1 Managing credit bubbles: online appendix

In the main body of the paper, we consider situations where the size of the bubble matters because it determines the collateral available to issue credit contracts. We have shown that this provides a rationale for macroprudential policy. A CMA can improve on market outcomes by guaranteeing credit when bubbly collateral is scarce and by taxing credit when bubbly collateral is excessive.

But there are also situations in which it is not just the amount of bubbly collateral, but also its type that matters. To illustrate this, we develop two simple extensions of the model. In the first one, we introduce financial intermediaries that sell deposits contracts to savers and purchase loan contracts form entrepreneurs. In this case, the economy needs different types of bubbly collateral to back deposit and loan contracts. In the second extension, we introduce a set of risk-averse savers. In this case, the economy needs different types of collateral to back safe and risky credit contracts. In these extensions, there is sometimes too much collateral of one type but too little collateral of the other type. Thus, to improve on market outcomes, the CMA must simultaneously subsidize some types of credit while taxing others.

1.1 Deposit vs. loan bubbles

We introduce now a third type of individual, the intermediary, that has the same preferences as the other two types. Now entrepreneurs and savers cannot trade directly. Instead, their trade must go be done through intermediaries. Just as entrepreneurs are the subset of individuals that have access to the production technology, one can think of intermediaries as the subset of individuals that have access to a screening or monitoring technology that is necessary to ensure that credit is profitable to all parties. To simplify the discussion, we assume that there are no productivity shocks, i.e. $A_t = 1,$; and that there is no fundamental collateral, i.e. $\phi = 0$.

The bubbly economy now contains two distinct credit markets. In the market for deposits, intermediaries sell credit contracts to savers: we use R^D_{t+1} to denote the gross, possibly state contingent interest rate paid by intermediaries on this credit. In the market for loans, intermediaries purchase credit contracts from entrepreneurs: we use R^L_{t+1} to denote the gross interest rate paid by entrepreneurs on these loans. We refer to the expected return on deposits, $E_t R^D_{t+1}$, as the deposit rate, and to the expected return on loans, $E_t R^L_{t+1}$, as the loan rate.

Now we have two types of credit bubbles, which we refer to as deposit and loan bubbles. Borrowers initiate and trade loan bubbles, while intermediaries initiate and trade deposit bubbles. Let B^D_t and B^L_t denote the value of deposit and loan bubbles in period t. Some of these bubbles are old and some of them are new. Thus, the aggregate bubbles evolve as follows:

$$B^D_{t+1} = R^{BD}_{t+1} \cdot B^D_t + B^{ND}_{t+1} \tag{1}$$

$$B^L_{t+1} = R^{BL}_{t+1} \cdot B^L_t + B^{NL}_{t+1} \tag{2}$$

where R^{BD}_{t+1} and R^{BL}_{t+1} are the returns to deposit and loan bubbles purchased from generation $t-1$; and B^{ND}_{t+1} and B^{NL}_{t+1} the value of the bubbles initiated generation t. We refer to B^{ND}_{t+1} and B^{NL}_{t+1} as deposit and loan bubble creation and we assume that they are random and non-negative, i.e. $B^{ND}_{t+1} \geq 0$ and $B^{NL}_{t+1} \geq 0$.

Just like entrepreneurs' loan contracts, deposit contracts must be collateralized. Since we have assumed that there is no fundamental collateral, this means that:

$$R^D_{t+1} \cdot D_t \leq B^D_{t+1} \tag{3}$$

$$R^L_{t+1} \cdot L_t \leq B^L_{t+1} \tag{4}$$

where D_t and L_t are the value of deposit and loan contracts, respectively.

With these assumptions, we can solve the optimization problem of all individuals and compute the equilibria in the labor, deposit, loan, and bubble markets. The analysis mirrors the one of Section 1, but it suffices to note that

$$E_t R^{BD}_{t+1} = E_t R^D_{t+1} \tag{5}$$

$$E_t R^{BL}_{t+1} = E_t R^L_{t+1} \tag{6}$$

in equilibrium, since otherwise demand for bubbles would be either zero or infinite and the markets for deposit and loan bubbles could not clear.

Taking all this into account, we can collapse the dynamics of the model to the following equations:

$$k_{t+1} \begin{cases} = \dfrac{(1-\alpha) \cdot k_t^\alpha - b_t^D - b_t^L}{\gamma} & \text{if } \beta \cdot E_t R^D_{t+1} > 1 \\ \in \left[0, \dfrac{(1-\alpha) \cdot k_t^\alpha - b_t^D - b_t^L}{\gamma}\right] & \text{if } \beta \cdot E_t R^D_{t+1} = 1 \end{cases} \tag{7}$$

$$E_t R^D_{t+1} = \min \left\{ E_t R^L_{t+1}, \frac{E_t n^D_{t+1} \cdot k^\alpha_{t+1}}{k_{t+1} + \gamma^{-1} \cdot b^L_t} \right\} \tag{8}$$

$$E_t R^L_{t+1} = \min \left\{ \alpha, E_t n^L_{t+1} \right\} \cdot k^{\alpha-1}_{t+1} \tag{9}$$

$$b^D_{t+1} = \frac{E_t R^D_{t+1} + u^D_{t+1}}{\gamma} \cdot b^D_t + n^D_{t+1} \cdot k^\alpha_{t+1} \tag{10}$$

$$b^L_{t+1} = \frac{E_t R^L_{t+1} + u^L_{t+1}}{\gamma} \cdot b^L_t + n^L_{t+1} \cdot k^\alpha_{t+1} \tag{11}$$

where $\left\{u^D_{t+1}, u^L_{t+1}, n^D_{t+1}, n^L_{t+1}\right\}$ are the bubble-return and the bubble-creation shocks of deposit and loan bubbles. Equations (7)-(11) generalize Equations (14)-(16) in the main body of the paper, and they provide a full description of the dynamics of the economy in the presence of deposit and loan bubbles.

The key innovation relative to our baseline model is that now it is both, the amount and the distribution of bubbly collateral that matter for investment. Equation (7) shows that regardless of their type, bubbles divert resources and crowd out investment. But whether loan and deposit bubbles have crowding-in effects depends on which collateral constraint binds, if any. Equation (8) represents intermediaries' demand for credit. If intermediaries have enough collateral, the deposit rate equals the loan rate. Otherwise intermediaries are constrained and the deposit rate is lower than the loan rate. Equation (9) represents borrower's demand for credit. If entrepreneurs have enough collateral, the loan rate equals the return to investment. Otherwise entrepreneurs are constrained and the loan rate is lower than the return to investment.

Thus, there are two reasons why the bubbly economy may find itself in the region of partial intermediation: (i) because intermediaries do not have the collateral to raise all wages through deposits; or (ii) because entrepreneurs do not have enough collateral to raise all deposits through loans. In the first case, deposit bubble creation ($E_t n_{t+1}^D > 0$) is expansionary because it raises the collateral of intermediaries and thus total deposits. Loan bubble creation ($E_t n_{t+1}^L > 0$), however, is contractionary because its only effect is to raise the loan rate. There is too little deposit collateral and too much loan collateral, so that the right policy requires subsidizing deposits while taxing loans. In the second case, the opposite is true. There is too little loan collateral, and the right policy requires subsidizing loans.

1.2 Safe vs. risky bubbles

We now extend the basic model to allow for risk-averse individuals. In particular, we assume that a fraction ρ of the savers have the following preferences:

$$U_t^i = C_{t,t}^i + \beta \cdot \min_t C_{t,t+1}^i \tag{12}$$

where the operator $\min_t \{\cdot\}$ indicates the lowest bound of the support of the corresponding variable. To simplify the discussion, we assume that there are no productivity shocks, i.e. $A_t = 1$; and that there is no fundamental collateral, i.e. $\phi = 0$.

The presence of risk-averse individuals induces entrepreneurs to issue two types of credit contracts, safe and risky. Let R_{t+1}^S and R_{t+1}^R respectively denote the ex-post interest rates on these contracts. Naturally, safe contracts promise a noncontingent interest rate but risky contracts do not. It follows that now entrepreneurs face two credit constraints:

$$R_{t+1}^R \cdot L_t^R + R_{t+1}^S \cdot L_t^S \leq B_{t+1} \tag{13}$$

$$R_{t+1}^S \cdot L_t^S \leq \min_t B_{t+1} \tag{14}$$

Since we have assumed that $\phi = 0$, there is only bubbly collateral in this economy. Equation (13) says that the total payments promised to savers cannot exceed the total amount of collateral. Equation (14) says that the amount of safe payments that are promised cannot exceed the minimum value of collateral. Otherwise, these payments would need to be state contingent and could not be riskless. Thus, while all collateral in this economy is bubbly, it comes in two types. The stock of safe collateral is given by $\min_t B_{t+1}$, while the stock of risky collateral is given by $B_{t+1} - \min_t B_{t+1}$.

If there is enough safe collateral to ensure that the credit constraint in Equation (14) is not binding, all the results are the same as in the baseline model. Basically, all credit is priced in a risk-neutral fashion: risk-averse savers purchase only safe credit and all risky credit is in the hands of risk-neutral savers. But if there is not enough safe collateral, a new and interesting situation can arise. Because risk-averse savers do not want to purchase risky credit, there may be full intermediation of risk-neutral savings but partial intermediation of risk-averse savings. In such a situation, the law of motion of the economy is given by:

$$k_{t+1} = \beta \cdot \min_t b_{t+1} + (1-\rho) \cdot \frac{1-\alpha}{\gamma} \cdot k_t^{\alpha} - \frac{b_t}{\gamma} \tag{15}$$

$$E_t R_{t+1}^R = \frac{E_t n_{t+1} \cdot k_{t+1}^{\alpha} - \min_t b_{t+1}}{k_{t+1} - \beta \cdot \min_t b_{t+1}} \tag{16}$$

$$b_{t+1} = \frac{E_t R_{t+1}^R + u_{t+1}}{\gamma} \cdot b_t + n_{t+1} \cdot k_{t+1}^{\alpha} \tag{17}$$

Equation (15) says that investment equals total savings minus the bubble: the difference with the basic model is that only risk neutral savers save their entire income, while risk-averse savers do not because there is not enough safe collateral. Thus, this economy has too much risky collateral and too little safe collateral.

It is straightforward to show that increases in $\min_t n_{t+1}$ are expansionary since they raise savings and crowd-in capital, while increases in $E_t n_{t+1} - \min_t n_{t+1}$ are contractionary since they reduce funds available for investment and crowd-out capital. In this example, the right policy would expand one bubble at the expense of the other. In terms of the notation used in section 3, the CMA would set a negative value for $E_t m_t - \min_t m_{t+1}$, but a positive value for $\min_t m_{t+1}$.