Currency Unions[*]

Alberto Alesina and Robert J. Barro

Harvard University

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1 Introduction

Traditionally, each country had its own currency, and only one currency circulated in each country. Monetary unions were rare, and, therefore, the surge in the number of countries in the post-war period generated a large increase in the number of currencies circulating in the world. In 1947 there were 76 countries in the world, today there are 193, and, with few exceptions, each country has its own currency.[1] Unless one believes that a country is, by de…nition, an "optimal currency area," either there were too few currencies in 1947 or there are too many today. In fact, the increasing integration of international markets implies that the optimal number of currencies would tend to decrease, rather than almost triple as it has.

Only recently, however, and perhaps as a result of this proliferation of currencies, the sanctity of "one country one money" has come into question. Eleven countries in Europe have adopted the same currency, dollarization is under active consideration in many countries in Latin America and is currently being implemented in Ecuador, and a currency union is being discussed in Central America. Countries in Eastern Europe and the former Soviet Union are considering adopting unilaterally the euro. In addition, several countries have adopted currency boards, including Hong Kong and Argentina with the dollar and Estonia and Bulgaria …rst with the German mark and later with the euro.

[1] See Rose (2000) and Joint Economic Committee (1999) for characterizations of countries that use currencies other than their own. The most important colonial related currency union is the French Franc Zone in Africa. Some other countries that use another nation's currency are Panama, Liechtenstein, Luxembourg, and San Marino.

Two factors have contributed to these trends. One is the increase in international trade in goods and services, expanded cross-border …nancial transactions, and heightened cross-country ‡ows of technology, in one word, "globalization." The second is the increased emphasis on price stability, as opposed to active macroeconomic stabilization, as a goal for monetary policy. This switch follows two decades (the seventies and eighties) with exceptionally high in‡ation rates in many developing countries and double-digit in‡ation in several industrial ones.

The basic framework for assessing common-currency zones comes from the optimal-currency-area analysis pioneered by Mundell (1961), who studied the choice between ‡exible and irrevocably …xed exchange rates in a monetary union. The bene…t of a common-currency area was its role in minimizing transaction costs and facilitating the ‡ow of information about relative prices.[2] The o¤setting force was that …xed exchange rates entailed the loss of independent monetary policies. In a bigger union with heterogeneous members, monetary policy would …t less well with the interests of each individual member. Mundell studied conditions under which the bene…ts of common currencies would outweigh their costs. He stressed factor mobility and price ‡exibility as key elements in this tradeo¤.

In this paper, we extend Mundell's framework to consider recent issues in monetary policy, especially the distinction between rules and discretion,

[2] Empirically, the e¤ect of …xed exchange rates on the volume of international trade is an unsettled issue. Several papers have investigated the bene…ts of exchange rate stability on trade ‡ows, reaching mixed results. See, in particular, Hooper and Kohlhagen (1978), Kenen and Rodrik (1986), and IMF (1984). Rose (2000) argues that the e¤ect of currency union on the volume of trade is large.

as emphasized by Barro and Gordon (1983).[3] Flexible exchange rates allow monetary independence, but the monetary authorities of many countries lack the ability to commit their policies to a stable and predictable rule. Policies carried out under these conditions may produce high and variable inflation. In addition, it is unclear that monetary discretion is efficiently used in many developing countries to reduce output fluctuations. In this situation, the cost of high inflation is not even compensated by less output variability.

A common currency reduces transaction costs in trade and financial transactions, and these benefits are larger the larger the size of the union. Money, like language, is more useful the greater the number of persons (and, in the case of money, transaction values) that share the same type. In addition, a system of irrevocably fixed exchange rates may be useful as a discipline device to assure price stability. However, this mechanism works effectively only if the domestic authority is willing to subordinate its monetary policy to the fixing of the exchange rate. Dollarization—or, less extreme, a currency board—is attractive as a way to ensure the credibility of a fixed-rate system.[4] However, even with a permanently fixed exchange rate, as guaranteed by full dollarization, a country would experience changes in prices relative to those of the anchoring country. These relative price movements reduce the

[3] There is now a large literature on the rules-versus-discretion trade off. An application of that framework that is especially related to the present paper is in Alesina and Grilli (1992).

[4] Some researchers, such as Hausmann, et al (1999), argue that a credible and stable fixed-rate regime, especially one involving dollarization, promotes the formation of long-term credit markets. This argument seems correct empirically, but the theoretical underpinnings have not been worked out.

desirability of fixed exchange rates. Therefore, countries would prefer to link to anchors with which they have small variations in relative prices.

The analysis is complicated by two factors that we take into account. First, the choice of regime tends itself to affect the variances of relative prices and output co-movements. Second, the anchor country's monetary policy may change as a function of which countries adopt the anchor's currency. This adjustment of policy may feature compensation schemes between "clients" and "anchors," possibly involving the amount of seignorage revenue received by the various governments.

After discussing the pros and cons of adopting another country's currency, we study how, given a distribution of independent countries, certain types of currency unions would emerge in equilibrium. Under a broad range of conditions, an increase in the number of countries (thus a reduction in their average size) would increase the desirability of currency unions. Hence, as the number of countries increases, the number of currencies should increase less than proportionately. In fact, under certain conditions, if one moves from, say, 100 countries to 200, the total number of currencies circulating may decrease in absolute terms. Consequently, in a world of small and highly integrated countries, where the benefits of low and stable inflation are highly valued, one should observe a collapse of the one-country one-money identity and a move toward a world with relatively few currencies.

The paper is organized as follows. The next section presents a model that highlights the pros and cons that a country faces when considering the adoption of a foreign currency. The following section discusses the endogenous formation of currency unions given a distribution of sizes of independent countries. The last section concludes.

2 A Model of Currency Unions

2.1 Output, Trade, and Country Size

We begin with a simple model of the real economy with a role for trade and country size. The text contains a sketch of the model with the main results. The details are in the appendix.

Suppose that the world consists of W individuals or economic regions, each of which has a fixed labor endowment, L. We can view these individuals as arrayed along a line segment, starting from the origin and then having equally spaced points at the positions $r = 1, \ldots, W$.

Each individual produces output, Y_r, using a varieties-type production function, which was originated by Spence (1976), Dixit and Stiglitz (1977), and Ethier (1982),

$$Y_r = A \cdot \left(\sum_{v=1}^{W} X_{vr}^{\alpha} \right) \cdot L^{1-\alpha}, \tag{1}$$

where $A > 0$ is a parameter, X_{vr} is the amount of nondurable intermediate input of type v used by individual r, and $0 < \alpha < 1$. Output, Y_r, can be used on a one-for-one basis for consumption, C_r, or to produce r-type intermediates, X_r. All consumer goods are identical, but each person produces a unique variety of intermediate. Prices of consumer goods are the same everywhere and are normalized to one. Person r is assumed to have monopoly power over the supply of his or her unique type of intermediate, X_r. The price set for this good is denoted by P_r, where $P_r > 1$ will apply.

We view a country as a collection of adjacent individuals. The size of country i, measured by the number of individuals, is denoted by N_i. The production function in equation (1) implies that every individual will want to use the intermediate input produced by all the others (as long as all of the prices are finite). Within each country, there is assumed to be complete free trade and no transaction costs for shipping goods. The shipping of an intermediate good across country borders entails transaction costs, which can reflect trade barriers and differences in language and currency. (For simplicity, we neglect any transaction costs for shipping consumer goods.) Specifically, we assume an iceberg technology, whereby, for each unit of intermediate good shipped from one country to another, only $1 - b$ units arrive, with $0 < b < 1$. The transaction costs would generally depend on the country pairs involved—for example, on distance and on differences in language—but we neglect these heterogeneities in the basic model even though we return to them later.[5]

Each producer of intermediates selects a single price, P_r, which applies at the point of origin for domestic purchasers and foreigners. Since foreigners receive only $1 - b$ units for each unit purchased, their effective price per unit

[5] A large empirical literature has shown that political borders matter greatly for the volume of trade. That is, regions of the same country trade with each other much more than they would if they were independent. See, for example, McCallum (1995) and Helliwell (1998). More generally, the "home-bias" effect is pervasive in various aspects of international economic relationships, as discussed in a unified framework by Obstfeld and Rogoff (2000).

of r-type intermediate employed in production is $P_r/(1-b)$. Thus, trade within a country faces monopoly pricing, whereas international trade faces monopoly pricing and shipping costs.

Each individual r chooses the quantity of intermediates to buy at home or abroad, X_{vr}, for $v = 1, \ldots, W$ $(\neq r)$; the quantity of own output to retain for use as an intermediate input, X_{rr}; and the price of its intermediate, P_r. The choice of the quantity of each type of intermediate to import takes as given the monopoly prices, P_v, set by $v \neq r$. Given the demand function for the r^{th} intermediate good, the setting of P_r determines the quantity of intermediate goods sold by r. The budget constraint determines consumption, C_r, as output, Y_r, less the amount of retained intermediates, X_{rr}, plus the net revenue from sales abroad and at home (the quantity sold multiplied by $[P_r - 1]$), less the amount paid for purchasing intermediates. The terms involving imports and exports take account of the iceberg losses on goods transported across country borders. The objective of each individual is to maximize C_r.

We show in the appendix that each producer of intermediates faces a demand curve with the constant elasticity $-1/(1-\alpha)$. This demand curve leads to the choice of the monopoly price or "markup ratio," $P_r = 1/\alpha > 1$, which is the same for all varieties of intermediate goods. The appendix also shows that the equilibrium level of output for individual r is given by

$$Y_r = \bar{A}L \cdot [1 + d_1 \cdot (N_i - 1) + d_1 d_2 \cdot (W - N_i)], \qquad (2)$$

where $\bar{A} \equiv A^{1/(1-\alpha)}\alpha^{\alpha/(1-\alpha)}$; $d_1 \equiv \alpha^{\alpha/(1-\alpha)}$, $d_2 \equiv (1-b)^{\alpha/(1-\alpha)}$, and N_i is the size of the country to which r belongs. The conditions $0 < \alpha < 1$

and $0 < b < 1$ imply $0 < d_1 < 1$ and $0 < d_2 < 1$. Note, inside the brackets in equation (2), that the production for own use counts as 1, the other $N_i - 1$ members of the same country count with the weight $d_1 < 1$ because of monopoly pricing of the traded intermediates, and the $W - N_i$ foreigners count with the even smaller weight $d_1 d_2 < 1$ because of monopoly pricing and shipping costs. From the perspective of incentives to produce, monopoly pricing and international trading costs have similar and reinforcing effects.

We show in the appendix that trades in intermediates between individuals in a country and across country borders are balanced. Hence, there are no net trades across borders in consumer goods. Note that the output concept given in equation (2) is gross of production of intermediates. In the case of balanced trade in intermediates, net output corresponds to consumption, which equals gross output less production of intermediates (including those that vanish due to the iceberg trading costs for international transactions). The appendix shows that the formula for consumption is

$$C_r = \bar{A}L \cdot (1 - \alpha) \cdot [1 + d_1 \cdot (1 + \alpha) \cdot (N_i - 1) + d_1 d_2 \cdot (1 + \alpha) \cdot (W - N_i)]. \tag{3}$$

The qualitative implications of equations (2) and (3) are intuitively reasonable and generalize beyond the specific model that we have adopted. The implications include the following:

- If international trading costs, b, were zero and pricing were competitive (which corresponds to $\alpha = 1$), so that $d_1 = d_2 = 1$, then Y_r/L and C_r/L would be proportional to the size of the world, W. This scale benefit arises because a larger world means more varieties of intermediate inputs. In this case, the size of the country, N_i, would not matter. More generally, for given N_i, a higher W raises $Y_r=L$ and $C_r=L$.

- If international trading costs exist, so that $d_1 > d_2$, then $Y_r=L$ and $C_r=L$ increase with N_i for given W. This effect arises because an increase in the size of the country expands the number of intermediate inputs for which the transaction costs in trade are nil.

- $Y_r=L$ and $C_r=L$ are decreasing in the international trading cost parameter, b (increasing in the parameter d_2 in equation [2]), because the costs paid for foreign imports of intermediate inputs are reduced.

- For given W, the larger the country, N_i, the smaller is the effect of international trading costs, b (or d_2), on $Y_r=L$ and $C_r=L$. Analogously, the lower b (the higher d_2), the smaller is the effect of country size, N_i, on $Y_r=L$ and $C_r=L$.

- The appendix shows that the ratio of foreign trade to output falls with international trading costs, b, and country size, N_i. The ratio of trade within a country to output rises with b and N_i.[6]

For given country sizes and trading costs, the distorting element in the model comes from the monopoly pricing of the intermediate goods. A social planner for the world would effectively price each of these goods at 1, rather

[6] If the production for own use is negligible, which holds, for example, if $N_i >> 1$, then these two effects are nearly offsetting. In this case, changes in international trading costs, b, and country size, N_i do not have a significant effect on the ratio of total trade to output.

than $P_r = 1/\alpha > 1$. Output, denoted by Y_r^*, would then be higher than before, corresponding to the replacement of the term d_1 in equation (2) by 1:

$$Y_r^* = \bar{A}L \cdot [N_i + d_2 \cdot (W - N_i)]. \tag{4}$$

This result assumes that the social planner takes as given the sizes of countries, N_i, and must pay the costs b for inter-country trades. If country i contains many individuals, so that $N_i >> 1$, then the shortfall of production due to monopoly pricing is given from equations (2) and (4) by

$$Y_r/Y_r^* \approx d_1 < 1. \tag{5}$$

In this model, consumption per person (and, hence, the utility of the representative consumer) would be maximized if the entire world consisted of one country, because cross-border transaction costs would then be eliminated. However, this conclusion arises only because we have neglected some costs that tend to rise with the size of the country. In particular, larger political jurisdictions typically have to deal with a more heterogeneous citizenry. The growing heterogeneity makes it increasingly difficult to agree on a set of polices and institutions. In addition, diseconomies of scale in public administration tend to emerge at some level of country size.

Suppose that the per capita costs of heterogeneity are an increasing function of country size and can be represented by the function $h(N_i)$, with $h'(\cdot) > 0$. Then, in an interior equilibrium, the optimal size of a country is determined by the condition that the marginal benefit of size, emerging from equation (2), equal the marginal cost of heterogeneity.[7] Given the symmetry

[7] This kind of tradeoff for determining country size is the one emphasized in Barro

of the model, this condition will tend to dictate that all countries be of the same size. However, if the heterogeneity costs—or the costs of trading across country borders—depend on the identity of the individuals, then we can have equilibria in which countries have di¤erent optimal sizes. In any event, we treat the country sizes, N_i, as exogenous in the present context.

2.2 Monetary Policy

To discuss currency unions, we have to enrich the model to introduce a role for monetary policy and in‡ation. One way is to assume that prices are set one period in advance by nominal contracts so that unexpected in‡ation reduces the real price. Another approach, represented by the Lucas supply function, assumes that nominal prices are mistakenly interpreted as real prices for a short period because of informational lags.

In the context of our model, the contracting approach implies that the nominal price of intermediates would be set one period in advance. Unexpected in‡ation in country i, which shows up as higher nominal prices of consumer goods throughout country i, would lower the relative prices of intermediates. This e¤ect applies to intermediate goods produced by individuals within the same country; it applies to intermediate goods produced in other countries only if the contract speci…ed a nominal price denominated in country i's currency. In any event, unexpected in‡ation in country i raises the quantity of intermediates demanded from other individuals in country i and would lead, thereby, to an expansion of output. Since the monopoly power of producers of intermediates keeps output below its …rst-best level,

(1991), Alesina and Spolaore (1997), and Alesina, Spolaore, and Wacziarg (2000).

this expansion in output caused by unexpected inflation tends to be efficient.

In the Lucas supply approach, the producers of intermediate goods tend to under-assess the rise in consumer prices and, therefore, set a nominal price that does not fully incorporate the country's general rise in prices. Hence, unexpected inflation in country i would again reduce the relative prices of intermediates and lead, accordingly, to a greater quantity of intermediates demanded.[8]

We can think of equation (2), when aggregated across individuals of a country, as determining a country's natural level of output, denoted by $\tilde{Y}_i$. The value $\tilde{Y}_i$ depends on trade costs, country size, endowments of labor, and technological parameters. Unexpected inflation lowers the relative price of intermediate goods, at least for those goods purchased from other individuals in the same country. The effect on output in equation (2) is analogous to that from a rise in the parameter d_1 attached to the term $N_i - 1$. As a log-linear approximation, we can write the formula for the country's log of output, $y_i \equiv \log(Y_i)$, as

$$y_i = \tilde{y}_i + \phi \cdot (\pi_i - \pi_i^e), \tag{6}$$

where $\tilde{y}_i \equiv \log(\tilde{Y}_i)$ and $\phi > 0$.

In the model, the monopoly pricing of intermediate goods implies that the target or efficient level of output exceeds the natural level and is given

[8] We could also introduce these inflation effects into the model by allowing for a variable quantity of labor, L_r, for each individual and then having unexpected inflation raise L_r for each person in the country. However, as the model stands, the resulting increases in output would not tend to be Pareto improving, because no distortion would apply directly to the determination of L_r. Such a distortion could be introduced, for example, by allowing for labor income taxation as a method for financing public goods.

by equation (4). We assume here that, in log terms, the target, y_i^*, equals the natural level, $\bar{y}_i$, plus a fixed amount z that does not vary by country, plus a country specific stochastic term, η_i:

$$y_i^* = \bar{y}_i + z + \eta_i, \tag{7}$$

where η_i is serially independent with zero mean and constant variance $\sigma^2_{\eta_i}$. In equation (5), η_i would correspond to country-specific, time-varying elements that affect the parameter d_1. Specifically, a higher η_i corresponds to a higher markup ratio, $1/\alpha_i$.

The gap between actual and target output is given by

$$y_i - y_i^* = \phi \cdot (\pi_i - \pi_i^e) - z - \eta_i. \tag{8}$$

Thus, $z > 0$ means that the monetary authority is typically motivated to engineer positive inflation surprises. This incentive is greater the higher is η_i.

2.3 Independent Monetary Policy under Discretion

Suppose for now that trading costs do not depend on currency choices. Then $\bar{y}_i$ is unaffected by the choice about dollarization. Thus, the policymaker cares about the gap between actual and target output, $y_i - y_i^*$(from equation [8]), but does not consider any effects on the natural level of output, $\bar{y}_i$. We bring in later effects of dollarization on trading costs. For given $\bar{y}_i$, the objective of monetary policy in country i can be described by the minimization of the expected net costs of inflation, ϖ_i, which we express as a fraction of country i's GDP in a simple functional form:

$$\varpi_i = (\gamma/2) \cdot (\pi_i)^2 + (\theta/2) \cdot [\phi \cdot (\pi_i - \pi_i^e) - z - \eta_i]^2 - a\pi_i. \qquad (9)$$

The first term, $(\gamma/2)(\pi_i)^2$, where $\gamma > 0$, captures deadweight losses from inflation (which we do not model formally). The second term, $(\theta/2) \cdot [\phi \cdot (\pi_i - \pi_i^e) - z - \eta_i]^2$, is an expectational Phillips curve effect. The use of surprise inflation to raise output is typically valued because $z > 0$. The condition $\theta > 0$ means that the loss function penalizes deviations of output from its target in either direction.[9] The final term in equation (9), $-a\pi_i$, where $a > 0$, represents seignorage revenue, which is taken to be linear in inflation.[10] Thus, the monetary authority values the seignorage revenue on a one-to-one basis. More generally, seignorage would be useful for a benevolent government because it expands the menu of taxes available.[11] For a discussion of currency unions, this term is interesting because it may be allocated in different ways among members of a currency union.

Country i has the choice of conducting monetary policy on its own or anchoring to another country. On its own, the inflation rate is determined

[9] An additional benefit of surprise inflation could reflect effects of surprise inflation on the real value of nominal obligations, for example, of government debt denominated in domestic currency. With distorting taxation, these kinds of capital levies would be valued, because they would reduce the distortions from other sources of revenue. In this case, a positive η_i would represent a situation in which this type of revenue is especially valuable, perhaps because of an emergency that motivates temporarily high levels of public spending.

[10] More complicated functional forms, including making seignorage a function of unexpected inflation, would not change the qualitative nature of our results.

[11] Had we adopted the formulation of a variable labor supply distorted by an income tax, the seignorage revenue would interact with the revenue from the income tax.

in a discretionary manner each period to minimize ϖ_i, as defined in equation (9). The authority cannot make commitments about inflation, and the rational formation of expectations, π_i^e—based on information from the previous period—takes this incapacity into account. The timing is as follows: first, expectations on inflation are set, then the shock is realized and publicly observed, then the policymaker chooses inflation.[12]

The solution for the discretionary equilibrium, which follows the approach of Barro and Gordon (1983), is

$$\hat{\pi}_i = \frac{a}{\gamma} + \frac{\theta\phi z}{\gamma} + \frac{\theta\phi\eta_i}{(\gamma + \theta\phi^2)}. \qquad (10)$$

The resulting expectation of the net costs of inflation can be calculated from equations (9) and (10) as

$$E\hat{\varpi}_i = \frac{1}{2} \cdot \left[-\frac{a^2}{\gamma} + \theta z^2 + \frac{(\theta\phi z)^2}{\gamma} + \frac{\theta\gamma\sigma_i^2}{\gamma + \theta\phi^2} \right]. \qquad (11)$$

If the monetary authority could commit inflation at least one period ahead, then the inflation rate in equation (10) would be reduced by the inflation-bias term, $\frac{\theta\phi z}{\gamma}$. The term $\frac{(\theta\phi z)^2}{\gamma}$ in equation (11) reflects these costs from the inflation bias. Note that the cost arises because the monetary authority cares about the departure of actual from target output ($\theta > 0$), because inflation surprises raise output ($\phi > 0$), and because target output typically exceeds actual output ($z > 0$).

[12] In practice, inflation is not set directly by the policymaker. However, as a vast literature including Cukierman (1992) has demonstrated, little insight is gained by complicating the model to include a monetary instrument that is linked imperfectly to inflation.

The monetary authority's reaction to the economic disturbance ´$_i$, as shown in equation (10), is a countercyclical policy. This reaction creates unexpectedly high or low in‡ation—and therefore e¤ects on output—in response to movements of ´$_i$. The ability of the monetary authority to tailor in‡ation to current economic conditions, as represented by ´$_i$, is valuable in the model, that is, $E\hat{\$}_i$ is lower than it would be if this ability were absent. This e¤ect provides the key bene…t from an independent monetary policy in the model. A monetary authority that can commit to an optimal contingent rule would also have ¼$_i$ responding to ´$_i$ in the manner shown in equation (10).

2.4 Outcomes under Dollarization

Consider now a potential anchor country, denoted by the subscript j. We assume that this country has the same underlying preference and cost parameters as country i, that is, the parameters in equation (9) are the same. However, the monetary authority of country j is able to commit its method for choosing in‡ation at least one period ahead. This authority picks an optimal contingent rule (a relation between ¼$_j$ and ´$_j$) to minimize the prior expectation of $\$_j$. The in‡ation rate in country j will be given by the form of equation (10), except that the in‡ation bias term is absent:

$$¼_j^{¤} = \frac{a}{°} + \frac{µÁ´_j}{(° + µÁ^2)}. \qquad (12)$$

Note that country j's monetary authority reacts to its own economic disturbance, ´$_j$, which is serially independent with zero mean and constant variance

$\sigma^2_{\eta_j}$. However, η_j need not be independent of η_i.

Suppose that country i irrevocably fixes the exchange rate of its currency to that of country j by adopting country j's currency. In what follows we assume that the decision to "dollarize" is irrevocable. That is, even though a country cannot make a binding commitment to a policy rule, it can make an irrevocable commitment to give up its currency. This assumption rests on the idea that it is institutionally much more costly to renege on a dollarization commitment than on a monetary policy rule.[13] In the case of a fixed exchange rate, the inflation rate in country i, π_i, would equal π_j^* plus the rate of change of the price of a market basket of goods in country i expressed relative to that in country j. We assume that this rate of change of relative prices is given by an exogenous, random error term, ε_{ij}. This shock is serially independent with zero mean, constant variance σ^2_ε, and is distributed independently of the shocks to economic activity, η_i and η_j, in the two countries. Hence, under dollarization, country i's inflation rate is given by

$$\pi_i^j = \frac{a}{\gamma} + \frac{\theta\chi\eta_j}{(\gamma + \theta\chi^2)} + \varepsilon_{ij}. \tag{13}$$

The j superscript indicates that the outcome applies for country i under anchoring to country j.

If country i no longer issues its own currency then it loses the seignorage income, given by $a\pi_i$. The corresponding income accrues instead to country j. Country j may or may not compensate country i for this transfer of

[13] In any event, a foreign monetary authority lacks the power to erode the real value of dollar bills. The foreign government may, however, be able to depreciate the real value of dollar denominated domestic obligations by formal defaults.

seignorage revenue. We assume, for now, that the anchor returns to country i the full amount of the seignorage obtained in country i. In this case the anchor country has no incentive to change its policy regardless of what country i chooses. We discuss below alternative arrangements. Country i's expected net costs of inflation are given from equation (9) by:

$$E\varpi_i^j = -\frac{a^2}{2\gamma} + \frac{\theta z^2}{2} + \frac{(\gamma + \theta\phi^2)\sigma_\epsilon^2}{2} + \frac{\theta^2\phi^2\sigma_{\eta_j}^2}{2(\gamma + \theta\phi^2)} + \frac{\theta\sigma_{\eta_i}^2}{2} - \frac{\theta^2\phi^2 COV(\eta_i;\eta_j)}{\gamma + \theta\phi^2}. \qquad (14)$$

The covariance between η_i and η_j appears in equation (14) because it determines the extent to which country j's adjustments to its own economic disturbances, η_j, are helpful for country i.

2.5 The Choice of Whether to Dollarize

The difference between $E\hat{\varpi}_i$ from equation (11) and $E\varpi_i^j$ from equation (14) is given by

$$\Delta\varpi^{ij} \equiv E\hat{\varpi}_i - E\varpi_i^j = \frac{(\theta\phi z)^2}{2\gamma} - \frac{1}{2}\cdot\left[(\gamma + \theta\phi^2)\cdot\sigma_\epsilon^2 + \left(\frac{\theta^2\phi^2}{\gamma + \theta\phi^2}\right)\cdot VAR(\eta_i - \eta_j)\right]. \qquad (15)$$

A positive value for $\Delta\varpi^{ij}$ indicates that the independent regime is more costly for country i than the system with anchoring to country j. Hence, anything that raises the terms on the right-hand side of the equation favors dollarization.

The first term, $\frac{(\theta\phi z)^2}{2\gamma}$, is the cost associated with the inflation bias under a discretionary regime in country i. The linkage to the committed country j avoids these costs and thereby favors dollarization. The second term, which involves σ_ε^2, derives from the random shifts in relative prices between countries i and j. Since country i receives country j's inflation rate only up to the random error, ε_{ij}, a higher value for σ_ε^2 makes dollarization less attractive. The third term, which contains $VAR(\eta_i - \eta_j)$, reflects the benefits from an independent monetary policy, in the sense that π_i can react to η_i in the autonomous regime. The extent of this benefit depends on how closely η_j moves with η_i. Equation (15) shows that the variance of $\eta_i - \eta_j$ is what matters for the comparison between the regimes.

Note from equation (15) that there are two senses in which greater comovement between countries i and j favors dollarization. One relates to the variance of relative prices, σ_ε^2. This effect arises even if the monetary authorities do not conduct countercyclical policies. An effect of σ_ε^2 in equation (15) applies even if inflation surprises do not affect output ($\phi = 0$). All that is necessary is that costs, ϖ_i, depend on inflation ($\gamma > 0$).

Second, a greater variance of relative economic disturbances, $\eta_i - \eta_j$, makes dollarization less attractive. The presence of this term depends on a number of assumptions in the model that make countercyclical monetary policy useful. First, unanticipated inflation raises output ($\phi > 0$) and the resulting expansions in output are valued ($\theta > 0$). Second, the effect depends on the monetary authority's ability to act while inflationary expectations are fixed—that is, π^e is based on the prior period's information.

2.6 Extensions

2.6.1 Dollarization affects the shocks

It is often argued that a common-currency link affects co-movements among countries, for example, by promoting trade and factor mobility. If we allow for an effect of the monetary system on the distributions of the shocks, then the criterion for dollarization is modified from equation (15) to

$$\Delta \$^{ij} = \frac{(\theta\phi z)^2}{2\gamma} - \frac{1}{2}\cdot\left[(\gamma+\theta\phi^2)\cdot\sigma_\epsilon^2 + \left(\frac{\theta^2\phi^2}{\gamma+\theta\phi^2}\right)\cdot VAR(\eta_i - \eta_j)\right] + \frac{\theta\gamma(\tilde{\sigma}^2_{\eta_i} - \sigma^2_{\eta_i})}{2(\gamma+\theta\phi^2)}, \quad (16)$$

where $\tilde{\sigma}^2_{\eta_i}$ is the variance of η_i in the autonomous regime, and the unmarked variances refer to the dollarized system. The last term indicates that dollarization would be favored if this linkage reduces the variance of disturbances in country i, that is, if $\tilde{\sigma}^2_{\eta_i} > \sigma^2_{\eta_i}$. This effect would be predicted if the currency linkage buffers the disturbances that impinge on country i (because of the easier adjustments of trade and factor flows). Dollarization is also more attractive the lower σ_ϵ^2 and $VAR(\eta_i - \eta_j)$—these values are the ones applicable in the dollarized setting. Hence, if linkage reduces these variances, then dollarization looks more favorable.

2.6.2 Simple rules

The analysis of dollarization has assumed that country j commits to the contingent rule for π_j that minimizes the prior expectation of $\$_j$. However, one may argue that commitment is difficult to verify and, hence, maintain when it involves these sorts of contingent reactions of π_j to η_j.[14] In our model,

[14] See, for example, the symposium on central bank independence in the 1995 NBER Macroeconomic Annual.

the contingent rule is easy to implement and verify, but matters become much more complicated if shocks are not immediately and universally observable.

The nature of the issue can be illustrated by assuming that country j can follow discretion or commit to a simple rule that precludes feedback from η_j to π_j. In this case π_j would be set to the constant a/γ.[15] If the anchor follows the simple rule, the next to last term in equation (16) becomes

$$-\frac{\theta^2\phi^2\sigma^2_{\eta_i}}{2(\gamma+\theta\phi^2)}.$$

This term is smaller in magnitude than the corresponding term in equation (16) if

$$\sigma_{\eta_j} > 2\rho_{ij}\sigma_{\eta_i},$$

where ρ_{ij} is the correlation (under the dollarized regime) between η_i and η_j. Thus, if $\sigma_{\eta_i} = \sigma_{\eta_j}$, then if $\rho_{ij} > 1/2$ country j is more attractive as an anchor for country i if country j follows an optimal contingent rule where π_j responds to η_j. If $\rho_{ij} < 1/2$, then country j is a more attractive anchor if it follows the simple rule in which π_j is constant. In other words, active countercyclical policy by the anchor country is attractive to linking countries only if their disturbances (η_i) are—under the dollarized system—highly correlated with those of the anchor (η_j). Thus, for some potential clients, the inability of the anchor to follow a contingent first-best rule is a plus.

[15] In this situation, country j might prefer discretion to the simple rule. Discretion allows for flexible responses of π_j to η_j, whereas the simple rule precludes these reactions.

2.6.3 The anchor keeps the seignorage

If country j's objective is to minimize the expectation of Ω_j less the seignorage revenue obtained from country i (with no allowance for the costs of inflation borne by country i), then the only difference from equation (12) is in the choice of intercept. The new coefficient is

$$\frac{a}{\gamma} \cdot \frac{1}{(1 - \tau_i)},$$

where $\tau_i \equiv Y_i/(Y_j + Y_i)$ is the shared of country i in the combined GDPs. Hence, the seignorage obtainable from country i motivates country j to select higher inflation than otherwise. The greater is τ_i the more inflation is raised above its previous level, a/γ. Thus, if the anchor country values the seignorage obtainable from clients but does not consider the costs that inflation imposes on these clients, then dollarization can be inflationary. The results are different, as discussed below, if the anchor takes account of the costs imposed on clients.

2.6.4 Adjustments by the anchor country with compensation

Another issue is whether the anchor country would be motivated to alter its policies to consider the interests of the linking countries, in effect, the clients of the anchor. We explore whether a system of transfers can make an adjustment of the anchor's policy mutually beneficial.[16]

The net cost of inflation, Ω_i, from equation (9) applies as a fraction of country i's GDP, Y_i. If we take the universe as the anchor country j plus

[16] A complex political game may be involved in the fixing and implementation of these schemes. This game is not modeled here.

one linking country i, then the total net cost due to in‡ation, expressed as a share of the combined GDPs, $Y_j + Y_i$, is

$$\$ = ¿_j \$_j + ¿_i \$_i, \tag{17}$$

where $¿_j ´ Y_j=(Y_j + Y_i)$ and $¿_i ´ Y_i=(Y_j + Y_i)$. One possibility is that the anchor country determines its policy rule to minimize the prior expectation of $\$$, rather than $\$_j$, as assumed before. The $\$$ objective weighs foreigners' net costs equally with those of domestic residents. Such an objective need not re‡ect global altruism by the anchor nation. Rather, this objective would emerge in equilibrium from competition among anchor countries, assuming that clients e¤ectively compensate the anchor for deviating from policies that are otherwise best for the anchor's domestic residents. One way that this compensation could occur, as part of a competitive equilibrium, is for each anchor country to retain the amount of seignorage that just compensates for the worsening of policy from a domestic perspective. If there is not enough seignorage revenue to compensate, then some other mechanism would have to be devised to allow international payments for monetary services.

Let the anchor's policy rule be designated by

$$¼_j = ¹ + º_j ´_j + º_i ´_i + º_2 ²_{ij}, \tag{18}$$

where $(¹, º_j, º_i, º_2)$ are the feedback coe¢cients chosen by the monetary authority. Equation (12) is the special case of equation (18) that arises when $\$$ depends only on $\$_j$. The inclusion of $\$_i$ as part of the revised objective will a¤ect the choice of some of the coe¢cients in equation (18), but the linear form will still be optimal in the present model.[17]

[17] Note that we have returned to the setting in which country j can commit to a contin-

If country j's objective is to minimize the prior expectation of $, then the optimal values of the coe¢cients that appear in equation (18) turn out to be

$$
\begin{aligned}
¹ &= a=°, \\
º_j &= ¿_j ¢ \frac{µÁ}{(° + µÁ^2)}, \\
º_i &= ¿_i ¢ \frac{µÁ}{(° + µÁ^2)}, \\
º_2 &= ¡ ¿_i.
\end{aligned}
\qquad (19)
$$

The constant term, ¹ = a=°, is the same as before. That is, the consideration of the broader universe that encompasses country i does not change the average in‡ation rate chosen by country j. Hence, dollarization is not in‡ationary when the anchor takes account of costs imposed on clients. Country j's response, $º_j$, to its own economic disturbance, $´_j$, is the same as before, except that the coe¢cient is attenuated by multiplication by the GDP share, $¿_j$. Correspondingly, the anchor's choice of in‡ation, $¼_j$, now reacts in accordance with the coe¢cient $º_i$ to country i's economic disturbance, $´_i$. This response depends on country i's GDP share, $¿_i$. The coe¢cient $º_2$ = ¡ $¿_i$ means that country j's monetary authority partly o¤sets an increase in relative prices in country i by lowering $¼_j$. The extent of the o¤set is given by $¿_i$, the share of country i's GDP.

>From the perspective of minimizing the expectation of its own net costs, $\$_j$, country j's reactions of $¼_j$ to $´_i$ and $²_{ij}$ and the insu¢cient reaction of

gent rule in the sense of committing to the coe¢cients shown in equation (18). We also neglect here, for simplicity, any e¤ect of dollarization on the distribution of the disturbances, as explored before.

π_j to η_j are, per se, unattractive. That is why this behavior by country j hinges on some sort of compensating payment from country i to country j. As already mentioned, one possibility is that country j retain part of the seignorage income associated with country i's use of country j's money.

On its own, country j chooses the inflation rate π_j^* given in equation (12). With the accommodation to country i, country j chooses the inflation rate π_j given by equations (18) and (19). The amount that country j loses from the accommodation can be calculated by looking at the difference in expected costs, $\$_j$, associated with the two choices of inflation. The result is

$$\begin{aligned}&\text{Cost of accommodation} \\ &= \frac{1}{2}(\tau_i)^2 \cdot \left\{ (\gamma + \theta\phi^2) \cdot \sigma_2^2 + \left(\frac{\theta^2\phi^2}{\gamma + \theta\phi^2}\right) \cdot VAR(\eta_i - \eta_j) \right\}.\end{aligned} \tag{20}$$

Thus, the cost to country j depends on the relative size of country i, τ_i, on the variance of the relative price shocks, σ_2^2, and on the variance of the difference in the economic disturbances, $\eta_i - \eta_j$. If there were no relative price shocks and no differences in economic disturbances, then it would be costless for country j to accommodate its inflation choice to country i.

Suppose now that country i can choose whether to link to country j, that country j accommodates its inflation choice to the presence of country i (as implied by equations [18] and [19]), and that country i pays the compensation corresponding to equation (20).[18] The criterion for country i to dollarize is then modified from equation (15) to

$$\Delta \$^{ij} = \frac{(\theta\phi z)^2}{2\gamma} - \frac{\tau_j}{2} \cdot \left\{ (\gamma + \theta\phi^2) \cdot \sigma_2^2 + \left(\frac{\theta^2\phi^2}{\gamma + \theta\phi^2}\right) \cdot VAR(\eta_i - \eta_j) \right\}. \tag{21}$$

[18] The level of compensation is the amount shown in equation (20) multiplied by Y_j.

The new element in equation (21) is that the terms involving σ_2^2 and $VAR(\epsilon_i - \epsilon_j)$ are smaller in magnitude than before because they are multiplied by τ_j, which is less than one. These terms are smaller because country j's partial adjustment of π_j for country i's disturbances makes these disturbances less costly for country i (even after considering the compensation that country i pays to country j). Thus, overall, the choice of dollarization looks more favorable because of the anchor country's accommodation of its clients.

Another result from equation (21) is that a smaller value for τ_j makes dollarization more attractive. The reason is that a smaller τ_j reduces the compensation that country i must pay to country j for its accommodations. In this model, the attraction of dollarization is that it buys a committed monetary policy. A small anchor country is, in this respect, as good as a large one, because the commitment technology is assumed to work as well in either case. However, for the large anchor country, the costs of accommodating to country i are greater (because the term in equation [20] applies over a larger scale, Y_j). Thus, for given values of σ_2^2 and $VAR(\epsilon_i - \epsilon_j)$, the small country is preferred as an anchor.

The conclusion about the desirable size of the anchor country may change if the capacity to maintain a commitment depends on the relative economic sizes of the anchor country and its customers. For example, consider a large country, such as Russia, using a small one, say Latvia, as an anchor. This arrangement may not work because ex-post pressure from Russia to create "unanticipated" inflation could be too much for Latvia to bear. In other words, anchors that are larger (in relation to their clients) may be more solid because they can better withstand pressures to be time inconsistent.

2.6.5 Sizes of currency unions: preliminary considerations

Suppose that there are M countries in the world, where M_1 have the capacity to make commitments and $M - M_1$ do not. The uncommitted countries are potential clients, who would be motivated to link up with committed anchors. For a given client, potential anchors would be more attractive the more they exhibited co-movements with the client, that is, the lower the variances of $\varepsilon_i - \varepsilon_j$ and η_{ij}. In addition, as linkages occur and currency unions grow larger, this size effect may influence the incentives for additional clients to join. We consider here the case in which the shocks ε_i for countries are either independent or perfectly correlated. We ignore the shocks η_{ij}, although effects analogous to those for ε_i would arise. We assume that all countries are the same size, but a modification to allow different sizes is straightforward. Finally, we assume that there are enough potential anchors so that each behaves competitively with respect to the fee charged to clients.

Suppose first that the ε_i for anchors and clients are all independent. In this specification, all committed countries are equally attractive as anchors. The only aspects of currency unions that can influence potential clients are the sizes of the unions, that is, the number of clients who have already attached themselves to a given anchor. Let K be the number of countries in a currency union, including the one committed country and $K - 1$ clients. We can then show the following results. As K rises, the expected inflation costs per country rise, because monetary policy is less well tailored to individual disturbances. However, the marginal effect of an increase from K to $K + 1$ on the total costs borne by the incumbent K members declines. Therefore, the competitive fee charged to a new entrant falls as K rises. However, the gross benefit to the entrant also falls (because monetary policy is less well suited to the new entrant's disturbances). This effect exactly offsets the declining entry fee, so that the net benefit to a new entrant is invariant with K. This net benefit may be positive or negative, that is, clients may prefer linkage or autonomy, depending on parameter values. If linkage is attractive, then the sizes of currency unions would be indeterminate. Small groupings of countries with independent disturbances work as well as large groupings.

In our model, where the only benefit from currency linkage is the securing of a commitment and where all of the M_1 committed countries are equally capable, none of the committed countries would join together in a common currency union. The uncommitted $M - M_1$ countries would link with one of the committed countries if the parameters were such that the net benefit from linkage were positive. With all disturbances independent (and all other parameters identical), this net benefit looks the same for each uncommitted country regardless of the identity of the anchor or the size of the union attached to a particular anchor. Hence, if the net benefit is positive, then all uncommitted countries link up with one of the committed countries. (Otherwise, all of the uncommitted countries remain on their own.) In an equilibrium where all of the uncommitted countries join unions, some of the M_1 committed countries would have large numbers of clients and some would have small numbers. The distribution of sizes is not pinned down.

Another way to understand the result is that—with the quadratic cost structure that we have assumed—costs arising in equilibrium in each currency union end up as a linear function of the number of uncommitted countries that belong to the union. Hence, equal or unequal sizes of unions end up generating the same aggregate of costs in the overall economy. It follows that any distribution of sizes of unions is equally efficient. Since the competitively determined sizes of unions (based on competitive fees charged by the anchors) correspond to efficient outcomes, it also follows that the sizes of the competitively determined unions are not pinned down.

Suppose now that the disturbances ϵ_i for the clients are perfectly correlated with that of a potential anchor, ϵ_j. In this case, the competitive entry fee is always zero, because the inclusion of a new client does not interfere with monetary policy. The gross benefit from joining a union is positive, because the inflation bias is avoided and no loss from relinquishing an independent monetary policy applies because of the perfect correlation of shocks. This benefit is also independent of K. Therefore, the net benefit of linkage is positive, and all clients would join a currency union with a committed anchor. However, small and large unions are equally attractive, so that the sizes of unions are again indeterminate.

Assume now that existing unions consist of K perfectly correlated or K uncorrelated countries. Suppose that a potential client country has disturbances that are independent of those in both unions. We can show that, for any K, the net benefit from joining the uncorrelated union exceeds that for the correlated one. The reason is that the inclusion of an uncorrelated newcomer contaminates the workings of the correlated union—therefore, the entry fee for joining the correlated union is higher than for the uncorrelated one. In addition, the gross benefit from joining the correlated union is smaller, because the correlated union is less accommodating to the interests of the newcomer. Hence, the equilibrium would tend to feature unions segregated by shocks—highly correlated countries would be grouped together and would tend to exclude uncorrelated countries.

An explicit consideration of trade benefits would introduce a well-defined trade-off between size and heterogeneity in a currency union. Assume, for a more general case than the ones consider thus far, that countries are arranged in decreasing order of correlation of shocks with a potential anchor. The more countries added to the currency union, the less correlated the next entrant is and the less well tailored monetary policy will be to the members. However, the larger the union, the greater the trade benefits. This trade off between size and heterogeneity will pin down the optimal size of currency unions. In order to analyze this issue more precisely, we now move to an explicit model of the "geography" of countries and currency unions.

3 Number of countries and of currencies

3.1 The setup

We now investigate the equilibrium number of currency unions in a world composed of an exogenous number of independent countries. To keep things simple, we return to the case of no compensation from clients to anchors. We also neglect the effect of dollarization on the variance of shocks, the issue addressed in section 2.6.1.

In this situation, equation (15) implies that the criterion for country i to prefer linkage to country j over autonomy is given by

$$\Delta \$^{ij} = \frac{(\theta\psi z)^2}{2\gamma} - \frac{1}{2}\left\{(\gamma + \theta\psi^2)\cdot\sigma_{\epsilon}^2 + \left(\frac{\theta^2\psi^2}{\gamma+\theta\psi^2}\right)\cdot VAR(\eta_i - \eta_j)\right\} > 0. \tag{22}$$

Recall that this criterion assumes that country j follows a committed policy, whereas country i would, on its own, follow a discretionary policy. Hence, the first element in the choice about currency unions is whether a country can make a commitment to a rule for monetary policy. We assume that there are two types of countries in this respect. The indicator β_i takes the value one if country i can make binding commitments and zero if it cannot. We treat this commitment ability as exogenous and do not allow for intermediate cases in which some form of partial commitment is feasible.

The second element concerns the distribution parameters for the disturbances in equation (22). Linkage is more attractive if σ_{ϵ}^2 and $VAR(\eta_i - \eta_j)$ are low under the dollarized system. We focus here on a key factor that would influence these distribution parameters—the extent to which countries i and j are linked by trade.[19]

Let T^0_{ij} be the volume of bilateral trade between countries i and j: The volume of trade depends on trading costs, which we represented by the parameter b in our initial model. Now we denote this parameter by b_{ij} and allow it to depend on the country pair. Specifically, the trading cost will depend on the distance between the countries. In the empirical gravity literature, the concept of distance captures physical distance and other factors, such

[19] See Imbs (1999) for a review of the literature on how trade affects co-movements of output.

as language, colonial history, sharing a border, being an island, etc. In our formalization, we assume that a country's position along the line segment that describes the world captures all these aspects of distance. Formally, let D_{ij} be the distance between the mid-points of countries i and j. We assume that b_{ij} is increasing in D_{ij}. Hence, the parameter $(d_2)_{ij} \equiv (1 - b_{ij})^{\alpha/(1-\alpha)}$, which enters into an extended version of equation (2), is decreasing in D_{ij}. A simple generalization of the trade model worked out in the appendix shows that the volume of trade between countries i and j (that is, imports of j from i plus imports of i from j) is given by

$$T'_{ij} = \tilde{A}\alpha^{\alpha/(1-\alpha)} L \cdot (d_2)_{ij} N_i N_j, \qquad (23)$$

where, as before, $\tilde{A} \equiv A^{1/(1-\alpha)}\alpha^{\alpha/(1-\alpha)}$.

The trade volume is increasing in the size of each country. However, the correlation between the shocks of the two economies will be related to the volume of trade scaled in some manner by country sizes. If T_{ij} is the trade volume scaled by size, then the larger T_{ij} the lower are σ_ε^2 and $VAR(\eta_i - \eta_j)$. Thus, a higher value of T_{ij} raises $\Delta\varpi^{ij}$ in equation (22).

If the adoption of a common currency reduces trading costs, then the adoption of a currency union also has a direct positive effect on output and consumption. Let $\Delta d_2 \equiv (d'_2)_{ij} - (d_2)_{ij} > 0$, where $(d'_2)_{ij}$ is the value of the d_2 parameter when the two countries share the same currency and $(d_2)_{ij}$ is the value otherwise. If N^u is the existing size of the currency union that country i is considering joining, then the gain in consumption for country i by joining the union is given (from an extension of equation [3]) by

$$\Delta C^{ij} = \psi L(1-\alpha)\cdot d_1(1+\alpha)N^u \cdot \Delta d_2 > 0. \quad (24)$$

The consumption gain is increasing in the size of the union and in the difference in the trading costs within and outside the union. Country i will now choose whether to join a currency union anchored to country j depending on whether the total benefit, given by $\Delta\$^{ij} + \Delta C^{ij}$, is positive. The country therefore cares about the expression

$$\Delta\$^{ij} + \Delta C^{ij} = \Gamma(\beta_j - \beta_i; T_{ij}; N^u), \quad (25)$$

where $\Gamma(\cdot)$ increases with $\beta_j - \beta_i$ and N^u and falls with D_{ij} (because of the reduction of T_{ij}).

We are interested in an equilibrium defined as follows:

Definition: An equilibrium is a configuration of currency unions in which no country belonging to a union would like to leave the union to have its own currency or to join another union. In addition, no country not belonging to a union would like to join one.

We begin by imposing some structure to the problem.

3.2 The case of equal country sizes

Assume first that the world consists of M countries of equal size $N = 1/M$. Obviously, countries for which $\beta = 1$ have a comparative advantage at providing the currencies used in multi-country currency unions. One can easily show that the largest D_{ij} for which country i would adopt the currency of country j is larger if $\beta_j = 1$ than if $\beta_j = 0$. Suppose that there are M countries, numbered from 1 to M from left to right. Assume that $\beta_k = \beta_h = 1$ with $1 \le k < h \le M$ and $\beta_i = 0$ for $i \ne k, h$. Then the following are all the possible configurations of equilibria:

Configuration of equilibria: If country j and $(j + 2)$ belong to the same currency union, so does country $(j + 1)$. If $-(0; T_{ij}; N^u) < 0$ for all i, j and any N^u, then the possible configurations are: 1) M currencies in the world; no currency unions; 2) 2 currencies in the world, those of country k and country h; if $(k - 1) = (M - h)$ then the two currency unions include an equal number of countries $m = M/2$; 3) two multi-country currency unions adopting currencies k and h, composed respectively of m_k and m_h countries. The remaining $(M - m_h - m_k)$ countries all have their own currency. If $(k - 1) = (M - h)$, then $m_k = m_h$.

If $-(0; T_{ij}; N^u) \gtrless 0$, depending on i, j and N^u, then the additional possible configurations are as follows: 4) all the countries adopt one currency, either the one of country k or of country h; 5) $\gamma > 2$ multi-country currency unions that include a total of $M' \le M$ countries.

The first statement implies that currency unions are formed by countries adjacent to each other. This result depends on all the countries having the same size. The sufficient condition that isolates the first three cases implies that the only countries that would want to adopt a currency other than their own are $\beta = 0$ countries, which may adopt the currency of a committed anchor. This condition tends to be satisfied if the main reason to enter a currency union is to obtain the commitment of the anchor country. That is, the first term on the left side of equation (25) is dominant. Also, if the benefits from trade arising from sharing the same currency are relatively low, then not much is gained by $\beta = 0$ countries (or $\beta = 1$ countries) in giving up an independent monetary policy. A third factor that would work in favor of satisfying this condition is a high value of $VAR(\epsilon_i - \epsilon_j)$ or σ_{ϵ}^2; for given trade shares. Case 2 is a situation in which all the countries belong to one of two currency unions. This outcome tends to emerge when country shocks are similar or the trade benefits from belonging to a union are high. In case 3, some of the countries with $\beta = 0$ are too far from countries k and h and their currency unions to join either union.

If $-(0; T_{ij}; N^u) > 0$ for some countries, then some countries may want to form a union even without the benefit of commitment. This outcome arises if the trade gains are sufficient to compensate for the loss of monetary autonomy. In this situation two or more non-committed countries may form a union, because they are too far from a $\beta = 1$ country. For instance, consider two countries with $\beta = 0$ bordering each other but far from any country with $\beta = 1$. These two countries may form a currency union if the trade benefits are sufficiently high and the benefit of commitment comes at too high a price because of the great distance of the closest $\beta = 1$ country. An analogous argument applies to countries with $\beta = 1$. Thus, two additional possibilities emerge. In case 4, all the countries adopt the same currency, either of country k or h. In case 5, some of the countries that do not belong to the currency unions of k or h in case 3 form their own multi-country currency union. A natural example is one in which countries k and h are close to the extremes of the line segment, so that a large range of countries in the middle of the line segment is far from a committed anchor country. A set of countries in the middle may then find it beneficial to form a currency union even without the benefits of commitment.[20]

3.3 Many countries and few currencies

As the number of countries increases, the equilibrium number of currencies may go up less than proportionally with the number of countries or may even decrease. Consider the following example with 3 countries of equal size—thus of size 1/3—numbered from 1 to 3 from left to right. Suppose that $\beta_1 = \beta_3 = 1$ and $\beta_2 = 0$ and that each country has its own currency. This configuration means that country 2 prefers autonomy, which implies, from equation (25), that[21]

$$-(1; T_{21}; 1/3) < 0 \text{ and } -(1; T_{23}; 1/3) < 0. \qquad (26)$$

Suppose now that country 2 splits (exogenously) into two equal-sized countries, labeled from left to right by 2a and 2b. In the new situation, countries 2a and 2b may find it attractive to adopt the currencies of countries 1 and 3, respectively. Consider, for instance, country 2a. This country prefers to use the currency of country 1 if

$$-(1; T_{2a;1}; 1/3) > 0. \qquad (27)$$

[20] An interesting example may be the discussion about a monetary union in Central America, as an alternative to dollarization.

[21] It follows immediately, if this condition holds, that it is not in the interest of countries 1 and 3 to form a currency union without country 2. A three-country currency union is also not an equilibrium.

Note, since $D_{1;2a} < D_{12}$, $T_{2a;1} > T_{21}$. Therefore, conditions (26) and (27) can both be satisfied. Furthermore, country 2a does not want to adopt the currency of 2b instead of that of 1 if

$$¡(1; T_{2a;1}; 1=3) > ¡(0; T_{2a;2b}; 1=6). \tag{28}$$

This condition can be satisfied together with the previous two, but it is not satisfied for all parameter values, because $D_{2a;2b} < D_{2a;1}$. Analogous considerations apply to country 2b and its decision to adopt the currency of country 3.

In summary, the example shows that a configuration of 3 countries/3 currencies can be an equilibrium and one with 4 countries/2 currencies can also be an equilibrium. Hence, as the number of countries increases, the number of currencies may fall. Two forces underlie this result. One is that smaller countries benefit more from currency unions because a larger fraction of their economy relies on foreign trade. The second is that a new country can be closer to an anchor than the original larger country to which the new one originally belonged.

By the same logic, consider the case of an initial 4 countries/4 currencies equilibrium. The two middle countries (2 and 3) are those with ¯ = 0. Suppose that the two middle countries split in half, becoming 2a and 2b and 3a and 3b, respectively. It is easy to verify that countries 2a and 3b may want to adopt the currencies of country 1 and 4, respectively. The other countries 2b and 3a may not adopt these anchor currencies because they are further away from the respective anchors. Hence, the equilibrium can move from 4 countries/4 currencies to 6 countries/4 currencies. It is also possible that countries 2b and 3a may want to form a currency union of their own even without a committed monetary policy. In this case, the new equilibrium would have 6 countries/3 currencies.

3.4 Countries of different size

Suppose now that countries come in two sizes, large and small, denoted by n and N, respectively. We can have four types of countries in terms of size and commitment ability: 1) Size N and $\beta = 1$; 2) Size N and $\beta = 0$; 3) Size n with $\beta = 1$; 4) Size n with $\beta = 0$.

Consider now the configuration of equilibria. A trivial case is one in which there are only countries of types 1 and 4, that is, the committed countries are also the large countries. The results of section 3.2 generalize immediately. A more interesting case is one in which all four types of countries exist. In this case, an important difference from before is that currency unions are not necessarily formed by countries adjacent to each other. For instance, suppose country j is of type 3 (small but committed), country $j + 1$ is of type 2 (large but not committed), and country $j + 2$ is of type 4 (small and not committed). It is possible that ¡$(1; T_{j+1;j}; n) < 0 <$ ¡$(1; T_{j+2;j}; n)$. That is, it may be in the interest of a small but relatively far country ($j + 2$) to adopt the currency of an anchor (j), although a closer but larger country ($j + 1$) may opt out. For example, it may be in the interest of Panama and El Salvador to adopt the dollar, although it may not be in the interest of Mexico; or it may be in the interest of Latvia and Estonia to link to the euro, although it may not be worthwhile for Poland. The intuition is clear: the small country may have a higher trade share with the anchor even though it is farther away, precisely because it is small.

Another dimension in which countries di¤er is in their location. A country at the extreme of the line segment is relatively far from more countries than a country located in the middle. Ceteris paribus, a country in the middle is a more likely anchor than a country at the extremes.[22] Therefore, a small uncommitted country at the "borders" of the world is the least likely anchor, whereas a large committed country in the middle is the most likely anchor. Obviously, the real world is not a line segment and these observations have to be interpreted cum grano salis, but the point is that New Zealand may be a less likely anchor than Switzerland, not only because of the di¤erent in‡ationary histories of the two countries but also because of their geographical locations.

4 Conclusions

Currency unions have several real and monetary e¤ects. To the extent that trade costs are lowered by a common currency, the latter lead to real output and consumption gains. The loss of monetary ‡exibility has costs and bene…ts. On the one hand, a country giving up its currency loses a stabilization device targeted to domestic shocks; on the other hand, it may gain credibility and thereby reduce undesired in‡ation. We have shown how the determination of optimal currency areas depends on a complex web of variables and interactions, including the size of countries, their "distance," the

[22] Note that the literature on the gravity model (e.g. Rose [2000]) accounts for the "remoteness" of a country with an appropriate empirical speci…cation.

size of the transaction costs of trade, the correlations between shocks, and on institutional arrangements that determine how the seignorage is allocated and whether transfers between members of a union are feasible. The type of country with the strongest incentive to give up its own currency is a small country with a history of high inflation that is close (in a variety of different ways) to a large and monetarily stable country.

As the number of countries increases, their average size decreases and the volume of international transactions rises. As a result, more and more countries will find it profitable to give up their independent currency. We have shown that it is possible that as the number of countries increases, the number of currencies may not only increase less than proportionally but may even fall.

5 Appendix: The Model of Output, Trade, and Country Size

Consumption for individual r satisfies the budget constraint

$$C_r = A \cdot \left(\sum_{v=1}^{N_i} X_{vr}^{\alpha} \right) \cdot L^{1-\alpha} - X_{rr} + (P_r - 1) \cdot (X_r - X_{rr}) \quad \text{(A1)}$$
$$- \sum_{v=1, v \neq r}^{N_i} P_v X_{vr} - \left(\frac{1}{1-b}\right) \cdot \sum_{v=N_i+1}^{N} P_v X_{vr},$$

where r belongs to country i that contains individuals $v = 1, ..., N_i$; X_r is the total of intermediates produced by r; and we used the expression for output from equation (1):

$$Y_r = A \cdot \left(\sum_{v=1} X_{vr}^{\alpha} \right) \cdot L^{1-\alpha}. \qquad \text{(A2)}$$

The first-order conditions for maximizing C_r relate the quantities of intermediate inputs employed by individual r, X_{vr}, to the price, P_v, in accordance with

$$\begin{aligned} A\alpha L^{1-\alpha} X_{rr}^{\alpha-1} &= 1, \qquad \text{(A3)} \\ A\alpha L^{1-\alpha} X_{vr}^{\alpha-1} &= P_v,\ v = 1, \ldots, N_i\ (\neq r), \\ A\alpha L^{1-\alpha} X_{vr}^{\alpha-1} &= \left(\frac{P_v}{1-b}\right),\ v = N_{i+1}, \ldots, W. \end{aligned}$$

The first-order condition for choosing P_r to maximize C_r is

$$\frac{(P_r - 1)}{P_r} \cdot \epsilon_{(X_r - X_{rr}), P_r} = -1, \qquad \text{(A4)}$$

where the ϵ term denotes the elasticity of demand for exports, $X_r - X_{rr}$, with respect to P_r.

Conditions of the form of equation (A3) determine the demand, X_{rv}, from the other producers for r's intermediate good. Each of these demands and (since the relative weights are fixed) the overall demand have constant price elasticities equal to $-1/(1-\alpha)$. Substitution of this result into equation (A4) determines the monopoly price of intermediates to be the constant

$$P_r = 1/\alpha. \qquad \text{(A5)}$$

This price is the same for all intermediate goods.

Substituting $P_v = 1/\alpha$ into equation (A3) determines the quantities of intermediates:

$$
\begin{aligned}
X_{rr} &= (A\alpha)^{1/(1-\alpha)} \cdot L, && \text{(A6)}\\
X_{vr} &= (A\alpha^2)^{1/(1-\alpha)} \cdot L,\ v = 1, \ldots, N_i\ (\neq r),\\
X_{vr} &= [A\alpha^2 \cdot (1-b)]^{1/(1-\alpha)} \cdot L,\ v = N_{i+1}, \ldots, W.
\end{aligned}
$$

Substitution of the results from equation (A6) into equation (A2) leads to the expression for output in equation (2):

$$Y_r = \tilde{A}L \cdot [1 + d_1 \cdot (N_i - 1) + d_1 d_2 \cdot (W - N_i)], \qquad \text{(A7)}$$

where $\tilde{A} \equiv A^{1/(1-\alpha)}\alpha^{\alpha/(1-\alpha)}$; $d_1 \equiv \alpha^{\alpha/(1-\alpha)}$, and $d_2 \equiv (1-b)^{\alpha/(1-\alpha)}$. The result for consumption can be obtained by substituting from Eqs. (A5)-(A7) into Eq. (A1) to get equation (3):

$$C_r = \tilde{A}L \cdot (1-\alpha) \cdot [1 + d_1 \cdot (1+\alpha) \cdot (N_i - 1) + d_1 d_2 \cdot (1+\alpha) \cdot (W - N_i)]. \qquad \text{(A8)}$$

An individual's total value of purchases of intermediates can be determined by multiplying the quantities from equation (A6) by the monopoly price, $P_v = 1/\alpha$, as

$$\text{Value of purchases} = \tilde{A}\alpha^{1/(1-\alpha)} \cdot L \cdot [N_i - 1 + d_2 \cdot (W - N_i)]. \qquad \text{(A9)}$$

This expression is gross of the losses from the iceberg transaction costs. The first term inside the brackets, $N_i - 1$, corresponds to purchases from individuals of the same country, whereas the second, $d_2 \cdot (W - N_i)$, corresponds to foreign imports. Equation (A6) can also be used to show that an individual's sales of intermediates—to persons in the same country and to foreigners—equals the value of purchases.

The ratio of the value of trade to output is given from equations (A7) and (A9) by

$$\text{Value of trade/Output} = \frac{\alpha^{1/(1-\alpha)} \cdot [N_i - 1 + d_2 \cdot (W - N_i)]}{1 + \alpha^{\alpha/(1-\alpha)} \cdot [(N_i - 1) + d_2 \cdot (W - N_i)]}. \quad \text{(A10)}$$

If $\alpha^{\alpha/(1-\alpha)} \cdot [(N_i - 1) + d_2 \cdot (W - N_i)] >> 1$, then this ratio is approximately equal to the constant α and is therefore roughly independent of N_i and d_2.

The total trade ratio breaks down into two parts:

$$\text{Value of domestic trade/Output} = \frac{\alpha^{1/(1-\alpha)} \cdot (N_i - 1)}{1 + \alpha^{\alpha/(1-\alpha)} \cdot [(N_i - 1) + d_2 \cdot (W - N_i)]} \quad \text{(A11)}$$

and

$$\text{Value of foreign trade/Output} = \frac{\alpha^{1/(1-\alpha)} \cdot d_2 \cdot (W - N_i)}{1 + \alpha^{\alpha/(1-\alpha)} \cdot [(N_i - 1) + d_2 \cdot (W - N_i)]}. \quad \text{(A12)}$$

Hence, the domestic trade ratio in equation (A11) rises with N_i and falls with d_2 (rises with the international trading cost, b). The foreign trade ratio in equation (A12) falls with N_i and rises with d_2 (falls with the international trading cost, b).

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