Bubbles and Private Liquidity*

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Abstract

We study an economy with limited enforcement of financial contracts where the punishment for defaulting agents is the loss of future borrowing privileges. Bulow and Rogoff (1989) have shown that this "soft punishment" is not sufficient to sustain any level of borrowing in an economy with a positive interest rate and no growth. In contrast, this paper shows that borrowing becomes self-sustaining in a competitive equilibrium, if the interest rate is lower than the growth rate of the economy. We apply this result to an economy with scarce liquidity, where liquid assets earn a liquidity premium and pay a rate of return lower than the economy growth rate. Comparing the roles of public and private liquid liabilities, we show that under limited enforcement and in the absence of lump-sum taxation, public liquidity provision in the form of fiat money is equivalent to private liquidity provision in the form of self-enforcing private debt contracts, both in steady-state, and when considering the transitional dynamics. Finally, we show, how in the presence of informational asymmetries, private liquidity provision in the form of credit lines may serve to screen those market participants with high liquidity needs, and hence leads to a Pareto-improvement over public liquidity provision. These results lead to novel insights regarding the coexistence and respective roles of public and private liquidity in competitive markets.

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1 Introduction

In this paper, we study self-enforced borrowing in economies with limited exclusion from financial markets, a la Bulow-Rogoff (1989). For such economies, we show that the possibility of borrowing sustained by a reputation for repayment, depends on the level of the interest-rate relative to the economy-wide growth rate. We then apply this result to compare the effects of private liquidity provision through the circulation of debt contracts with public liquidity provision in the form of outside money.

The literature on borrowing with limited enforcement appears divided in two major strands. On the one hand, we have the negative result of Bulow and Rogoff (1989) that show in a partial equilibrium setting that if, upon default, agents are excluded from borrowing but are allowed to save, then no positive amount of debt can be sustained in equilibrium. A second strand of literature, starting with Kehoe and Levine (1993) and Kocherlakota (1996), has considered the outright exclusion from asset markets as a punishment after default. When this harsher type of punishment is available, a certain amount of debt can be sustained in equilibrium. Based on this, Alvarez and Jermann (2000) build a theory of endogenous borrowing constraints.[1]

This paper provides a partial reconciliation of these approaches. We use Bulow and Rogoff's notion of limited punishment to study economies where positive debt can be sustained in equilibrium. In particular, we show that in economies, where the equilibrium interest rate is lower than the economy-wide growth rate, Bulow and Rogoff's impossibility result no longer holds and positive amounts of debt can be sustained by the threat of losing borrowing privileges.[2] In equilibrium, the value of borrowing privileges is determined by an implicit flow of transfers to the issuer of liabilities, which provides a form of private seignorage. In Bulow and Rogoff, this flow of transfers is negative for any non-zero level of debt, so the punishment is not effective. However, in an economy with interest rates below the growth rate, this flow of transfers is positive and lending can be sustained in equilibrium. We refer to this flow of transfers as the *lending franchise value*.

[1] Yet another punishment mechanism for default is discussed by Lustig (2001). He considers collateralizable and non-collateralizable assets, and assumes that in case of a default, only collateralizable assets can be seized, but a consumer in default is not prevented from further full participation in credit markets. Hence, no reputational mechanisms are at play.

[2] An implicit assumption in Bulow and Rogoff (1989), that the interest rate exceeded the growth rate, ruled out our case, hence the difference in results.

We apply this result to economies in which some agents are willing to accept a low enough rate of return on highly liquid liabilities. In particular, our result suggests that agents who issue highly liquid liabilities (such as banks) can sustain their debt only by reputation, and do not need to hold collateral on the asset side of their balance sheet. To illustrate this idea, we consider a classic example of a currency and credit economy in which consumers use different forms of liquidity to smoothe their consumption profile over time. For such an environment, we compare equilibria with fiat money with equilibria with self-enforced private debt. Our no-default condition implies that in order to be sustainable in equilibrium, private lending has to be a bubble. In the private debt environment, the maximum liquidity provision is achieved, when the interest rate equals the growth rate of the economy. The equilibrium allocations are then identical to the ones that arise, if households are simply allowed to trade a fiat money.

This equivalence is a general property of a competitive market with private liquidity and limited market exclusion: Comparing a public liquidity environment, in which the government has no power to raise lump sum taxes to finance a more efficient deflationary policy with an environment, in which the private sector has no power to extort payment from a borrower in default, we show that the allocations of any equilibrium in the public liquidity environment (starting from an arbitrary distribution of fiat money) can be replicated by equilibrium allocations in a private liquidity equilibrium (with some endogenously determined set of borrowing limits), and vice versa. In the absence of enforcement powers and under complete information regarding the households' endowment streams, the sets of equilibrium allocations implementable with public liquidity is hence identical to the one implementable with private liquidity.

We then relax the assumption of complete information about consumer types and endowments, and instead consider an environment, in which this information is private. Here, we show that public and private liquidity have quite different properties: in contrast with public liquidity, private liquidity in the form of credit lines can be used to screen those consumers who have most of an immediate liquidity need. The use of private liquidity then leads to a redistribution of resources towards those who have the highest liquidity needs, relative to an equilibrium with public liquidity provision that does not allow such screening. From an ex ante perspective (before uncertainty about types is resolved), such a redistribution provides insurance and is hence welfare-improving.

Finally, we analyze the equivalent of hyperinflations in our setting, i.e. equilibria in which an adverse shift in market expectations about future credit market conditions becomes self-fulfilling and leads to a collapse in private liquidity provision. These equilibria have the features of a "real liquidity trap" and provide a possible explanation for a collapse of intermediation driven by expectations and associated to a long-lasting reduction of the real interest rate. As the deposit base is contracting over time, the willingness to provide loans diminishes, since the punishment in case of a strategic default is weakening. This further undermines the confidence in the borrowers, generating a self-sustaining collapse in the provision of liquidity services. As we converge towards the low private liquidity equilibrium, the interest rate drops, reflecting the scarcity of liquidity supply.

We want to emphasize several implications of our analysis. First, our model links the coexistence of public and private liquidity, i.e. different components of the monetary base, to contract enforcement issues and to the government's ability to raise lump sum taxes. If neither is given, the two forms of liquidity provision are equivalent, and may actually coexist as perfect substitutes - we conjecture that their respective use in transactions could be pinned down by technological considerations, as well as a more concrete modelling of contract enforcement costs. Second, our model suggests that if the provision of private credit can freely adjust to the credit market conditions (provided that it is self-enforcing), then in the absence of nominal frictions, a monetary authority has little control over the real interest rate, which in equilibrium is tightly linked to the economy-wide growth rate. Controling the supply of private credit restores a central bank's ability to conduct an active monetary policy, however, such controls necessarily lead to welfare losses.[3] Related to the previous comments, it follows that the view that banks as "liquidity multipliers", who expand an existing quantity of base liquidity by issuing deposits that are only partially backed by reserves, can only be upheld in an environment, where (i) the government can raise lump sum taxes, and (ii) the private sector has some power to enforce repayments.[4]

This paper is connected to a large literature on liquidity provision in competitive markets with limited enforcement. We already commented on the relation to Bulow and Rogoff (1989), and alternative models of competitive lending by Kehoe and Levine (1993) and Alvarez and Jermann (2000). In this context, our paper relates debt market equilibria to equilibria with rational asset

[3] This theme is the subject of Sargent and Wallace (1982), and is further explored below.

[4] This idea appears in models with multiple layers of liquidity. See for example the discussion in Kiyotaki and Moore (2001).

pricing bubbles. The existence of liquidity bubbles in models with borrowing constraints *à la* Bewley (1980) was recognized in Scheinkman and Weiss (1986) and in Woodford (1990). This paper shows that these borrowing constraints can be derived endogenously from a theory of self-enforcing debt contracts. Moreover, the condition for the existence of bubbles is closely connected to the condition that makes debt contracts self-enforcing. In reference to the literature on bubbles in Bewley models, Santos and Woodford (1997) make the distinction between rational asset pricing bubbles and the mispricing inherent in the liquidity premium on highly liquid assets, which is interpreted as the relaxation of some transactional constraint. Here, we show that these two interpretations are inherently connected, i.e. the no-default condition that private liquidity must satisfy is directly reinterpretable as a condition that self-enforcing private liquidity has to be a bubble. We think our theory can be extended to models in which more liquid and less liquid assets coexist and in which some institutions specialize in the issuance and management of highly liquid liabilities. We believe that our central result will carry over to that type of environment: the rate of growth of these liabilities and their rate of return will determine the value of the bank franchise (i.e. the present value of a banks' seigniorage). The value of the bank franchise will provide reputational collateral for this type of borrowing. A Bewley model with bubbles makes this conceptual point in a simple framework, in which we do not need to make additional assumptions to justify the transactional advantages of banks' liabilities.

Private note circulation and liquidity provision has also been studied in the context of matching environments. In particular, Cavalcanti and Wallace (1999) study a random matching model with indivisible outside money where the use of bank debit is sustained by reputation. Cavalcanti, Erosa and Temzelides (1998) study private bank debit in a random matching model with explicit note-clearing. In contrast, our competitive framework abstracts from explicitly modelling the circulation and clearing mechanisms of private IOUs and shows that the loss of borrowing privileges can be sufficient to sustain private liquidity even in the absence of market frictions. Our framework leads to a simple characterization of the no-default condition, and also allows us to address transitional dynamics and imperfect information, which turn out to be crucial for the welfare comparison of inside and outside liquidity. We emphasize that our model is relevant for any kind of liability that is backed only by the issuing privilege, and whose supply grows at a rate higher than their rate of return, not only to privately issued bank-notes. Our primary interest is to understand the theory of self-enforcing private liabilities in a decentralized market, without modelling the specific exchange frictions that separate private bills from checking accounts or from certificates of deposit.

Finally, this paper is related to a number of recent papers that have studied the creation and allocation of liquidity in economies with scarce collateral.[5] In particular, Holmstrom and Tirole (1998) and Woodford (1990) argue that government taxation may substitute for commitment in the provision of private liquidity; here we show that in the absence of lump sum taxation and contract enforcement powers, the two forms of liquidity are actually equivalent.

In section 2, we introduce our set-up and derive some general results on the sustainability of debt by reputation in a deterministic setting. We revisit Bulow and Rogoff, and highlight the no-debt-bubble condition, under which the theorem holds. We then provide a partial converse that states general conditions under which private debt is self-sustaining in equilibrium. In section 3, we study a simple credit and currency economy in which private liquidity is sustained by reputation, and we compare public and private liquidity provision. In section 4, we establish our general equivalence result. We then illustrate the difference between public and private liquidity under incomplete information. Finally, we apply our result to study non-stationary equilibria with a collapse in private lending.

2 Reputations and bubbles

Consider an infinite-horizon, discrete-time exchange economy populated by a continuum of infinitely-lived consumers. There is one non-storable consumption good. There are J different types of consumers, characterized by their endowment sequences $\left\{y_t^j\right\}_{t=0}^{\infty}$ of the consumption good. There is a unit mass of each consumer type. Consumer preferences over consumption sequences $\{c_t\}_{t=0}^{\infty}$ are represented by the utility function

$$\sum_{t=0}^{\infty} \beta^t u(c_t).$$

A consumer's net financial position a_t evolves according to the flow of funds constraint

$$c_t + q_t a_{t+1} = y_t^j + a_t \tag{1}$$

[5] By economies with scarce collateral we mean economies in which not all future income flows can be credibly promised by agents to back their financial commitments.

where q_t is the price of period $(t+1)$ consumption in terms of period t consumption. Define $p_t = q_1 q_2 ... q_{t-1}$, the price at date 0 of a unit of consumption at date t.

In a world where consumers are able to fully commit to repayments, debt contracts enable consumers to smooth idiosyncratic endowment fluctuations over time. Here we introduce imperfections in the enforcement of credit contracts: in particular, we assume that a consumer may default on any of his past financial commitments. If he chooses to do so, the fact that he defaulted becomes public information, as is the history at time t of a consumer's net financial positions, $\{a_\tau\}_{\tau=1}^{t}$. In addition, we assume for the time being that each consumer's type, and hence his endowment sequence, is public information. However, due to the inalienability of human capital, creditors have access only to his current financial wealth, but not to human wealth, i.e. his future stream of endowments $\left\{y_\tau^j\right\}_{\tau=t}^{\infty}$. Furthermore, creditors are unable to exclude a consumer in default from *lending* in the future.

The only reason why a consumer could possibly keep a negative financial position $a_t < 0$ and not default is that he will loose his reputation for repayment, and therefore the ability to borrow in the future. The simple punishment mechanism we study is the following: every period, a consumer effectively faces a borrowing constraint ϕ_t^j. From the individual's point of view, these borrowing limits are taken as exogenous, however, in equilibrium, the are required to be self-enforcing, i.e. satisfy a no-default constraint. If a consumer fails to repay at any point in time, i.e. renegotiates on his current financial obligations, he loses all his current financial assets and loses the ability to hold a negative financial position in the future, i.e. his borrowing constraint is set to zero from that period onward.

De facto, the description of our bankruptcy procedure restricts strategies with respect to bilateral lending in such a way, that individual actions are contingent only on publicly observed events, i.e. the individual asset positions, and past renegociation decisions, but not on private bilateral lending histories. The restriction to public strategies allows us to abstract from this more explicit lending game, and hence enables us to carry out our analysis within a Walrasian set-up with borrowing constraints, since all liabilities issued by agents who did not default in the past are perfect substitutes, as long as the issuer stays within his borrowing limit. Thus, we can safely maintain a price-taking assumption. However, as we shall briefly discuss below, we believe that our restriction to Walrasian equilibrium with borrowing constraints can also be motivated as a subgame-perfect equilibrium in a more general game that explicitly considers the exchange and circulation of bilateral IOU's in this economy, and then defines strategies in terms of individual rules for bilateral acceptance and repayment. Hence, our notion of a competitive equilibrium with self-enforced borrowing satisfies sequential rationality.

For a given sequence of borrowing constraints $\Phi_t^j = \{\phi_\tau^j\}_{\tau=t+1}^{\infty}$, the period t optimization problem of a type j consumer who never defaults then takes the form

$$\begin{aligned} V_t(a_t, \Phi_t^j) &= \max_{\{c_\tau\}_{\tau=t}^{\infty}} \sum_{\tau=t}^{\infty} \beta^\tau u(c_\tau) \\ s.t. \quad & c_\tau + q_\tau a_{\tau+1} \leq y_\tau^j + a_\tau \\ & a_\tau \geq \phi_\tau^j \\ & a_t \text{ given} \end{aligned} \tag{2}$$

A consumer's utility after default at time t is derived from the problem:

$$\begin{aligned} V_t^d &= \max_{\{c_\tau\}_{\tau=t}^{\infty}} \sum_{\tau=t}^{\infty} \beta^\tau u(c_\tau) \\ s.t. \quad & c_\tau + q_\tau a_{\tau+1} \leq y_\tau^j + a_\tau \\ & a_\tau \geq 0 \\ & a_t = 0. \end{aligned} \tag{3}$$

Using the solutions to (2) and (3) it is to straightforward to show that the following condition is necessary and sufficient to ensure that consumers never default [6]:

$$V_t(\phi_t^j, \Phi_t^j) \geq V_t^d \text{ for all } t. \tag{4}$$

We next define our notion of competitive equilibrium with endogenous issue of private debt:

Definition 1 *A competitive equilibrium with self-enforcing private debt is defined by a sequence* $\{c_t^j, a_{t+1}^j\}_{t=0}^{\infty}$ *for each consumer type* j*, a sequence of borrowing limits* $\{\phi_{t+1}^j\}_{t=0}^{\infty}$ *for each consumer type, and a price sequence* $\{q_t\}_{t=0}^{\infty}$*, such that:*

[6] This claim is proved in Lemma 2 in the appendix.

(i) $\{c_t^j, a_{t+1}^j\}_{t=0}^{\infty}$ *solves (2) at* $T = 0$*, and the value functions defined in (2) and (3) satisfy the no-default condition (4);*

(ii) goods and asset markets clear in every period, i.e.

$$\sum_j c_t^j = \sum_j y_t^j \text{ and } \sum_j a_{t+1}^j = 0 \text{ for all } t.$$

Definition 2 *A competitive equilibrium with self-enforcing private debt is said to satisfy* maximum credit expansion*, if and only if*

$$V_t(\phi_t^j, \Phi_t^j) = V_t^d,$$

whenever the constraint $a_t^j \geq \phi_t^j$ *is binding for consumer* j *at time* $t-1$ *(i.e. whenever* $u'(c_t^i) > \beta u'(c_{t+1}^i)$*).*

While our definition of a competitive equilibrium with self-enforcing private debt takes the sequences of borrowing constraints as given, the definition of maximum credit expansion leads to an endogenous determination of these borrowing constraints, which is similar to Alvarez and Jermann (2000). The rationale behind maximum credit expansion is that if that condition is violated, an individual lender has no reason to refuse credit at the going interest rate to an agent who can credibly commit to repay, and the latter can always offer a slightly larger interest rate to attract funds. In an equilibrium with maximum credit expansion, the no-default condition then leads to an endogenous determination of the sequence of borrowing constraints Φ_0^j through market forces. Formally, the borrowing limits are treated in the same way as prices, i.e. individuals optimize taking prices and the sequence of borrowing constraints as given; and prices and borrowing limits adjust to satify market-clearing and maximum credit expansion. The idea that the competitive market adjusts the provision of liquidity is the view taken historically by advocates of the real bills doctrine. Alternatively, one might argue that a central monetary authority should partially control the provision of private liquidity, for instance through reserve requirements on banks; a policy supported by advocates of the quantity theory. Our discussion on the comparison of public and private liquidity provision will touch, among others, on the effects of different ways of determining the borrowing constraints.

We conclude our description of the environment by arguing that the individual decision rules for the exchange and acceptance of private debt are indeed optimal. We make this claim based on the following observations:

(i) No household ever has an incentive to lend to a consumer who previously defaulted: if a consumer who previously defaulted is expected not to be able to borrow at any point in the future, then he has no incentive to repay any debt, and hence the zero borrowing limit on a consumer in default is self-enforcing.

(ii) No household ever has an incentive to refuse to borrow from a consumer who previously defaulted at or below the market interest rate.

(iii) If the borrowing limits satisfy maximum credit expansion, no household ever has an incentive to lend to a consumer in excess of the consumer's current borrowing limit; since he expects that any borrowing beyond the current limit would lead to a default (Note that the motivation of maximum credit expansion above was already couched in terms of individual incentives to provide funds).

In a competitive equilibrium with maximum credit expansion, households therefore have no incentives to deviate from the individual acceptance rules for bilateral credit that are specified above in our description of the market environment.

2.1 Bulow and Rogoff revisited

Having introduced our general environment, we first derive a negative result that connects our analysis to the result of Bulow and Rogoff (1989). In particular, we show that if the value of a consumer's debt grows at a rate slower than the interest rate, then the consumer must find it optimal to default on any debt at some point in time.

Proposition 1 *(Bulow-Rogoff) If a consumer's asset holdings satisfy*

$$\lim_{t\to\infty} p_t a_t = 0 \tag{5}$$

then $a_t < 0$ at some t implies that there exists a period in which the no-default condition is violated.

Proof. Consider a consumer whose solution to the no-default problem (2) leads to consumption path $\{c_t\}$ and a corresponding sequence of financial positions $\{a_t\}$. Suppose that for some T',

$a_{T'} < 0$. Then there exists $T \geq T'$ such that $p_T a_T < -\epsilon$ and $p_{T+j}a_{T+j} > -\epsilon$ for all $j > 0$, for some $\epsilon > 0$. If the consumer defaults at time T and chooses the alternative sequence of financial positions $\{\tilde{a}_t\}_{t=T}^{\infty}$, where

$$p_t\tilde{a}_t = p_t a_t + \varepsilon > 0,$$

he can finance his original consumption sequence from $T+1$ onward, since for $t \geq T+1$

$$\begin{aligned} p_t\tilde{c}_t &= p_t y_t - p_{t+1}\left(a_{t+1} + \frac{\epsilon}{p_{t+1}}\right) + p_t\left(a_t + \frac{\epsilon}{p_t}\right) = \\ &= p_t y_t - p_{t+1}a_{t+1} + p_t a_t = p_t c_t \end{aligned}$$

At T, his consumption is

$$\begin{aligned} p_T\tilde{c}_T &= p_T y_T - p_{T+1}\left(a_{T+1} + \frac{\epsilon}{p_{T+1}}\right) \\ &= p_T y_T - p_{T+1}a_{T+1} - \varepsilon \\ &> p_T y_T - p_{T+1}a_{T+1} + p_T a_T = c_T, \end{aligned}$$

since $p_T a_T < -\epsilon$, and hence it is profitable to default in period T. ■

This proposition is a generalization of Bulow and Rogoff (1989) to environments where the value of debt is not necessarily bounded by the net present value of income, or where the net present value of income (at the prices p_t) is infinite. In order to have repayment sustained by reputation the total debt $-a_t$ must asymptotically grow at a rate at least as high as the interest rate. Essentially a no-bubble condition makes debt non-sustainable from the reputational point of view: if debt has to be sustained by the threat of losing borrowing privileges, it has to be a bubble.

It is important to notice that if the condition

$$\lim_{t\to\infty} p_t a_t = 0$$

is violated, this does not necessarily imply that the transversality condition for the consumer problem is violated. The transversality condition is

$$\lim_{t\to\infty} \beta^t u'(c_t) a_t = 0$$

and, as discussed in Kocherlakota (1990) and in Santos and Woodford (1997), in the presence of borrowing constraints the latter can hold independently of the former because the Euler equation does not hold at all points in time as an equality. In the next section, we will study equilibria in which the consumer problem is well defined and the transversality condition is satisfied, even though condition (5) is violated. We call (5) a "no debt-bubble" condition.

Our next proposition provides a partial converse to proposition 1, stating a condition under which the no-default condition is satisfied for every type in every period.

Proposition 2 *Suppose that the private debt limits for a type j consumer satisfy*

$$\left|\phi_t^j\right| p_t \leq \left|\phi_{t+1}^j\right| p_{t+1} \tag{6}$$

for all $t \geq 0$. Then the no-default condition is satisfied for type j in every period.

Proof. Suppose that a consumer of type j considers default in some period T, holding a net financial position a_T. Let $\left\{c_{T+s}^{j,D}\right\}_{s=0}^{\infty}$ denote his optimal consumption path if he decides to default. For $t \geq T$, the associated asset path $\left\{a_{t+1}^{j,D}\right\}_{t=T}^{\infty}$ satisfies

$$p_{t+1} a_{t+1}^{j,D} = p_t \left[y_t^j - c_t^{j,D}\right] + p_t a_t^{j,D}$$

and iterating this solution, along with the fact that, in default, $a_T^{j,D} = 0$, yields

$$p_{t+1} a_{t+1}^{j,D} = \sum_{s=T}^{t} p_s \left[y_s^j - c_s^{j,D}\right]$$

Consider now an agent, who, without defaulting, intends to replicate the same consumption plan, starting from some asset level a_{T-1}. His asset profile $\left\{a_{t+1}^{j,N}\right\}_{t=T}^{\infty}$ has to follow

$$\begin{aligned} p_{t+1} a_{t+1}^{j,N} &= \sum_{s=T}^{t} p_s \left[y_s^j - c_s^{j,D}\right] + p_T a_T \\ &= p_{t+1} a_{t+1}^{j,D} + p_T a_T \end{aligned}$$

which, to be feasible, has to satisfy the debt constraint $a_{t+1}^{j,N} \geq \phi_{t+1}^j$. Since post default, $a_{t+1}^{j,D} \geq 0$, this condition is satisfied, if

$$a_T p_T \geq p_{t+1} \phi_{t+1}^j$$

for all $t \geq T$. But that is guaranteed by $\phi_t^j p_t \geq \phi_{t+s}^j p_{t+s}$, for all $t \geq 0$, $s \geq 1$. ■

Proposition 2 provides conditions under which a consumer never has an incentive to default. Note that (6) is not necessary to sustain lending in equilibrium: In periods, in which $a^{j,D}_{T+s} > 0$, i.e. a consumer's borrowing constraint after a default is not binding, (6) can be relaxed to account for the consumer's post-default lending. (6) states that the present value of a consumer's borrowing capacity has to grow from each period to the next, hence, it is a condition that explicitly allows a consumer to run a Ponzi game: Starting from any initial level of borrowing ϕ^j_t, the consumer can roll over his debt and finance interest payments by new debt issues, without ever repaying. (6) implies that (5) is violated, but not the other way round: An optimal no-default consumption path will lead to the debt constraint being binding infinitely often, which in turn implies that $\liminf_{t\to\infty} a_t p_t$ is bounded away from 0. We conclude our discussion by remarking that we can construct sequences of borrowing constraints satisfying (6) such that the no-default consumption plan violates (5), yet the consumer will find it optimal to default at some point - it thus follows that a violation of (5) is necessary, but not sufficient for the sustainability of borrowing in equilibrium. We defer a brief formal discussion of this statement to the next section, where we can easily illustrate it in the context of a simple example.

Finally, we discuss equilibria with maximum credit expansion:

Proposition 3 *An equilibrium with self-enforced borrowing satisfies maximum credit expansion if and only if for all j there exists $\widehat{\phi}_j \leq 0$ such that (i) $\phi^j_t p_t = \widehat{\phi}_j$ in any period in which the borrowing constraint is binding, and (ii) asset positions in all periods satisfy $a^j_t p_t \geq \widehat{\phi}_j$.*

Proof. The proof proceeds in three steps: Define $\{t_k\}_{k=0}^{\infty}$ as the sequence of periods in which $a^j_t = \phi^j_t$, i.e. the no-default borrowing constraints are binding. We first show that $\left|\phi^j_{t_k}\right| p_{t_k} \leq \left|\phi^j_{t_{k+1}}\right| p_{t_{k+1}}$. We then show that if $\left|\phi^j_t\right| p_t = \left|\phi^j_{t+1}\right| p_{t+1}$ for all t, then sets of feasible consumption profiles after default and without default are identical whenever the borrowing limit is binding, and hence $V_{t_k}(\phi^j_{t_k}, \Phi^j_{t_k}) = V^d_{t_k}$ for all k. It then follows that if ever $\left|\phi^j_{t_k}\right| p_{t_k} < \left|\phi^j_{t_{k+1}}\right| p_{t_{k+1}}$, $V_{t_k}(\phi^j_{t_k}, \Phi^j_{t_k}) > V^d_{t_k}$, and maximum credit expansion must be violated.

(*i*) $\left|\phi^j_{t_k}\right| p_{t_k} \leq \left|\phi^j_{t_{k+1}}\right| p_{t_{k+1}}$ for all k in an equilibrium with maximum credit expansion: Suppose for some type j, $V^j_{t_k}(\phi^j_{t_k}, \Phi^j_{t_k}) = V^{j,d}_{t_k}$ for all k, but $\left|\phi^j_{t_k}\right| p_{t_k} > \left|\phi^j_{t_{k+1}}\right| p_{t_{k+1}}$ for some k. Then the no-default condition is violated time $t' = \arg\min_\tau \{p_\tau a_\tau : t \leq \tau < s\}$. If type j defaults at t', he can follow the same consumption path of a non-defaulting agent up to time s. This is feasible and allows him to end up with positive wealth $\hat{a}_s = \left(a_{t'} p_{t'} - \phi_s^j p_s\right)/p_s > 0$.Therefore, his utility from time s on will be $W > V_s^{j,d} = V_s^j(\phi_s^j, \Phi_s^j)$ and hence the no-default constraint is violated.

Incidentally, this step also proves that in an equilibrium with maximum credit expansion $p_\tau a_\tau \geq p_s \phi_s^j$ for any τ such that $t \leq \tau \leq s$.

(*ii*) Next, we show that if $\left|\phi_t^j\right| p_t = \left|\phi_{t+1}^j\right| p_{t+1}$ for all t, for any k, defaulting in period t_k leads to the same consumption possibility set as never defaulting: Consider an arbitrary consumption profile $\left\{c_{t_k+\tau}^j\right\}_{\tau=0}^{\infty}$. This consumption profile is feasible after defaulting in period t_k, if and only if

$$p_{t_k+\tau} a_{t_k+\tau}^{j,d} = \sum_{s=0}^{\tau-1} p_{t_k+\tau}\left(y_{t_k+\tau}^j - c_{t_k+\tau}^j\right) \geq 0 \text{ for all } \tau \geq 1.$$

Without ever defaulting, the same consumption profile is feasible, if and only if

$$p_{t_k+\tau} a_{t_k+\tau}^j = \sum_{s=0}^{\tau-1} p_{t_k+\tau}\left(y_{t_k+\tau}^j - c_{t_k+\tau}^j\right) + p_{t_k}\phi_{t_k}^j \geq p_{t_k+\tau}\phi_{t_k+\tau}^j \text{ for all } \tau \geq 1.$$

Hence, the sets of feasible consumption profiles after a default in period t_k and without ever defaulting are identical, whenever (6) holds with equality between any pair of periods. But then, $V_{t_k}(\phi_{t_k}^j, \Phi_{t_k}^j) = V_{t_k}^d$, and the equilibrium satisfies maximum credit expansion.

(*iii*) If an equilibrium with self-enforced borrowing satisfies (6) everywhere, but $\left|\phi_t^j\right| p_t < \left|\phi_{t+1}^j\right| p_{t+1}$ for some t, then $V_{t_k}(\phi_{t_k}^j, \Phi_{t_k}^j) > V_{t_k}^d$, where t_k is the last period before t in which the borrowing constraint binds: Let $\left\{c_{t_k+\tau}^{j,d}\right\}_{\tau=0}^{\infty}$ denote the optimal consumption path after a default at time t_k. $\left\{c_{t_k+\tau}^{j,d}\right\}_{\tau=0}^{\infty}$ satisfies

$$\sum_{s=0}^{\tau-1} p_{t_k+\tau}\left(y_{t_k+\tau}^j - c_{t_k+\tau}^j\right) \geq 0 \text{ for all } \tau \geq 1.$$

Financing the same consumption profile without default requires asset positions

$$a_{t_k+\tau}^{j,n} p_{t_k+\tau} = \sum_{s=0}^{\tau-1} p_{t_k+\tau}\left(y_{t_k+\tau}^j - c_{t_k+\tau}^j\right) + p_{t_k}\phi_{t_k}^j \geq p_{t_k}\phi_{t_k}^j$$

But then, without ever defaulting, a consumer can raise his consumption by $\left(\left|\phi_{t_{k+1}}^j\right| p_{t_{k+1}} - \left|\phi_{t_k}^j\right| p_{t_k}\right)/p_{t_{k+1}}$ in period t_{k+1} without changing his consumption in any other period. Thus, $V_{t_k}(\phi_{t_k}^j, \Phi_{t_k}^j) > V_{t_k}^d$, and maximum credit expansion must be violated. ■

Proposition 3 discusses maximum credit expansion. It shows that the consumption paths that are implementable under maximum credit expansion with no default are equivalent to the consumption plan that arises, if a consumer defaults in the period after his borrowing constraint was binding. Proposition 3 has three major consequences: First, if an equilibrium has borrowing limits such that (i) the post-default consumption path is always implementable without default, and (ii) the borrowing capacity is constant in net present value between any pair of periods, in which the borrowing limit is binding, then all borrowing and lending is self-enforced, and furthermore, the equilibrium is one of maximum credit expansion (i.e. the above conditions are both necessary and sufficient). Second, a consumer's consumption allocation is invariant in the sequence of borrowing limits and initial asset level. More precisely, for each initial net financial position, there exists a different sequence of self-sustaining borrowing limits that also satisfy maximum credit expansion, and implement the same consumption plan. As we shall see in the next section, it follows as a consequence of this latter property that consumption allocations are invariant to the distribution of borrowing privileges across the population in any steady-state equilibrium with maximum credit expansion. Finally, maximum credit expansion endogenously determines borrowing constraints in such a way that condition (6) is equivalent to a violation of condition (5). In particular, we have the following converse of proposition 1:

Corollary 1 *In an equilibrium with maximum credit expansion, if a consumer's net asset position satisfies* $\liminf_{t\to\infty} a_t p_t < 0$*, then the consumer's no default constraint is never violated.*

We conclude this section with a brief remark about the arguments that led to propositions 1-3: In all three propositions, our main argument rested on the comparison of budget sets, i.e. the sets of feasible consumption plans with or without a default. In other words, our analysis mostly relied on the wealth effects and implicit redistribution of funds from lenders to borrowers. We define this implicit transfer as the lending franchise value. This lending franchise can be valued by $p_{t+1}\phi^j_{t+1} - p_t\phi^j_t$, i.e. as the implicit loosening of the budget constraint that follows from an increase in the borrowing capacity in any given period.

3 A simple model with scarce liquidity

In this section, we consider a simple example of a deterministic exchange economy with fluctuating endowments, and we study equilibria in which non-collateralized private debt is sustained by the threat of losing borrowing privileges. There are two types of consumers, odd and even. Odd consumers are endowed with the income sequence $\{y_t^o\}$ and even consumers with $\{y_t^e\}$, where:

$$\begin{aligned}\{y_t^o\} &= \{\theta_0\overline{e}, \theta_1\underline{e}, \theta_2\overline{e}, ...\} \\ \{y_t^e\} &= \{\theta_0\underline{e}, \theta_1\overline{e}, \theta_2\underline{e}, ...\}\end{aligned}$$

The productivity parameter θ_t grows at the exogenously given rate g, with $\theta_0 = 1$. We assume that $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$ to derive a balanced growth path, and assume

$$Q^* \equiv \beta\,(1+g)^{1-\sigma} < 1 \tag{7}$$

to guarantee that the consumers' utility is bounded in equilibrium. We further assume

$$\overline{e} > \frac{1}{(Q^*)^{\frac{1}{\sigma}}}\underline{e} \tag{8}$$

which implies that the fluctuations in the endowment sequence are sufficiently important.

It is useful to recall that in the absence of enforcement frictions, the market equilibrium of this economy achieves a Pareto-optimal allocation in which the consumption of all consumers grows at a rate g and in which the interest rate is given by

$$q^* = \frac{1}{1+r^*} = \beta\,(1+g)^{-\sigma}\,.$$

Because of assumption (7), we have

$$1 + r^* > 1 + g.$$

This type of economy with deterministic fluctuations is analyzed in detail by Woodford (1990) and by Ljungqvist and Sargent (2000, Chap. 18), and it is a useful example of an economy where liquid assets are traded to smooth idiosyncratic income fluctuations and where a liquidity premium arises in equilibrium. Here we will show that in this economy, liquid assets can be privately issued and sustained by reputation.

3.1 Steady states with private liquidity

We now consider steady-state equilibria in which consumption levels and asset positions oscillate in odd and even periods between two trend levels that grow at the same rate g. To be precise, we study a steady-state in which the asset position of even (odd) consumers evolves according to

$$-\theta_t d \quad \theta_{t+1} a \quad -\theta_{t+2} d \quad ...$$

with t even (odd). The sequence of borrowing constraints $\{\phi_t\}$ is identical for all consumers, and is given by $\phi_t = -d\theta_t$. Consumers issue debt every other period, when they have a low income realization and repay it the next period. Essentially, consumers have access to a credit line that grows at the same rate as the economy, as long as they haven't defaulted in the past. If they ever default, their credit line is set to zero. Consumption levels evolve according to

$$\underline{c}\theta_t \quad \overline{c}\theta_{t+1} \quad \underline{c}\theta_{t+2} \quad ...$$

with t even (odd) for even (odd) consumers. The intertemporal price is constant and equal to q.

For a steady-state value of q, $(\underline{c}, \overline{c}, a)$ are found as the steady-state solution to the following recursive problem, defined for an initial asset position w:

$$V(w) = \max_{\underline{c}, \overline{c}, a, w'} \left\{ \frac{\overline{c}^{1-\sigma}}{1-\sigma} + \beta (1+g)^{1-\sigma} \frac{\underline{c}^{1-\sigma}}{1-\sigma} + \left[\beta (1+g)^{1-\sigma} \right]^2 V(w') \right\}$$

where

$$\overline{c} = \overline{e} + w - aq(1+g) \tag{9}$$

$$\underline{c} = \underline{e} + a - w'q(1+g) \tag{10}$$

and

$$w' \geq -d$$

If $q \geq \beta (1+g)^{-\sigma} = q^*$, the borrowing constraint is binding every other period, which implies $w = w' = -d$ in steady-state. If high endowment consumers are unconstrained, the first-order condition for a and w' are

$$q(1+g)\overline{c}^{-\sigma} = \beta (1+g)^{1-\sigma} \underline{c}^{-\sigma} \tag{11}$$

$$q(1+g)\underline{c}^{-\sigma} \geq \beta (1+g)^{1-\sigma} \overline{c}^{-\sigma} \tag{12}$$

and the transversality condition is $\lim_{t\to\infty}\beta^t u'(c_t)a_t = 0$. Given (11), (12) is satisfied, if and only if $q \geq \beta(1+g)^{-\sigma} = q^*$. Together with the steady-state condition $w = w' = -d$ and the flow budget constraints, (11) determines the steady-state values of $\overline{c}$, $\underline{c}$ and a, in the consumers' optimization problem, as a function of the growth-adjusted price of a one-period bond $Q \equiv q(1+g)$ and d:

$$\overline{c} = \frac{(Q/Q^*)^{\frac{1}{\sigma}}}{Q+(Q/Q^*)^{\frac{1}{\sigma}}}\left[\overline{e}+Q\underline{e}+d\left(Q^2-1\right)\right] \tag{13}$$

$$\underline{c} = \frac{1}{Q+(Q/Q^*)^{\frac{1}{\sigma}}}\left[\overline{e}+Q\underline{e}+d\left(Q^2-1\right)\right] \tag{14}$$

$$a = \frac{1}{Q+(Q/Q^*)^{\frac{1}{\sigma}}}\left[\overline{e}-(Q/Q^*)^{\frac{1}{\sigma}}\underline{e}-d\left(1+Q\cdot(Q/Q^*)^{\frac{1}{\sigma}}\right)\right] \tag{15}$$

To check the transversality condition, note that $|a_t| \leq \max\{|a|, d\}(1+g)^t$ and $c_t^{-\sigma} \leq \underline{c}^{-\sigma}(1+g)^{-\sigma t}$, and hence $\lim_{t\to\infty}\beta^t u'(c_t)a_t \leq \lim_{t\to\infty}\max\{|a|, d\}\,\underline{c}^{-\sigma}\left[\beta(1+g)^{1-\sigma}\right]^t = 0$ by assumption (7). The steady-state solution for the post-default problem (3) is identical to (13)-(15), setting $d = 0$. We hence observe:

Lemma 1 *In any competitive equilibrium, debt contracts are self-sustaining, if and only if*

$$Q \geq 1 \tag{16}$$

Since $Q = q(1+g) = \frac{1+g}{1+r}$, the no-default condition thus leads to the requirement that in steady-state, the real interest rate cannot exceed the economy-wide growth rate.

The solution to the consumers' intertemporal optimization problem leads to a simple definition of the "lending franchise value": substituting (9) into (10), we observe that in steady-state, consumers choose $\overline{c}$ and $\underline{c}$ in the high-endowment period to maximize in the two-period utility $u(\overline{c})+\beta u(\underline{c}(1+g))$, subject to the inter-temporal budget constraint

$$\overline{c}+Q\underline{c} = \left[\overline{e}+Q\underline{e}+d\left(Q^2-1\right)\right]$$

If $Q \geq 1$, a consumer's ability to borrow effectively leads to an increase in his wealth level, and we can define $d\left(Q^2-1\right)$ as the *"lending seigniorage"* the consumer earns, or as the *lending franchise value.*

To complete the steady-state analysis, we have to consider the market-clearing condition: asset market clearing requires $a = d$. Therefore, given d, the equilibrium price Q is defined implicitly by

$$d\,(1+Q) = \frac{\overline{e} - (Q/Q^*)^{\frac{1}{\sigma}}\,\underline{e}}{1 + (Q/Q^*)^{\frac{1}{\sigma}}} \tag{17}$$

The LHS of (17) is the economy-wide supply of liquidity, while the RHS is the demand for liquidity. Both sides are monotonic in Q, and if $d < \frac{\overline{e}-\underline{e}}{2(1+Q^*)}$, (17) admits exactly one solution, for which $Q > Q^*$.[7] Defining the quantity

$$E = \frac{\overline{e} - (1/Q^*)^{\frac{1}{\sigma}}\,\underline{e}}{1 + (1/Q^*)^{\frac{1}{\sigma}}}$$

which represents the aggregate liquidity demand at $Q = 1$, we have the following proposition:

Proposition 4 *There exists a competitive steady-state equilibrium with self-sustaining debt contracts, whenever*

$$2d \leq E \tag{18}$$

If (18) is violated, Q clears the market below 1, and some consumers find it optimal to default before a steady-state is reached. Since Q is decreasing in the economy-wide borrowing capacity, defaults will raise Q, until in steady-state, $Q \geq 1$.

In equilibrium the following inequalities hold:

$$Q \equiv \frac{1+g}{1+r} \geq 1 > Q^*$$

[7] If d is larger, there is no economy-wide liquidity constraint, and we attain the first-best equilibrium, in which $Q = Q^*$.

so there can be no full consumption smoothing and the equilibrium will not be able to replicate the Pareto Optimal allocation achieved by the competitive equilibrium with no financial frictions.

At this point, we briefly return to the remark made in the discussion of proposition 2, that a violation of (5) is not sufficient to sustain borrowing in equilibrium: In the example of this section, we observe that, as long as the borrowing limit in the low endowment period satisfies $2d = E$ and the constraint in the high endowment period isn't binding, $Q = 1$ (hence (5) is violated), and no consumer has an incentive to default in high endowment periods. If we relax the borrowing capacity in the high endowment period, i.e. expand it beyond $E/2$, we will not affect equilibrium allocations, however, a consumer may eventually find it optimal to borrow as much as he can to boost his immediate consumption in the high endowment period, and then default one period later, which provides a consumption profile of $\left\{\overline{e} + \overline{d}, \underline{e}, \overline{c}, \underline{c}, \overline{c}, \underline{c}, ...\right\}$, where $\overline{d}$ denotes the borrowing in the high endowment period. Provided that

$$u\left(\overline{c}\right) + \beta u\left(\underline{c}\left(1+g\right)\right) \leq u\left(\overline{e} + \overline{d}\right) + \beta u\left(\underline{e}\left(1+g\right)\right)$$

this alternative consumption profile is preferred and hence leads to a default of a consumer in the low endowment period. Note that this sequence of borrowing constraints violates the condition of proposition 1, but also does not satisfy (6). Also, one observes that such a problem is per se irrelevant in an equilibrium with maximum credit expansion, since such an equilibrium endogenously determines the borrowing limits, and hence restricts the borrowing limits on high endowment consumers.

In the equilibrium with maximum credit expansion, $Q = 1$: the market always drives Q to 1, and then determines the borrowing capacity endogenously. In this equilibrium,

$$2d = E \tag{19}$$

and it follows from (13)-(15) that consumer allocations in this equilibrium do not depend on any individual consumer's borrowing limit. In particular we could set different borrowing constraints for even and odd consumers

$$\begin{array}{cccc} -\theta_t d^e & \theta_{t+1} a^e & -\theta_{t+2} d^e & ... \\ \theta_t a^o & -\theta_{t+1} d^o & \theta_{t+2} a^o & ... \end{array}$$

and have the same steady-state competitive allocation for any pair d^e, d^o that satisfies

$$d^e + d^o = E.$$

Hence, in this equilibrium, the distribution of borrowing privileges across the population does not affect welfare levels, and any distribution of borrowing limits, such that $Q = 1$ implements the same allocation of consumption.

3.2 Steady-states with public liquidity

In this section, we introduce outside liquidity in the form of a government-issued liquid liability. We show that in steady-state outside liquidity does at least as well as a system of private liabilities - in fact, the circulation of a constant supply of public liquidity leads to steady-state allocations that are equivalent to the ones in a competitive equilibrium with private liquidity and maximum credit expansion. For the moment we concentrate on a fixed supply of fiat money, i.e. a government liability that pays zero returns and is fully self-financing. Then, all the government can do is to run a Ponzi scheme by issuing a bond, rolling it over forever, and finance interest payments by new bond issues - equivalently, we can think of the government as injecting fiat money. In that case, the growth-adjusted steady-state bond price Q is bounded below by 1, and is equal to 1 if the government does not make any lump-sum transfers of new bonds to the private sector. If $Q = 1$, the equilibrium allocations are identical to those obtained in a system of private liquidity. Normalizing the equilibrium supply of money (or bonds) to 1, we find a growth-adjusted equilibrium price of money P,[8]

$$P = E,$$

i.e. the steady-state supply of liquidity exactly equals the the private sector demand for liquidity. We thus find

Proposition 5 *The competitive steady-state equilibrium in an economy with public supply of liquidity and no money growth is equivalent to the competitive steady-state equilibrium with self-sustaining private debt that maximizes the aggregate supply of private liquidity.*

Hence, when the government has no power to tax consumers, and the private sector has no power to enforce contracts, other than through the withdrawal of borrowing privileges, public and private liquidity lead to exactly the same equilibrium allocations. Along the same lines, private

[8] To avoid confusion: P is the price of the money stock, which is the inverse of the nominal price level.

liquidity can implement first-best allocations as a market equilibrium, if a sufficiently high fraction of future endowments is pledgeable, and it is well-known that, when the government is able to raise non-distortionary taxes, public liquidity also achieves the first best; Woodford (1990) and Holmstrom and Tirole (1998) interpret the role of public liquidity in this sense. Overall, our analysis suggests that welfare comparisons between the effects of public and private liquidity provision are linked to their respective ability to enforce payment of either taxes or private debt contracts.

3.3 Public and private liquidity

We next consider the coexistence of public with private liquidity. If the supply of public liquidity is constant, then $Q = 1$ in equilibrium, and the price level and the borrowing capacity together satisfy

$$P + 2d = E$$

Hence, we encounter an indeterminacy: to implement the first-best equilibrium, the central bank may want to control the price level, and let the market participants adjust borrowing limits, or it may control borrowing limits and let the steady-state price level be determined by the market. In any case, our notion of equilibrium is not able to determine both; welfare levels, however, remain determinate. If the supply of public liquidity is inflationary, on the other hand, public liquidity is dominated as an asset by private borrowing and lending, and hence is driven out of the market.

The equivalence between public and private provision of liquidity in the absence of external enforcement by either the public or the private sector has several implications. First, our results provide new insights on the coexistence of public and private liquidity, and, related to that, the role of banks as providers of private liquidity: we have observed that there are two dimensions along which competitive steady-state equilibria are indeterminate: On the one hand, the equilibrium with maximum credit expansion is indeterminate with respect to the distribution of borrowing privileges across agents, on the other hand, the equilibrium is indeterminate with respect to the part of transactions that is carried out using private, as opposed to public liquidity. These indeterminacies could be overcome, if some transactional frictions are added to the model which break the perfect substitutability of credit and of public and private liquidity. For instance, the cost of monitoring private balance sheets and transactions is a prime factor determining the optimal allocation of borrowing privileges, and in particular, if there are non-convexities and increasing returns to size in the monitoring of portfolios, then it might be optimal to concentrate borrowing on a small number of agents who provide large amounts of liquidity.[9] Similarly, the indeterminacy between public and private liquidity can also be resolved by considerations regarding the transactions technology. To summarize, in an environment with no external taxation or contract enforcement, the equilibrium structure of liquidity provision is entirely driven by technological considerations. Compared with an equilibrium in which only fiat money circulates, the introduction of private credit does not create additional liquidity.

This leads to our next comment regarding the effects of monetary policy: First, if public and private liquidity are perfect substitutes for transaction purposes, then a competitive equilibrium with maximum credit expansion leaves no room for an active monetary policy, since credit expansion in equilibrium sets the rate of return on money equal to the growth rate of the economy. In such an environment, an active monetary policy has to be accompanied by policy measures that directly influence the private credit expansion, such as reserve ratios for banks, i.e. policy measures that limit the supply of credit by the private sector. We further observe that such measures are necessary, if nominal price stability is the ultimate objective of the central bank.[10] Our model however provides no theoretical basis for the claim that such a policy should be desirable. Quite to the contrary, the equilibrium with maximum credit expansion that lets the market freely determine the supply of credit that is optimal, even if this leads to fluctuations in the price level.[11]

The discussion of the previous two paragraphs is in contrast with the view that (i) private credit serves to expand the existing amount of public liquidity through "multiplication", and (ii) monetary policy has an active role to play in regulating the supply of private credit. Through its "negative" results, our analysis also provides new insights with respect to the role of banks as "liquidity multipliers", i.e. as agents who multiply an existing stock of base liquidity by issuing claims that are only partially backed by reserves. Our results suggest that external enforcement powers, both for the public sector and for the private sector, are necessary to sustain liquidity

[9] Alternatively, when monitoring costs increase with the portfolio size, it would be optimal to spread borrowing privileges across the population.

[10] Recall that the nominal price level is determined by technological considerations. Inflation is then linked to changes in technological factors on the one hand, and the real growth rate on the other.

[11] In a very different environment, that entirely abstracts from issues of contract enforcement, Sargent and Wallace (1982) come to similar conclusions regarding the desirability of a laissez-faire policy with respect to the credit market, as opposed to an interventionist one.

multiplication: we have already observed that liquidity expansion by private credit is possible only if the interest rate exceeds the growth rate. That in turn implies that there has to be some deflation, if public liquidity is to be valued in equilibrium, and hence the public sector has to have some power to impose lump sum taxes (i.e. reduce the monetary base over time). But if the interest rate exceeds the growth rate, private liquidity is not sustainable by reputation alone, and hence some external enforcement of contracts becomes necessary. Once banks operate as liquidity multipliers, however, it follows immediately that the central bank can affect the supply of private credit through changes in the supply of base liquidity.

We conclude the discussion of steady-state equilibria with public and/or private liquidity with a brief remark regarding the informational assumptions that sustain the monitoring of balance sheets, and hence the provision of private credit. So far, we have maintained the assumption, that all information regarding a consumer's type, his past balance sheets, and his past default decisions is public. Obviously, in an environment with public liquidity, these informational assumptions are not per se necessary, since there is no need for balance sheet monitoring. In the presence of private liquidity, some of these informational assumptions are also redundant (even though these informational assumptions determine what distributions of borrowing privileges can be sustained): since equilibrium borrowing privileges are not contingent on an individual's net financial positions $\{a_\tau\}_{\tau=1}^{t}$, it is not necessary to maintain a memory of the entire history - rather, it is sufficient to monitor past defaults and the *current* net asset positions. Similarly, it is not necessary to monitor a consumer's *type*, since some uniform borrowing limit ϕ will always implement the equilibrium with maximum credit expansion. Hence, we conclude that once past defaults and current balance sheets are actively monitored, no additional information is needed to sustain private lending with maximum credit expansion.

4 Public vs. private liquidity: a general equivalence result

After studying the steady-state properties of public and private liquidity provision, we turn to issues of transitional dynamics. we show that the equivalence between public and private liquidity is much more general than what was shown for steady-states in the context of the previous example. Namely, we return to our initial, general set-up and show that *for any initial allocation of fiat money, there exist sequences of self-enforcing borrowing limits that satisfy maximum credit expansion and lead to* *the same equilibrium consumption allocations, and vice versa.* Hence, under complete information, any equilibrium allocation that can be implemented by initial lumpsum transfers of fiat money can also be implemented by the use of private liquidity, and vice versa. We begin the analysis by a formal definition of public liquidity equilibria, given an initial allocation of fiat money $\left\{M_0^j\right\}_{j\in J}$, such that $\sum_{j\in J} M_0^j = 1$:

Definition 3 *For given initial balances $\left\{M_0^j\right\}_{j\in J}$, a competitive equilibrium with public liquidity is given by sequences $\left\{\widetilde{c}_t^j, \widehat{M}_{t+1}^j\right\}_{t=0}^{\infty}$ for all j and $\{P_t\}_{t=0}^{\infty}$ of consumption allocations, nominal balances and the real value of money, respectively, such that*

(i) given the sequence of prices $\{P_t\}_{t=0}^{\infty}$, for all j, the sequence $\left\{\widetilde{c}_t^j, \widehat{M}_{t+1}^j\right\}_{t=0}^{\infty}$ maximizes expected discounted life-time utility

$$\sum_{t=0}^{\infty} \beta^t u(c_t^j).$$

subject to the sequence of flow-of-funds constraints

$$c_t^j + P_t M_{t+1}^j = y_t^j + P_t M_t^j$$

and the non-negativity constraint on nominal balances

$$M_{t+1}^j \geq 0,$$

with M_0^j given, and

(ii) goods and money markets clear in every period, i.e.

$$\sum_j \widetilde{c}_t^j = \sum_j y_t^j \text{ and } \sum_j \widehat{M}_{t+1}^j = 1 \text{ for all } t.$$

We further observe that in the private liquidity environment with maximum credit expansion, $\phi_{t+1}^j p_{t+1} = \phi_t^j p_t = \phi_0^j$ implies that for any price sequence $\{p_t\}_{t=0}^{\infty}$, the present discounted value of a type's debt capacity, ϕ_0^j, determines the entire sequence of self-sustaining borrowing limits. We summarize these by $\left\{\phi_0^j\right\}_{j\in J}$, and then state our main proposition in this section as an equivalence result between $\left\{M_0^j\right\}_{j\in J}$ and $\left\{\phi_0^j\right\}_{j\in J}$:

Proposition 6 *Suppose* $p_t = \frac{P_0}{P_t}$ *for all* t*, and* $\phi_0^j = P_0 M_0^j$*, for all* j*. Then,* $\{P_t\}_{t=0}^{\infty}$ *is a sequence of competitive equilibrium prices for the public liquidity environment following initial money holdings* $\left\{M_0^j\right\}_{j\in J}$*, if and only if* $\{p_t\}_{t=0}^{\infty}$ *is a sequence of competitive equilibrium prices for the private liquidity environment with maximum credit expansion, for debt capacities* $\left\{\phi_0^j\right\}_{j\in J}$*.*

Proof. (i) We first show that if (and only if) $p_t = \frac{P_0}{P_t}$ for all t, and $\phi_0^j = P_0 M_0^j$, for all j, then the budget sets of the public liquidity environment, and that of the private liquidity environment are identical, and hence will lead to the choice of identical consumption profile: Call a consumption plan $\left\{c_t^j\right\}_{t=0}^{\infty}$ admissible for a public or private liquidity environment, if it satisfies the relevant flow-of-funds and inequality constraints. For the public liquidity environment, these are $c_t^j + P_t M_{t+1}^j = y_t^j + P_t M_t^j$ and $M_{t+1}^j \geq 0$. Substituting out M_t^j, we find that a consumption plan $\left\{c_t^j\right\}_{t=0}^{\infty}$ is admissible with public liquidity, if and only if

$$P_0 M_{t+1}^j = \sum_{s=0}^{t} \frac{P_0}{P_s}\left(y_s^j - c_s^j\right) + P_0 M_0^j \geq 0, \text{ for all } t \geq 0$$

$\left\{c_t^j\right\}_{t=0}^{\infty}$ is admissible with private liquidity and maximum credit expansion, if and only if it satisfies the flow of funds constraints $p_t c_t + p_{t+1} a_{t+1}^j = p_t y_t^j + p_t a_t^j$, and $p_{t+1} a_{t+1}^j \geq \phi_0^j$, for all t. Substituting out a_t^j, $\left\{c_t^j\right\}_{t=0}^{\infty}$ is admissible with private liquidity, if and only if

$$p_{t+1} a_{t+1}^j = \sum_{s=0}^{t} p_s\left(y_s^j - c_s^j\right) \geq \phi_0^j, \text{ for all } t \geq 0$$

Hence, the set of admissible consumption plans $\left\{c_t^j\right\}_{t=0}^{\infty}$ is equivalent, if and only if $p_t = \frac{P_0}{P_t}$ for all t, and $\phi_0^j = P_0 M_0^j$, for all j. Hence, j's optimal consumption plan, given prices $\{P_t\}_{t=0}^{\infty}$ and initial nominal balances M_0^j is identical to the optimal consumption plan of the private liquidity environment given ϕ_0^j and prices $\{p_t\}_{t=0}^{\infty}$.

(ii) If $\left\{\widehat{c}_t^j, \widehat{M}_{t+1}^j\right\}_{t=0}^{\infty}$ and $\{P_t\}_{t=0}^{\infty}$ are competitive equilibrium allocations and prices, respectively, for the public liquidity environment with initial money holdings $\left\{M_0^j\right\}_{j\in J}$, then $\sum_j \widehat{c}_t^j = \sum_j y_t^j$ by virtue of market-clearing. But then, $\sum_j \widehat{c}_t^j = \sum_j y_t^j$ for all t, for prices $\{p_t\}_{t=0}^{\infty}$, where $p_t = \frac{P_0}{P_t}$ in the private liquidity environment, given $\phi_0^j = P_0 M_0^j$, and it follows from Walras' Law that markets also clear in the private liquidity environment. ■

This proposition establishes the equivalence of equilibria with public liquidity and those with private liquidity and maximum credit expansion at the most general possible level. This equivalence is further discussed below, when we discuss the equivalent of hyperinflations in this environment - equilibria, in which the provision of private liquidity collapses for reasons purely driven by expectations. Before we discuss this, however, we want to highlight the importance of the complete information assumption. Within the context of our example, we therefore discuss the set of allocations implementable with public and private liquidity, first under complete information, then under the assumption that a household's type is private information.

4.1 Implementable allocations with complete information

Let us return to the previous example, setting $g = 0$ for simplicity. We start by studying two polar cases; in the first, all liquidity is initially allocated to the even consumers. With public liquidity, this is achieved by a one time transfer of money to the even period consumers, and prices adjust so that $P = E$. The same steady-state allocations are implemented with private liquidity by letting only consumers borrow a total amount of $d^e = E$, and odd-period consumers do not borrow at all.

Alternatively, we consider the case, where borrowing privileges are constant across types - or equivalently, where the planner makes an initial money-transfer to all households. In this case, the equilibrium converges to the steady state allocation described in the previous section in one period. The price of fiat money at date 0, P_0 is implicitly determined as the solution to the the odd consumers' Euler equation in period 0, or

$$u'(\overline{e} - \frac{1}{2}P_0) = \frac{P}{P_0}\beta u'\left(\underline{e} + E\right)$$

where $E = P$ is the steady-state value of money. Note that $P < P_0$, since $\beta u'\left(\underline{e} + E\right) < u'\left(\underline{e} + E\right) < u'(\overline{e} - E)$, and since $u'(\overline{e} - E) = \beta u'\left(\underline{e} + E\right) > \frac{1}{2}\beta u'\left(\underline{e} + E\right)$, it follows that $P_0 < 2P$. We thus have $P < P_0 < 2P$, and the consumption of even and odd consumers satisfies

$$c_0^e < \underline{c} < \overline{c} < c_0^o.$$

In this equilibrium even consumers start out with only half the money stock, which they sell entirely to the odd consumers. Since odd consumers already have half the money stock they are richer than in steady state at date 0 and therefore they are willing to receive a rate of return $\frac{P}{P_0} < 1$. This equilibrium can be replicated by a sequence of private borrowing constraints and maximum credit expansion. This requires that in steady-state, $\phi_t^e = \phi_{t+2}^e = d^e$ between even periods (other than period 0), and $\phi_t^o = \phi_{t+2}^o = d^0$ between odd periods, and furthermore that $d^e + d^o + E$. Equilibrium prices are then $q_t = 1$, for all $t > 1$. The period 0 borrowing limit d_0^e and one-period return q_0 together satisfy

$$u'(\overline{e} - q_0 d_0^e) = \frac{\beta}{q_0} u'\left(\underline{e} + E\right), \tag{20}$$

so that a one-period price equal to $q_0 = \frac{P}{P_0}$ is achieved by setting $d_0^e = \frac{1}{2}E$. To satisfy maximum credit expansion, the steady-state borrowing limits then solve $d^e q_0 = d_0^e$, or $d^e = \frac{1}{2}P_0$ and $d^o = E - \frac{1}{2}P_0$. Hence, the public liquidity equilibrium allocation is exactly replicated by setting the borrowing constraints $\left\{\phi_t^j\right\}$ of even and odd agents equal to, respectively, $\left\{\frac{1}{2}E, 0, \frac{1}{2}P_0, 0, \frac{1}{2}P_0, ...\right\}$ and $\left\{0, E - \frac{1}{2}P_0, 0, E - \frac{1}{2}P_0, ...\right\}$, in which case $q_0 = \frac{P}{P_0}$. The initial allocation of liquidity into the hands of the odd consumers leads to a utility transfer from the even to the odd consumers.

More generally, depending on the initial allocation of borrowing privileges or fiat currency, there is a continuum of transition paths, in which consumption allocations reach a steady-state after the first period. These allocations can be reached either by an initial injection of fiat money, or by an appropriate design of borrowing limits that also satisfies maximum credit expansion. Starting from a value of $d_0^e \in [0, E]$, we derive q_0 from (20). Note that this guarantees a value of $q_0 \geq 1$. The steady-state borrowing limits are then determined by $d^e = d_0^e / q_0$ and $d^o = E - d^e$. This same allocation is implemented through public liquidity by transfering a proportion $\frac{d_0^e}{E}$ of the fiat money injection to the even agents and $\frac{E - d_0^e}{E}$ to the odd type, which leads to a first-period price level $P_0 = q_0 E$. We summarize the findings in the following proposition:

Proposition 7 *There is a continuum of transition paths on which steady-state allocations are reached within one period, that are characterized by $q_0 \geq 1$ and $q_t = 1$ for $t \geq 1$. These allocations can be implemented with self-enforcing private liquidity characterized by a sequence of borrowing constraints*

$$0 \quad d^o \quad 0 \quad d^o \quad ...$$

for odd agents and

$$d_0^e \quad 0 \quad d^e \quad 0 \quad ...$$

for even agents. For $d_0^e \in [0, E]$, (d^e, d^o, q_0) *are implicitly defined as the solution to*

$$\begin{aligned} d^e + d^o &= E \\ u'(\overline{e} - q_0 d_0^e) &= \frac{\beta}{q_0} u'\left(\underline{e} + E\right) \\ d^e q_0 &= d_0^e \end{aligned}$$

The same equilibrium allocations can be implemented through a period 0 *money injection which allocates a share* $\frac{d_0^e}{E}$ *of the money injection to even agents, leading to equilibrium prices* $P_0 = q_0 E$ *in the first period,* $P = E$ *in all following periods.*

We conclude this section with a brief welfare comparison of the set of equilibria of the previous proposition. We already observed that shifts in the period one allocation of liquidity from even types to odd types leads to a welfare increase of odd types at the expense of the even types (and inverse). Hence the allocation of liquidity has ex post redistributive effects. However, we can consider as an ex ante welfare criterion

$$W = \frac{1}{2} V_0^o + \frac{1}{2} V_0^e$$

i.e. the ex ante expected lifetime utility, before a consumer learns his type. We observe that the equilibrium which allocates all liquidity to the initially constraint (even) households is ex ante efficient; more generally, the more liquidity is allocated to the initially constrained household, the higher the ex ante welfare. the initial allocation of liquidity thus serves to implicitly provide insurance against the endowment risk.

4.2 Implementable allocations with incomplete information

From a social planner's perspective, one of the implications of proposition 7 is that the sets of allocations that can be sustained through a onetime money injection is identical to the set of allocations that can be sustained by the distribution of private borrowing privileges. This result crucially depends on the assumption that all information is public, *including the borrowers' types.* In this section, we show that private liquidity implements strictly more allocations, when each household's type is private information. The reason is that the planner/central bank can design the borrowing limits in such a way that they screen between types - this of course is not feasible when the central planner's policy consists of a one-time money injection.

Consider the following modification of our setup. At date -1 all consumers are identical and each consumer becomes an even or an odd consumer at date 0 with equal probability. The information about a consumers' type is private. Consider an economic system in which each consumer declares his identity and then is allowed to borrow and lend according to $\{\phi_t^k\}$[12]. The system of borrowing constraints $(\{\phi_t^o\}, \{\phi_t^e\})$ now must satisfy an additional incentive compatibility condition, that we can write, in short notation as

$$\begin{aligned} W^o(\{\phi_t^o\}) &\geq W^o(\{\phi_t^e\}) \\ W^e(\{\phi_t^e\}) &\geq W^e(\{\phi_t^o\}) \end{aligned} \tag{21}$$

where the functions W^j denote a consumer's life-time utility at date 0 in the competitive equilibrium when he faces a sequence of borrowing constraints $\{\phi_t\}$. Note that W^j includes the possibility of default. Even though the equilibrium conditions guarantee that the value $W^j\left(\{\phi_t^j\}\right)$ corresponds to a solution of the consumer maximization problem with no default, the value $W^j\left(\{\phi_t^{k'}\}\right)$ may involve the possibility of a strategic default (which will never be observed in equilibrium).

We immediately observe that a one-time injection of public liquidity has to be symmetric across all agents, and hence can only implement the corresponding equilibrium allocation. The same is true if borrowing limits allow for Ponzi schemes, i.e., if they satisfy $\left|\phi_t^j\right| p_t \leq \left|\phi_{t+1}^j\right| p_{t+1}$. However, we note that maximum credit expansion only implies that $\left|\phi_t^j\right| p_t = \left|\phi_{t+s}^j\right| p_{t+s}$ is necessary between periods in which the constraint is binding, but not, when the constraint is not binding. Self-selection then results from a sequence of borrowing limits that work like a "credit line", where the outstanding balance has to be cleared on a regular basis. Alternatively, these self-selecting borrowing constraints can be interpreted as consisting of (i) a maximum outstanding balance (the borrowing limit), and (ii) an average balance requirement, which requires that a consumer's average balance exceeds a certain lower bound.

More specifically, we consider sequences of borrowing limits of the form $\{d_0^e, 0, d^e, 0, d^e, ...\}$ and $\{0, d^o, 0, d^o, 0, ...\}$. The restriction of borrowing to be zero every other period incorporates the requirement that outstanding balances be cleared on a regular basis (note that this requirement will not be binding in a steady-state). Since the economy reaches a steady state after one period only, we further observe that $d^e + d^o = E$, as a consequence of maximum credit expansion. As we

[12] As private information is restricted to date zero the revelation principle holds in its simplest form.

observed previously, for any value of d_0^e, there exists a pair of values q_0 and d^e, such that markets clear and maximum credit expansion is satisfied between periods 0 and 2. We have to check that this equilibrium also satisfies the incentive compatibility conditions (21). The second inequality is easy to check, since every even consumer would want to borrow immediately. The first inequality depends on the optimal behavior of an odd consumer facing the borrowing constraint $\{d_0^e, 0, d^e, 0, d^e, ...\}$. This consumer has the option to borrow d_0^e immediately and default after one period to consume $\underline{e}$, after which he obtains his steady-state consumption profile. $\{\overline{e}-E, \underline{e}+E, \overline{e}-E, \underline{e}+E, ...\}$, giving him a life-time utility

$$u(\overline{e}+q_0 d_0^e) + \beta u(\underline{e}) + \frac{\beta^2}{1-\beta^2}\left[u(\overline{e}-E) + \delta u(\underline{e}+E)\right].$$

The life-time utility of not defaulting is given by

$$u(\overline{e}-q_0 d_0^e) + \beta u(\underline{e}+E) + \frac{\beta^2}{1-\beta^2}\left[u(\overline{e}-E) + \delta u(\underline{e}+E)\right].$$

Therefore, the self-selection constraint (21) is satisfied if

$$u(\overline{e}+q_0 d_0^e) + \beta u(\underline{e}) \leq u(\overline{e}-q_0 d_0^e) + \beta u(\underline{e}+E). \tag{22}$$

Since q_0 is increasing in d_0^e, there exists a maximum level of d_0^e, at which (22) is just binding. If

$$u(\overline{e}+E) + \beta u(\underline{e}) \leq u(\overline{e}-E) + \beta u(\underline{e}+E) \tag{23}$$

we find that the steady-state can be implemented from date 0. This also corresponds to our ex ante optimal allocation. On the other hand, the consumption path resulting from a symmetric injection of money is replicated by a value of $d_0^e = \frac{1}{2}E$, in which case $q_0 = \frac{P_0}{P} \leq 1$. It is easy to check that (22) is then always satisfied. Thus, even if (23) is violated, private credit still implements a welfare improvement over public liquidity, but it falls short of the first best. We summarize the result in the following proposition:

Proposition 8 *The optimal self-enforcing and incentive compatible equilibrium corresponds to* $d_0^e = E$ *if (23) is satisfied, or to the value of* $d_0^e \in \left[\frac{1}{2}E, E\right]$ *that satisfies (22) with equality.*

The basic difference between a public liability and a system of self-enforcing private liabilities is in terms of the date 0 transfers that are feasible and incentive compatible. If the government has full information it can transfer the funds created at date 0 to the consumers with a low income and replicate the transitional dynamics studied in the preceding section. However, if the government has no information on the consumers' identities, a transfer of public liquidity at date 0 cannot target the low income consumers, while a system of private liabilities can induce self-selection. Therefore if we adopt an ex ante welfare criterion, the system of private liabilities is superior to the issuance of outside liquidity.

Notice that in order to implement equilibria better than the pure public liquidity equilibrium, the exact form that the credit constraints take matters. Had we allowed consumers to run pure Ponzi games (i.e. we set $\{\phi_t^e\} = \{d, d, d, ...\}$), then there is no way of separating even from odd consumers, and the system of private liabilities would have no advantage over a pure transfer of public liquidity ex-ante. Therefore, the presence of balance sheet monitoring is important, and the optimal borrowing contract takes the form of a "credit line" that requires the regular clearing of the outstanding balance.

A related observation concerns the implementation of the best equilibrium with private liquidity using public liquidity. The government could, in principle, implement the best equilibrium described in proposition 8 by offering a transfer of public liquidity at date 0 contingent on future net asset positions of a consumer and then monitor these asset positions. However, the need to monitor individual balance sheets makes this essentially a system of private liabilities. What the government cannot do is to replicate the best equilibrium with private liabilities by simply issuing bonds at date 0 and then letting people trade in a full anonymity.

The central message of this section is that a system of private credit lines can have beneficial effects in terms of selecting those agents who need liquidity the most. It is important to notice that the transition is crucial in the model presented because date 0 is the only moment when new credit lines are opened. This explains why the crucial difference between private and public liquidity only appears when we take the transition into account. The essence of this result should carry over to a more general setup with random shocks, where transfers of outside liquidity cannot be made contingent on individual shocks (possibly for the same informational asymmetries introduced here). In such a setup we will expect to observe that the private credit lines can dominate public liquidity in a stationary equilibrium. On the other hand, we have not modelled the costs of keeping track of private balance sheets. As we already observed previously, introducing explicitly the costs of running a system of private liabilities should bring us closer to a theory of the optimal composition of public and private liquidity.

Finally, we note the relation between our results here, and the discussion of Holmstrom and Tirole (1998/2000) on the shortage and waste of liquidity. The inefficiency of the allocations with a symmetric liquidity injection, as well as the inefficiency of private liquidity equilibria that do not immediately implement the steady-state is the result of a misallocation of liquidity away from those who need it most. The inefficiency is not the consequence of a shortage of liquidity - as in the equilibria that do not satisfy maximum credit expansion. Rather, it comes from a waste of existing liquidity. Our results suggest that in the presence of informational frictions, the use of private liquidity is essential in limiting the waste of liquidity, and optimizing its allocation.

4.3 Fragility

We now turn to the study of non-stationary dynamics. In discussing our previous example, we focused on equilibria where the economy converges to a (periodic) steady state. Here we consider equilibria that do not converge to a steady state. In these equilibria, the borrowing constraints are self-enforcing *and* maximum credit expansion holds. Borrowing constraints get tighter over time and lenders are not willing to lend more to borrowing agents since they forecast that their borrowing constraints will get tighter in the future and therefore the punishment for default will be weaker over time. These type of equilibria are equivalent to hyperinflations in models of fiat money.

Let us turn to the construction of non-stationary equilibria for the simple model of section 3. Let d_t denote the growth-adjusted borrowing limit of a consumer who is borrowing-constrained in period t (i.e. the odd or even type, depending on whether t is odd or even). For a consumer with high endowment in an arbitrary period t, growth-adjusted asset holdings in $t+1$, a_{t+1}, as well as $\overline{c}_t$ and $\underline{c}_{t+1}$ i.e. growth-adjusted consumption in period t (the high-endowment period) and period $t+1$ (the low-endowment period) have to satisfy the flow budget constraints

$$\begin{aligned}\overline{c}_t &= \overline{e} - d_t - Q_t a_{t+1} \\ \underline{c}_{t+1} &= \underline{e} + a_{t+1} + Q_{t+1} d_{t+2}\end{aligned}$$

as well as the first-order condition

$$\overline{c}_t^{-\sigma} = \frac{Q^*}{Q_t} \underline{c}_{t+1}^{-\sigma},$$

where Q_t is the growth-adjusted one-period return between periods t and $t+1$. Note that maximum credit expansion implies that

$$d_t = Q_t Q_{t+1} d_{t+2}. \tag{24}$$

We then solve for $\overline{c}_t$, $\underline{c}_{t+1}$, and a_{t+1}:

$$\begin{aligned}
\overline{c}_t &= \frac{(Q_t/Q^*)^{\frac{1}{\sigma}}}{Q_t + (Q_t/Q^*)^{\frac{1}{\sigma}}} \left[\overline{e} + Q_t \underline{e}\right] \\
\underline{c}_{t+1} &= \frac{1}{Q_t + (Q_t/Q^*)^{\frac{1}{\sigma}}} \left[\overline{e} + Q_t \underline{e}\right] \\
a_{t+1} &= \frac{1}{Q_t + (Q_t/Q^*)^{\frac{1}{\sigma}}} \left[\overline{e} - (Q_t/Q^*)^{\frac{1}{\sigma}} \underline{e}\right] - \frac{d_t}{Q_t}
\end{aligned}$$

Define the savings function $F(Q)$:

$$F(Q) \equiv \frac{1}{Q + (Q/Q^*)^{\frac{1}{\sigma}}} \left[\overline{e} - (Q/Q^*)^{\frac{1}{\sigma}} \underline{e}\right]$$

Using the market-clearing condition $a_{t+1} = d_{t+1}$, we can then express d_{t+1} as a function of Q_t and d_t:

$$d_{t+1} = F(Q_t) - \frac{d_t}{Q_t}. \tag{25}$$

The same equation has to hold for d_{t+2} as a function of d_{t+1} and Q_t, and substituting in, we find

$$d_{t+2} = F(Q_{t+1}) - \frac{F(Q_t)}{Q_{t+1}} + \frac{d_t}{Q_t Q_{t+1}}$$

It then follows from (24) that

$$F(Q_t) = Q_{t+1} F(Q_{t+1}) \tag{26}$$

It is easy to show that if we take any sequence $\{Q_t\}$ of prices $Q_t > 1$ the borrowing constraint will be binding for odd agents in odd periods and for even agents in even periods. Therefore, for any sequence $\{Q_t\}$ that satisfies (26) and $Q_t > 1$ is a competitive equilibrium with self enforcing credit limits and maximum credit expansion. Again, the credit limit are indeterminate across type, but depend on the initial allocation of borrowing privileges. There are two stationary equilibria that correspond to the two solutions of the following equation:

$$(Q-1)\,F(Q)=0 \tag{27}$$

The first solution to equation 27, corresponds to $Q = 1$ and has been studied in detail in the previous sections. The second solution is the Q' such that $F(Q') = 0$ (i.e. $\overline{e} = (Q'/Q^*)^{\frac{1}{\sigma}}\,\underline{e}$) and it corresponds to an equilibrium with zero private liquidity. Such an equilibrium has maximum credit expansion at every date: since future credit lines will always be zero, the consumers will have no incentive to repay and therefore lending is followed by certain default.

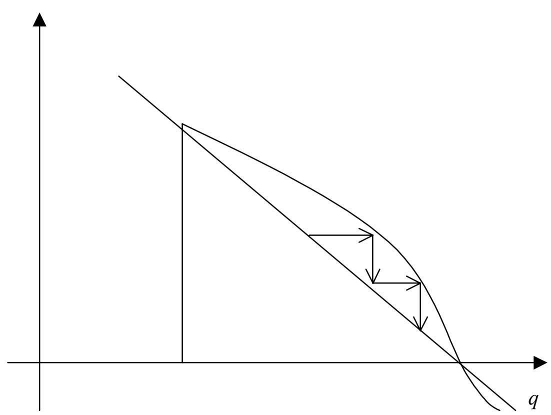

Figure 1

Given the presence of these two solutions, there are a continuum of sequences $\{Q_t\}_{t=0}^{\infty}$ with $Q_0 \in (1, Q')$ that converge asymptotically to the equilibrium with no borrowing Q'. This type of equilibria are illustrated in figure 1. The economy can start arbitrarily close to the good equilibrium and converge to the bad equilibrium. Since the only thing sustaining repayment in this economy is future access to credit lines there are rational expectations equilibria in which credit lines contract over time. Therefore, along the equilibrium path the supply of liquid assets shrinks and the liquidity premium grows over time. At the same time, the large liquidity premium increases the present value of future borrowing and therefore makes the current borrowing constraints enforcable. Notice that the dynamics of our 'disintermediation trap' are similar to the dynamics of a hyperinflation in a model with fiat money. However, while in a hyperinflation the supply of the outside asset is constant and its price is contracting, here it is the borrowing limit of private agents that is contracting over time.

The transition path of our model is characterized by a decline in the interest rate on highly liquid assets and a contraction of the mass of private liquidity. These two features appear in various historical examples of collapses in intermediation. A recent one is that of Japan in the 90s, where an aggregate measure of money supply such as M2+CD (that is made up mostly of private money) moved from a growth rate of around 10% in the second half of the 80s to a growth rate of around 3% in the 90s. At the same time nominal interest rates went from 7-8% to close to 0% (for realistic values of the expected inflation/deflation, real interest rates also declined considerably). The mechanism at work according to our 'disintermediation trap' is the following: as the deposit base is contracting over time, lenders are not willing to provide more loans because the punishment in case of a strategic default is too weak. This undermines confidence in the banking system and generate a self-sustaining collapse in the provision of liquidity services. As we converge towards the low private liquidity equilibrium the interest rate drops, reflecting the scarcity of liquidity supply. We have thus shown, how the equivalence between public and private liquidity extends to non-stationary equilibria. In these equilibria, a heavy contraction in the amount of real liquidity in an economy can take place due to a perverse dynamics in the reputation for repayment that are purely driven by expectations.

Notice that if one introduces fiat money in the model as in section 3.2, the intermediation collapse has to be accompanied by a hyperinflation. But this is not a special problem of our model, as in any model in which money and interest bearing liquid instruments are perfect substitutes, the real interest rate must be equal to the negative of the rate of inflation. No such a model can account for a situation in which the real interest rate declines (the liquidity premium increases) and the inflation rate declines at the same time. To construct such a model we need to move away from a world in which outside money and liquid private liabilities are perfect substitutes, and we need to incorporate nominal features in more realistic ways, and possibly allow for positive risks of default. Although we leave this to future work, we suspect that such a model could accurately capture the "flight to quality" aspect of disintermediation traps (given default risks, households prefer to hold fiat money rather than provide private loans).

5 Conclusion

In this paper, we studied an economy, in which debt contracts have to be self-enforcing, i.e. contracts have to be structured in such a way that it is in the debtors interest to repay any debt. Departing from earlier models by Kehoe and Levine (1993), Kocherlakota (1996) and Alvarez and Jermann (2000), we consider an environment where individuals can only be excluded from future borrowing, but retain the ability to save. In contrast to Bulow and Rogoff's (1989) partial equilibrium analysis, we show that some lending is sustained in equilibrium, in such a way that the present value of an individual's borrowing capacity is non-decreasing over time. In a general equilibrium setting, this implication of the no-default condition links the equilibrium with private liquidity to a dynamic inefficiency condition, and hence that an equilibrium with maximum credit expansion, in which the market generates as much liquidity as is possible given the enforcement constraint, is equivalent to an equilibrium in which consumers trade a fiat money in fixed supply to smooth consumption. This equivalence holds quite generally for environments, in which (i) the government has no power to extort lump sum taxes to implement a negative money growth rate, (ii) the private sector has no power to extort any payment on defaulted contracts. One critical assumption in this equivalence result is complete information, and we show within the context of an example that when individual liquidity needs are private information, private liquidity will generally lead to better consumption smoothing, since it enables a social planner/central banker to screen out those households that have the highest immediate liquidity needs.

Although our model is admittedly simple, we believe that its implications may be of interest for several applications. Following Bulow and Rogoff (1989), our enforcement mechanism may be particularly appealing for thinking about international capital flows - indeed, Ventura (2002) recently suggested that bubbles may be sustainable in the presence of contracting frictions, and may then substitute for international capital flows. Our paper suggests that such a bubble may actually be competitively supplied in the form of private borrowing and lending contracts - whether the rates of return on sovereign lending actually satisfy the no-default condition that this paper suggests is an open empirical question. With respect to the dynamic efficiency tests by Abel et al. (1989), our paper suggests an alternative test specification that tests whether or not the return to monetary assets satisfies dynamic efficiency. Preliminary results suggest that in the US, the return on M3 falls indeed below the economy growth rate. Finally, we point to theoretical issues that we haven't addressed yet: Given our focus on a simple deterministic economy with perfect foresight; the obvious next step would extend our analysis to environments with uncertainty in the endowment process. We leave these and other possible extentions to future work.

References

[1] A. Abel, G. Mankiw, L. H. Summers and R. J. Zeckhauser (1989), "Assessing Dynamic Efficiency: Theory and Evidence", *Review of Economic Studies* 56, 1, 1-20.

[2] F. Alvarez and U. Jermann (2000), "Efficiency, Equilibrium, and Asset Pricing with Risk of Default", *Econometrica*, 68, 4, 775-797.

[3] T. Bewley (1980), "The Optimum Quantity of Money", in *Models of Monetary Economics*, ed. by John Kareken and Neil Wallace. Minneapolis, Minnesota: Federal Reserve Bank, 1980.

[4] J. Bulow and K. Rogoff (1989), "Sovereign Debt: Is to Forgive to Forget?" *American Economic Review*, 79, 1, 43-50.

[5] R. Cavalcanti, A. Erosa, and T. Temzelides (1999), "Private Money and Reserve Management in a Random Matching Model," *Journal of Political Economy* 107, 5, 929-945.

[6] R. Cavalcanti and N. Wallace (1999), "Inside and Outside Money as Alternative Media of Exchange," *Journal of Money, Banking, and Credit*, 31, 3, part 2, 443-57.

[7] T. Kehoe and D. Levine (1993), "Debt-constrained Asset Markets," *Review of Economic Studies* 60, 4, 865-888.

[8] N. Kiyotaki and J. H. Moore (2001), "Evil is the Root of all Money", forthcoming in *the Clarendon Lectures*, Oxford University Press.

[9] N. Kocherlakota (1992), "Bubbles and Constraints on Debt Accumulation," *Journal of Economic Theory*, 57, 1, 245-256.

[10] N. Kocherlakota (1996), "Implications of Efficient Risk-Sharing without Commitment," *Review of Economic Studies* 63, 4, 595-609.

[11] B. Holmstrom and J. Tirole (1998), "Public and Private Supply of Liquidity" *Journal of Political Economy,* 106, 1, 1-40.

[12] B. Holmstrom and J. Tirole (2000), "Shortage and Waste of Liquidity," mimeo, MIT.

[13] L. Ljungqvist and T. J. Sargent (2000), *Recursive Macroeconomic Theory*, MIT Press.

[14] H. Lustig (2001), "The Market Price of Aggregate Risk and the Wealth Distribution," mimeo, Stanford University.

[15] M. S. Santos and M. Woodford (1997), "Rational Asset Pricing Bubbles," *Econometrica*, 65, 1, 19-57.

[16] T. J. Sargent and N. Wallace (1982),"The Real-Bills Doctrine versus the Quantity Theory: a Reconsideration", *Journal of Political Economy,* 90, 6, 1212-1236.

[17] J. A. Scheinkman and L. Weiss (1986), "Borrowing Constraints and Aggregate Economic Activity", *Econometrica*, 54, 1, 23-46.

[18] J. Ventura (2002), "Bubbles and Capital Flows", mimeo, MIT.

[19] M. Woodford (1990), "Public Debt as Private Liquidity," *American Economic Review*, 80, 2, 382-388.

6 Appendix

Lemma 2 *The sequence of borrowing constraints $\{\phi_t\}$ is self enforcing iff condition (4) is satisfied.*

Proof. Consider an optimal asset plan that involves no default $\{a_t^*\}_{t=0}^{\infty}$. We prove first the "if" part. For any possible deviation $\{\tilde{a}_t\}_{t=0}^{\infty}$ that involves default at some time T we can construct a deviation that involves no default and makes the consumer at least as well off. This follows from the fact that the function V_t is increasing in its first argument and therefore

$$V_T(\tilde{a}_T, \Phi_T) \geq V_T\left(\phi_T, \Phi_T\right) \geq V_T^d.$$

To prove the "only if" part suppose that at time T $V_T\left(\phi_T, \Phi_T\right) < V_T^d$. Then the consumer can deviate at time $T-1$ setting $a_T' = \phi_T$, consume the amount $a_T^* - a_T' \geq 0$ at time $T-1$ and then default. ■