Border effects and urban structure*

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Abstract
I propose a general model of economic geography to investigate the effect of border changes on the spatial distribution of population. I decompose the total effect into a standard "local effect" related to the change in distance from borders, and a novel "global effect" related to centrality before the border change. The global effect is especially strong in economies with a dominant central region that is home to a large fraction of the country’s population. Conforming to this prediction, I show that the global effect played an important role in the population reallocation in Hungary after border changes in 1920.

1 Introduction

Borders, be them either man-made or natural barriers, affect the spatial distribution of economic activity. The spatial distribution of economic activity, in turn, has important consequences on aggregate economic variables such as productivity, efficiency, and welfare.1 The effect that borders have on the

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1Papers highlighting mechanisms through which the spatial distribution of population affects productivity and welfare include Krugman (1991), Redding and Venables (2004), Rossi-Hansberg (2005), Desmet and Rossi-Hansberg (2014), and many others. For the empirical side (that is, the quantitative relevance of these effects), see Head and Mayer (2004) or Ciccone and Hall (1996).
spatial distribution of economic activity is, however, far from obvious. On the one hand, the intensity of economic activity, and, in particular, population density, tends to be low near borders.\(^2\) This implies that a change in borders leads to population increases or decreases in areas close to the border change, depending on whether these areas become more or less isolated than before. This effect of border changes is well-documented in the empirical economic geography literature.\(^3\)

On the other hand, the location of borders is likely to affect the entire spatial distribution of population, not just population levels near borders. It is an obvious fact that the spatial distribution of population is very uneven in space, most of the population being concentrated in and near cities. Besides differences in natural advantage and the endowments of immobile factors, agglomeration economies are usually held responsible for this phenomenon.\(^4\) Since the location of borders might influence the strength of agglomeration (and dispersion) forces, a change in borders might lead to more or less dispersed spatial distribution of population around its center(s). This, in turn, influences population density at a specific location, irrespectively of how far it is from the change in borders.\(^5\)

This paper proposes a model of economic geography which offers a natural way to decompose the effect of border changes into two distinct components. One of these components, which I call the local effect, is related to the classical mechanism presented in the economic geography literature: locations that are close to the change in borders are much more affected (because they become more or less isolated) than locations far away. The other component, which I call the global effect, acts through changes in the entire spatial distribution: since agglomeration and dispersion forces become stronger or weaker as a

\(^2\)Theoretical models of economic geography are successful at predicting that population density is lower at borders. In Rossi-Hansberg (2005), this result is due to border regions obtaining less productivity spillovers because they are relatively isolated. In new economic geography models such as Krugman (1991) or Helpman (1998), the effect is due to the fact that regions near the border have little market access relative to interior regions.

\(^3\)See, for instance, Redding and Sturm (2008).


\(^5\)Krugman and Livas Elizondo (1996) make a closely related point. The literature review part of the introduction describes this paper in more detail.
result of the border change, the distribution of population becomes either
more or less dispersed around its center(s). The strength of the global effect
at a given location, as opposed to the local effect, does not depend on distance
from changes in borders: it only depends on the centrality of a location before
the border change. The model is general in that it allows for any geographical
structure, such as continuous space or a discrete number of locations. Also,
the model incorporates many models of economic geography such as Helpman
(1998), as well as Krugman (1991)-type, Eaton and Kortum (2002)-type, and
Armington-type setups as special cases.

The following empirical example sheds light on the fact that centrality
might be even more important than distance from borders. In the empirical
part of the paper, I fit the theoretical model to Hungarian data, and use the
fact that Hungary lost more than two-thirds of its area in 1920 to identify
the magnitudes of local and global effects. Instead of using the model, I only
run two regressions for now. In the first specification, I regress the change
in log population between 1910 and 1930 for cities that remained part of
Hungary on their distance from the new border. As can be seen in Table 1,
the estimated coefficient is positive and significant – that is, regions far away
from the new borders gained in terms of population relative to those that
were close. However, the fact that the coefficient is only slightly significant
suggests that other important forces might be at play. This is confirmed
by the second specification, in which I also include distance from capital city
Budapest as a proxy of centrality. Column (2) of Table 1 shows that distance
from the new border became insignificant, whereas our variable measuring
centrality is highly significant. Locations that were close to the center gained
relative to others. Although the estimated coefficients might be easily biased
(by misspecification of functional forms, for instance), the results suggest
that the global effect might have played an important role here. Taking the
model to the data will confirm this prediction.

The theoretical model in the paper suggests a relationship between a
country’s urban structure and the relative magnitude of local and global
effects. According to the model, the global effect is especially important in
shaping the reallocation of population if the country has a dominant central city, or region. Border changes that are nearly symmetric, that is, leave the center of the economy unchanged, tend to be characterized by strong global effects as well.\footnote{The number and locations of cities (or, more precisely, central business districts) are treated as exogenous in the model. This is an obvious drawback as these might change as a result of changes in borders. However, the relocation of city infrastructure is likely to happen very slowly. Hence, the model can be thought of as describing the short-run effect of border changes, i.e., population reallocations across existing cities.}

In the second part of the paper, I use the theoretical model to measure the extent of local and global effects in early-20th century Hungary. Hungary underwent dramatic and unexpected changes in its borders in 1920. I fit the model to census data on the population of Hungarian cities in 1910, simulate the actual border changes in the model, and compare the population of cities predicted by the model to their actual population in 1930. I find a correlation coefficient of 0.975 between actual and predicted log city sizes. When calculating the correlation of log changes in city sizes, I obtain a value of 0.245, which is a much smaller number, but still suggests that the model captures an essential part of the reallocation of population that took place between 1910 and 1930. Moreover, the model is able to replicate the main qualitative features of the population reallocation: it predicts that cities in the center and in the Northwest gained the most in terms of population, cities in the far East and South experienced the smallest gains, and the fraction of people living in cities increased by about 50%. All these predictions are verified by the data.

Finally, I turn to measuring the magnitudes of the two effects. The results suggest that the local effect was present and hurt especially those regions that were located near the new border between Hungary and Romania. However, the global effect, hence centrality of locations, was about as important in shaping the reallocation of population as the local effect. In light of the theoretical results discussed above, this should not come as a surprise. Hungary had been and remained an economy with a dominant and very densely populated center (Budapest and its surroundings). Thus, the country had an urban structure that, according to the model’s predictions, should be
characterized by significant global effects.

Robustness of the results is checked by considering alternative ways of taking the model to the data. Most important, I address the issue that Hungary was part of the Austro-Hungarian Empire before 1920, forming a free trade zone with Austria. To resolve this problem, I fit the model to the whole empire before the border change. The global effect turns out to be even stronger in this case than in the baseline specification.

This paper is related to four strands of the literature. First, the idea that border changes might change the spatial concentration of economic activity is similar to the idea that opening up to trade affects agglomeration, a point first made by Krugman and Livas Elizondo (1996). Krugman and Livas Elizondo’s model involves discrete space with three regions, two domestic and one foreign. They assume that domestic regions are the same distance from the rest of the world, which rules out the local effect, leaving only the global effect in action. Following Krugman and Livas Elizondo (1996), a series of theoretical and empirical papers, thoroughly surveyed by Brülhart (2011), have investigated the effect of trade openness on agglomeration. To ensure tractability, all these papers assume a small number of regions and many symmetries. By contrast, my model is tractable even though I hardly make any assumptions on geographical structure. Also, to my knowledge, mine is the first paper looking at the effects of border changes, as opposed to opening up to trade with no labor mobility across borders, on the concentration of population.

The paper is also related to the part of the theoretical economic geography literature that uses more realistic assumptions on space than classical models do. Rossi-Hansberg (2005) solves for the spatial distribution of economic activity over a line segment. The line segment possibly consists of many "countries," that is, connected subsets. The location of borders across countries is exogenously given. If a border is put in, we can observe a reordering in the internal spatial distribution of economic activity. This reordering is likely to be large because it is amplified by adjustments in endogenous productivity. Chapters 9 to 11 in Fujita et al. (1999) constitute another
example of using one-dimensional continuous space. Although they do not investigate border effects, they assume the existence of \textit{cities} at specific locations (that are endogenous in their model). The number and locations of cities are a key determinant of the spatial distribution of population in my model too. It is, however, impossible to separate local and global effects in the models discussed in this paragraph. The reason for this is that distance from the center is perfectly correlated with distance from the border in symmetric one-dimensional space. Thus, my main contribution to this literature is that I make even less restrictive assumptions on space, which allows me to investigate questions that cannot be investigated in the above mentioned setups.

The setup I apply is most closely related to the class of structural economic geography models presented in Helpman (1998), Allen and Arkolakis (2014), and Redding (2015). Helpman (1998) presents a monopolistic competition framework that incorporates increasing returns to scale in production, trade costs, and labor mobility. Redding and Sturm (2008) extend the framework to multiple locations, while Allen and Arkolakis (2014) show that this model is isomorphic to a special case of an Armington model of economic geography. This paper generalizes the multiple-location Helpman framework by separating the concept of a \textit{location}, of which there are many, from the concept of a \textit{market}, or \textit{central business district}: a special location at which the exchange of tradable goods can take place. This separation leads to the endogenous formation of cities around markets, thus allowing me to talk about \textit{urban structure} within the context of the model, and obtain results about the relationship between urban structure and the relative importance of local and global effects.

Finally, my work is also related to papers emphasizing the role of border changes in changing the market access of locations. Redding and Sturm (2008) use the division of Germany after World War II as a natural experiment to identify the effect of the new border on locations nearby versus locations far away. They find substantial declines in the population growth of cities near the new border, and relate this result to the fact that border cities suffered large losses in their access to markets. Wolf (2007) and Brülhart et
al. (2012) conduct similar analyses for Poland and Austria, respectively. In line with this literature, my paper also finds that the change in market access, which, as shown in Section 2, corresponds to the local effect, is an important determinant of population changes after a change in borders. However, I find that the global effect is also present, and might be as important as the local effect in shaping the reallocation of population.

The structure of the paper is as follows. The theoretical model is presented in Section 2.1. In Section 2.2, I derive a formula that can be used to separate local and global effects in the model. Section 2.3 provides a way to assess the relative strength of local and global effects, and presents a set of examples which shed light on the relationship between border effects and urban structure. The data and the empirical strategy are described in Section 3, while the results are presented in Section 4. Section 5 concludes.

2 Model

2.1 Setup

A country is represented by a set $S$. Elements of the set are called locations, and are indexed by $r \in S$. Space can be, but is not necessarily, continuous – I use continuous space notation, but all theoretical results carry over to a discrete set of locations. Also, there exist a finite number of exogenously given locations $\eta_1, \ldots, \eta_K \in S$ that serve as markets, or central business districts. There are two types of goods in the economy: a continuum of tradable heterogenous goods, and a homogenous nontradable good called housing. There is one factor of production, labor. Markets are the only places where (1) firms can operate, (2) the exchange of tradable goods and labor can take place. As we will see, assumption (2) is crucial because it leads to the formation of cities around markets. Assumption (1), on the other hand, does not play an important role in the analysis, and is made only for simplicity.

The country is populated by $\bar{L}$ workers. Workers are freely mobile across
locations, and are endowed with one unit of labor which they supply inelastically. They simultaneously choose a location \( r \) at which they live, and a market \( k \) at which they shop and work.

### 2.1.1 Consumption

The representative worker at location \( r \) chooses her consumption of heterogeneous goods and housing by maximizing the utility function

\[
u (r) = a (r) \left[ \int_0^{n(r)} x^\omega (r)^\rho \, d\omega \right]^{\frac{\mu}{\rho}} h (r)^{1-\mu},
\]

where \( a (r) > 0 \) is the level of amenities at location \( r \), \( n (r) \) is the mass of heterogeneous goods available at \( r \), \( x^\omega (r) \) is the worker’s consumption of heterogeneous good \( \omega \), and \( h (r) \) is her consumption of housing. I follow Redding and Sturm (2008) by assuming that (1) housing is available in fixed supply \( H (r) > 0 \) at location \( r \), (2) rents are redistributed to local residents with equal shares.

Since \( u (r) \) is Cobb–Douglas, the consumer spends a \((1 - \mu)\) fraction of her income on housing:

\[
P^H (r) h (r) = (1 - \mu) y (r),
\]

where \( P^H (r) \) is the rental rate, and \( y (r) \) is per capita income at \( r \). Finally, as the subutility function of heterogeneous goods is CES, the demand for heterogeneous good \( \omega \) takes the standard constant-elasticity form:

\[
x^\omega (r) = \mu p^\omega (r)^{-\sigma} P (r)^{\sigma-1} y (r),
\]

where \( \sigma = \frac{1}{1-\rho} \) is the elasticity of substitution, and \( P (r) \) is the CES price index.
2.1.2 Shipping and production

Shipping heterogenous goods across markets is subject to iceberg trade costs. If \( \tau (\eta_m, \eta_k) \geq 1 \) units are sent from market \( m \) to market \( k \), only one unit arrives. To keep the model tractable, I assume that cross-market trade costs are symmetric: \( \tau (\eta_k, \eta_m) = \tau (\eta_m, \eta_k) \) for all \( k \) and \( m \). The consumer is also subject to shipping costs when bringing the good home from the market. If she buys \( \zeta (\eta_k, r) \geq 1 \) units at market \( k \), she is left with one unit at her home location \( r \).

As discussed already, firms producing heterogenous goods can operate in central business districts only. Heterogenous goods are produced under monopolistic competition with free entry. As a consequence, each good is produced in one CBD. The firms’ cost function is linear: they have to use one unit of labor in order to produce one unit of output, plus they have to pay a fixed startup cost \( f \) (in units of labor). Consider the problem of a firm producing good \( \omega \) at market \( k \). Since preferences are CES and trade costs are of the iceberg type, the firm sets a mill price that is a constant markup over its marginal cost:

\[
p_k^\omega = p_k = \frac{\sigma}{\sigma - 1} w_k,\]

and, by free entry, each firm produces \( x = f (\sigma - 1) \) units. Let \( n_k \) denote the mass of firms at \( k \). Labor market clearing then yields

\[
L_k = n_k (x + f) = n_k f \sigma,
\]

where \( L_k \) is the number of people commuting to market \( k \) (the "size" of the market). Thus, the mass of firms operating at \( k \) is \( n_k = \frac{L_k}{f \sigma} \), and the total mass of firms (and heterogenous goods) in the economy is \( \frac{L}{f \sigma} \). Assuming finite

\[
7 \zeta (\eta_k, r) \text{ can also be interpreted as commuting costs for the consumer, to be paid either in } \text{utils or as a fraction of income, as in Ahlfeldt et al. (2015). Since } \zeta (\cdot, \cdot) \text{ is calibrated to match the concentration of population in the empirical part of the paper, the results do not depend on the specific interpretation one chooses.}
\]

\[
8 \text{The choice of a monopolistic competition environment is somewhat arbitrary. It is possible to write down – either Armington-type, or Eaton and Kortum (2002)-type – models that are characterized by perfect competition in all markets, and yield the exact same equilibrium conditions as this model. The next subsection describes these equivalences in more detail.}
\]
Trade costs, this is the mass of goods available at any location: \( n(r) = \frac{L}{f_r} \).

### 2.1.3 Equilibrium

Consumers who decide to live at a given location \( r \) are identical in this model. Therefore, they all choose the same market to commute to. Let us denote this market by \( \kappa(r) \). Given this, I define the equilibrium of the economy below.

**Definition 1** Given parameters \( \sigma, \mu, f, L, \) geography \( S, \{ \eta_1, \ldots, \eta_K \} \), and functions \( a(\cdot), H(\cdot), \tau(\cdot, \cdot) \) and \( \zeta(\cdot, \cdot) \), an equilibrium is the spatial distribution of population \( L(r) \),\(^9\) housing prices \( P^H(r) \), heterogenous good prices \( p_k \), wages \( w_k \), housing consumption \( h(r) \), consumption of heterogenous goods \( x^e(r) \), production of heterogenous goods \( x^e_k \), market sizes \( L_k \), a market assignment function \( \kappa(r) \), and a utility level \( \bar{u} \) such that

1. consumers maximize utility, and utility at each location equals \( \bar{u} \) by free labor mobility,
2. firms maximize profits, and profits are driven down to zero by free entry,
3. markets for heterogenous goods and labor clear at each CBD,
4. housing markets clear at each location,
5. the national labor market clears,
6. there is no possibility of arbitrage.

The Appendix shows that the spatial distribution of population is governed by the following system of equations in equilibrium:

\[
\lambda_k = \nu \left[ \sum_m \lambda_m \left( \frac{\sigma-1}{(2\sigma-1)\mu} \right)^{\frac{(2\sigma-1)\mu}{\sigma(\sigma-1)(1-\rho)}} \right]^{(\sigma-1)^{\mu}}
\]

\[^9\]If space is discrete, \( \{L(r)\}_{r \in S} \) is a set of non-negative numbers that sum to \( L \). If space is continuous, \( L(\cdot) \) is a density, with the area below it being equal to \( L \).
\[ L(r) = \bar{a}(r) \varsigma(\eta_{\varsigma(r)}, r)^{-\frac{1}{1-\mu}} \lambda_{\varsigma(r)} \]  \hfill (2)

\[ L_k = \int_{\{r: k = \varsigma(r)\}} L(r) \, dr \]  \hfill (3)

\[ \varsigma(\eta_{\varsigma(r)}, r)^{-\frac{1}{1-\mu}} \lambda_{\varsigma(r)} \geq \varsigma(\eta_k, r)^{-\frac{1}{1-\mu}} \lambda_k \quad \forall k, \]  \hfill (4)

where \( \nu \) is a combination of parameters and \( \bar{a}(r) = a(r) \frac{1}{1-\mu} H(r) \) is a combination of amenities and the housing stock at location \( r \), and \( \lambda_k \) is a positive number that I call the *attractiveness* of market \( k \). Attractiveness is an increasing function of the wage, and a decreasing function of the price index at the market.\(^{10}\) Hence, if market \( k \) offers higher wages and lower prices than market \( m \), then \( \lambda_k \) is higher than \( \lambda_m \).

The intuition for equations (1) to (4) is as follows. According to equation (1), attractiveness of each market \( \lambda_k \) depends on the sizes \( L_m \) and attractiveness \( \lambda_m \) of neighboring markets. Equation (2) determines population at location \( r \) as a function of amenities \( \bar{a}(r) \), shipping costs from the market which consumers at \( r \) choose to commute to \( \varsigma(\eta_{\varsigma(r)}, r) \), and the attractiveness of this market \( \lambda_{\varsigma(r)} \). Equation (3) says that the size of each market is equal to the total number of people who commute to the market. Finally, equation (4) shows how the choice of markets takes place in equilibrium. Consumers at \( r \) choose the market that offers them the best combination of "proximity" \( \varsigma(\eta_k, r)^{-\frac{1}{1-\mu}} \) and attractiveness \( \lambda_k \). When facing two markets with the same \( \lambda_k \), they choose the one that offers lower shipping costs. When facing two markets from which shipping is equally costly, they choose the one that has a higher level of attractiveness.

Equation (2) sheds light on an important feature of the equilibrium. Take two locations \( r \) and \( s \) that have the same level of amenities, and from which consumers commute to the same market. Assume that consumers’ shipping costs are an increasing function of distance. Then, if \( r \) is closer to the market than \( s \), we have \( \varsigma(\eta_{\varsigma(r)}, r) < \varsigma(\eta_{\varsigma(r)}, s) \), hence equation (2) implies \( L(r) > \)

\(^{10}\)See the Appendix for the exact definition of \( \lambda_k \).
$L(s)$. That is, population decreases with distance from the market. On the other hand, if $r$ and $s$ are equidistant from the market, then $\zeta(\eta_{r(r)}, r) = \zeta(\eta_{s(r)}, s)$, implying $L(r) = L(s)$: level curves of $L(\cdot)$ are circles around the market. Thus, circular cities with a negative population gradient form around markets.\footnote{Another feature of equations (1) to (4) is their scale-independence. Changing total population $\tilde{L}$ does not affect locations’ relative population levels $\nu \tilde{L}$ and markets’ “relative attractiveness” $\frac{\lambda_m}{\tilde{L}}$. In the case of equations (2) to (4), this can be seen by dividing both sides by $\tilde{L}$. As for equation (1), it can be shown that the value of $\nu$ adjusts in a way that leads to the result.}

It is also important to note that, if every location $r \in S$ is a market and consumers’ shipping costs are large enough that no one commutes to a market that is distinct from her location, then the model reduces to the multiple-location version of the Helpman (1998) model presented in Redding and Sturm (2008). As Allen and Arkolakis (2014) show, the Helpman setup is isomorphic to a class of economic geography models that include Eaton and Kortum (2002)-type, as well as Armington-type models. Hence, the special case of our model in which every location is an operating market is isomorphic to these models too. Finally, Krugman (1991) without an agricultural sector is an even more special case in which $\mu = 1$, and there are no differences in amenities across locations.

The next subsection uses equations (1) and (2) to explore how border changes affect the spatial distribution of population in the model.

### 2.2 The effect of a change in borders

In this subsection I examine the effect of border changes – i.e., changes in the set $S$ – on the equilibrium spatial distribution of population, $L(\cdot)$. Use equations (1) and (2) to express population at location $r$ as

$$L(r) = \nu \tilde{u}(\zeta(\eta_{r(r)}, r))^\frac{\mu}{1-\mu} \sum_m \tilde{\lambda}_m \frac{(\sigma-1)^2(1-\mu)}{(2\sigma-1)^\mu} \tilde{L}_m \tau(\eta_{r(r)}, \eta_m)^{1-\sigma}.$$
Now change borders in any way such that $S$ changes to $S' \neq S$, and denote the new population distribution by $L'(\cdot)$. Assume that markets do not change their location, and amenities do not change either – the reason for this might be that city infrastructure (roads, buildings, etc.) would be extremely costly to relocate, or its relocation would take more time than the time span captured by the model. Then we have

$$\frac{L'(r)}{L(r)} = \left[ \frac{\nu'}{\nu} \right]^{\sigma(\sigma-1)(1-\mu)/(2\sigma-1)^\mu} \left[ \frac{k'(\eta_{m^c}(r), r)}{k(\eta_{m}(r), r)} \right]^{-\frac{\sigma(\sigma-1)}{2\sigma-1}} \left[ 1 - \frac{l(r)}{g(r)} \right], \quad (5)$$

where

$$l(r) = \sum_m \lambda_m^{-(\sigma-1)^2(1-\mu)/(2\sigma-1)^\mu} L_{m^c}(\eta_{m^c}(r), \eta_m)^{1-\sigma} - \sum_m (\lambda'_m)^{-(\sigma-1)^2(1-\mu)/(2\sigma-1)^\mu} L'_m(\eta_{m^c}(r), \eta_m)^{1-\sigma}$$

and

$$g(r) = \sum_m \lambda_m^{-(\sigma-1)^2(1-\mu)/(2\sigma-1)^\mu} L_m(\eta_{m^c}(r), \eta_m)^{1-\sigma}.$$

Equation (5) says that the effect of a change in borders can be decomposed into four components. The first one, $\left[ \frac{\nu'}{\nu} \right]^{\sigma(\sigma-1)(1-\mu)/(2\sigma-1)^\mu}$, is common across locations – the role of this term is to guarantee that total population changes from $L$ to $L'$. The second term, $\left[ \frac{k'(\eta_{m^c}(r), r)}{k(\eta_{m}(r), r)} \right]^{-\frac{\sigma(\sigma-1)}{2\sigma-1}}$, arises because the border change might lead to consumers at $r$ changing the market to which they commute. I do not focus on this term in the analysis because it turns out to be quantitatively unimportant. The third term, the denominator $g(r)$ is large for locations whose market was close to other markets (and, in particular, to markets with a large number of people) before the change in borders. Thus, $g(r)$ is a measure of the global effect, or centrality of a location before the border change – but it is independent of changes in distance from the border.

The opposite is true for the fourth term, i.e., $l(r)$. This term is large for locations that are far away from markets which experienced large increases in their size, but close to markets which had large drops in their $L_m$’s – and turns out to be a measure of the local border effect on location $r$, that is,
effects whose magnitudes depend on the change in the distance between $r$ and the border. The intuition for this is as follows. Suppose that borders change such that some regions secede from the country, and these regions were close to market $k$. This results in an "exogenous" decrease in $L_k$ and the $L_m$ of markets that surround market $k$. At the same time, markets far away from the seceding regions – i.e., far away from the border change – must gain since market sizes $L_m$ necessarily sum to total population. That is, we can observe population increases in cities far away from market $k$, and decreases in cities close to it, which drives up the value of $l(r)$, hence it decreases the value of the right-hand side of equation (5). This then results in an increase in $L(r)$ for locations surrounding market $k$ – thus increasing $L_k$ even further.

Another interpretation of $l(r)$ is as follows. By a slight generalization of the traditional market access formula $MA_{\text{traditional}}(s) = \sum_m L_m \tau (s, \eta_m)^{-\varepsilon}$, one can define the market access of a location $s$ before and after the border change as

$$MA(s) = \sum_m \lambda_m^{-(\sigma-1)^2(1-\mu)/(2\sigma-1)\mu} L_m \tau (s, \eta_m)^{1-\sigma}$$

and

$$MA'(s) = \sum_m (\lambda'_m)^{-(\sigma-1)^2(1-\mu)/(2\sigma-1)\mu} L'_m \tau (s, \eta_m)^{1-\sigma},$$

respectively. Since $l(r) = MA(\kappa(r)) - MA'(\kappa'(r))$, the (absolute value of the) local effect equals the change in the market access of location $r$’s CBD.

To sum up, the model offers a way to decompose the effects of changes in borders into two key components: (1) a global effect that is due to the reordering of the entire distribution, and is a function of distance from the "center" of the country’s economy, and (2) a local effect that is large for locations near the border change, but small for those far away from it – put differently, this is the classical mechanism of locations facing different changes in their market access as a result of changes in borders.
2.3 The importance of global and local effects

This subsection offers a way to assess the importance of global and local effects in shaping the redistribution of population due to a border change, and presents stylized examples that shed light on the relationship between urban structure and the importance of the two effects. Rearranging equation (5) yields

$$\Lambda (r) = \frac{l (r)}{g (r)},$$

where

$$\Lambda (r) = 1 - \left[ \frac{\nu}{\nu'} \right]^{\frac{\sigma (\sigma - 1) (1 - \mu)}{(2 \sigma - 1) \mu}} \left[ \frac{\zeta (\eta \kappa (r), r)}{\zeta (\eta \kappa' (r), r)} \right]^{-\frac{\sigma (\sigma - 1)}{2 \sigma - 1}} \left[ \frac{L' (r)}{L (r)} \right]^{\frac{\sigma (\sigma - 1) (1 - \mu)}{(2 \sigma - 1) \mu}}.$$  

Take logs in equation (6), and apply the variance operator to both sides:

$$Var [\log \Lambda (r)] = Var [\log l (r)] + Var [\log g (r)] - 2 Cov [\log l (r), \log g (r)],$$  

from which

$$1 = \frac{Var [\log l (r)]}{Var [\log \Lambda (r)]} + \frac{Var [\log g (r)]}{Var [\log \Lambda (r)]} - 2 \frac{Cov [\log l (r), \log g (r)]}{Var [\log \Lambda (r)]}.$$  

Equation (7) can be used to decompose the spatial variation in \(\Lambda (r)\) into the contribution of local and global effects, as well as their covariance. \(\frac{Var[\log l(r)]}{Var[\log \Lambda (r)]}\) and \(\frac{Var[\log g(r)]}{Var[\log \Lambda (r)]}\) are the fractions of variation in \(\Lambda (r)\) that can be attributed to the local and the global effect, respectively. As a consequence, \(\frac{Var[\log g(r)]}{Var[\log l(r)]} = \frac{Var[\log l(r)]}{Var[\log \Lambda (r)]}/\frac{Var[\log g(r)]}{Var[\log \Lambda (r)]}\) tells us how important the global effect was relative to the local effect in shaping the variation in \(\Lambda (r)\).

If no residents switch from one market to another, then we have...
\[
\frac{\varsigma(\eta_{r}(r),r)}{\varsigma(\eta_{r},r)} - \frac{\sigma(\sigma-1)}{2\sigma-1} = 1 \text{ for any } r, \text{ hence the spatial variation in } \Lambda (r) \text{ is solely due to the spatial variation of population changes } \frac{L'(r)}{L(r)}. \text{ Hence, } \frac{\text{Var}[\log g(r)]}{\text{Var}[\log l(r)]} \text{ can be interpreted as the importance of the global effect relative to the local effect in shaping the reallocation of population. If some locations switch between markets, this is no longer the case as variation in } \Lambda (r) \text{ might also stem from the spatial variation of } \frac{\varsigma(\eta_{r}(r),r)}{\varsigma(\eta_{r},r)}. \text{ However, it turns out that, both in the examples below and in the empirical application of Sections 3 and 4, the share of market-switching locations is so small that they hardly influence the results. Therefore, I keep interpreting } \frac{\text{Var}[\log g(r)]}{\text{Var}[\log l(r)]} \text{ as the relative contribution of the global effect, even though it is, strictly speaking, just an approximation of the true contribution.}
\]

In what follows, I present two results that link the strength of local and global effects to urban structure of the country, as well as to the geography – in particular, the "symmetry" – of border changes. Unfortunately, the complex structure of the model does not allow me to derive analytical predictions. Therefore, I have to rely on a simple and stylized geography, namely, a country that has the shape of a square. However, I argue that the results I obtain are general as they follow from the underlying mechanisms of the model, not from the specific choice of geography, structural parameters, and shipping costs. In this sense, I choose a specific environment only for the sake of illustration.

Imagine a country that has the shape of a 200 by 200 km square on the Euclidean plane. The country has 10 million inhabitants, and nine markets: one in the midpoint of the square, four in the vicinity of the midpoint, and four near borders (see Figure 1). Amenities are distributed uniformly in space. The second graph in the first row of Figure 2 depicts the equilibrium distribution of population for the country.12 Now suppose that borders change in the way shown in the first graph of the first row: the country is still a square, but its area shrinks to 100 by 100 km, and only the five cen-

---

12 I use $\sigma = 9$, $\mu = \frac{3}{4}$, $r(r,s) = e^{0.0001 \text{dist}(r,s)}$ and $\varsigma(r,s) = e^{0.02 \text{dist}(r,s)}$ in all of the simulations of Section 2.3. $\text{dist}(r,s)$ denotes the Euclidean distance between locations $r$ and $s$. 

---

16
trally located markets remain. The last figure in the first row depicts the new distribution of population. The relative importance of the global effect is equal to 0.764.

What if the country had a dominant center? Increase the level of amenities in the 5 km wide neighborhood of the market in the midpoint by a factor of ten. The results of this exercise are presented in the second row of Figure 2. Not surprisingly, the central city now attracts a larger fraction of population both before and after the change in borders. The relative importance of the global effect has also increased to 0.828. Hence, this stylized example conveys the following result.

**Result 1.** Global effects tend to be more important in the case of countries which have a dominant central city, or region.

The intuition for Result 1 is as follows. In a country with a dominant center, a large fraction of interactions take place through the center. This implies that a border change mainly affects a location through the center, hence a sufficient statistic for the effect on the specific location is its centrality before the border change. But this is just the global effect by definition.

Notice that the change in borders done in the previous exercise was not symmetric in the sense that the midpoint of the square did not remain the midpoint after the border change; instead, the midpoint shifted to the East (see Figure 2). Repeating the exercise with a symmetric border change (that is, such that the central city remains in the middle of the country) and uniform amenities, I find that the relative importance of the global effect goes up from 0.764 to 1.060. With the amenity level of the central city being ten times as high, the importance of the global effect equals 1.197 instead of the value of 0.828 that I found for the asymmetric border change. We thus have the following result.

**Result 2.** Global effects tend to be more important for symmetric border changes, especially in countries with a dominant center.

Result 2 is also intuitive. If borders change in an asymmetric way, then the center of the economy is likely to relocate, resulting in increases in the market access of locations toward which the center moves, and decreases in the market access of locations which get farther away from the center.
This leads to more variation in the local effect. Although the center plays a crucial role in this "relocation-of-the-center effect" (and this is the reason why this effect is stronger with a dominant center), it cannot be captured by the global effect because the global effect is influenced by pre-border change characteristics only.

Taking the model to the data, it becomes possible to measure the magnitudes of global and local effects for the 1920 border changes of Hungary, examine how the two effects shaped the reallocation of population, and see the relevance of Results 1 and 2 in that specific case. This is what I do in the next two sections of the paper.

3 Data and empirical strategy

Hungary underwent dramatic changes in its borders due to the Treaty of Trianon in 1920: its area shrunk from 325,000 km$^2$ to 93,000 km$^2$, and its population fell from 20.9 million to 7.6 million (see Figure 3). These border changes were unexpected until 1918, the end of the First World War. Although Hungary was a multi-ethnic state before 1920 and ethnic minorities occasionally claimed for autonomy or even independence, new borders were largely unrelated to ethnic boundaries; see Kontler (2002) or Teleki (1923).

I use data from the 1910 and 1930 population censuses to examine the effects of changes in borders on the distribution of population. These censuses provide the number of inhabitants in each settlement of the country, as well as each settlement’s area in square kilometers. Every location in space belongs to one and only one settlement – in other words, settlements’ areas constitute a partition of the country’s territory. Unfortunately, there is no data on settlements’ actual geographic boundaries. Although the censuses themselves classify settlements into cities (város), towns (nagyközség), and villages (kisközség), I do not rely on this classification because (1) it is largely based on history, with settlements with one or two thousand inhabitants that had once been important places being called cities but some with much larger population classified as towns or even as villages, (2) for political reasons, this classification changed substantially before the 1930 census. Instead, I
regard any settlement with more than 10,000 inhabitants in 1910 as a city. According to the model presented in Section 2, a city is different from other locations because agents can use its market, or CBD, to exchange tradable goods. It is likely that only settlements above some population threshold have such a marketplace, and the 10,000-inhabitant threshold seems to be a reasonable approximation. Still, Section 4 presents results for a 20,000-inhabitant threshold as well. Hungary had 154 cities with population above 10,000 in 1910, and 84 of them remained in the country after the border changes in 1920.13

I regard Budapest and its suburbs as one large city instead of a collection of cities in the analysis. Even though these cities were not united formally until 1950, they were largely integrated economically already in 1910 (Hanák, 1988). Treating them as separate cities does not lead to a significant change in the results. Lacking data on the exact location of CBDs, I assume that they were located at the present geographic center of each city.

Some comments on the period of investigation are in order. In Hungary, starting from 1869, one population census was carried out in a decade. Therefore, the 1910 census is the last one providing a picture of the population distribution in Hungary before the border change. The 1920 census, which was carried out right after the change in borders, is not likely to reflect all the effects of border changes. This is because cities' relative population levels—abstracting from differences in birth and death rates—can only change through migration, and it is unlikely that migration fully took place over a period of months. Moreover, 1920 city populations are distorted by the fact that about 350,000 ethnic Hungarian refugees, who fled to the country in the previous years, were given temporary accommodation in school buildings and railway cars around railway stations of the largest cities. These "railway car towns" gradually disappeared by 1930 (Kontler, 2002). Population data in the 1941 census, on the other hand, is likely to be influenced by other factors including the Great Depression, the increase in trade between Hungary and

13In 1910, 24.9% of the total population lived in cities above 10,000 inhabitants. This number rose to 38.7% by 1930. This dramatic increase in urban population is almost fully captured by the model; for further discussion, see Section 4.
Austria, Germany and Italy in the 1930s (Kosáry, 1941), the first two years of World War II, and significant new border changes between 1938 and 1940. Thus, the twenty-year period between 1910 and 1930 seems to be the best choice if one tries to measure the effects of border changes on the spatial distribution of population.

Finally, note that the model assumes that Hungary was a closed economy both before and after the change in its borders. Trade with foreign countries, factor mobility, or flows of ideas through the Hungarian border could hence bias the results. This is not really a concern after the border change as Hungary essentially went into autarky after the Treaty of Trianon (Teleki, 1923), and foreign trade started to grow substantially in the 1930s only (Kosáry, 1941). On the other hand, Hungary was part of the Austro-Hungarian Empire before the border change, forming a free trade zone with the Austrian part since 1868. Kontler (2002) reports that trade with Austria accounted for approximately 20% of Hungarian GDP, a fact that cannot be ignored in the analysis. As a consequence, Section 4 involves a robustness check in which the model is fit to the whole empire instead of only the Hungarian part before the border change.

Having the data on hand, I apply the following empirical strategy. Using the fact that the amenity function, \( \widetilde{a}(r) \), can take any form, I match the model exactly to cities’ population in 1910. I assume the following amenity function: \( \widetilde{a}(r) = a_k \) if location \( r \) belongs to the area of city \( k \), and \( \widetilde{a}(r) = 1 \) otherwise. Since I do not have data on cities’ geographic boundaries but I do have their area in square kilometers, I assume that each city had a circular shape around its CBD, with the area of the circle being equal to the city’s actual area, as reported in the data. Then I run the model with the cities in 1910 at their actual locations, and search for values of \( a_k \) that are consistent with each city having the same population as in the data.\(^{14}\) In order to be

\(^{14}\)In principle, there could exist different sets of amenity levels that are consistent with city sizes observed in the data. However, running the searching procedure with many different initial values has always resulted in the same values of \( a_k \). This suggests that the set of amenity levels leading to the observed city sizes is a singleton, at least for the specific geography and values of structural parameters used in this paper.
close to continuous space, a very fine discretization of space is used when calculating the equilibrium on the computer: I split the territory of Hungary into 0.01° by 0.01° grid cells. This means about 400,000 grid cells in total. Despite the large number of locations, the relatively simple structure of the model leads to quick calculations: finding the values of $a_k$ that rationalize the data takes about a minute on a typical personal computer.

Besides $a(\cdot)$, I have to choose the values of two structural parameters: $\sigma$ and $\mu$, as well as two functions: cross-market trade costs $\tau(\cdot, \cdot)$ and consumers’ shipping costs $\zeta(\cdot, \cdot)$. I choose the value of $\sigma$ based on the fact that the elasticity of trade with respect to variable costs is $1 - \sigma$ in the model. Following Eaton and Kortum (2002), I set the value of this elasticity to negative eight, which implies $\sigma = 9$. Concerning $\mu$, I set the share of housing expenditures in the consumer’s budget to one fourth, which implies $\mu = \frac{3}{4}$. This is consistent with the estimates of housing expenditure shares by Davis and Ortalo-Magné (2007), but it is somewhat higher than the value used by Redding and Sturm (2008). However, the spatial unit of observation is the city in Redding and Sturm (2008), while it is smaller (a 0.01° by 0.01° grid cell) in this paper. Therefore, the set of traded goods includes those traded within the city in my analysis, but not in theirs. As a consequence, it is reasonable to assume that the expenditure share on traded goods is larger in my case.\textsuperscript{15}

I assume that both types of shipping costs are exponential functions of distance:\textsuperscript{16}

$$
\tau(r, s) = e^{\phi \text{dist}(r, s)}
$$
$$
\zeta(r, s) = e^{\phi \text{dist}(r, s)}
$$

\textsuperscript{15}Section 4 includes robustness checks with lower values of $\sigma$ and $\mu$.

\textsuperscript{16}The exponential formulation of trade costs is a frequently used assumption in the economic geography literature. Examples are Fujita et al. (1999), Rossi-Hansberg (2005), and Desmet and Rossi-Hansberg (2012). Section 4 includes a robustness check with shipping costs that are power functions of distance, as in Redding and Sturm (2008). Distances are measured "as the crow flies." It is unlikely that using road and rail distances instead would lead to a significant change in the results. This is because the Hungarian road and railroad network was very dense already in 1910 – comparable in density to the networks of developed countries (Kontler, 2002).
It remains to choose the values of shipping cost elasticities $\phi$ and $\psi$. I match $\phi$ to evidence on trade costs. The 1910 Yearbook of the Hungarian Statistical Office reports average prices of the three main products imported through the port of Fiume (now Rijeka, Croatia) both in Fiume and in Budapest. Wheat was 25% more expensive in Budapest than in Fiume, coffee was 15% more expensive, and rice was 8% more expensive. With $\phi = 3.51 \cdot 10^{-4}$, trade costs between the CBDs of Fiume and Budapest are equal to 16.2%, which corresponds to the average of the above three numbers.

Finally, the value of $\psi$ is chosen such that the model replicates the concentration of rural population in 1910, measured by the Theil index.\(^{17}\) This is calculated from the data as

$$T = \sum_{s=1}^{S} \frac{pop_s}{pop} \ln \left( \frac{pop_s \ area_s}{pop \ area} \right),$$

where $s \in \{1, \ldots, S\}$ indexes non-city settlements, $pop$ and $area$ are the total population and area of Hungary in 1910, and $pop_s$ and $area_s$ are the population and area of settlement $s$, respectively. Since the geographic boundaries of settlements are not known, the exact same statistic cannot be calculated from the model. Instead, I merge grid cells neighboring each other (excluding grid cells that belong to cities) so that the number of the spatial units created in this way is close to the number of non-city settlements in the data. Then I calculate the Theil index using these spatial units’ population levels. Since the population density gradient around markets is strictly increasing in consumers’ shipping costs – see equation (2) –, the Theil index is strictly increasing in $\psi$ in the model. This implies that the parameter is identified. The procedure pins down a value of $\psi = 1.52 \cdot 10^{-2}$.

Once I have this, I can use the model to simulate the distribution of population within the new borders and (1) compare the simulated distribution to what we see in the data in 1930,\(^{18}\) (2) calculate the magnitudes of local

\(^{17}\)Using other indices of concentration does not change the results substantially.

\(^{18}\)Just like in the case of recovering cities’ amenity levels, multiplicity is a potential issue here. If there were multiple equilibria, then it would not be clear which one we should compare to 1930 city sizes. Multiplicity of equilibria tends to arise when agglomeration forces are strong. As already discussed in Section 2.1, the model becomes identical to the
and global effects, (3) assess the relative importance of the two effects in reordering the spatial distribution of economic activity. The next section reports the results of the analysis.

4 Results

Figure 4 presents the change in population levels implied by the model in the baseline calibration. The correlation between log city sizes predicted by the model and log city sizes in the 1930 data is as high as 0.975. This means that the model is successful at predicting city sizes after the change in borders. If one calculates the correlation of log changes in city sizes between the model and the data, one obtains a value of 0.245 (see the first row of Table 2), which is a much smaller number, but still suggests that the model captures an essential part of the reallocation of population that took place between 1910 and 1930. Moreover, the model is able to replicate the main qualitative features of the population reallocation. First, the model predicts that locations in the center and in the West gained the most in terms of population. Out of the fifteen cities that experienced the highest population growth between 1910 and 1930, nine were indeed located in this region. Second, the far East and South were hurt the most by the border change according to the model; ten out of the fifteen cities with lowest population growth were in fact located here. Finally, the model predicts that the fraction of people living in cities above 10,000 inhabitants rose to 37.0% by 1930. This is very close to the fraction we can see in the data (38.7%).

The relative importance of the global effect is 0.797. This number shows that centrality played a role in the spatial reallocation of population that is comparable to the role of the local effect. I obtain a very similar number (0.812) if I calculate the variances of local and global effects from the subset of locations that do not switch from one market to another. Figure 5 shows the multiple-location version of Helpman (1998) if every location is a market, and consumers’ shipping costs are high enough. In this case, the equilibrium is guaranteed to be unique whenever the famous "no-black-hole" condition is satisfied. Ruling out multiplicity in the general version of the model is a much harder task. However, simulations suggest that the equilibrium is unique in all the exercises conducted in this paper.
values of (log) local and global effects for each location. As can be seen on the left map, locations in the Southeast, i.e., near the new Hungarian-Romanian border, were significantly more hurt by the local effect than others – the intuition for this is that they have got close to the border after 1920, whereas they had been very far from borders before. If only the local effect had been in action, regions in the Western part of the country would have gained the most in terms of population; the reason for this is that these regions have always been close to the border, so they did not have to suffer large decreases in their market access after 1920. However, this is not what happens since the global effect is also at play. As one would expect, the global effect was large in regions that were centrally located before the border change. If only the global effect had been present, these regions would have gained the most – however, the local effect led to people moving away from these locations as they were relatively close to the Hungarian-Romanian border. At the end of the day, locations that were characterized by medium levels of both effects, that is, locations in the Northwest and in the center, saw the largest increases in their population.

The relatively large importance of the global effect is in line with the theoretical results of Section 2.3. To see this, notice first that Hungary had been and remained an economy with a dominant and very densely populated center. In 1910, capital city Budapest and its suburbs constituted 6% of total population but only 0.1% of the country’s area. These numbers rose to 15% and 0.5% by 1930. The population of the second-largest city was 12.6% of the capital’s population in both years. Road and railway networks were spider-web-like with Budapest as the center. This promoted most trade and other interactions taking place through Budapest, thus increasing the capital’s dominance in the Hungarian economy even further; see Teleki (1923).

Second, although border changes due to the Treaty of Trianon were not completely symmetric, regions that were centrally located in 1910 remained in the middle of the country even after 1920 (check Figure 3).19 Hence, one can argue that Hungary was an economy (1) with a dominant center, and (2)

19The geographic center of the country shifted from Szarvas to Pusztavaes, which means a shift of only 87 kilometers to the Northwest.
in which border changes took place in a relatively symmetric fashion. As I showed in Section 2.3, both cases are characterized by a strong global effect (Results 1 and 2).

In order to check whether Result 1 indeed holds in this case, I do an exercise that is similar to the one conducted in Section 2.3. In particular, I increase the amenity levels of cities that are in the 100 km wide neighborhood of Budapest by a factor of ten relative to the amenity levels recovered from the data. This shift in amenities makes the central region of Hungary even more dominant than it actually was, hence, by Result 1, it should increase the importance of the global effect even further. The relative importance of the global effect is in fact 1.087 in this hypothetical case, almost 40% higher than our baseline estimate of 0.797.

The estimate of the relative importance of the global effect could be biased if the discrepancies between actual population changes and those predicted by the model were systematically related to the magnitudes of local and global effects. If, for instance, the model overestimated the magnitudes of population changes at locations which experienced a large global effect but underestimated population changes at locations for which \( g(r) \) is small, then the global effect would play a more important role in the model than in the data, leading to an upward bias in the relative importance estimate. To see whether this is the case, I compute the (squared) error between actual and predicted log city sizes in 1930, and calculate the correlation between the error and (log) local and global effects. The correlation coefficients for local and global effects are \(-0.059\) and \(-0.081\), respectively. Neither of the two is significantly different from zero. Thus, I could not find evidence for the above mentioned bias in the relative importance estimate.

Finally, Table 2 also presents the results of seven alternative calibrations that are conducted in order to check for robustness of the results. For each calibration, the first, second, and third columns show the correlation between log city size changes implied by the model and the data, the relative importance of the global effect for the entire set of locations, and the relative importance of the global effect without market-switchers, respectively. In calibration (2), only those cities are considered that had at least 20,000 in-
habitants in 1910. In calibrations (3) and (4), I experiment with lower values of the elasticity of substitution $\sigma$ and tradables’ budget share $\mu$. Instead of exponential shipping costs, power-function costs of the form

$$
\tau(r, s) = [1 + t \text{dist}(r, s)]^\phi \\
\zeta(r, s) = [1 + t' \text{dist}(r, s)]^\psi
$$

are used in calibration (5). Following Redding and Sturm (2008), the value of $\phi$ is set to one third; the value of $t$ is calibrated to match the average price differential between Budapest and Fiume; and the value of $t'$ is chosen such that the model replicates the concentration of rural population, measured by the Theil index. Calibrations (6) and (7) again work with exponential shipping costs, but the maximum price differential (wheat, 25%) is matched to cross-market trade costs for (6), while the minimum price differential (rice, 8%) is captured in the case of (7). Finally, calibration (8) addresses the concern that Hungary was not a closed economy before 1920, but part of the Austro-Hungarian Empire, and fits the model to the whole empire, as already discussed in Section 3. As can be seen in rows (2) to (8), the results do not change substantially under the alternative calibrations.

This section showed that the theoretical model presented in Section 2 is successful in two respects. First, it is able to predict the total effect of border changes on the population distribution with relatively large confidence. Second, it is able to explain the fact that not just distance from the border, but also centrality played an important role in determining how population levels changed at different locations.

5 Conclusion

This paper proposed a model of economic geography that has a handful of advantages: it incorporates many existing models as special cases; despite the fact that it hardly makes any assumptions on space, the distribution of population is governed by a relatively simple system of equations in the model; and it yields a decomposition of the effect of border changes into a standard
local effect (changes in population are related to changes in distance from the border), and a novel global effect (changes in population are related to centrality before the border change). As already discussed in the introduction, higher-dimensional space is necessary to separate the two effects from one another – distance from the border and distance from the center are perfectly correlated in a symmetric one-dimensional economy. Thus, using higher-dimensional space in economic geography models might be important not just because it is closer to reality, but also because it can unveil forces that would remain hidden otherwise.

The model yields predictions on the relationship between urban structure and border effects. The global effect is likely to be strong in economies with a dominant central city, or region. The global effect also tends to be more important for border changes that are symmetric, i.e., leave the location of the center unchanged.

The second part of the paper took the model to Hungarian data from 1910 and 1930, and quantified the effects of border changes in 1920. It found that the global effect, that is, centrality of locations, played a role in shaping the reallocation of population that was about as important as the local effect, that is, changes in market access. This is in line with the predictions of the theoretical model since Hungary was an economy with a dominant center both before and after the change in borders, and central regions remained centrally located even after 1920.

The theoretical framework presented in the paper might constitute a basis for further investigations. An especially interesting task would be endogenizing the number and locations of markets. This would make it possible to ask questions such as: Where do new cities form as the economy grows, or borders change? What is the difference in growth between old and new cities? How does the city size distribution evolve over time, and how does it depend on the location of borders? Answering these questions would help us better understand how economic fundamentals and geographical structure shape the spatial distribution of economic activity.
References


Appendix: Derivation of equations (1) to (4)

Notice first that the CES price index at location \( r \) can be written as

\[
P(r) = \left[ \int_0^{\tau_r} p^\sigma(r)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} = \\ = \sum_m n_m \left[ p_m \tau(\eta_m, \eta_{\kappa(r)}) \varsigma(\eta_{\kappa(r)}, r) \right]^{\frac{1}{1-\sigma}} = \\ = \varsigma(\eta_{\kappa(r)}, r) \int_{\sigma-1}^{\frac{1}{\sigma-1}} \frac{\sigma - 1}{\sigma - 1} \left[ \sum_m w_m^{1-\sigma} L_m \tau(\eta_m, \eta_{\kappa}) \right]^{\frac{1}{1-\sigma}}.
\]

The first term is the only one that directly depends on \( r \). Hence, one can write \( P(r) = \varsigma(\eta_{\kappa(r)}, r) P_{\kappa(r)} \), where

\[
P_k = \int_{\sigma-1}^{\frac{1}{\sigma-1}} \frac{\sigma - 1}{\sigma - 1} \left[ \sum_m w_m^{1-\sigma} L_m \tau(\eta_m, \eta_k) \right]^{\frac{1}{1-\sigma}}. \quad (A1)
\]

Market clearing for any good produced at \( k \) implies

\[
f(\sigma - 1) = \tau = \int_s \mu P_k^{-\sigma} \left[ \tau(\eta_k, \eta_{\kappa(r)}) \varsigma(\eta_{\kappa(r)}, r) \right]^{\frac{1}{1-\sigma}} P(r)^{\sigma-1} Y(r) \, dr,
\]

where \( Y(r) \) is total income at location \( r \). Recall that housing rents are redistributed to local residents with equal shares. Then total income is the sum of labor and rental income:

\[
Y(r) = w_{\kappa(r)} L(r) + P^H(r) H(r) = w_{\kappa(r)} L(r) + (1 - \mu) Y(r),
\]

from which

\[
Y(r) = \frac{1}{\mu} w_{\kappa(r)} L(r).
\]
Therefore,

\[
 f (\sigma - 1) = \int_S p_k^{-\sigma} \left[ \tau (\eta_k, \eta_{\kappa(r)}) \zeta (\eta_{\kappa(r)}, r) \right]^{1-\sigma} P (r)^{\sigma-1} w_{\kappa(r)} L (r) \, dr = \\
 = \left[ \frac{\sigma}{\sigma - 1} \right]^{-\sigma} w_k^{-\sigma} \int_S \tau (\eta_k, \eta_{\kappa(r)})^{1-\sigma} P_k^{\sigma-1} \omega_{\kappa(r)} L (r) \, dr = \\
 = \left[ \frac{\sigma}{\sigma - 1} \right]^{-\sigma} w_k^{-\sigma} \sum_m \tau (\eta_k, \eta_m)^{1-\sigma} P_m^{\sigma-1} w_m L_m,
\]

so

\[
 w_k^\sigma = f^{-1} \sigma^{-1} \sum_m P_m^{\sigma-1} w_m L_m \tau (\eta_k, \eta_m)^{1-\sigma}. \tag{A2}
\]

Free mobility of consumers ensures that utility is equalized across locations. Although, in principle, there could exist uninhabited locations at which utility is lower, this never happens in equilibrium because these locations would attract consumers by offering a housing price of zero. Due to homotheticity of \( u (r) \), the representative consumer’s utility at \( r \) is equal to amenities times the real wage:

\[
 u (r) = a (r) \frac{y (r)}{P (r)^\mu P^H (r)^{1-\mu}}.
\]

Using the facts \( y (r) = \frac{Y (r)}{L (r)} = \frac{1}{\mu} w_{\kappa(r)} \), \( P (r) = \zeta (\eta_{\kappa(r)}, r) P_{\kappa(r)} \), and \( P^H (r) = (1-\mu) \frac{Y (r)}{H (r)} = \frac{1-\mu}{\mu} \frac{w_{\kappa(r)} L (r)}{H (r)} \), utility equalization implies

\[
 \pi = \mu^{-\mu} (1-\mu)^{-1-\mu} \bar{a} (r)^{1-\mu} \zeta (\eta_{\kappa(r)}, r)^{-\mu} P_{\kappa(r)}^{-\mu} w_{\kappa(r)} \mu \bar{L} (r)^{-1-\mu}, \tag{A3}
\]

where \( \bar{a} (r) = a (r)^{1-\mu} \). \( H (r) \) is a combination of amenities and the housing stock at \( r \). Also, if the representative consumer at \( r \) chooses any market \( k \), then her utility cannot be higher:

\[
 \pi \geq \mu^{-\mu} (1-\mu)^{-1-\mu} \bar{a} (r)^{1-\mu} \zeta (\eta_k, r)^{-\mu} P_{k}^{-\mu} w_k \mu L (r)^{-1-\mu} \quad \forall k,
\]

from which, using (A3), we obtain

\[
 \zeta (\eta_{\kappa(r)}, r)^{-\mu} P_{\kappa(r)}^{-\mu} w_{\kappa(r)}^{\mu} \geq \zeta (\eta_k, r)^{-\mu} P_k^{-\mu} w_k^{\mu} \quad \forall k.
\]
Now let \( \lambda_k = \overline{w}^{-\frac{1}{1-\mu}} \mu^{-\frac{1}{1-\mu}} (1 - \mu)^{-1} P_k^{-\frac{1}{1-\mu}} \overline{w}^{-\frac{1}{1-\mu}} P_k^\mu \) for each \( k \), and call \( \lambda_k \) the attractiveness of market \( k \). Express \( P_k \) as a function of attractiveness:

\[
P_k = \overline{w}^{-\frac{1}{1-\mu}} \mu^{-\frac{1}{1-\mu}} (1 - \mu)^{-1} \overline{w}^{-\frac{1}{1-\mu}} P_k^\mu \lambda_k^{-\frac{1}{1-\mu}}, \tag{A4}
\]

and use this to rewrite the previous inequality as

\[
\zeta (\eta_{\nu(r)}, r)^{-\frac{1}{1-\mu}} \lambda_{\nu(r)} \geq \zeta (\eta_k, r)^{-\frac{1}{1-\mu}} \lambda_k \quad \forall k. \tag{4}
\]

Also use (A4) in (A1) and (A2) to get

\[
w_k^{1-\sigma} \lambda_k^{\frac{(\sigma-1)(1-\mu)}{\mu}} = \tilde{\nu} \sum_m w_m^{1-\sigma} L_m \tau (\eta_m, \eta_k)^{1-\sigma} \tag{A5}
\]

and

\[
w_k^{\sigma} = \tilde{\nu} \sum_m w_m^{\sigma} \lambda_m^{\frac{(\sigma-1)(1-\mu)}{\mu}} L_m \tau (\eta_k, \eta_m)^{1-\sigma}, \tag{A6}
\]

where

\[
\tilde{\nu} = f^{-1} \sigma^{-\sigma} (\sigma - 1)^{\sigma-1} \mu^{1-\sigma} (1 - \mu)^{-\frac{(\sigma-1)(1-\mu)}{\mu}} \overline{w}^{-\frac{\sigma-1}{\mu}}.
\]

Now recall that cross-market trade costs are assumed to be symmetric: \( \tau (\eta_k, \eta_m) = \tau (\eta_m, \eta_k) \). In this case, (A5) and (A6) can be reduced to one equation. This is done using the trick by Allen and Arkolakis (2014): guess that wages at market \( k \) take the form

\[
w_k = \overline{w} \lambda_k^{\frac{1}{2}}. \tag{20}
\]

Then notice that for \( \iota = \frac{(\sigma-1)(1-\mu)}{2(\sigma-1)} \), both (A5) and (A6) imply

\[
\lambda_k^{\frac{\sigma(\sigma-1)(1-\mu)}{2(\sigma-1)^2(1-\mu)}} = \tilde{\nu} \sum_m \lambda_m^{\frac{(\sigma-1)^2(1-\mu)}{(2\sigma-1)^2}} L_m \tau (\eta_k, \eta_m)^{1-\sigma}.
\]

Hence,

\[
\lambda_k = \nu \left[ \sum_m \lambda_m^{\frac{(\sigma-1)^2(1-\mu)}{(2\sigma-1)^2}} L_m \tau (\eta_k, \eta_m)^{1-\sigma} \right]^{\frac{(2\sigma-1)(1-\mu)}{\sigma(\sigma-1)(1-\mu)}}, \tag{1}
\]

\[20\] Given that we have not normalized any price yet, we can set \( \overline{w} = 1 \).
where \( \nu = \nu^{(2r-1)\mu}(r-1)(1-r) \).

Now plug (A4) into (A3) and rearrange to obtain

\[
L(r) = \tilde{a}(r) \gamma \left( \eta_{k(r)}, r \right)^{-\frac{\mu}{1-r}} \lambda_{k(r)}. \tag{2}
\]

Finally, equation (3) simply follows from the fact that \( L_k \) equals the number of people commuting to market \( k \):

\[
L_k = \int_{\{r=k(r)\}} L(r) \, dr \tag{3}
\]

We have thus derived equations (1) to (4).

**Tables and figures**

**Table 1**

<table>
<thead>
<tr>
<th>Independent variable</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(distance from new border)</td>
<td>0.020</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>(0.011)*</td>
<td>(0.015)</td>
</tr>
<tr>
<td>log(distance from Budapest)</td>
<td>-0.111***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.037)***</td>
<td></td>
</tr>
<tr>
<td>Number of observations</td>
<td>84</td>
<td>83</td>
</tr>
</tbody>
</table>

Heteroskedasticity-robust standard errors in parentheses. *: significant at 10%; ***: significant at 1%.
Table 2

<table>
<thead>
<tr>
<th>Calibration</th>
<th>Corr</th>
<th>$\frac{Var[\log(g(r))]}{Var[\log(l(r))]}$</th>
<th>$\frac{Var[\log(g(r))] - Var[\log(l(r))]}{\kappa'(r) - \kappa(r)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Baseline</td>
<td>0.245</td>
<td>0.797</td>
<td>0.812</td>
</tr>
<tr>
<td>2 Cities &gt; 20K inhabitants</td>
<td>0.322</td>
<td>0.696</td>
<td>0.752</td>
</tr>
<tr>
<td>3 $\sigma = 5$</td>
<td>0.246</td>
<td>0.364</td>
<td>0.382</td>
</tr>
<tr>
<td>4 $\mu = \frac{2}{3}$</td>
<td>0.252</td>
<td>0.930</td>
<td>0.936</td>
</tr>
<tr>
<td>5 Power-fct shipping costs</td>
<td>0.228</td>
<td>0.874</td>
<td>0.896</td>
</tr>
<tr>
<td>6 Max trade cost matched</td>
<td>0.234</td>
<td>0.815</td>
<td>0.840</td>
</tr>
<tr>
<td>7 Min trade cost matched</td>
<td>0.255</td>
<td>0.787</td>
<td>0.794</td>
</tr>
<tr>
<td>8 Austro-Hungarian Empire</td>
<td>0.189</td>
<td>1.124</td>
<td>2.226</td>
</tr>
</tbody>
</table>

Figure 1: A rectangular economy with nine markets
Figure 2: Border change in the rectangular economy: with no dominant center (above), with a dominant center (below)

Figure 3: The Hungarian Kingdom before (green) and after (brown) the Treaty of Trianon, 1920
Figure 4: Change in log population levels implied by the model

Figure 5: Log local (left) and global (right) effects implied by the model