CREI Lectures in Macroeconomics: The Economics of Sovereign Debt and Default Part II

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November 2-4, 2016

One-Period Eaton-Gersovitz Model

In the last lecture...

- Presented some empirical facts
  - Scope for shocks other than output
  - Default payoffs
  - Beliefs
- Discussed the competitive equilibrium of the one-period bond model
- Proved a welfare theorem:
  - The competitive equilibrium solves a planning problem
  - Fixed point of a contraction operator

Overview

In this lecture...

- Review one-period bond planning problem
- Present stripped down model to discuss debt dynamics
- Link to "exogenous" default models a la Cole and Kehoe

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- Review one-period bond planning problem
- Present stripped down model to discuss debt dynamics
- Link to "exogenous" default models a la Cole and Kehoe
- Discuss long-maturity bonds
  - Multiplicity
  - Inefficiency

The EG Planning Problem

$$
\begin{aligned}
B(s, v) = \max_{c, v(s'), b'} \; & y(s) - c + R^{-1} b' \sum_{s' \in S} \pi(s'|s) \mathbb{1}_{\{v(s') \geq V^D(s')\}} \\
& \text{subject to:} \\
& v \leq u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max\langle v(s'), V^D(s') \rangle \\
& b' \leq B(s', v(s')) \text{ for } s' \in S \text{ such that } v(s') \geq V^D(s')
\end{aligned}
$$

The EG Planning Problem
Frictions

- Two (related) frictions:
    1. Incomplete Markets
        - Cannot insure fluctuations in $y(s)$

The EG Planning Problem

Incomplete Markets

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The EG Planning Problem

Frictions

- Two (related) frictions:
    1. Incomplete Markets
        - Cannot insure fluctuations in $y(s)$
    2. Deadweight Costs of Default

The EG Planning Problem

Costs of Default

$$B(s,v) = \max_{c, v(s'), b'} y(s) - c + R^{-1} b' \sum_{s' \in S} \pi(s'|s) \mathbb{1}_{\{v(s') \geq V^D(s')\}}$$

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Implications of Inefficiency

- Inability to insure $y(s)$ generates precautionary savings
  - Well understood with or without default

Implications of Inefficiency

- Inability to insure $y(s)$ generates precautionary savings
  - Well understood with or without default
- Focus on second friction:
  - Set $y(s) = y$ for all s
  - Only risk: $V^D(s)$
  - V^D *iid* over time

The EG Planning Problem

$$B(s,v) = \max_{c, v(s'), b'} y - c + R^{-1} b' \sum_{s' \in S} \pi(s'|s) \mathbb{1}_{\{v(s') \geq V^D(s')\}}$$

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- *iid* shocks

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- s does not appear on RHS

The EG Planning Problem

$$B(v) = \max_{c, v(s'), b'} y - c + R^{-1} b' \sum_{s' \in S} \pi(s') \mathbb{1}_{\{v(s') \geq V^D(s')\}}$$

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$$b' \leq B(v(s')) \text{ for } s' \in S \text{ such that } v(s') \geq V^D(s')$$

- Optimal to set $B(v(s')) = B(v(s'')) = B(v')$

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- Can use final constraint to substitute out b' with $B(v')$ in objective

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Dynamics

- Two cases:
  1. $V^D(s) \in [\underline{V}^D, \overline{V}^D]$ and $V^D \sim F(v^D)$
  2. $V^D(s) \in \left\{\underline{V}^D, \overline{V}^D\right\}$ and $\Pr(V^D = \overline{V}^D) = \lambda$

Dynamics

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  1. $V^D(s) \in [\underline{V}^D, \overline{V}^D]$ and $V^D \sim F(v^D)$
  2. $V^D(s) \in \left\{\underline{V}^D, \overline{V}^D\right\}$ and $\Pr(V^D = \overline{V}^D) = \lambda$
- Assume $B(\overline{V}^D) \geq 0 \Leftrightarrow \overline{V}^D \leq V(0)$
- Assume $\beta R = 1$: Only dynamics due to default costs

Dynamics

Continuous Distribution

$$B(v) = \max_{c,v'} y - c + R^{-1} B(v') F(v')$$

subject to:

$$v \leq u(c) + \beta F(v') v' + \beta \int_{v'}^{\overline{V}^D} v^D dF(v^D)$$

Dynamics

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- First-Order Conditions:

$$\frac{1}{u'(c)} = \mu = -B'(v)$$

$$B'(v') + \frac{f(v')}{F(v')} B(v') + \mu = 0$$

Dynamics

Continuous Distribution

- Inverse Euler Equation

$$\frac{1}{u'(c')} = \frac{1}{u'(c)} + \frac{f(v')}{F(v')} B(v')$$

- Default probability places "wedge" in inter-temporal decision

Dynamics

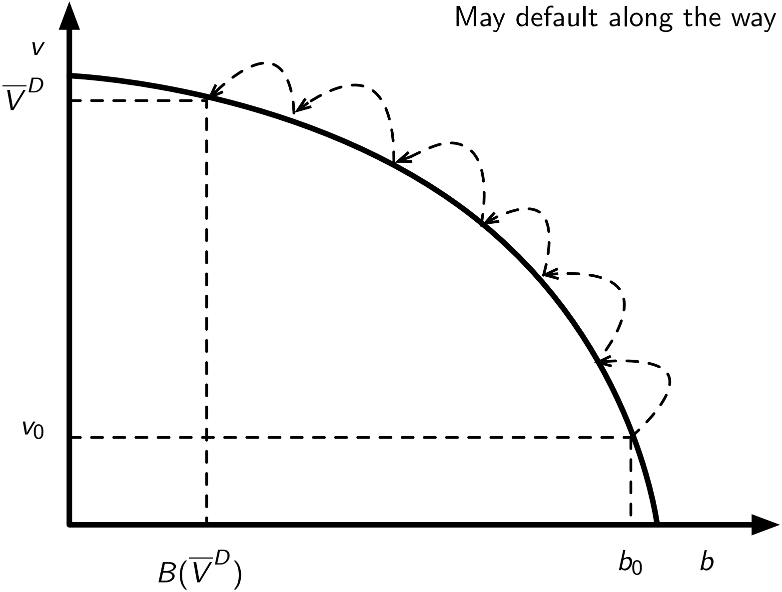
Continuous Distribution

- Inverse Euler Equation

$$\frac{1}{u'(c')} = \frac{1}{u'(c)} + \frac{f(v')}{F(v')}B(v')$$

- Default probability places "wedge" in inter-temporal decision
- Backloading:
  - $c' > c$ as long as $v \in [\underline{V}^D, \overline{V}^D)$
  - $v \to \overline{V}^D$
  - Save to point where default is ruled out

Move up continuously
Speed determined by $f(v')$
May default along the way



Dynamics
Continuous Distribution

- Same implication as complete markets case with output shocks
- Not driven by desire for insurance

Dynamics

- Same implication as complete markets case with output shocks
- Not driven by desire for insurance
- Seems like a local intuition, but holds more generally

Dynamics

Two Shock Case

- V^D takes two values $V^D \in \{\underline{V}^D, \overline{V}^D\}$
- Poisson probability of high-payoff state λ

Dynamics

Two Shock Case

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- Poisson probability of high-payoff state λ
- Split state space into three "zones":
  1. Safe Zone: $v \geq \overline{V}^D$
  2. Crisis Zone: $v \in [\underline{V}^D, \overline{V}^D)$
  3. Default Zone: $v < \underline{V}^D$

Dynamics

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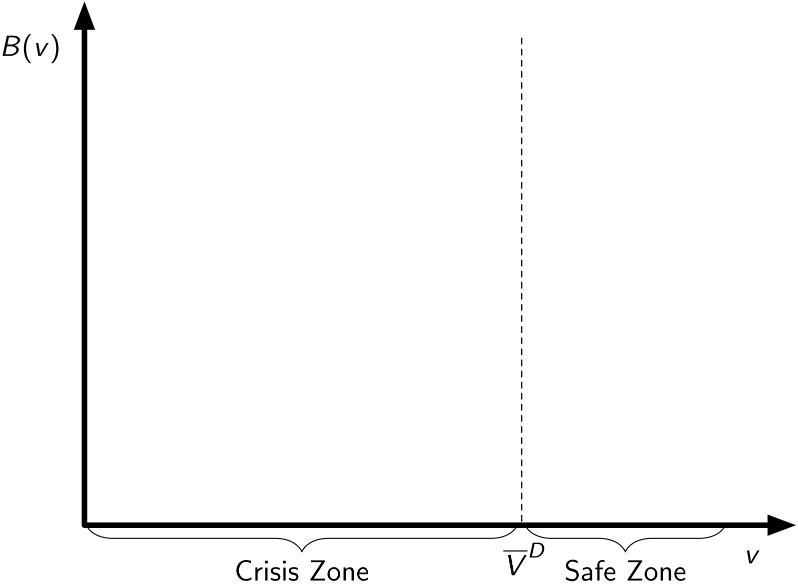
- Ignore Default Zone


$B(v)$
$\overline{V}^D$
Crisis Zone
Safe Zone
v

Two Shock Case

Continuous Time Limit

- Useful to let the time period become small:

$$\beta = R^{-1} = e^{-\rho \Delta t}$$

- Let T denote first realization of $\overline{V}^D$

$$\Pr(T > t + \Delta t | T > t) = e^{-\lambda \Delta t}$$

Safe Zone

Hamilton-Jacobi-Bellman Equation

- Safe Zone Bellman Equation

$$\rho B(v) = \max_c y - c + B'(v) \underbrace{(\rho v - u(c))}_{\dot{v}}$$

Safe Zone

Hamilton-Jacobi-Bellman Equation

- Safe Zone Bellman Equation

$$\rho B(v) = \max_{c} y - c + B'(v)\left(\rho v - u(c)\right)$$

- First-Order Condition:

$$\begin{aligned} & -1 - B'(v)u'(c) = 0 \\ & \Rightarrow \\ & -B'(v) = \frac{1}{u'(c)} \end{aligned}$$

Safe Zone

Conjectured Solution

- In Safe Zone there is no risk of default
- Lenders and government both discount at rate ρ
- Conjectured solution: $v = \frac{u(C(v))}{\rho}$

$$C(v) = u^{-1}(\rho v)$$

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$$B(v) = \frac{y - C(v)}{\rho}$$

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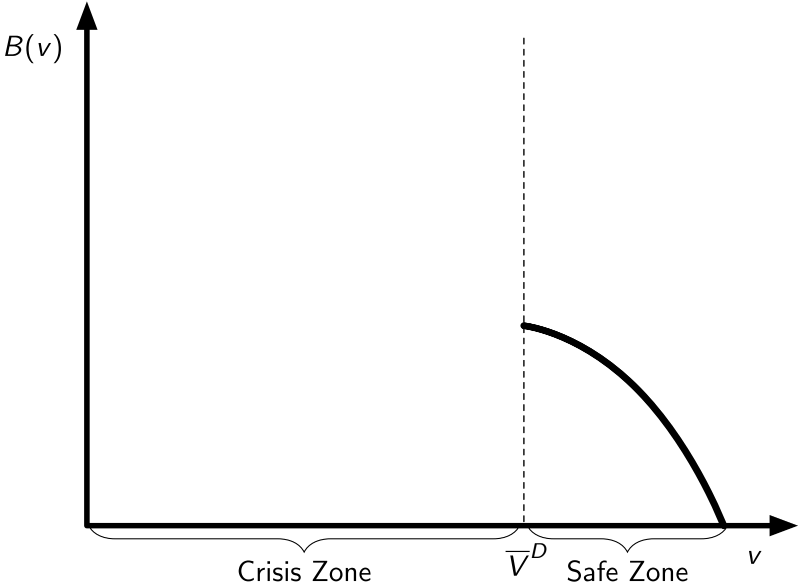
- Note: $B'(v) = -\frac{C'(v)}{\rho} = \frac{-1}{u'(C(v))}$



$B(v)$
$\overline{V}^D$
Crisis Zone
Safe Zone
v

Crisis Zone

Hamilton-Jacobi-Bellman Equation

- Crisis Zone: Probability of default λ

$$(\rho + \lambda)B(v) = \max_{c} y - c + B'(v)\dot{v}$$

- With

$$\rho v = u(c) + \dot{v} + \lambda \left(\overline{V}^D - v \right)$$

Crisis Zone

Hamilton-Jacobi-Bellman Equation

- Crisis Zone: Probability of default λ

$$(\rho + \lambda)B(v) = \max_c y - c + B'(v)\dot{v}$$

- With

$$\rho v = u(c) + \dot{v} + \lambda\left(\overline{V}^D - v\right)$$

or

$$(\rho + \lambda)v = u(c) + \dot{v} + \lambda\overline{V}^D$$

- Note: As if both discount at $(\rho + \lambda)$

Crisis Zone

Conjectured Solution

- Discount at actuarially fair prices
- Conjecture constant c:

$$v = \frac{u\left(\tilde{C}(v)\right)}{\rho + \lambda} + \frac{\lambda}{\rho + \lambda}\overline{V}^D$$

Crisis Zone

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$$v = \frac{u\left(\tilde{C}(v)\right)}{\rho + \lambda} + \frac{\lambda}{\rho + \lambda}\overline{V}^D$$

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Crisis Zone

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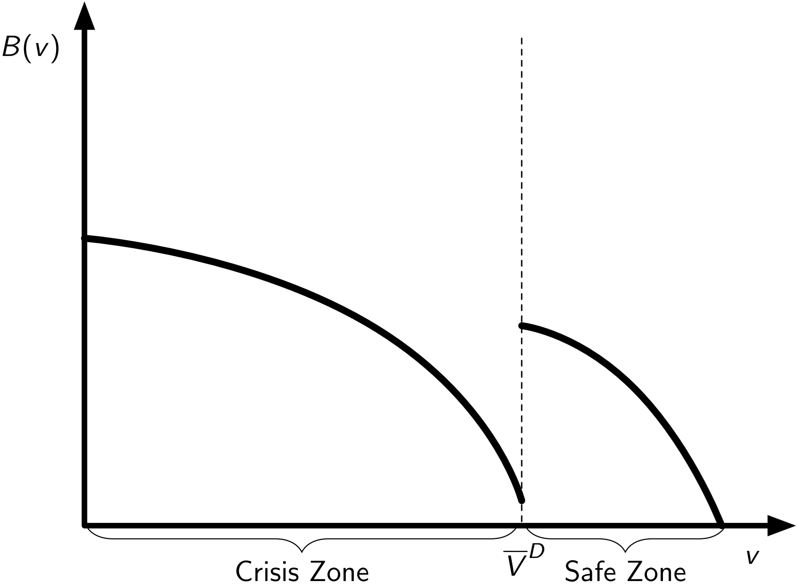
$$v = \frac{u\left(\tilde{C}(v)\right)}{\rho + \lambda} + \frac{\lambda}{\rho + \lambda}\overline{V}^D$$

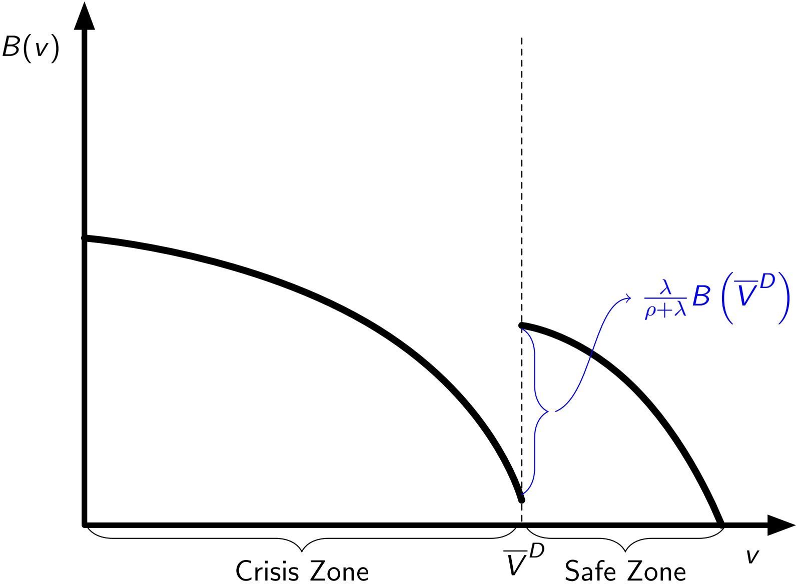
- Payoff to lender:

$$B(v) = \frac{y - \tilde{C}(v)}{\rho + \lambda}$$

- Note: $B'(v) = \frac{-\tilde{C}'(v)}{\rho+\lambda} = \frac{-1}{u'\left(\tilde{C}(v)\right)}$



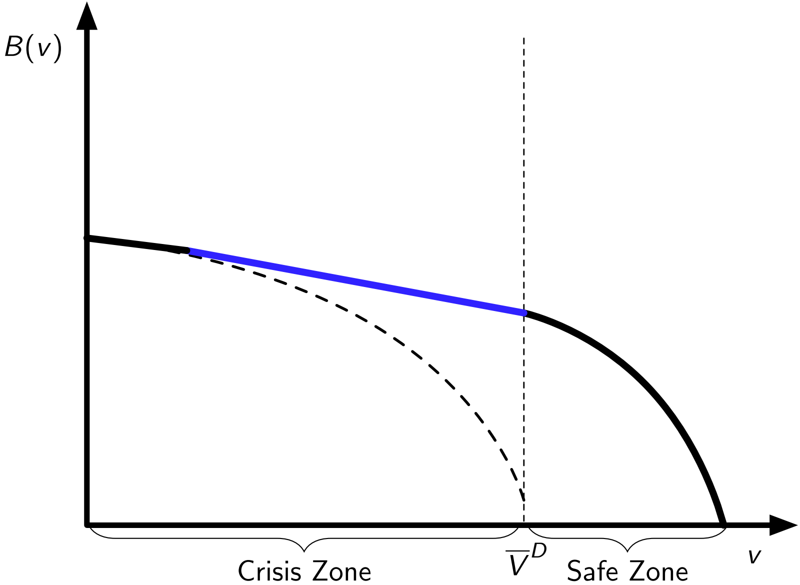
$B(v)$
v
Crisis Zone
$\overline{V}^D$
Safe Zone


$B(v)$
$\frac{\lambda}{\rho+\lambda} B\left(\overline{V}^D\right)$
Crisis Zone
$\overline{V}^D$
Safe Zone
v

Inefficiency

Role for Backloading

- Discontinuity cannot be part of the solution
- Inefficiency:
  - To the left of $\overline{V}^D$, small decrease in c (increase in $\dot{v}$) generates discrete gain to lender with second order costs
  - Optimal to backload in neighborhood below $\overline{V}^D$


$B(v)$
v
Crisis Zone
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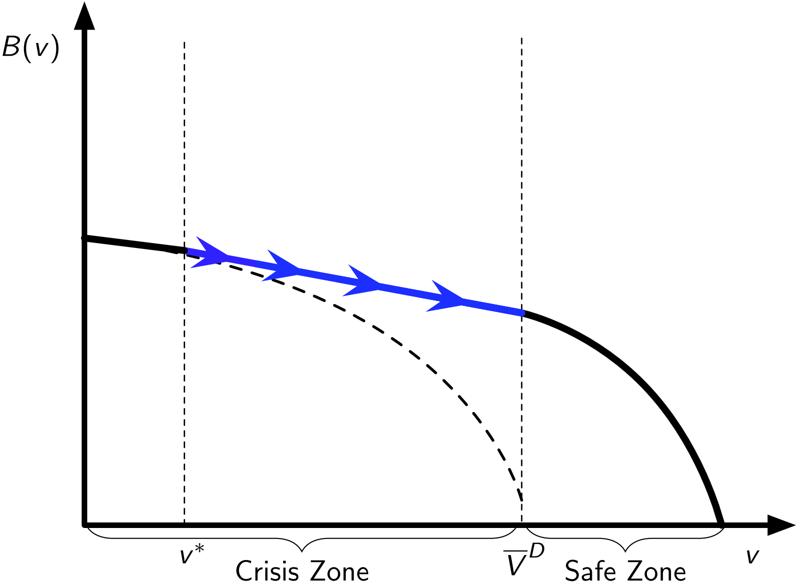

$B(v)$
v^*
Crisis Zone
$\overline{V}^D$
Safe Zone
v

Two Shock Case

Role for Backloading

- In neighborhood of $v < \overline{V}^D$:
  - Set $c = \bar{c}$
  - $\bar{c}$ solves Bellman equation to the left of $\overline{V}^D$:

$$(\rho + \lambda)B(\overline{V}^D) = y - \bar{c} - \frac{1}{u'(\bar{c})}\left[(\rho + \lambda)v - u(\bar{c}) - \lambda\overline{V}^D\right]$$

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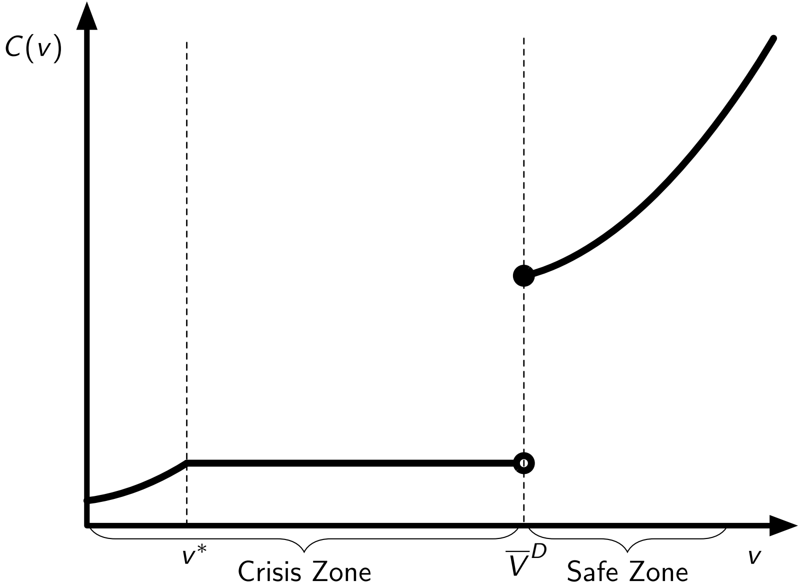
- Threshold for saving:

$$v^* = \frac{u(\bar{c})}{\rho + \lambda} + \frac{\lambda}{\rho + \lambda}\overline{V}^D$$


$C(v)$
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Crisis Zone
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Safe Zone
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- Planner's solution:
  - If $v \geq \overline{V}^D$: keep consumption and v constant
  - If $v \in (v^*, \overline{V}^D)$ back load until $v = \overline{V}^D$
  - If $v \leq v^*$: keep consumption and v constant and default will eventually happen

Dynamics

Two Shock Case

- Key is that Planner delays consumption until reach Safe Zone
- Efficient from perspective of lender: Saves $\lambda B(\overline{V}^D)/(\rho + \lambda)$

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- How is this decentralized in a competitive equilibrium?

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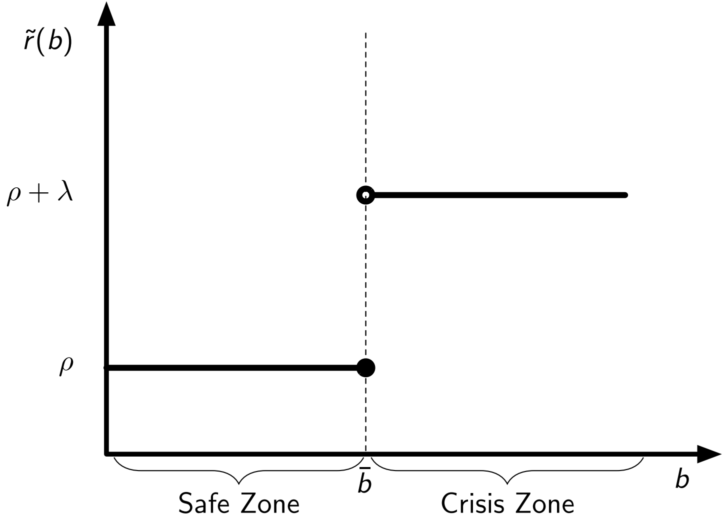
Two Shock Case

- Key is that Planner delays consumption until reach Safe Zone
- Efficient from perspective of lender: Saves $\lambda B(\overline{V}^D)/(\rho+\lambda)$
- How is this decentralized in a competitive equilibrium?
  - Remember that default occurs when payoff V^D is high
  - In Crisis Zone, $\overline{V}^D$ is greater than value of repayment
  - Why not just rollover bonds until high payoff and then default?

Decentralization



Dynamics

Decentralization

- In Crisis Zone:
  - To left of $\bar{b} \equiv B(\overline{V}^D)$, pays $\rho + \lambda$ to roll over bonds
  - By saving to $\bar{b}$, pays only ρ
  - Saves $\frac{\lambda}{\rho+\lambda}\bar{b} = \frac{\lambda}{\rho+\lambda}B(\overline{V}^D)$
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  - Saves $\frac{\lambda}{\rho+\lambda}\bar{b} = \frac{\lambda}{\rho+\lambda}B(\overline{V}^D)$
  - Completely internalizes efficiency cost via prices
- Important that government rolls over entire stock of debt each period
  - Otherwise, only internalizes fraction that is rolled over

Exogenous Default

Role of V^D

- Consider arbitrary Crisis Zone: $v < \bar{v}$
- Consider arbitrary default payoff $V^D \leq \bar{v}$

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- Consumption level in Crisis Zone solves:

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- Note: $\frac{d\bar{c}}{dV^D} = \frac{u''(\bar{c})}{u'(\bar{c})^2}\dot{v} > 0$ as $\dot{v} < 0$.

Exogenous Default

Role of V^D

- Consider arbitrary Crisis Zone: $v < \bar{v}$
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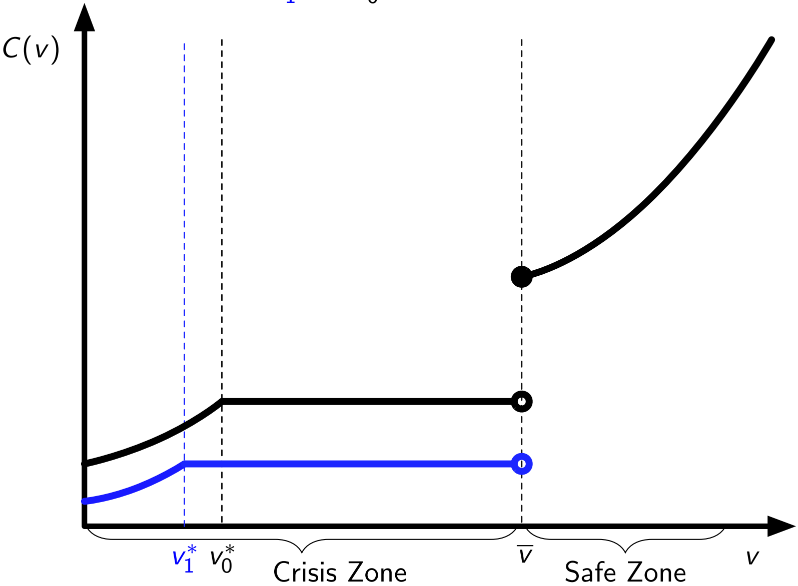
- Note: $\frac{d\bar{c}}{dV^D} = \frac{u''(\bar{c})}{u'(\bar{c})^2}\dot{v} > 0$ as $\dot{v} < 0$.
- A decrease in V^D implies faster convergence in "saving" region

$V_1^D < V_0^D$



Default

A Tale of Two Dragons

- Conventional Wisdom: Save to avoid costly default state

Default

A Tale of Two Dragons

- How the model works: Save even if default is a windfall

Two Dragons

Why does it matter?

- Many papers argue government's save to avoid (exogenous) costly default
  - Example: Cole-Kehoe's "run" model

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- Role of maturity?
  - Cole-Kehoe: Maturity restores efficiency
  - Eaton-Gersovitz?

Two Dragons

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  - Example: Cole-Kehoe's "run" model
- Endogenous default environment:
  - Efficiency implies sovereign should save
- Role of maturity?
  - Cole-Kehoe: Maturity restores efficiency
  - Eaton-Gersovitz?
- Quantitative implications
  - Quantitative models take default costs as free parameter
  - Nonlinear costs a la Arellano reduce the incentive to save

Longer Maturity

Next Steps

- One-period bond model:
  - Constrained efficient
  - Equilibrium is unique

Longer Maturity
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Next Steps

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  - Constrained efficient
  - Equilibrium is unique
- Longer maturities:
  - Observed in practice
  - Improve quantitative fit of EG model
- How does longer maturity change lessons from one-period bond environment?

Longer Maturity

Environment

- Continue with simplified environment
  - No output shocks: $y(s) = y$
  - Two default states: $V^D \in \{\underline{V}^D, \overline{V}^D\}$
  - *iid* transition: $\Pr(V^D = \overline{V}^D) = \lambda$
  - Safe Zone and Crisis Zone
  - Continuous time limit

Longer Maturity

Environment

- Random maturity (perpetual youth) bonds
  - Probability of maturity δ
  - *iid* across bonds and time
  - $\delta \to \infty$: Short-term debt
  - $\delta \to 0$: Perpetuitities
- Normalize coupon to r
- Assume $\rho > r$: Incentive to borrow

Longer Maturity
Environment

- Solve for equilibrium using "primal" approach:
  - Equilibrium is no longer solution to planning problem
- Let b denote face value of bonds
- Let $q(b)$ denote price per bond given face value b
- Let $V(b)$ denote value of repayment given b

Longer Maturity

Government's Problem

- Faced with price schedule q:

$$\rho V(b) = \max_{c} \left\{ u(c) + V'(b)\dot{b} + \lambda \left(\max\langle V(b), \overline{V}^{D} \rangle - V(b) \right) \right\}$$

- Subject to:

$$c = y - (r + \delta)b + q(b)\left(\dot{b} + \delta b\right)$$

Longer Maturity

Government's Problem

- Faced with price schedule q:

$$\rho V(b) = \max_{c} \left\{ u(c) + V'(b)\dot{b} + \lambda \left(\max\langle V(b), \overline{V}^{D} \rangle - V(b) \right) \right\}$$

- Subject to:

$$c = y - (r+\delta)b + q(b)\left(\dot{b} + \delta b\right)$$

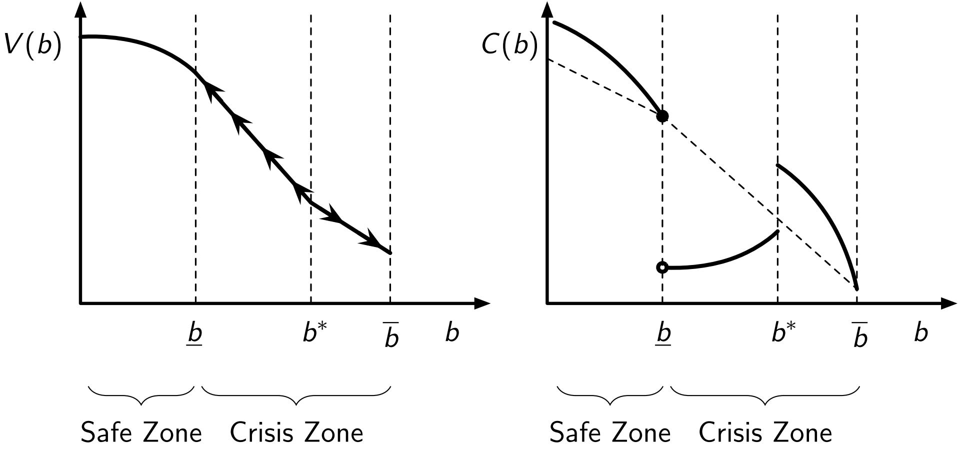
- Lenders' Break-Even Condition:

$$rq(b) = r + (1-q(b))\delta + q'(b)\dot{b} - \lambda q(b)\mathbb{1}_{\{V(b)<\overline{V}^{D}\}}$$

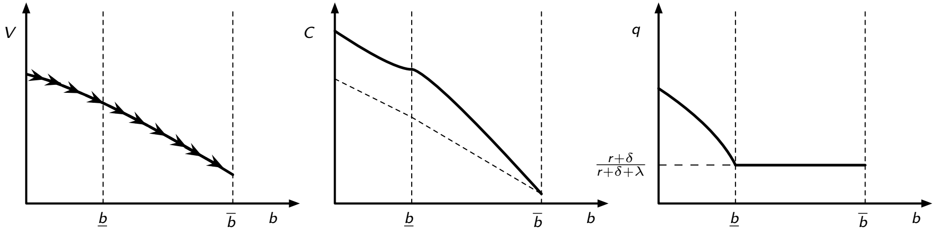
Constructing Equilibria

1. $\delta \to \infty$ (Uniqueness)
2. $\delta = 0$ (Uniqueness)
3. Intermediate case: $\delta \in (0, \infty)$ (Multiplicity)

Short-term Bonds: $\delta = \infty$



Perpetuities: $\delta = 0$



Short vs. Long

- Short-term bonds are "efficient" as government faces correct incentives to reduce default risk
  - At boundary of $\underline{b}$, government recognizes a small reduction in c lowers rollover costs
  - Prices correctly align incentives
  - Like a variable cost

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  - At boundary of $\underline{b}$, government recognizes a small reduction in c lowers rollover costs
  - Prices correctly align incentives
  - Like a variable cost
- Perpetuities provide no incentives to economize on default costs
  - When issued, price reflects future default probabilities
  - Never rolled over, so no incentive to reduce debt once issued
  - Like a sunk cost

Intermediate Maturity

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  - Need to roll over bonds in the future makes reducing debt worthwhile
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- "Perpetuity" type of equlibrium:
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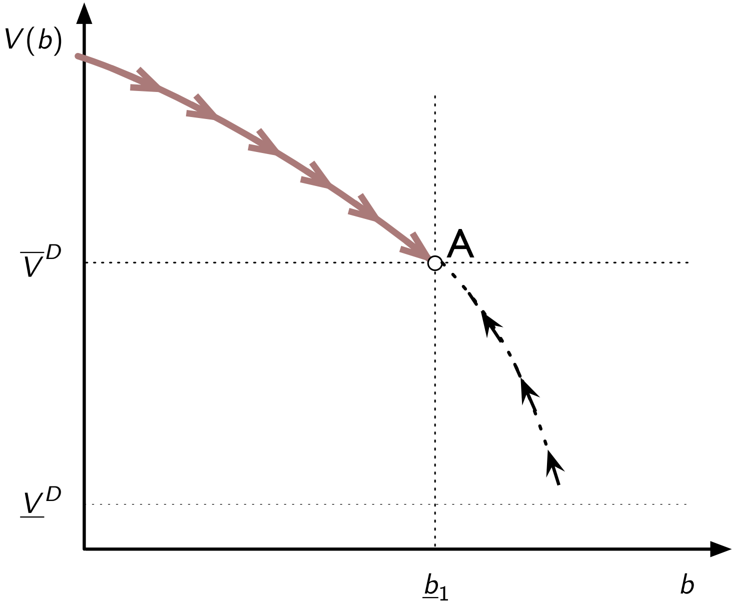
  - $\overline{b}$ a stationary point

Constructing Equilibria with $\delta \in (0, \infty)$



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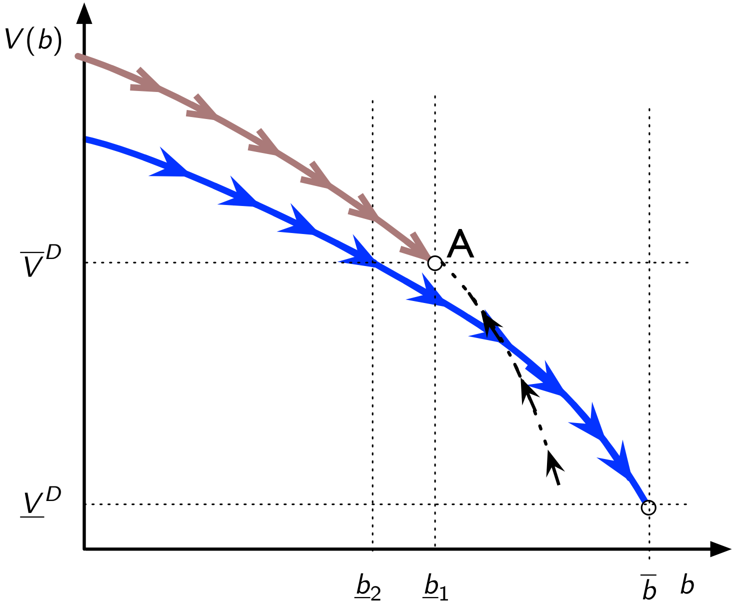

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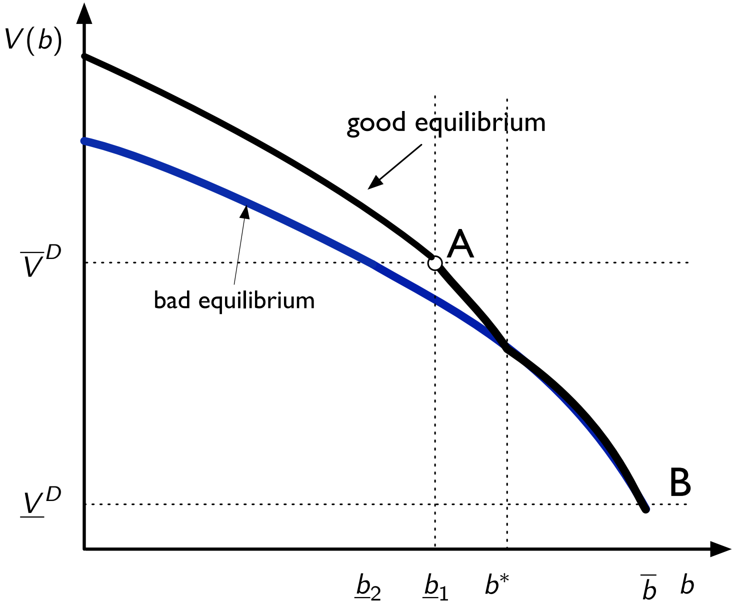

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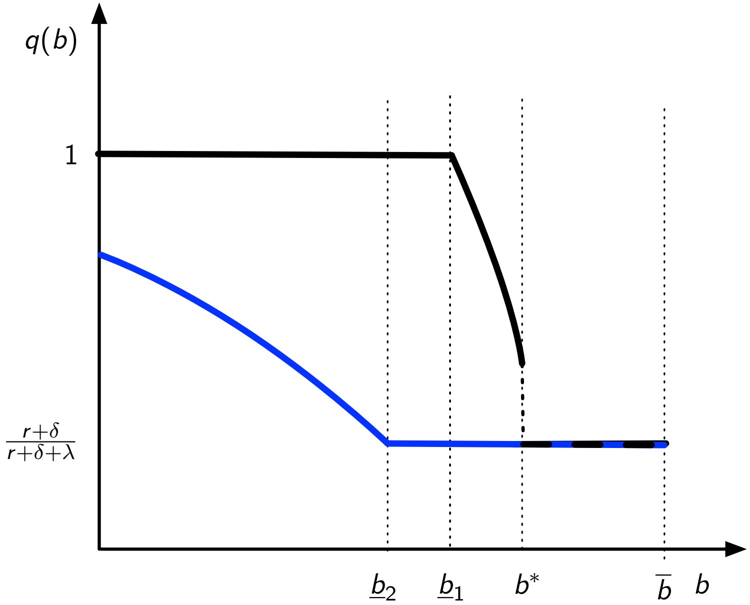

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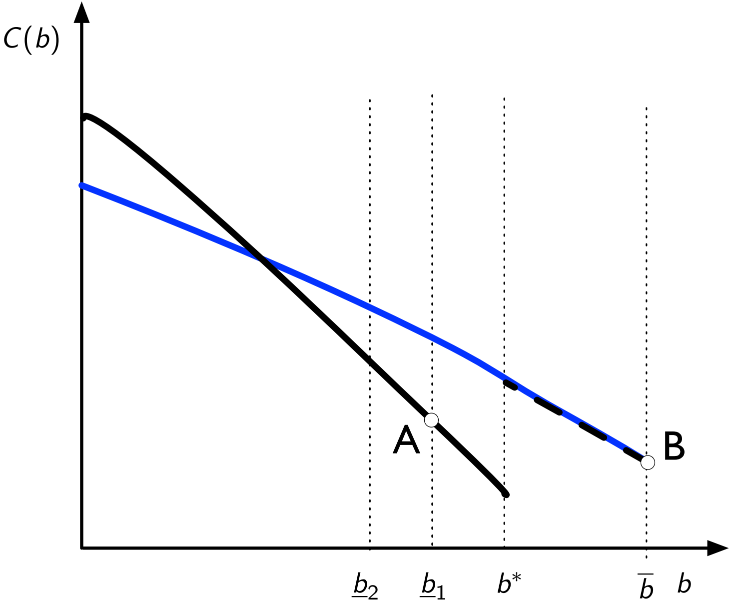

Incentives behind Multiplicity

- Multiplicity due to creditor beliefs about future fiscal policy
  - Prices reflect creditor beliefs
  - Value functions reflect shape of price schedule
- Role of maturity:
  - With one-period debt, future fiscal policy irrelevant
  - With perpetuities, cannot support an interior stationary point (no need to roll over debt at stationary points)
- With endowment shocks same forces at work, but greater incentive to save due to precaution

Policy Implications

- How can an outside institution rule out bad equilibrium?
- Traditional policy: Price floor
  - Kills feedback from budget sets (Calvo)
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- Traditional policy: Price floor
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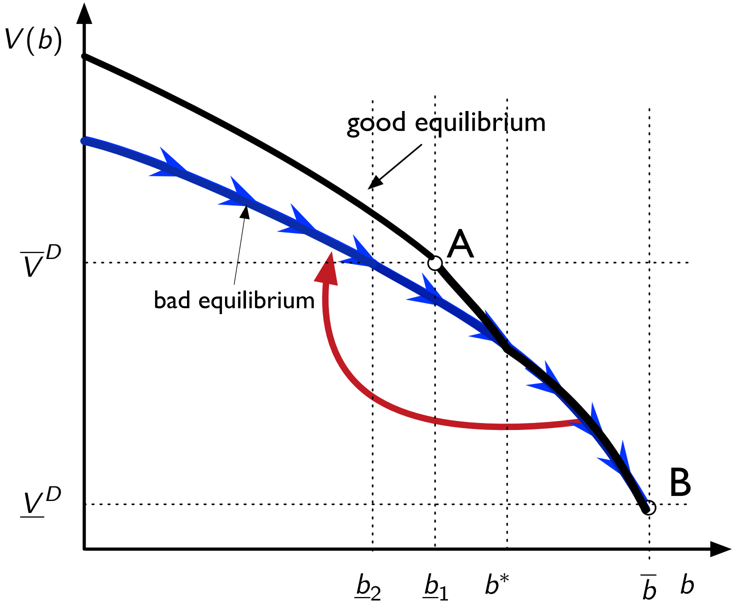
- In our version of EG model, price floor *selects* bad equilibrium
  - Kills incentive to save
  - "Flattens" price schedule
  - Sovereign borrows to limit
  - Requires third-party resources on equilibrium path

Other Policies

- Debt Forgiveness:
  - As long as sovereign relatively impatient, will resume borrowing
  - Does not rule out eventual default

Debt Forgiveness



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  - How to enforce non-market limits on debt?
- Costs to delay:
  - If b too high, unique equilibrium
  - Point emphasized by Lorenzoni-Werning in their framework

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- How to interpret episodes like Draghi's speech?
  - Were debt limits crucial to its success?

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- Even if default is a relatively positive outcome for government
- Long-term debt does not provide same incentive
  - Costs of default "sunk" into original prices
  - Leads to multiple equilibria in canonical EG model
- How do these considerations affect optimal maturity choice?
- Rolling over debt provides correct incentives:
  - What about rollover "risk"?