# APPENDIX to "Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?"

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# 1. Unit Root and Cointegration Tests

## 1.1. U.S. Data

Panel (a) in Table A-1 [Table 1 in Gali (1996)] reports the results of Augmented Dickey Fuller (ADF) unit root tests applied to the levels and first differences of the U.S. time series used in the different specifications of the bivariate model discussed in the text. The test fails to reject the null of a unit root in the levels of all the series (at a 5% significance level), but it systematically reject the same null when applied to their first differences. Those results suggest a characterization of $\{[x_t, n_t]'\}$ as an $I(1)$ process (regardless of whether hours or employment is used), thus motivating the benchmark VAR specification used. Nevertheless, and in order to check the robustness of the results, a VAR model with detrended hours (or employment) was also estimated (see discussion of results in the main text).

The results of ADF unit root tests applied to the remaining time series used in the five-variable VAR are reported in Panel (b) of Table A-1. They are consistent with the hypothesis of a unit root in the nominal rate ($r_t$), the growth rate of the money supply ($\Delta m_t$), and inflation ($\Delta p_t$). Furthermore, the tests do not reject the null that money growth ($\Delta m$) and inflation ($\Delta p$) are cointegrated with cointegrating vector $[1, -1]$, implying a stationary process for the rate of growth of real balances, $\{\Delta m_t - \Delta p_t\}$. Analogous properties also seem to hold for the nominal rate $r$ and inflation ($\Delta p_t$), implying a stationary (ex-post) real interest rate process $\{r - \Delta p_{t+1}\}$. Such a characterization implies that those three variables have a single common trend (as would be predicted by many plausible monetary models), and is consistent with the findings of many other authors (e.g., Shapiro and Watson (1988), Gali (1992)). As is well known, the presence of cointegration prevents us from estimating a VAR "in first differences," since such a representation does not exist. That leads me to estimate a VAR for $[\Delta x_t,\ \Delta n_t,\ \Delta m_t - \Delta p_t,\ r_t - \Delta p_t,\ \Delta^2 p_t]'$ (with $\widehat{n}_t$ replacing $\Delta n_t$, when noted).

I also checked the possibility of alternative cointegrating relationships involving the nominal variables, as well as the real variables $x$ and/or $n$, by means of a Johansen test. the results turned out to be rather ambiguous and often difficult to interpret. When implemented on the five variable vector $[x_t,\ n_t,\ \Delta m_t, \Delta p_t, r_t]'$ using hours as a labor input measure the Johansen procedure (based on the trace statistic) pointed to the presence of a cointegration rank equal to 2 (in a way consistent with the results above), but a test of the joint hypothesis that vectors $[0, 0, 1, -1, 0]$ and $[0, 0, 0, -1, 1]$ belong to the cointegration space is rejected at conventional significance levels. On the other hand, when employment was used as a labor input measure the Johansen procedure did not reject the hypothesis of a cointegration rank equal to one, corresponding to a cointegrating vector $[0, 0, 1, -1, 0]$ (i.e., stationarity of the growth rate of real balances).

Given the previous findings, and in order to make sure that none of the qualitative results obtained for the five-variable model hinged on the cointegration assumptions implicit in the specification of the VAR, I repeated the exercise using the estimates of the VAR "in first differences" (as would be appropriate in the absence of cointegration), i.e., a VAR for the five-variable vector $[\Delta x_t,\ \Delta n_t,\ \Delta^2 m_t,\ \Delta r_t,\ \Delta^2 p_t]'$ (with $\widehat{n}_t$ replacing $\Delta n_t$, when noted). The estimates of productivity-employment conditional correlations, reported in table A-3 of this appendix, are very similar to those reported and discussed in the main text for the five-variable VAR, both in terms of the signs and the sizes of the estimated correlations (the same was true for the implied impulse responses).

### 1.2. International Data

Table A-2 in this appendix [Table 3 in Gali (1996)] reports the results of ADF unit root tests on the levels and first differences of (log) employment ($n$), (log) productivity ($x$), and (log) GDP ($y$), for each the G7 countries other than the U.S.. Description of the data can be found in the main text. With a few exceptions the test results are consistent with those obtained in the U.S.: they point to the presence of a unit root in the levels of employment, productivity and output series, but they tend to reject a unit root in their first differences (at a 5% significance level). The exceptions lie in the results for some of the employment series. Thus, for France the test rejects a unit root in $n$, a result that leads me to adjust the VAR specification accordingly for that country . On the other hand, U.K., Germany, and Italy I cannot reject (marginally) the null of a unit root in $\Delta n$. The latter result, however, is likely to reflect the low power of the test, since it is not consistent with the parallel rejection of a unit root in $\Delta y$ and $\Delta x$ in the same countries. Most importantly for my identification strategy, the characterization of the productivity series as being integrated of order one holds in each of the six countries considered.

## 2. A Monte Carlo Simulation

In this section I report the results from a Montecarlo simulation which aims to answer the following question: how (un)likely would it be for a standard RBC model to generate equilibrium time series for hours and productivity (with a number of observation similar to my sample size) that could lead, when decomposed following the procedure discussed in the text, to estimates of conditional correlations with sign and size similar to the ones I obtained ?

As a data-generating model I used Christiano and Eichenbaum (1992) model, which is a standard one-sector RBC model with two sources of fluctuations: exogenous variations in technology (which follows an random-walk) and government purchases (which follows a stationary AR(1) process when expressed in terms of efficiency units of labor). After setting the values of all parameters according to their benchmark calibration (which is based, in part, on GMM estimates), I generated 500 equilibrium time series for the vector $[\Delta x_t, \widehat{n}_t]'$, each containing 180 observations, and computed estimates of conditional correlation for each of them, based on the bivariate VAR model discussed in the main text.[1] Table A-4 in this appendix summarizes the results of that exercise. The mean (across the 500 replications) of the productivity-hours correlation conditioned on technology is positive and high (0.75), with a 5th centile of 0.46. On the other hand, the mean of the correlation conditioned on govenrment purchases as only sources of fluctuations is negative (−0.43), with a 90th centile of 0.13 and a 95th centile of 0.30.

The previous Monte Carlo results suggest that the estimates of the productivity-labor input correlations conditional on technology found in the paper (which, with the exception of Japan, are always negative and larger than 0.5 in absolute value) would extremely unlikely (if not impossible) if the data had been generated by the RBC model. On the other hand, positive (but low) values for the estimated correlations conditional on non-technology shocks fall within the bounds of the confidence interval for that statistic. Yet, for a majority of countries and specifications my estimates lie either outside of that interval or very close to the 95th centile (and with the exception of Japan estimates and the five-variable model using detrended hours, they are always above the 90th centile), an outcome which would also be very unlikely if if the data had been generated by the RBC model.

In light of the previous Monte Carlo results I conclude that, were the data to have been generated by a standard RBC economy like Christiano and Eichenbaum's, the probability of obtaining conditional correlation estimates with the magnitude and sign patterns of those reported in my paper would be extremely

[1] The Christiano-Eichenbaum model implies a stationary equilibrium process for hours (in percent deviations from its steady state value). Accordingly, I used the VAR specification that includes "detrended" hours.

low.

## 3. Structural MA Estimator

Let the reduced form VAR representation for $\{[\Delta x_t, \Delta n_t]'\}$ be given by

$$B(L) \begin{bmatrix} \Delta x_t \\ \Delta n_t \end{bmatrix} = \nu_t \tag{3.1}$$

where $B(0) \equiv I$ , $E\ \nu_t \nu_t' = \Sigma$, and $E\ \nu_t\ \Delta x_{t-j} = 0$, for $j = 1, 2, 3, \ldots$ . We assume that each reduced form innovation is an (independent) linear combination of the structural shocks, i.e., $\nu_t = S\ \epsilon_t$ for some non-singular matrix $S$, a condition that guarantees that the structural shocks are "fundamental."

Under the previous assumptions one can show that $SS' = \Sigma$ and $C(L) = E(L)S$, where

$$C(1)C(1)' \quad E(1)\ \Sigma\ E(1)'$$

where $E(L) \equiv B(L)^{-1}$.The identifying restriction implies that $C(1)$ is lower triangular and, hence, the Choleski factor of $E(1)\ \Sigma\ E(1)'$. Given consistent estimates for $E(L)$ and $\Sigma$, the matrix of impulse responses $C(L)$ can be estimated using

$$\begin{aligned} C(L) \quad & E(L)\ S \\ & E(L)\ E(1)^{-1}\ chol[E(1)\ \Sigma\ E(1)'] \end{aligned}$$

## 4. Figures

The present appendix also contains the following Figures not available in the main text.

- Figure A-1: Productivity-labor input scatterplots, using employment.
- Figures A-2 and A-3: Estimated impulse responses from bivariate U.S. model, using employment (first-differenced and detrended).
- Figures A-4 (b-d): Estimated impulse responses from five-variable U.S. model, using detrended hours, first-differenced and detrended employment
- Figures A-4 (e-h): Estimated impulse responses from five-variable U.S. model, using the alternative specification in "first differences" (and first-differenced hours)
- Figure A-5: Decomposition of the Business Cycle using detrended hours
- Figure A-6: Estimated impulse responses from bivariate model for each of the remaining G7 countries (first-differenced, with the exception of France).

## REFERENCES


Christiano, Lawrence J. and Martin Eichenbaum (1992): "Current Real Business Cycle Theories and Aggregate Labor Market Fluctuations," *American Economic Review* 82, 430-450.

Galí, Jordi (1992): "How Well does the IS-LM Model Fit Postwar U.S. Data ?" *Quarterly Journal of Economics,* 709-738.

Galí, Jordi (1996): "Technology, Employment, and the Business Cycle: Do Technology Shocks Explain aggregate Fluctuations," NBER Working Paper #5721 (also CEPR Discussion Paper #1499).

Shapiro, Matthew D., and Mark Watson (1988): "Sources of Business Cycle Fluctuations," in Stanley Fischer ed., *NBER Macroeconomics Annual 1988*, 111-148.

**Table A-1: Unit Root Tests**

*U.S. Data*

| (a) | level | $\Delta$ |
|---|---|---|
| $n$ (hours) | | -7.36* |
| $n$ (employment) | | -6.50* |
| $x$ (hours) | | -6.79* |
| $x$ (employment) | | -5.67* |
| $y$ | | -6.04* |
| (b) | level | $\Delta$ |
| $m$ | 0.90 | -2.76 |
| $p$ | 1.30 | -3.27 |
| $m-p$ | -2.12 | -3.77* |
| $i$ | -2.36 | -4.66* |
| $i-\Delta p$ | -3.76* | -7.47* |

Note: t-statistic for the null hypothesis of a unit root in the level or the first difference of each time series, based on an ADF test with 4 lags, intercept and time trend. 5% significance critical value: -3.41 (lower values denoted with asterisk). Sample period: 49:1-94:4, with the exception of $m$ and $m-p$ (59:1-94:4). Source: Citibase.

**Table A-2: Unit Root Tests**

*International Data*

| | $n$ | $x$ | $y$ | $\Delta n$ | $\Delta x$ | $\Delta y$ |
|---|---|---|---|---|---|---|
| *Canada* | -1.75 | -1.22 | -1.67 | -4.26* | -4.49* | -4.67* |
| *U.K.* | -3.16 | -2.39 | -2.39 | -3.32 | -4.90* | -4.40* |
| *Germany* | -1.74 | -3.30 | -3.33 | -3.32 | -4.86* | -4.40* |
| *France* | -3.48* | -2.66 | -3.07 | -3.83* | -4.41* | -3.93* |
| *Italy* | 0.04 | -1.15 | -2.63 | -3.25 | -4.64* | -5.28* |
| *Japan* | -2.69 | -0.00 | -2.17 | -3.78* | -3.98* | -3.90* |

Note: t-statistic for the null hypothesis of a unit root in the level or the first difference of each time series, based on an ADF test with 4 lags, intercept and time trend. 5% significance critical value: -3.41 (lower values denoted with asterisk). Sample period: Canada (62:1-94:4), U.K. (62:1-94:3), Germany (70:1-94:4), France (70:1-94:4), Italy (70:1-94:3), and Japan (62:1-94:4). Data source: OECD Quarterly National Accounts.

**Table A-3: Conditional Correlation Estimates**
**U.S. Five Variable Model ("First Difference Specification")**

| | *Technology* | *Non-Technology* |
|---|---|---|
| $\Delta n$ *model* | | |
| *Hours* | -0.73** | 0.26** |
| | (0.03) | (0.10) |
| *Employment* | -0.74** | 0.44** |
| | (0.08) | (0.14) |
| $\hat{n}$ *model* | | |
| *Hours* | -0.55** | 0.13 |
| | (0.06) | (0.13) |
| *Employment* | -0.82** | 0.45** |
| | (0.04) | (0.09) |

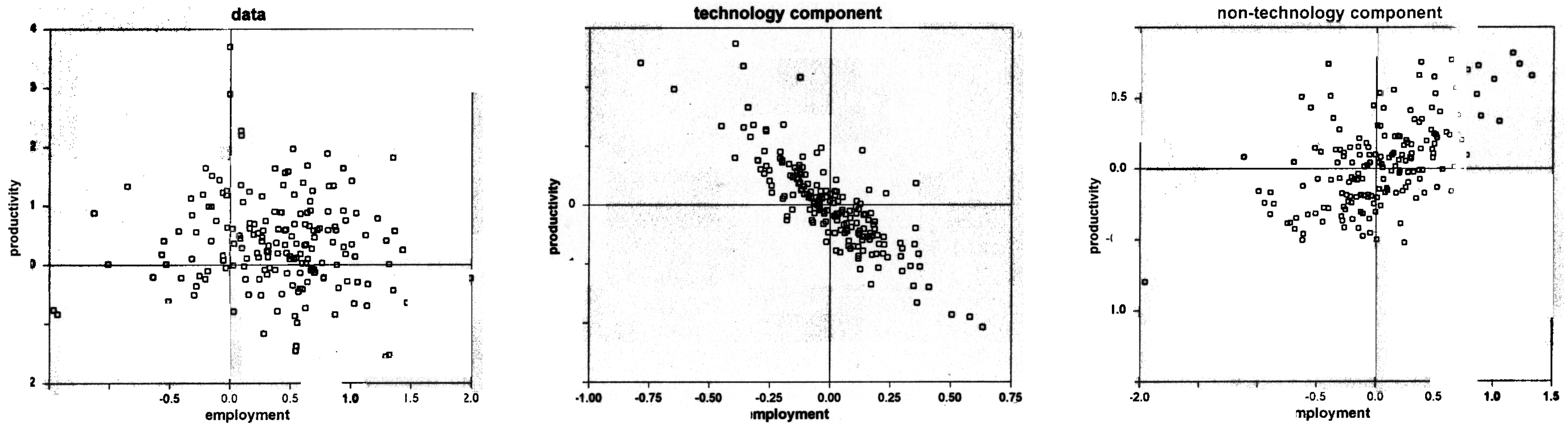
NOTE: The Table reports estimates of conditional correlations between the growth rates of productivity and labor input (hours or employment) in the U.S..Standard errors are shown in brackets. Significance at conventional levels is indicated by one (10% level) or two asterisks (5% level). The conditional correlation estimates are based on the partially-identified estimated five-variable VAR described in the text. The VAR is estimated using quarterly data for the period 1959:1-1994:4, and includes series for productivity growth, hours (or employment), first-differenced M2 growth,. real interest rates, and first-differenced inflation. The top panel displays the results for the specification that includes labor input growth. The results using detrended labor input are shown in the bottom panel.. Data sources and exact definition of variables can be found in the text.

**Table A-4: Montecarlo Simulation**
**Distribution of Conditional Correlation Estimates**

| | *Technology* | *Government* |
|---|---|---|
| *mean* | 0.75 | -0.43 |
| *5th centile* | 0.46 | -0.92 |
| *10th centile* | 0.60 | -0.86 |
| *90th centile* | 0.92 | 0.13 |
| *95th centile* | 0.93 | 0.30 |

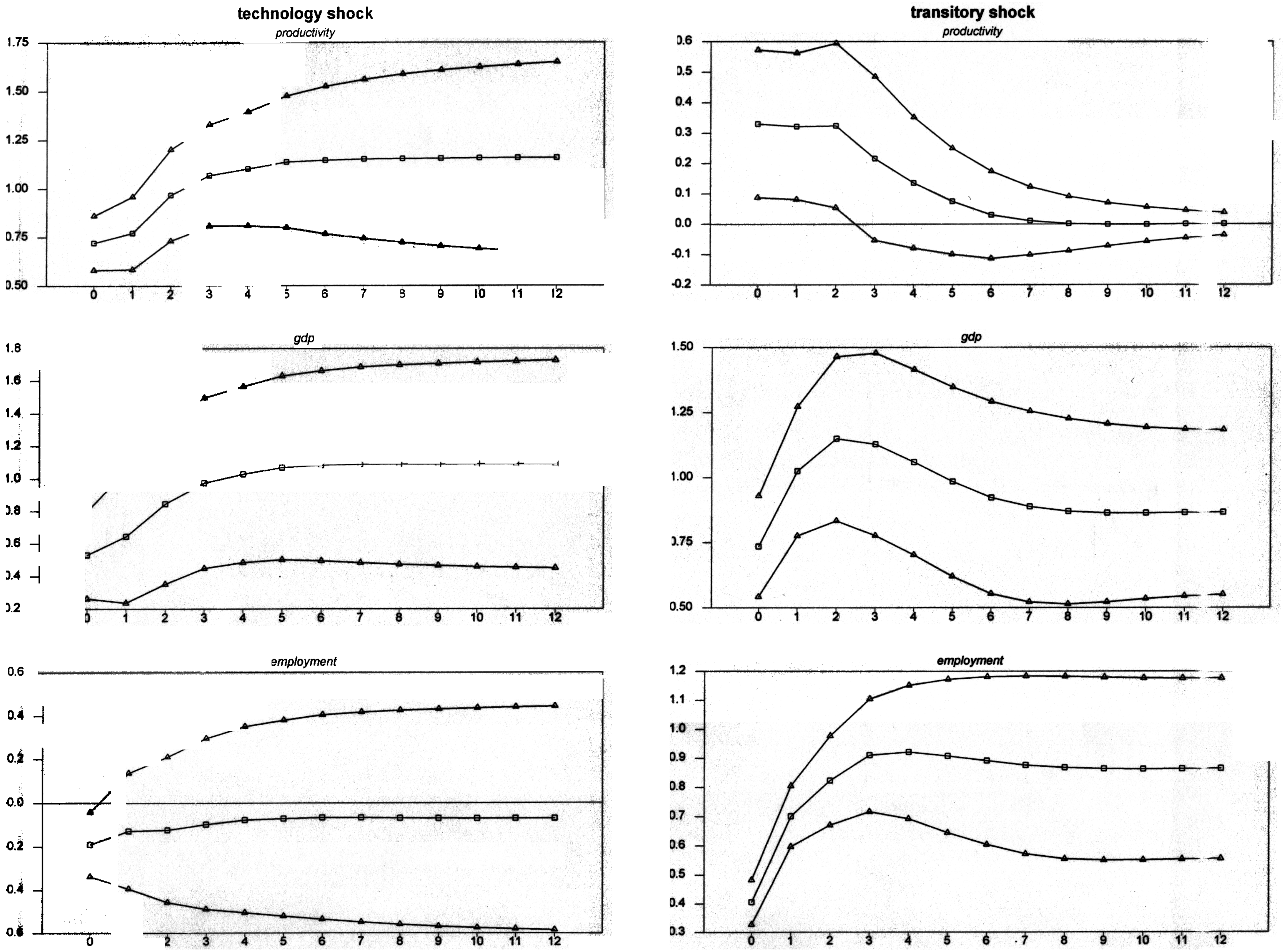
## Figure A-1 : productivity-employment comovements

*US data, employment, first differences*



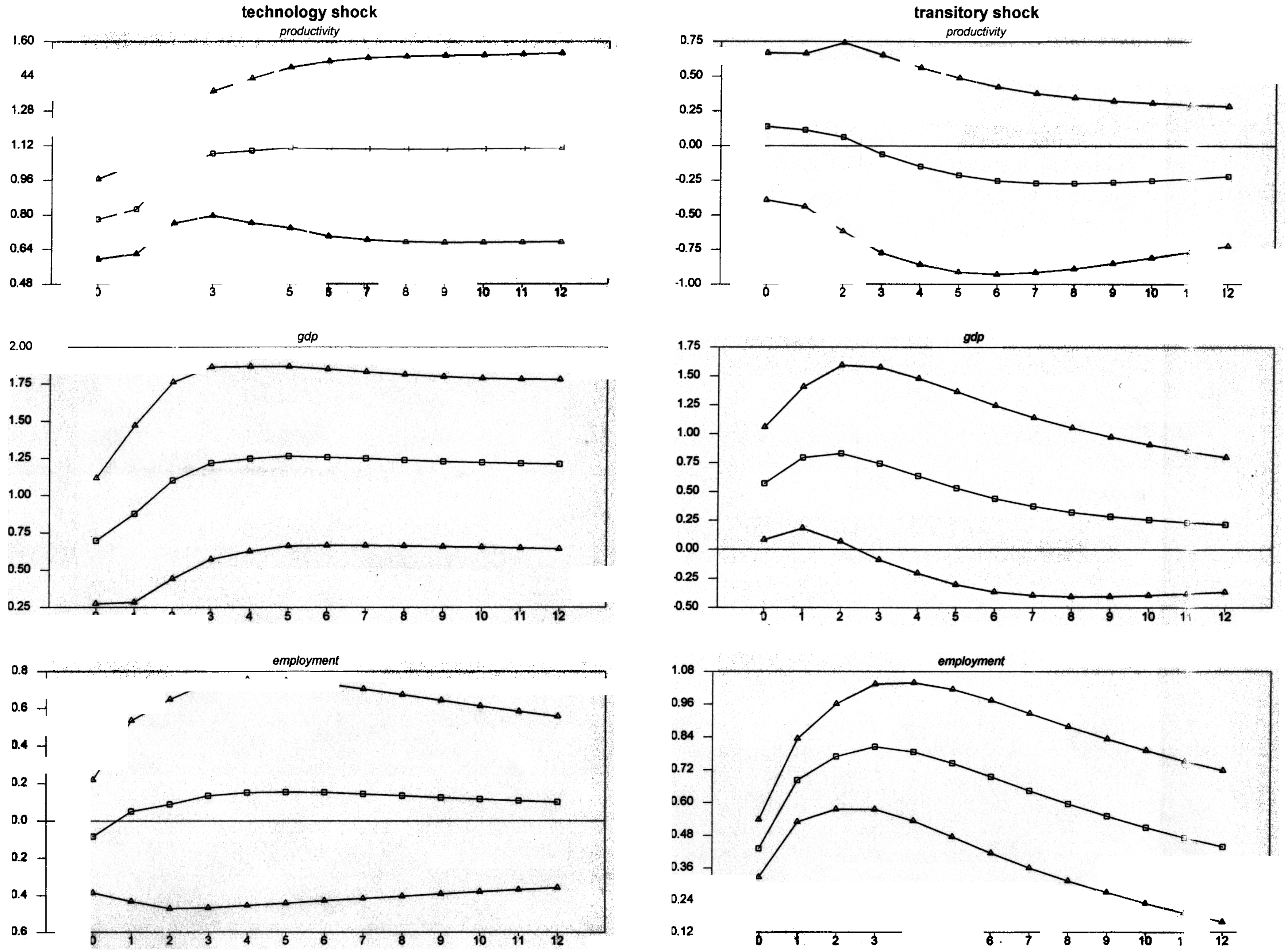
# Figure A-2 : Estimated Impulse Responses

*US data,employment,first differences*



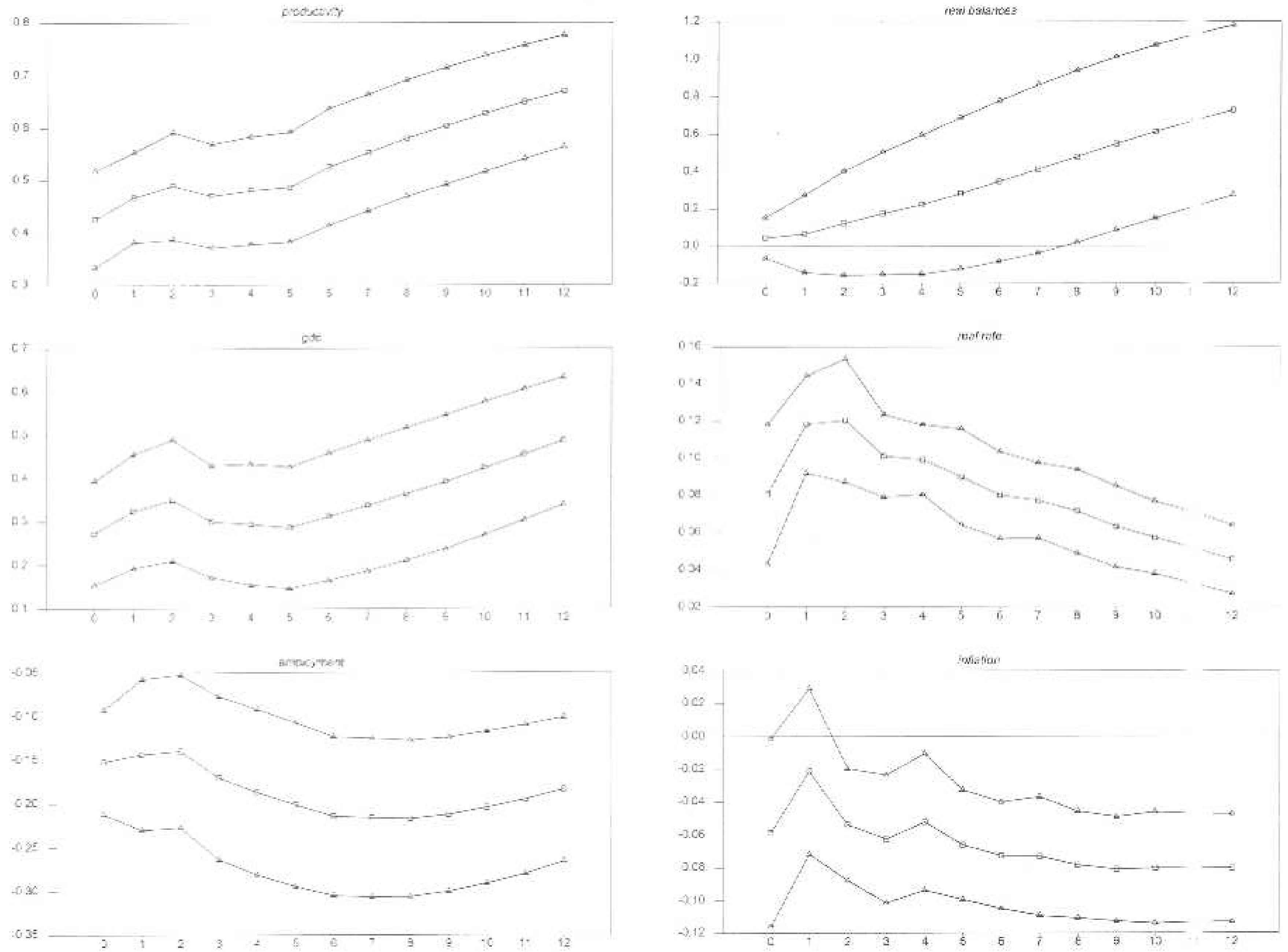
# Figure A-3 : Estimated Impulse Responses

*US data,employment,detrended*



# Figure A-4d : Estimated Impulse Responses: Five Variable VAR

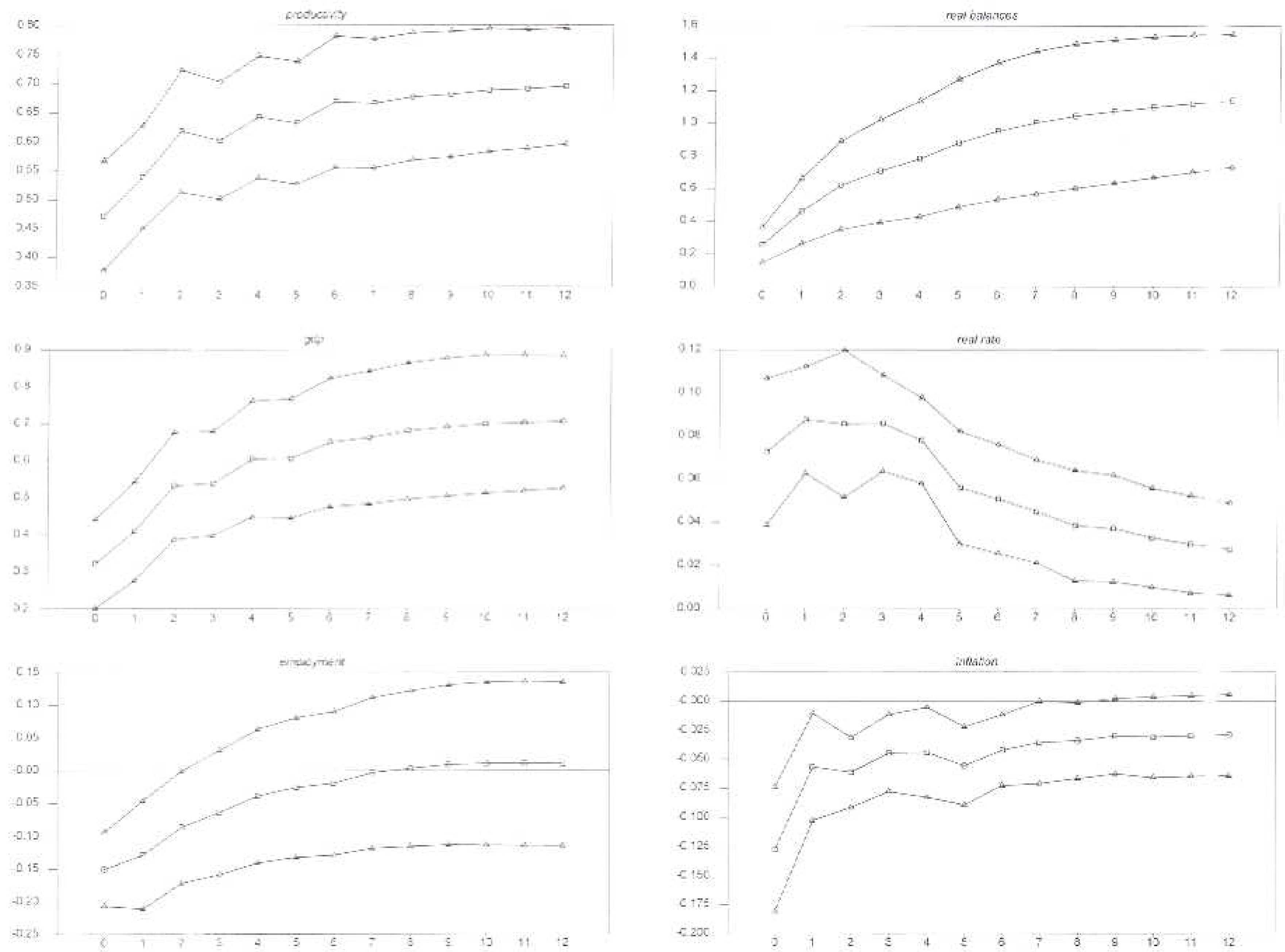
*US data, employment, detrended*



# Figure : Estimated Impulse Responses: Five Variable VAR

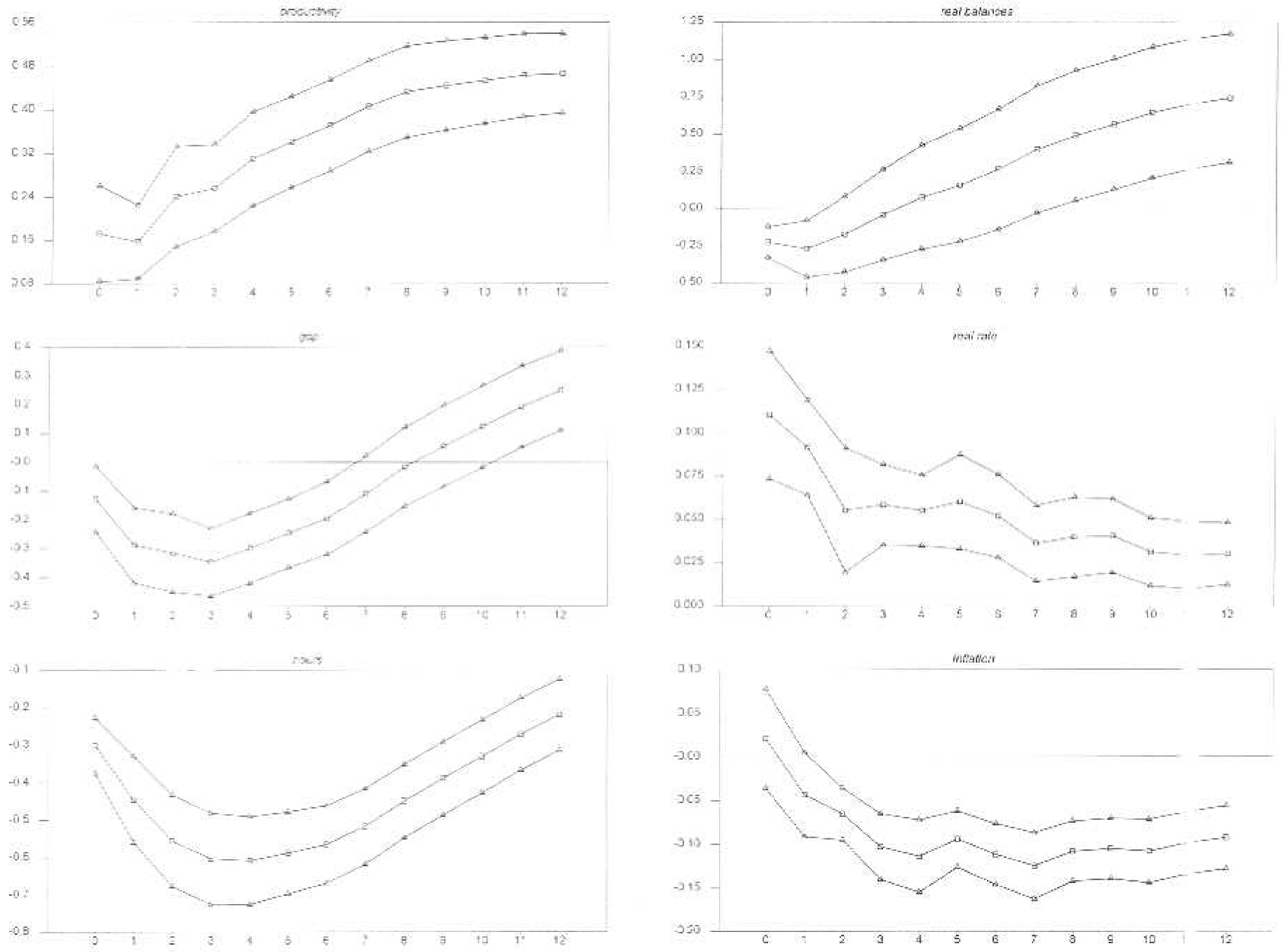
A-4c

*US data, employment, first differences*

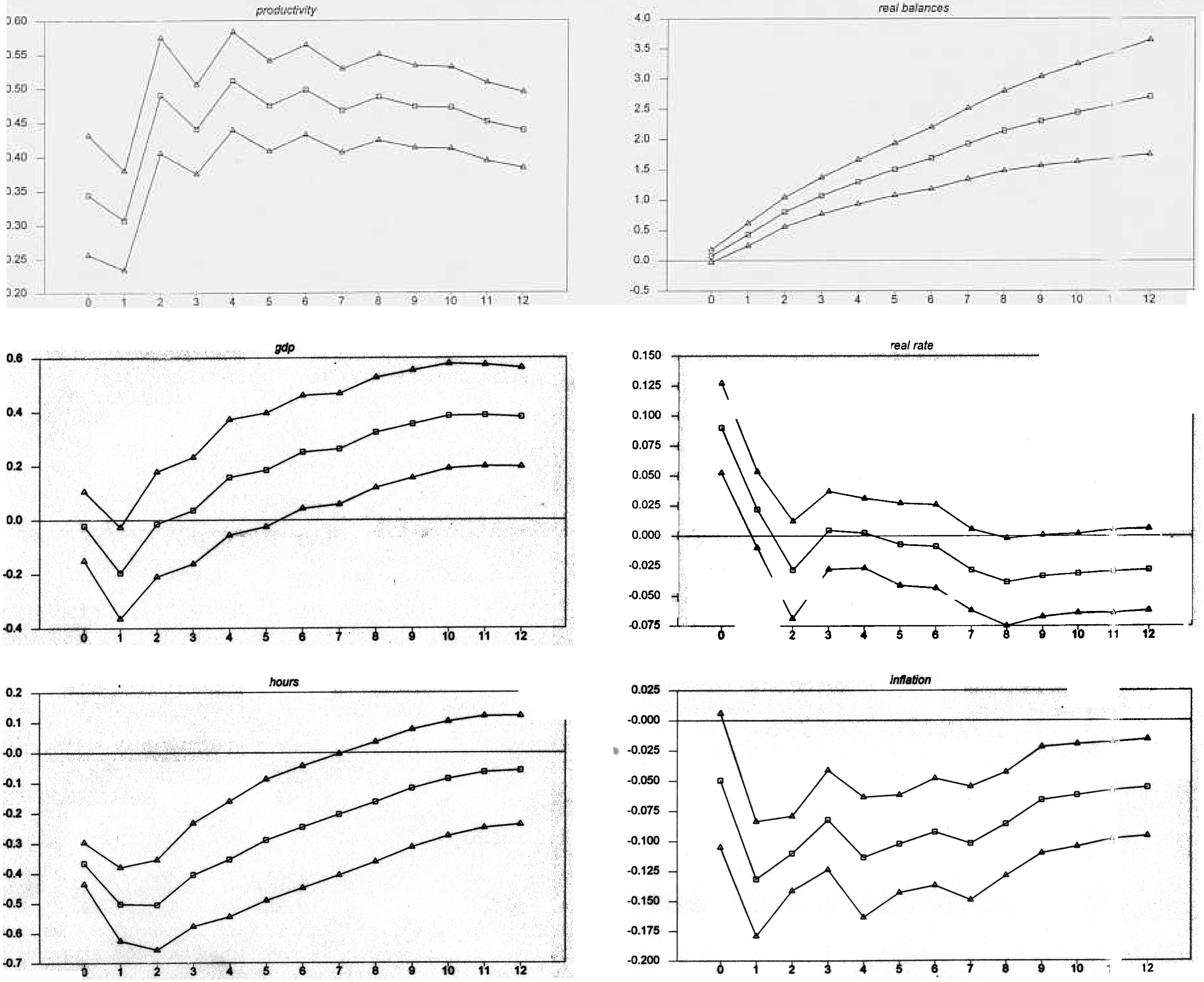


## Figure A-4b : Estimated Impulse Responses: Five Variable VAR

*US data, hours, detrended*

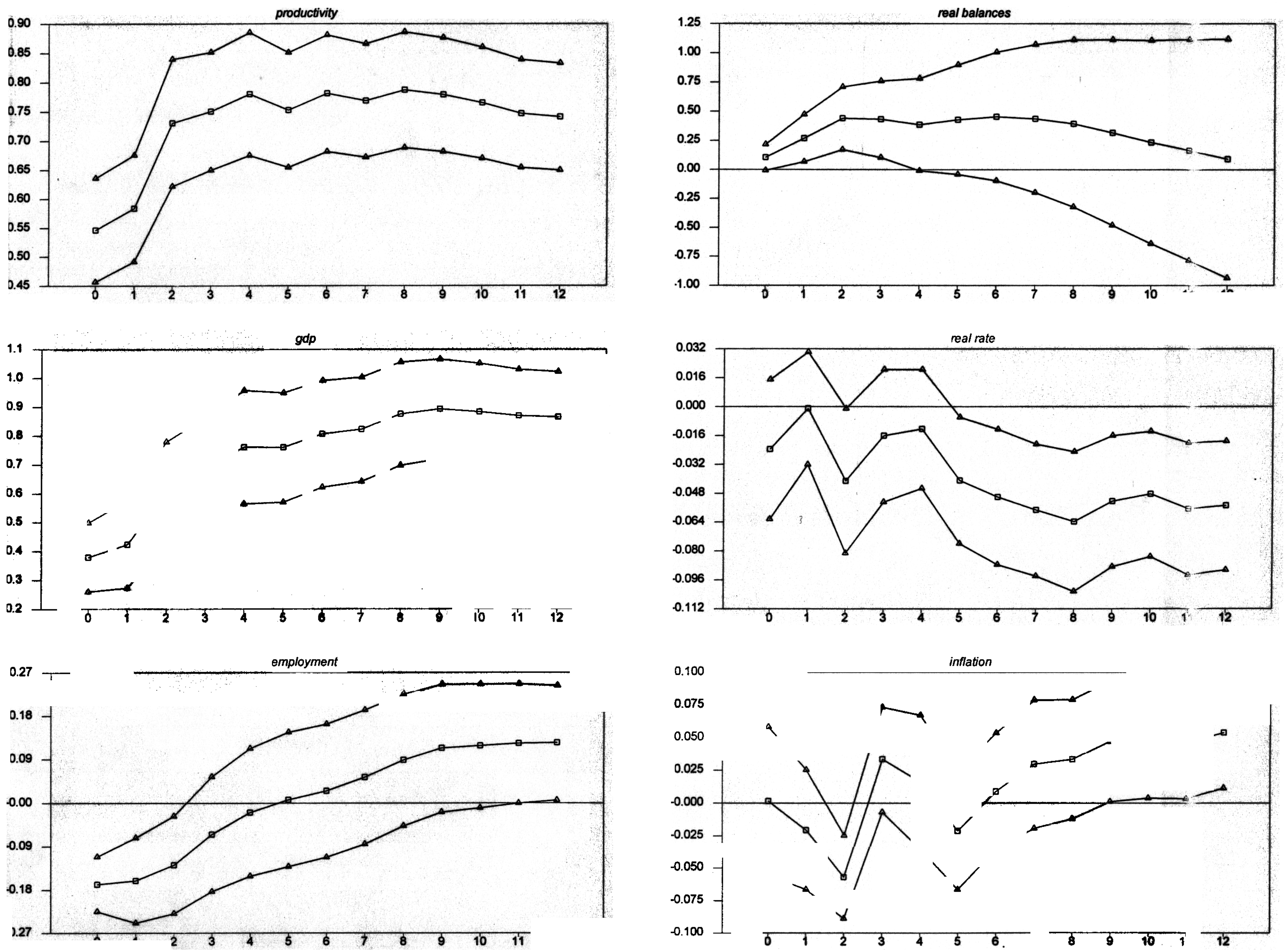


A-4e ... FIVE-VARIABLE MODEL (ALTERNATIVE SPECIFICATION)



# Figure A-4F : Estimated Impulse Responses: Five Variable VAR

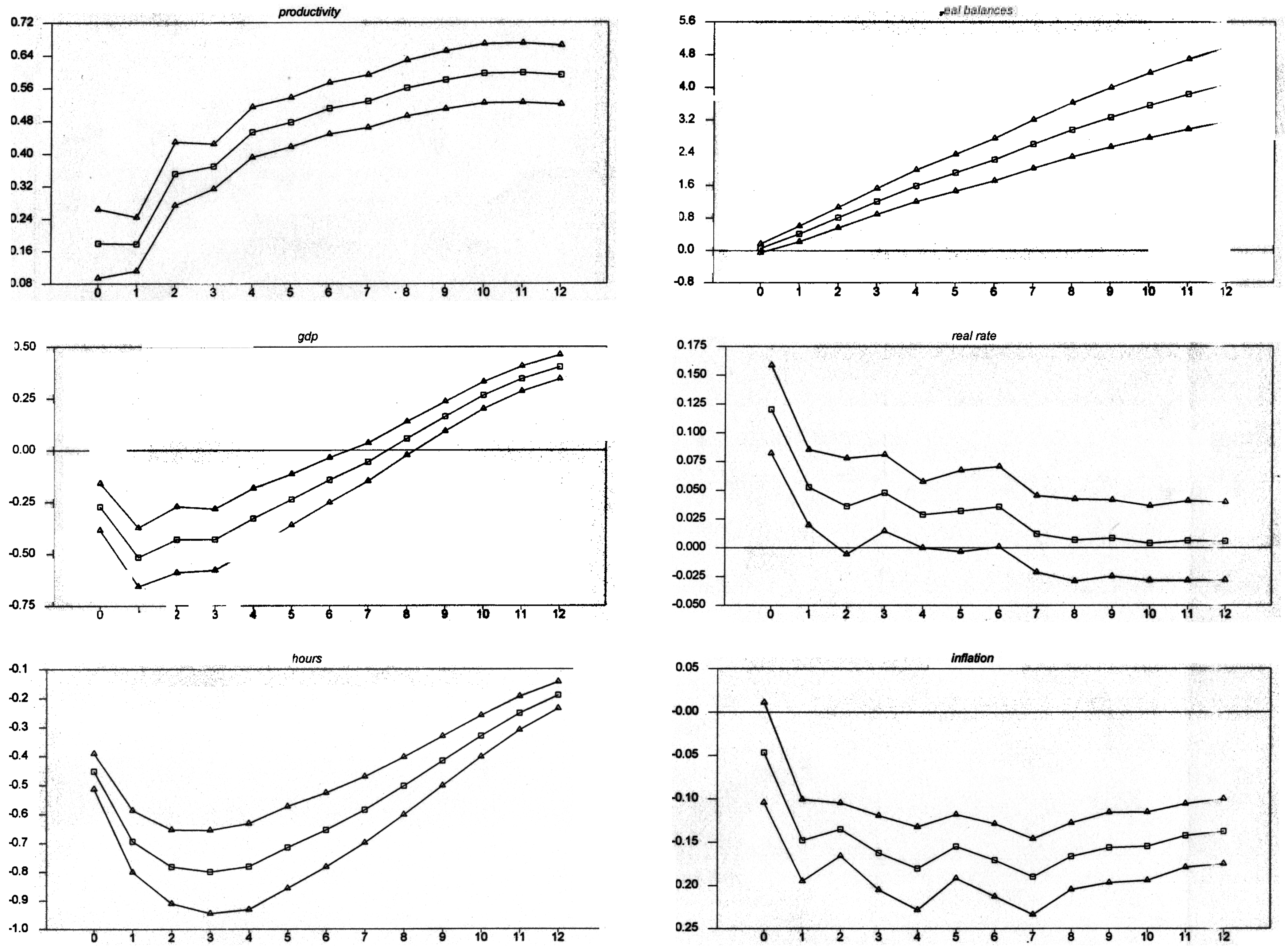
*US data,employment,first differences* (ALTERNATIVE SPECIFICATION)



# Figure : Estimated Impulse Responses: Five Variable VAR

A-4g

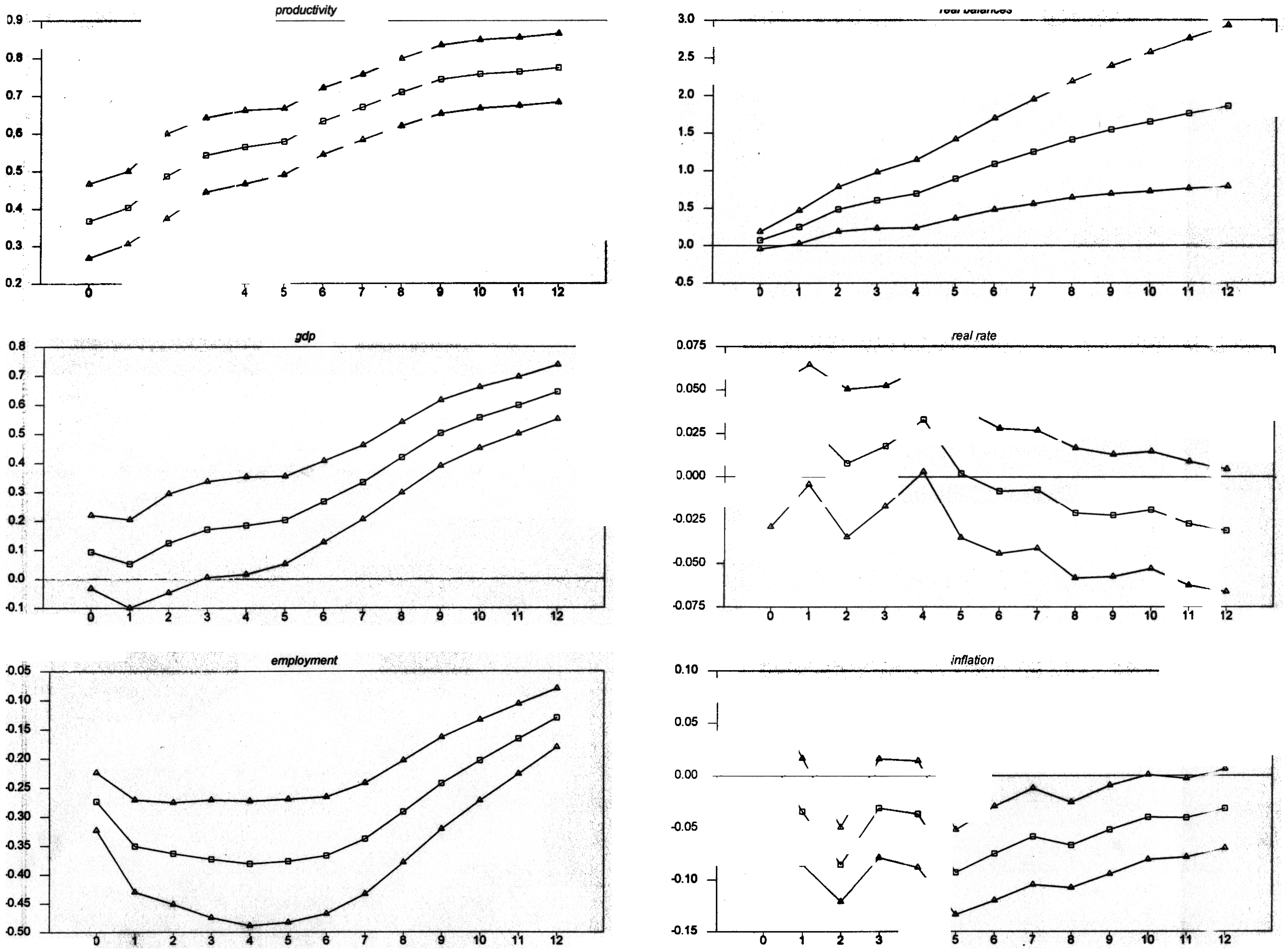
*US data,hours,detrended* (ALTERNATIVE SPECIFICATION)



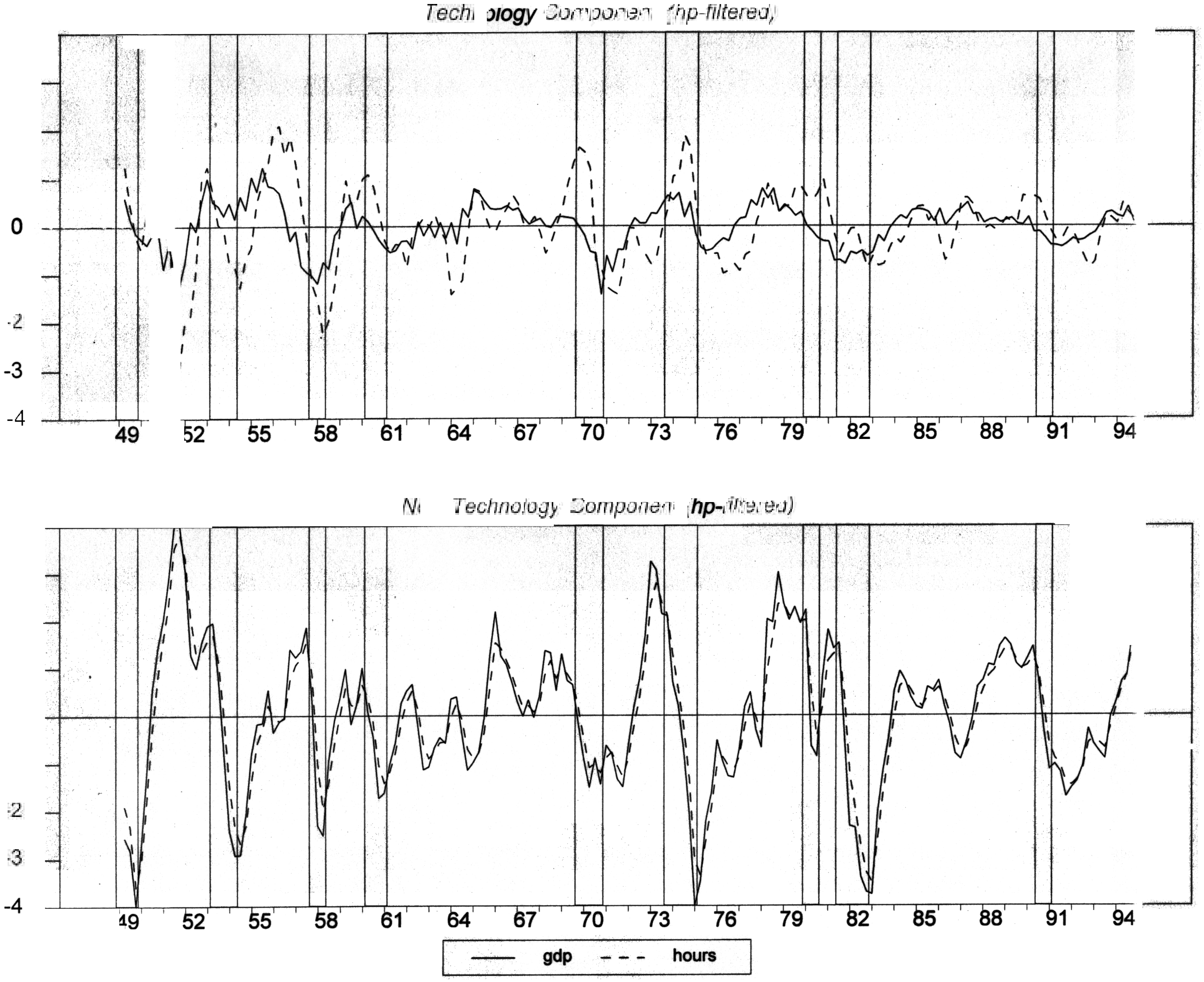
# Figure : Estimated Impulse Responses: Five Variable VAR

A-4h

*US data,employment,detrended* (ALTERNATIVE SPECIFICATION)

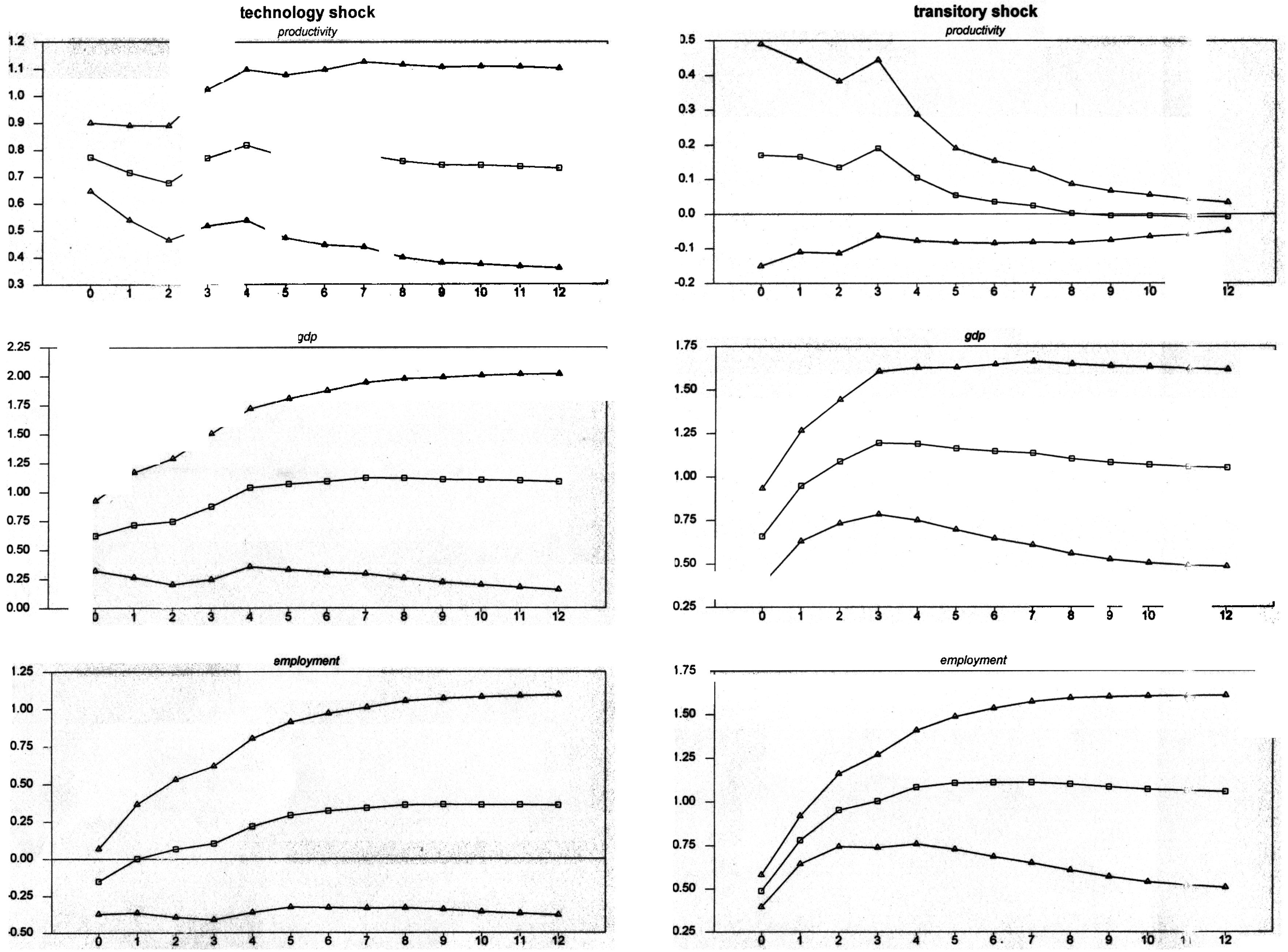


## Figure A-5 Decomposing the Business Cycle: Detrended Hours



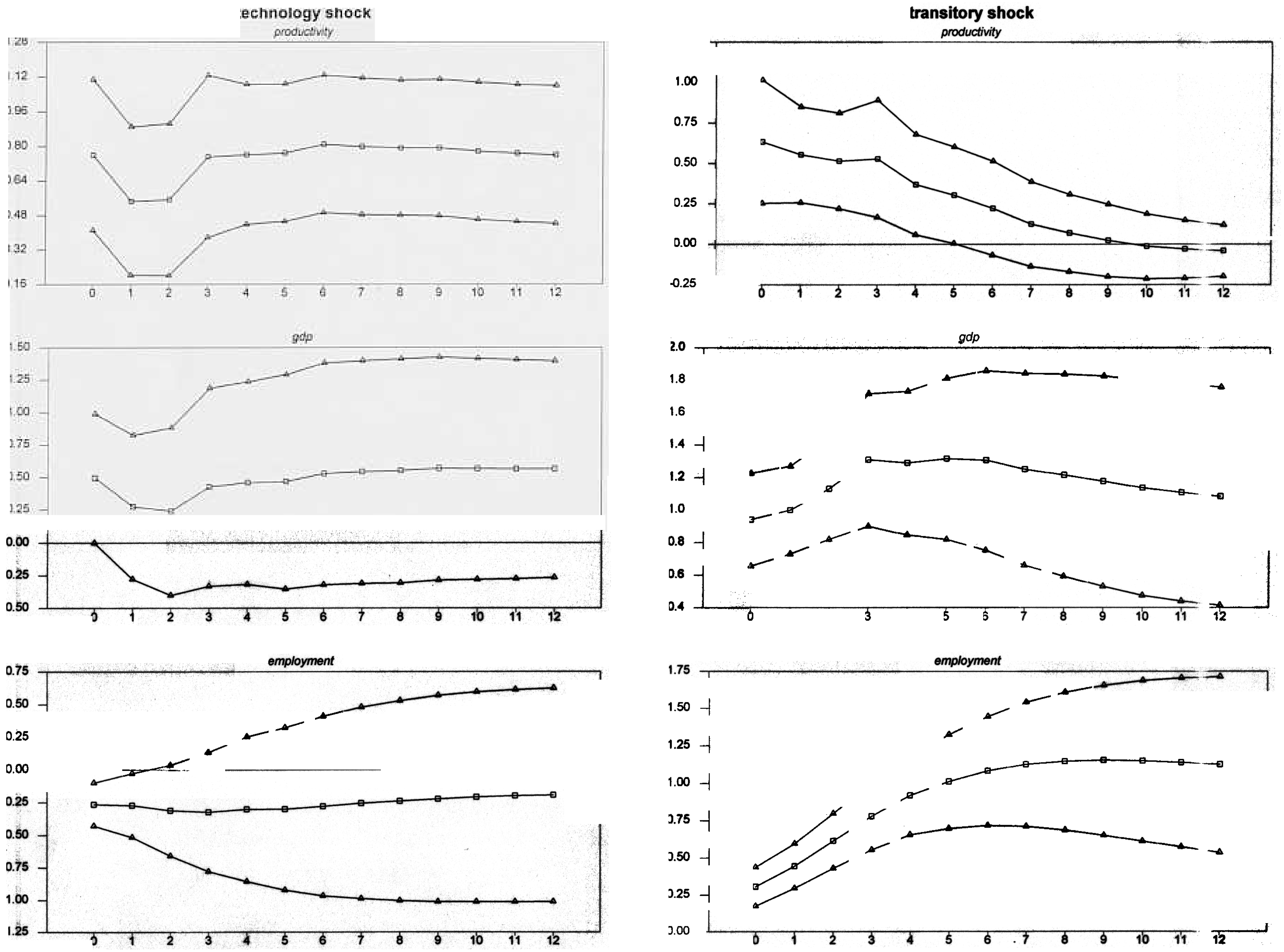
# Figure A-6a : Estimated Impulse Responses

CANADA ,employment,first differences



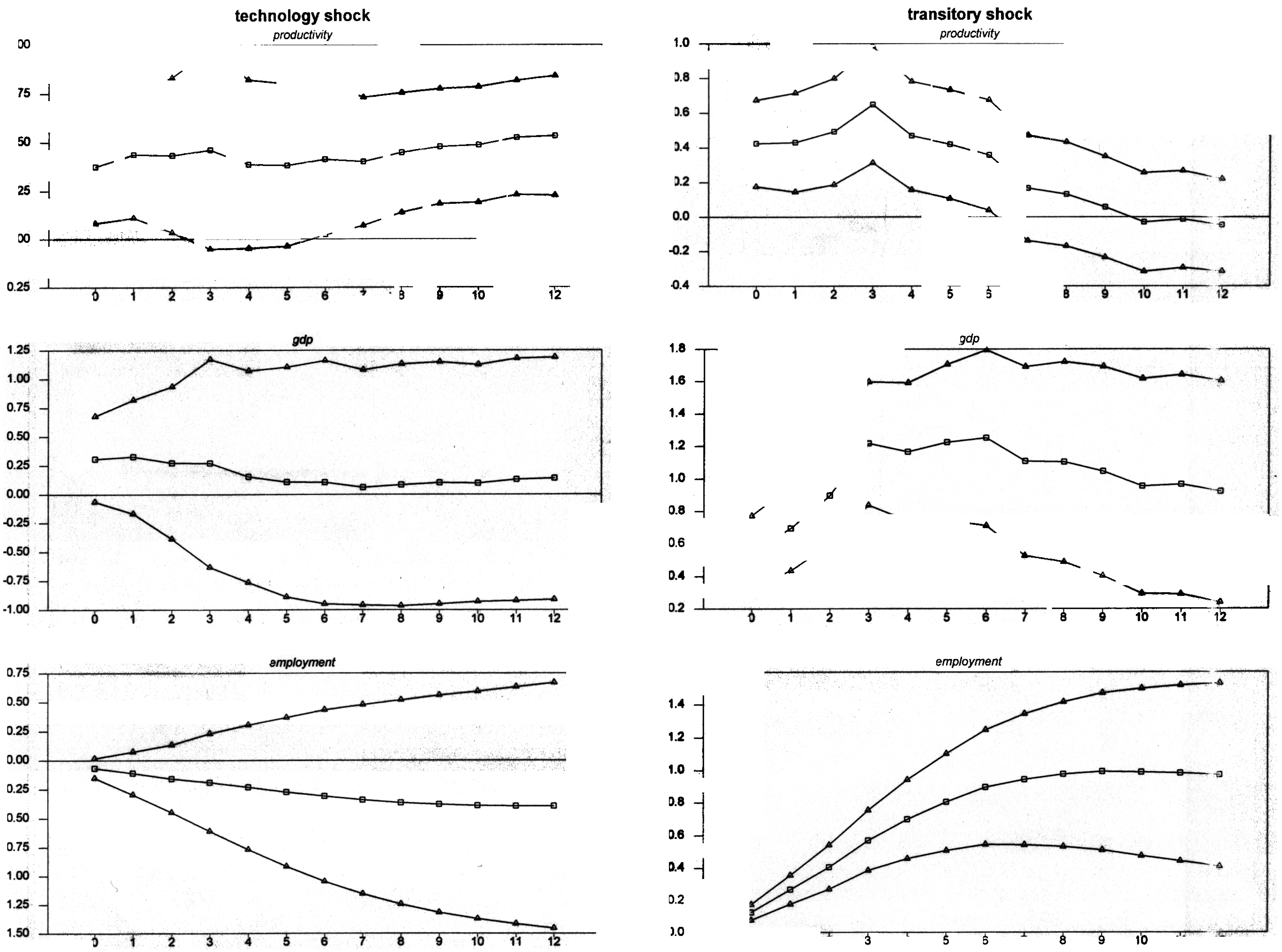
# Figure A-6b : Estimated Impulse Responses

*UK ,employment,first differences*



# Figure : Estimated Impulse Responses

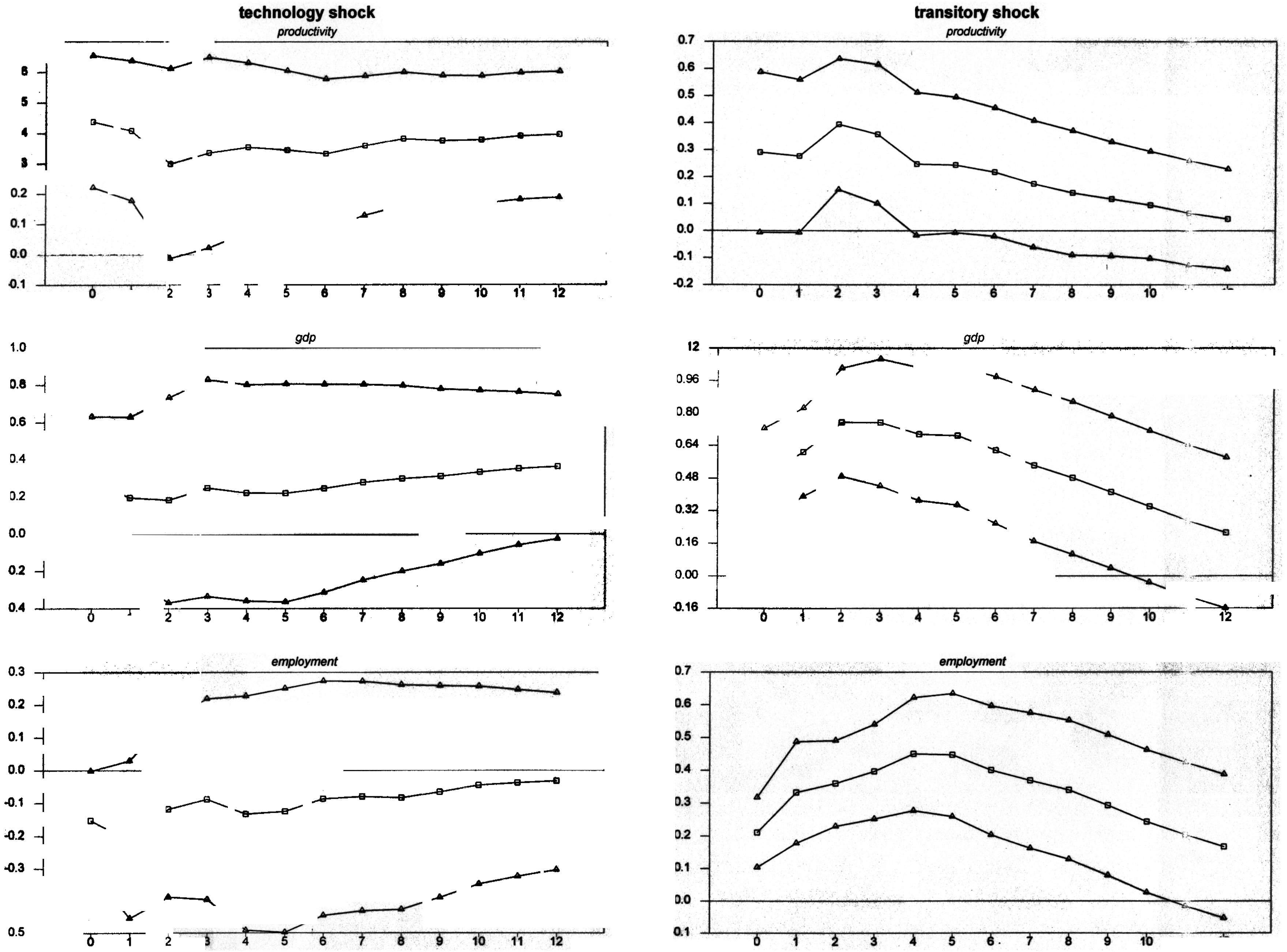
A-6c GERMANY ,employment,first differences



# Figure : Estimated Impulse Responses

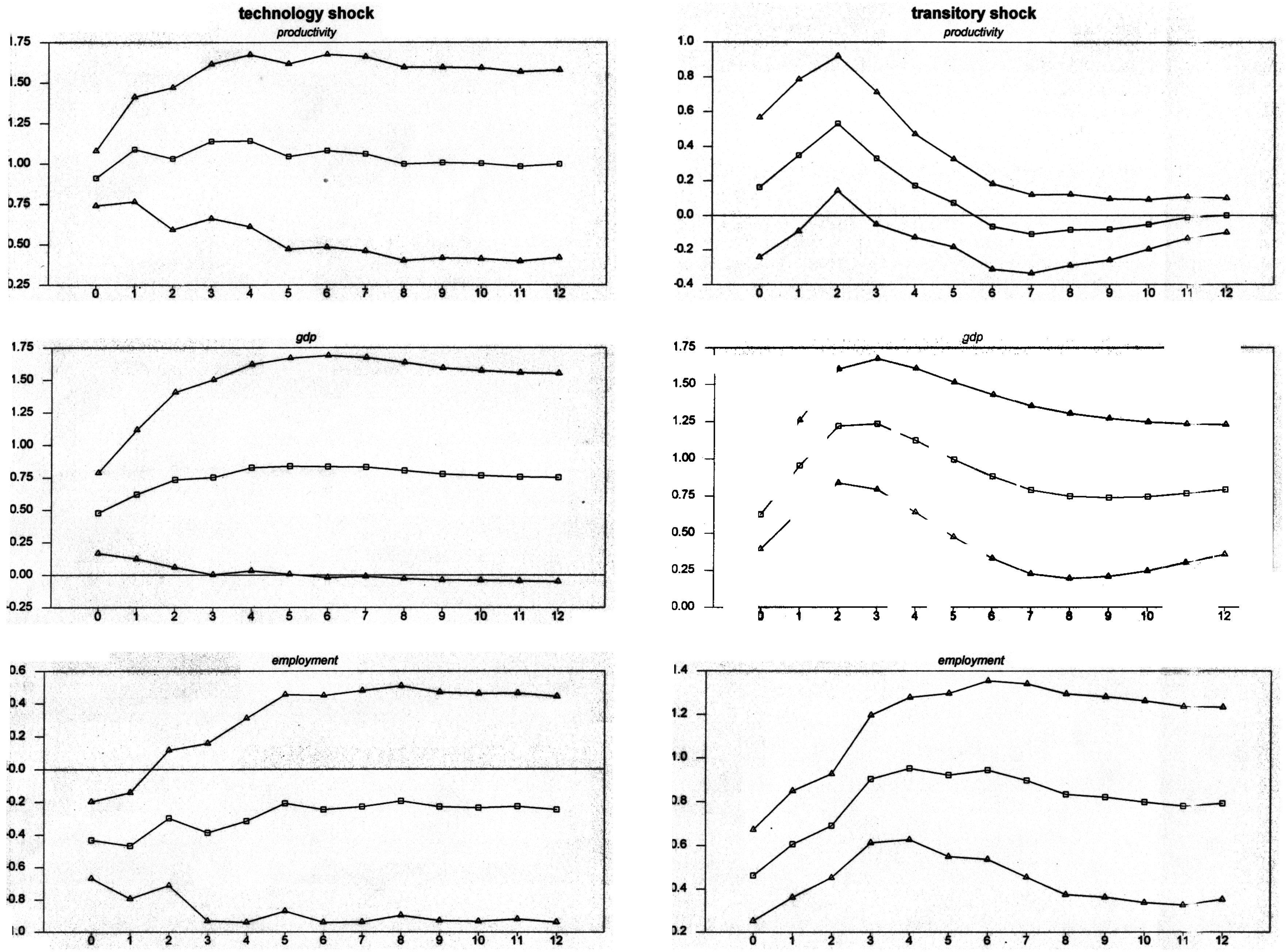
A-6d

*FRANCE ,employment,detrended*



# Figure A-6e : Estimated Impulse Responses

*ITALY ,employment,first differences*



# Figure : Estimated Impulse Responses

A-6f *JAPAN ,employment,first differences*