# CREI Lectures in Macroeconomics: The Economics of Sovereign Debt and Default Part I

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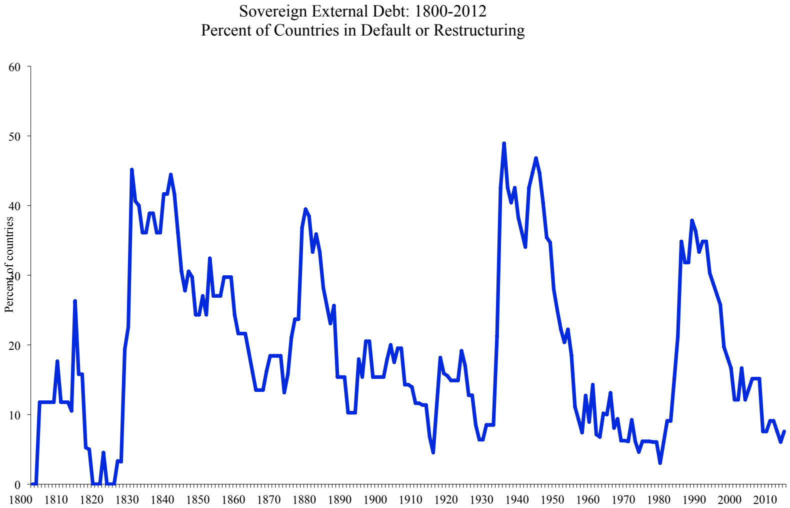
# Introduction




Source: Reinhart and Rogoff

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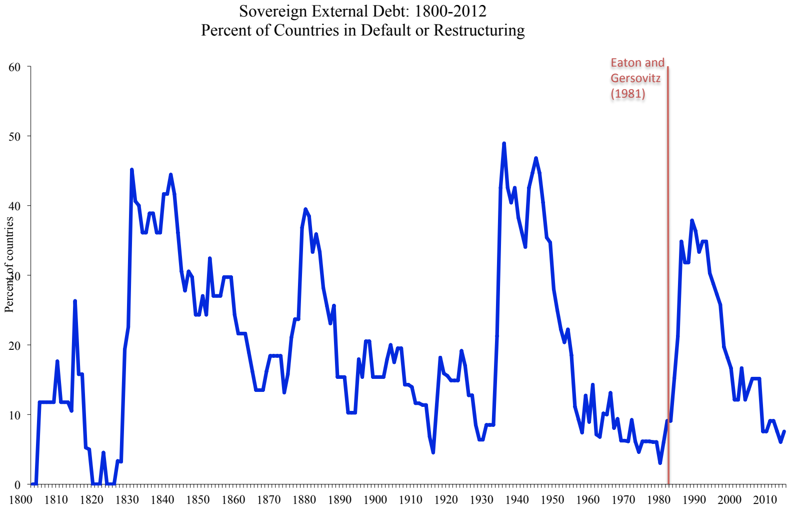


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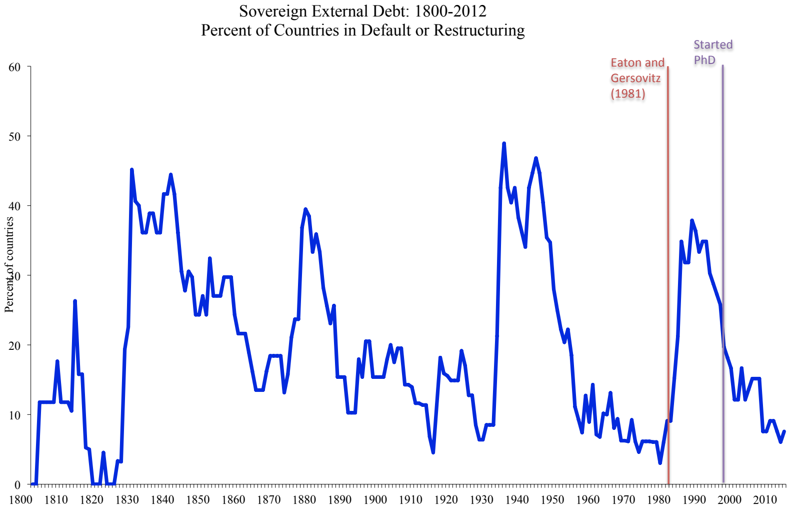


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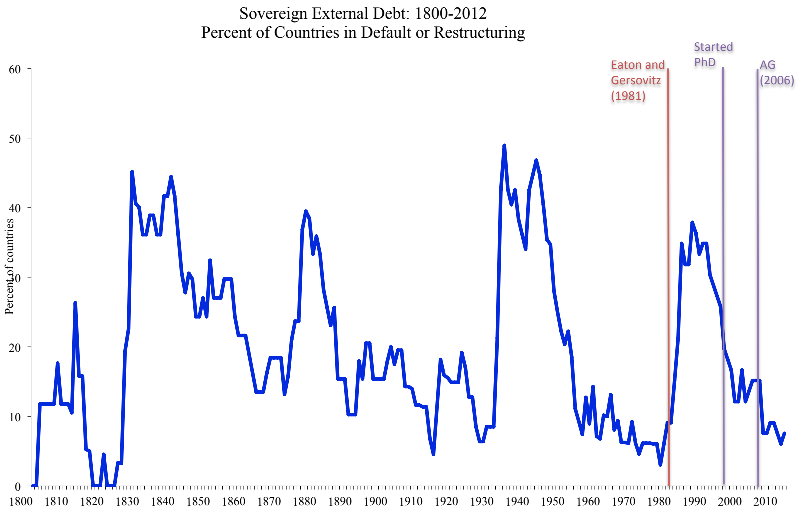


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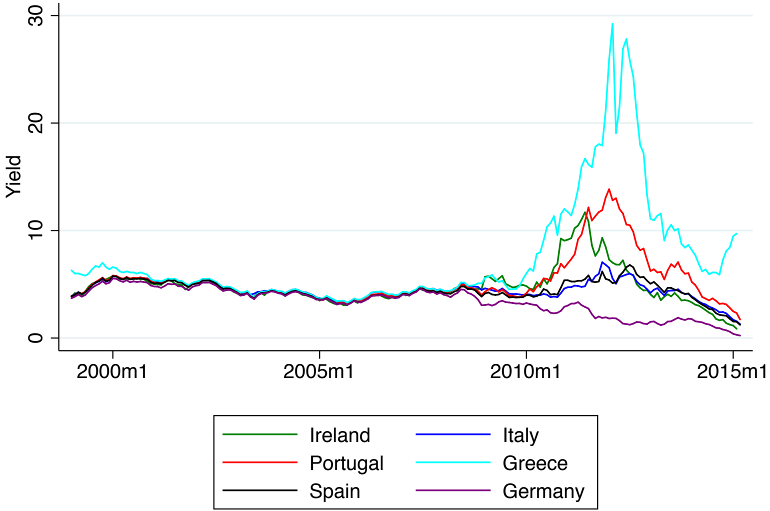

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What are we trying to understand

1. Default
   - Traditional focus of the literature
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3. Equilibrium debt dynamics and maturity choice
4. Role for third-party policies or institutions

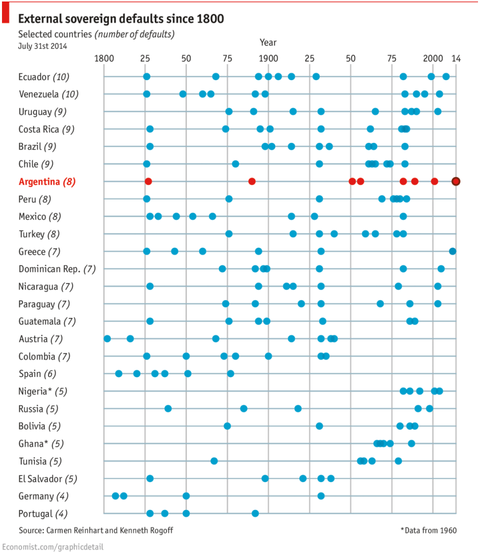
# Motivating Facts

## Major Defaults

- Sturzenegger and Zettlemeyer list 29 countries that defaulted or restructured between 1980 and 1983
- Major defaults in late 1990s/early 2000s
  - Russia - 1998
  - Ecuador - 1999
  - Argentina -2001
  - Uruguay - 2003
- More recent examples
  - Ecuador - 2008
  - Greece - 2012
  - Argentina - 2014
  - Venezuela - ?

# Motivating Facts

## Serial Defaulters



# Motivating Facts

## Why Default?

- Low output
  - Tomz and Wright document 62% of defaults start when output is below trend
  - Average deviation of output is only -1.6%
  - Correlation of output and default status is only -0.08

# Motivating Facts

Why Default?

- External Fundamentals
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## Why Default?

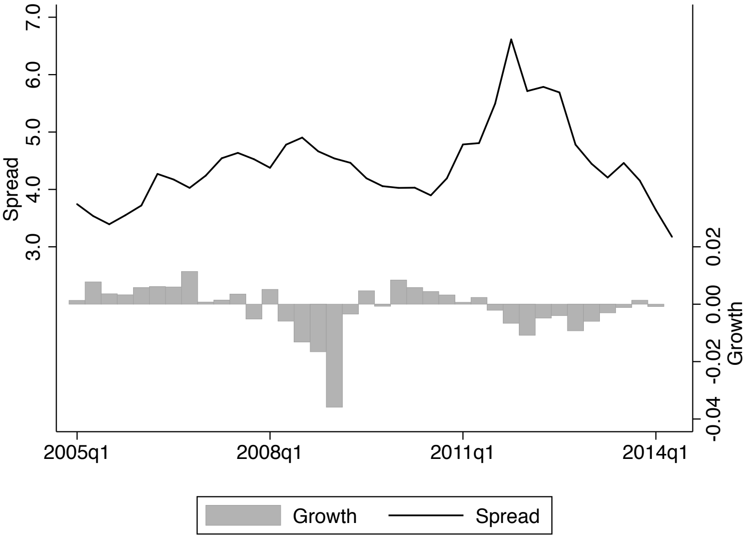
- External Fundamentals
  - Latin American Debt Crisis of 1980s
  - Global Financial Crises/Risk Premia
- Self-fulfilling Runs
  - Mexico 1994/95
  - Europe 2012 ("Whatever it takes...")

# Motivating Facts
## Why Default?

- Political Shocks
  - Ecuador default in 2008
    - Oil prices high
    - President argued foreign debt was "illegitimate" and bondholders "monsters"
    - Contrast with repayment in 2015 when oil prices were low
  - Greece near default in 2015
    - Syriza elected in January 2015
    - Referendum in July 2015 rejects Troika's proposed bailout terms
    - Agreement reached a week later averting default

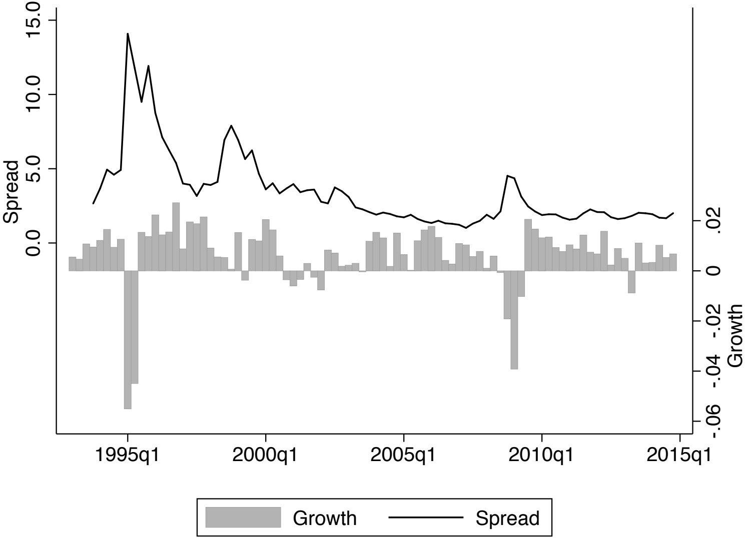
# Motivating Facts

Spreads: Italy



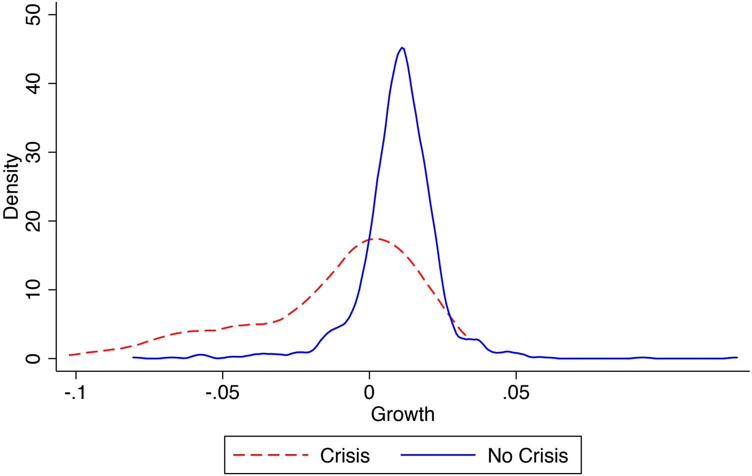
# Motivating Facts

Spreads: Mexico



# Motivating Facts

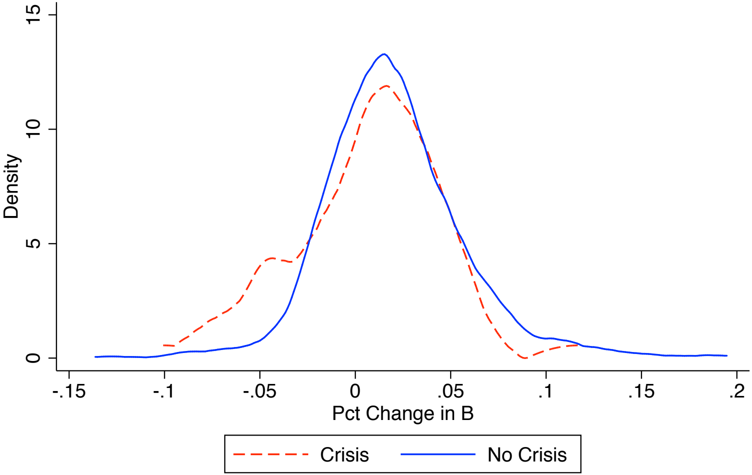
## Spreads and Growth: Emerging Markets




Crisis: Contemporaneous with $\Delta EMBI > 158bp$
Median Growth: $-0.4$ and 1.1, resp

# Motivating Facts

Spreads and Deleveraging



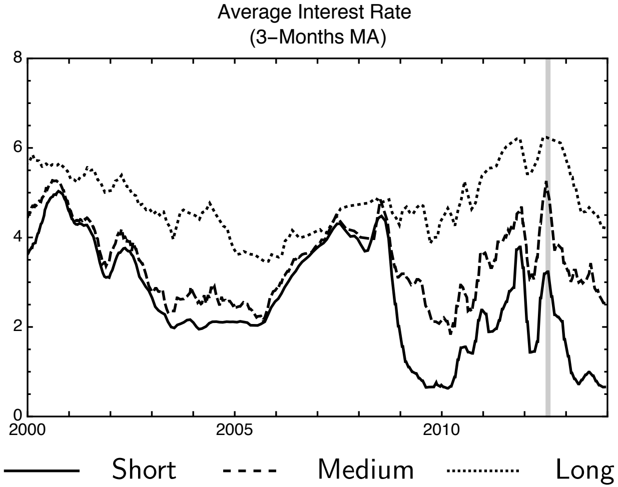
# Motivating Facts

Maturity Choice

- Issuances shorten in crises
- Yield curve flattens or inverts
  - Keep in mind: Secondary market yield curve is not marginal yield

# Motivating Facts

Maturity Choice (Spain)



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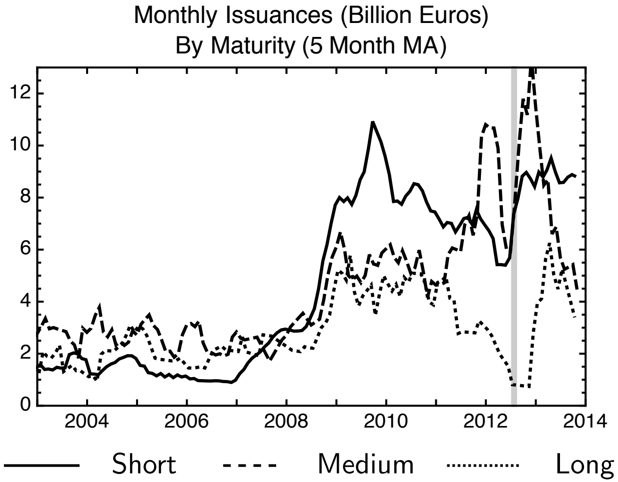
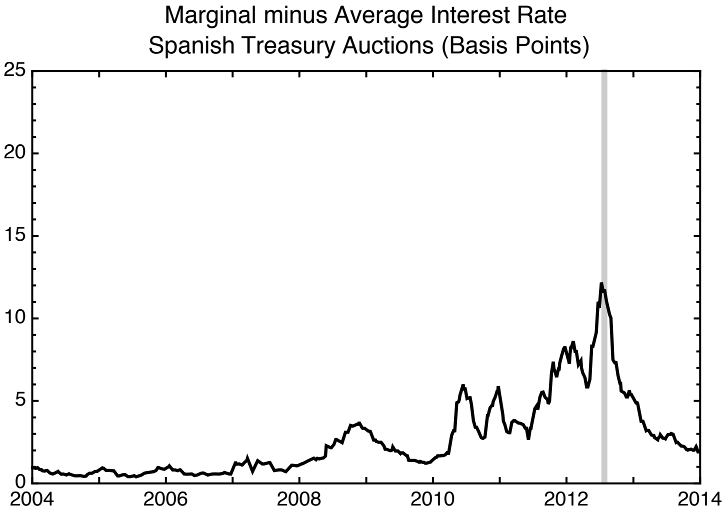


# Motivating Facts

Marginal vs. Average Yields



# Taking Stock

- Defaults and spikes in spreads occur regularly
  - But only mildly correlated with output
  - Plausibly some role for self-fulfilling beliefs
  - Political risk important
- Some evidence that high spreads associated with deleveraging
- Maturity choice shifts during crises

# Road Map

1. Discuss general framework

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2. Analyze one-period bond economy
    - Efficiency and uniqueness
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   - Multiplicity

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   - Efficiency and uniqueness
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3. Long-term bonds
   - Inefficiency
   - Debt dynamics
   - Multiplicity
4. Maturity choice
   - With and without rollover risk

# Framework

Nests Key Variations:

- Complete Markets (Arrow-Debreu, Thomas-Worral, Kehoe-Levine)
- Eaton-Gersovitz and descendants (Arellano, Aguiar-Gopinath, Chatterjee-Eygingur, Hatchondo-Martinez, etc.)
- Cole-Kehoe and descendants (e.g. Aguiar-Chatterjee-Cole-Stangebye)
- As well as the models Manuel and I have used in various papers
  - Aguiar-Amador
  - Aguiar-Amador-Gopinath-Farhi
  - Aguiar-Amador-Hopenhayn-Werning

# Framework

## Basic Environment

- Study a small open economy (SOE) – pins down world risk-free rate
- A single, freely traded good – numeraire
- Benchmark: Time is discrete
  - Some extensions will be easier to discuss in continuous time

# Framework

Notation: Exogenous States

- Denote the exogenous state at time $t$ by $s_t$
  - Endowment
  - Punishments
  - Sunspots
- $s^t = \{s_0, s_1, ..., s_t\}$
- Date zero probability of history $s^t$: $\pi(s^t)$

# Framework

## Government

- A single decision maker: Government or Sovereign
- Not necessarily benevolent
- Benchmark preferences:

$$U(\boldsymbol{c}) = \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t) u(c(s^t))$$
$$= \mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

# Framework

Lenders

- Atomistic – competitive asset markets
- Discount at $R^{-1} = (1 + r)^{-1}$
- Risk Neutral
  - Have explored extensions with risk-averse lenders
- Have full commitment

# Framework

## Asset Markets

1. Complete Markets

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   - Discount bond: Pays one in all states next period

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1. Complete Markets
2. One-period non-contingent bond
   - Discount bond: Pays one in all states next period
3. Random maturity bond
   - Poisson process for maturity: $\lambda$
   - Independent across units: LLN implies fraction $\lambda$ matures each period
   - Non-maturing bonds are identical – "perpetual youth" property
   - Special cases:
     - $\lambda = 1$: One-period bonds
     - $\lambda = 0$: Perpetuities

# Framework

## Asset Markets

4. Arbitrary portfolio of non-contingent bonds
    - Random maturity or time dependent

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   - Random maturity or time dependent
5. Nominal bonds
   - Mention only in passing
   - Interesting to the extent punishment for "default" differs
   - Adds some additional state contingency but brings in additional commitment issues

# Framework

Endowment

- Endowment $y_t = y(s^t)$
- Endowment fluctuations dominate discussion of sovereign default models

# Framework

## Default Payoffs

- How to support repayment is crucial in this class of models
  - Eaton-Gersovitz: Default triggers financial autarky
  - Bulow-Rogoff: Reputation “not enough”
  - Quantitative models: Combination of temporary autarky and direct punishments (endowment loss)

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- Short-hand for this is value of default: $V^D(s)$
  - Incorporates stochastic punishments (endowment loss, political consequences, etc.)
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  - Incorporates stochastic punishments (endowment loss, political consequences, etc.)
  - We will treat as a primitive of environment
  - Our main source of risk

# Framework

## Timing

- Timing of actions within a period important
  - Does an auction occur before or after default decision?
  - Does choice of amount of debt occur before or after auction begins?
  - Can there be more than one auction per period?

# Framework

## Timing

- Canonical "Eaton-Gersovitz" Timing
  1. Exogenous states realized (endowment, default cost, sunspot)
  2. Government decides (commits) to repay or default that period
  3. If repay, decides (commits) how much (face value) new debt to auction that period
  4. Auction occurs
  5. Repayment and consumption

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  5. Repayment and consumption
- Will introduce "Cole-Kehoe" timing later

# Framework

Taking Stock

- What our framework captures:
  - Uninsurable risk and default
  - Limited commitment to repayment and fiscal plans more generally
  - Multiplicity and self-fulfilling crises

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## Taking Stock

- What our framework captures:
  - Uninsurable risk and default
  - Limited commitment to repayment and fiscal plans more generally
  - Multiplicity and self-fulfilling crises
- Some things we are missing:
  - Richer post-default environments (renegotiation, hold outs, haircuts, etc)
  - Information frictions (other than default payoff)
  - Richer political economy frictions (other than default payoffs)
  - Richer domestic economic environment (private agents, externalities)

# Our Approach
## Planning Problems

- To the extent possible, analyze planning problems
- Establish equivalence between competitive equilibrium and a dynamic contract
  - Representative lender as Principal
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# Our Approach

Planning Problems

- To the extent possible, analyze planning problems
- Establish equivalence between competitive equilibrium and a dynamic contract
  - Representative lender as Principal
  - Government as Agent
- Useful to highlight in what sense efficiency holds or fails in competitive equilibria
- Requires "flipping" between primal and dual problems
- Approach taken in Aguiar-Amador-Hopenhayn-Werning

# Two Planning Problems

1. Complete Markets with Limited Commitment
   - Implication of limited commitment: saving
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1. Complete Markets with Limited Commitment
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   - Motivation for saving: Improve insurance
2. One-period non-contingent bonds
   - Modified welfare theorem for the competitive equilibrium
   - Where inefficiencies arise relative to CM benchmark
   - What incompleteness does to equilibrium allocation relative to CM
   - Incentives to save

# Complete Markets Benchmark

## Primal Problem

- Government begins with some initial debt $b$
- Trades contingent assets with risk-neutral lenders
- Cannot commit to contracts
  - If reneges, receives $V^D(s)$ in state $s$
  - Gains from trade: Cheaper to provide $V^D(s)$ within relationship

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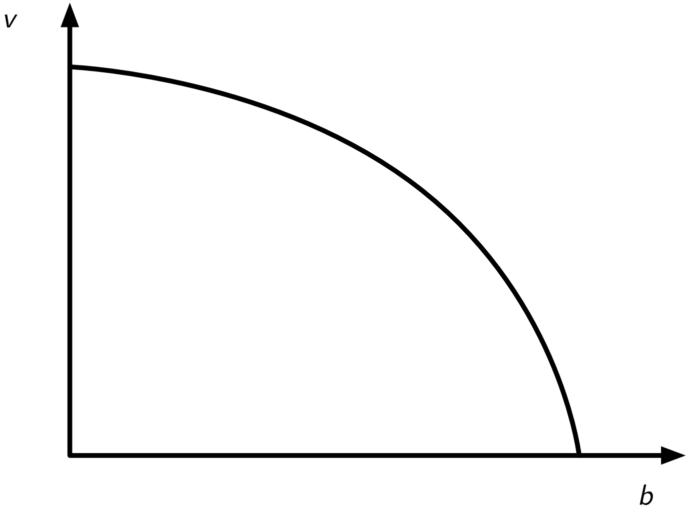
- Appeal to Welfare Theorems and solve a planning problem

# Pareto Frontier



# Complete Markets Benchmark

## Pareto Planning Problem

$$B(s_0, v) = \max_{\boldsymbol{c}} \sum_{t=0}^{\infty} R^{-t} \sum_{s^t} \pi(s^t) \left( y(s^t) - c(s^t) \right)$$

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$$V^D(s_t) \leq \sum_{k=0}^{\infty} \beta^k \sum_{s^{t+k}} \pi(s^{t+k} | s^t) u(c(s^{t+k})) \text{ for all } t, s^t$$

# Complete Markets Benchmark

- Let $\mu_0$ be multiplier on promised utility
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0 = & -R^{-t}\pi(s^t) + \mu_0 \beta^t \pi(s^t) u'(c(s^t)) \\
& + \mu_0 \beta^t \pi(s^t) u'(c(s^t)) \sum_{s^{t-k} \in s^t} \lambda(s^{t-k})
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$$

- Rearranging:

$$
\frac{1}{\mu_0} = R^t \beta^t u'(c(s^t)) \left( 1 + \sum_{s^{t-k} \in s^t} \lambda(s^{t-k}) \right)
$$

# Complete Markets Benchmark

## Backloading

- Suppose $\beta R = 1$:

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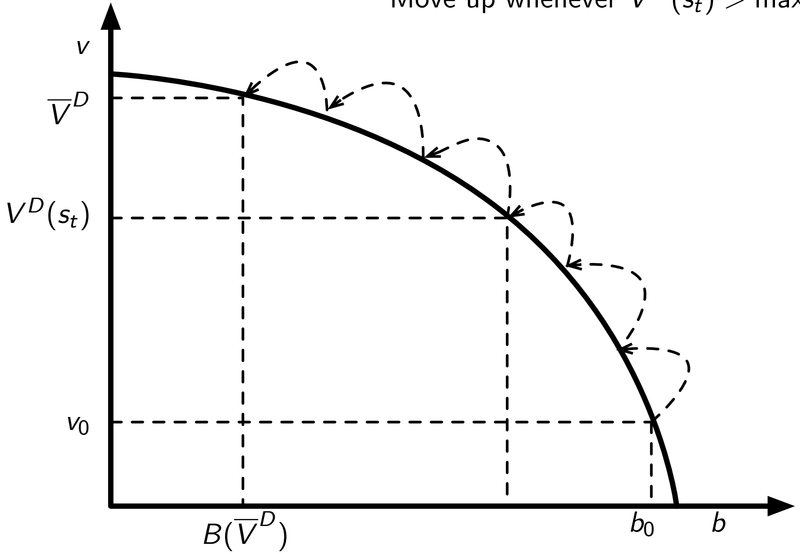
- $\lambda(s^t) \geq 0$
- $\sum \lambda(s^{t-k})$ converges $\Rightarrow \lim \lambda(s^t) \to 0$.
- $c(s^t)$ weakly increases over time and converges to a constant
- Full risk sharing after first realization of $\overline{V}^D = \max_{s \in S} V^D(s)$

Move up whenever $V^D(s_t) > \max_{j<t} V^D(s_j)$



# Complete Markets Benchmark

## Key Implications

- Risk-neutral foreign lenders insuring a risk-averse government
- Limited commitment is the only friction in the model
- Promising consumption in the future relaxes participation constraints along the path
- Extra return to saving: Improves insurance
- No "default" in this environment given complete markets
  - Never exercise outside option $V^D(s)$

# Eaton-Gersovitz

## Incomplete Markets Planning Problem

- One period non-contingent bond
- Canonical Eaton-Gersovitz (Arellano, Aguiar-Gopinath, etc.) model

# Eaton-Gersovitz

## Incomplete Markets Planning Problem

- One period non-contingent bond
- Canonical Eaton-Gersovitz (Arellano, Aguiar-Gopinath, etc.) model
- Recast competitive equilibrium as a constrained planning problem
- Highlight how it contrasts with complete markets planning problem
- Shed light on aspects of the equilibrium

# Eaton-Gersovitz

Recursive Competitive Equilibrium

- One period discount bond: $b$

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- One period discount bond: $b$
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- Equilibrium objects:
  - Price schedule: $q(s, b, b')$
  - Value of repayment: $V^R(s, b)$
  - Default if $V^R(s, b) < V^D(s)$

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  - Default if $V^R(s, b) < V^D(s)$
- EG timing:
  1. $s$
  2. Default or Repay Decision
  3. Choose $b'$
  4. Auction

# Eaton-Gersovitz



- Lender's break-even condition:

$$q(s,b,b') = \begin{cases} R^{-1} \text{ if } b' \leq 0 \\ R^{-1} \sum_{s' \in S} \pi(s'|s) \mathbb{1}_{\{V^R(s,b') \geq V^D(s')\}} \end{cases}$$

- First row: Risk-free rate if NFA$>$0
- Second row: Repayment only if optimal for government

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- First row: Risk-free rate if NFA$>$0
- Second row: Repayment only if optimal for government
- Inherited debt $b$ irrelevant: $q(s, b, b') \to q(s, b')$

# Eaton-Gersovitz

## Recursive Competitive Equilibrium

- Government's problem if Repay:

$$V^R(s,b) = \sup_{c,b'} u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max\langle V^R(s',b'), V^D(s')\rangle$$

subject to:

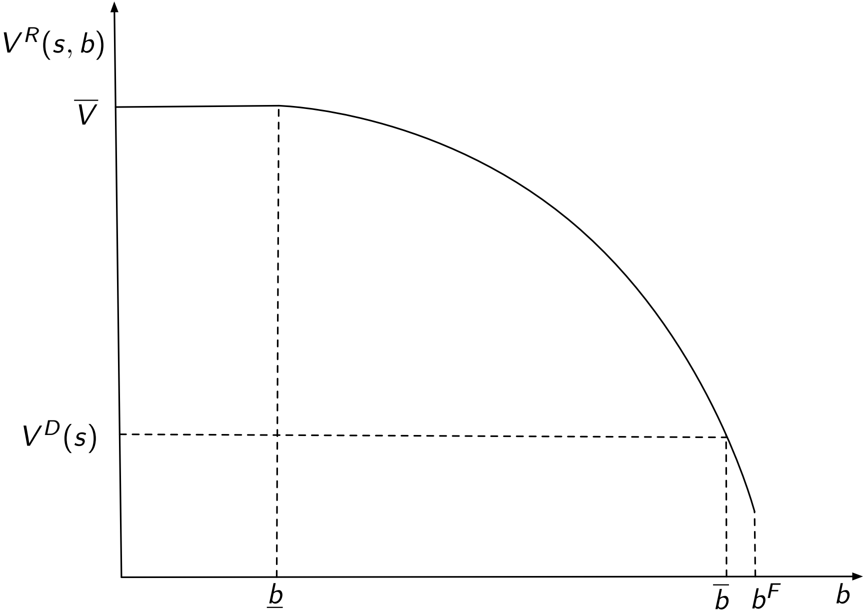
$$c \leq y(s) - b + q(s,b')b'$$

**Definition 1**

An **equilibrium** consists of functions $V^R : S \times \mathbb{R} \to \mathbb{R}$ and $q : S \times R \to [0, R^{-1}]$ such that:

(i) Given $q$, $V^R$ solves government's problem

(ii) Given $V^R$, $q$ satisfies lenders' break-even condition and NPC


$V^R(s,b)$
$\overline{V}$
$V^D(s)$
$\underline{b}$
$\overline{b}$
$b^F$
$b$

# Eaton-Gersovitz

## A Dual Problem

- Show that the competitive equilibrium is solution to a "planning" problem
- Highlight how incompleteness changes the complete-markets planning problem

# Eaton-Gersovitz

## A Dual Problem

- Show that the competitive equilibrium is solution to a "planning" problem
- Highlight how incompleteness changes the complete-markets planning problem
- Additionally:
  - Show equilibrium is fixed point of a contraction operator
  - Existence, uniqueness, and a fast method of computation

# Eaton-Gersovitz

A Dual Problem

- Road Map:
  1. Introduce an operator of a planning problem

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  2. Argue operator is a contraction
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A Dual Problem

- Road Map:
  1. Introduce an operator of a planning problem
  2. Argue operator is a contraction
  3. Show that the inverse of $V^R$ is a fixed point of the operator
  4. Result is that equilibrium is solution to a planning problem
  5. Discuss similarities and differences with complete markets planning problem

# Inverse Value Function

- Start with an equilibrium pair $\{q, V^R\}$ and define the inverse of $V^R$ as $B$:

$$B(s, V^R(s, b)) = b$$

for any $b \leq \overline{b}(s)$

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- Start with an equilibrium pair $\{q, V^R\}$ and define the inverse of $V^R$ as $B$:

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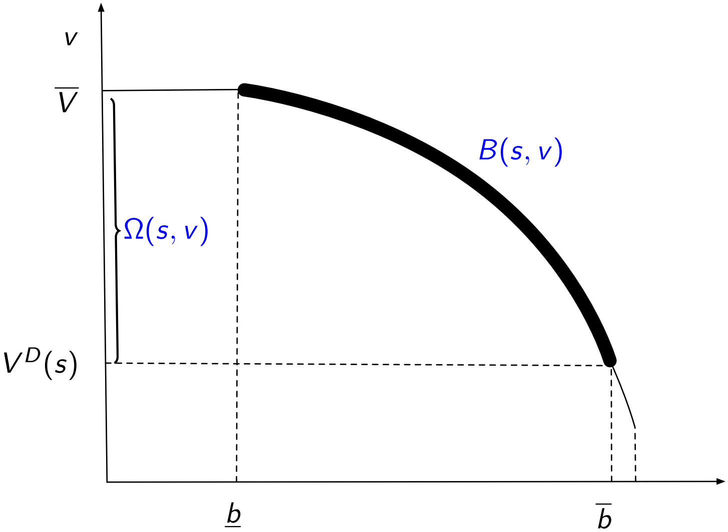
- Given monotonicity, can move between $V^R$ and its inverse

# Inverse Value Function



- Will argue $B$ is fixed point of operator $T$:

$$
\begin{aligned}
[TB](s,v) = \max_{c, v(s'), b'} & \; y(s) - c \\
& + R^{-1} \max\langle 0, b' \rangle \sum_{s' \in S} \pi(s'|s) \mathbb{1}_{\{v(s') \geq V^D(s')\}} \\
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# Eaton-Gersovitz

## A Dual Problem

- Will argue $B$ is fixed point of operator $T$:

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# Eaton-Gersovitz

## A Dual Problem

- Blackwell's Sufficient Conditions: Monotonicity
  - $B$ shows up on the right-hand side only in an inequality constraint
  - Objective must be weakly increasing in $B$

# Eaton-Gersovitz

A Dual Problem

- Blackwell's Sufficient Conditions: Discounting
  - $B + a$ for $a > 0$:

$$b' \leq B(s', v(s')) + a$$

  - Rewrite choice as $\hat{b} \equiv b' - a$:

$$\begin{aligned}[T(B+a)](s,v) \leq \max_{c, v(s'), \hat{b}} \; & y(s) - c \\ & + R^{-1} \max\langle 0, \hat{b} \rangle \sum_{s' \in S} \pi(s'|s) \mathbb{1}_{\{v(s') \geq V^D(s')\}} \\ & + R^{-1} \min\langle 0, \hat{b} \rangle + R^{-1} a\end{aligned}$$

subject to $\hat{b} \leq B(s', v(s'))$

  - Identical problem with an added constant: $\Rightarrow T(B+a) = TB + R^{-1}a$

# Eaton-Gersovitz

## A Dual Problem

- Contraction Mapping Theorem gives us existence and uniqueness of a fixed point
  - Alternative to Auclert-Rognlie

# Eaton-Gersovitz

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# Eaton-Gersovitz

A Dual Problem

- Contraction Mapping Theorem gives us existence and uniqueness of a fixed point
  - Alternative to Auclert-Rognlie
  - Contrast with Passadore-Xandri
- Next key step is to show that the equilibrium can be mapped into this planning problem

# Eaton-Gersovitz

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- Show the inverse of $V^R$ if fixed point

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- Note that objective is weakly increasing in $b'$ and strictly if $b' > 0$

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- To substitute out $v(s')$ need to rule out: $V^R(s', b') \geq V^D(s') > v(s')$

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- But $V^R(s', b') \geq V^D(s') > v(s')$ is never optimal

# Eaton-Gersovitz

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- Substitute out $v(s')$ using $V^R(s', b') = v(s')$ constraint

# Eaton-Gersovitz

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- Lender's break-even condition:

$$
q(s,b') = \begin{cases} R^{-1} \text{ if } b' \leq 0 \\ R^{-1}\sum_{s'} \pi(s'|s)\mathbb{1}_{\{V^R(s',b')\geq V^D(s')\}} \text{ if } b' \geq 0 \end{cases}
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# Eaton-Gersovitz

## A Dual Problem

$$[TB](s, v) = \max_{c,b'} y(s) - c + q(s, b')b'$$

subject to:

$$v \leq u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max\langle V^R(s', b'), V^D(s') \rangle$$

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# Eaton-Gersovitz

## A Dual Problem

- This is the dual of the government's problem:

$$b = \max_{c,b'} y(s) - c + q(s, b')b'$$

subject to

$$v \leq u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max\langle V^R(s', b'), V^D(s')\rangle$$

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- Hence $B(s,v) = b = TB(s,v)$
- Thus an equilibrium pair $\{q, V^R\}$ generates an inverse value that is a fixed point of our operator
- From the Contraction Mapping Theorem existence and uniqueness follows

# The Eaton-Gersovitz Planning Problem

## Two Key Steps

- Choosing continuation values resembles complete markets planning problem
- How can $v(s')$ be a state-by-state choice in an incomplete markets environment?

# The Eaton-Gersovitz Planning Problem

## Two Key Steps

- Choosing continuation values resembles complete markets planning problem
- How can $v(s')$ be a state-by-state choice in an incomplete markets environment?
- Role of the constraint:

$$b' = B(s', v(s')) \text{ for all } s' \text{ such that } v(s') \geq V^D(s')$$

- Restricts freedom to allocate utility across states

# The Eaton-Gersovitz Planning Problem

Two Key Steps

- How to replace $V^R(s', b')$, an equilibrium object, with a choice $v(s')$?

# The Eaton-Gersovitz Planning Problem

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# The Eaton-Gersovitz Planning Problem

Two Key Steps

- How to replace $V^R(s', b')$, an equilibrium object, with a choice $v(s')$?
- How is planning problem independent of $q(s, b')$?
- Both are related:
  - $q(s, b')$ uniquely pinned down by $V^R(s', b')$ – really only one equilibrium object
  - Constraint $b' = B(s', v(s'))$ ensures that $v(s') = V^R(s', b')$ as long as $B$ is the inverse of the equilibrium value

# The EG Planning Problem

## Frictions

- Planning problem suggests cannot find a better allocation that satisfies limited commitment to repay and incompleteness of markets

# The EG Planning Problem

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- Planning problem suggests cannot find a better allocation that satisfies limited commitment to repay and incompleteness of markets
- Two (related) frictions:
  1. Incomplete Markets
     - Cannot insure fluctuations in $y(s)$

# The EG Planning Problem

## Incomplete Markets

$$B(s,v) = \max_{c,v(s'),b'} y(s) - c + R^{-1} b' \sum_{s' \in S} \pi(s'|s) \mathbb{1}_{\{v(s') \geq V^D(s')\}}$$

subject to:

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# The EG Planning Problem

## Frictions

- Planning problem suggests cannot find a better allocation that satisfies limited commitment to repay and incompleteness of markets

- Two (related) frictions:
    1. Incomplete Markets
        - Cannot insure fluctuations in $y(s)$
    2. Deadweight Costs of Default

# The EG Planning Problem

## Costs of Default

$$B(s,v) = \max_{c, v(s'), b'} y(s) - c + R^{-1} b' \sum_{s' \in S} \pi(s'|s) \mathbb{1}_{\{v(s') \geq V^D(s')\}}$$

subject to:

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# The EG Planning Problem

Frictions

- Planning problem suggests cannot find a better allocation that satisfies limited commitment to repay and incompleteness of markets
- Two (related) frictions:
  1. Incomplete Markets
     - Cannot insure fluctuations in $y(s)$
  2. Deadweight Costs of Default
     - Moving from $v(s') = V^D(s') - \epsilon$ to $v(s') = V^D(s')$
     - Second-order costs
     - First-order gain: $b'\pi(s'|s)$
     - Cannot avoid due to IM restriction on $v(s')$

# The Eaton-Gersovitz Planning Problem

Inefficiency

- Both frictions provide an incentive to save
- Lack of insurance generates precautionary saving
  - Close parallel to CM benchmark: More wealth implies better insurance

# The Eaton-Gersovitz Planning Problem



- Both frictions provide an incentive to save
- Lack of insurance generates precautionary saving
  - Close parallel to CM benchmark: More wealth implies better insurance
- Deadweight loss of default also generates saving
  - But how is this internalized in equilibrium when...
    - Prices are actuarially fair
    - Government chooses default because it is optimal

# Taking Stock and Next Steps

- One-period bond model is solution to planning problem
- Equilibrium unique
  - Not true with long-term bonds
  - Will show examples in next lecture
- Has some nice efficiency properties
  - This will be important when we discuss maturity choice
- All of these issues will be related to the incentives to save