The Conduct of Monetary Policy in the Face of Technological Change: Theory and Postwar U.S. Evidence

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Abstract

The present paper analyzes the implications of technological change for the design and conduct of monetary policy, using an optimizing sticky price model as a reference framework. I show how the optimal policy seeks to insulate the price level from the effects of changes in productivity. I provide some evidence that suggests that the Federal Reserve has responded to technological change in a way consistent with that rule in the Volcker-Greenspan era, but not during the pre-Volcker period. The second part of the paper discusses the conceptual difference between the notion of output gap arising in the new sticky price paradigm, and conventional measures of that variable. I also provide some evidence, based on postwar U.S. data, of the quantitative significance of that discrepancy. Finally, I perform a simple exercise to illustrate how a well intentioned monetary policy designed to stabilize inflation and the output gap could lead to unnecessary instability in both variables if a conventional (but incorrect) measure of the output gap was used.

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I. Introduction

A number of developments taking place in recent years have brought technological change to the forefront of the economic policy debate. The extraordinary performance of the U.S. economy in the second half of the nineties is, for example, commonly associated with the IT revolution.[2] High productivity growth is often credited with the fact that the rate of inflation in the U.S. economy has remained quite stable, at levels unheard of since the early 60s, despite that fast pace of economic activity, record low unemployment rates, and the unusually long duration of the current expansion. Such observations have led many economic commentators to proclaim the death of the Phillips curve, and the beginning of a new era of unlimited prosperity accompanied by nominal stability-a "new economy" indeed.[3]

In the present paper I discuss some of the implications of technological change for the design and conduct of monetary policy, using an optimizing sticky price model as a reference framework. In the first part of the paper, I discuss how a central bank should respond to an exogenous change in productivity, and provide some evidence on the actual response of the Federal Reserve to such changes during the postwar period. I start by laying a tractable version of a model economy with staggered price setting, in which technology disturbances are the main source of fluctuations. In that economy, the pattern of the optimal interest rate response to a technology shock is shown to depend critically on the dynamic properties of technology. Yet, and regardless of the latter, I show that the optimal policy always seeks to stabilize prices, i.e., to insulate the price level from the effects of the shock. With that result

[2] Some statistics help underpin that assertion. Hence, for instance, labor productivity has grown at an annual rate of 2.9 percent over the period 1995-99, compared to an average growth rate of 1.4 percent between 1973 and 1995. About two thirds of that acceleration can be traced the increase in the rate of TFP growth. See, e.g., Table 2-3 in the Economic Report of the President (2000).

[3] The recent episode marks the second death of the Phillips curve in recent history. Its first death was proclaimed in the mid-seventies, under more somber circumstances. At that time "stagflation" was the term coined to designate the killer.

as a benchmark, I describe recent work by López-Salido, Javier Vallés and myself which assesses the extent to which the Federal Reserve has come close to implementing an optimal policy in the face of technological change. The verdict turns out to be quite favorable when we look at the past two decades, corresponding to the Volcker-Greenspan era, but not so good for the earlier period.

The second part of the paper focuses on the notion of output gap that emerges in the new sticky price paradigm. That notion is one of its most novel aspects, as well as one that is strongly linked to technology variations. After stressing the conceptual differences with conventional measures of the output gap, I also provide some evidence suggesting that significant quantitative differences are likely to exist in practice: using postwar U.S. data I construct a measure of the output gap consistent with the underlying theory, and show its significant differences with more conventional measures. Then, I perform a simple exercise to illustrate how a well intentioned monetary policy designed to stabilize inflation and the output gap could lead to unnecessary instability in both variables if a conventional (but incorrect) measure of the output gap were used.

II. Technological Change in a New Keynesian Model

II.1 Technology, Money, and Business Cycles Models: Some Background

The idea of assigning a central role to technology in accounting for short run macroeconomic phenomena is not new in macroeconomics. It was the central tenet of Real Business Cycle (RBC) theory, the research program associated with the names of Prescott and collaborators.[4] The RBC conception of the business cycle implied a dramatic departure from the traditional, dichotomous approach to macroeconomics. According to the latter, technological change was a source of long-term growth, while monetary, fiscal, and other demand factors, interacting with

[4] See, e.g., Kydland and Prescott (1982) and Prescott (1986) for early examples of RBC models.

rigidities in wages and prices, were the main forces behind business cycles and inflation developments.

In monetary versions of RBC models, that dichotomy was replaced by a classical one, according to which real variables are determined (largely) independently of monetary factors, to the extent that no nominal rigidities or other imperfections are introduced.[5] That neutrality property renders them little useful as a reference framework to guide or evaluate monetary policy. Furthermore, it contrasts with the observation of a key role played by central banks all over the world in fine-tuning the economy, as well as the existing formal evidence of strong and persistent real effects of identified exogenous monetary policy shocks.[6]

Some of those shortcomings of the RBC program have motivated a considerable effort in recent years to integrate Keynesian-type elements into the class of dynamic stochastic general equilibrium models generally associated with RBC theory. The new class of models has two key ingredients: nominal rigidities and imperfect competition. Nominal rigidities constitute the main source of monetary non-neutralities. Imperfect competition takes the form of firms setting prices optimally, given the constraints on frequency and cost of that adjustment. The existence of a positive markup guarantees their willingness to accommodate small changes in demand through changes in the quantity produced and sold, at unchanged prices.

The development of that new generation of models has led to an explosion of research on the effects of alternative monetary policy rules, and other aspects of monetary economics which had been put aside during the era of RBC hegemony. It has also generated a renewed interest by many central banks in academic research on monetary economics, as well as improved communication and cooperation between central bank

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[5] See, e.g., Cooley and Hansen (1989).

[6] See Christiano, Eichenbaum and Evans (1998) for a recent survey of that evidence.

economists and academic researchers.[7]

In the remainder of this section, I lay out a version of the Calvo (1983) model with staggered price setting, which I take as representative of the new generation of dynamic sticky price models.[8]). In order to keep the exposition as simple as possible, the model below abstracts from capital accumulation and has no external sector. I will also ignore sources of fluctuations other than technology. Next I describe the model's key building blocks.[9]

II.2 Households

The representative consumer is infinitely-lived and seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \qquad (2.1)$$

subject to a (standard) sequence of budget constraints and a solvency condition. N_t denotes hours of work. C_t is a CES aggregator of the quantities of the different goods consumed:

$$C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Let $P_t \equiv \left(\int_0^1 P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$ represent the aggregate price index, where $P_t(i)$ denotes the price of good *i* Î [0,1]. The solution to the consumer's problem can be summarized by means of three optimality conditions (two static

[7] Examples of natural applications of the new models can be found in the recent work by Rotemberg and Woodford (1999) and Clarida, Galí, and Gertler (1999), among others.

[8] Alternative approaches to modelling price rigidities have been used in the literature. Those include (a) models with staggered price setting à la Taylor (with a certain time between price adjustments), as exemplified by the work of Chari, Kehoe, and McGrattan (1998), and (b) models with convex costs of price adjustment (but no staggering), as in Hairault and Portier (1993) and Rotemberg (1996).

[9] See, e.g., King and Wolman (1996), Yun (1996) and Woodford (1996) for a detailed derivation of the model's equilibrium conditions

and one intertemporal), which I represent in log-linearized form (henceforth, lower case letters denote the logarithm of the original variables).

First, the optimal allocation of a given amount of expenditures among the different goods generates the set of demand schedules:

$$c_t(i) = -\varepsilon\left(p_t(i) - p_t\right) + c_t \tag{2.2}$$

for all $i \in [0,1]$.

Second, and under the assumption of a perfectly competitive labor market, the supply of hours must satisfy:

$$w_t - p_t = \sigma\, c_t + \varphi\, n_t \tag{2.3}$$

where w is the (log) nominal wage.

Finally, the intertemporal optimality condition is given by the Euler equation:

$$c_t = -\frac{1}{\sigma}\left(r_t - E_t\{\pi_{t+1}\} - \rho\right) + E_t\{c_{t+1}\} \tag{2.4}$$

where r_t is the yield on a nominally riskless one period bond (the nominal interest rate, for short), π_{t+1} is the rate of inflation between t and $t+1$, and $\rho \equiv -\log\beta$ represents the time discount rate.

In equilibrium, $c_t = y_t$ all t, implying:

$$rr_t = \rho + \sigma\, E_t\{\Delta y_{t+1}\} \tag{2.5}$$

where $rr_t \equiv r_t - E_t\{\pi_{t+1}\}$ denotes the ex-ante real rate.

II.3 Firms

We assume a continuum of firms, each producing a differentiated good with a technology represented by:

$$y_t(i) = a_t + n_t(i) \tag{2.6}$$

where (log) productivity a_t follows some exogenous stochastic process. The demand schedule facing each firm is given by:

$$y_t(i) = -\varepsilon\left(p_t(i) - p_t\right) + c_t \tag{2.7}$$

For reasons discussed below, let me assume that employment is subsidized at a constant rate *v*. Hence, all firms face a common *real* marginal cost, which in equilibrium is given by

$$\begin{aligned} mc_t &= w_t - p_t - a_t - v \\ &= (\sigma + \varphi)\, y_t - (1 + \varphi)\, a_t - v \end{aligned} \tag{2.8}$$

In addition, and letting $n_t = \log \int_0^1 N_t(i)\, di$, one can derive the following mapping between labor input and output aggregates:[10]

$$n_t = y_t - a_t \tag{2.9}$$

II.4 Flexible Price Equilibrium

Suppose that all firms adjust prices optimally each period, taking the path of aggregate variables as given. The assumption of an isoelastic demand implies that they will choose a markup (defined as the ratio of price to marginal cost) given by $\frac{\varepsilon}{\varepsilon-1}$. That markup will be common across firms, and constant over time. Hence, it follows that the real marginal cost (i.e., the inverse of the markup) will also be constant, and given by

$$mc_t^* = -\mu \tag{2.10}$$

for all t, where $\mu \equiv \log\left(\frac{\varepsilon}{\varepsilon-1}\right)$.[11] Furthermore, given identical prices and

[10] For nondegenerate distributions of prices across firms the previous equation holds only up to a first-order approximation. More generally, we have $n_t = y_t - z_t^a + \xi_t$, where $\xi_t \equiv \log \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di$ can be interpreted as an indicator of relative price distortions. See Yun (1996) and King and Wolman (1996) for a detailed discussion.

[11] Henceforth, an asterisk is used to denote the equilibrium value of a variable under flexible prices

demand conditions, the same quantities of all goods will be produced and consumed.

Combining (2.10) with (2.8), (2.5), and (2.9) we can derive the equilibrium processes for output, hours, and the expected real rate under flexible prices:

$$\overline{y_t} = \gamma + \psi_a a_t \tag{2.11}$$

$$\overline{n}_t = \gamma + (\psi_a - 1) a_t \tag{2.12}$$

$$\overline{rr}_t = \rho + \sigma\psi_a E_t\{\Delta a_{t+1}\} \tag{2.13}$$

where $\psi_a \equiv \frac{1+\varphi}{\sigma+\varphi}$, and $\gamma \equiv \frac{\nu-\mu}{\sigma+\varphi}$. Henceforth, I refer to the above equilibrium values as the *natural* levels of (log) output, (log) hours, and the real interest rate, respectively.

Notice that, in the absence of nominal rigidities, the equilibrium behavior of the above variables is independent of monetary policy. Furthermore, if $\gamma = 0$, the equilibrium allocation under flexible prices coincides with the *efficient* allocation, i.e., the one that would obtain under flexible prices, perfect competition, and no distortionary taxation of employment (i.e., $v = \mu = 0$). Henceforth I assume that the government sets $v = \mu$, i.e. an employment subsidy that exactly offsets the distortion associated with monopolistic competition, thus making the flexible price equilibrium allocation correspond to the efficient one.[12]

II.5 Equilibrium with Staggered Price Setting

Following the Calvo formalism, each firm resets its price in any given period only with probability $1-\theta$ independently of other firms and of the time elapsed since the last adjustment. Let p_t^* denote the log of the price set by firms adjusting prices in period *t*.[13] One can show that a firm seeking to

[12] A similar assumption can be found in Woodford (1999), Obstfeld and Rogoff (1999), and Gali and Monacelli (1999).

[13] Notice that they will all be setting the same price, since they face an identical problem.

maximize its value, subject to (2.6) and (2.7), will choose the price of its good according to the (approximate) log-linear rule

$$p_t^* = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{mc_{t+k}^n\right\} \tag{2.14}$$

where $\mu \equiv \log\left(\frac{\varepsilon}{\varepsilon-1}\right)$; i.e., prices are set as a markup over a weighted average of current and expected future nominal marginal costs $\left\{mc_{t+k}^n\right\}$.[14]

By the law of large numbers, a measure 1-θ of producers reset their prices each period, while a fraction θ keep their prices unchanged. The evolution of the price level over time can be approximated by the log-linear difference equation:

$$p_t = \theta\, p_{t-1} + (1-\theta)p_t^* \tag{2.15}$$

If firms do not adjust prices optimally each period, real marginal costs will no longer be constant. On the other hand, in a perfect foresight steady state with zero inflation, all firms will be charging their desired markup. Hence, the steady state marginal cost, *mc*, will be equal to its flexible price counterpart (i.e., −μ). Let $\hat{mc}_t \equiv mc_t - mc$ denote the percent deviation of marginal cost from its steady state level. We can then combine (2.15) and (2.14), and after some algebra, obtain a simple stochastic difference equation describing the dynamics of inflation, with marginal costs as a driving force:

$$\pi_t = \beta\, E_t\left\{\pi_{t+1}\right\} + \lambda\, \hat{mc}_t \tag{2.16}$$

where $\lambda \equiv \theta^{-1}(1-\theta)(1-\beta\theta)$.

[14] In order to get some intuition for the form of that rule, let $\mu_{t,t+k} \equiv p_t^* - mc_{t+k}^n$ denote the markup in period t + k of a firm that last set its price in period t. We can rewrite (2.14) as $\mu = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\mu_{t,t+k}\right\}$ which yields a simple interpretation of the pricing rule: firms set prices at a level such that a (suitable) weighted average of anticipated future markups matches the optimal frictionless markup μ.

Furthermore, firms' inability to adjust prices optimally every period will generally imply the existence of a wedge between output and its natural level. Let me denote that wedge by $x_t \equiv y_t - \overline{y}_t$, and refer to it as the *output gap*. It follows from (2.8) that the latter will be related to marginal cost according to

$$\hat{mc}_t = (\sigma + \varphi)\, x_t \, . \tag{2.17}$$

Combining (2.16) and (2.17) yields a version of the so called New Phillips Curve (NPC):

$$\pi \;= \beta\, E_t \{ \pi_{t+1} \} + k\, x_t \tag{2.18}$$

where $k \equiv \lambda(\sigma + \varphi)$.

Using the goods market clearing condition $y_t = c_t$ we can rewrite Euler equation (2.4) in terms of the output gap and the natural rate of interest:

$$x_t = -\frac{1}{\sigma}\left(r_t - E_t\{\pi_{t+1}\} - \overline{rr}_t\right) + E_t\{x_{t+1}\} \tag{2.19}$$

Equations (2.18) and (2.19), together with a specification of monetary policy (i.e., of how the interest rate evolves over time), and of the exogenous process $\{ a_t \}$ (which determines the natural rate of interest), fully describe the equilibrium dynamics of the baseline model economy.

Having laid out the equations of the baseline sticky price model, I turn to a discussion of the consequences and desirability of alternative monetary policy responses to technology shocks.

III. Optimal Monetary Policy Responses to Changes in Technology

III.1 The Objectives of Monetary Policy

Modern economies are plagued by a variety of distortions: market power in goods and labor markets, the need to use (non-interest bearing) monetary assets in many transactions, distortionary taxes, uncorrected externalities of all sorts, etc. Many of those distortions are "aggregate" in nature, and are thus likely to have an impact on macroeconomic variables. But it would be naive and unrealistic to expect the central bank to seek to correct all those distortions. Attaining a first-best allocation is generally beyond the mandate of modern central banks; most important, however, is that it is likely to be unfeasible anyway. The principle of division of labor would seem to be desirable in policymaking as well, and other branches of government are likely to have more suitable tools than those under the control of the central bank to handle many of those distortions. Hence, it would seem desirable to assign the central bank with the task of correcting the distortions of a monetary nature.

Here I follow a number of recent papers and assume that the monetary authority's mandate is to correct the distortion associated with the presence of staggered price setting.[15] Notice that, once the government sets the employment subsidy at the appropriate level ($\nu = \mu$), the existence of nominal rigidities is the only explicit distortion left uncorrected in the model above. In that context, the natural level of output and employment coincide, by definition, with the efficient levels of those variables. In such an environment monetary policy should aim at attaining the allocation associated with the flexible price equilibrium. In that case the monetary authority focuses on correcting a distortion that is monetary in nature. The optimal policy thus requires that

$$x_t = \pi_t = 0$$

all t.

[15] See, e.g., Rotemberg and Woodford (1999), and Galí and Monacelli (1999), among others.

III.2 Optimal Interest Rate Rules

Given the price stability requirement, the path for the nominal rate consistent with the optimal policy will correspond to that of the real rate under the flexible price equilibrium, i.e., the natural real rate. Hence, and given (2.13) the optimal policy implies:

$$\begin{aligned} r_t &= \overline{rr}_t \\ &= \rho + \sigma \psi_a E_t\{\Delta a_{t+1}\} \end{aligned} \quad (3.1)$$

where we recall that $\psi_a \equiv \frac{1+\varphi}{\sigma+\varphi}$.

The behavior of the equilibrium interest rate can be easily grasped by considering the response of consumption to a technology shock in the flexible price case. The anticipation of higher productivity growth (i.e., $E_t\{\Delta a_{t+1}\} > 0$) brings about the expectation of gradual adjustment of consumption to a new, higher plateau; supporting that response pattern requires a higher interest rate.

Interestingly, however, (3.1) cannot be interpreted as a monetary policy rule that the central bank could follow mechanically, and which would guarantee that the optimal allocation is attained. In other words, even though in that case the desired outcome, $x_t = \pi_t = 0$, for all t, always satisfies all the equilibrium conditions, there exist many other sequences for $\{x_t, \pi_t\}$ which are also consistent with the set of equilibrium conditions, including genuinely random sequences that involve fluctuations in inflation and the output gap driven by self-fulfilling revisions in expectations (stationary sunspot fluctuations).

Fortunately, and a shown formally in the appendix, the previous indeterminacy problem can be avoided, and the uniqueness of the equilibrium allocation restored, by having the central bank follow a rule which would make the interest rate respond to inflation and/or the output gap were those variables to deviate from their (zero) target values. More precisely, suppose that the central bank commits itself to following the rule:

$$r_t = \overline{rr}_t + \phi_\pi \pi_t + \phi_x x_t \quad (3.2)$$

If we restrict ourselves to non-negative values of ϕ_π and ϕ_x, a necessary and sufficient condition for uniqueness of the efficient allocation is given by.[16]

$$k(\phi_\pi - 1) + (1 - \beta)\phi_x > 0 \quad (3.3)$$

Notice that, once uniqueness is restored and the efficient allocation is attained, the term $\phi_\pi \pi_t + \phi_x x_t$ appended to the interest rate rule vanishes, implying that $r_t = rr_t$ all t. Hence, we see that stabilization of both the output gap and inflation requires that the central bank credibly threatens to vary the interest rate sufficiently in response to any deviations of inflation and/or the output gap from target; yet, the very existence of that threat makes its effective application unnecessary.

III.3 Optimal Interest Rate Responses to Technology Shocks

Under the optimal policy, the equilibrium response of output, employment, and the interest rate to a technology shock will match that of their corresponding natural levels. Hence, the monetary policy response to such a shocks will hinge critically on the dynamic properties of productivity and its anticipated path. Let us next consider several possibilities.

III.3.1 Permanent Changes in the Growth Rate of Productivity

Suppose that the process followed by the technology parameter $\{a_t\}$ is non-stationary in first differences, so that innovations have a permanent effect on the rate of growth of productivity and potential output. For simplicity let me assume the following process:

$$\Delta a_t = \Delta a_{t-1} + \varepsilon_t \quad (3.4)$$

where $\{\varepsilon_t\}$ is white noise. That specification would seem to capture several episodes in recent U.S. macroeconomic history, including the productivity slowdown of the 70s, as well as the more recent arrival of

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[16] See, e.g., Bullard and Mitra (1999) for a formal proof.

the "new economy" era. The distinctive feature in all these cases is the existence of a sudden, unanticipated change in the pace of productivity growth which is expected to last indefinitely.

The consequences for the interest rate under the optimal policy can be easily derived; thus, combining (3.1) and (3.4):

$$\begin{aligned} r_t &= \rho + \sigma\psi_a \Delta a_t \\ &= r_{t-1} + \sigma\psi_a \varepsilon_t \end{aligned}$$

i.e., the interest rate–both nominal and real–inherits the random walk property assumed for productivity growth, with an acceleration (deceleration) in the latter bringing about a permanent increase (decrease) in interest rates.

Though the existence of a unit root in either productivity growth or real interest rates is rather questionable,[17] it is of interest to see to what extent the previous variables display positive comovements at low frequencies. Figure 1 displays the corresponding HP-filtered series for the real rate and labor productivity. Though the correlation between the two is far from perfect, their low frequency patterns show many similarities. In particular, they both appear to experience most significant turning points at about the same dates.

III.3.2 Permanent Shocks to the Level of Productivity

Innovations in technology may instead have a permanent effect on the level of productivity, but not on its growth rate. Suppose, as an illustration of that case, that the technology parameter $\{a_t\}$ follows the process:

$$\Delta a_t = (1-\rho_a)\gamma_a + \rho_a \Delta a_{t-1} + \varepsilon_t \qquad (3.5)$$

where $\{\varepsilon_t\}$ is white noise, $\rho_a \in [0,1)$ and γ_a denotes the average growth

[17] When applied to postwar U.S. data, standard unit root tests generally generally reject the null of a unit root in either variable at conventional significance levels.

rate of productivity. The implications of such a specification for potential output would seems to be more in line with the time series properties of U.S. GDP in the postwar period, for the latter variable is often characterized as having a unit root in its (log) level, but seldom in its growth rate. Combining (3.1) and (3.5), and letting $\gamma_y \equiv \psi_a \gamma_a$ denote the average output growth rate, we obtain:

$$\begin{aligned} r_t &= \rho + \sigma\gamma_y + \sigma\psi_a \rho_a (\Delta a_t - \gamma_a) \\ &= (1-\rho_a)(\rho + \sigma\gamma_y) + \rho_a r_{t-1} + \sigma\psi_a \rho_a \varepsilon_t \end{aligned}$$

Thus, we see that the stationarity of the interest rate—both nominal and real—is restored here. Its unconditional mean is given by

$$E\{r_t\} = \rho + \sigma\psi_a \gamma_a$$

and is thus increasing in average productivity growth. Given the positive serial correlation of productivity growth–a property inherited by output and consumption changes under the optimal policy–a positive (negative) technology shock requires an increase (decrease) in the interest rate. The persistence of that increase coincides with the persistence of the underlying technology process, and is required to support the gradual adjustment of consumption to its new permanent level.

III.3.3 Transitory Shocks to the Level of Productivity

Next I consider the case in which productivity displays stationary fluctuations around an upward deterministic trend, a characterization of the technology process often found in the RBC literature. Formally let me assume that the technology parameter $\{a_t\}$ follows the process:

$$a_t = a_0 + \gamma_a t + \hat{a}_t \qquad (3.6)$$

with

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_t \qquad (3.7)$$

where $\{\varepsilon_t\}$ is white noise, and $\rho_a \in [0,1)$. Such a specification of technology implies a trend stationary potential output, a characterization favored by many empirical macroeconomists. Combining (3.1) and (3.6), we can derive the implied equilibrium response of the interest rate to a technology shock under the optimal policy:

$$\begin{aligned} r_t &= \rho + \sigma\gamma_y - \sigma\psi_a(1-\rho_a)\hat{a}_t \\ &= (1-\rho_a)(\rho + \sigma\gamma_y) + \rho_a r_{t-1} - \sigma\psi_a(1-\rho_a)\varepsilon_t \end{aligned}$$

Thus, we see that under the assumption of a trend stationary technology, the sign of the optimal interest rate response to a productivity shock is reversed, relative to the previous case. Hence, a positive (negative) technology shock now requires a decrease (increase) in the interest rate, in a way consistent with the corresponding adjustment path of output and consumption.[18]

III.3.4 A Common Feature: Price Stability

The analysis above is meant to illustrate, by means of three examples, a more general characteristic of optimal monetary policy: the sign and pattern of the optimal interest rate response to changes in productivity depends critically on the dynamic properties of the latter variable and, in particular, of its forecastable component. This will also be generally true in versions of the staggered price setting model less abridged than the one developed above, and which may include capital accumulation, and external sector, etc. The reason is simple: under our assumptions, in the face of productivity changes, an optimizing central bank will seek to replicate the economy's response associated with the flexible price equilibrium, which will require setting the interest rate equal to its natural level; the latter, in turn, will generally depend in a non-trivial fashion on the properties of the model as well as the underlying technology process. Furthermore, recent work by Khan, King and Wolman (2000), and Goodfriend and Wolman (2000) has shown that, even in an environment in which the presence of uncorrected markup distortions and/or

[18] As in the previous case, the unconditional mean of the interest rate is given by

$$E\{r_t\} = \rho + \sigma\psi_a\gamma_a$$

and is thus increasing in average productivity growth.

transactions frictions prevent the flexible price allocation from being optimal, the optimal policy response to technology shocks involves near price stabilization.

That feature, together with the difficulties inherent in measuring total factor productivity at high frequencies, make it hard to assess the extent to which a central bank has responded optimally to changes in productivity in a given historical period. Interestingly, however, there is a dimension in which the optimal monetary policy response to technological change is independent of the nature and properties of the latter and which can be used to assess the extent to which a central bank has come close to the optimal policy: in all cases the interest rate response should seek to insulate the aggregate price level from any macroeconomic developments caused by the change in aggregate productivity. Intuitively, this requires choosing an interest rate pattern that would sustain the level of economic activity (and associated marginal costs) which is compatible with firms maintaining their desired markups at the existing prices (hence making unnecessary an adjustment of the latter).[19]

In the next section I provide some evidence, based on my recent work with López-Salido and Vallés, of the extent to which the Federal Reserve has acted in accordance to that principle in the postwar era.

IV. Price Stabilization in Response to Productivity Changes: Some Evidence for the Postwar U.S.

The purpose of the empirical exercise described in the present section is to estimate the effects of technology shocks on prices and inflation. The methodology used is based on a structural VAR in which technology shocks are identified as the only sources of the unit root in labor productivity; in other words, (permanent) technology shocks are the only

[19] In a recent paper King and Goodfriend (2000) argue forcefully that the requirement of price stability in response to productivity shocks will carry over, at least in an approximate sense, to more environments in which other distortions may be present (transctions friction, sticky wages, etc.). That finding also appears to be consistent with the analysis of Khan, King and Wolman (2000).

shocks which may have a permanent effect on the level of labor productivity. That identifying assumption was originally proposed in Galí (1999) and, as argued in that paper, it is likely to be satisfied for a broad range of business cycle models under relatively standard assumptions. In recent work with López-Salido and Vallés, I have applied a similar methodology in order to assess the nature of the Fed's response to an identified technology shock, using several simple policy rules as benchmarks.

Here I estimate a simple VAR model with two variables: (average) labor productivity, and inflation. I specify labor productivity in log first differences, in accordance with the maintained hypothesis of a unit root in that variable.[20]

The results below pertain to the sample period 1954:I-1999:IV. A number of authors have argued, on the basis of estimated interest rate rules, that U.S. monetary policy has experienced important structural changes over that period.[21]The existing evidence, as well as the related findings in my recent work with López-Salido and Vallés, suggest splitting the sample into two subperiods: the pre-Volcker years (54:I-79:II) and the more recent Volcker-Greenspan era (79:III-99:4).

Figure 2 displays the estimated dynamic responses of labor productivity growth and the rate of inflation to an identified technology shock, using data for the pre-Volcker era.[22]

In addition to the point estimates for each impulse response, the graphs also show a 95% confidence band for the null hypothesis that the response is zero at all horizons. The responses reported correspond to an adverse shock, whose size is normalized to one-standard deviation. That shock is shown to

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[20] Labor productivity is constructed as the ratio of GDP (GDPQ) to employee-hours in nonagricultural establishments (LPMHU). The price series used is the GDP deflator. All the series used are quarterly and were drawn from CITIBASE.

[21] See, e.g. Taylor (1993) and Clarida, Galí and Gertler (2000).

[22] The sample period 54:I-79:II encompasses the tenures of W.M. Martin, A. Burns, and Miller.

lead to a significant decline in average labor productivity on impact, with no further significant changes later on. Hence, the shock appears to have a permanent negative effect on the level of labor productivity. Most interestingly, and as shown in the bottom graph, the estimated response of inflation is significantly positive and highly persistent. In other words, the Fed failed to insulate the price level from the effects of the technology shock during that period, thus deviating from the optimal policy prescription derived in section 3. It is also clear from the estimates that, far from stabilizing prices, the Fed accommodates about half of the initial inflationary burst, with inflation remaining about 50 basis points higher three years after the shock. That finding appears to be consistent with the empirical interest rate rules that have been estimated for that period, and which invariably point to a weak response by the Fed to changes in inflationary pressures.[23]

The corresponding evidence for the Volcker-Greenspan era is shown in Figure 3. The pattern of the labor productivity response to an adverse technology shock appears to be very similar to the one estimated for the pre-Volcker period, though the initial impact of the shock is now slightly smaller. The key difference lies in the estimated response of inflation, which is hardly detectable. Thus, we clearly cannot reject the null hypothesis of full price stabilization in the face of a technology shock during the Volcker-Greenspan period. That pattern is consistent with the optimal policy in the presence of staggered price-setting. It also reinforces recent results in the literature on interest rate rules and which point to the adoption of a more anti-inflationary stance by the Fed in the past two decades.

V. Technological Change, the Output Gap, and the Design of Monetary Policy

V.1 Conceptual Issues

In the present section I turn to a discussion on the usefulness of output gap measures as an indicator for monetary policy, as well as the potential dangers associated with that use. Throughout, I use the simple model

[23] See, e.g., Clarida, Galí and Gertler (2000), Judd and Rudebusch (1999), and Taylor (1999).

developed in Section 2 as a reference framework, but it should be clear that the basic analysis, suitably modified, should also apply to more general environments.

The concept of "output gap" has traditionally played an important role in macroeconomics, both as a measure of the economy's cyclical position and as an indicator of potential inflationary pressures. That dual role is also present in the new Keynesian paradigm, as exemplified by the model laid out in Section 2. Thus, as shown in (2.18), inflation fluctuations are associated with variations in the output gap.[24] In addition, the output gap and its volatility also play an important role in welfare evaluation exercises: as shown in Rotemberg and Woodford (1997), the variance of the output gap is one of the key terms of a second order approximation to the equilibrium utility of the representative consumer, in the context of a model similar to the one sketched above.

Nevertheless, the concept of output gap associated with the new optimizing sticky price models entails substantial differences with the traditional one and, in particular, with the one used in most empirical applications. That discrepancy will be particularly relevant for economies that are subject to significant real turbulence. The latter may be the result, among other factors, of the sort of variations in the pace of technological change which are the focus of this paper. Consequently, and as argued below, the use of traditional output gap measures as an input in the monetary policy process may lead to unintended, inefficient outcomes. Let me next explain the nature of that discrepancy, and illustrate it using evidence for the postwar U.S. period.

In most existing empirical applications, the concept of output gap used would be better characterized as a measure of detrended output, i.e., devia-

[24]As should be clear from the derivation of (2.18) that relationship is not a primitive one. It arises from the proportionality between the output gap and markups (or real marginal costs) which holds under some standard, though by no means general, assumptions.

tions of log GDP from a *smooth* trend.[25] That trend is computed using one of a number of available procedures, but the main properties of the resulting series do not seem to hinge critically on the exact procedure used. This is illustrated in Figure 4, which plots three "output gap" series for the U.S. economy commonly used in empirical work. Those gap measures correspond to three alternative estimates of the trend: (a) a fitted quadratic function of time, (b) a Hodrick-Prescott filtered GDP series, and the (c) Congressional Budget Office's estimate of potential output. The fact that the resulting trend is a very smooth series has two implications. First, the bulk of the fluctuations in output at business cycle frequencies are attributed to fluctuations in the output gap. Second, the correlation among the three output gap measures is very high; which is chosen by a researcher is not likely to affect the outcome of any empirical exercise in a significant manner.

But, as argued in Galí and Gertler (1999), the use of detrended GDP as a proxy for the output gap does not seem to have a clear theoretical justification, at least in the context of the new optimizing sticky price models. In effect, the implicit assumption underlying traditional output gap measures is that the natural level of output $\{y_t^*\}$ can be well approximated by a smooth function of time. Yet, the theory implies that any shock (other than monetary shocks) will be a source of fluctuations in that natural level of output which, as a result, may be quite volatile and thus not well approximated by any smooth function of time.[26]

V.2 A Quantitative Example: Two Views of the U.S. Output Gap

Let me illustrate the previous point by means of a simple quantitative example. Under the assumptions made in the baseline model above, and as shown in equation (2.17), the true output gap is proportional to the log deviations of real marginal cost from steady state. But, the latter is easily

[25]Traditional output gap measures are used in a variety of contexts, including the estimation of empirical interest rate rules (e.g., Taylor (1993)), and the characterization of inflation dynamics (e.g., Fuhrer and Moore (1985)).

[26]See, e.g., Rotemberg and Woodford (1999) for an illustration of the quantitative importance of this point in the context of a calibrated version of a sticky price model.

shown to correspond, up to an additive constant, to the (log) labor income share s_t.[27]

$$mc_t = (w_t - p_t) - (y_t - n_t)$$
$$= (w_t + n_t) - (y_t + p_t) \equiv s_t$$

thus implying

$$s_t = \frac{s_t}{\sigma + \varphi}$$

Accordingly, we can use a time series for the (log) labor income share to approximate (up to a scalar factor and an additive constant) the "true", model-based, output gap. Figure 5 displays the corresponding measure for the U.S. output gap, under the assumption that $\sigma + \varphi = 2$. This is consistent, e.g., with parameter settings $\sigma = 1$ and $\varphi = 1$, values which arguably fall within a reasonable range. In addition, Figure 5 also shows the deviation of log GDP from a fitted quadratic trend, a popular proxy for the output gap in empirical applications.

Let me not emphasize here the apparent differences in volatility between the two series, since the model pins down the output gap only up to a scale factor (determined by the size of $\sigma + \varphi$). Instead I want to focus on their comovement: if detrended GDP was a good proxy for the output gap, we should observe a strong positive comovement between the two series. But a look at Figure 5 makes it clear that no obvious relationship exists; in fact, the contemporaneous correlation between them is slightly negative.

V.3 Output Gap Measures and Interest Rate Rules

The existence of variations in the pace of technological change is likely to be a source of large and persistent measurement error in traditional

[27] The key assumptions are: (a) firms are wage takers, (b) no labor adjustment costs, and (c) a constant elasticity of output with respect to labor input. All those assumptions can be relaxed, leading to some modification of the expression for marginal cost. See, e.g., Rotemberg and Woodford (1999) and Galí and Gertler (1999) for details.

output gap measures, since a smooth trend will hardly capture variations in potential output. As a result, sustained periods of unusually high output growth will sooner or later lead to positive values in conventional output gap measures. Most importantly, the latter are likely to be interpreted as a warning signal of overheating and forthcoming inflationary pressures, and thus lead to a tightening of policy. That response may be carried out independently of the underlying evolution of potential output, which is ignored by "statistical" measures of the output gap. As a result, a well intentioned stabilizing policy may turn against itself, and become a source of macroeconomic instability.

In a recent paper, Orphanides (1999) argues that a dramatic mismeasurement of potential output may have been at the root of the Great Inflation of the 70s in the U.S., warning us of the potential dangers of mechanistic responses to real-time data. In a similar vein, and using a calibrated version of the baseline model developed in Section 2 as a reference framework, I try to quantify next the consequences of having the central bank follow a simple interest rate rule, but using the wrong "output gap" measure.

V.3.1 The Performance of Alternative Interest Rate Rules

Suppose that the monetary authority follows a rule that makes the nominal rate respond systematically to the contemporaneous values of inflation and the output gap. Specifically, let me assume

$$r_t = rr + \phi_\pi \pi_t + \phi_x x_t \tag{5.1}$$

for all t, where $\overline{rr} \equiv \rho + \sigma\gamma_y$ corresponds to the natural interest rate evaluated at the steady state. Notice that this is a version of the rule put forward by John Taylor as a good characterization of U.S. monetary policy, and analyzed in numerous recent papers.[28]

If condition (6.2) is satisfied, equations (2.18), (2.19) and (5.1), together with a specification of the process followed by $\{a_t\}$, fully determine

[28] See Taylor (1993, 1999).

the economy's equilibrium. In general, (5.1) will differ from the optimal rule (3.2) and, hence, will lead to fluctuations in both the output gap and inflation. In order to quantify the magnitude of those fluctuations I calibrate the model as follows. For the coefficients in the interest rate rule I use the values suggested in Taylor (1993) for the inflation response, namely, $\phi_\pi = 1.5$ and $\phi_x = 0.5$, though below I will also report results using $\phi_x = 1$ and $\phi_x = 0$. For the price stickiness parameter θ I choose a setting of 0.75, which is consistent with the existing evidence (econometric and otherwise) of an average duration of prices of about one year. Preference parameters are set at the following values: $\sigma = 1$, $\beta = 0.99$, $\varphi = 1$. Productivity is assumed to follow a trend-stationary AR(1) process, as in equations (3.6)-(3.7). Calibrations of RBC models based on the properties of measured TFP tend to select a high value for the autoregressive coefficient; here I choose $\rho_a = 0.9$. The variance of the technology innovation is selected so that, given the remaining parameters, the standard deviation of the natural level of output is 1 percent, a convenient normalization.

The first row in Table 1 shows the implied standard deviations of a number of variables, expressed in percentage points, as implied by the model's equilibrium under the optimal policy rule (3.2), which we take as a benchmark.[29] The second row displays the corresponding statistics when the central bank follows a Taylor rule given by (5.1), with the baseline coefficient settings. As is clear from a quick glance at the numbers, the differences in volatility between the two rules are not large, which constitutes a further illustration of a result found in a variety of papers: the Taylor rule appears to provide a good approximation to the optimal policy in economies like the one considered here.[30]

The last column of Table 1 reports the loss in welfare, expressed as a percent of steady state consumption, which result from the central bank's adherence to a suboptimal rule. Such losses can be approximated by the expression.[31]

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[29]For ease of interpretation, I report the standard deviation of the *annualized* rate of inflation.

[30]See, e.g., the contributions to the Taylor (1999) volume.

[31]See Woodford (1999) for a derivation.

$$\frac{1}{2}\left((\sigma+\varphi)\operatorname{var}(x_t)+\frac{\varepsilon}{\lambda}\operatorname{var}(\pi_t)\right) \tag{5.2}$$

Clearly, under a Taylor rule of the form given by (5.1), and as a direct consequence of the implied low variability of both inflation and the output gap, the magnitude of the welfare losses is almost negligible.

The third row in Table 1 shows the corresponding second moments under the assumption that the monetary authority follows a rule of the form:

$$r_{tt}=\overline{rr}+\phi_{\pi}\pi_t+\phi_x\hat{y}_t \tag{5.3}$$

where $\hat{y}_t$ denotes the deviations of (log) output from its steady state (or balanced growth path) level, i.e. detrended output, for short. This is precisely how a central bank that meant to follow a Taylor rule would behave, were it to respond to a "conventional" measure of the output gap (detrended output), instead of the "true" one (which is not directly observable, anyway). As shown in Table 1, the alternative rule turns out to have rather destabilizing effects for both inflation and the output gap. On the other hand, and as one could have anticipated, that rule leads to a dampening of output fluctuations. But in an environment like the one considered here in which real shocks are dominant, all attempts to stabilize output (instead of accommodating the changes in its natural level) will tend to amplify nominal instability and will have a negative impact on welfare. In the example under consideration, the latter amount to a permanent decrease in the level of consumption of one quarter of a percent, a loss that is roughly 40 times larger than under the output gap-based rule.

Not surprisingly, the magnitude of the discrepancy in the performance of the two rules, as well as the associated welfare losses are only magnified if the monetary authority chooses a higher output coefficient, i.e., if it decided to respond more aggressively to fluctuations in the output gap or detrended output. This is illustrated in the subsequent panel of Table 1 which reports statistics for the case $\phi_x=1$. While the output gap-based rule gets the equilibrium allocation even closer to the first best, the destabilizing effects of a rule that ignores the changes in potential output, as well as the associated welfare losses, are now significantly larger.

The difficulties in measuring the true output gap, and thus the potential non-operationality of a rule of the form (5.1) do not provide a justification for following a rule based on detrended output. As illustrated by the results reported in the bottom panel of Table 1, a pure inflation targeting rule (corresponding to $\phi_x = 0$) will generally dominate a rule based on detrended output.[32] The advantages of an inflation targeting rule are manifold: it is easy to understand and communicate, and it can be easily implemented in real time.

VI. Concluding Remarks

The present paper has analyzed the implications of technological change for the design and conduct of monetary policy, using an optimizing sticky price model as a reference framework. I have shown how the optimal policy seeks to insulate the price level from the effects of changes in productivity and described some evidence that suggests that the Federal Reserve has responded to technological change in a way consistent with that rule in the Volcker-Greenspan era, but not during the pre-Volcker period. The second part of the paper has discussed the conceptual difference between the notion of output gap arising in the new sticky price paradigm, and conventional measures of that variable. I have also provided some evidence, based on postwar U.S. data, of the quantitative significance of that discrepancy. Finally, I have performed a simple exercise to illustrate how a well intentioned monetary policy designed to stabilize inflation and the output gap could lead to unnecessary instability in both variables if a conventional (but incorrect) measure of the output gap was used.

[32] A similar conclusion is reached in Rotemberg and Woodford (1999) and Orphanides (1999), among others.

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APPENDIX

Notice that, after plugging (3.1) into (2.19), the equilibrium dynamics for inflation and the output gap can be represented by means of the stochastic difference equation:

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \mathrm{A}_{\mathrm{O}} \begin{bmatrix} E_t\{x_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} \qquad (6.1)$$

where

$$\mathrm{A}_{\mathrm{O}} \equiv \begin{bmatrix} 1 & \sigma^{-1} \\ k & \beta + k\sigma^{-1} \end{bmatrix}$$

The desired outcome of an inflation targeting policy, $x_t = \pi_t = 0$, for all *t*, always constitutes a solution to (6.1). Yet, a necessary and sufficient condition for the uniqueness of such a solution in a system with no predetermined variables like (6.1) is that the two eigenvalues of $\mathbf{A}_{\mathrm{O}}$ lie inside the unique circle.[33] It is easy to check, however, that such a condition is *not* satisfied in our case. More precisely, while both eigenvalues of $\mathbf{A}_{\mathrm{O}}$ can be shown to be real and positive, only the smallest one lies in the [0,1] interval. As a result there exists a continuum of solutions in a neighborhood of (0,0) that satisfy the equilibrium conditions (local indeterminacy). Furthermore, one cannot rule out the possibility of equilibria displaying fluctuations driven by self-fulfilling revisions in expectations (stationary sunspot fluctuations).

Under the rule (3.2), the equilibrium is described by a stochastic difference equation like (6.1), with $\mathbf{A}_{\mathrm{O}}$ replaced with

$$\mathrm{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma k & k + \beta(\sigma + \phi_x) \end{bmatrix}$$

where $\Omega \equiv \frac{1}{\sigma + \phi_x + k\phi_\pi}$ If we restrict ourselves to non-negative values of

[33] See, e.g., Blanchard and Kahn (1982).

ϕ_π and ϕ_x , a necessary and sufficient condition for A_T to have both eigenvalues inside the unit circle (thus implying uniqueness of the (0,0) solution to (6.1) is given by[34]

$$k(\phi_\pi - 1) + (1 - \beta)\phi_x > 0 \qquad (6.2)$$

Substituting (5.1) into expressions (2.18) and (2.19) the equilibrium dynamics are represented by the system:

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = A_R \begin{bmatrix} E_t\{x_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + B_R \Delta a_t$$

where

$$A_R \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma k & k + \beta(\sigma + \phi_x) \end{bmatrix} \quad ; \quad B_R \equiv -\sigma(1 - \rho_a)\psi\Omega \begin{bmatrix} 1 \\ k \end{bmatrix}$$

and $\Omega \equiv \frac{1}{\sigma + \phi_x + k\phi_\pi}$.

[34] See, e.g., Bullard and Mitra (1999) for a formal proof.

Table 1
Performance of Alternative Interest Rate Rules

	$\sigma(\pi)$	$\sigma(x)$	$\sigma(y)$	*Welfare Loss*
Optimal Rule	0.00	0.00	1.00	0.00
Taylor Rule				
$\phi_x = 0.5$				
Output Gap	0.40	0.06	0.93	0.006
Detrended Output	2.44	0.38	0.61	0.241
$\phi_x = 1$				
Output Gap	0.30	0.04	0.95	0.003
Detrended Output	3.38	0.53	0.46	0.465
Inflation Targeting	0.60	0.09	0.90	0.014

Figure 1
Productivity Growth and the Real Interest Rate

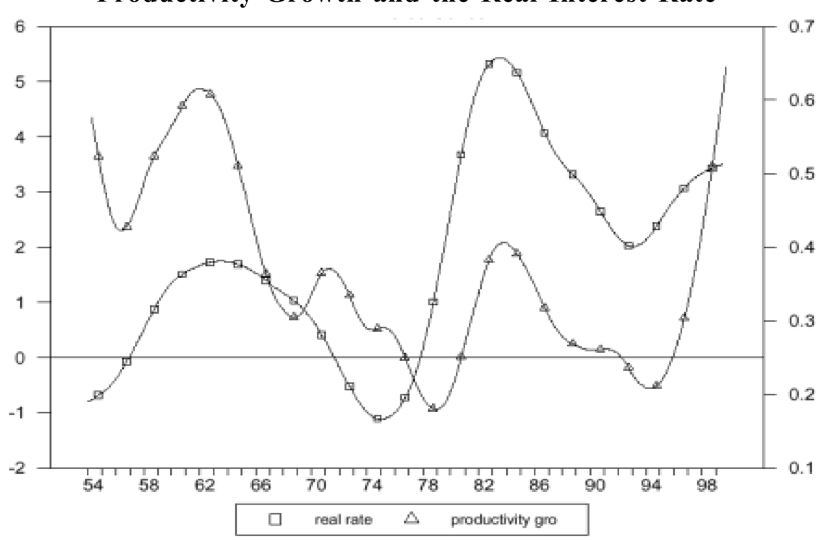

Figure 2
Productivity Shocks and Price Stability in the Pre-Volcker Era

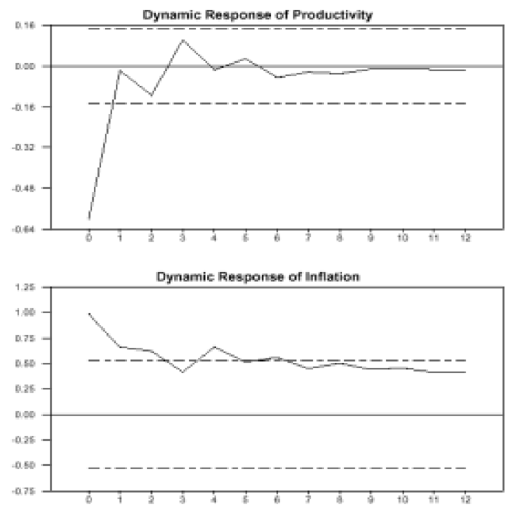

Figure 3
Productivity Shocks and Price Stability in the Volcker-Greenspan Era

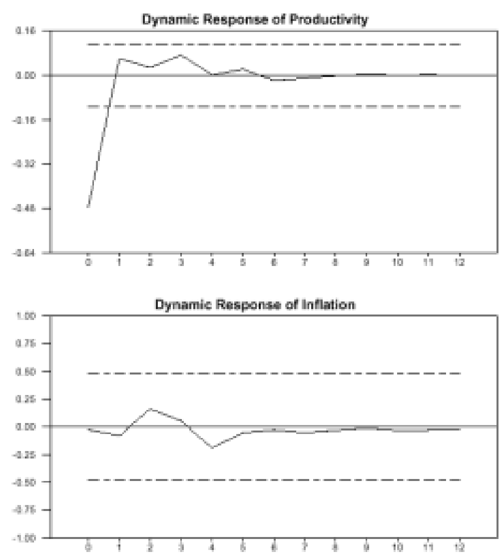

Figure 4
Three Conventional Output Gap Measures

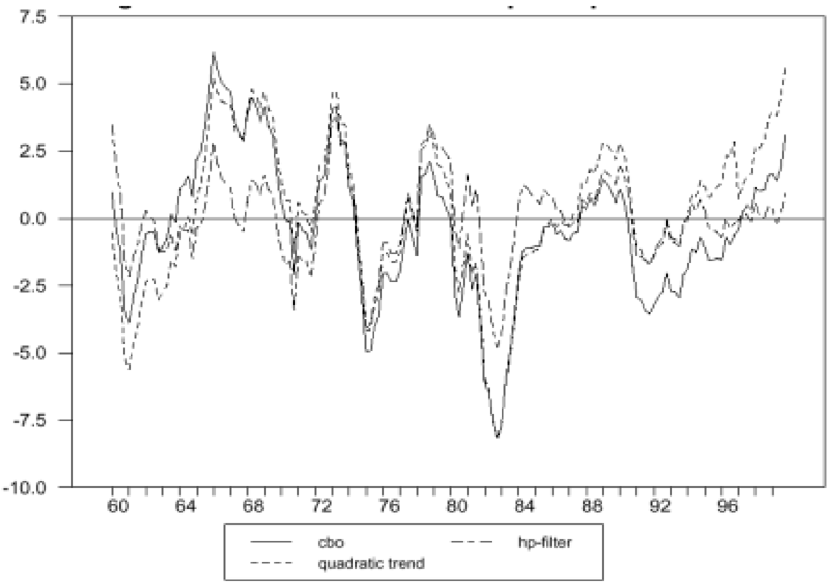

Figure 5
Model-Based Output Gap Vs. Detrended GDP