

# **Monetary Policy and the Open Economy**

by

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## Motivation

- The basic new Keynesian model for the closed economy
  - equilibrium dynamics: simple three-equation representation
  - ability to match much of the evidence on the effects of monetary policy and technology shocks
  - monetary policy: optimality of inflation targeting
- How does the introduction of open economy elements affect that analysis and prescriptions?
- What role should the exchange rate play in the design of policy? What is the optimal degree of exchange rate volatility?
- Issue: many possible combinations of assumptions on size, market completeness, pass-through, etc.

# A Baseline New Keynesian Model of the Small Open Economy (Galí 2015, GM 2005)

## Households

Representative household maximizing

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t; Z_t)$$

where

$$\begin{aligned} C_t &\equiv \left( (1 - v)^{\frac{1}{\eta}} C_{H,t}^{1-\frac{1}{\eta}} + v^{\frac{1}{\eta}} C_{F,t}^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \\ C_{H,t} &\equiv \left( \int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \end{aligned}$$

subject to

$$\int_0^1 P_{H,t}(i) C_{H,t}(i) di + P_{F,t} C_{F,t} + E_t \{ Q_{t,t+1} D_{t+1} \} \leq D_t + W_t N_t$$

Assumption:

$$U(C_t, N_t; Z_t) = \begin{cases} \left( \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) Z_t & \text{for } \sigma \neq 1 \\ \left( \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right) Z_t & \text{for } \sigma = 1 \end{cases}$$

with  $z_t \equiv \log Z_t = \rho_z z_{t-1} + \varepsilon_t^z$

- Optimal allocation of expenditures

(i) Domestic goods:

$$C_{H,t}(i) = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon} C_{H,t} \quad (1)$$

for all  $i \in [0, 1]$ , where  $P_{H,t} \equiv \left( \int_0^1 P_{H,t}(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$

$$\Rightarrow \int_0^1 P_{H,t}(i) C_{H,t}(i) di = P_{H,t} C_{H,t}$$

(ii) Domestic vs. Foreign

$$C_{H,t} = (1 - v) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \quad ; \quad C_{F,t} = v \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \quad (2)$$

where  $P_t \equiv [(1 - v)(P_{H,t})^{1-\eta} + v(P_{F,t})^{1-\eta}]^{\frac{1}{1-\eta}}$

$$\Rightarrow P_{H,t} C_{H,t} + P_{F,t} C_{F,t} = P_t C_t$$

- Other optimality conditions

$$Q_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{Z_{t+1}}{Z_t} \right) \left( \frac{P_t}{P_{t+1}} \right)$$

$$Q_t = E_t \{ Q_{t,t+1} \} = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{Z_{t+1}}{Z_t} \right) \left( \frac{P_t}{P_{t+1}} \right) \right\}$$

$$C_t^\sigma N_t^\varphi = \frac{W_t}{P_t}$$

- Some definitions and identities

Terms of trade:

$$s_t \equiv \log \mathcal{S}_t = p_{F,t} - p_{H,t}$$

CPI:

$$\begin{aligned} p_t &= (1 - v)p_{H,t} + vp_{F,t} \\ &= p_{H,t} + vs_t \end{aligned}$$

CPI vs. Domestic inflation:

$$\pi_t = \pi_{H,t} + v\Delta s_t$$

where  $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$

- Law of one price

$$p_{F,t} = e_t + p_t^*$$

- Real exchange rate

$$\begin{aligned} q_t &\equiv p_{F,t} - p_t \\ &= s_t + p_{H,t} - p_t \\ &= (1 - v)s_t \end{aligned}$$

- International risk sharing

$$\begin{aligned} \beta \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \left( \frac{P_t^*}{P_{t+1}^*} \right) \left( \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right) &= Q_{t,t+1} \\ \Rightarrow C_t &= \vartheta C_t^* Z_t^{\frac{1}{\sigma}} Q_t^{\frac{1}{\sigma}} \end{aligned}$$

$$\begin{aligned} c_t &= y_t^* + \frac{1}{\sigma} (z_t + q_t) \\ &= y_t^* + \frac{1}{\sigma} z_t + \left( \frac{1-v}{\sigma} \right) s_t \end{aligned}$$

given  $c_t^* = y_t^*$

## Firms

- Technology

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

where  $a_t \equiv \log A_t = \rho_a a_{t-1} + \varepsilon_t^a$ .

- Optimal price setting

$$\bar{p}_{H,t} = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t\{\psi_{t+k|t}\}$$

- Domestic inflation dynamics

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} - \lambda \hat{\mu}_t$$

where  $\hat{\mu}_t \equiv p_{H,t} - \psi_{t+k} - \mu$  and  $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$

## Exports

$$X_t(i) = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon} X_t$$

where  $X_t \equiv \left( \int_0^1 X_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$  given by

$$\begin{aligned} X_t &= v \left( \frac{P_{H,t}}{\mathcal{E}_t P_t^*} \right)^{-\eta} Y_t^* \\ &= v \mathcal{S}^\eta Y_t^* \end{aligned}$$

## Equilibrium

- Goods market clearing

$$\begin{aligned} Y_t(i) &= C_{H,t}(i) + X_t(i) \\ &= \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon} \left[ (1 - v) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + v \mathcal{S}_t^\eta Y_t^* \right] \end{aligned}$$

for all  $i \in [0, 1]$  and all  $t$ . Combined with  $Y_t \equiv \left( \int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$

$$Y_t = (1 - v) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + v \mathcal{S}_t^\eta Y_t^*$$

Log-linearized version:

$$y_t = (1 - v)c_t + v(2 - v)\eta s_t + vy_t^*$$

Euler equation (in terms of  $\pi_{H,t+1}$ ):

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{H,t+1}\} - \rho) + \frac{v}{\sigma}E_t\{\Delta s_{t+1}\} + \frac{1}{\sigma}(1 - \rho_z)z_t$$

Terms of trade (using IRS condition):

$$s_t = \sigma_v(y_t - y_t^*) - (1 - v)\Phi z_t$$

where  $\sigma_v \equiv \sigma\Phi > 0$  with  $\Phi \equiv \frac{1}{1+v(\varpi-1)} > 0$  and  $\varpi \equiv \sigma\eta + (1 - v)(\sigma\eta - 1)$ .

Combining them all

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma_v}(i_t - E_t\{\pi_{H,t+1}\} - r_t^n)$$

where

$$r_t^n = \rho + \sigma_v E_t\{\Delta y_{t+1}^n\} + \sigma_v v(\varpi - 1) E_t\{\Delta y_{t+1}^*\} + \Phi(1 - v)(1 - \rho_z) z_t$$

- Labor market clearing

$$N_t \equiv \int_0^1 N_t(i) di = \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\frac{\epsilon}{1-\alpha}} di$$

Up to a first-order approximation:

$$y_t = a_t + (1 - \alpha)n_t$$

- Price markups and the output gap

$$\begin{aligned} \mu_t &= p_{H,t} - (w_t - a_t + \alpha n_t) \\ &= -(w_t - p_t) - (p_t - p_{H,t}) + a_t - \alpha n_t \\ &= -(\sigma c_t + \varphi n_t) - v s_t + a_t - \alpha n_t \\ &= -\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) y_t + v(\varpi - 1) s_t + \left(1 + \frac{\varphi + \alpha}{1 - \alpha}\right) a_t - v z_t \end{aligned}$$

Accordingly,

$$\hat{\mu}_t = -\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) \tilde{y}_t + v(\varpi - 1) \tilde{s}_t$$

and

$$\begin{aligned} y_t^n &= \Gamma_a a_t + \Gamma_z z_t + \Gamma_* y_t^* \\ s_t^n &= \sigma_v (y_t^n - y_t^*) - (1 - v) \Phi z_t \end{aligned}$$

with  $\Gamma_a \equiv \frac{1+\varphi}{\sigma_v(1-\alpha)+\varphi+\alpha} > 0$ ,  $\Gamma_* \equiv -\frac{v(\varpi-1)\sigma_v(1-\alpha)}{\sigma_v(1-\alpha)+\varphi+\alpha}$  and  $\Gamma_z \equiv -\frac{v\varpi\Phi(1-\alpha)}{\sigma_v(1-\alpha)+\varphi+\alpha}$ .

Using the fact that  $\tilde{s}_t = \sigma_v \tilde{y}_t$ ,

$$\begin{aligned} \hat{\mu}_t &= - \left( \sigma_v + \frac{\varphi + \alpha}{1 - \alpha} \right) \tilde{y}_t \\ \pi_{H,t} &= \beta E_t \{ \pi_{H,t+1} \} + \kappa_v \tilde{y}_t \end{aligned}$$

where  $\kappa_v \equiv \lambda \left( \sigma_v + \frac{\varphi + \alpha}{1 - \alpha} \right)$ . Also

$$r_t^n \equiv \rho - \sigma_v \Gamma_a (1 - \rho_a) a_t + \Psi_* E_t \{ \Delta y_{t+1}^* \} + \Psi_z (1 - \rho_z) z_t$$

with  $\Psi_* \equiv \sigma_v (v(\varpi - 1) + \Gamma_*)$  and  $\Psi_z \equiv (1 - v) \Phi - \sigma_v \Gamma_z$ . Note that  $\lim_{v \rightarrow 0} \Psi_* = 0$  and  $\lim_{v \rightarrow 0} \Psi_z = 1$ .

## Equilibrium Dynamics under a Taylor-type Rule

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa_v \tilde{y}_t$$

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma_v}(i_t - E_t\{\pi_{H,t+1}\} - r_t^n)$$

$$i_t = \rho + \phi_\pi \pi_{H,t} + \phi_y \hat{y}_t + v_t$$

where  $v_t = \rho_v v_{t-1} + \varepsilon_t^v$  and  $r_t^n \equiv \rho - \sigma_v \Gamma_a (1 - \rho_a) a_t + \Psi_* E_t\{\Delta y_{t+1}^*\} + \Psi_z (1 - \rho_z) z_t$   
 Equivalently

$$\begin{bmatrix} \tilde{y}_t \\ \pi_{H,t} \end{bmatrix} = \mathbf{A}_v \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\pi_{H,t+1}\} \end{bmatrix} + \mathbf{B}_v u_t$$

where

$$u_t \equiv \hat{r}_t^n - \phi_y \hat{y}_t^n - v_t$$

$$\mathbf{A}_v \equiv \Omega_v \begin{bmatrix} \sigma_v & 1 - \beta \phi_\pi \\ \sigma_v \kappa_v & \kappa_v + \beta(\sigma_v + \phi_y) \end{bmatrix} \quad ; \quad \mathbf{B}_v \equiv \Omega_v \begin{bmatrix} 1 \\ \kappa_v \end{bmatrix}$$

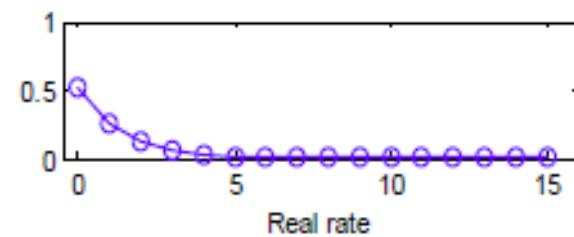
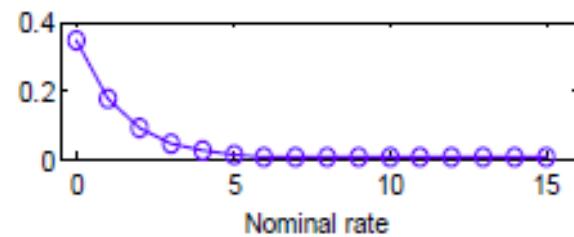
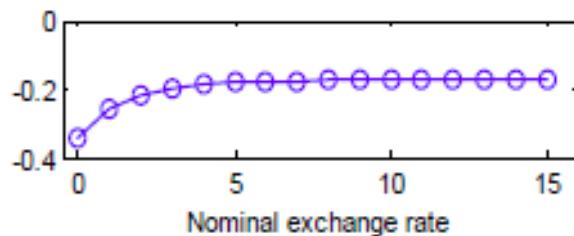
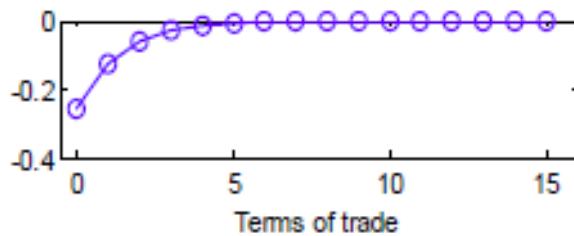
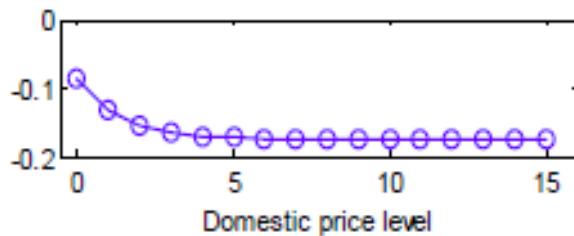
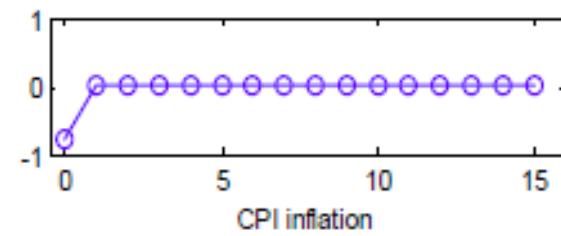
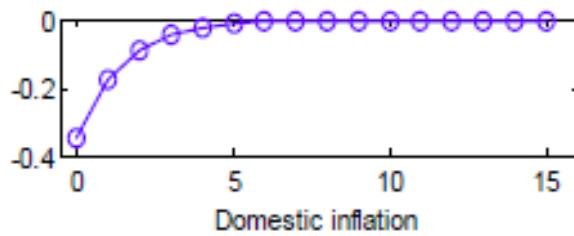
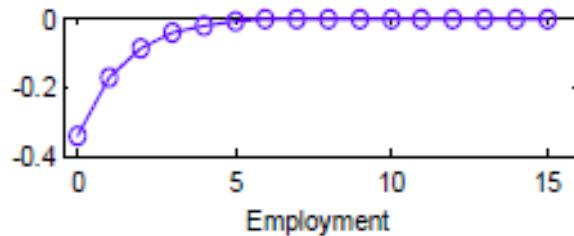
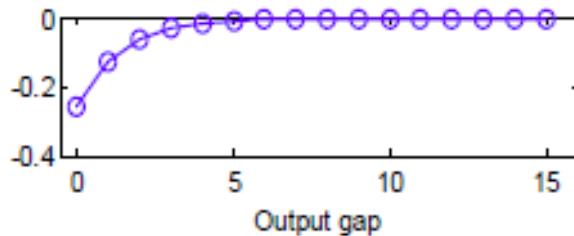
with  $\Omega_v \equiv \frac{1}{\sigma_v + \phi_y + \kappa_v \phi_\pi}$ .

- *Uniqueness condition*

$$\kappa_v(\phi_\pi - 1) + (1 - \beta)\phi_y > 0$$

- Effects of a monetary policy shock

## Dynamic responses to a monetary policy shock



## Optimal Monetary Policy: A Special Case

- Assumptions:

$$Z_t = 1 \text{ all } t$$

$$\sigma = \eta = 1$$

- Social Planner's Problem

$$\max U(C_t, N_t; 1)$$

subject to the consumption/output possibilities set

$$C_t = Y_t^{1-v} (Y_t^*)^v$$

Optimal allocation

$$-\frac{U_n(C_t, N_t)}{U_c(C_t, N_t)} = (1 - v)(1 - \alpha) \frac{C_t}{N_t}$$

Under the assumed preferences:

$$\begin{aligned} C_t N_t^\varphi &= (1 - v)(1 - \alpha) \frac{C_t}{N_t} \\ \Rightarrow N &= [(1 - v)(1 - \alpha)]^{\frac{1}{1+\varphi}} \end{aligned}$$

- Flexible price equilibrium

$$\mathcal{M} = \frac{P_{H,t}}{W_t / MPN_t} = \frac{(1 - \alpha)(Y_t/N_t)}{(1 - \tau)C_t N_t^\varphi S_t^v} = \frac{(1 - \alpha)}{(1 - \tau)N_t^{1+\varphi}}$$

Optimal subsidy:

$$(1 - \tau)(1 - v)\mathcal{M} = 1$$

- Implementation

$$\tilde{y}_t = \pi_{H,t} = 0$$

$$i_t = r_t^n$$

Optimal interest rate rule:

$$i_t = r_t^n + \phi_\pi \pi_{H,t}$$

- Macroeconomic implications of domestic inflation targeting

$$\begin{aligned} s_t^n &= \sigma_v(y_t^n - y_t^*) - (1 - v)\Phi z_t \\ &= \sigma_v \Gamma_a a_t + \sigma_v(\Gamma_* - 1)y_t^* + [\sigma_v \Gamma_z - (1 - v)\Phi]z_t \end{aligned}$$

$$e_t^{DIT} = s_t^n - p_t^*$$

$$\begin{aligned} p_t^{DIT} &= v s_t^n \\ &= v(e_t^{DIT} + p_t^*) \end{aligned}$$

## Evaluation of Alternative Policies

- Welfare losses (under special case)

$$\begin{aligned}\mathbb{W} &= \frac{(1-v)}{2} \sum_{t=0}^{\infty} \beta^t \left( \left( \frac{1+\varphi}{1-\alpha} \right) \tilde{y}_t^2 + \frac{\epsilon}{\lambda} \pi_{H,t}^2 \right) \\ \mathbb{V} &= \frac{(1-v)}{2} \left[ \left( \frac{1+\varphi}{1-\alpha} \right) \text{var}(\tilde{y}_t) + \frac{\epsilon}{\lambda} \text{var}(\pi_{H,t}) \right]\end{aligned}$$

- Four suboptimal rules

$$\pi_t = 0$$

$$e_t = 0$$

$$i_t = 0.01 + 1.5\pi_{H,t} + 0.125\hat{y}_t$$

$$i_t = 0.01 + 1.5\pi_t + 0.125\hat{y}_t$$

**Table 8.1 Properties of Simple Policy Rules**

	<i>SDIT</i>	<i>SCIT</i>	<i>PEG</i>	<i>FDIT</i>	<i>FCIT</i>
$\sigma(y)$	2.29	1.92	1.72	1.85	1.76
$\sigma(\tilde{y})$	0	0.66	0.92	0.44	0.62
$\sigma(\pi_H)$	0	0.19	0.35	0.69	0.58
$\sigma(\pi)$	0.41	0	0.21	0.70	0.52
$\sigma(s)$	2.29	1.92	1.72	1.85	1.76
$\sigma(\Delta e)$	1.02	0.29	0	0.95	0.59
$L$	0	0.05	0.17	0.61	0.43