CREI Lectures in Macroeconomics: The Economics of Sovereign Debt and Default Part III

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Recap

- One-period bonds in Eaton-Gersovitz model
  - Constrained efficient
  - Align incentives of lenders and government
  - Minimize deadweight costs of default

Recap

- One-period bonds in Eaton-Gersovitz model
  - Constrained efficient
  - Align incentives of lenders and government
  - Minimize deadweight costs of default
- Longer maturity bonds
  - Do not correctly align incentives
  - Support multiple equilibria

Road Map for Today's Lecture

1. Discuss maturity choice
2. Introduce coordination failures
3. Hedging versus incentives
4. Debt overhang

“Drying up” of Long Term Bond Markets

- In crisis times, countries do not issue long-term bonds
- Yield curve flattens or inverts
  - Broner et al. argue risk premium of long-term bonds increases

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- There is also an efficiency argument that favors short-term bonds
  - Short-term bonds are better at minimizing costs of default
- Question: If issuing long-term debt is a bad idea, why not buy back?
- Builds on recent work with Hopenhayn and Werning

Extending the Framework

- Maintain from one-period bond analysis:
  - No output risk (kills spanning motive)
  - Outside option risk $v^D \sim F(v^D)$ and *iid*
  - One-period bond b

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- Add:
  - Arbitrary portfolio of long-term bonds:

$$\boldsymbol{I}^t = \{\boldsymbol{I}_0, \boldsymbol{I}_1, \dots\}$$

where $\boldsymbol{I}_k$ is payment due in k periods $(t + k)$

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    where $\boldsymbol{l}_k$ is payment due in k periods $(t+k)$
- Note: $b + \boldsymbol{l}_0$ is amount of debt due today

Competitive Equilibrium

Equilibrium Objects

- State variables at start of period $(b, \boldsymbol{I})$
- v^D realized
- If government does not default:
  - Value $V(b, \boldsymbol{I})$ if repays
  - Chooses $(b', \boldsymbol{I}')$
  - Faces prices $q(b', \boldsymbol{I}')$ and $Q(b', \boldsymbol{I}', \boldsymbol{I})$

Competitive Equilibrium

Equilibrium Objects

- One-period bond "break-even condition"

$$q(b', \boldsymbol{I}') = R^{-1} \Pr\left[v^{D\prime} < V(b', \boldsymbol{I}')\right]$$
$$= R^{-1} F\left(V(b', \boldsymbol{I}')\right)$$

Competitive Equilibrium

Equilibrium Objects

- Moving from $\boldsymbol{I}^t = \boldsymbol{I}$ to $\boldsymbol{I}^{t+1} = \boldsymbol{I}'$ raises net revenue:

$$Q(b', \boldsymbol{I}', \boldsymbol{I}) = \sum_{k=1}^{\infty} \rho_k(b', \boldsymbol{I}')(\boldsymbol{I}'_{k-1} - \boldsymbol{I}_k)$$

- $\boldsymbol{I}_k$ is k^{th} element of $\boldsymbol{I}$
- $\rho_k(b', \boldsymbol{I}')$ is the price of a unit promised in k periods (A "k-period zero coupon bond")

Competitive Equilibrium

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- $\rho_k(b', \boldsymbol{I}')$ is the price of a unit promised in k periods (A "k-period zero coupon bond")
- Revenue generated from *net* issuance of $t + k$ promises: $\rho_k \left(\boldsymbol{I}'_{k-1} - \boldsymbol{I}_k\right)$

Competitive Equilibrium

Equilibrium Objects

- Break-even condition for long-term bonds:

$$\rho_k(b', \boldsymbol{I}') = R^{-k}\Pi_{i=1}^{k} F(V_{t+i})$$

where V_{t+k} is value in period $t+k$ conditional on non-default through $t+k$

Competitive Equilibrium

Equilibrium Objects

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- Expectations hypothesis holds:

$$\begin{aligned}\rho_k &= \rho_{k-1}q_k \\ &= \Pi_{i=1}^{k}q_i \\ &= R^{-k}\Pi_{i=1}^{k}F\left(V_{t+k}\right)\end{aligned}$$

where q_i is one-period price in $t+i-1$ for delivery in $t+i$

Competitive Equilibrium

Government's Problem

$$V(b, \boldsymbol{I}) = \sup_{c, b', \boldsymbol{I}'} \left\{ u(c) + \beta \int \max \left\langle V(b', \boldsymbol{I}'), v^D \right\rangle dF_t(v^D) \right\}$$

subject to:

$$c \leq y - b - \boldsymbol{I}_0 + q(b', \boldsymbol{I}')b' + Q(b', \boldsymbol{I}', \boldsymbol{I})$$

Competitive Equilibrium

Welfare Theorem

- Presence of long-term bonds makes the CE inefficient in the usual sense
- Nevertheless, CE solves a modified planning problem:
  - A government enters period t with legacy liabilities $(b, \boldsymbol{l})$
  - Contracts with a new set of lenders:
    - Receives $\{c_{t+k}\}_{k\geq 0}$
    - Pays $y_{t+k} - \boldsymbol{l}_k - c_{t+k}$ conditional on no default through $t + k$
    - Contract maximizes joint surplus between government and "new lenders"

Planning Problem

$$B^\star(v, \boldsymbol{I}) = \sup_{\{c_{t+k}, V_{t+k}\}} \sum_{k=0}^{\infty} \left(\prod_{i=1}^{k} R^{-1} F(V_{t+i}) \right) (y_{t+k} - \boldsymbol{I}_k - c_{t+k})$$

subject to:

$\{V_{t+k}\}_{k=0}^{\infty}$ solves recursion given $\{c_{t+k}\}_{k=0}^{\infty}$

$V_t \geq v$

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$V_t \geq v$

- Objective is payments net of consumption and payments to long-term bonds
- Respects legacy promises
- Does not maximize market value of long-term bonds

Dual Problem

- Will show that the CE solves the planning problem

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- As before, consider the dual of the government's problem:

$$b = B(v, \boldsymbol{I}) \equiv \sup_{c, b', \boldsymbol{I}', v'} \left\{ y_t - \boldsymbol{I}_0 - c + q(b', \boldsymbol{I}')b' + Q(b', \boldsymbol{I}', \boldsymbol{I}) \right\}$$

subject to:

$$v = u(c) + \beta F(v')v' + \beta \int_{v^D \geq v'} v^D dF(v^D)$$

$$v' = V(b', \boldsymbol{I}')$$

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- Need to show $B = B^{\star}$

Dual Problem

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- Complication is the presence of long-term bond prices in Q
- Prices depend on path of $\{V_{t+i}\}$ not just next period's value
  - In CE: can commit to $(b', \boldsymbol{I}')$ today, but not future debt choices
  - Planning problem chooses entire sequence at start of contract
- The lack of commitment to fiscal trajectories featured in our previous discussion of long-term bonds

Toward's a welfare theorem...

- $B(v, \boldsymbol{I}) \leq B^{\star}$ is easy:
  - Given allocation $\{c_{t+i}, V_{t+i}\}$ define:

$$p_k \equiv R^{-k} \Pi_{i=1}^{k} F_{t+i}(V_{t+i})$$

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  - Now consider a CE allocation: $\{c_{t+i}, V_{t+i}\}$
  - Government's budget constraint:

$$\begin{aligned} B(v, \boldsymbol{I}) = b &\leq \sum_{k=0}^{\infty} \rho_k \left(y_{t+k} - \boldsymbol{I}_k - c_{t+k} \right) \\ &\leq \sup_{\{c_{t+k}, V_{t+k}\}} \sum_{k=0}^{\infty} p_k \left(y_{t+k} - \boldsymbol{I}_k - c_{t+k} \right) \\ &= B^{\star}(b, \boldsymbol{I}) \end{aligned}$$

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- Planning problem unconstrained by prices
- Always feasible to choose ocmpetitive allocation

Toward's a welfare theorem...

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Toward's a welfare theorem...

Other way: $B(v, \boldsymbol{I}) \geq B^{\star}$:

- Always feasible for government to not trade long-term bonds: $Q(b', \boldsymbol{I}, \boldsymbol{I}) = 0$

$$B(v, \boldsymbol{I}) = \left\{ y_t - \boldsymbol{I}_0 - c + q(b_{t+1}, \boldsymbol{I}')b' + Q(b', \boldsymbol{I}', \boldsymbol{I}) \right\}$$

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- Always feasible for government to not trade long-term bonds: $Q(b', \boldsymbol{I}, \boldsymbol{I}) = 0$

$$B(v, \boldsymbol{I}) \geq \sup_{c, b', v'} \left\{ y_t - \boldsymbol{I}_0 - c + q(b', \boldsymbol{I}) b' \right\}$$

Toward's a welfare theorem...

Other way: $B(v, \boldsymbol{I}) \geq B^{\star}$:

- Substitute in for $q(b', \boldsymbol{I}')b'$:

$$B(v, \boldsymbol{I}) \geq \sup_{c, b', v'} \left\{ y_t - \boldsymbol{I}_0 - c + F(v')B(v', \boldsymbol{I}_{\geq 1}) \right\}$$

Toward's a welfare theorem...

Other way: $B(v, \boldsymbol{I}) \geq B^{\star}$:

- Let $\{c_{t+k}, V_{t+k}\}$ be a choice for the planning problem
- Feasible in CE to choose $c = c_t$ and $v' = V_{t+1}$

$$B(v, \boldsymbol{I}) \geq y_t - \boldsymbol{I}_0 - c_t + F(V_{t+1}) B(V_{t+1}, \boldsymbol{I}_{\geq 1})$$

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- Iterating forward:

$$\begin{aligned} B(v, \boldsymbol{I}) \geq & \sum_{k=0}^{\infty} p_k (y_{t+k} - \boldsymbol{I}_k - c_{t+k}) \\ & + \lim_{k \to \infty} p_k B(V_{t+k}, \boldsymbol{I}_{\geq k}) \end{aligned}$$

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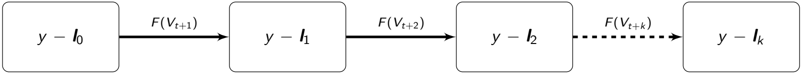
- Last term $\to 0$
- As $\{c_{t+k}, V_{t+k}\}$ was arbitrary, we have $B(v, \boldsymbol{I}) \geq B^\star(v, \boldsymbol{I})$

Modified Welfare Theorem

- CE implements a modified planning problem
- Trading only one-period bonds is enough to do this
- One-period bonds “solve” the time consistency problem regarding fiscal trajectories
- Legacy debt is a drag on efficiency, but must be respected absent default

Distortion due to Long-term Bonds



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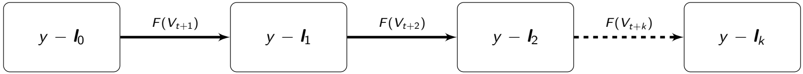

- Lower c_t and higher V_{t+k}:
  - Raises likelihood of "reaching" $t+k$ (and preceding path)
  - Raises cost of delivering initial v (as consumption not smooth)

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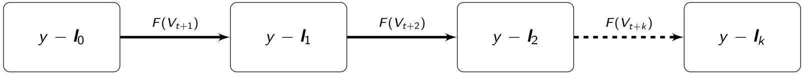

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  - Raises likelihood of "reaching" $t+k$ (and preceding path)
  - Raises cost of delivering initial v (as consumption not smooth)
- $\boldsymbol{I}_k \uparrow$:
  - Less surplus to split in $t+k$
  - Less incentive to reach that period
  - Default more likely

Convexity

- Let $\{c_{t+k}, V_{t+k}\}$ be the optimal allocation given $(v, \boldsymbol{I})$

$$B^{\star}(v, \boldsymbol{I}) = \sum_{k=0}^{\infty} p_k \left(y_{t+k} - \boldsymbol{I}_k - c_{t+k}\right)$$

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- Envelope condition wrt $\boldsymbol{I}$:

$$\nabla B^{\star} = \{-p_k\}_{k=0}^{\infty}$$

Convexity

- Compare $\boldsymbol{l}$ and $\boldsymbol{l}'$ given v

Convexity

- Compare $\boldsymbol{I}$ and $\boldsymbol{I}'$ given v
- Can always deliver v with same sequence $\{c_{t+k}, V_{t+k}\}$

$$
\begin{aligned}
B^{\star}(v, \boldsymbol{I}') &\geq B^{\star}(v, \boldsymbol{I}) - \sum_{k=0}^{\infty} p_k(\boldsymbol{I}'_k - \boldsymbol{I}_k) \\
&= B^{\star}(v, \boldsymbol{I}) + \nabla B^{\star}(v, \boldsymbol{I}) \bullet \left(\boldsymbol{I}' - \boldsymbol{I}\right)
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- $B^\star(v, \boldsymbol{I})$ is weakly convex in $\boldsymbol{I}$

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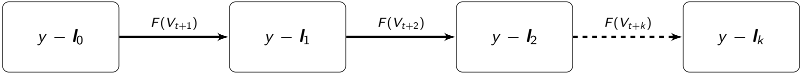
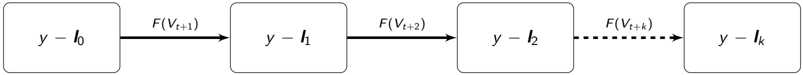
- $B^\star(v, \boldsymbol{I})$ is weakly convex in $\boldsymbol{I}$
- Strictly convex if default probability interior

Incentives and Prices

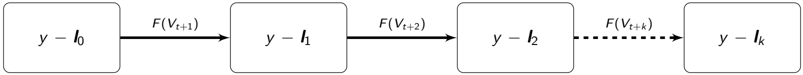


Incentives and Prices



- $\boldsymbol{I}_k \uparrow$:

Incentives and Prices



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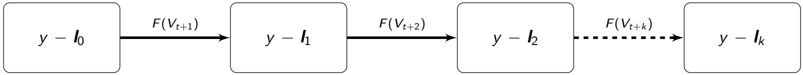
Incentives and Prices



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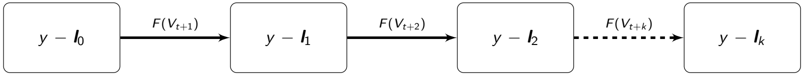

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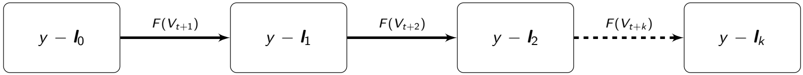
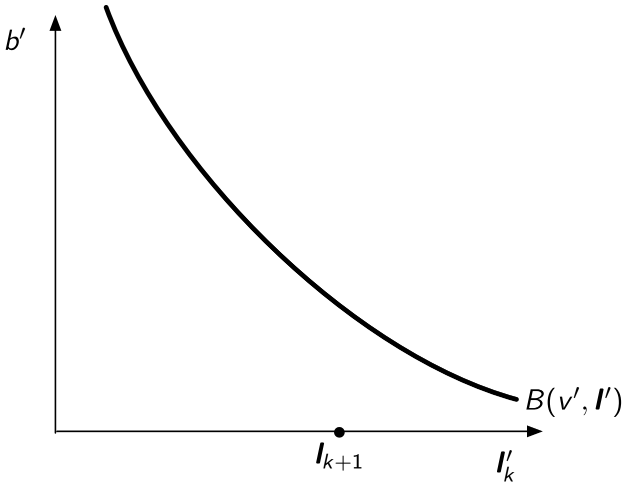
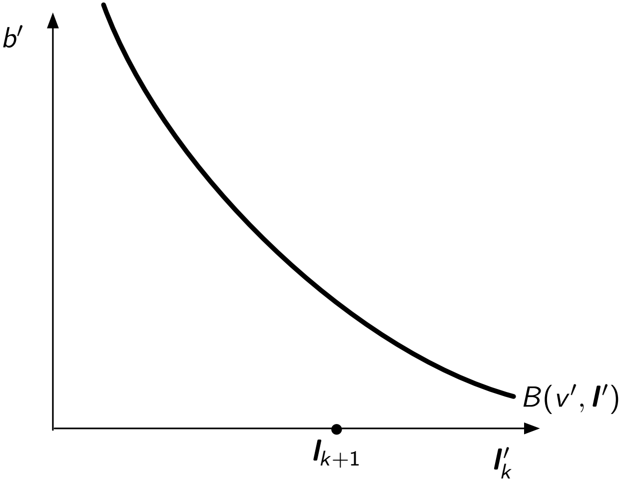


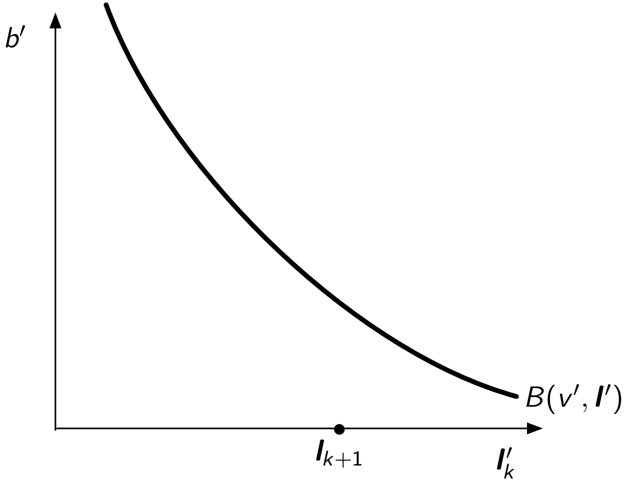
- $\boldsymbol{l}_k \uparrow$:
  - Less incentive to reach $t + k$
  - Lower V_{t+k} and move consumption ahead
  - Lowers $F(V_{t+k}) \Rightarrow p_k \downarrow$
- Issuing long-term debt lowers price
- Repurchasing long-term debt raises price



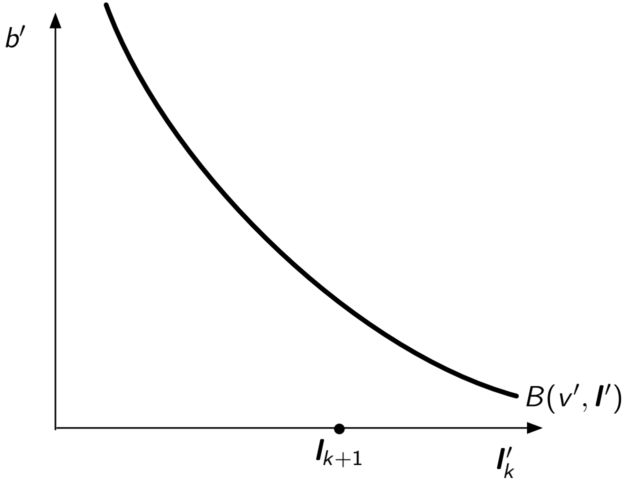
Consider starting out with $(b, \boldsymbol{I})$



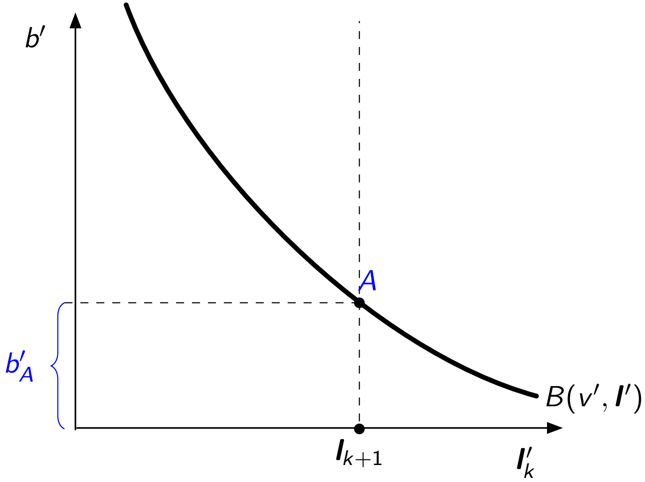
Choose to go to v' next period



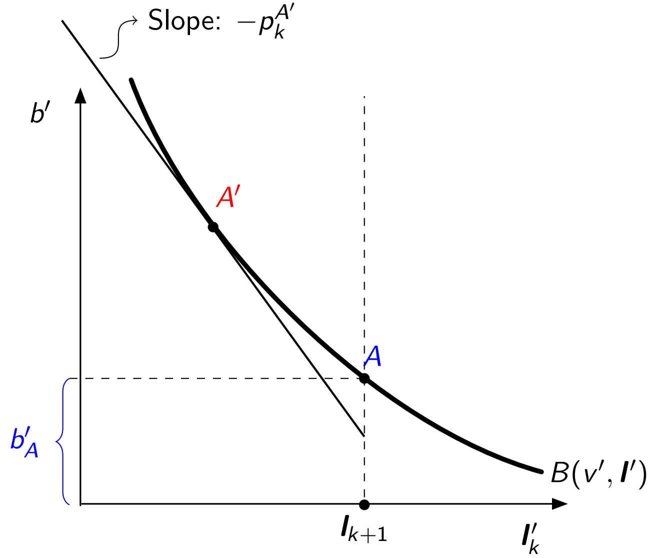
Which path $(b', \boldsymbol{l}'_k)$ maximizes today's consumption?



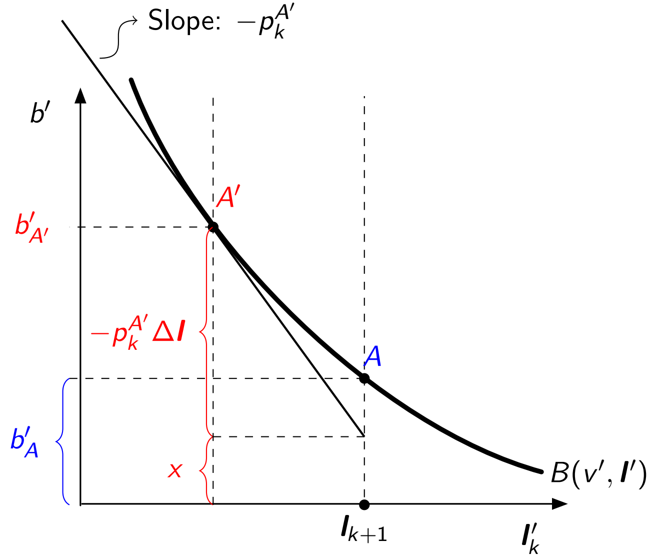
$$c = y - b + R^{-1} F(v') \left[b' + p_{k'} (\boldsymbol{I}_k' - \boldsymbol{I}_{k+1}) \right]$$



No LT Debt Trade: $c = y - b + R^{-1}F(v')\,[b'_A]$



Alternative: $c = y - b + R^{-1}F(v')\left[\underbrace{b'_{A'} + p_k^{A'}(\boldsymbol{I}'_{A'} - \boldsymbol{I}_{k+1})}_{x}\right]$



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Decentralization

- Prices deter government from trading long-term bonds
  - Buy backs generate greater incentive to save, ex post
  - Can implement same faster savings path without buying back debt
  - Same for issuances in reverse: Deter savings going forward
  - Raise more funds by issuing one-period bonds
- Reminiscent but different mechanism than “Buy Back Boondoggle”

Dynamics

- Dynamics are as in one-period bond economy
  - As if income were $y - \boldsymbol{I}_t$

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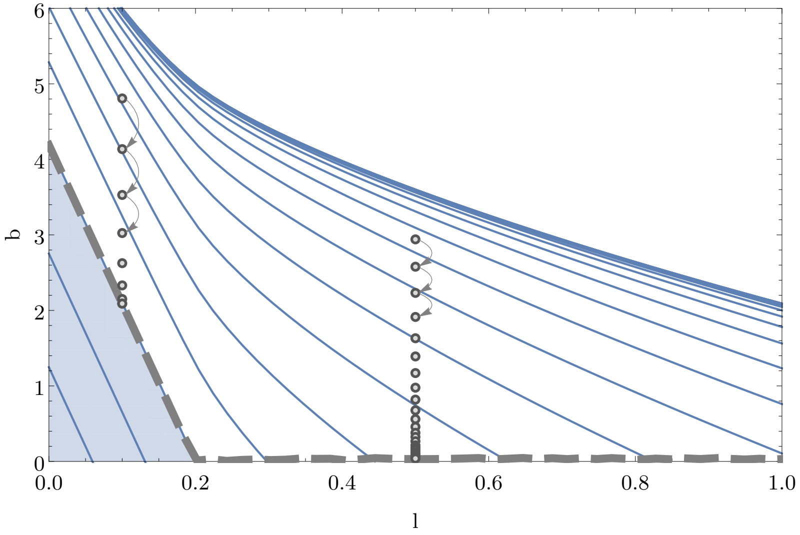
- Consider case of one-period bond and perpetuity:
  $\boldsymbol{l} = (l, l, l, l, \dots)$

Dynamics

$\beta R = 1$



Dynamics

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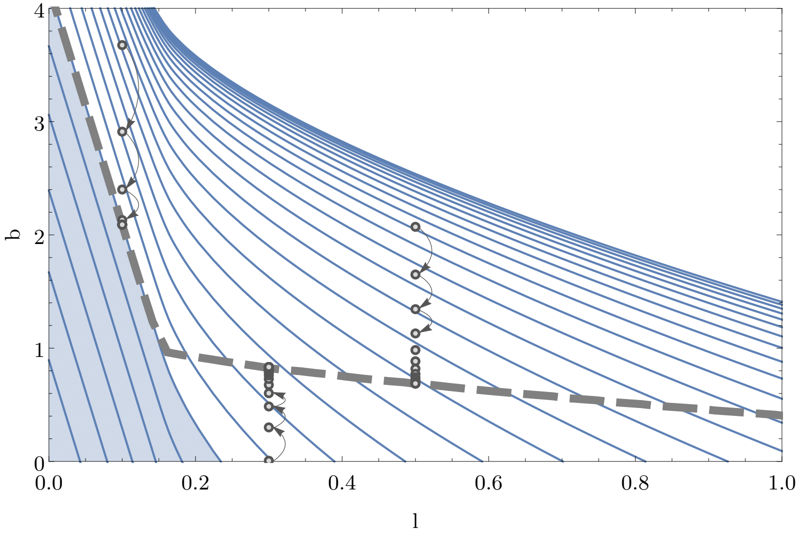

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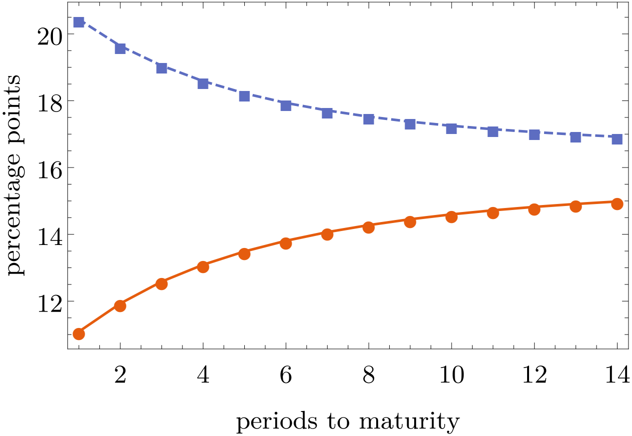
- Yield curve says nothing about *marginal* price

Dynamics

Yield Curve



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- Incentive to hold out
- Restructuring cannot be done via competitive trades

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- In model, market prices provide correct incentives
- No worries about enforcement

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- Default occurred when value of default exceeds value of repayment
- What about a coordination failure that induces inefficient default

Rollover Risk

No-Default Region

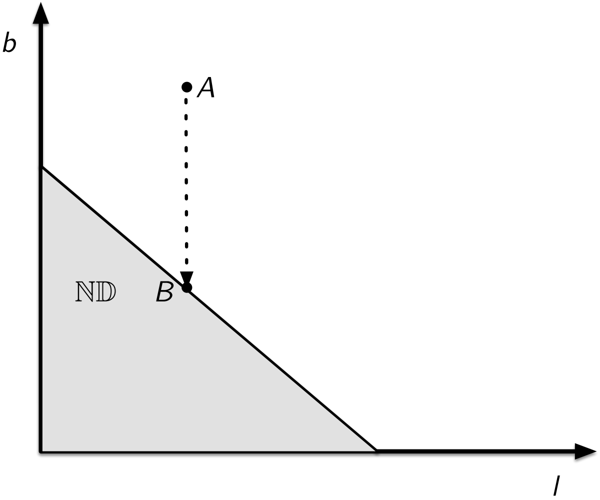
- When $\beta R = 1$, government saves to region of no default
- In benchmark model:

$$\frac{u\left(y - rb - l\right)}{1 - \beta} \geq \overline{V}^{D}$$

- Fundamental No-Default Region $\mathbb{ND}$: $rb + l \leq \overline{b}$

Fundamental Risk

Dynamics



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- Restrict to two-state case: $v^D \in \{\underline{V}^D, \overline{V}^D\}$
- Consider an equilibrium in which coordination failure is perfectly correlated with realization of $\overline{V}^D$

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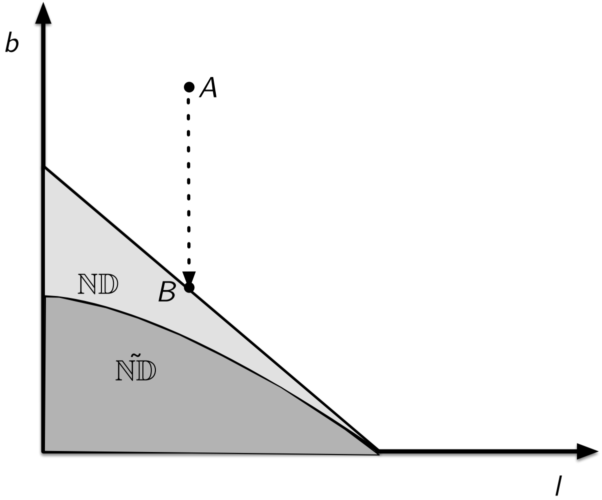
- Strictly inside if $b > 0$

Fundamental Risk plus Rollover Risk

Dynamics



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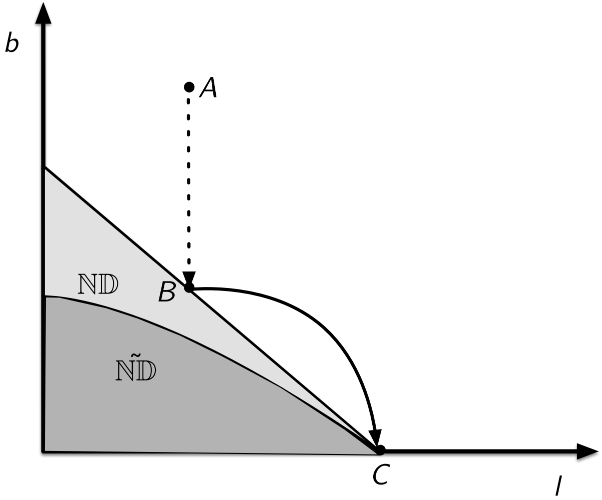

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- In our environment, clear sequencing:
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- More generally, solve a "stopping time" problem:
  - Reduce debt to mitigate fundamental risk
  - At some point, lengthen maturity to address rollover risk
  - Then continue to delever at slower pace to remove remaining fundamental risk

Hedging

- So far, downplayed insurance as a motive for longer maturities
- Well known that multiple maturities can help span business cycle risk
  - Not clear how important in practice
  - Quantitative models with commitment imply extremely large positions

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- Issuing perpetuities insures against the risk of a "bad" draw
  - Do not need to roll over debt
  - Consumption: $c = y - l$ until default regardless of F
- However, lenders internalize the bad incentives of long-term debt
  - Optimum maturity choice generally will include one-period debt
  - Provides commitment that if F is bad, government will mitigate default risk by saving

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- Long maturity
  - Generates an efficiency wedge between lenders and the government
  - Opens door to multiple equilibria
  - Issue is fiscal trajectory, not prices per se (as in a rollover crisis)

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- Additional questions:
  - Spillover from debt to other economic outcomes (output)?
  - Why don't governments save?

Debt Overhang

- Straightforward to incorporate a link between debt and output:

$$y \to f(k)$$
$$F(v^D) \to F(v^D|k)$$

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- Non-default requires: $W(b) \geq v^D(k)$

Planning Problem

$$B(v, w) = \sum_{t=0}^{\infty} R^{-t} \left(f(k_t) - (r+\delta)k - c_t\right)$$

subject to:

$$v \leq \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$w \leq \theta u(c_0) + \sum_{t=1}^{\infty} \beta^t u(c_t)$$

$$v^D(k_t) \leq \theta u(c_t) + \sum_{k=1}^{\infty} \beta^k u(c_{t+k}) \text{ for all } t$$

Planning Problem

Dynamics

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- First-order conditions:

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- As before, $\lambda_t \to 0$
- Implies $k_t \to k^*$ despite political economy frictions

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- Along path, debt falls and capital increases
- Consistent with data from developing economies

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  - Mirrored complete-markets approach
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- Emphasized costs of debt
  - Minimize loss to joint surplus from default
  - Motivation for debt reduction

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- Debt overhang links debt dynamics with economic outcomes

Thank You