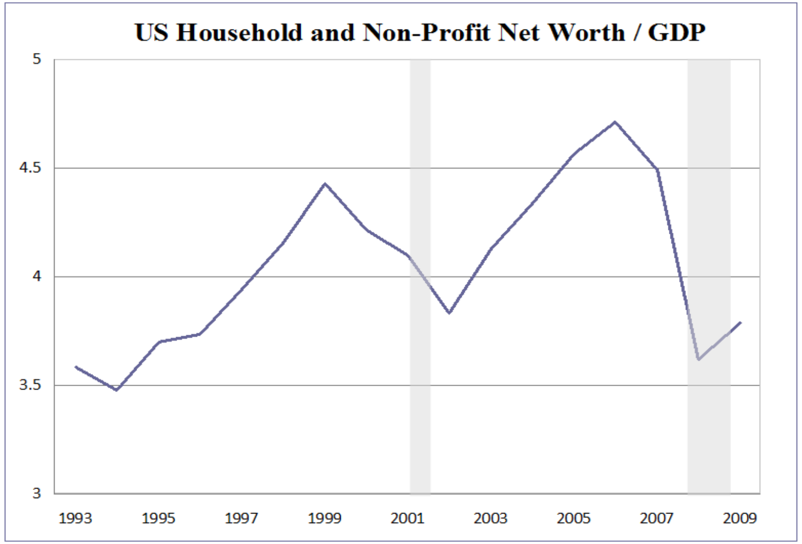
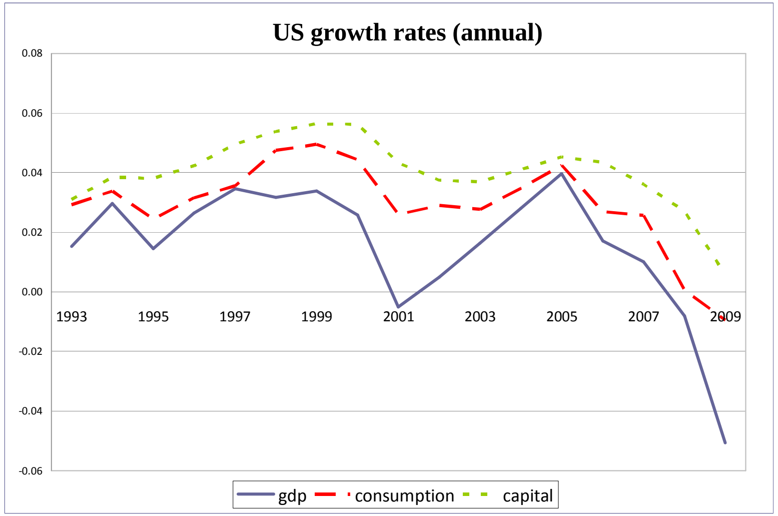
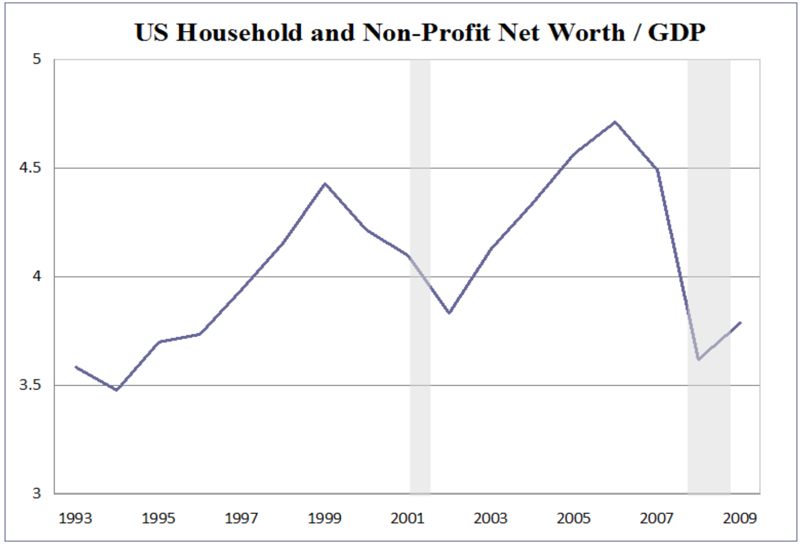
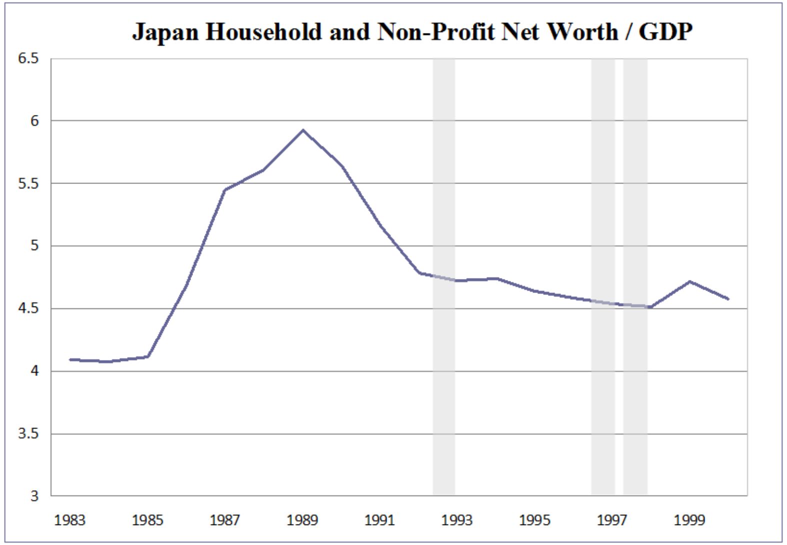

US Household and Non-Profit Net Worth / GDP
5
4.5
4
3.5
3
1993
1995
1997
1999
2001
2003
2005
2007
2009

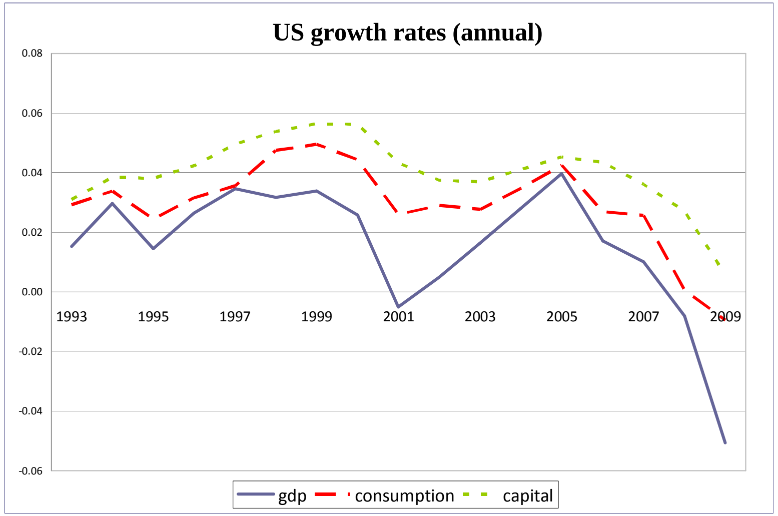

US growth rates (annual)
0.08
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
1993
1995
1997
1999
2001
2003
2005
2007
2009
gdp
consumption
capital



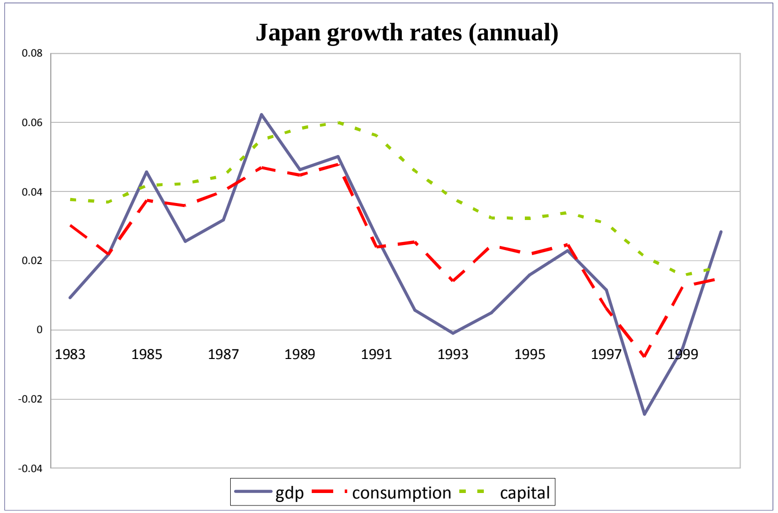
US Household and Non-Profit Net Worth / GDP
5
4.5
4
3.5
3
1993
1995
1997
1999
2001
2003
2005
2007
2009



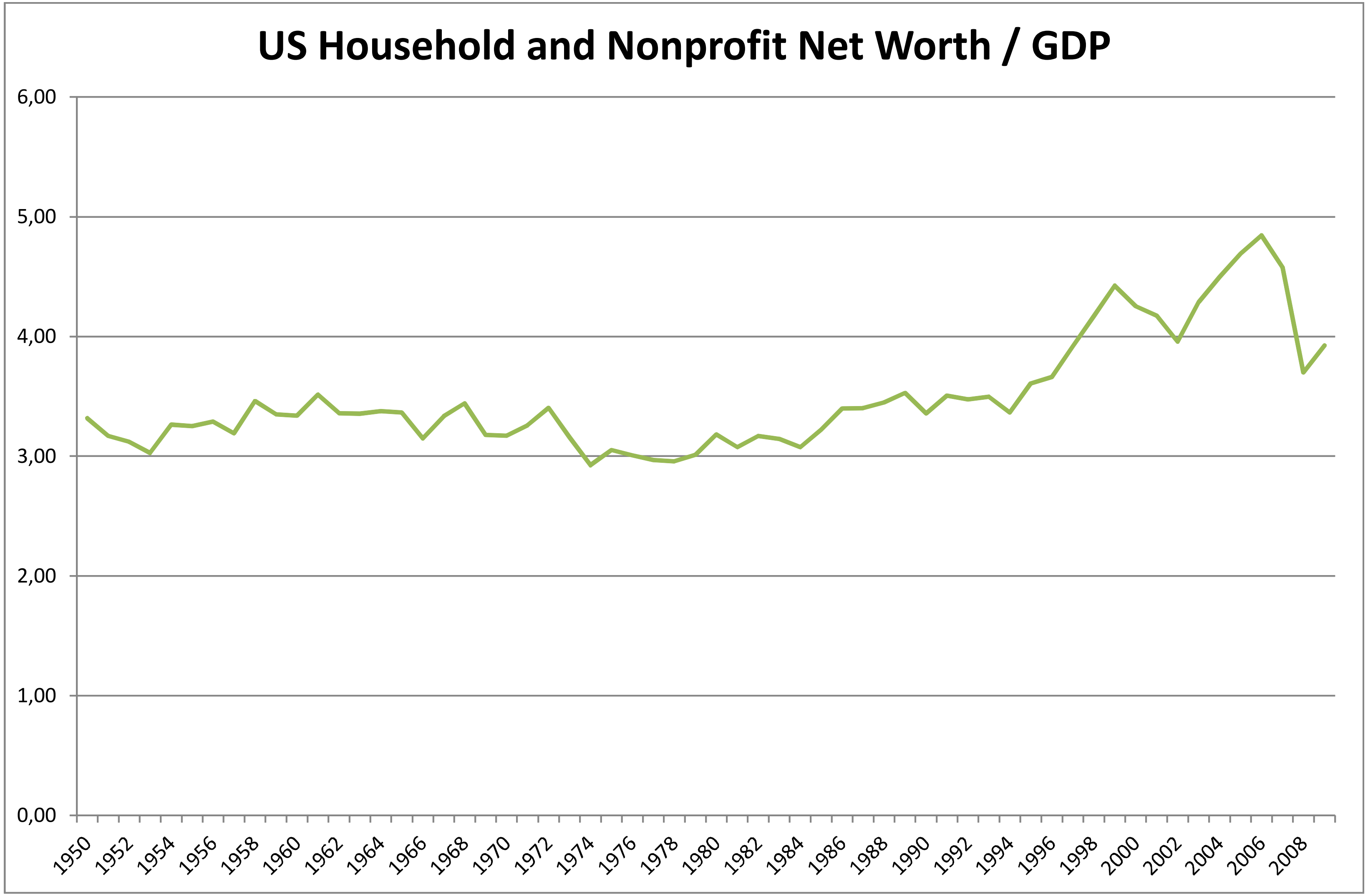
Japan Household and Non-Profit Net Worth / GDP
6.5
6
5.5
5
4.5
4
3.5
3
1983
1985
1987
1989
1991
1993
1995
1997
1999



US growth rates (annual)
0.08
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
1993
1995
1997
1999
2001
2003
2005
2007
2009
gdp
consumption
capital



Japan growth rates (annual)
0.08
0.06
0.04
0.02
0
-0.02
-0.04
1983
1985
1987
1989
1991
1993
1995
1997
1999
gdp
consumption
capital



US Household and Nonprofit Net Worth / GDP
6,00
5,00
4,00
3,00
2,00
1,00
0,00
1950
1952
1954
1956
1958
1960
1962
1964
1966
1968
1970
1972
1974
1976
1978
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008

Calculations:

- Define:
  - $W_t =$ value of all assets assets located in the US, which mostly consist of its capital stock and land
  - $D_t =$ dividend or payoffs that these assets generate
  - $I_t =$ addition to the stock of productive assets
  - $N_t =$addition to the stock of bubbles
  - $r_t =$ required expected return

- Equilibrium requires that:

$$(1 + r_{t+1}) \cdot W_t = E_t \{D_{t+1} + W_{t+1} - I_{t+1} - N_{t+1}\}$$

- Iterating forward:

$$W_t = F_t + B_t$$

where the fundamental and bubble components are:

$$F_t \equiv E_t \left\{ \sum_{\tau=1}^{\infty} \frac{D_{t+\tau+1} - I_{t+\tau+1}}{\prod_{i=1}^{\tau} (1 + r_{t+i})} \right\}$$

$$B_t \equiv E_t \left\{ \lim_{\tau \to \infty} \frac{W_{t+\tau+1}}{\prod_{i=1}^{\tau} (1 + r_{t+i})} - \sum_{\tau=1}^{\infty} \frac{N_{t+\tau+1}}{\prod_{i=1}^{\tau} (1 + r_{t+i})} \right\}$$

How to compute this value?

1. Measure the cash-flows that US productive assets generate as capital income, net of taxes and investment.
2. Compute the expected present discounted value of these cash-flows by following Robert J. Shiller (2005) in making two assumptions:
   (a) The expected return, r_{t+i}, is constant for all time horizons i; and well approximated by the average real return on wealth over the 1950-2010 period; and
   (b) Out-of-sample cash-flows grow at a constant rate - given by the historical average of their real growth rate - and we resort to perfect foresight for within-sample cash-flows.

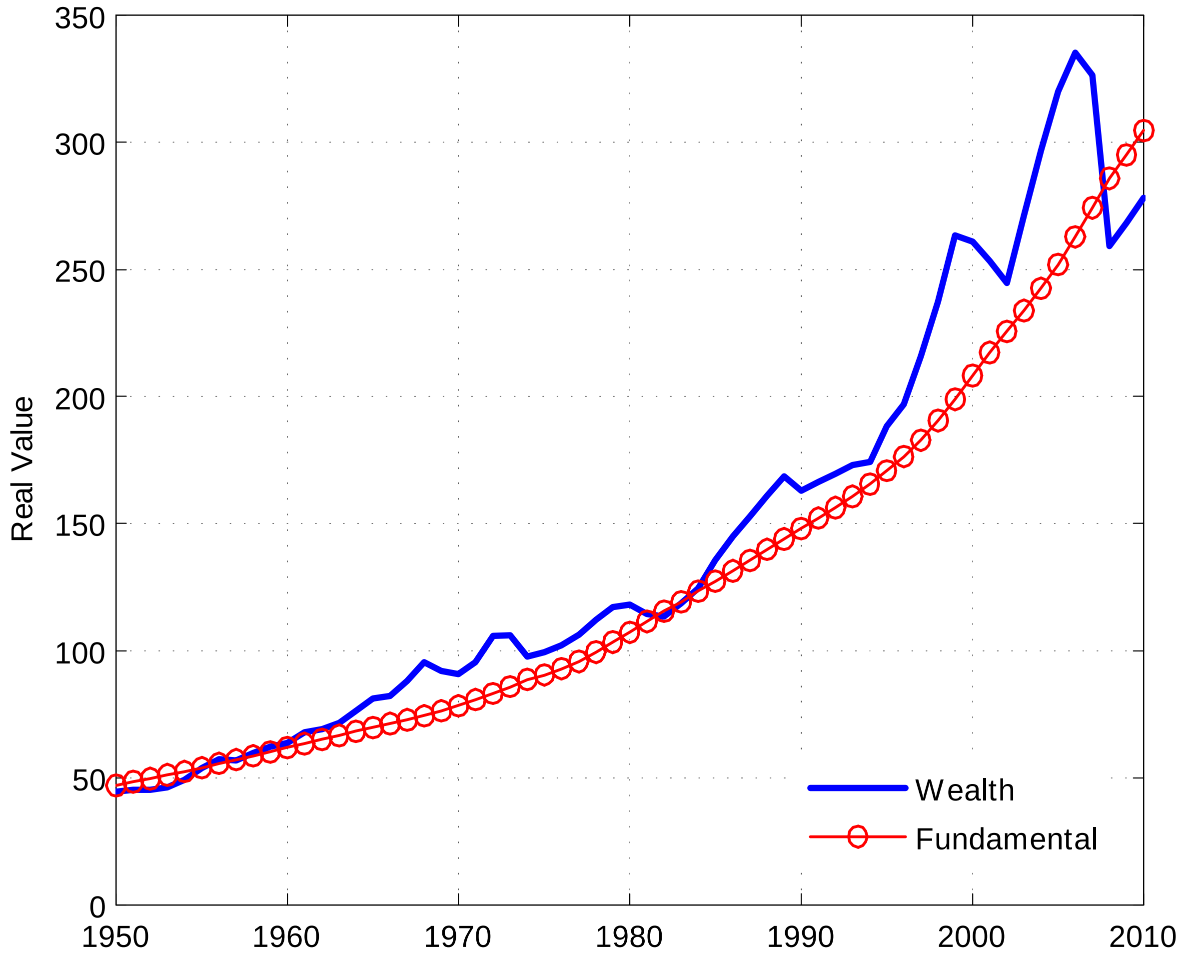


Figure 1: Real value of U.S. wealth and its fundamental, 1960:2010.

Two facts are immediately apparent:

1. Up until the early 1990s wealth has remained remarkably close to its fundamental.

2. The two boom-and-bust episodes of the last two decades constitute unprecedented deviations from the fundamental.

How do we detect bubbles in the data?

1. The best approach is to perform simple NVP calculations, as above. See Leroy (2004) for an excellent application to the dotcom bubble.

2. (GE approach) Test directly whether the conditions for existance are satisfied. Later we will look at Abel et al. (1989) who follow this approach.

3. (PE approach) There are some alternative tests based on PE analysis that we describe next

Variance bound tests

1. Start with general model: $P_t = \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^i \cdot E_t\left(d_{t+i}\right) + \lim_{i\to\infty} \left(\frac{1}{1+r}\right)^i \cdot P_{t+i}$

2. Null hypothesis: $P_t = \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^i \cdot E_t\left(d_{t+i}\right)$

3. Define: $P_t^* = \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^i \cdot d_{t+i}$

4. Then: $P_t^* = \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^i \cdot \left[E_t\left(d_{t+i}\right) + \varepsilon_{t+i}\right] = P_t + \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^i \cdot \varepsilon_{t+i}$

5. Under some conditions: $V\left(P_t^*\right) = V\left(P_t\right) + \varphi \cdot V\left(\varepsilon_t\right) \geq V\left(P_t\right)$ with $\varphi = \frac{1/(1+r)^2}{1 - 1/(1+r)^2}$

6. Problems:
   (a) Implementation problems:
      i. We do not have an infinite sequence for d_{t+i}.
      ii. This is a joint test of bubbles and dividend process.
   (b) Conceptual problems:
      i. It is possible to find theoretical assumptions about dividends such that the test fails without bubbles, or it works with bubbles

Other tests:

1. *West's two-step test:* Estimates the dividend process and the FOC separately, and tests cross-equation restrictions. On top of serious implementation issues, it can only detect some class of bubbles.

2. *Integration/co-integration tests*: Based on the equation

$$E_t (B_{t+1}) = (1 + r) \cdot B_t$$

   But are based on the incorrect assumption that bubbles cannnot be created.

3. *Intrinsic bubbles*: Froot and Obstfeld (1991) test for bubbles of this type:

$$B_t = c \cdot d_t^{\lambda}$$

   They find strong evidence in favor of such bubbles in the US stock market.