Basic model (Samuelson-Tirole)

- OLG: young and old, generations of size one
- Preferences: all generations maximize expected consumption when old

$$U_t = E_t\left\{c_{t+1}\right\}$$

- Technology:

$$F\left(l_t, k_t\right) = l_t^{1-\alpha} \cdot k_t^{\alpha}, \;\; \alpha \in (0,1)$$

  – labor: supplied by young

$$l_t = 1 \text{ for all } t$$

  – capital: supplied by old, from investment during youth

$$k_{t+1} = s_t \cdot k_t^{\alpha}$$

    * s_t: investment rate, i.e. the fraction of output that is devoted to capital formation

- Competitive markets:

$$w_t = (1-\alpha) \cdot k_t^{\alpha} \quad \text{and} \quad r_t = \alpha \cdot k_t^{\alpha-1}$$

- Note: $s_t = s = 1 - \alpha$ if investment is only way of saving

Bubbly Equilibria

- Consider young can also purchase bubbles
  - Intrinsically useless assets: only held for resale, do not promise any payments
- In any period t:
  - b_t: stock of old bubbles, pre-existing
  - b_t^N: stock of new bubbles, created by the young
- Bubbles created randomly and without cost:
  - pure rent to originator
- With capital and bubbles

$$s_t = s - \frac{b_t}{k_t^\alpha}$$

  - Note: bubbles (old) lower investment rate

Bubbly Equilibria (II)

- Can $b_t \geq 0$ and/or $b_t^N > 0$ in equilibrium?
  - Income of the young

$$s \cdot k_t^{\alpha} + b_t^N$$

  - Young purchase entire bubble

$$b_t + b_t^N$$

- Bubble must be rational / feasible:

$$E_t \left\{ \frac{b_{t+1}}{b_t + b_t^N} \right\} = \alpha \cdot k_{t+1}^{\alpha-1} \quad \text{if } b_t > 0$$

$$b_t \leq s \cdot k_t^{\alpha}$$

- Evolution of capital stock

$$k_{t+1} = s \cdot k_t^{\alpha} - b_t$$

- Equilibrium: given $k_0 > 0$ and b_0, sequence $\left\{k_t, b_t, b_t^N\right\}_{t=0}^{\infty}$ satisfying previous 3 equations
  - b_t, b_t^N possibly stochastic
- In general: multiple equilibria (always $b_t = b_t^N = 0$)

Anatomy of Bubbly Episodes

- Denote:
  - fundamental state: $b_t = b_t^N = 0$
  - bubbly episode: $b_t > 0$ and/or $b_t^N > 0$

- In this model:
  - bubbly episodes are possible if and only if $\alpha < s$ $(\alpha < 0.5)$
    - chain of all investments is dynamically inefficient

$$s \cdot k_t^{\alpha} \cdot \left(\alpha \cdot k_{t+1}^{\alpha-1}\right) < s \cdot k_{t+1}^{\alpha}$$

    - bubbles play intertemporal role: alternative store of value

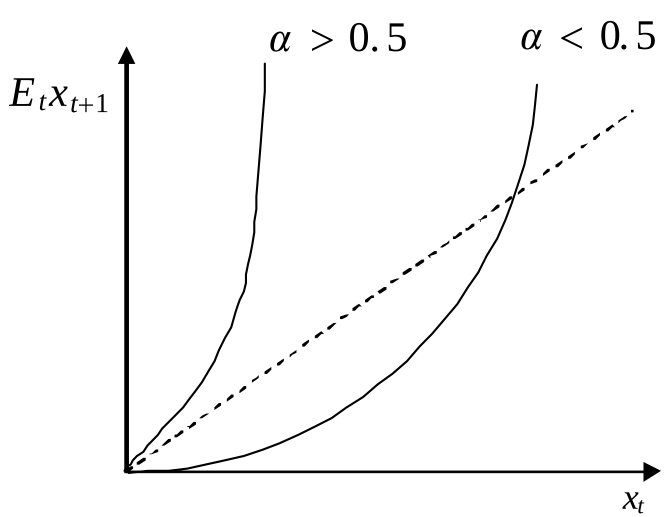
Anatomy of Bubbly Episodes (II)

- Sketch of proof:
  - let $x_t \equiv \frac{b_t}{s \cdot k_t^{\alpha}}$.
  - find $\left(x_t, x_t^N\right)$ for which

$$E_t x_{t+1} = \frac{\alpha}{s} \cdot \frac{x_t + x_t^N}{1 - x_t} < x_t$$

  - if not, bubble eventually exceeds wages!

- Graphically:

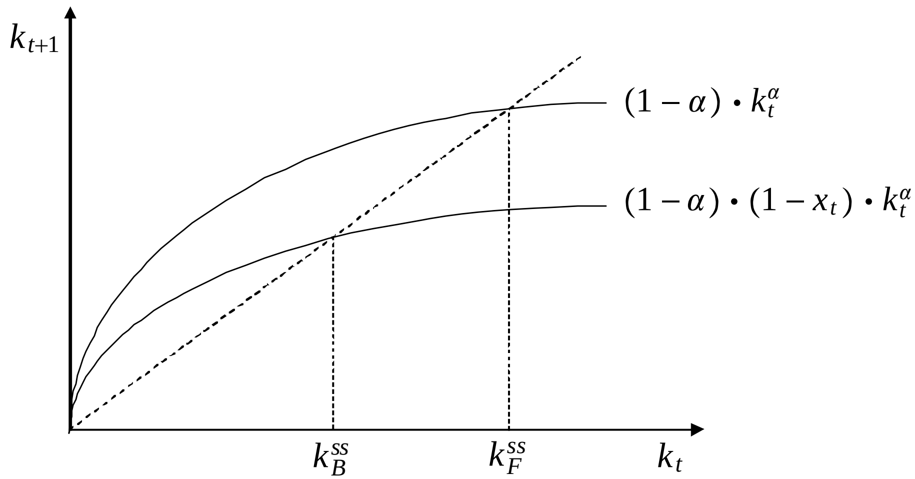


- Note: intragenerational transfers ($x_t^N > 0$) useless and costly

Macroeconomic effects of bubbly episodes

- Evolution of capital stock:

$$k_{t+1} = (1 - x_t) \cdot s \cdot k_t^{\alpha}$$



- Beautiful and insightful model, but disappointing fit to reality
  - Bubbly episodes:
    - ∗ low investment and output
    - ∗ high consumption
  - Dynamic inefficiency empirically questioned (Abel et al. argue $s < \alpha$)

Model with financial frictions

- Introduce heterogeneity: two groups of individuals
  - Productive investors: produce one unit of k per unit invested (measure ε)
  - Unproductive investors: produce $\delta < 1$ units of k per unit invested (measure $1-\varepsilon$)
  - No financial markets: no borrowing/lending
- Capital dynamics now depend on amount (s_t) and average productivity (A_t) of investment

$$k_{t+1} = s_t \cdot A_t \cdot k_t^{\alpha}$$

- If young can only save by investing in capital

$$s_t = s = 1 - \alpha$$

$$A_t = A = \varepsilon + (1-\varepsilon) \cdot \delta$$

- Evolution of capital stock

$$k_{t+1} = A \cdot s \cdot k_t^{\alpha}$$

Main effects of bubbles

- **Bubble demand:** who buys market bubble?
- **Bubble supply:** who creates new bubbles?
  - Now $b_t^N = b_t^{NP} + b_t^{NU}$
  - b_t^{NP} and b_t^{NU}: new bubble creation by efficient and inefficient investors, respectively
- Both affect amount (s_t) and composition (A_t) of investment

Equilibrium

- Given $k_0 > 0$ and b_0, sequence $\left\{k_t, b_t, b_t^{NP}, b_t^{NU}\right\}_{t=0}^{\infty}$ satisfying

$$E_t\left\{\frac{b_{t+1}}{b_t + b_t^{NP} + b_t^{NU}}\right\} = \begin{cases} \delta \cdot \alpha \cdot k_{t+1}^{\alpha-1} & \text{if } \dfrac{b_t + b_t^{NP}}{(1-\varepsilon) \cdot s \cdot k_t^{\alpha}} < 1 \\ \left[\delta \cdot \alpha \cdot k_{t+1}^{\alpha-1}, \alpha \cdot k_{t+1}^{\alpha-1}\right] & \text{if } \dfrac{b_t + b_t^{NP}}{(1-\varepsilon) \cdot s \cdot k_t^{\alpha}} = 1 \quad \text{if } b_t > 0 \\ \alpha \cdot k_{t+1}^{\alpha-1} & \text{if } \dfrac{b_t + b_t^{NP}}{(1-\varepsilon) \cdot s \cdot k_t^{\alpha}} > 1 \end{cases}$$

$$0 \leq b_t \leq s \cdot k_t^{\alpha}$$

- As before, bubble rational / feasible

Equilibrium (II)

$$k_{t+1} = \begin{cases} A \cdot s \cdot k_t^\alpha + (1-\delta) \cdot b_t^{NP} - \delta \cdot b_t & \text{if } \dfrac{b_t + b_t^{NP}}{(1-\varepsilon) \cdot s \cdot k_t^\alpha} < 1 \\ \\ s \cdot k_t^\alpha - b_t & \text{if } \dfrac{b_t + b_t^{NP}}{(1-\varepsilon) \cdot s \cdot k_t^\alpha} \geq 1 \end{cases}$$

- Effects of bubble on capital accumulation
  - b_t : crowding out effect (AS BEFORE)
  - b_t^{NU} no effect on investment (AS BEFORE)
  - b_t^{NP} reallocation effect (NEW)
- Bubbly episode could be expansionary, i.e. $b_t^{NP} > \dfrac{\delta}{1-\delta} \cdot b_t$
  - additional intratemporal role of bubble

Existence of bubbly episodes (I)

- Formally, triplet $\left\{x_t, x_t^{NP}, x_t^{NU}\right\}$ satisfying optimality and feasibility:

$$E_t x_{t+1} \begin{cases} = \dfrac{\alpha}{s} \cdot \dfrac{\delta \cdot \left(x_t + x_t^{NP} + x_t^{NU}\right)}{A + (1-\delta) \cdot x_t^{NP} - \delta \cdot x_t} & \text{if } \dfrac{x_t + x_t^{NP}}{1-\varepsilon} < 1 \\ \in \left[\dfrac{\alpha}{s} \cdot \dfrac{\delta \cdot \left(x_t + x_t^{NP} + x_t^{NU}\right)}{A + (1-\delta) \cdot x_t^{NP} - \delta \cdot x_t}, \dfrac{\alpha}{s} \cdot \dfrac{x_t + x_t^{NU} + x_t^{NP}}{1 - x_t}\right] & \text{if } \dfrac{x_t + x_t^{NP}}{1-\varepsilon} = 1 \\ = \dfrac{\alpha}{s} \cdot \dfrac{x_t + x_t^{NP} + x_t^{NU}}{1 - x_t} & \text{if } \dfrac{x_t + x_t^{NP}}{1-\varepsilon} > 1 \end{cases},$$

$$0 \leq x_t \leq 1.$$

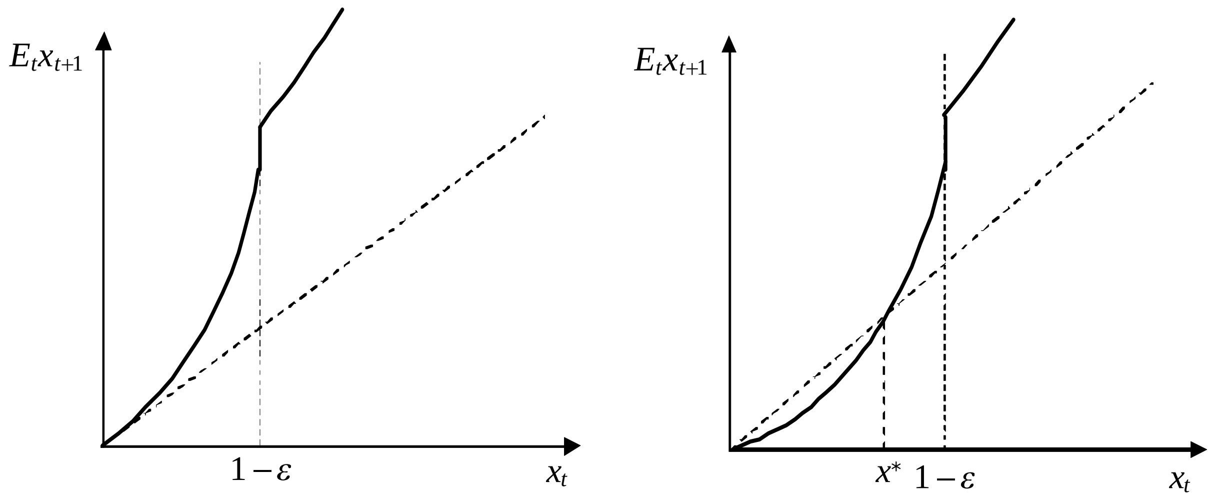
- Sources of randomness:
  - shocks to bubble creation x_t^{NP} and x_t^{NU}
  - shocks to value of existing bubble x_t

- Result: Bubbly episodes are possible iff $\alpha < \begin{cases} s \cdot \dfrac{A}{\delta} & \text{if } A > 1-\varepsilon \\ s \cdot \dfrac{A}{\delta} \cdot \max\left\{1, \dfrac{1}{4 \cdot (1-\varepsilon) \cdot A}\right\} & \text{if } A \leq 1-\varepsilon \end{cases}.$

Existence of bubbly episodes (II):

- No bubble creation: $x^{NP} = x^{NU} = 0$

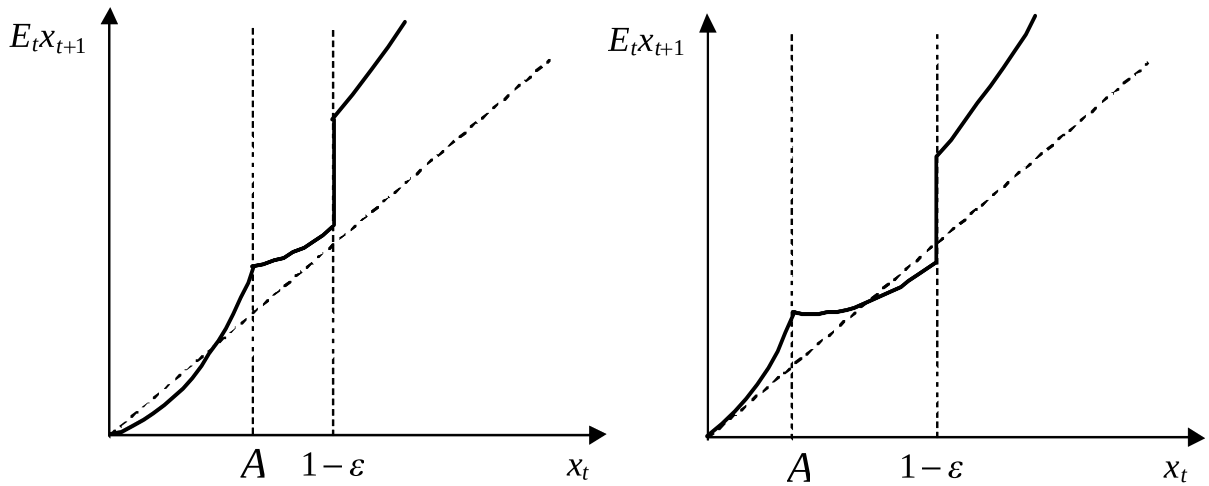
- Graphically:



- Slope of $E_t x_{t+1} < 1$ at the origin $\Longleftrightarrow \alpha < s \cdot \dfrac{A}{\delta}$

Existence of bubbly episodes (III):

- Can bubble creation help?
- In principle, NO: x^{NP}, $x^{NU} > 0$ decrease return of bubble (increase $E_t x_{t+1}$)
- BUT....with $x^{NP} > 0$ reallocation effect lowers return to capital (decrease $E_t x_{t+1}$)
- If $A \leq 1 - \varepsilon$, $x^{NP} > 0$ relaxes existence
- Graphically,

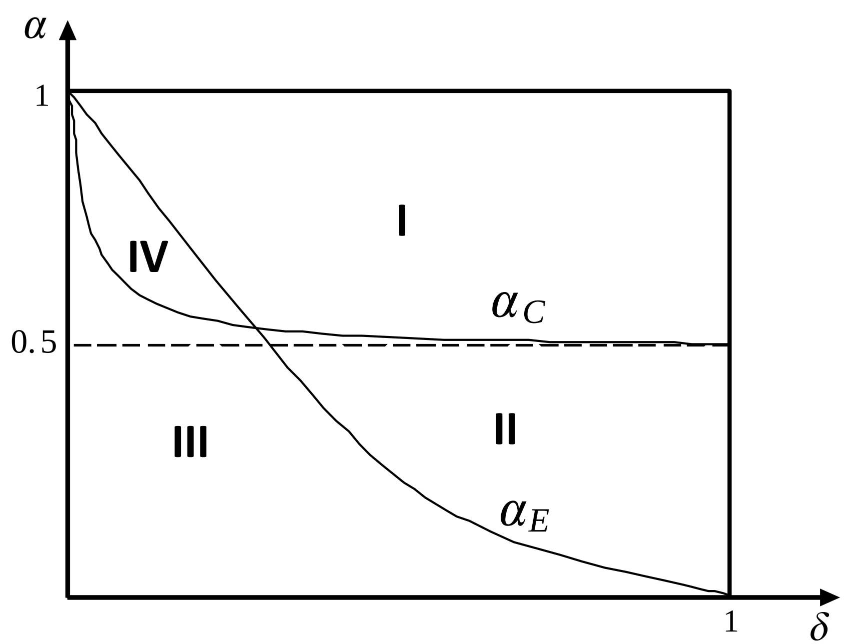


Existence of bubbly episodes according to type

- Classify episodes through effects on k

- Proposition: contractionary bubbly episodes are possible iff $\alpha < \alpha_C \equiv s \cdot \frac{A}{\delta}$. Expansionary bubbly episodes are possible iff $\alpha < \alpha_E \equiv s \cdot \frac{A}{\delta} \cdot \begin{cases} (1-\delta) & \text{if } A > 0.5 \\ \frac{1}{4 \cdot (1-\varepsilon) \cdot A} & \text{if } A \leq 0.5 \end{cases}$.

- Graphically,



Existence of bubbly episodes and dynamic inefficiency

- Existence requires dynamically inefficient chain of investments

$$E_t\left\{I_{t+1} - R_{t+1} \cdot I_t\right\} = E_t\left\{D_{t+1}\right\} \geq 0$$

  where R_{t+1} is *equilibrium* rate of return

- With $x^{NP} = 0$: check unproductive investments

$$E_t x_{t+1} \cdot s \cdot k_{t+1}^{\alpha} \geq \delta \cdot \alpha \cdot k_{t+1}^{\alpha-1} \cdot x_t \cdot s \cdot k_t^{\alpha} \quad \Leftrightarrow \quad s \cdot \frac{A}{\delta} \geq \alpha$$

  – dynamically inefficient chain in fundamental equilibrium (regions II and III)

- With $x^{NP} > 0$:

  – bubble absorb investments with $D_t > 0$

  – but equilibrium R_{t+1} also decreases if bubble expansionary

  – bubble creates dynamically inefficient investments that it replaces (region IV)

- In the absence of heterogeneity (or financial frictions): all investments must be dynamically inefficient

  – only then does Abel et al. (1989) make sense

Macroeconomic effects of bubbles revisited

- Evolution of capital stock

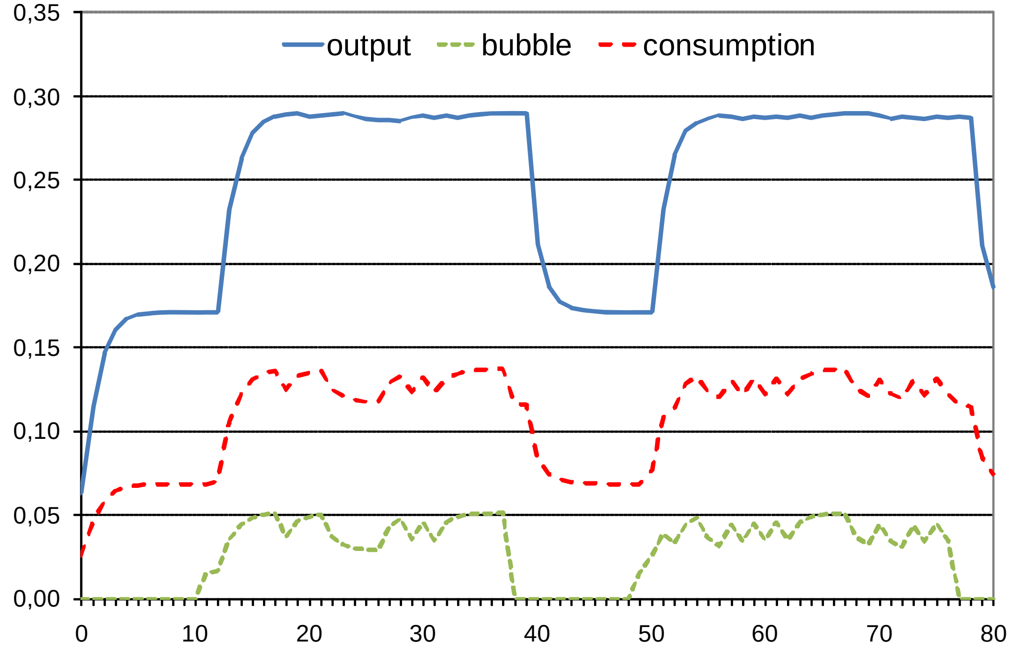
$$k_{t+1} = \left[A + (1-\delta)\cdot x_t^{NP} - \delta\cdot x_t\right]\cdot s\cdot k_t^{\alpha} \text{ if } \frac{x_t + x_t^{NP}}{1-\varepsilon} < 1.$$

- Consumption:

$$c_t = (\alpha + x_t\cdot s)\cdot k_t^{\alpha}.$$

- Expansionary episodes raise capital and consumption

- Shocks to investor sentiments as sources of business cycles: simulation



Bubbles and Long-run Growth

- In previous model: all effects of bubbles are transitory
- General production structure:
  - Continuum of intermediate inputs $m \in [0, m]$
- Final good production: competitive

$$y_t = \widehat{\psi} \cdot \left(\int_0^{m_t} q_{tm}^{\frac{1}{\mu}} \cdot dm \right)^{\mu}$$

  - $\widehat{\psi}$: constant
  - q_{tm}: units of intermediate m
  - $\mu/(1-\mu)$: elasticity of substitution

- Intermediate good production: monopolistic competition with free entry

$$q_{tm} = (l_{tm,v})^{1-\alpha} \cdot (k_{tm,v})^{a},$$

with fixed costs

$$f_{tm} = \begin{cases} 1 = (l_{tm,f})^{1-\alpha} \cdot (k_{tm,f})^{a} & \text{if } q_{tm} > 0 \\ 0 & \text{if } q_{tm} = 0 \end{cases},$$

Bubbles and Long-run Growth (II)

- With this production structure:

$$y_t = k_t^{\alpha \cdot \mu}$$

  - where μ captures market size effects
  - opposing effects of growth k: diminishing returns $(\alpha < 1)$ vs. market-size $(\mu > 1)$

- With competitive factor markets

$$w_t = (1 - \alpha) \cdot (k_t)^{\alpha \cdot \mu}$$
$$r_t = \alpha \cdot (k_t)^{\alpha \cdot \mu - 1}$$

- Equilibrium as before with

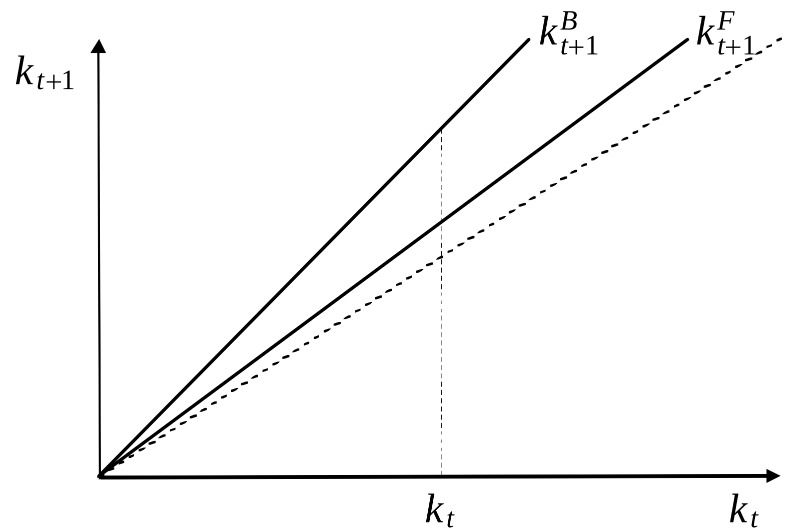
$$k_{t+1} = s_t \cdot A_t \cdot (k_t)^{\alpha \cdot \mu},$$

$$E_t \left\{ \frac{b_{t+1}}{b_t + b_t^{NP} + b_t^{NU}} \right\} = \begin{cases} \delta \cdot \alpha \cdot (k_t)^{\alpha \cdot \mu - 1} & \text{if } b_t + b_t^{NP} < (1 - \varepsilon) \cdot w_t \\ \left[\delta \cdot \alpha \cdot (k_t)^{\alpha \cdot \mu - 1}, \alpha \cdot (k_t)^{\alpha \cdot \mu - 1} \right] & \text{if } b_t + b_t^{NP} = (1 - \varepsilon) \cdot w_t \\ \alpha \cdot (k_t)^{\alpha \cdot \mu - 1} & \text{if } b_t + b_t^{NP} > (1 - \varepsilon) \cdot w_t \end{cases}$$

- Formally: even if $\alpha \cdot \mu \geq 1$ nothing in the analysis changes!

Constant returns to capital ($\alpha \cdot \mu = 1$)

- All equations as before
- Formally: dynamics in x driven by changes in growth rate, not in r
- Bubbles have long-lasting impact on k and y
- Graphic example:

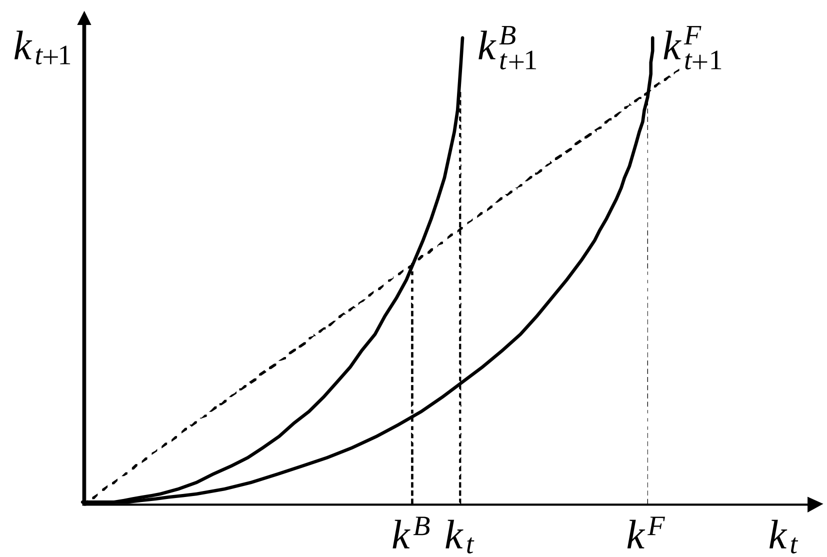


Increasing returns to capital $(\alpha \cdot \mu > 1)$

- Exactly as before

- Bubbles have long-lasting impact on growth rates

- Graphic example:



Where is the market for bubbles?

- In our model
  - reallocation of resources in market for bubbles
  - where is this market in reality?
- With minor variations: stock and credit markets!
- Extension:
  - production and investment take place in firms: owned/managed by entrepreneurs
    - young can buy firms or create them at zero cost
    - v_t: price of all firms (stock market)
  - firms inherit productivity of entrepreneurs
  - entrepreneurs can borrow to purchase firms and/or to invest
    - firms are pledged as collateral

Where is the market for bubbles? (II)

- In modified model
  - borrowing today depends on expected firm value tomorrow
- Bubbles in stock market raise value of firms
- Effects of bubbly episode:
  - existing firms become more expensive → reduction in capital accumulation
  - productive firms expand borrowing and investment → increase in capital accumulation
- All results go through, but...
  - transfer of resources through credit and stock markets
  - reasoning extends to real estate
  - Martin Ventura (2011)

Where is the market for bubbles?

- In our model
  - reallocation of resources in market for bubbles
  - where is this market in reality?
- Same economy with stock and credit markets
  - production and investment take place in firms: owned/managed by entrepreneurs
    - young can buy firms or create them at zero cost
    - v_t: price of all firms (stock market)
  - firms inherit productivity of entrepreneurs
    - $v_t^{P_t}$ and $v_t^{U_t}$: price of firms managed by productive and unproductive of generation t
  - entrepreneurs can obtain credit to purchase firms and/or to invest
    - empty firms are only pledgeable collateral
    - credit contracts last for one period and specify ex-post payment in each possible history

Equilibrium of the modified economy

- Individuals have access to three savings options
  - purchase firms, build capital, lend in credit market

- Return to investment

$$R^K_{i,t+1} = \begin{cases} \delta \cdot \alpha \cdot k_{t+1}^{\alpha-1} & \text{if } \; i \in U_t \\ \alpha \cdot k_{t+1}^{\alpha-1} & \text{if } \; i \in P_t \end{cases}$$

- R_{t+1}: ex-post average return on credit contracts

$$E_t R_{t+1} = \begin{cases} \delta \cdot \alpha \cdot k_{t+1}^{\alpha-1} & \text{if } \; \dfrac{E_t v_{t+1}^{P_t}}{(1-\varepsilon) \cdot s \cdot k_t^{\alpha} - v_t^{U_t}} < \delta \cdot \alpha \cdot k_{t+1}^{\alpha-1} \\ \dfrac{E_t v_{t+1}^{P}}{(1-\varepsilon) \cdot s \cdot k_t^{\alpha} - v_t^{U}} & \text{if } \; \dfrac{E_t v_{t+1}^{P_t}}{(1-\varepsilon) \cdot s \cdot k_t^{\alpha} - v_t^{U_t}} \in \left[\delta \cdot \alpha \cdot k_{t+1}^{\alpha-1}, \alpha \cdot k_{t+1}^{\alpha-1}\right] \\ \alpha \cdot k_{t+1}^{\alpha-1} & \text{if } \; \dfrac{E_t v_{t+1}^{P_t}}{(1-\varepsilon) \cdot s \cdot k_t^{\alpha} - v_t^{U_t}} > \alpha \cdot k_{t+1}^{\alpha-1} \end{cases}$$

- Reflects identity of marginal lender is
  - unproductive if $\dfrac{E_t v_{t+1}^{P_t}}{E_t R_{t+1}} < (1-\varepsilon) \cdot s \cdot k_t^{\alpha} - v_t^{U_t}$
  - productive otherwise

Equilibrium of the modified economy (II)

- Law of motion of the capital stock

$$k_{t+1} = \begin{cases} A \cdot s \cdot k_t^\alpha + (1-\delta) \cdot \left(\dfrac{E_t v_{t+1}^{P_t}}{(1-\varepsilon) \cdot s \cdot k_t^\alpha - v_t^{U_t}} - v_t^{P_t} \right) - \delta \cdot v_t & \text{if } \frac{E_t v_{t+1}^{P_t}}{(1-\varepsilon) \cdot s \cdot k_t^\alpha - v_t^{U_t}} < \delta \cdot \alpha \cdot k_{t+1}^{\alpha-1} \\ s \cdot k_t^\alpha - v_t & \text{if } \frac{E_t v_{t+1}^{P_t}}{(1-\varepsilon) \cdot s \cdot k_t^\alpha - v_t^{U_t}} \geq \delta \cdot \alpha \cdot k_{t+1}^{\alpha-1} \end{cases}$$

  is now a function of stock prices

- Formally, exactly as benchmark model with

$$b_t = v_t, \quad b_t^P = \frac{E_t v_{t+1}^{P_t}}{E_t R_{t+1}} - v_t^{P_t} \text{ and } b_t^U = \frac{E_t v_{t+1}^{U_t}}{E_t R_{t+1}} - v_t^{U_t}$$

- Firms bundle capital and bubbles
  - reallocation of resources in stock/credit markets
  - bubble creation: growth rate of firm value exceeds interest rate
  - entrepreneur can borrow against it to invest
  - note: no need for firm creation

Discussion and final thoughts

- Re-interpretation of our model
  - connect abstract theory with episodes in introduction
  - bubbly episodes: growth in stock prices, collateral, credit and productive investment
  - efficiency of investment rises, interest rate declines
- Connection with models of the financial accelerator (Martin Ventura 2011)
  - like them: higher asset prices raise credit, efficiency and growth
  - unlike them: asset prices decoupled from fundamentals
- Next step
  - embed investor sentiment shock into quantitative model
    - sophisticated model with rich demographics / preferences
  - can we distinguish between TFP and investor sentiment shocks in recent past?
  - Carvalho et al. (2011)

Bubbly episodes

- Interpretation: investor sentiment shocks $v_{s^t} \in \{F, B\}$
- Economy oscillates between:
  - Fundamental state: $b_{s^t} = 0$
  - Bubbly episodes: $b_{s^t} > 0$
- For analytical convenience: focus on particular class of examples
  - Constant probability of beginning /end
    * $\Pr\left(v_{s^{t+1}} = B \left| v_{s^t} = F\right.\right) = q$ and $\Pr\left(v_{s^{t+1}} = B \left| v_{s^t} = F\right.\right) = p$
  - Constant rate of bubble creation
    * during bubbly episode: $b^N_{s^t} = n \cdot b_{s^t}$
  - Full depreciation

Bubbly episodes (II): recursive representation

- Define $x_{s^t} \equiv \dfrac{b_{s^t}}{(1-\alpha)\cdot A^Q_{s^t}\cdot l^{1-\alpha}\cdot k^{\alpha}_{i,s^t}}$

- Equilibrium: sequence of x_{s^t} satisfying

$$\frac{\displaystyle\sum_{s^{t+1}\in S_{t+1}} \pi_{s^t s^{t+1}}\cdot\left(\frac{\alpha}{1-\alpha}+x_{s^{t+1}}\right)^{-\gamma}}{\displaystyle\sum_{s^{t+1\prime}\in S_{t+1}} \pi_{s^t s^{t+1\prime}}\cdot\left(\frac{\alpha}{1-\alpha}+x_{s^{t+1\prime}}\right)^{-\gamma}}\cdot\frac{x_{s^{t+1}}}{x_{s^t}}=\frac{\frac{\alpha}{1-\alpha}\cdot(1+n)}{1+\left(\left(A^K_{s^t}-1\right)\cdot n-1\right)\cdot x_{s^t}},$$

  and

$$x_{s^t}\leq\frac{1}{1+n}.$$

- Intuition: bubble must be attractive and feasible

Bubbly episodes (III)

- Law of motion of capital stock:

$$k_{s^{t+1}} = \left[1 + \left(\frac{A^K_{s^t} - 1}{1 - \phi A^K_{s^t}} \cdot n - 1\right) \cdot x_{s^t}\right] \cdot (1-\alpha) \cdot A^Q_{s^t} \cdot k^{\alpha}_{s^t} \cdot l^{1-\alpha}$$

- Two benchmark episodes:
  - **Conventional bubbles** (Samuelson-Tirole)

$$\left(A^K_{s^t} - 1\right) \cdot n < 1$$

    * Contractionary (raise the interest rate and crowd out k)
    * Do not require financial frictions
    * Require dynamic inefficiency
  - **Non-conventional bubbles** (Martin-Ventura 2011)

$$\left(A^K_{s^t} - 1\right) \cdot n > 1$$

    * Expansionary (lower interest rate and crowd in k)
    * Require financial frictions
    * *Do not require dynamic inefficiency.*

Parametrization

Table 1: Parameter values for figures

Parameter	Description	Value	Shock
α	Capital Share	$2/3$	-
ε	Measure of entrepreneurs	0	-
γ	Risk aversion coefficient	2	$\gamma' = 8$
A^Q	Total factor productivity	3	$[-0.005\%, 0.005\%]$
A^k	Investment efficiency	3.77	$[-0.005\%, 0.005\%]$
$\overline{x}$	Initial bubble	0.02	
n	Growth Rate of Bubble	0.14	
μ	Shocks to existing bubbles	$--$	$\pm\ 0.005$
q	Probability of bubble episode starting	0.15	-
p	Probability of bubble bursting	0.5	-

Example 1: deterministic economy

- No technology shocks: $A^K_{s^t} = \overline{A^K}$ and $A^Q_{s^t} = \overline{A^Q}$
- Bubbly episode that never ends: $p = 0$
- With bubbly episode
  - High investor sentiment sustain bubble / bubble creation
  - Helps overcome contracting friction
    * higher borrowing by new firms
    * higher efficient investment
  - In example: $b_{s^t} \approx 12\%$ of wages sustains six-fold increase in k and c
- Expansion and dynamic inefficiency
  - Existence requires dynamically inefficient chain of investments
  - In fundamental equilibrium: savings $>$ capital income

$$
\begin{aligned}
(1-\alpha) \cdot \overline{A^Q} \cdot l^{1-\alpha} \cdot k^{\alpha}_{s^{t+1}} &> \alpha \cdot \overline{A^Q} \cdot l^{1-\alpha} \cdot k^{\alpha}_{s^{t+1}} \\
0.5 &> \alpha
\end{aligned}
$$

  - If not satisfied, bubbly episode must generate dynamic inefficiency: expansionary!

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0.5 &> \alpha
\end{aligned}
$$

  - If not satisfied, bubbly episode must generate dynamic inefficiency: expansionary!

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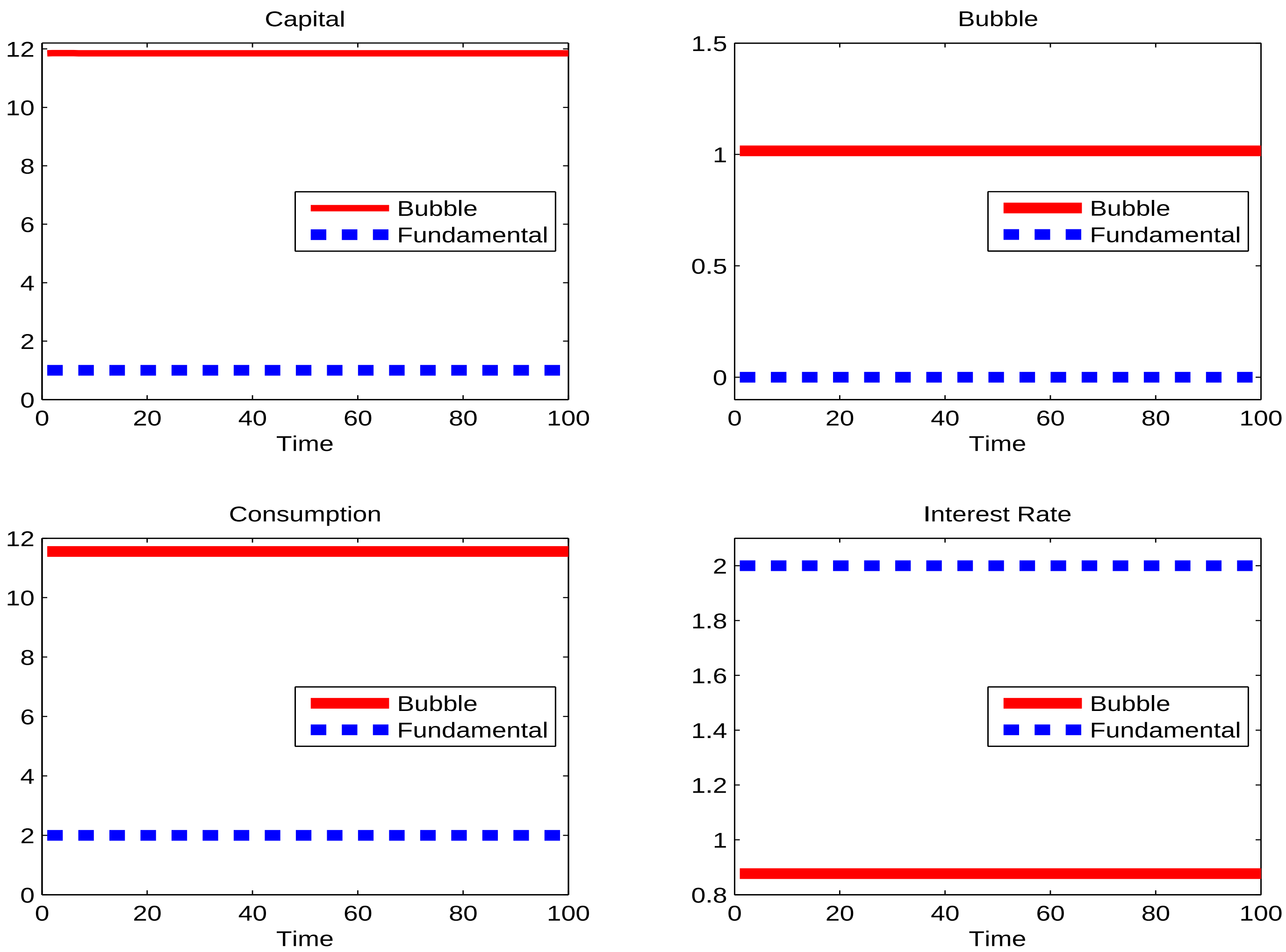


Example 2: stochastic economy with deterministic bubble

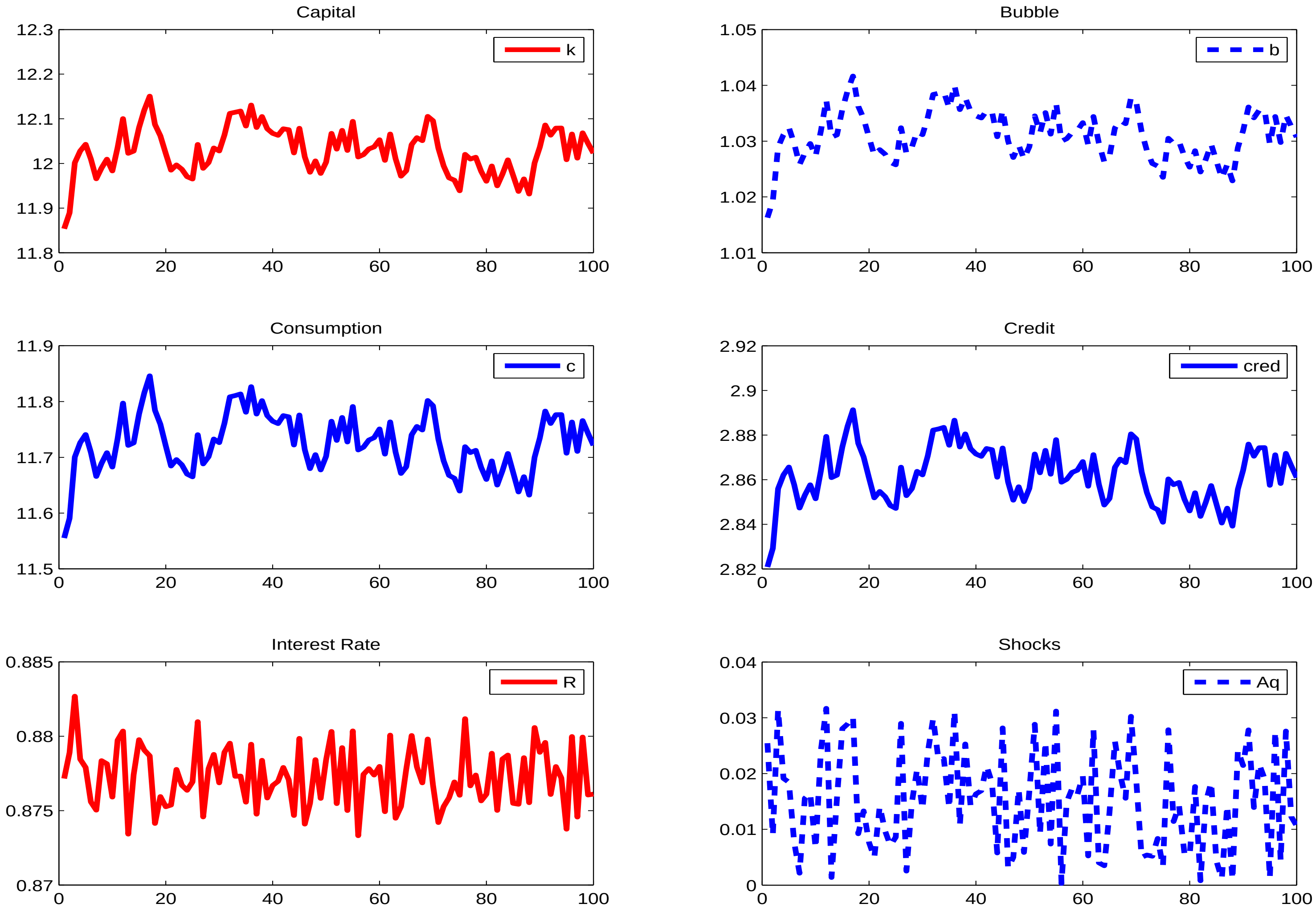
- Technology shocks: $A^Q_{s^t} \in \left[A^Q_L, A^Q_H\right]$ and $A^K_{s^t} \in \left[A^K_L, A^K_H\right]$

- Bubbly episode that never ends: $p = 0$

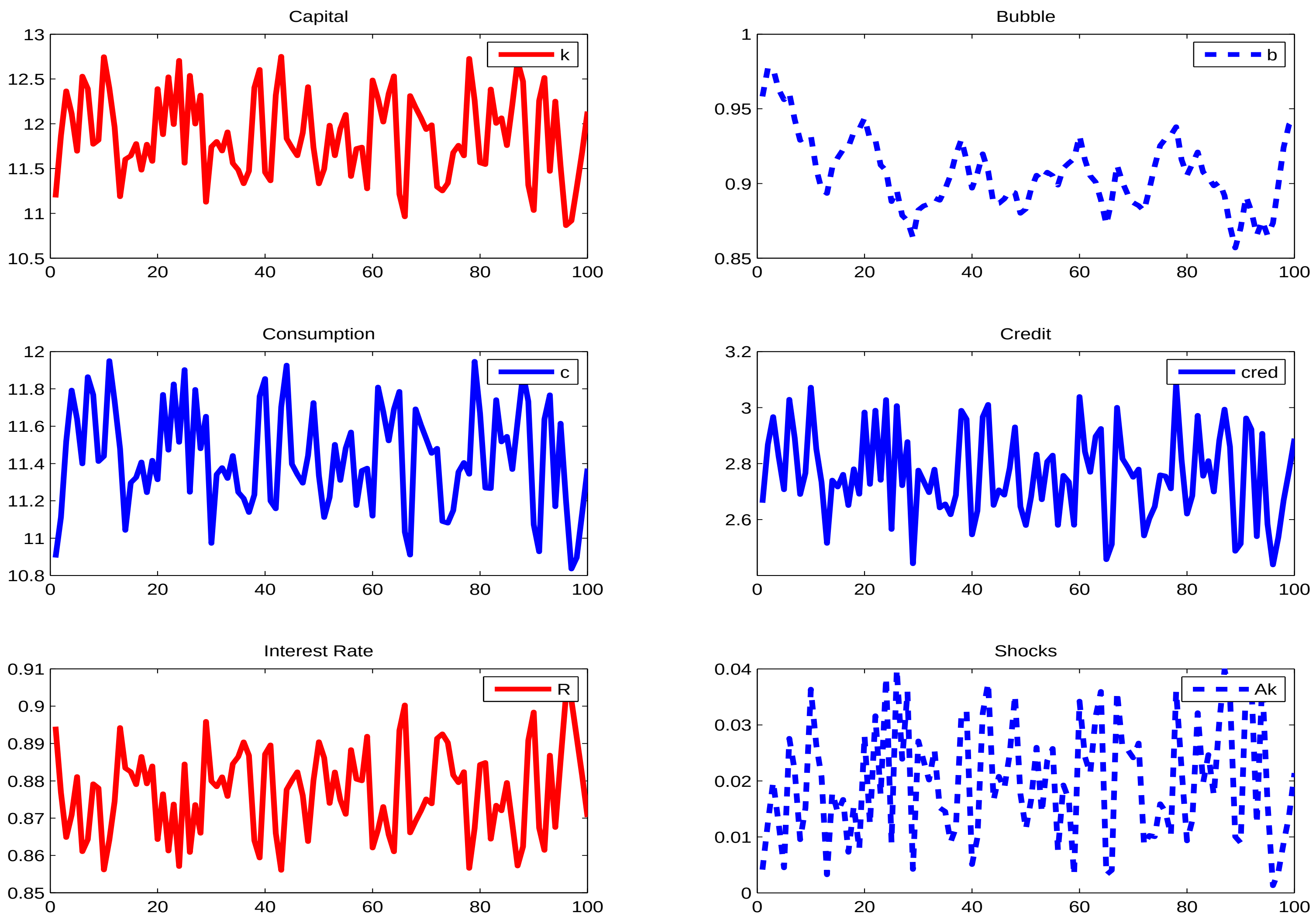
- Fundamental shocks have the usual effects
  - High values of $A^Q_{s^t}$
    - * Raise output, consumption, capital accumulation
    - * Lower interest rate: raise borrowing and investment by new firms
  - High values of $A^K_{s^t}$
    - * Raise output and consumption with a lag
    - * Raise borrowing and investment by new firms

- Interaction with bubble
  - Shocks to $A^Q_{s^t}$: proportional effect on output and bubble (b_{s^t} constant as share of wages)
  - Shocks to $A^K_{s^t}$: lower interest rate and growth rate of bubble
  - Bubble amplifies effects of technology shocks (↑ volatility)
    - * aggregate effects proportional to intermediation
    - * intermediation proportional to aggregate bubble creation

Example 2: stochastic economy with deterministic bubble



Example 2: stochastic economy with deterministic bubble



Example 3: bubbly business cycles

- No technology shocks: $A^K_{s^t} = \overline{A^K}$ and $A^Q_{s^t} = \overline{A^Q}$

- Stochastic bubbly episodes: $p > 0$, $q > 0$

  – shocks to b_{s^t} and to $b^N_{s^t}$

- Huge effects of investor sentiment shocks

  – Bubbly episodes of approx. 20 periods

  – Bubble peaks at approx. 8% of wages

  – Increase of capital stock, consumption, efficient investment: $> 500\%$

  – When episode ends: increases disappear in two periods

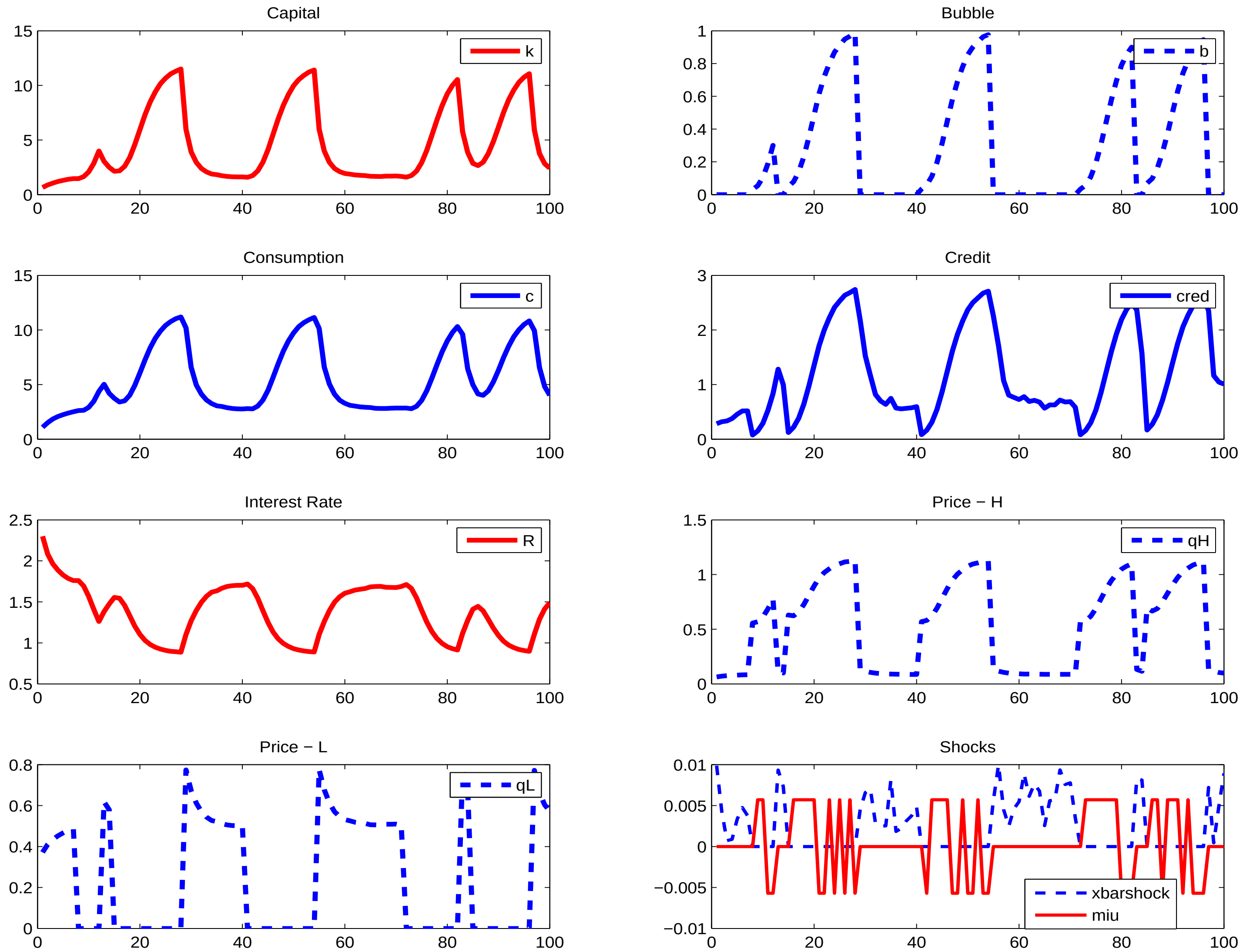
- Main insight

  – Large equilibrium effects of investor sentiment shocks

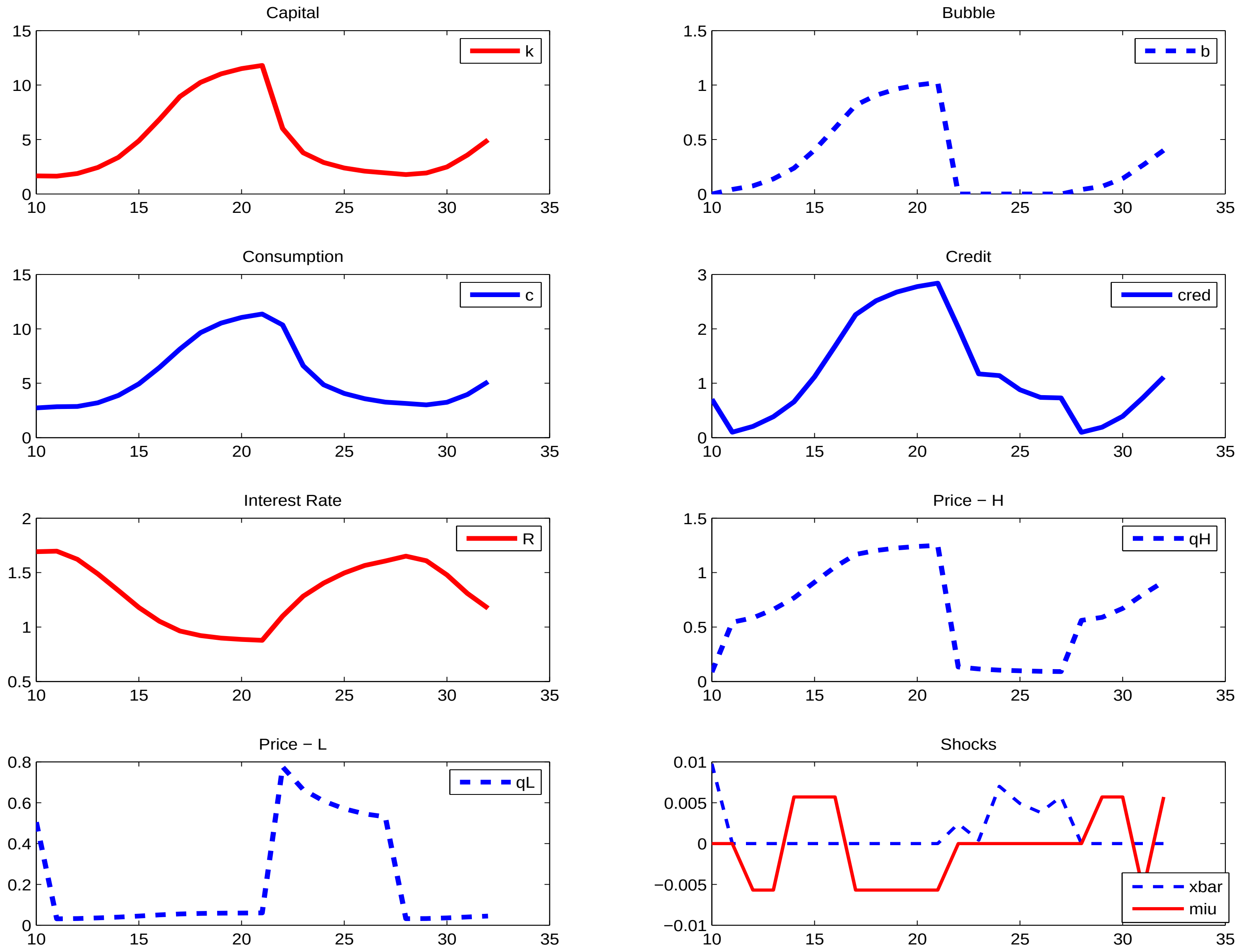
  – Despite rationality and risk aversion

    * risk aversion increases the effects

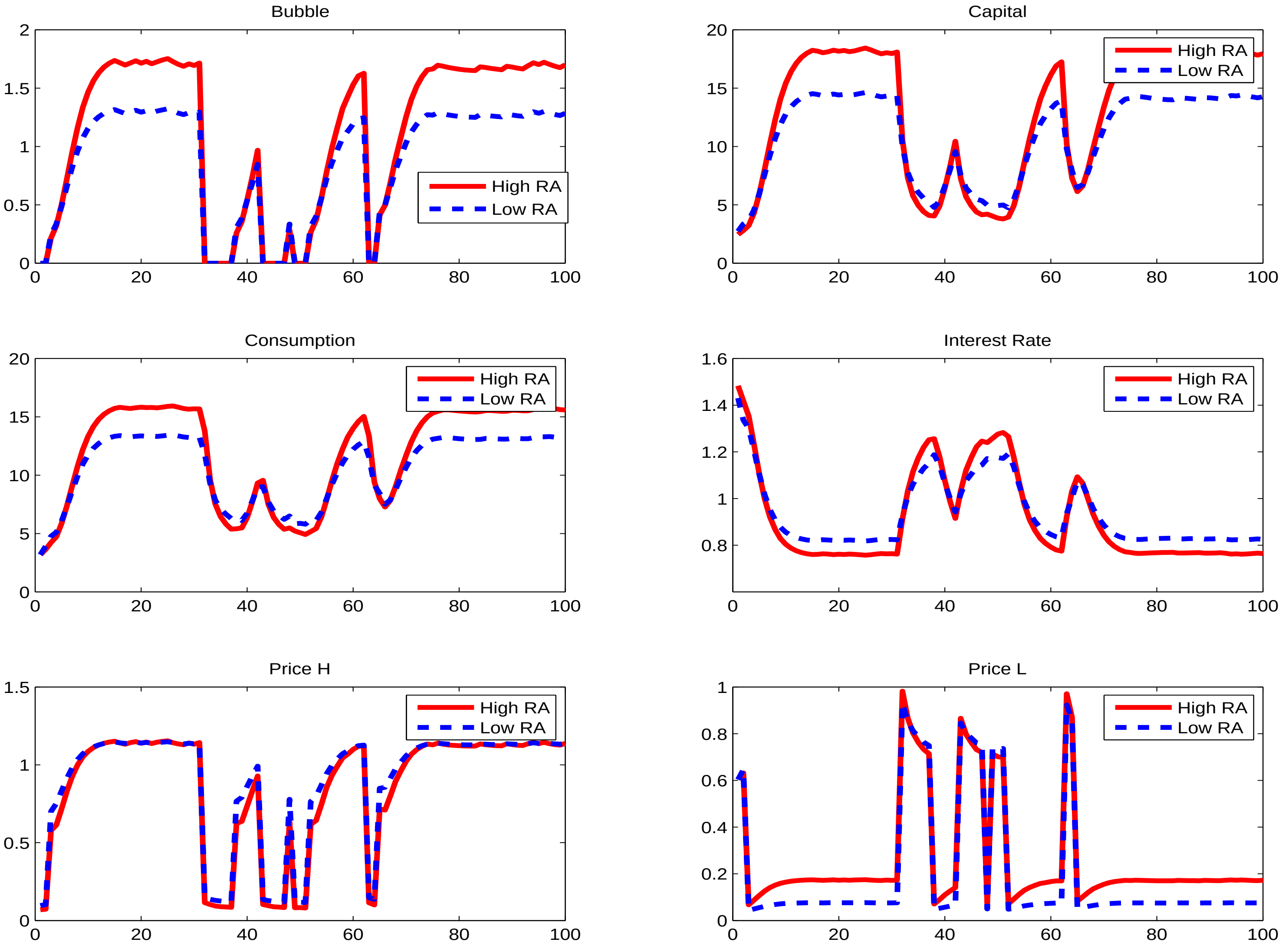
Example 3: bubbly business cycles



Example 3: a closer look at an episode



Example 3: role of risk aversion



Example 4: types of bubble shocks

- No technology shocks: $A^K_{s^t} = \overline{A^K}$ and $A^Q_{s^t} = \overline{A^Q}$

- Bubbly episode that never ends: $p = 0$

  – shocks to b_{s^t} and to $b^N_{s^t}$

- Shocks to existing bubble b_{s^t}

  – Contractionary

  – Crowding-out of capital

  – Decrease in consumption and intermediation

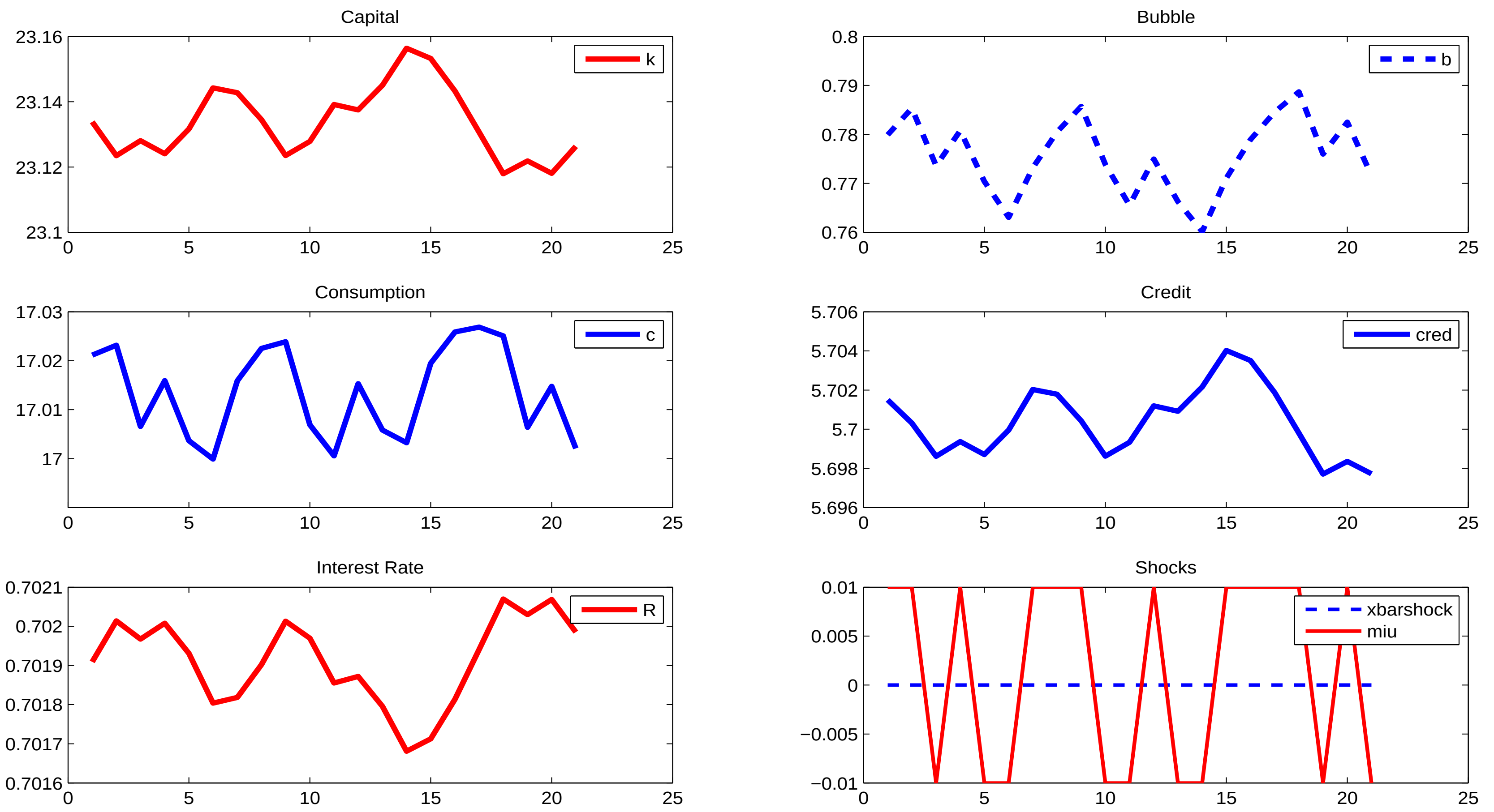
- Shocks to bubble creation $b^N_{s^t}$

  – Expansionary

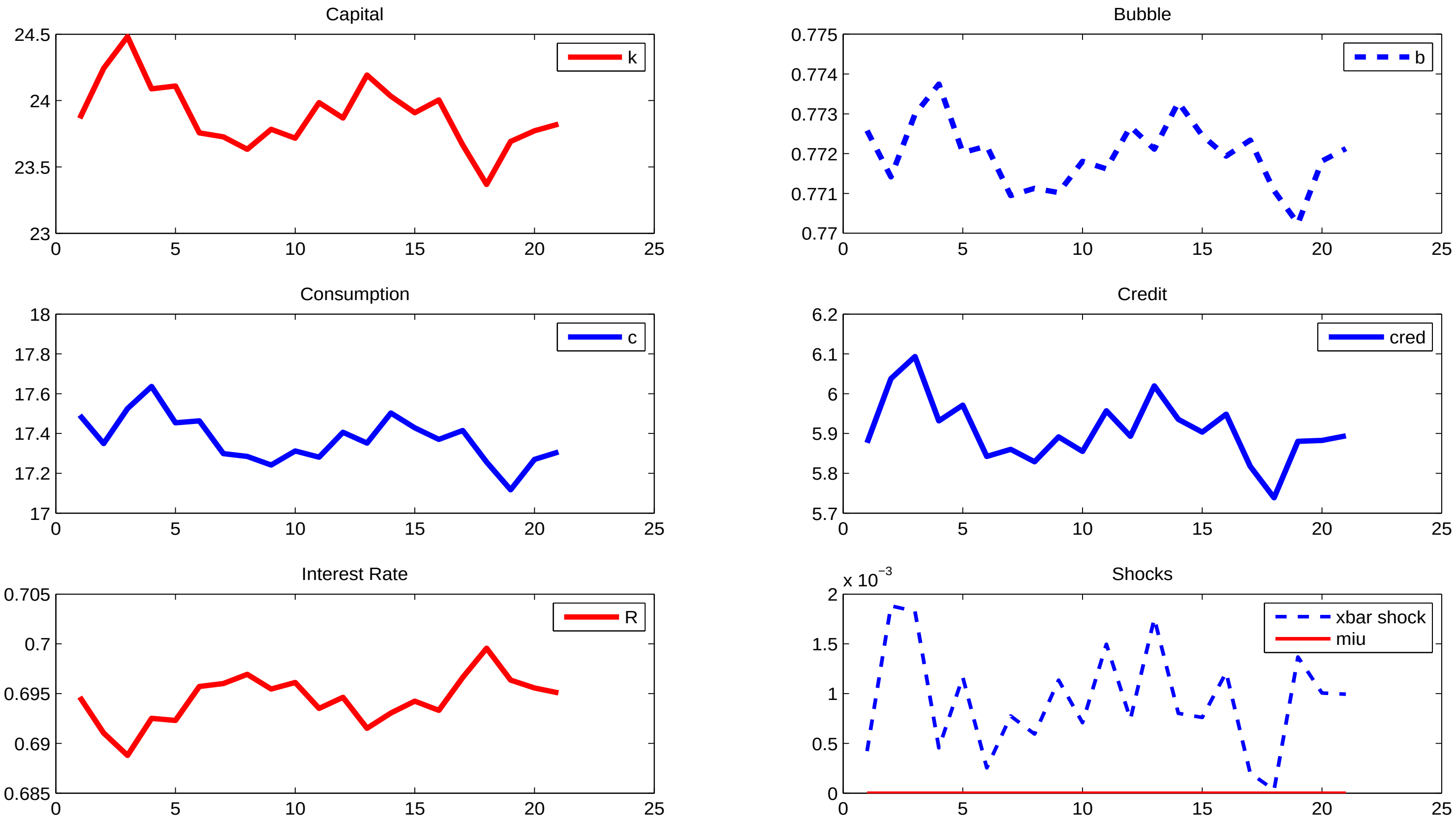
  – Reallocation of resources towards efficient investment

  – Increase in consumption

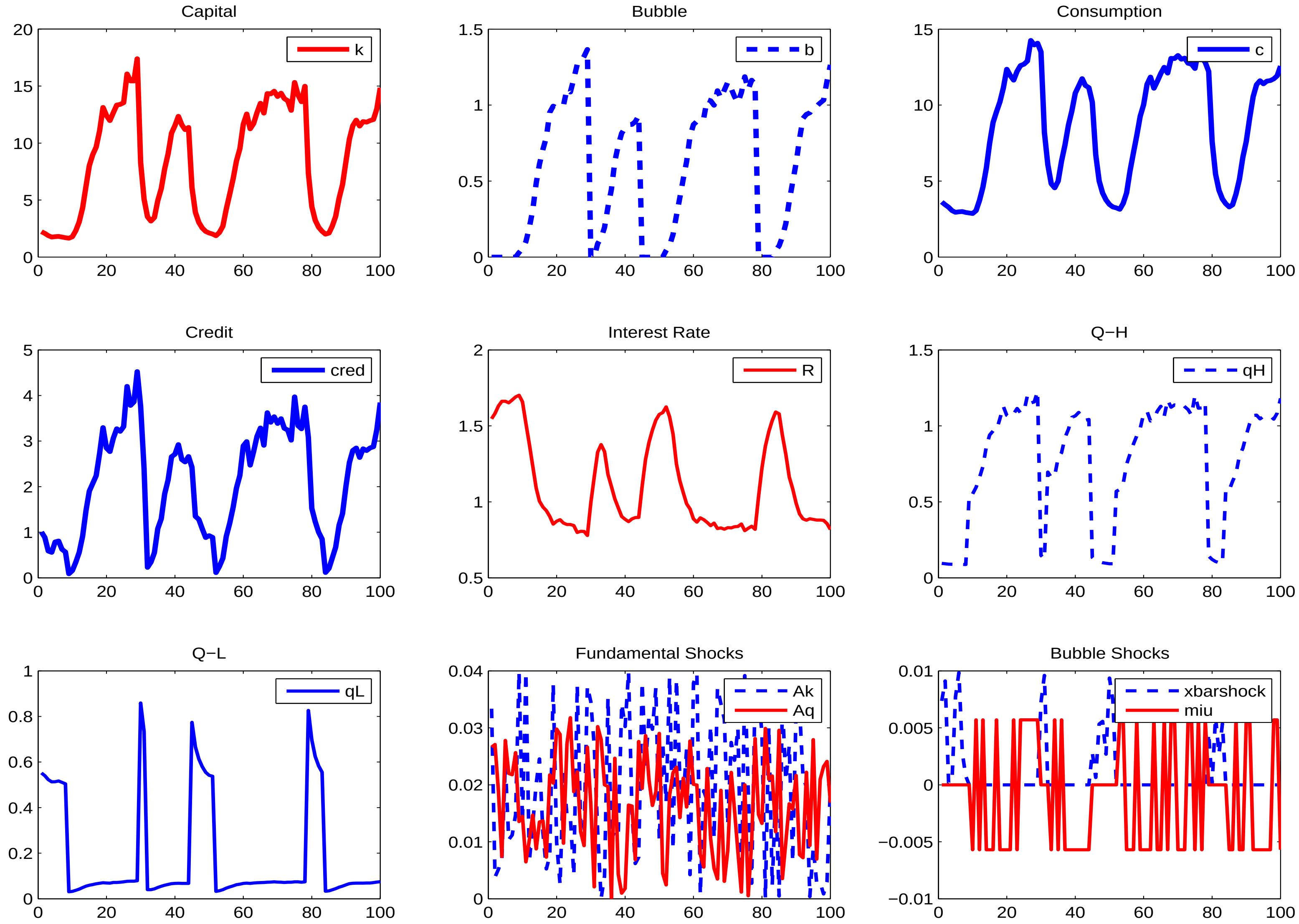
Example 4: shocks to b_{s^t}



Example 4: shocks to $b^N_{s^t}$



Example 5: the full economy



A benchmark model

- Two period OLG model with constant population size.
- Young receive a wage W_t and maximize $E_t\left\{U\left(C_{t+1}\right)\right\}$ with $U\left(C_{t+1}\right) = \dfrac{C_{t+1}^{1-\theta} - 1}{1-\theta}$.
- Investment options: (1) real investments with return R_{t+1}; and (2) speculative investments with return $\dfrac{B_{t+1}}{B_t}$.
- Bubbles are possible if:

  1. They grow fast enough to create their own demand:
  $$E_t\left\{C_{t+1}^{-\theta} \cdot \left(\frac{B_{t+1}}{B_t} - R_{t+1}\right)\right\} \geq 0$$
  2. They do not grow too fast to outgrow their own demand:
  $$B_t \leq W_t$$

- To determine whether asset bubbles are possible we need to specify labor productivity and the return to real investments, i.e. W_t and R_t.

Samuelson-Tirole case

- $W_t = 1$ and $R_t = \rho < 1$ for all t.

- Without asset bubbles (pessimistic beliefs):

$$C_t^F = \rho \text{ for all } t$$

- With asset bubbles that pop up and burst with probabilities δ and π, with $\pi > \rho > \delta$ (alternance of optimistic and pessimistic beliefs):

$$C_{t+1}^B = \begin{cases} B^* + \rho \cdot (1 - B^*) & \text{if } B_t = B_{t+1} = B^* \\ \rho \cdot (1 - B^*) & \text{if } B_t = B^* \text{ and } B_{t+1} = 0 \\ \rho & \text{if } B_t = B_{t+1} = 0 \\ \rho + B^* & \text{if } B_t = 0 \text{ and } B_{t+1} = B^* \end{cases} \quad \text{with } B^* = \frac{\rho \cdot \left[\left(\frac{\pi \cdot (1-\rho)}{(1-\pi) \cdot \rho} \right)^{\frac{1}{\theta}} - 1 \right]}{1 + \rho \cdot \left[\left(\frac{\pi \cdot (1-\rho)}{(1-\pi) \cdot \rho} \right)^{\frac{1}{\theta}} - 1 \right]} \leq 1$$

  If $\pi - \delta \geq \rho$, the young prefer to arrive to the world when there is an asset bubble $E_t \left\{ U \left(C_{t+1}^B \right) | B_t = B^* \right\} \geq E_t \left\{ U \left(C_{t+1}^F \right) | B_t = 0 \right\}$.

- *Result:* Asset bubbles are welfare-improving $E_t \left\{ U \left(C_{t+1}^B \right) \right\} \geq E_t \left\{ U \left(C_{t+1}^F \right) \right\}$.

  Why? The economy without bubbles is dynamically inefficient and (at the margin) consuming is better than investing. The bubble eliminates investments, creates consumption and restores efficiency.

Beyond the standard case: financial frictions

- With financial frictions, existing investments might be efficient relative to consumption but inefficient relative to other potential investments.

- In such a context bubbles can stop the inefficient investments and raise the efficient investments:

  1. (Intragenerational imperfections) Bubbles can stop inefficient investments and channel funds to efficient investments.

  2. (Intergenerational imperfections) Bubbles can help current generations appropriate future returns to their investments.

- We focus next on an example of the second type.

Beyond Samuelson-Tirole

- $W_{t+1} = \begin{cases} \gamma \cdot W_t & \text{if } L_t = \mu \\ W_t & \text{if } L_t = 0 \end{cases}$ with $W_0 = 1$ and $R_t = \rho > 1$ for all t.

- Productivity growth requires μ workers to do a public investment (basic research, improvement in institutions, ...)

- *Financial friction*: the returns to the public investment cannot be appropriated.

- Without asset bubbles, generations have no incentive to make the public investment, $L_t = 0$, and the equilibrium is:
$$C_t^F = \rho \quad \text{for all } t$$

- With asset bubbles that pop up and burst with probabilities δ and π, with $\pi \cdot \gamma > \rho > \delta$ generations make the public investment if the bubble is large enough:
$$L_t = \begin{cases} \mu & \text{if } B_t = B^* \cdot W_t \\ 0 & \text{if } B_t = 0 \end{cases} \quad \text{iff} \quad B^* \geq \frac{\rho}{\pi \cdot \gamma - \rho} \cdot \mu$$
Why? The bubble depends on *future* labor productivity and this provides incentives for the present generation to make the investment:
  1. Expected gain from public investment: $(\pi \cdot \gamma - \rho) \cdot B^* \cdot W_t$.
  2. Cost of public investment: $\rho \cdot \mu \cdot W_t$

- With asset bubbles, the equilibrium is:

$$C_{t+1}^{B} = \begin{cases} [\gamma \cdot B^* + \rho \cdot (1 - B^* - \mu)] \cdot W_t & \text{if } B_t = B_{t+1} = B^* \cdot W_t \\ \rho \cdot (1 - B^* - \mu) \cdot W_t & \text{if } B_t = B^* \cdot W_t \text{ and } B_{t+1} = 0 \\ \rho \cdot W_t & \text{if } B_t = B_{t+1} = 0 \\ (\rho + B^*) \cdot W_t & \text{if } B_t = 0 \text{ and } B_{t+1} = B^* \cdot W_t \end{cases}$$

$$\text{with } B^* = (1 - \mu) \cdot \frac{\rho \cdot \left[\left(\frac{\pi \cdot (\gamma - \rho)}{(1 - \pi) \cdot \rho} \right)^{\frac{1}{\theta}} - 1 \right]}{\gamma + \rho \cdot \left[\left(\frac{\pi \cdot (\gamma - \rho)}{(1 - \pi) \cdot \rho} \right)^{\frac{1}{\theta}} - 1 \right]}$$

- *Result:* Asset bubbles are welfare-improving $E_t \left\{ U \left(C_{t+1}^{B} \right) \right\} \geq E_t \left\{ U \left(C_{t+1}^{F} \right) \right\}$.

  Why? The economy without bubbles has no incentives to make the public investment and there is no efficient economic growth. The bubble creates these incentives and restores growth and efficiency.

Channels through which a bubble expands investment and output:

1. By providing collateral: Martin-Ventura (2011, 2012, 2013) and Carvalho-Martin-Ventura (2012a, 2012b)

2. By solving intergenerational problems: example above

3. By lowering the cost of liquidity: Caballero-Krishnamurty (2006), Farhi-Tirole (2012)

4. By lowering the cost of capital: Ventura (2012)