominal rigidity.

However, there are other directions in which the main policy prescription of this paper is not robust. In the presence of other sources of rigidity, e.g. wage stickiness as in Erceg et al. (1999), monetary policy should take in consideration wage inflation in formulating its optimal policy. Instead if the monopolistic distortions are time varying, it is the case that monetary policy should target an appropriate output gap as well as inflation. Moreover, if the liquidity services from holding money are not marginal, monetary policy should also pay attention to the volatility of the nominal interest rate. This is also the case if the zero-floor bound on the nominal interest rate is embedded in the analysis, as in Woodford (1999a).

These qualifications affects our final conclusions but not our emphasis on the distortions arisen because relative prices are sticky. In a more general setting monetary policy should balance the latter distortions with others in order to formulate the optimal policy.

Finally, this paper has only focused on the optimal path of the economy without analyzing how this optimal plan can be implemented. Further research on how monetary policy should use all it information in order to implement the optimal policy has to be conducted.

References

[1] Aoki, Kosuke [1998], "Optimal Monetary Responses to Relative Price Changes", unpublished, Princeton University.

[2] Ball, Laurence [1994], "What Determines the Sacrifice Ratio?", in Mankiw, N.G., ed., *Monetary Policy,* Chicago: Chicago University Press.

[3] Bayoumi, T., and B. Eichengreen [1996], "Operationalizing the Theory of Optimum Currency Areas", CEPR Discussion Paper No. 1484.

[4] Bean, Charles [1999], "Monetary Policy Under EMU", *Oxford Review of Economic Policy*, vol.14 No.3.

[5] Bernanke, Ben and Michael Woodford [1997], "Inflation Forecast and Monetary Policy", *Journal of Money, Credit and Banking,* 24: 653-684.

[6] Blinder, Alan S. [1998], *Central Banking in Theory and Practice*, Cambridge: MIT Press.

[7] Blinder, Alan S., Elie R.D. Canetti, David E. Lebow, and Jeremy B. Rudd [1998], *Asking About Prices: A New Approach to Understanding Price Stickiness,* New York: Russell Sage Foundation.

[8] Bryan, Michael F. and Stephen G. Cecchetti [1994], "Measuring Core Inflation", in Mankiw, N.G., ed., *Monetary Policy,* Chicago: Chicago University Press.

[9] Brock, William A. [1974], "Money and Growth: The Case of Long-Run Perfect Foresight", *International Economic Review* 15: 750-777.

[10] Calmfors, Lars [1998], "Macroeconomic Policy, Wage Setting and Employment — What Difference Does the EMU Make?", Institute for International Economic Studies Seminar Paper No.657, Stockholm University.

[11] Calvo, Guillermo A. [1983], "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics* 12: 383-398.

[12] Cardia, Emanuela [1991], "The Dynamic of a Small Open Economy in Response to Monetary, Fiscal and Productivity Shocks", *Journal of Monetary Economics* 28: 411-434.

[13] Chamie, N., Deserres, A., and Lalonde, R. [1994], "Optimum Currency Areas and Shock Asymmetry: A Comparison of Europe and the United States", Bank of Canada Working Paper 94/1.

[14] Clarida, Richard and Jordi Gal [1995], "Sources of Real Exchange Rate Movements: How Important are Nominal Shocks?", *Carnegie Rochester Conference Series on Public Policy,* 41.

[15] Clarida, Richard, Jordi Galí and Mark Gertler [1997], "Monetary Policy Rules in Practice: Some International Evidence", *Quarterly Journal of Economics*, forthcoming.

[16] Clarida, Richard, Jordi Galí and Mark Gertler [1999], "The Science of Monetary Policy", *Journal of Economic Literature*, forthcoming.

[17] Cole, Harold and Maurice Obstfeld [1991], "Commodity trade and international risk sharing: How much do financial markets matter?", *Journal of Monetary Economics*, 28: 3-24.

[18] Corsetti, Giancarlo and Paolo Pesenti [1997], "Welfare and Macroeconomic Interdependence", NBER Working Paper No. 6307.

[19] Dornbusch Rudiger, C.Favero and F.Giavazzi [1998], "The Immediate Challenges for the European Central Bank", *Economic Policy*, 26, 15-64.

[20] ECB [1999], *Monthly Bulletin*, European Central Bank, January 1999.

[21] Erceg, Christopher J., Dale W. Henderson, and Andrew T. Levin [1999], "Optimal Monetary Policy with Staggered Wage and Price Contracts", Federal Reserve Board of Governors Discussion Paper no. 1999.640.

[22] Galí, Jordi and Mark Gertler [1998], "Inflation Dynamics: A Structural Econometric Analysis", *Journal of Monetary Economics*, forthcoming.

[23] Ghironi, Fabio [1999], "Alternative Monetary Rules for a Small Open Economy: The Case of Canada", unpublished, Federal Reserve Bank of New York.

[24] Goodfriend, Marvin S., and Robert G. King [1997], "The New Neoclassical Synthesis and the Role of Monetary Policy", in B.S. Bernanke and Rotemberg (eds.), *NBER Macroeconomic Annual 1997*, Cambridge, MA: MIT Press, 231-82.

[25] Hall, Simon, M. Walsh and Anthony Yates [1997], "How Do UK Companies Set Prices?", unpublished, Bank of England.

[26] Kerr, William and Robert G. King [1996], "Limits on Interest Rate Rules in the IS Model", Federal Reserve Bank of Richmond, *Economic Quarterly* volume 82/2 Spring 1996.

[27] King, Robert G. and Alexander L. Wolman [1998], "What Should Monetary Authority Do When Prices are Sticky?" in J.B.Taylor (ed.), *Monetary Policy Rules.* University of Chicago Press for NBER.

[28] Kollmann, R., [1996], "The Exchange Rate in a Dynamic-Optimizing Current Account Model with Nominal Rigidities: A Quantitative Investigation", unpublished, Université de Montréal.

[29] Layard, P.R.G., Stephen Nickell and Richard Jackman [1991], *Unemployment: macroeconomic performance and the labour market,* Oxford University Press.

[30] Lane, Philip R.[1998], "The New Open Economy Macroeconomics: A Survey", *Journal of International Economics,* forthcoming.

[31] Mendoza, Enrique G. [1991], "Real Business Cycles in a Small Open Economy", *American Economic Review*, 81: 797-818.

[32] Mundell, Robert A. [1961], "A Theory of Optimum Currency Areas", *American Economic Review* 51: 657-675.

[33] Nickell, S. [1997], "Unemployment and Labor Market Rigidities: Europe versus North America", *Journal of Economic Perspectives,* 11.

[34] Obstfeld, Maurice [1985], "Floating Exchange Rates: Experience and Prospectus", *Brookings Papers on Economics Activity, 2,* 369-450.

[35] Obstfeld, Maurice, and Kenneth Rogoff [1995], "Exchange Rate Dynamics Redux", *Journal of Political Economy* 103, 624-660.

[36] Obstfeld, Maurice, and Kenneth Rogoff [1996], *Foundations of International Macroeconomics.* Cambridge, MA: MIT Press.

[37] Obstfeld, Maurice, and Kenneth Rogoff [1998], "Risk and Exchange Rates", NBER Working Paper no. 6694.

[38] Peersman, Gert and Frank Smets [1999], "Uncertainty and the Taylor Rule in a Simple Model of the Euro Area Economy", unpublished.

[39] Roberts, John M. [1998], "Inflation Expectations and the Transmission of Monetary Policy", FEDS Discussion Paper no. 1998-43, Federal Reserve Board.

[40] Rotemberg, Julio J., and Michael Woodford [1998], "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy", in B.S. Bernanke and Rotemberg (eds.), *NBER Macroeconomic Annual 1997*, Cambridge, MA: MIT Press. 297-346.

[41] Rotemberg, Julio J., and Michael Woodford [1998], "Interest Rate Rules in an Estimated Sticky Price Model", in J.B.Taylor (ed.), *Monetary Policy Rules.* University of Chicago Press for NBER.

[42] Rudebush, Glenn D. and Lars E.O. Svensson [1999], "Eurosystem Monetary Targeting: Lessons from U.S. Data", unpublished, Federal Reserve Bank of San Francisco.

[43] Sidrauski, Miguel [1967], "Rational Choice and Patterns of Growth in a Monetary Economy", *American Economic Review* 57(2): 534-544.

[44] Smets, Frank [1995], "Central Bank Macroeconometric Models and the Monetary Policy Transmission Mechanism," in *Financial Structure and the Monetary Policy Transmission Mechanism,* Bank for International Settlements (Basle, Switzerland), 225-266.

[45] Svensson, Lars E.O. [1999a], "Monetary Policy Issues for the Eurosystem", *Carnegie-Rochester Conference Series on Public Policy,* forthcoming.

[46] Svensson, Lars E.O. [1999b], "How Should Monetary Policy Be Conducted in an Era of Price Stability", unpublished, IIES.

[47] Taylor, John [1993], *Macroeconomic Policy in World Economy: From Econometric Design to Practical Operation*, New York: W.W.Norton.

[48] Uzawa, Hirofumi [1968], "Time Preference, the Consumption Function and Optimum Asset Holdings," in J.N.Wolfe, ed., *Value, Capital and Growth: Papers in Honor of Sir John Hicks,* Edinburgh: Edinburgh University Press, 1968, pp.485-504.

[49] Weerapana, Akila [1998], "The Performance of Simple Monetary Policy Rules in a Large Open Economy", unpublished, Stanford University.

[50] Woodford, Michael [1996], "Control of the Public Debt: A Requirement for Price Stability?", NBER Working Paper no. 5684.

[51] Woodford, Michael [1999a], "Inflation Stabilization and Welfare", unpublished, Princeton University.

[52] Woodford, Michael [1999b], "Optimal Monetary Policy Inertia", NBER Working Paper no. 7261.

[53] Woodford, Michael [1999c], "Commentary: How Should Monetary Policy Be Conducted in an Era of Price Stability?", unpublished, Princeton University.

[54] Yun, Tack [1996], "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles," *Journal of Monetary Economics* 37: 345-370.

Appendix A

In this appendix we describe the model in details. The whole area is populated by a continuum of agents on the interval $[0, 1]$. The population on the segment $[0, n)$ belongs to the region H, while the segment $[n, 1]$ belongs to F. There is no possibility of migration across regions. A generic agent j belonging to the currency area is both producer and consumer: a producer of a single differentiated product and a consumer of all the goods produced in both region H and F. Thus all goods produced are traded between regions. Preferences of the generic household j are given by

$$U_t^j = \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s^j) + L\left(\frac{M_s^j}{P_s^i}, \xi^i \right) - V(y_s^j, z_s^i) \right],$$

where the upper index j denotes a variable that is specific to agent j, while the upper index i denotes a variable that is specific to region i. We have that $i = H$ if $j \in [0, n)$, while $i = F$ if $j \in [n, 1]$. E_t denotes the expectation conditional on the information set at date t, while β is the intertemporal discount factor, with $0 < \beta < 1$.

Agents obtain utility from consumption and from the liquidity services of holding money, while they receive disutility from producing goods. The utility function is separable in these three factors. We have that U is an increasing concave function of the index C^j defined as

$$C^j \equiv \frac{(C_H^j)^n (C_F^j)^{1-n}}{n^n (1-n)^{1-n}} \tag{A.1}$$

and C_H^j and C_F^j are indexes of consumption across the continuum of differentiated goods produced respectively in region H and F. Specifically,

$$C_H^j \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_o^n c^j(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}}, \qquad C_F^j \equiv \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c^j(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}. \tag{A.2}$$

We have that σ, which is assumed greater than one, is the elasticity of substitution across goods produced within a region, while the elasticity of substitution between the bundles C_H and C_F is 1. The parameter n denotes both the population size and the 'economic' size of region H, where the 'economic size' is the share of the bundle of goods produced within that region in the consumption index.

As in the models of Sidrauski (1967) and Brock (1974), L is an increasing concave function of the real money balances, while ξ^i is a region-specific shock to the liquidity preference; we will interpret it as an exogenous disturbance to money demand. Agents derive utility from the real purchasing power of money, where M_t^j is the agent j's money balance at the end of date t, while P^i is the appropriate region-specific price index used to deflate M_t^j. Here P^i is defined as

$$P^i \equiv (P_H^i)^n (P_F^i)^{1-n},$$

$$P_H^i \equiv \left[\left(\frac{1}{n}\right)\int_o^n p^i(h)^{1-\sigma}dh\right]^{\frac{1}{1-\sigma}}, \qquad P_F^i \equiv \left[\left(\frac{1}{1-n}\right)\int_n^1 p^i(f)^{1-\sigma}df\right]^{\frac{1}{1-\sigma}},$$

where $p^i(h)$ is the price of good h sold in the market of region i. The price index P^i is properly defined as the minimum expenditure in region i required to purchase goods resulting in the consumption index of C^j, such that $C^j = 1$. Similar definitions are given for P_H^i and P_F^i. Here we assume that there are no transaction costs in transporting goods across regions; furthermore prices are set considering the whole area as a common market. It follows that $p^H(h) = p^F(h)$ and $p^H(f) = p^F(f)$. Given these assumptions and given the structure of the preferences, it is also the case that purchasing power parity holds, i.e. $P^H = P^F$. We can then drop the index i from the consumption-based price indexes.

Here we define the terms of trade T of region F as the ratio of the price of the bundle of goods produced in region F relative to the price of the bundle imported from region H. We have then $T \equiv P_F/P_H$.

Finally V is an increasing convex function of agent j's supply of its product y^j. Assuming that agents have disutility of working $g(N^j)$, where N^j is the number of hours worked by agent j, and that the production function is $y^j = f(N^j)$, we can interpret $V(y^j)$ as being equal to $g(f^{-1}(y^j))$. We have that z^i is region-specific stochastic disturbances. We will interpret it as a productivity shock.

Given a decision on C^j, household j allocates optimally the expenditure on C_H^j and C_F^j by minimizing the total expenditure PC^j under the constraint given by (A.1). Then given the decisions on C_H^j and C_F^j, household j allocates the expenditure among the differentiated goods by minimizing $P_H C_H^j$ and $P_F C_F^j$ under the constraints given by (A.2). The demands of the generic good h, produced in region H, and of the generic good f, produced in region F are

$$c^j(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma} T^{1-n} C^j, \qquad c^j(f) = \left(\frac{p(f)}{P_F}\right)^{-\sigma} T^{-n} C^j. \qquad \text{(A.3)}$$

Furthermore we assume that each fiscal authority allocates a level of public expenditure only among the goods produced in the region of its sovereignty. The public expenditure production functions are respectively for the fiscal authority of regions H and F

$$G^H = \left[\frac{1}{n}\int_0^n g(h)^{\frac{\sigma-1}{\sigma}} dh\right]^{\frac{\sigma}{\sigma-1}}, \qquad G^F = \left[\frac{1}{1-n}\int_n^1 g(f)^{\frac{\sigma-1}{\sigma}} df\right]^{\frac{\sigma}{\sigma-1}};$$

and they imply the following demands of the generic goods h and f

$$g(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma} G^H, \qquad g(f) = \left(\frac{p(f)}{P_F}\right)^{-\sigma} G^F. \tag{A.4}$$

Combining (A.3) with (A.4) we can write total demand of good h and f as

$$y^d(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma} [T^{1-n}C^W + G^H], \qquad y^d(f) = \left(\frac{p(f)}{P_F}\right)^{-\sigma} [T^{-n}C^W + G^F] \tag{A.5}$$

where the union aggregate consumption C^W is defined as

$$C^W \equiv \int_0^1 C^j dj.$$

Regions differ from one another also in the structure of the assets traded. Within each region agents can trade in a set of securities that is sufficient to completely span all the states of nature. Instead, in trading securities across regions, agents are allowed to take positions only in a nominal non-contingent bond denominated in units of the union currency. Markets are complete within a region, but incomplete at an interregional level. The budget constraint of the household j in region i (expressed in real terms with respect to the price index) is for each state s_t at date t, and for each date t

$$\mathrm{E}_t\{q_t^i B_t^{i,j}\} + \frac{B_t^j}{P_t(1+R_t)} + \frac{M_t^j}{P_t} \leq W_{t-1}^j + (1-\tau^i)\frac{p_t(j)y_t(j)}{P_t} - C_t^j + \frac{TR_t^{i,j}}{P_t},$$

with

$$W_{t-1}^j \equiv B_{t-1}^{i,j} + \frac{M_{t-1}^j + B_{t-1}^j}{P_t}.$$

where $B_t^{i,j}$ is the real value at time $t+1$ of the portfolio held by agent j composed by contingent securities issued in region i denominated in units of the consumption-based price index with maturity one-period while q_t^i is the vector of the security prices.[22] We have that B_t^j is the household j's holding of the nominal one-period non-contingent bond denominated in the union currency, that is traded among all the households belonging to the union. The nominal interest rate on this bond which is certain at the issuing date is R_t; $Q_t^{i,j}$ are nominal lump sum transfers from the fiscal authority of region i in which j resides to the household j, while τ^i is a regional proportional tax on nominal income. The budget constraint at date t of the fiscal authority of region i for $i = H$ or F is

$$\tau^i \int_{j \in i} p_t(j) y_t(j) dj = \int_{j \in i} M_t^j - \int_{j \in i} M_{t-1}^j + G_t^i + \int_{j \in i} Q_t^{i,j},$$

where we have assumed that seignorage is returned to each region according to its source; M^U, the level of money supplied by the common central bank, is equal to the aggregate demand of money

$$M^U = \int_0^1 M_t^j dj.$$

We set the initial conditions $B_{-1}^{i,j} = B_{-1}^j = 0 \ \forall j \in [0,1]$. Given the sequences of prices and incomes, and given the initial conditions, the problem of allocation of consumption is completely characterized by the utility function and the resource constraint. The latter is derived by combining an appropriate borrowing limit with the budget constraint of the households. Because households have identical preferences and because markets are complete within each region, the assumption that the initial wealth is identical among agents belonging to the same region implies that there is perfect risk sharing of consumption within each region. We can then synthesize the optimal plan of consumption by focusing only on the consumption of the "representative agents" of regions H and F. The exhaustion of the intertemporal resource constraint and the Euler equations (if we assume an interior optimum) describe the optimal allocation. We have the following optimality conditions: (i) that

$$\beta^{T-t} \frac{U_C(C_T^i(s_T))}{U_C(C_t^i)} = R_{t,T}^i(s_T) \tag{A.6}$$

at each state $s_T \in S_T$, for each date t and every T, with $T > t$ and for $i = H$ or F; this is an optimality condition which equates the marginal rate of substitution, between future consumption in a particular state and present consumption, to the

[22] At each date t the economy faces one of finitely many states ($s_t = 1, 2, 3 ... S_t$). With h_t we denote the history of the states up to date t.

appropriate stochastic discount factor $R^i_{t,T}$; (ii) that

$$L_{M/P}\left(\frac{M^i_t}{P_t},\xi^i\right)=\frac{R_t}{1+R_t}U_C(C^i_t) \tag{A.7}$$

at each date t and for each $i=H$ or F, where $L_{M/P}$ is the derivative of L with respect to the real money balance; here the marginal rate of substitution between real money balances and consumption is equated to the user cost in terms of the consumption good index of holding an extra unit of real money balances for one period; (iii) that the resource constraint holds with equality at each date t and in every history h_t. We can use the optimality conditions to price the internationally traded bond obtaining at each date t and for $i=H$ or F

$$U_C(C^i_t)=(1+R_t)\beta\mathrm{E}_t\left\{U_C(C^i_{t+1})\frac{P_t}{P_{t+1}}\right\}. \tag{A.8}$$

A no arbitrage implication of (A.8) is that

$$\mathrm{E}_t\left[\frac{U_C(C^H_{t+1})}{U_C(C^H_t)}\frac{P_t}{P_{t+1}}\right]=\mathrm{E}_t\left[\frac{U_C(C^F_{t+1})}{U_C(C^F_t)}\frac{P_t}{P_{t+1}}\right] \tag{A.9}$$

at each date t. In the equilibrium, the contingent region-specific real bonds are in zero-net supply within each region, while the global holdings of the nominal bond should satisfy at each date t

$$nB^H_t+(1-n)B^F_t=0.$$

Once we integrate the budget constraints among all the households belonging to the same region, by using the government budget constraint and equation (A.5), we can write the aggregate resource constraint of each region i at date t as

$$\frac{B^i_t}{P_t(1+R_t)}=\frac{B^i_{t-1}}{P_t}+C^W_t-C^i_t, \tag{A.10}$$

where all the variables in (A.10) are per-capita variables. Given the initial condition $B^i_{t-1}=0$ for each $i=H$ and F the interregional traded bond is redundant.

Lemma 1 Given (A.10) and the optimality conditions describing the optimal path of consumption (i),(ii),(iii), and given the initial condition $B^i_{t-1}=0$ for each $i=H$, F it follows that $B^i_T=0$ $\forall T\geq t$ for each $i=H$, F.

Proof. We use a proof by induction where the proposition M_T has been defined as $M_T := B_T^i = 0$ for each $i = H, F$ and $\forall T \geq t-1$.

By using the assumption that $B_{t-1}^i = 0$ for each i, we have that $M_{t-1} = 0$. It remains to prove that for a $T > t$ if M_{T-1} is true, M_T is also true. If M_{T-1} is true, $B_{T-1}^i = 0$ for each i. By using (A.10) we have that in a generic state $s_T \in S_T$ at date T

$$\begin{aligned} C_T^H &= C_T^W - \frac{B_T^H}{P_T(1+R_T)}, \\ C_T^F &= C_T^W - \frac{B_T^F}{P_T(1+R_T)}. \end{aligned}$$

At date $T+1$ in each state $s_{T+1} \in S_{T+1}$ the optimality conditions of the representative household i for each $i = H$ and F are

$$U_C(C_{T+1}^i(s_{T+1})) = (1+R_{T+1})\beta \mathrm{E}_{T+1}\left\{U_C(C_{T+2}^i)\frac{P_{T+1}}{P_{T+2}}\right\}, \tag{A.11}$$

$$\{U_C(C_{T+s}^i)\} = \mathrm{E}_{T+s}\left\{(1+R_{T+s})\beta U_C(C_{T+s+1}^i)\frac{P_{T+s}}{P_{T+s+1}}\right\} \text{ for } s > 1, \tag{A.12}$$

$$\left\{\sum_{k=T+1}^{\infty} R_{T+1,k}^r C_k^i(s_k)\right\} = \frac{B_T^i}{P_{T+1}} + \left\{\sum_{k=T+1}^{\infty} R_{T+1,k}^r C_k^W(s_k)\right\}, \tag{A.13}$$

where the discount factor has been defined as

$$\begin{aligned} R_{T+1,k}^r &= \frac{P_k}{P_{T+1}\prod_{s=T+1}^{k-1}(1+R_s)} \text{ for } k > T+1, \\ R_{T+1,T+1}^r &= 1. \end{aligned}$$

and where (A.13) has been obtained after iterating (A.10) and it consists of a set of conditions corresponding to any possible history starting from each state $s_{T+1} \in S_{T+1}$ at date $t+1$. If $B_T^H = B_T^F$ the optimal allocations of consumptions of households of region H and F are exactly the same looking ahead from period $T+1$ in each state $s_{T+1} \in S_{T+1}$. Thus we can write $C_{T+1}^H(s_{T+1})$ and $C_{T+1}^F(s_{T+1})$ as they were implicitly defined by the same indirect function, which is state-dependent, of respectively B_T^H and B_T^F,

$$\begin{aligned} C_{T+1}^H(s_{T+1}) &= \Gamma_{s_{T+1}}(B_T^H), \\ C_{T+1}^F(s_{T+1}) &= \Gamma_{s_{T+1}}(B_T^F), \end{aligned}$$

and this is true for each $s_{T+1} \in S_{T+1}$. Moreover $\Gamma_{s_{T+1}}$ is a non-decreasing monotone function of the initial level of assets, B_T^H or B_T^F. From the equilibrium condition at time T we have $nB_T^H + (1-n)B_T^F = 0$. If $B_T^H > 0$, it follows that $C_{T+1}^H(s_{T+1}) > C_{T+1}^F(s_{T+1})$ for each $s_{T+1} \in S_{T+1}$, while $C_T^H < C_T^F$. But this violates the optimality condition[23]

$$\mathrm{E}_T \left\{ \left[\frac{U_C(C_{T+1}^H)}{U_C(C_T^H)} - \frac{U_C(C_{T+1}^F)}{U_C(C_T^F)} \right] \frac{P_T}{P_{T+1}} \right\} = 0, \tag{A.14}$$

because the term in the square bracket is negative across all the states and prices are always positive. It should be then that $B_T^H = B_T^F = 0$. ■

A corollary of this conclusion is that there is perfect risk sharing of consumption between regions, i.e. $C^H = C^F = C^W$ at any time and at any state.

To complete the demand side of the economy we compute aggregate demand in both regions by using the appropriate Dixit-Stiglitz aggregators related to (A.2)

$$Y^H \equiv \left[\left(\frac{1}{n} \right) \int_o^n y^d(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}}, \qquad Y^F \equiv \left[\left(\frac{1}{1-n} \right) \int_n^1 y^d(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}. \tag{A.15}$$

After applying (A.15) to (A.5) we obtain

$$Y^H = T^{1-n}C + G^H, \qquad Y^F = T^{-n}C + G^F. \tag{A.16}$$

While consumption is completely insured, aggregate production can vary between regions. From (A.16), it follows that changes in the terms of trade explain divergences in output. Moreover the response of a regional output to changes in the terms of trade is bigger the smaller the size of that region, i.e. the higher the degree of openness.

The model is closed by identifying the instrument of monetary policy. In this model we let the common central bank set its instrument in terms of the one-period risk free nominal interest rate on the nominal bond denominated in the common currency.

Firms and Price Setting

Sellers are monopolists in selling their products. Demand (A.5) is not taken as given, but it can be affected by different price decisions $p(z)$. On the other hand sellers are small with respect to the overall market and they take as given the indexes P, P_H, P_F and C. Monopolistic competition does not imply price rigidity,

[23] I am grateful to Cedric Tille for pointing out this last observation

but it creates the environment in which price rigidity can exist without violating any individual rationality participation constraint, assuming that the sequences of shocks is bounded. Prices are subjected to changes at random intervals as in Calvo (1983). In each period a seller faces a fixed probability $1-\alpha$ of adjusting its price, irrespective on how long it has been since the seller had changed its price. In this event the price is chosen to maximize the expected discounted profits under the circumstance that the decision on the price is still maintained; in fact the seller also considers that the price chosen at a certain date t will apply in the future at date $t+k$ with probability α^k. It is important to note that all the sellers that belong to the same region and that can modify their price at a certain time will face the same discounted future demands and future marginal costs under the hypothesis that the new price is maintained. Thus they will set the same price. We denote with $\widetilde{p}_t(j)$ the price of the good j chosen at date t and with $\widetilde{y}_{t,t+k}(j)$ the total demand of good j at time $t+k$ under the circumstances that the price $\widetilde{p}_t(j)$ still applies. We have that j can be equal to h or f according to notation in (A.5), if the seller is respectively in region H or F. The function to maximize is

$$\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha^i \beta)^k \left[\lambda_{t+k}(1-\tau^i)\widetilde{p}_t(j)\widetilde{y}_{t,t+k}(j) - V(\widetilde{y}_{t,t+k}(j), z^i_{t+k}) \right], \qquad \text{(A.17)}$$

where revenues are evaluated using the marginal utility of nominal income $\lambda_{t+k} = U_C(C_{t+k})/P_{t+k}$ which is the same for all the consumers belonging to the union, because of both the hypothesis of complete markets within each region and of the result of redundancy of the interregional bond. The upper index i, as in the previous section, denotes region-specific variables, with $i = H$ if $j = h$ and $i = F$ if $j = f$. From (A.5) $\widetilde{y}_{t,t+k}(h)$ and $\widetilde{y}_{t,t+k}(f)$ are

$$\begin{aligned} \widetilde{y}_{t,t+k}(h) &= \left(\frac{\widetilde{p}_t(h)}{P_{H,t+k}} \right)^{-\sigma} \left[T^{1-n}_{t+k} C_{t+k} + G^H_{t+k} \right], \\ \widetilde{y}_{t,t+k}(f) &= \left(\frac{\widetilde{p}_t(f)}{P_{F,t+k}} \right)^{-\sigma} \left[T^{-n}_{t+k} C_{t+k} + G^F_{t+k} \right]. \end{aligned}$$

The seller maximizes (A.17) with respect to $\widetilde{p}_t(j)$ taking as given the sequences $\{P_{H,t}, P_{F,t}, P_t, C_t, G^i_t\}$, the optimal choice of $\widetilde{p}_t(j)$ is

$$\widetilde{p}_t(j) = \frac{\sigma}{(\sigma-1)(1-\tau^i)} \frac{\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha^i \beta)^k V_y(\widetilde{y}_{t,t+k}(j), z^i_{t+k}) \widetilde{y}_{t,t+k}(j)}{\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha^i \beta)^k \lambda_{t+k} \widetilde{y}_{t,t+k}(j)}. \qquad \text{(A.18)}$$

Calvo-price setting implies the following state equation for $P_{H,t}$ and $P_{F,t}$

$$P^{1-\sigma}_{H,t} = \alpha^H P^{1-\sigma}_{H,t-1} + (1-\alpha^H)\widetilde{p}_t(h)^{1-\sigma}, \qquad \text{(A.19)}$$

$$P_{F,t}^{1-\sigma} = \alpha^F P_{F,t-1}^{1-\sigma} + (1-\alpha^F)\tilde{p}_t(f)^{1-\sigma}, \tag{A.20}$$

because in each region the fraction $(1-\alpha)$ of sellers, that is chosen to adjust the price, sets the same price.

Equilibrium

We can describe the equilibrium of this model by combining the aggregate demand block with the aggregate supply. Our model is not solvable in a closed form solution. However we focus on equilibria where the state variables follow paths that are close to a deterministic stationary equilibrium, in which the inflation rates are zero. In this steady state, we interpret the stochastic shocks $\{\xi_t^i, G_t^i, z_t^i\}$ for $i = H$ or F as zero at all dates. The instrument of monetary policy, the interest rate rule, is set in order to anchor the nominal interest rate to the inverse of the intertemporal discount factor in the consumer preferences. We have then a stationary equilibrium in which

$$1 + \overline{R} = \frac{1}{\beta}.$$

As it is common in models with monopolistic competition, the marginal utility of consumption is not equated to the marginal disutility of producing output. From the pricing decision in region H we obtain

$$(1-\tau^H)U_C(\overline{C}) = \frac{\sigma}{\sigma-1}\overline{T}^{1-n}V_y\left(\overline{T}^{1-n}\overline{C}, 0\right), \tag{A.21}$$

while in region F we have

$$(1-\tau^F)U_C(\overline{C}) = \frac{\sigma}{\sigma-1}\overline{T}^{-n}V_y\left(\overline{T}^{-n}\overline{C}, 0\right). \tag{A.22}$$

If $\tau^H = \tau^F$ from (A.21) and (A.22) it follows that $\overline{T} = 1$ and that $\overline{Y}^H = \overline{Y}^F = \overline{C}$. In this deterministic equilibrium prices are well determined by the initial conditions $P_{H,-1}$, $P_{F,-1}$. In fact the decision of the policy makers to set its instrument to $1/\beta$ and the consequent stationary equilibrium with zero inflation rates imply the determination of the paths of $\overline{P}_H$, $\overline{P}_F$. By using the money demand conditions (A.7) and the equilibrium condition in the money market we can pin down the level of money.

Here we describe the stochastic equilibrium which arises from perturbations around the deterministic equilibrium identified above. Given a broad class of interest rate rules, the aggregate demand block groups conditions (A.8), (A.9), (A.16).[24]

[24] The transversality conditions have to hold in our rational expectations equilibrium because we are considering small pertubations around the initial deterministic steady state.

We can interpret these conditions as determining the sequences $\{C_t, R_t, M_t\}$ given the initial conditions $P_{H,-1}$, $P_{F,-1}$, the sequences of prices $\{P_{H,t},P_{F,t}\}$ and the sequences of stochastic shocks $\{\xi_t^i,G_t^i,z_t^i\}$ for $i = H$ or F. It is worth noting that with an interest rate rule the money-market equilibrium determines only the level of money in the union. Moreover shocks to the liquidity preference have only repercussions on the path of money while they do not affect the other variables.

We turn to the aggregate supply blocks which is composed by conditions (A.18), (A.19), (A.20). Given the sequences of consumption $\{C_t\}$, the initial conditions $P_{H,-1}$, $P_{F,-1}$, and the sequences of shocks, it determines the sequence of prices $\{P_{H,t},\widetilde{p}_t(h),P_{F,t},\widetilde{p}_t(f)\}$.

Appendix B

In this appendix, we derive the log-linear approximation of region H's AS equation, equation (4) in the text. The derivation of the region F's supply side follows in a specular way. Given the sequences $\{C_t\}$, the sequences of shocks and the initial conditions, the optimal paths of prices $\{\widetilde{p}_t(h), P_{H,t}\}$ is described by the following conditions

$$\widetilde{p}_t(h) = \frac{\sigma}{(\sigma-1)(1-\tau^H)} \frac{\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha^H \beta)^k V_y(\widetilde{y}^d_{t,t+k}(h), z^H_{t+k}) \widetilde{y}^d_{t,t+k}(h)}{\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha^H \beta)^k \lambda_{t+k} \widetilde{y}^d_{t,t+k}(h)}, \quad \text{(B.1)}$$

$$P^{1-\sigma}_{H,t} = \alpha^H P^{1-\sigma}_{H,t-1} + (1-\alpha^H)\widetilde{p}_t(h)^{1-\sigma}, \quad \text{(B.2)}$$

where

$$\widetilde{y}^d_{t,t+k}(h) = \left(\frac{\widetilde{p}_t(h)}{P_{H,t+k}}\right)^{-\sigma} [T^{1-n}_{t+k} C_{t+k} + G^H_{t+k}]. \quad \text{(B.3)}$$

We can write (B.1) as

$$0 = \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha^H \beta)^k \{[(1-\sigma)(1-\tau^H)\lambda_{t+k}\widetilde{p}_t(h) + \\ + \sigma V_y(\widetilde{y}^d_{t,t+k}(h), z^H_{t+k})]\widetilde{y}^d_{t,t+k}(h)\},$$

and after substituting the expression for λ_{t+k}

$$\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha^H \beta)^k \left\{ \left[\begin{array}{c} (1-\sigma)(1-\tau^H) U_C(C_{t+k}) \frac{\widetilde{p}_t(h)}{P_{t+k}} + \\ + \sigma V_y(\widetilde{y}^d_{t,t+k}(h), z^H_{t+k}) \end{array} \right] \right\} \widetilde{y}^d_{t,t+k}(h) = 0,$$

or

$$\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha^H \beta)^k \left\{ \left[\begin{array}{c} (1-\sigma)(1-\tau^H) U_C(C_{t+k}) \frac{\widetilde{p}_t(h)}{P_{H,t+k}} T^{n-1}_{t+k} + \\ + \sigma V_y(\widetilde{y}^d_{t,t+k}(h), z^H_{t+k}) \end{array} \right] \widetilde{y}^d_{t,t+k}(h) \right\} = 0, \quad \text{(B.4)}$$

where $T_{t+k} = P_{F,t+k}/P_{H,t+k}$. We take a log-linear approximation of this equilibrium condition around a steady state in which $C_t = \overline{C}$, $T_t = 1$, $\widetilde{p}_t(h)/P_{H,t} = 1$, $G^H_t = 0$, $z^H_t = 0$ and $(1-\tau^H)U_C(\overline{C}) = \frac{\sigma}{\sigma-1} V_y(\overline{C}, 0)$ at all times, obtaining

$$0 = \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha^H \beta)^k \{(1-\sigma)(1-\tau^H) U_C(\overline{C}) \widehat{p}_{t,t+k} +$$

$$+(1-\sigma)(1-\tau^H)U_C(\overline{C})[-(1-n)\widehat{T}_{t+k}]$$
$$+(1-\sigma)(1-\tau^H)U_{CC}(\overline{C})\overline{C}\widehat{C}_{t+k}+\sigma\overline{C}V_{yy}(\overline{C},0)[-\sigma\widehat{p}_{t,t+k}+$$
$$+(1-n)\widehat{T}_{t+k}+\widehat{C}_{t+k}+g^H_{t+k}]+\sigma V_{yz}(\overline{C},0)\widehat{z}^H_{t+k}\}$$

where $\widehat{p}_{t,t+k}=\ln(\widetilde{p}_t(h)/P_{H,t+k})$. We can further simplify the equation above to

$$0=\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha^H\beta)^k\{(\widehat{p}_{t,t+k}-(1-n)\widehat{T}_{t+k}-\rho\widehat{C}_{t+k}-\eta[-\sigma\widehat{p}_{t,t+k}+(1-n)\widehat{T}_{t+k}$$
$$+\widehat{C}_{t+k}+g^H_{t+k}-\overline{Y}^H_t]\},$$

where $\rho\equiv -U_{CC}(\overline{C})\overline{C}/U_C(\overline{C})$ and $\eta\equiv V_{yy}(\overline{C},0)\overline{C}/V_y(\overline{C},0)$, while we have define $\overline{Y}^H_t$ such that $V_{yz}(\overline{C},0)\widehat{z}^H_{t+k}\equiv -\overline{C}V_{yy}(\overline{C},0)\overline{Y}^H_t$. We note that

$$\widehat{p}_{t,t+k}=\widehat{p}_{t,t}-\sum_{s=1}^{k}\pi_{H,t+s}$$

we can then simplify to

$$\frac{\widehat{p}_{t,t}}{1-\alpha^H\beta}=\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha^H\beta)^k[\frac{1+\eta}{1+\sigma\eta}(1-n)\widehat{T}_{t+k}+\frac{\rho+\eta}{1+\sigma\eta}\widehat{C}_{t+k}$$
$$+\frac{\eta}{1+\sigma\eta}(g^H_{t+k}-\overline{Y}^H_{t+k})]+\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha^H\beta)^k\left[\sum_{s=1}^{k}\pi_{H,t+s}\right]. \tag{B.5}$$

Log-linearizing (B.2), we obtain

$$\widehat{p}_{t,t}=\frac{\alpha^H}{1-\alpha^H}\pi^H_t$$

Thus we can simplify (B.5) further to

$$\frac{\pi^H_t}{1-\alpha^H\beta}\frac{\alpha^H}{1-\alpha^H}=\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha^H\beta)^k[\frac{1+\eta}{1+\sigma\eta}(1-n)\widehat{T}_{t+k}+\frac{\rho+\eta}{1+\sigma\eta}\widehat{C}_{t+k}+$$
$$+\frac{\eta}{1+\sigma\eta}(g^H_{t+k}-\overline{Y}^H_{t+k})]+\mathrm{E}_t\sum_{k=1}^{\infty}(\alpha^H\beta)^k\frac{\pi^H_{t+k}}{1-\alpha\beta}$$

We obtain

$$\pi_t^H = (1-\alpha^H\beta)\frac{1-\alpha^H}{\alpha^H}\frac{1+\eta}{1+\sigma\eta}(1-n)\widehat{T}_t + (1-\alpha^H\beta)\frac{1-\alpha^H}{\alpha^H}\frac{\rho+\eta}{1+\sigma\eta}\widehat{C}_t$$
$$+(1-\alpha^H\beta)\frac{1-\alpha^H}{\alpha^H}\frac{\eta}{1+\sigma\eta}(g_t-\overline{Y}_t)+\beta \mathrm{E}_t\pi_{t+1}^H \qquad \text{(B.6)}$$

noting that the natural rate of world consumption and of the terms of trade, which will arise when prices are flexible, are defined as

$$\widetilde{C}_t \equiv \frac{\eta}{\rho+\eta}(\overline{Y}_t^W - g_t^W),$$
$$\widetilde{T}_t \equiv \frac{\eta}{1+\eta}(g_t^R - \overline{Y}_t^R).$$

we can simplify the equation above to

$$\pi_t^H = (1-n)k_T^H(\widehat{T}_t - \widetilde{T}_t) + k_C^H(\widehat{C}_t - \widetilde{C}_t) + \beta E_t\pi_{t+1}^H, \qquad \text{(B.7)}$$

which corresponds to equation (3) in the text (note that $\widehat{C}_t = \widehat{C}_t^W$) where

$$k_T^H \equiv (1-\alpha^H\beta)\frac{1-\alpha^H}{\alpha^H}\frac{1+\eta}{1+\sigma\eta}$$
$$k_C^H \equiv (1-\alpha^H\beta)\frac{1-\alpha^H}{\alpha^H}\frac{\rho+\eta}{1+\sigma\eta}.$$

Appendix C

In this appendix we derive the utility-based loss function, equation (14) in the text. We follow Rotemberg and Woodford (1997,1998) and Woodford (1999a). The average utility flow among all the households belonging to region H is

$$w_t^H = U(C_t) - \frac{\int_0^n v(y_t(h), z_t^H)dh}{n}, \tag{C.1}$$

while that of region F is

$$w_t^F = U(C_t) - \frac{\int_{1-n}^1 v(y_t(f), z_t^F)df}{1-n}. \tag{C.2}$$

The welfare criterion of the Central Bank in the currency area is the discounted value of a weighted average of the average utility flows of the regions,

$$W = \mathrm{E}_0 \sum_{j=0}^{\infty} \beta^j (n w_{t+j}^H + (1-n) w_{t+j}^F). \tag{C.3}$$

We take a Taylor expansion of each term of the utility function. Taking a second-order linear expansion of $U(C_t)$ around the steady state value $\overline{C}$ defined by equation (A.22), we obtain

$$U(C_t) = U(\overline{C}) + U_C(C_t - \overline{C}) + \frac{1}{2}U_{CC}(C_t - \overline{C})^2 + o(\|\xi\|^3), \tag{C.4}$$

where in $o(\|\xi\|^3)$ we group all the terms that are of third or higher order in the deviations of the various variables from their steady-state values. Furthermore expanding C_t with a second-order Taylor approximation we obtain

$$C_t = \overline{C}(1 + \widehat{C}_t + \frac{1}{2}\widehat{C}_t^2) + o(\|\xi\|^3), \tag{C.5}$$

where $\widehat{C}_t = \ln(C_t/\overline{C})$. Substituting (C.5) into (C.4) we obtain

$$U(C_t) = U_C\overline{C}\widehat{C}_t + \frac{1}{2}(U_C\overline{C} + U_{CC}\overline{C}^2)\widehat{C}_t^2 + \text{t.i.p.} + o(\|\xi\|^3), \tag{C.6}$$

which can be written as

$$U(C_t) = U_C\overline{C}[\widehat{C}_t + \frac{1}{2}(1-\rho)\widehat{C}_t^2] + \text{t.i.p.} + o(\|\xi\|^3),$$

where we have defined $\rho \equiv -U_{CC}\overline{C}/U_C$ and where in t.i.p. we include all the terms that are independent of monetary policy. Similarly we take a second-order Taylor expansion of $v(y_t(h), z_t^H)$ around a steady state where $y_t(h) = \overline{Y}^H$ for each h, and at each date t, and where $z_t^H = 0$ at each date t. We obtain

$$\begin{aligned} v(y_t(h), z_t^H) &= v(\overline{Y}^H, 0) + v_y(y_t(h) - \overline{Y}^H) + v_z z_t^H + \frac{1}{2} v_{yy}(y_t(h) - \overline{Y}^H)^2 \\ &\quad + v_{yz}(y_t(h) - \overline{Y}^H) z_t^H + \frac{1}{2} v_{zz}(z_t^H)^2 + o(\|\xi\|^3), \end{aligned} \tag{C.7}$$

where $\widehat{y}_t(h) = \ln(y_t(h)/\overline{Y}^H)$. Here we recall that

$$y(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma} \left[(T)^{1-n} C^W + G^H\right],$$

which can be rewritten as

$$y(h) = y^d(h) + y^g(h),$$

where we have defined

$$\begin{aligned} y^d(h) &\equiv \left(\frac{p(h)}{P_H}\right)^{-\sigma} (T)^{1-n} C^W, \\ y^g(h) &\equiv \left(\frac{p(h)}{P_H}\right)^{-\sigma} G^H. \end{aligned}$$

Here we take a second order Taylor expansion of $y_t^d(h)$ and $y_t^g(h)$ obtaining

$$\begin{aligned} y_t^d(h) &= \overline{Y}^H \cdot (1 + \widehat{y}_t^d(h) + \frac{1}{2} \cdot [\widehat{y}_t^d(h)]^2) + o(\|\xi\|^3), \\ y_t^g(h) &= \overline{Y}^H \cdot (\widehat{y}_t^g(h) + \frac{1}{2} \cdot [\widehat{y}_t^g(h)]^2) + o(\|\xi\|^3). \end{aligned}$$

We note that $y_t^g(h)$ can be neglected because in its expansion, the term of order less than $o(\|\xi\|^3)$ are independent of monetary policy, being the shock G^H equal to zero in the steady state. We can simplify (C.7) to

$$\begin{aligned} v(y_t(h), z_t^H) &= v_y \overline{Y}^H \cdot [\widehat{y}_t^d(h) + \frac{1}{2} \cdot \widehat{y}_t^d(h)^2 + \frac{\eta}{2} \cdot \widehat{y}_t(h)^2 \\ &\quad - \eta \cdot \widehat{y}_t(h) \overline{Y}_t^H] + \text{ t.i.p.} + o(\|\xi\|^3), \end{aligned} \tag{C.8}$$

where $\overline{Y}_t^H$ has been defined by the relation $v_{yz}z_t^H \equiv -v_{yy}\overline{Y}^H\overline{Y}_t^H$ and we have that $\eta \equiv V_{yy}(\overline{Y}^H, 0)\overline{Y}^H / V_y(\overline{Y}^H, 0)$. Our steady state with zero inflation implies the following conditions, respectively for region H

$$(1-\tau^H)U_C(\overline{C}) = \frac{\sigma}{\sigma-1}\overline{T}^{1-n}V_y\left(\overline{T}^{1-n}\overline{C}, 0\right), \tag{C.9}$$

and for region F

$$(1-\tau^F)U_C(\overline{C}) = \frac{\sigma}{\sigma-1}\overline{T}^{-n}V_y\left(\overline{T}^{-n}\overline{C}, 0\right), \tag{C.10}$$

which can be rewritten as

$$(1-\Phi^H)U_C(\overline{C}) = \overline{T}^{1-n}V_y\left(\overline{T}^{1-n}\overline{C}, 0\right), \tag{C.11}$$

$$(1-\Phi^F)U_C(\overline{C}) = \overline{T}^{-n}V_y\left(\overline{T}^{-n}\overline{C}, 0\right), \tag{C.12}$$

after having defined

$$\begin{aligned} (1-\Phi^H) &\equiv (1-\tau^H)\frac{\sigma-1}{\sigma}, \\ (1-\Phi^F) &\equiv (1-\tau^F)\frac{\sigma-1}{\sigma}. \end{aligned}$$

In the efficient equilibrium, we have that $\Phi^H = \Phi^F = 0$. As outlined in Woodford (1999a), we have to restrict our attention on steady state in which the deviations of Φ^H and Φ^F are of order at least $o(\|\xi\|)$. We also restrict the analysis to the case in which $\Phi^H = \Phi^F$. In this case we have that $\overline{Y}^H = \overline{Y}^F = \overline{C}$. In the neighbor of the efficient level of production and consumption we can write the steady state term of trade and consumption, by using conditions (C.11) and (C.12), as

$$\begin{aligned} \overline{T} &= 1, \\ \ln\overline{C}/C^* &= -\frac{n\Phi^H + (1-n)\Phi^F}{\rho+\eta}, \end{aligned} \tag{C.13}$$

where C^* is the efficient level of consumption. By using (C.11) we can write (C.8) as

$$\begin{aligned} v(y_t(h), z_t^H) &= U_C\overline{C} \cdot [(1-\Phi)\cdot \widehat{y}_t^d(h) + \frac{1}{2}\cdot \widehat{y}_t^d(h)^2 + \frac{\eta}{2}\cdot \widehat{y}_t(h)^2 \\ &\quad -\eta \cdot \widehat{y}_t(h)\overline{Y}_t^H] + \text{ t.i.p.} + o(\|\xi\|^3). \end{aligned} \tag{C.14}$$

Here we integrate (C.14) across the households belonging to region H, obtaining

$$\begin{aligned}
\frac{\int_0^n v(y_t(h), z_t^H)dh}{n} &= U_C\overline{C}\cdot\{(1-\Phi)\cdot E_h\widehat{y}_t^d(h)+\frac{1}{2}\cdot[\mathrm{var}_h\widehat{y}_t^d(h)+[E_h\widehat{y}_t^d(h)]^2] \\
&+\frac{\eta}{2}\cdot[\mathrm{var}_h\widehat{y}_t(h)+[E_h\widehat{y}_t(h)]^2]-\eta E_h\widehat{y}_t(h)\overline{Y}_t^H\} \\
&+\text{t.i.p.}+o(\|\xi\|^3). \qquad \text{(C.15)}
\end{aligned}$$

Using the aggregator (A.15) we can write

$$Y_{H,t}=Y_{H,t}^d+Y_{H,t}^g,$$

where

$$\begin{aligned}
Y_{H,t}^d &= T_t^{1-n}C_t^W, \\
Y_{H,t}^g &= G^H.
\end{aligned}$$

We take a second-order approximation of the aggregators obtaining

$$\begin{aligned}
\widehat{Y}_{H,t} &= E_h\widehat{y}_t(h)+\frac{1}{2}\left(\frac{\sigma-1}{\sigma}\right)\mathrm{var}_h\widehat{y}_t(h)+o(\|\xi\|^3), \\
\widehat{Y}_{H,t}^d &= E_h\widehat{y}_t^d(h)+\frac{1}{2}\left(\frac{\sigma-1}{\sigma}\right)\mathrm{var}_h\widehat{y}_t^d(h)+o(\|\xi\|^3). \qquad \text{(C.16)}
\end{aligned}$$

Finally substituting (C.16) into (C.15) we obtain

$$\begin{aligned}
\frac{\int_0^n v(y_t(h), z_t)}{n} &= U_C\overline{C}\cdot[(1-\Phi^H)\cdot\widehat{Y}_{H,t}^d+\frac{1}{2}\cdot[\widehat{Y}_{H,t}^d]^2+\frac{\eta}{2}\cdot[\widehat{Y}_{H,t}]^2 \\
&+\frac{1}{2}(\sigma^{-1}+\eta)\cdot\mathrm{var}_h\widehat{y}_t(h)-\eta\widehat{Y}_{H,t}^d\overline{Y}_t^H] \\
&+\text{t.i.p.}+o(\|\xi\|^3) \qquad \text{(C.17)}
\end{aligned}$$

where we have used the fact that $\mathrm{var}_h\widehat{y}_t(h)=\mathrm{var}_h\widehat{y}_t^d(h)$.

Combining (C.17) and (C.6) into (C.1), we obtain

$$\begin{aligned}
w_t^H &= U_C\overline{C}[\widehat{C}_t+\frac{1}{2}(1-\rho)\widehat{C}_t^2-(1-\Phi^H)\cdot\widehat{Y}_{H,t}^d-\frac{1}{2}\cdot[\widehat{Y}_{H,t}^d]^2-\frac{\eta}{2}\cdot[\widehat{Y}_{H,t}]^2 \\
&-\frac{1}{2}(\sigma^{-1}+\eta)\cdot\mathrm{var}_h\widehat{y}_t(h)+\eta\widehat{Y}_{H,t}^d\overline{Y}_t^H] \\
&+\text{t.i.p.}+o(\|\xi\|^3), \qquad \text{(C.18)}
\end{aligned}$$

while for region F we have

$$\begin{aligned} w_t^F &= U_C\overline{C}[\widehat{C}_t + \frac{1}{2}(1-\rho)\widehat{C}_t^2 - (1-\Phi^F)\cdot \widehat{Y}_{F,t}^d - \frac{1}{2}\cdot[\widehat{Y}_{F,t}^d]^2 - \frac{\eta}{2}\cdot[\widehat{Y}_{F,t}]^2 \\ &\quad -\frac{1}{2}(\sigma^{-1}+\eta)\cdot \mathrm{var}_f\widehat{y}_t(f) + \eta\widehat{Y}_{F,t}^d\overline{Y}_t^F] \\ &\quad +\mathrm{t.i.p.} + o(\|\xi\|^3). \end{aligned} \tag{C.19}$$

Taking a linear combination of (C.18) and (C.19) with weight n, we obtain

$$\begin{aligned} w_t &= U_C\overline{C}\{\widehat{C}_t\cdot[n\Phi^H + (1-n)\Phi^F] + \frac{1}{2}(1-\rho)\widehat{C}_t^2 \\ &\quad -\frac{1}{2}\cdot[n(\widehat{Y}_{H,t}^d)^2 + (1-n)(\widehat{Y}_{F,t}^d)^2] - \frac{1}{2}\eta\cdot[n\widehat{Y}_{H,t}^2 + (1-n)\widehat{Y}_{F,t}^2] \\ &\quad +\eta\cdot[n\widehat{Y}_{H,t}\overline{Y}_t^H + (1-n)\widehat{Y}_{F,t}\overline{Y}_t^F] + \\ &\quad -\frac{1}{2}(\sigma^{-1}+\eta)\cdot[n\mathrm{var}_h\widehat{y}_t(h) + (1-n)\mathrm{var}_f\widehat{y}_t(f)]\} \\ &\quad +\mathrm{t.i.p.} + o(\|\xi\|^3), \end{aligned} \tag{C.20}$$

and after substituting the expressions for $\widehat{Y}_{H,t}$, $\widehat{Y}_{F,t}$, $\widehat{Y}_{H,t}^d$, $\widehat{Y}_{F,t}^d$ we get

$$\begin{aligned} w_t &= U_C\overline{C}\{\widehat{C}_t\cdot[n\Phi^H + (1-n)\Phi^F] + \frac{1}{2}(1-\rho)\widehat{C}_t^2 \\ &\quad +\eta[\widehat{C}_t\overline{Y}_t^W + n(1-n)\widehat{T}_t\overline{Y}_t^R] - \frac{1}{2}[\widehat{C}_t^2 + n(1-n)\widehat{T}_t^2] \\ &\quad -\frac{1}{2}\eta\cdot[\widehat{C}_t^2 + n(1-n)\widehat{T}_t^2 + 2\widehat{C}_t g_t^W - 2n(1-n)\widehat{T}_t g_t^R] \\ &\quad -\frac{1}{2}(\sigma^{-1}+\eta)\cdot[n\mathrm{var}_h\widehat{y}_t(h) + (1-n)\mathrm{var}_f\widehat{y}_t(f)]\} \\ &\quad +\mathrm{t.i.p.} + o(\|\xi\|^3), \end{aligned} \tag{C.21}$$

which can be written as

$$\begin{aligned} w_t &= -U_C\overline{C}\{-\widehat{C}_t\cdot[n\Phi^H + (1-n)\Phi^F] \\ &\quad +\frac{1}{2}(\rho+\eta)[\widehat{C}_t - \widetilde{C}_t]^2 + \frac{1}{2}(1+\eta)n(1-n)[\widehat{T}_t - \widetilde{T}_t]^2 \\ &\quad +\frac{1}{2}(\sigma^{-1}+\eta)\cdot[n\mathrm{var}_h\widehat{y}_t(h) + (1-n)\mathrm{var}_f\widehat{y}_t(f)]\} \\ &\quad +\mathrm{t.i.p.} + o(\|\xi\|^3). \end{aligned} \tag{C.22}$$

Where the natural rate of world consumption and of the term of trade, which will arise when prices are flexible, are defined as

$$\begin{aligned}\widetilde{C}_t^W &\equiv \frac{\eta}{\rho+\eta}(\overline{Y}_t^W - g_t^W),\\ \widetilde{T}_t &\equiv \frac{\eta}{1+\eta}(g_t^R - \overline{Y}_t^R).\end{aligned}$$

By using equations (C.13) and after having defined $c_t^W \equiv \widehat{C}_t^W - \widetilde{C}_t$ we obtain

$$\begin{aligned}w_t &= -U_C\overline{C}\{\frac{1}{2}(\rho+\eta)[c_t^W - \overline{c}^W]^2 + \frac{1}{2}(1+\eta)n(1-n)[\widehat{T}_t - \widetilde{T}_t]^2\\ &+\frac{1}{2}(\sigma^{-1}+\eta)\cdot[n\text{var}_h\widehat{y}_t(h) + (1-n)\text{var}_f\widehat{y}_t(f)]\}\\ &+\text{t.i.p.} + o(\|\xi\|^3), \qquad \text{(C.23)}\end{aligned}$$

where $\overline{c}^W \equiv -\ln\overline{C}/C^*$.

Here we derive $\text{var}_h\widehat{y}_t(h)$ and $\text{var}_f\widehat{y}_t(f)$. We have that

$$\text{var}_h\{\log y_t(h)\} = \sigma^2\text{var}_h\{\log p_t(h)\}.$$

Defining $\overline{p}_t \equiv \text{E}_h\log p_t(h)$, we have

$$\begin{aligned}\text{var}_h\{\log p_t(h)\} &= \text{var}_h\{\log p_t(h) - \overline{p}_{t-1}\} = \text{E}_h\{[\log p_t(h) - \overline{p}_{t-1}]^2\} - (\Delta\overline{p}_t)^2\\ &= \alpha^H\text{E}_h\{[\log p_{t-1}(h) - \overline{p}_{t-1}]^2\} + (1-\alpha^H)[\log\widetilde{p}_t(h) - \overline{p}_{t-1}]^2 +\\ &\quad -(\Delta\overline{p}_t)^2\\ &= \alpha^H\text{var}_h\{\log p_{t-1}(h)\} + (1-\alpha^H)[\log\widetilde{p}_t(h) - \overline{p}_{t-1}]^2 - (\Delta\overline{p}_t)^2.\end{aligned}$$

We have also that

$$\overline{p}_t - \overline{p}_{t-1} = (1-\alpha^H)[\log\widetilde{p}_t(h) - \overline{p}_{t-1}], \qquad \text{(C.24)}$$

from which we obtain that

$$\text{var}_h\{\log p_t(h)\} = \alpha^H\text{var}_h\{\log p_{t-1}(h)\} + \frac{\alpha^H}{1-\alpha^H}(\Delta\overline{p}_t)^2.$$

But

$$\overline{p}_t = \log P_{H,t} + o(\|\xi\|^2),$$

which implies

$$\text{var}_h\{\log p_t(h)\} = \alpha^H\text{var}_h\{\log p_{t-1}(h)\} + \frac{\alpha^H}{1-\alpha^H}(\pi_t^H)^2 + o(\|\xi\|^3),$$

after integration of the above equation we obtain

$$\text{var}_h\{\log p_t(h)\} = (\alpha^H)^{t+1}\text{var}_h\{\log p_{-1}(h)\} + \sum_{s=0}^{t}(\alpha^H)^{t-s}\frac{\alpha^H}{1-\alpha^H}(\pi_t^H)^2 + o(\|\xi\|^3)$$

where we note that the first term in the right hand side is independent of the policy chosen after period $t \geq 0$. After taking the discounted value, with the discount factor β, we obtain

$$\sum_{t=0}^{\infty}\beta^t\text{var}_h\{\log p_t(h)\} = \frac{\alpha^H}{(1-\alpha^H)(1-\alpha^H\beta)}\sum_{t=0}^{\infty}\beta^t(\pi_t^H)^2 + \text{t.i.p.} + o(\|\xi\|^3)$$

The same derivations apply also for the Foreign country. We define

$$\begin{aligned} d^H &\equiv \frac{\alpha^H}{(1-\alpha^H)(1-\alpha^H\beta)}, \\ d^F &\equiv \frac{\alpha^F}{(1-\alpha^F)(1-\alpha^F\beta)}. \end{aligned}$$

We can simplify (C.3) to

$$W_t = -\Omega\sum_{j=0}^{\infty}\beta^j L_{t+j} \tag{C.25}$$

where

$$L_{t+j} = \Lambda[c_{t+j}^W - \overline{c}^W]^2 + n(1-n)\Gamma[\widehat{T}_{t+j} - \widetilde{T}_{t+j}] + \gamma(\pi_{t+j}^H) + (1-\gamma)(\pi_{t+j}^F) + \text{t.i.p.} + o(\|\xi\|^3),$$

which corresponds to equation (14) in the text, where $c^W = y^W$ and $\overline{y}^W \equiv \overline{c}^W$. Furthermore

$$\begin{aligned} \Omega &\equiv \frac{1}{2}U_C\overline{C}(nd^H + (1-n)d^F)\sigma(1+\sigma\eta) \\ \Lambda &\equiv \frac{k_C^H k_C^F}{\sigma}\frac{1}{nk_C^F + (1-n)k_C^H}, \\ \Gamma &\equiv \frac{k_T^H k_T^F}{\sigma}\frac{1}{nk_T^F + (1-n)k_T^H}, \\ \gamma &\equiv \frac{nd^H}{nd^H + (1-n)d^F}. \end{aligned}$$

We note that when the degrees of rigidity are the same, i.e $d^H = d^F$, γ coincides with n.

Appendix D

This appendix contains the proofs of some propositions of section 5.

Proposition 4

Proof. (i) By inspection of equation (11); (ii) the stochastic difference equation (11) has always one eigenvalue with modulus less than 1 and one which is bigger than $1/\beta$. The unique and stable solution is given by

$$\widehat{T}_t = \lambda_1 \widehat{T}_{t-1} + \lambda_1 k_T \mathrm{E}_t \sum_{s=t}^{\infty} \left(\frac{1}{\lambda_2}\right)^{s-t} \widetilde{T}_s,$$

where λ_1 is a positive eigenvalue of the second order difference equation (11), with λ_1 less than 1; (iii) given the initial condition $\widehat{T}_{t-1} = 0$, if $\widehat{T}_t = \widetilde{T}_t$ at all dates t then from (10) it follows that $\widehat{T}_t = 0$ which contradicts $\widehat{T}_t = \widetilde{T}_t$. ■

Proposition 5

Proof. We have that the AS equation of the region F, in the case its prices are sticky, is

$$\pi_t^F = -k_T^F[n\widehat{T}_t - \frac{k_C^F}{k_T^F}(\widehat{C}_t^W - \widetilde{C}_{F,t})] + \beta \mathrm{E}_t \pi_{t+1}^F, \tag{D.1}$$

where we have that

$$\widetilde{C}_{F,t} \equiv \frac{\eta}{\rho + \eta}(\overline{Y}_t^F - g_t^F).$$

Under the assumption that prices in region F are flexible, we can observe that the term of trade is implied by the term in the square brackets of (D.1), namely

$$n\widehat{T}_t = \frac{k_C^F}{k_T^F}(\widehat{C}_t^W - \widetilde{C}_{F,t}). \tag{D.2}$$

Similarly rearranging the AS equation of region H we obtain

$$\pi_t^H = k_T^H[(1-n)\widehat{T}_t + \frac{k_C^H}{k_T^H}(\widehat{C}_t^W - \widetilde{C}_{H,t})] + \beta \mathrm{E}_t \pi_{t+1}^H, \tag{D.3}$$

where we have that

$$\widetilde{C}_{H,t} \equiv \frac{\eta}{\rho + \eta}(\overline{Y}_t^H - g_t^H).$$

After plugging (D.2) into (D.3) we obtain

$$\pi_t^H = \frac{k_C}{n}[\widehat{C}_t^W - n\widetilde{C}_{H,t} - (1-n)\widetilde{C}_{F,t}] + \beta \mathrm{E}_t \pi_{t+1}^H.$$

By noting that $\widetilde{C}_t^W = n\widetilde{C}_{H,t} + (1-n)\widetilde{C}_{F,t}$, we reach the conclusion that by stabilizing the inflation rate in region H at all date t monetary authority reaches a path of consumption consistent with its efficient level at all dates t. Moreover if $\widehat{C}_t = \widetilde{C}_t$ it follows from (D.2) that $\widehat{T}_t = \widetilde{T}_t$. ■

Proposition 6

Proof. Rearranging (D.2) we obtain

$$n(\widehat{T}_t - \widetilde{T}_t) = \frac{k_C^F}{k_T^F}(\widehat{C}_t^W - \widetilde{C}_t^W). \tag{D.4}$$

In this case the welfare function is

$$W = -\Omega \mathrm{E}_0 \left\{ \sum_{t=0}^{+\infty} \beta^t L_t \right\}, \tag{D.5}$$

with

$$L_t = \Lambda \cdot [y_t^W - \overline{y}^W]^2 + n(1-n)\Gamma \cdot [\widehat{T}_t - \widetilde{T}_t]^2 + (\pi_t^H)^2 + \text{t.i.p} + o(\|\xi\|^3).$$

Noting that

$$\pi_t^H = \frac{k_C}{n} \cdot [y_t^W] + \beta \mathrm{E}_t \pi_{t+1}^H,$$

we can simplify the welfare to

$$W = -\Omega \mathrm{E}_0 \left\{ \sum_{t=0}^{+\infty} \beta^t [\theta^y (\pi_t^H - \beta \pi_{t+1}^H)^2 + (\pi_t^H)^2 \right\} + 2\Omega\Lambda \overline{y}^W \frac{n}{k_C^H} \pi_0^H \tag{D.6}$$

where we have used (D.4) and where

$$\theta^y \equiv \Lambda \left[1 + \frac{1-n}{n} \frac{\rho + \eta}{1+\eta} \right] \frac{n^2}{k_C^H}.$$

The first order condition of (D.6) with respect to π_t^H are at each date $t > 0$

$$\beta \pi_{t+1}^H - (1 + (\theta^y)^{-1} + \beta)\pi_t^H + \pi_{t-1}^H = 0, \tag{D.7}$$

while at date 0

$$\pi_0(1+(\theta^y)^{-1}) - \beta\pi_1 = \Lambda\overline{y}^W n(k_C^H\theta^y)^{-1}.$$

A stable solution for π_t^H is given by

$$\begin{aligned}\pi_t^H &= \lambda_1^t \pi_0 \\ \pi_0 &= \frac{\Lambda\overline{y}^W n}{(\lambda_1^{-1}-\beta)k_C^H\theta^y}\end{aligned}$$

where λ_1 is the stable eigenvalue of the difference equation (D.7) with $0 < \lambda_1 < 1$. ■

Proposition 7

Proof. We can write relative and union inflation as, respectively

$$\pi_t^R = -\psi[\widehat{T}_t - \widetilde{T}_t] + \omega[\widehat{C}_t^W - \widetilde{C}_t^W] + \beta \mathrm{E}_t \pi_{t+1}^R, \quad \text{(D.8)}$$

$$\pi_t^W = \theta[\widehat{T}_t - \widetilde{T}_t] + \kappa[\widehat{C}_t^W - \widetilde{C}_t^W] + \beta \mathrm{E}_t \pi_{t+1}^W. \quad \text{(D.9)}$$

By contradiction if $\widehat{T}_T = \widetilde{T}_T$ and $\widehat{C}_T^W = \widetilde{C}_T^W$ at all dates $T \geq t$, we have that $\pi_T^R = 0$ at all dates $T \geq t$, implying that $\widehat{T}_t = \widehat{T}_{t-1} = 0$ (given the initial condition $\widehat{T}_{t-1} = 0$), which contradicts $\widehat{T}_t = \widetilde{T}_t$ unless $\widetilde{T}_t = 0$ and this is for each date t. ■

Proposition 10

Proof. The optimal plan can be obtained by taking the first order condition of the following Lagrangian[25]

$$\begin{aligned}&\mathrm{E}_0\{\textstyle\sum_{t=0}^{\infty}\beta^t\{L_t + 2\phi_{1,t}[\pi_t^W - \theta(\widehat{T}_t - \widetilde{T}_t) - kc_t^W - \beta\pi_{t+1}^W] + \\ &+2\phi_{2,t}[\widehat{T}_t - \widehat{T}_{t-1} + \psi(\widehat{T}_t - \widetilde{T}_t) - \omega c_t^W - \beta(\widehat{T}_{t+1} - \widehat{T}_t)]\}.\end{aligned}$$

The first-order conditions are

$$\Lambda c_t^W - k\phi_{1,t} - \omega\phi_{2,t} = 0, \quad \text{(D.10)}$$

$$\pi_t^W + 2b\pi_t^R - \phi_{1,t-1} + \phi_{1,t} = 0, \quad \text{(D.11)}$$

[25] Note that we have omitted the term Ω, by normalizing the Lagrange multiplier. We have also multiplied the Lagrange multiplier by a factor of two in order to eliminate a recurrent factor of two from the first-order conditions.

$$
\begin{aligned}
0 &= n(1-n)\Gamma(\widehat{T}_t - \widetilde{T}_t) + a(\pi_t^R - \beta\pi_{t+1}^R) + 2b(\pi_t^W - \beta\pi_{t+1}^W) + \\
&\quad -\theta\phi_{1,t-1} - \phi_{2,t-1} + (1+\beta+\psi)\phi_{2,t} - \beta\phi_{2,t+1},
\end{aligned} \quad \text{(D.12)}
$$

obtained by minimizing the Lagrangian with respect to c_t^W, π_t^W, $\widehat{T}_t$.[26] These conditions hold at each date t with $t \geq 1$. They also hold at time 0, given the initial conditions on the absence of commitment at time 0

$$
\phi_{1,-1} = \phi_{2,-1} = 0,
$$

and the initial condition on $\widehat{T}_{-1}$ which is imposed to be equal to zero. The optimal bounded plan is a set of bounded processes $\{\pi_t^W, \pi_t^R\; c_t^W, \widehat{T}_t, \phi_{1,t}, \phi_{2,t}\}$ that satisfy conditions (D.8), (D.9), (D.10), (D.11) and (D.12), given the initial conditions[27] and given the process for $\widetilde{T}_t$. Noting that each of the first-order conditions hold at each date t, they should hold under commitment also conditional upon the information set at each date t. We can rearrange the conditions characterizing the optimal plan as

$$
Q\mathrm{E}_t x_{t+1} = Mx_t + v\widetilde{T}_t, \quad \text{(D.13)}
$$

where Q and M are 9×9 matrices, $x_t^{'} \equiv [c_t^W, \pi_t^W, \pi_t^R, \widehat{T}_t, \phi_{1,t}, \phi_{2,t}, p_t, s_t, w_t]$ while $p_t \equiv \widehat{T}_{t-1}$, $s_t \equiv \phi_{1,t-1}$, $w_t \equiv \phi_{2,t-1}$, v is a 9×1 vector. Considering a bounded stochastic process for the shock $\widetilde{T}_t$, a bounded optimal plan exist and it is unique if and only if the matrix $(Q)^{-1}M$ has exactly three eigenvalues inside the unit circle; in fact in the system of stochastic difference equations (D.13) there are three predetermined variables. If we assume an autoregressive process for the shock $\widetilde{T}_t = \rho\widetilde{T}_{t-1} + \varepsilon_t$, where ε_t is a white noise and $0 \leq \rho < 1$, the unique bounded solution can be written as

$$
\begin{aligned}
y_t &= Gz_t + h\widetilde{T}_t, \\
z_t &= Nz_{t-1} + n\widetilde{T}_t, \\
\widetilde{T}_t &= \rho\widetilde{T}_{t-1} + \varepsilon_t
\end{aligned}
$$

where $y_t \equiv [c_t^W, \pi_t^W, \pi_t^R]$ $z_t' \equiv [\widehat{T}_t, \phi_{1,t}, \phi_{2,t}]$, N and G are $3{\times}3$ matrices while h and n are 3×1 vectors. Furthermore we can write the solution of the terms of

[26] We are considering only bounded solutions, thus we can neglect the set of transversality conditions.

[27] This optimal plan is not time-consistent. Time consistency would have required that the Lagrange multiplier were zero at all dates, but this is not feasible as a solution.

trade as

$$\begin{aligned}\det[I - NL]\widehat{T}_t &= n_1\widetilde{T}_t + [N_{13}n_3 + N_{12}n_2 - N_{22}n_1 - N_{33}n_1]\widetilde{T}_{t-1} \\ &\quad +[N_{22}N_{33}n_1 - N_{23}N_{32}n_1 - N_{12}N_{33}n_2 + N_{12}N_{23}n_3 + \\ &\quad +N_{13}N_{32}n_2 - N_{13}N_{22}n_3]\widetilde{T}_{t-2},\end{aligned}$$

or more compactly as

$$G(L)\widehat{T}_t = V(L)\widetilde{T}_t, \tag{D.14}$$

where L is the lag operator and $G(L)$ and $V(L)$ are polynomials in the lag operators respectively of the third order and of the second order. Furthermore it can be shown that also π_t^R, π_t^W, c_t^W and the nominal interest rate i_t have the same representations as in (D.14) with different polynomials but of the same orders ■

Proposition 11

Proof. Our welfare function can be written omitting the term Ω as

$$W = \mathrm{E}_0\left\{2\frac{\Lambda}{\omega\theta + \psi\kappa}(\theta\pi_0^R + \psi\pi_0^W)\overline{y}^W\right\} - \mathrm{E}_0\left\{\sum_{t=0}^{+\infty}\beta^t L_t\right\}.$$

At each date t with $t \geq 1$, the first order conditions are exactly the same as in Proposition 10, while at date $t = 0$ we have that (D.11) and (D.12) are changed in

$$0 = \pi_0^W - \frac{\Lambda}{\omega\theta + \psi\kappa}\psi\overline{y}^W + 2b\widehat{T}_0 + \phi_{1,0} \tag{D.15}$$

$$\begin{aligned}0 &= n(1-n)\Gamma(\widehat{T}_0 - \widetilde{T}_0) - \frac{\Lambda}{\omega\theta + \psi\kappa}\theta\overline{y}^W + a(\widehat{T}_0 - \beta\pi_1^R) + \\ &\quad +2b(\pi_0^W - \beta\pi_1^W) - \theta\phi_{1,0} + (1 + \beta + \psi)\phi_{2,0} - \beta\phi_{2,t+1} \qquad \text{(D.16)}\end{aligned}$$

It easy to see, that the responses of the variables to the shock on the terms of trade, are the same as in the case in which $\overline{y}^W = 0$. The only difference is only in a deterministic component at time 0, that asymptotically approaches zero. ■

Appendix E

In this appendix we sketch the main characteristic of the K-region extension. The whole economy is populated by a continuum of agents on the interval $[0,1]$. Each agent is both consumer and producer. Consumer of all the goods produced within the economy, producer of a single differentiated product. In each sector a measure n_i of goods is produced, with $i = 1, 2, ..., K$. We have that $\sum_{i=1}^{K} n_i = 1$. Preferences of the generic household j are given by

$$U_t^j = \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s^j) + L\left(\frac{M_s^j}{P_s}, \xi^i\right) - V(y_s^j, z_s^i) \right],$$

where everything has the same interpretation as in the model of the main text, except that C^j is defined as

$$C^j \equiv \frac{\prod_{i=1}^{K} (C_i^j)^{n_i}}{\prod_{i=1}^{K} n_i}$$

and C_i^j is an index of goods produced in region i. Specifically,

$$C_i^j \equiv \left[\left(\frac{1}{n^i}\right)^{\frac{1}{\sigma}} \int_{u \in i} c^j(u)^{\frac{\sigma-1}{\sigma}} du \right]^{\frac{\sigma}{\sigma-1}}.$$

for $i = 1, 2, ..., K$. In this case we can write total demand of good h produced in region k as

$$y_k^d(h) = \left(\frac{p(h)}{P_k}\right)^{-\sigma} [(P_k^R)^{-1} C^W + G_k]$$

where the union aggregate consumption C^W is defined as

$$C^W \equiv \int_0^1 C^j dj,$$

and the relative price of region k with respect to the overall price index has been defined as $P_k^R \equiv P_k/P$ for $k = 1, 2, ..., K$. The supply side of the model is the same except that we have to deal with K regions.

Here we note that Lemma 1 can be extended to this general context by observing that the first order conditions are the same as in the previous case as well as the aggregate budget constraint of each region.

In the log-linear approximation we use the following notation. Given a generic variable X, a world variable X^W is defined as the weighted average of the region's variables with weights n_i

$$X^W \equiv \sum_{i=1}^{K} n_i X_i,$$

while a relative variable X_i^R is defined as

$$X_i^R \equiv X_i - X^W$$

while $X_{i,j}^R$ as

$$X_{i,j}^R \equiv X_i - X_j.$$

The flexible-price solution is

$$\begin{aligned}
\widetilde{C}_t^W &= \frac{\eta}{\rho+\eta}(\overline{Y}_t^W - g_t^W), \\
\widetilde{Y}_t^W &= \frac{\eta}{\rho+\eta}\overline{Y}_t^W + \frac{\rho}{\rho+\eta} g_t^W, \\
\widetilde{P}_{i,t}^R &= \frac{\eta}{1+\eta}(g_{i,t}^R - \overline{Y}_{i,t}^R).
\end{aligned}$$

Here we discuss how the log-linear approximation of the equilibrium will behave under the hypothesis of sticky prices. We obtain the log-linear version of the Euler equation and of aggregate outputs as

$$E_t \widehat{C}_{t+1}^W = \widehat{C}_t^W + \rho^{-1}(\widehat{R}_t - E_t \pi_{t+1}^W),$$

$$\widehat{Y}_{i,t} = -\widehat{P}_{i,t}^R + \widehat{C}_t^W + g_t^i,$$

for each $i = 1, 2, ...K$. Our set of AS equations will be

$$\pi_t^i = -k_T^i(\widehat{P}_{i,t}^R - \widetilde{P}_{i,t}^R) + k_C^i(\widehat{C}_t^W - \widetilde{C}_t^W) + \beta E_t \pi_{t+1}^i, \qquad \text{for } i = 1, 2...K$$

Furthermore the definitions of relative price imply

$$P_{i,t}^R = \widehat{P}_{i,t-1}^R + \pi_t^i - \pi_t^W, \qquad \text{for } i = 1, 2...K$$

The welfare criterion of the Central Bank is again the discounted value of a weighted average of the average utility flows of all the households,

$$W = \mathrm{E}_0 \sum_{j=0}^{\infty} \sum_{i=1}^{K} \beta^j n_i w_{t+j}^i.$$

In this case we obtain

$$
\begin{aligned}
w_t^i &= U_C\overline{C}[\widehat{C}_t^i + \frac{1}{2}(1-\rho)(\widehat{C}_t^i)^2 - (1-\Phi)\cdot\widehat{Y}_{i,t}^d - \frac{1}{2}\cdot[\widehat{Y}_{i,t}^d]^2 - \frac{\eta}{2}\cdot[\widehat{Y}_{i,t}]^2 \\
&\quad -\frac{1}{2}(\sigma^{-1}+\eta)\cdot \mathrm{var}_h\widehat{y}_{i,t}(h) + \eta\widehat{Y}_{i,t}^d\overline{Y}_t^i] + \text{t.i.p.} + o(\|\xi\|^3). \qquad \text{(E.1)}
\end{aligned}
$$

Taking a linear combination of (E.1) with weights n_i, we have

$$
\begin{aligned}
w_t &= U_C\overline{C}\{\widehat{C}_t^W\cdot[\Phi] + \frac{1}{2}(1-\rho)(\widehat{C}_t^W)^2 \\
&\quad -\frac{1}{2}\cdot[\sum_{i=1}^K n_i(\widehat{Y}_{i,t}^d)^2] - \frac{1}{2}\eta\cdot[\sum_{i=1}^K n_i(\widehat{Y}_{i,t})^2] \\
&\quad +\eta\cdot[\sum_{i=1}^K n_i\widehat{Y}_{i,t}\overline{Y}_t^i] - \frac{1}{2}(\sigma^{-1}+\eta)\cdot[\sum_{i=1}^K n_i \mathrm{var}_h\widehat{y}_{i,t}(h)]\} \\
&\quad +\text{t.i.p.} + o(\|\xi\|^3),
\end{aligned}
$$

Noting that

$$
\begin{aligned}
\sum_{i=1}^K n_i(\widehat{Y}_{i,t}^d)^2 &= (\widehat{C}_t^W)^2 + \sum_{i=1}^K n_i(\widehat{P}_{i,t}^R)^2, \\
\sum_{i=1}^K n_i(\widehat{Y}_{i,t})^2 &= (\widehat{C}_t^W)^2 + \sum_{i=1}^K n_i(\widehat{P}_{i,t}^R)^2 - 2\sum_{i=1}^K n_i\widehat{P}_{i,t}^R g_{i,t} + 2\widehat{C}_t^W g_t^W, \\
\sum_{i=1}^K n_i\widehat{Y}_{i,t}\overline{Y}_t^i &= \widehat{C}_t^W\overline{Y}_t^W - \sum_{i=1}^K n_i\widehat{P}_{i,t}^R\overline{Y}_t^i, \\
\sum_{i=1}^K n_i\widehat{P}_{i,t}^R &= 0,
\end{aligned}
$$

we obtain

$$
\begin{aligned}
w_t &= -U_C\overline{C}\{\frac{1}{2}(\rho+\eta)[c_t^W - \overline{c}^W]^2 + \frac{1}{2}(1+\eta)[\sum_{i=1}^K n_i(\widehat{P}_{i,t}^R - \widetilde{P}_{i,t}^R)^2] + \\
&\quad +\frac{1}{2}(\sigma^{-1}+\eta)\cdot[\sum_{i=1}^K n_i \mathrm{var}_h\widehat{y}_{i,t}(h)]\} + \text{t.i.p.} + o(\|\xi\|^3), \qquad \text{(E.2)}
\end{aligned}
$$

where $\overline{c}^W \equiv -\ln\overline{C}/C^*$.

Moreover we have that

$$
\sum_{t=0}^{\infty}\beta^t var_h\widehat{y}_{i,t}(h) = \sigma^2\frac{\alpha^i}{(1-\alpha^i)(1-\alpha^i\beta)}\sum_{t=0}^{\infty}\beta^t(\pi_t^i)^2 + t.i.p. + o(\|\xi\|^3).
$$

Defining

$$d^i \equiv \frac{\alpha^i}{(1-\alpha^i)(1-\alpha^i\beta)},$$

we can simplify the welfare function to

$$W_t = -\Omega \sum_{j=0}^{\infty} \beta^j L_{t+j} \tag{E.3}$$

where

$$L_{t+j} = \Lambda[c^W_{t+j} - \overline{c}^W]^2 + \Gamma[\sum_{i=1}^{K} n_i(\widehat{P}^R_{i,t} - \widetilde{P}^R_{i,t})^2] + \sum_{i=1}^{K} \gamma_i(\pi^i_{t+j})^2 + \text{t.i.p.} + o(\|\xi\|^3),$$

and

$$\begin{aligned}
\Omega &\equiv \frac{1}{2}U_C\overline{C}(\sum_{i=1}^{K} n_i d^i)\sigma(1+\sigma\eta) \\
\Lambda &\equiv \frac{1}{\sigma}\left(\sum_{i=1}^{K} n_i (k^i_C)^{-1}\right)^{-1} \\
\Gamma &\equiv \frac{1}{\sigma}\left(\sum_{i=1}^{K} n_i (k^i_T)^{-1}\right)^{-1}, \\
\gamma_i &\equiv \frac{n_i d^i}{(\sum_{i=1}^{K} n_i d^i)}.
\end{aligned}$$

We note that when the degrees of rigidity are the same, γ_i coincides with n_i.

Given this structure, some generalizations of the results of the main text follow directly. Efficiency can be obtained only if $K-1$ regions have flexible prices. In this case monetary policy should target the inflation rate in the sticky price region. If all the regions have the same degree of nominal rigidity, then the optimal policy is to target to zero π^W_t. If we restrict the attention to the inflation targeting class of policies, regions with equal degree of nominal rigidity should have the same weight. For example, if only one sector has flexible prices, and the others have identical degree of nominal rigidity, then it is optimal to target a weighted average of the sticky-price inflations with equal weights.

Table 1: Weights in Euro-GDP(%)

Germany	34.3
France	22.3
Italy	17.5
Spain	8.5
Netherland	5.7
Be-Lux	4.0
Austria	3.3
Finland	1.8
Portugal	1.5
Ireland	1.0

Source: European Economy, No.63, 1997

Table 2: Wage Flexibility: The percentage Increase in Wages in Response to a One Percentage Point Fall in the Unemployment Rate

Germany	0.55
France	2.22
Italy	2.07
Spain	0.17
Netherland	0.66
Be-Lux	0.65
Austria	1.43
Finland	0.48
Portugal	missing
Ireland	0.80

Source: Nickell (1997)

Table 3: Optimal 'Inflation Targeting' Policy

Average Duration	RD = $\frac{1-\alpha^F}{1-\alpha^H}$	δ	$1-\delta$	DR (%)
2 quarters	2	0.82	0.18	98.9
	1.5	0.70	0.30	98.8
	1.2	0.59	0.41	98.8
	1	0.5	0.5	-
	0.83	0.41	0.59	98.8
	0.66	0.30	0.70	98.8
	0.5	0.18	0.82	98.9
3 quarters	2	0.80	0.20	97.8
	1.5	0.69	0.31	98.0
	1.2	0.59	0.41	98.3
	1	0.5	0.5	-
	0.83	0.41	0.59	98.3
	0.66	0.31	0.69	98.0
	0.5	0.20	0.80	97.8
4 quarters	2	0.79	0.21	97.9
	1.5	0.69	0.31	98.2
	1.2	0.59	0.41	98.3
	1	0.5	0.5	-
	0.83	0.21	0.59	97.9
	0.66	0.31	0.69	98.2
	0.5	0.41	0.79	98.3
6 quarters	2	0.79	0.21	98.7
	1.5	0.69	0.31	98.9
	1.2	0.59	0.41	99.0
	1	0.5	0.5	-
	0.83	0.41	0.59	99.0
	0.66	0.31	0.69	98.9
	0.5	0.21	0.79	98.7
8 quarters	2	0.79	0.21	99.2
	1.5	0.69	0.31	99.3
	1.2	0.59	0.41	99.4
	1	0.5	0.5	-
	0.83	0.41	0.59	99.4
	0.66	0.31	0.69	99.3
	0.5	0.21	0.79	99.2

Figure 1: Optimal Choice of the Weight δ in the Inflation Targeting Class of Policies

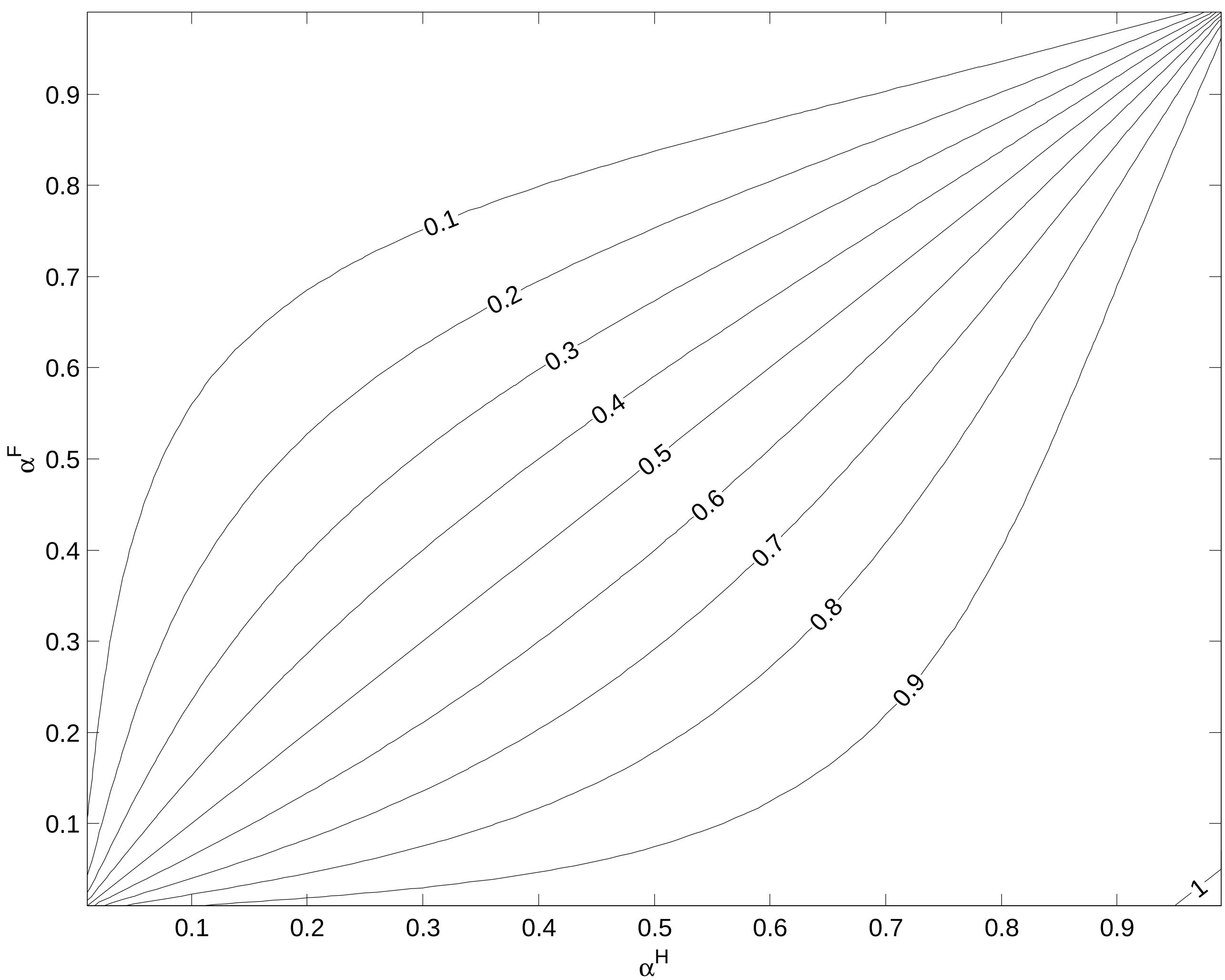

Figure 2: Welfare Comparisons, Average Duration = 2

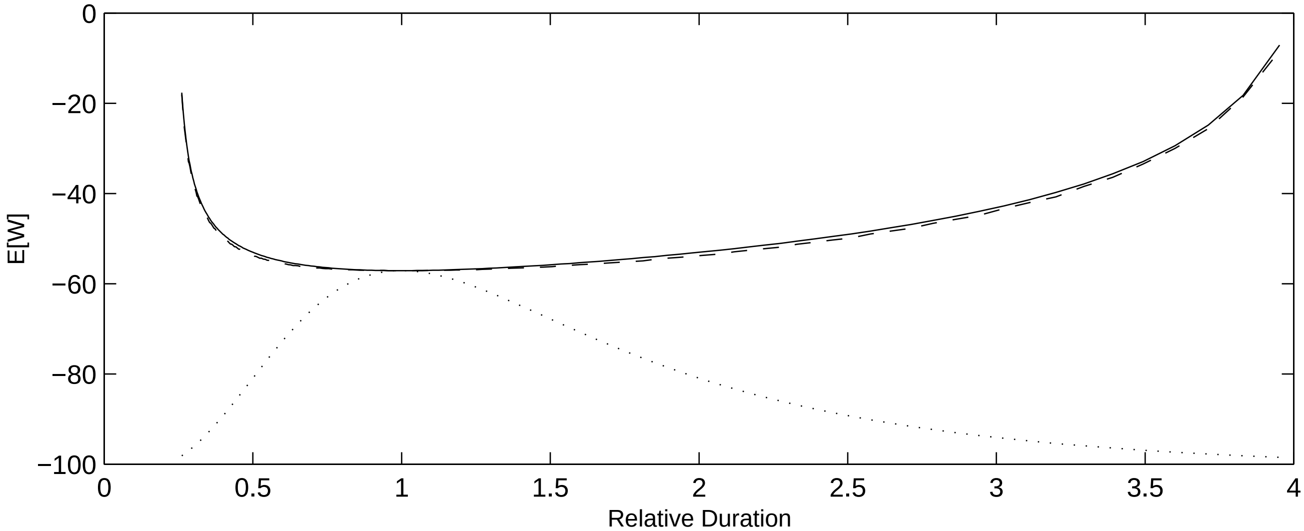

Figure 3: Welfare Comparisons, Average Duration = 3

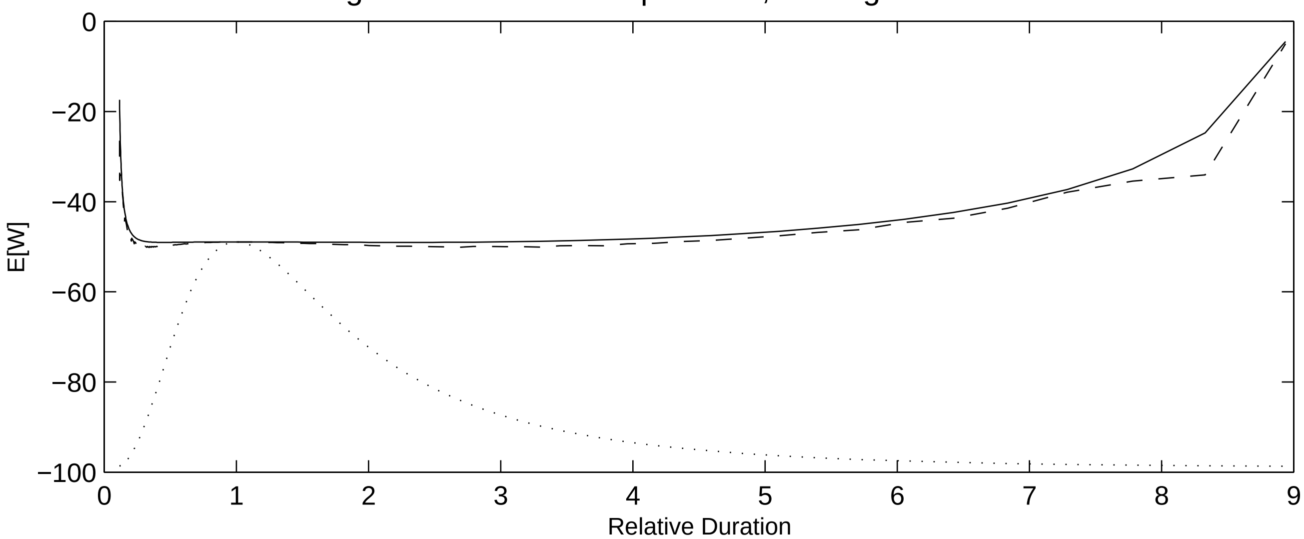

Figure 4: Welfare Comparisons, Average Duration = 4

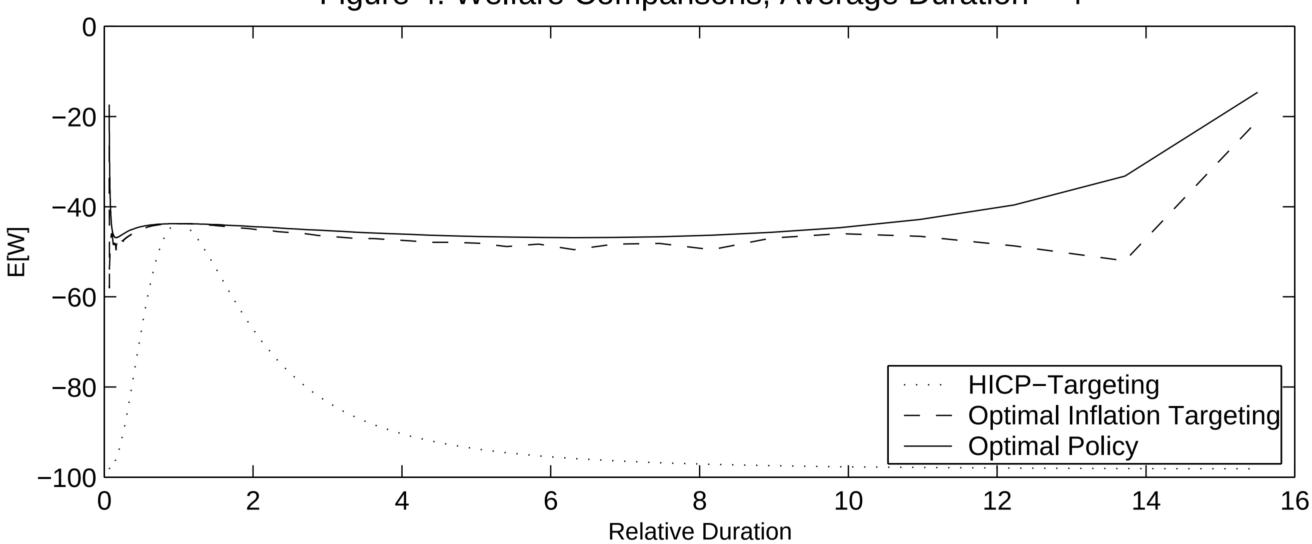

Figure 5: Welfare Comparisons, Average Duration = 6

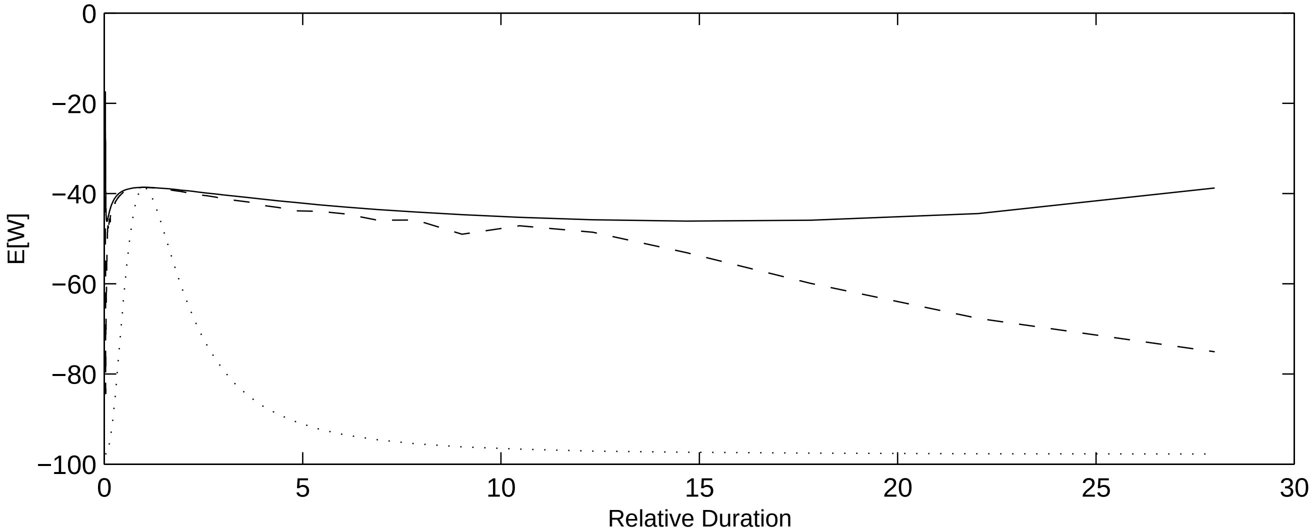

Figure 6: Welfare Comparisons, Average Duration = 8

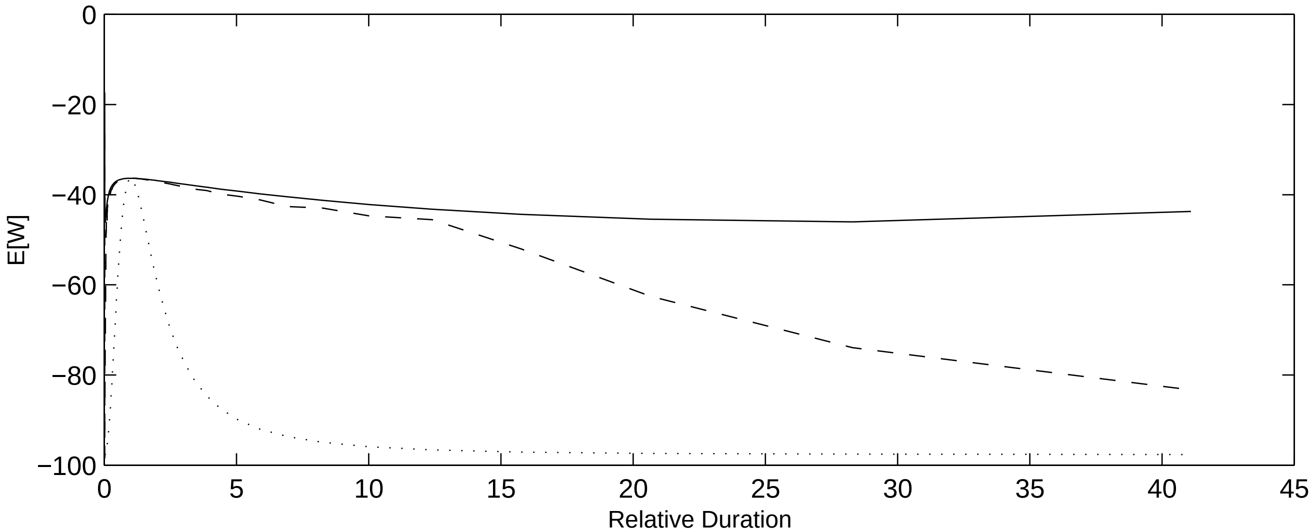

Figure 7: Deadweight Loss Reduction (%): Average Duration = 2

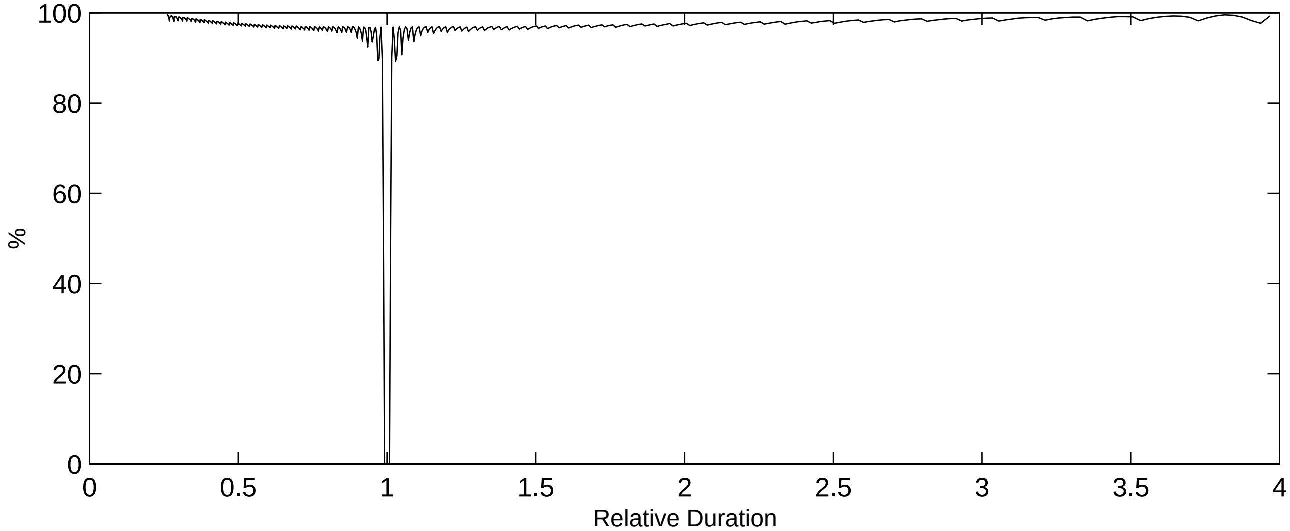

Figure 8: Deadweight Loss Reduction (%): Average Duration = 3

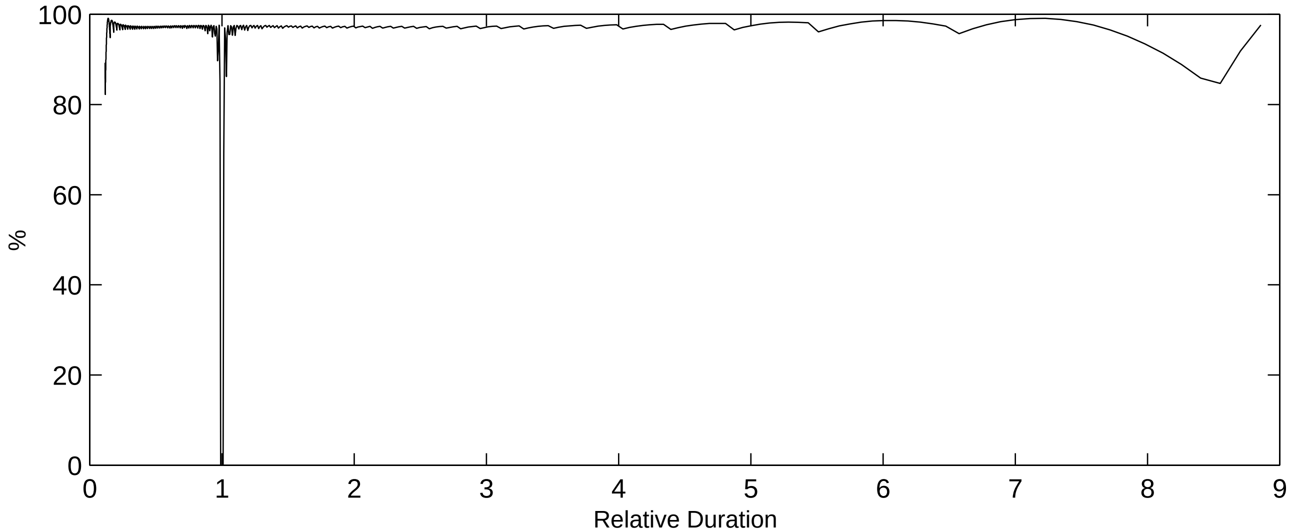

Figure 9: Deadweight Loss Reduction (%): Average Duration = 4

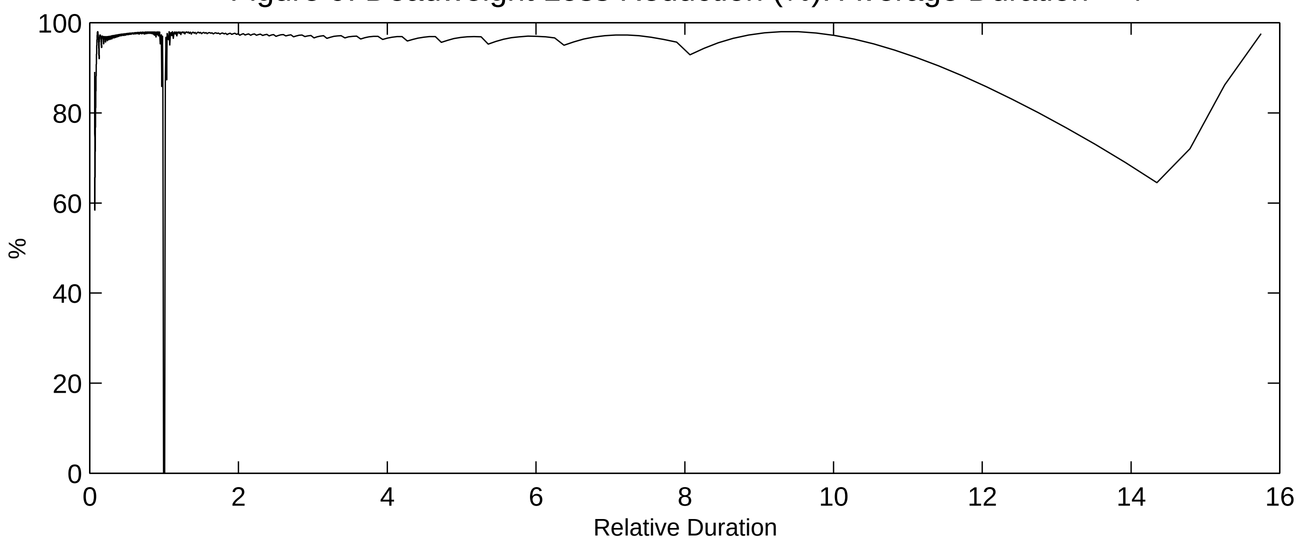

Figure 10: Deadweight Loss Reduction (%): Average Duration = 6

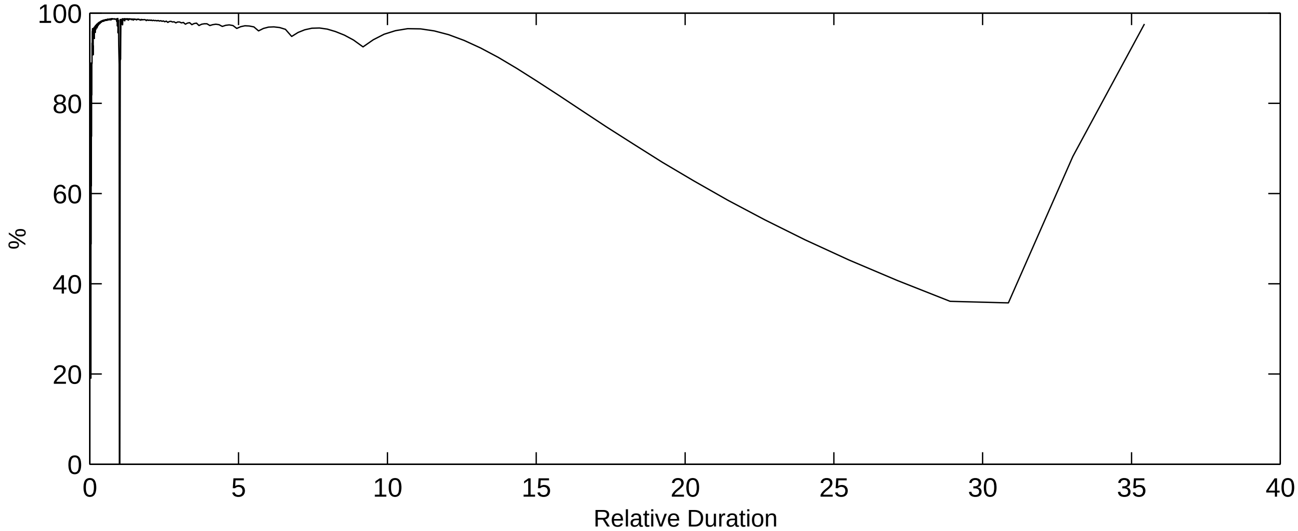

Figure 11: Deadweight Loss Reduction (%): Average Duration = 8