Monetary Policy for a Bubbly World

Vladimir Asriyan, Luca Fornaro, Alberto Martin and Jaume Ventura*

July 2016

Abstract

We propose a model of money, credit and bubbles, and use it to study the role of monetary policy in managing asset bubbles. In this model, bubbles pop up and burst, generating fluctuations in credit, investment and output. Two key insights emerge from the analysis. First, the growth rate of bubbles, which is driven by agents' expectations, can be set in real or in nominal terms. This gives rise to a novel channel of monetary policy, as changes in the inflation rate affect the real growth rate of bubbles and their effect on economic activity. Crucially, this channel does not rely on contract incompleteness or price rigidities. Second, there is a natural limit on monetary policy's ability to control bubbles: the zero-lower bound. When a bubble crashes, the economy may enter into a liquidity trap, a regime in which agents shift their portfolios away from bubbles - and the credit that they sustain - to money, reducing intermediation, investment and growth. We explore the implications of the model for the conduct of "conventional" and "unconventional" monetary policy, and we use the model to provide a broad interpretation of salient macroeconomic facts of the last two decades.

JEL classification: E32, E44, O40

Keywords: bubbles, monetary policy, liquidity traps, financial frictions

*Asriyan: vasriyan@crei.cat. Fornaro: lfornaro@crei.cat. Martin: amartin@crei.cat. Ventura: jventura@crei.cat. All authors: CREI, Universitat Pompeu Fabra and Barcelona GSE, Ramon Trias Fargas 25-27, 08005-Barcelona, Spain. We thank seminar participants at Stanford University, Santa Clara University, Nova Business School, Humboldt Universitat, and participants at the Barcelona GSE Summer Forum, the Conference on Banking, Monetary Policy and Macroeconomic Performance at Goethe Universitat Frankfurt, the Research Forum on Macro-Finance at the Bank of England, the Annual Research Conference at Sciences Po, the NBER Conference on Capital Flows and Debt in Emerging Markets, the Transpyrenean Macro Workshop, the 2015 SED Meeting, the Conference on Monetary Policy and Financial Instability at the Banque de France and the joint BoE, ECB, CEPR and CFM Conference on Credit Dynamics and the Macroeconomy. We thank Philippe Bacchetta, Wenxin Du, Marcus Hagedorn and Gregory Thwaites for very helpful discussions. Youel Rojas provided excellent research assistantship. We acknowledge support from the Spanish Ministry of Science and Innovation (grants ECO2011-23197), the Generalitat de Catalunya (grant 2014SGR-830 AGAUR), and the Barcelona GSE Research Network. In addition, both Martin and Ventura acknowledge support from the ERC (Consolidator Grant FP7-615651–MacroColl and Advanced Grant FP7-249588-ABEP, respectively), and Martin thanks the IMF Research Fellowship.

1 Introduction

What is the role of monetary policy in managing asset bubbles? This question has become increasingly prominent over the last thirty years, as business cycles have become tightly linked with large fluctuations in asset prices and credit. The decades-long slump that has characterized Japan since the early 1990s, for instance, is commonly interpreted as the result of the collapse of a real estate and equity bubble. In the United States, the recent recession has also been associated with the development and subsequent burst of a large bubble in the real estate and equity markets. In the same vein, some of the most severe recessions experienced in the Eurozone in the aftermath of the global financial crisis – such as those in Ireland and Spain – have coincided with the bursting of real estate bubbles. In all of these cases, bursting bubbles have been associated with deep and protracted recessions and reductions in credit, as well as with declining interest rates and liquidity traps.

These episodes have sparked a heated debate about whether and how monetary policy should stabilize asset prices and credit markets. Some have argued that central banks should take an active role, for instance by including an asset price and credit stability objective among their targets (Cecchetti, 2000; Borio et al., 2001). Others have argued that such policies can be destabilizing and counterproductive, and have recommended a more traditional focus on price stability (Bernanke and Gertler, 2001). Even among the activist camp, moreover, there is no clear consensus on what monetary policy should aim for. Should monetary policy intervene ex ante to restrain the rise of asset prices and credit during the boom, or should it intervene ex post to contain the collapse of economic activity during the bust? This debate has been further complicated by the recent emergence of liquidity traps, in which the conventional toolkit of monetary policy is no longer effective. As a result, many central banks have stepped into uncharted territories and resorted to novel "unconventional" policies, with limited historical precedents.

Despite the importance of this debate for macroeconomics, there is no analytical framework to address the various questions that it raises. To do so, we need a framework that connects bubbles, credit and monetary policy, and the goal of this paper is to provide one. We study an environment in which entrepreneurs need to borrow from savers to invest in physical capital. Crucially, entrepreneurs' borrowing is constrained by the amount of collateral that they can credibly pledge. This friction plays two key roles. First, it gives rise to a link between collateral, credit and investment. Second, it produces a low interest rate environment that makes bubbles possible.

In our economy, collateral can be either fundamental or bubbly. Fundamental collateral is the part of a borrower's pledgeable income that corresponds to future output, i.e. it consists of a borrower's rights to future production. Bubbly collateral is instead the part of a borrower's pledgeable income that corresponds to future credit, i.e. it consists of a borrower's rights to future borrowing. We call this type of collateral bubbly because it constitutes a rational bubble or pyramid scheme, in which present contributions (present credit) purchase future contributions (future credit): as long as the return to these bubbles or pyramid schemes is no lower than the interest rate, lenders will be willing to accept them as collateral.

At any point in time, there are two sources of bubble growth. The first is the creation of new bubbles, which provide collateral to entrepreneurs, alleviate the financing friction and expand credit. In this regard, bubble creation has a wealth effect that raises the net worth of entrepreneurs and enables them to expand their borrowing and investment. The second source is the growth of bubbles that were created in the past, which in essence need to be rolled over every period. In this regard, the growth of old bubbles absorbs credit and diverts it away from investment, creating a debt overhang effect that reduces investment in productive capital. Depending on which source of bubble growth dominates, larger bubbles may lead to higher or lower investment. Since bubbles are driven by investor sentiment, the economy is prone to business cycle fluctuations solely due to changes in expectations. Moreover, markets are generically unable to coordinate on the best possible equilibrium, and this creates a role for policy. In this paper, we focus on monetary policy.

The model delivers the following main results. First, bubbles can give rise to nominal credit contracts. This happens if investors' expectations about the future value of the bubble are set in nominal terms. Intuitively, this is a situation in which an entrepreneur borrows today against the *units of money* that she is expected to borrow in the future. The credit contracts backed by such expectations are optimally written in nominal terms, and they will therefore be affected by changes in inflation. A rise in inflation, for instance, reduces the real value of nominal credit contracts that were issued in the past, weakening their debt overhang effect and freeing up resources for investment in physical capital. Very much in the spirit of Fisher's debt deflation theory (Fisher, 1933), changes in inflation can thus generate fluctuations in asset prices, credit, and investment.

This leads to a second result, which is that monetary policy can – through its control over inflation – manage the debt overhang effect of bubbles. We refer to this novel transmission mechanism of monetary policy as the *bubble channel of monetary policy*. Crucially, this channel is sustained only by agents' expectations: it does not require money illusion, and it operates even if prices are fully flexible and there are no restrictions on the state-contingency of contracts that agents can write. Importantly, the strength of this channel depends on the mix of credit contracts present in the economy, so that monetary policy is most powerful when expectations about the bubble are such that a large fraction of the credit contracts are nominal.

A third key result is that the interaction between bubbles and monetary policy also determines whether and when the economy falls into a liquidity trap or not. When a bubble crashes, entrepreneurial collateral falls and so does the demand for credit. This leads to a fall in the real interest rate. Once the interest rate equals the return to holding money, however, it can fall no further. At this point, the economy is at the zero lower bound and savers are indifferent between holding money and lending in the credit market. Consequently, they demand money as a store of value and this increase in money holdings crowds out investment and aggravates the fall in output. Exactly when the economy hits the zero lower bound depends on the monetary authority, though, which controls the return to holding money by setting expected inflation.

What implications do these results have for the aforementioned debate on monetary policy and bubbles? The first is that monetary policy can react to expectation shocks and exploit the bubble channel to stabilize the economy. Doing so requires raising inflation in the aftermath of a bubble crash and reducing it during periods of high bubble growth. By overshooting the inflation target after the burst of a bubble, monetary policy can inflate away pre-existing credit contracts and weaken their debt overhang effect, freeing up resources for investment in physical capital. The flip-side of such a policy is that it must undershoot the inflation target in the boom phase, when bubble growth is already high, strengthening the bubble's overhang effect and dampening investment and growth. This type of monetary policy thus stabilizes investment and output but – contrary to conventional wisdom – it does so by destabilizing credit. We show that, besides having various redistributive effects across generations and types of agents, such a stabilization policy may also reduce average growth, i.e., the output that is gained during the busts may not be enough to compensate for the output that is lost during the booms. Therefore, its desirability ultimately depends on the objective function of the policy maker.

A second implication refers to the role of monetary policy inside the liquidity trap. At that point, the monetary authority has two options. The first is to increase expected inflation, reducing the real interest rate until the economy exits the liquidity trap. By reducing money holdings, this policy is successful at stimulating investment. It has two drawbacks, however: it hurts savers by lowering the return on their savings, and its effectiveness is limited by the lack of collateral, in the sense that investment is bound to remain low until a new bubble emerges in the credit market. The monetary authority's second option is to make use of seigniorage revenues, which can be substantial inside the liquidity trap, to fund credit-market interventions. We show that, by purchasing private assets at above-market price, the monetary authority can effectively transfer this seigniorage to entrepreneurs and thus mitigate the fall in investment during the liquidity trap. Our model thus illustrates why the monetary authority might choose to stay inside the liquidity trap, even if it could exit at will, as long as bubbly collateral remains low.

The theory thus provides a rich view of the interaction between credit, bubbles and monetary policy. But it also provides a stylized account of the salient macroeconomic developments of the last few decades. In fact, these decades have been characterized by a substantial decline in real and nominal interest rates, along with large fluctuations in asset prices, credit, and holdings of money and other liquid assets. One interpretation is that these low interest rates are the result of excess savings in the global economy, which are constantly looking for alternative stores of value. Our framework shows formally how this may have opened the door both for expectation-driven bubbles and liquidity traps to arise. In such a scenario, credit bubbles sustain investment and growth while they last. When they collapse, however, the real interest rate falls until the economy experiences a liquidity trap during which agents substitute private credit for money in their portfolios. According to this view, the enormous fluctuations in asset prices and money holdings of the recent past are different manifestations of the same phenomenon and should not come as a surprise. In fact, should such circumstances persist, we are likely to see more of these fluctuations in the future.

Related literature. Our paper is related to different strands of literature. We build on the recent work that has connected rational bubbles and credit, such as Caballero and Krishnamurthy (2006), Martin and Ventura (2012, 2015, 2016) and Farhi and Tirole (2011). Although we share many similarities with these models, they have no money and thus no role for monetary policy. Galí (2014) has recently explored the relationship between monetary policy and bubbles in a New-Keynesian framework, although credit plays no role in his model. Our approach differs from that work both in the key role of bubbles as drivers of credit and in our introduction of nominal bubbles, which create a novel transmission channel for monetary policy, beyond the usual one of nominal price rigidities.

Our work is also closely related to the "financial accelerator" literature, in which borrowers' net worth in general – and asset prices in particular – play a key role in determining the level of financial intermediation and economic activity (Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997). Our theory differs from these models because net worth and asset prices are not just a transmission channel for fundamental shocks: instead, they are driven by expectations and can therefore be a source of shocks themselves. In this regard, our work is also related to Bernanke and Gertler (2001), who allow asset prices to deviate from their fundamental value in a financial accelerator framework and analyze the implications for various monetary policy rules.

More broadly, we contribute to the literature identifying the channels through which monetary policy affects economic activity. This literature has typically focused on contractual restrictions to generate nominal rigidities in prices or wages (Galí, 2009). Instead, we show that the combination of bubbles and credit frictions creates the possibility of nominal rigidities in credit contracts. When this happens, changes in inflation have real effects because they change the real value of existing credit contracts. While the idea that nominal credit contracts open the door to real effects of monetary policy goes back at least to Fisher (1933), these are often imposed by the literature through exogenous restrictions on the contracts that agents can sign. Instead, we provide microfoundations for the existence of nominal rigidities in the credit markets.

The paper also contributes to the vast literature on liquidity traps (Eggertsson and Woodford, 2003; Krugman, 1998). In particular, our paper is connected with the work identifying financial shocks as the source of liquidity traps (Eggertsson and Krugman, 2012; Guerrieri and Lorenzoni, 2011). Different from existing work, we provide a framework in which financial shocks arise because of changes in expectations. Our paper is also related to Hansen's secular stagnation hypothesis

(Hansen, 1939), that is the idea that a drop in the natural interest rate might push the economy in a long-lasting liquidity trap, characterized by the absence of any self-correcting force to restore full employment. Hansen formulated this concept inspired by the US Great Depression, but recently some commentators, most notably Summers (2013) and Krugman (2013), have revived the idea of secular stagnation to rationalize the long duration of the Japanese liquidity trap and the slow recoveries characterizing the US and the Euro area after the 2008 financial crisis. A recent literature has formalized the secular stagnation hypothesis in microfounded frameworks (Benhabib et al., 2001; Eggertsson and Mehrotra, 2014; Caballero and Farhi, 2014; Benigno and Fornaro, 2015; Bacchetta et al., 2016). We contribute to this literature by showing that a long-lasting liquidity trap can be the outcome of a bubble crash.

2 A model of credit bubbles

In this section, we develop a model of credit bubbles. In this model, entrepreneurs take on past debts and also incur new debts of their own. But they do not do so with the intention of paying these debts out of their future income. Instead, they rationally expect that future entrepreneurs will take on their debts. It is in this specific sense that this is a model of credit bubbles.

As it is well known, bubbly equilibria are possible in environments in which the interest rate does not exceed the growth rate. The classic way of generating such an environment is to assume that the economy is dynamically inefficient. The interest rate is low because the supply of funds is high and there is overinvestment. Bubbles absorb funds and reduce unproductive investment allowing the economy to sustain a higher level of consumption and welfare.

We do not take this route, however. We assume instead that financial frictions limit the stock of available collateral. The interest rate is low because the demand for funds is low and, if anything, there might be underinvestment. Bubbles raise collateral and the demand for funds. Most of the times, but not always, this allows the economy to sustain a higher level of investment and consumption. We explain when and how this happens.

2.1 Basic setup

Consider an economy populated by overlapping generations of size one that live for two periods. Time is discrete and infinite, $t \in \{0, ..., \infty\}$. The economy might be subject to various shocks, which will be discussed later. We define h_t as the realization of the shocks in period t; h^t as a history of shocks until period t, that is, $h^t = \{h_0, h_1, ..., h_t\}$; and H_t as the set of all possible histories until period t. This economy does not experience technology or preference shocks, but it displays stochastic equilibria with bubble and monetary-policy shocks.

All members of generation t maximize the following utility function:

$$U\left(C_{1t}^{i}, C_{2t+1}^{i}\right) = \frac{\left(C_{1t}^{i}\right)^{1-1/\theta^{i}} - 1}{1-1/\theta^{i}} + \beta^{i} \cdot \frac{E_{t}\left\{\left(C_{2t+1}^{i}\right)^{1-\sigma^{i}}\right\}^{\frac{1-1/\theta^{i}}{1-\sigma^{i}}} - 1}{1-1/\theta^{i}}, \tag{1}$$

where C_{1t}^{i} and C_{2t+1}^{i} are the consumptions of individual i in the first and second periods of his/her life, respectively. Naturally, $C_{1t}^{i} \geq 0$ and $C_{2t+1}^{i} \geq 0$.[1] The preferences in Equation (1) are often called Epstein-Zin-Weil preferences, and they are defined by three parameters: the coefficient of risk aversion, $\sigma^{i} \in [0, \infty)$; the intertemporal elasticity of substitution, $\theta^{i} \in (0, \infty)$; and the discount factor, $\beta^{i} \in (0, \infty)$.[2] To simplify the exposition, we assume throughout that individuals are risk-neutral, i.e. $\sigma^{i} = 0$ for all i.[3]

Goods are produced with labor and capital using a standard Cobb-Douglas technology:

$$Y_{t} = \left(\gamma^{t} \cdot L_{t}\right)^{1-\alpha} \cdot K_{t}^{\alpha}, \tag{2}$$

with $\alpha \in [0, 1]$, where Y_{t}, L_{t} and K_{t} denote output, the labor force and the capital stock in the economy. Labor productivity grows at the rate $\gamma \geq 1$. Each generation supplies one unit of labor during youth so that $L_{t} = 1$. To produce one unit of capital in period $t+1$, one unit of goods is needed in period t. Capital is reversible and depreciates at rate δ. Competition implies that factors are paid their marginal products:

$$w_{t} = (1-\alpha) \cdot \gamma^{(1-\alpha) \cdot t} \cdot K_{t}^{\alpha} \quad \text{and} \quad r_{t} = \alpha \cdot \gamma^{(1-\alpha) \cdot t} \cdot K_{t}^{\alpha-1}, \tag{3}$$

where w_{t} and r_{t} are the wage and rental, respectively.

The economy contains also a market for bubbles. Let B_{t} denote the value of all bubbles started by earlier generations. Let N_{t} be the value of all bubbles started by the current generation. Thus, the value of all bubbles in period t is given by $B_{t} + N_{t}$. And the return to holding these bubbles from t to $t+1$ is given by:

$$R_{t+1}^{B} = \frac{B_{t+1}}{B_{t} + N_{t}}. \tag{4}$$

Free-disposal implies that old and new bubbles must be non-negative: $B_{t} \geq 0$ and $N_{t} \geq 0$ for all t. Equation (4) determines the evolution of B_{t} given a sequence for R_{t}^{B} and N_{t}. We refer to these two variables as bubble-return and bubble-creation shocks, respectively. As we shall see, there are many specifications for these shocks that are consistent with maximization and market clearing.

[1] All variables are indexed by h^{t}. For instance, C_{2t+1}^{i} depends on the particular history being considered. We could be more explicit about this dependence by writing $C_{2t+1,h^{t+1}}^{i}$. We prefer to streamline the notation, however, and omit the history sub-index.

[2] The usual isoelastic case applies when the coefficient of risk aversion equals the inverse of the elasticity of intertemporal substitution, i.e. $\sigma^{i} = 1/\theta^{i}$.

[3] See Martin and Ventura (2015) for a related model where this assumption is relaxed.

The economy also contains money. Let M_t and p_t be the quantity of money and the price level. For the time being, we assume that the monetary authority transfers the profits/losses from money creation to the fiscal authority. The latter raises a proportional tax τ on labor income, and spends all its revenues on useless government spending X_t.[4] Thus, the budget constraint of the fiscal authority is given by:

$$X_t = \tau \cdot w_t + \frac{M_t - M_{t-1}}{p_t}. \tag{5}$$

Naturally, we must check that $X_t \geq 0$ for all histories h^t. The key assumption here is that the monetary authority does not keep the seigniorage. Thus, monetary policy consists only of setting the money supply in order to achieve its desired target for inflation $\pi_{t+1} = \frac{p_{t+1}}{p_t}$. One can think of this as 'conventional' monetary policy. In Section 4, we will allow the monetary authority to keep some or all of the seigniorage and use it for credit bailouts and/or asset purchases. One can then think of this additional policy tool as 'unconventional' monetary policy.

Absent any additional assumptions, each individual chooses whether to hold money by comparing their returns to the available alternatives. One problematic implication is that, whenever money is dominated by these alternatives, the total demand for money is exactly equal zero and – as we show in Appendix B – the monetary authority loses control of the price level. We avoid this technical problem by imposing the constraint that all individuals must hold a small amount of real balances:

$$\frac{M_t^i}{p_t} \geq \upsilon \cdot \gamma^t, \tag{6}$$

where M_t^i are the money holdings of individual i. This assumption guarantees that there is a positive demand for money at all times, and that this demand grows at the long-run growth rate of the economy. But we make this forced demand for money arbitrarily small by assuming that $\upsilon \to 0$. One interpretation of this demand is that money helps agents fulfill some "small" transactions need (e.g., shopping needs, taxes).

2.2 Savers, entrepreneurs and the credit market

Each generation contains a representative saver and entrepreneur. The key difference between these types is that entrepreneurs can hold capital and bubbles, while savers cannot do this.[5] There are also two additional differences between these types. First, the representative saver has $1 - \varepsilon$ units of labor while the representative entrepreneur has ε units. Second, unlike savers, entrepreneurs are arbitrarily patient: $\beta^E \to \infty$. This assumption simplifies the exposition without affecting the results, as we show in Appendix A.

[4] We could assume instead that government spending is useful and enters the utility function in an additive way. None of what follows would be affected by this.

[5] Actually, the important assumption here is that savers do not hold capital. As it will become clear later, in equilibrium savers are indifferent between holding bubbles or not.

In the credit market, entrepreneurs sell credit contracts to savers. These contracts cost Q_t in period t, and promise a contingent payment F_{t+1} in period $t+1$. Define R^F_{t+1} as the return to this credit contract: $R^F_{t+1} = \frac{F_{t+1}}{Q_t}$. Define R_{t+1} as the expected return to a credit contract: $R_{t+1} \equiv E_t R^F_{t+1}$. Then, it follows that:

$$R_{t+1} = \frac{E_t F_{t+1}}{Q_t}. \tag{7}$$

We shall describe the equilibrium in terms of R_{t+1} rather than Q_t.

The representative saver supplies $1-\varepsilon$ units of labor when young, pays taxes, saves a fraction of her labor income, and uses it to hold money and/or to provide credit to entrepreneurs. Let S_t be her savings. Then, her budget constraints are given by:

$$C^S_{1t} = (1-\tau)\cdot(1-\varepsilon)\cdot w_t - S_t \tag{8}$$

$$C^S_{2t+1} = R^F_{t+1}\cdot\left(S_t - \frac{M^S_t}{p_t}\right) + \frac{M^S_t}{p_{t+1}}. \tag{9}$$

Equation (8) simply states that the young saver's consumption equals her after-tax labor income minus savings. Equation (9) contains a series of constraints, one for each history h^{t+1}, stating that the old saver consumes the return to her portfolio of credit and money. Maximization implies that:

$$S_t = \frac{\beta^\theta}{\beta^\theta + R^{1-\theta}_{t+1}}\cdot(1-\tau)\cdot(1-\varepsilon)\cdot w_t \tag{10}$$

$$\frac{M^S_t}{p_t}\begin{cases} = 0 & \text{if } R_{t+1} > E_t\pi^{-1}_{t+1} \\ \in [0, S_t] & \text{if } R_{t+1} = E_t\pi^{-1}_{t+1} \\ = S_t & \text{if } R_{t+1} < E_t\pi^{-1}_{t+1}. \end{cases} \tag{11}$$

Equations (10) and (11) define the optimal savings and portfolio choice of the saver. Equation (10) shows that total savings are increasing in the real interest rate if the intertemporal elasticity of substitution is higher than one, i.e. $\theta > 1$. We assume this throughout. Equation (11) shows that the young saver uses none (all) of their savings to hold money if the expected return to credit exceeds (falls short of) the expected return to holding money. If both returns are equal, the young saver is indifferent between credit and money.[6] Naturally, the purchases of credit contracts by the young saver are given by $S_t - \frac{M^S_t}{p_t}$.

The representative entrepreneur purchases capital, bubbles, and money during youth, and finances these purchases by supplying ε units of labor and selling credit contracts.[7] The budget

[6]Implicit in Equation (11) is the assumption that savers cannnot borrow to hold money in excess of their savings. This assumption is not important since savers never want to do this in equilibrium.

[7]When we refer to purchases of capital, we include both: (i) actual purchases of used capital from old entrepreneurs: $(1-\delta)\cdot K_t$, and (ii) the production of new units of capital by young entrepreneurs: $K_{t+1} - (1-\delta)\cdot K_t$. Since capital

constraints of the entrepreneur can be written as follows:

$$C_{1t}^E = (1-\tau) \cdot \varepsilon \cdot w_t + \frac{E_t F_{t+1}}{R_{t+1}} - K_{t+1} - B_t - \frac{M_t^E}{p_t} \tag{12}$$

$$C_{2t+1}^E = (r_{t+1} + 1 - \delta) \cdot K_{t+1} + B_{t+1} + \frac{M_t^E}{p_{t+1}} - F_{t+1}. \tag{13}$$

Equation (12) says that the young entrepreneur uses his after-tax labor income and the funds raised by selling credit contracts to consume and purchase capital, old bubbles and money.[8] Equation (13) contains a set of constraints, one for each possible history h^{t+1}, saying that the old entrepreneur uses the return to capital and the proceeds from selling used capital, bubbles and money to repay credit contracts and consume.

We introduce now a key restriction on the credit contracts offered by entrepreneurs:

$$F_{t+1} \leq \phi \cdot K_{t+1} + \frac{M_t^E}{p_{t+1}} + B_{t+1}. \tag{14}$$

Equation (14) contains a set of collateral constraints, one for each possible history h^{t+1}, saying that entrepreneurs cannot promise payments that exceed a fraction ϕ of their capital plus their money and bubbles. Since $\phi \leq r_{t+1} + 1 - \delta$, only a part of the return to capital can be used as collateral to issue credit contracts. We think of the first two terms in the right-hand side as the fundamental collateral of entrepreneurs, and the last term as their bubbly collateral. We refer to Equation (14) as the credit or collateral constraints. One interpretation of this set of constraints is based on the notion that courts can only seize part of the capital income from entrepreneurs.

We focus throughout on equilibria in which the return to capital is higher than both the expected return to credit and the return to money holdings, i.e. $r_{t+1} + 1 - \delta > \max\left\{R_{t+1}, E_t \pi_{t+1}^{-1}\right\}$ for all h^{t+1} and t. Thus, the money and collateral constraints in Equations (6) and (14) are both always binding,[9] which implies that:

$$\frac{M_t^E}{p_{t+1}} = 0 \quad \text{and} \quad E_t F_{t+1} = \phi \cdot K_{t+1} + E_t B_{t+1}. \tag{15}$$

Since the young entrepreneur does not consume, $C_{1t}^E = 0$, he uses all his savings and the credit obtained to purchase capital and bubbles:

$$K_{t+1} = \frac{R_{t+1}}{R_{t+1} - \phi} \cdot \left[(1-\tau) \cdot \varepsilon \cdot w_t + \frac{E_t B_{t+1}}{R_{t+1}} - B_t\right]. \tag{16}$$

is reversible, the production of new units of capital could be negative.

[8] Notice that N_t does not appear in Equation (12). This happens because we assume, without loss of generality, that entrepreneurs do not sell newly created bubbles until they are old.

[9] When the interest rate is equal to the return on money, $R_{t+1} = E_t \pi_{t+1}^{-1}$, it is costless for entrepreneurs to hold money as money balances are fully pledgeable. Our assumption that entrepreneurs do not hold money balances in this case is without loss of generality.

Equation (16) is obtained by combining Equations (12) and (15). It says that the purchases of capital are the product of two terms. The first one is a financial multiplier that indicates how many units of capital can be purchased for each unit of entrepreneurial wealth. The intuition behind this multiplier is well known. One additional unit of wealth allows the entrepreneur to purchase one unit of capital. This allows the entrepreneur to borrow and raise the capital stock by $\frac{\phi}{R_{t+1}}$ additional units. And this allows him to borrow and raise the capital stock by $\left(\frac{\phi}{R_{t+1}}\right)^2$ additional units. And so on. Thus, one unit of wealth allows the entrepreneur to purchase $1+\frac{\phi}{R_{t+1}}+\left(\frac{\phi}{R_{t+1}}\right)^2+...=\frac{R_{t+1}}{R_{t+1}-\phi}$ units of capital.

The second term in Equation (16) is entrepreneurial wealth and it consists of the sum of after-tax wages and the gains obtained from holding bubbles. It follows from Equation (4), that these gains have two sources:

$$\frac{E_t B_{t+1}}{R_{t+1}} - B_t = N_t + \left(\frac{E_t R^B_{t+1}}{R_{t+1}} - 1\right) \cdot (B_t + N_t). \tag{17}$$

That is, these gains consist of the value of new bubbles started by the entrepreneur and the expected profits from purchasing and selling bubbles in the market. But there cannot be profits from purchasing and selling bubbles since these can be fully collateralized. If the expected return to holding bubbles exceeds the expected return to credit contracts, the demand for bubbles would be unlimited as this allows the entrepreneur to attain unbounded capital and consumption. If the expected return from holding bubbles falls short of the expected return to credit contracts, there would be no demand for bubbles because holding bubbles reduces capital holdings and the consumption attainable to the entrepreneur. Thus, the expected return to holding bubbles in equilibrium must be given by:

$$E_t R^B_{t+1} = R_{t+1}, \tag{18}$$

for all t. This not only ensures that the entrepreneur is willing to purchase existing bubbles, but it also ensures that he is able to borrow enough to finance these purchases. It also implies that only starting new bubbles generates net wealth for the entrepreneur.

We now turn to the credit market. The supply of credit by savers equals their savings minus money holding: $S_t - \frac{M^S_t}{p_t}$. Since collateral constraints are binding, the demand for credit by entrepreneurs equals the discounted value of their collateral: $\frac{E_t B_{t+1} + \phi \cdot K_{t+1}}{R_{t+1}}$. Equilibrium in the credit market implies that:

$$R_{t+1} = \max\left\{\frac{\phi \cdot K_{t+1} + E_t B_{t+1}}{\dfrac{\beta^\theta}{\beta^\theta + R_{t+1}^{1-\theta}} \cdot (1-\tau) \cdot (1-\varepsilon) \cdot w_t}, E_t \pi_{t+1}^{-1}\right\}. \tag{19}$$

Equation (19) describes the equilibrium real interest rate. The key observation is that collateral is given by $\phi \cdot K_{t+1} + E_t B_{t+1}$. If there is enough collateral, the expected return to credit contracts is high enough to induce young savers to convert all their savings into credit: $R_{t+1} > E_t \pi_{t+1}^{-1}$. In this situation, credit dominates money as a store of value. If there is not enough collateral, some savings are allocated into money: $R_{t+1} = E_t \pi_{t+1}^{-1}$. Now money and credit are both used as a store of value, and we say that the economy is inside the liquidity trap.

2.3 Equilibrium dynamics

To study the dynamics of our economy, we work with quantity variables expressed in efficiency units and denote them with lowercase letters. For instance, we refer to k_t and b_t as the capital stock and bubbles, and we define them as $k_t \equiv \gamma^{-t} \cdot K_t$ and $b_t \equiv \gamma^{-t} \cdot B_t$.

To construct a competitive equilibrium we propose a joint stochastic process for monetary policy and bubble shocks that generates $h_t = \left\{\pi_t, R_t^B, n_t\right\}$ for all $h^t \in H_t$. This process must be such that for all j and $h^t \in H_t$: (i) $\pi_t > 0$, (ii) $E_t R_{t+1}^B = R_{t+1}$, and (iii) $n_t \geq 0$. To determine whether this process is an equilibrium, we compute the evolution of the capital stock and bubbles for all $h^t \in H_t$ from a given initial condition using the following equations:

$$b_{t+1} = \frac{R_{t+1}^B}{\gamma} \cdot (b_t + n_t) \tag{20}$$

$$k_{t+1} = \frac{1}{\gamma} \cdot \frac{R_{t+1}}{R_{t+1} - \phi} \cdot \left[(1-\tau) \cdot \varepsilon \cdot (1-\alpha) \cdot k_t^\alpha + n_t\right] \tag{21}$$

$$R_{t+1} = \max\left\{\frac{\gamma \cdot (\phi \cdot k_{t+1} + E_t b_{t+1})}{\dfrac{\beta^\theta}{\beta^\theta + R_{t+1}^{1-\theta}} \cdot (1-\tau) \cdot (1-\varepsilon) \cdot (1-\alpha) \cdot k_t^\alpha}, E_t \pi_{t+1}^{-1}\right\}. \tag{22}$$

If all sequences generated in this way are such that $k_t \geq 0$ and $b_t \geq 0$ for all j and $h^t \in H_t$, the proposed stochastic process for monetary policy and bubbles constitutes an equilibrium. Otherwise, it does not.[10]

[10]Equation (20) is the law of motion of bubbles, and it follows directly from Equation (4). Equation (21) is the law of motion of capital, and it follows from Equations (16), (17) and (18). The evolution of both the capital stock and bubbles depends on the expected return to credit, and the latter is described in Equation (22), which follows directly from Equation (19).

Bubbles are debts that are never paid, debts that are rolled over forever. Young entrepreneurs purchase the debts of old entrepreneurs, i.e. b_t; and they also incur new debts of their own, i.e. n_t. But they do not do so with the intention of paying these debts with their capital income. Instead, they rationally expect to sell their debts, b_{t+1}, to the next generation of entrepreneurs. Thus, bubbles capture the real-world notion of a credit chain or Ponzi scheme. This is why we refer to them as credit bubbles or, for short, bubbles.

Credit bubbles have two effects on capital accumulation. The first one is a "wealth" effect. When young entrepreneurs incur new debts or start a bubble, they receive a windfall equal to n_t. As Equation (21) shows, this windfall provides additional funds for investment. Through this channel, bubbles crowd in capital.

The second effect is some sort of "debt overhang". Young entrepreneurs are expected to sell or pass their debts to the next generation. Equation (22) shows that these looming debts, whose expected value is $E_t b_{t+1}$, raise the interest rate. This lowers the discounted value of collateral, and reduces the funds available for investment. This effect is captured in Equation (21) by a reduction in the financial multiplier. Through this channel, bubbles crowd out capital. This debt overhang effect is not operative inside the liquidity trap because there the interest rate is equal to expected inflation and the credit supply is perfectly elastic.

We use Equations (20), (21) and (22) to generate equilibria and study their properties. Each equilibrium corresponds to a specific stochastic process for monetary policy and bubble shocks, i.e. h_t. There are, in principle, many stochastic processes that satisfy the requirements for equilibrium. In Sections 2 and 3, we shall consider inflation processes that encapsulate alternative monetary policy choices. In the reminder of this section, we develop first some intuitions and then tackle the important issue of picking a process for the bubble shocks.

2.4 Understanding credit bubbles and their effects

To provide some intuition on the workings of the model and the effects of credit bubbles, we consider now a simple equilibrium in which: (i) bubble creation takes place only in period $t = T$: $n_t = 0$ for all $t \neq T$ and $n_T = n > 0$; and (ii) bubble returns are certain: $R^B_{t+1} = R_{t+1}$. For many parameter values, this equilibrium exists if n is not too large.

Before period T, there is only fundamental collateral, and all credit is paid from capital income. This is the bubbleless economy. Let R be implicitly determined by:

$$\frac{\beta^\theta}{\beta^\theta + R^{1-\theta}} \cdot (1-\varepsilon) = \frac{\phi}{R-\phi} \cdot \varepsilon. \tag{23}$$

Then, Equation (22) implies that $R_{t+1} = \max\left\{R, E_t \pi^{-1}_{t+1}\right\}$. If the inflation rate is high, i.e. $R_{t+1} =$

$R > E_t\pi_{t+1}^{-1}$, the return to credit or real interest rate is R.[11] In this case, the interest rate is increasing in ϕ and decreasing in β. This is intuitive, as increases in collateral raise the demand for credit while increases in savings raise the supply of credit. Let R_{t+1}^N be the nominal interest rate.[12] Outside the liquidity trap, monetary policy determines the nominal interest rate but it cannot affect the real one. If the inflation rate is low, i.e. $R_{t+1} = E_t\pi_{t+1}^{-1} > R$, the real interest rate is $E_t\pi_{t+1}^{-1}$. Inside the liquidity trap, monetary policy determines the real interest rate but it cannot affect the nominal interest rate which equals one.

Before period T, the capital stock evolves according to:

$$k_{t+1} = \gamma^{-1} \cdot \frac{R_{t+1}}{R_{t+1} - \phi} \cdot (1-\tau) \cdot \varepsilon \cdot (1-\alpha) \cdot k_t^{\alpha} \qquad \text{for all } t < T.$$

This follows from Equation (21). Since we have ruled out shocks to preferences and technology, the bubbleless economy only experiences monetary shocks. When inflation is high and the economy is outside the liquidity trap, the financial multiplier is large and both credit and capital accumulation are maximized. When inflation is low and the economy is inside the liquidity trap, the financial multiplier is small and both credit and capital accumulation are depressed. This describes the behavior of the bubbleless economy, and it is the starting point of our inquiry about the effects of credit bubbles.

In period T, a credit bubble pops up. Now everyone expects (with probability one) young entrepreneurs to sell or pass their debts to future generations. This allows young entrepreneurs to borrow n in period T and create a debt equal to $b_{T+1} = \frac{R_{T+1}}{\gamma} \cdot n$ in period $T+1$. Since expectations are rational or self-fulfilling, young entrepreneurs in period $T+1$ borrow b_{T+1} to purchase this debt from old entrepreneurs. This generates a new debt equal to $b_{T+2} = \frac{R_{T+2}}{\gamma} \cdot b_{T+1}$ in period $T+2$. Young entrepreneurs in period $T+2$ borrow b_{T+2} to purchase this new debt. This generates yet another debt equal to $b_{T+3} = \frac{R_{T+3}}{\gamma} \cdot b_{T+2}$ in period $T+3$. And so on.

The start of a credit bubble constitutes a positive wealth shock for generation T, since this generation borrows n and it never has to pay back this debt. This allows young entrepreneurs to finance additional investment and raise the capital stock, as Equation (21) shows:

$$k_{T+1} = \frac{1}{\gamma} \cdot \frac{R_{T+1}}{R_{T+1} - \phi} \cdot \left[(1-\tau) \cdot \varepsilon \cdot (1-\alpha) \cdot k_T^{\alpha} + n\right].$$

This is the wealth effect of the credit bubble. Where do the resources that finance this bubble come from? Outside the liquidity trap, this debt raises the real interest rate and savings. Thus, the resources that finance the bubble come from a decline in the consumption of young savers. The

[11] The real interest rate is the return promised by a one-period non-contingent bond.

[12] The nominal interest rate is the return promised by a one-period nominal bond. This bond delivers an ex-post real return equal to $\pi_{t+1}^{-1} \cdot R_{t+1}^N$. Since its expected return must equal R_{t+1}, we have that $R_{t+1}^N = \frac{R_{t+1}}{E_t\pi_{t+1}^{-1}}$.

increase in the real interest rate lowers the financial multiplier. This is the debt overhang effect of the credit bubble. Inside the liquidity trap, the increase in expected debt replaces money in the portfolios of savers. Thus, the resources that finance the credit bubble come from a reduction in seigniorage and government spending. Since the real interest rate is not affected, the financial multiplier remains constant. Within the liquidity trap, there is no debt overhang effect.[13]

After period T, the credit bubble has no additional wealth effects. Later generations of young entrepreneurs borrow $\frac{\gamma \cdot b_{t+1}}{R_{t+1}}$, but this is just enough to pay for the debt b_t of old entrepreneurs. Thus, Equation (21) becomes again:

$$k_{t+1} = \frac{1}{\gamma} \cdot \frac{R_{t+1}}{R_{t+1} - \phi} \cdot (1-\tau) \cdot \varepsilon \cdot (1-\alpha) \cdot k_t^{\alpha} \qquad \text{for all } t \geq T.$$

The wealth effect of the bubble is gone. But the debt overhang effect remains, as Equation (22) shows. Rolling over the credit bubble still requires financing. In those periods in which the economy is outside the liquidity trap, the credit bubble raises the real interest rate and bubble is financed through a combination of increased savings and reduced investment. This debt overhang effect is absent in those periods in which the economy is in the liquidity trap, as the bubble is financed entirely with a reduction in money holdings.

This example illustrates the two effects of a credit bubble: (i) a temporary wealth effect when it pops up; and (ii) a permanent debt overhang effect throughout its lifetime. The former raises the resources available for investment, while the latter lowers them. Indeed, the overall impact of a credit bubble on the capital stock can always be interpreted as the result of the dynamic interplay between these two effects.

Figure 1 shows two simulations.[14] In both of them, the economy starts in the steady state before period T, and it returns to it as the credit bubble shrinks and vanishes asymptotically.[15] This steady state is outside the liquidity trap. The key difference between the two simulations is the intertemporal elasticity of substitution θ.[16] The higher is this elasticity, the more elastic is the credit supply. Since the two simulations feature the same path of bubble creation, the wealth effect is exactly the same. Any difference between them can be traced back to the debt overhang effect.

The dashed lines show the case in which the credit supply is elastic and the bubble has a moderate effect on the real interest rate. There is a large initial increase in the capital stock, as the wealth effect is much stronger than the debt overhang effect. As the bubble shrinks the capital

[13]The credit bubble can also take the economy outside of the liquidity trap. That is, it is possible that the economy be inside the liquidity trap before T, and outside in $T+1$. This happens when the credit bubble is larger than money holdings. In this case, both savings increase and seignorage declines.

[14]All the simulations are meant to illustrate the qualitative properties of the model. See Appendix C for the parameters used to construct each figure.

[15]This equilibrium exist for a set of credit bubbles indexed by $n \in (0, \bar{n}]$. All bubbles, except for the maximal one $\bar{n}$, vanish asymptotically.

[16]The two economies differ also in the discount factor β, which is set so that both economies have the same steady state capital stock.

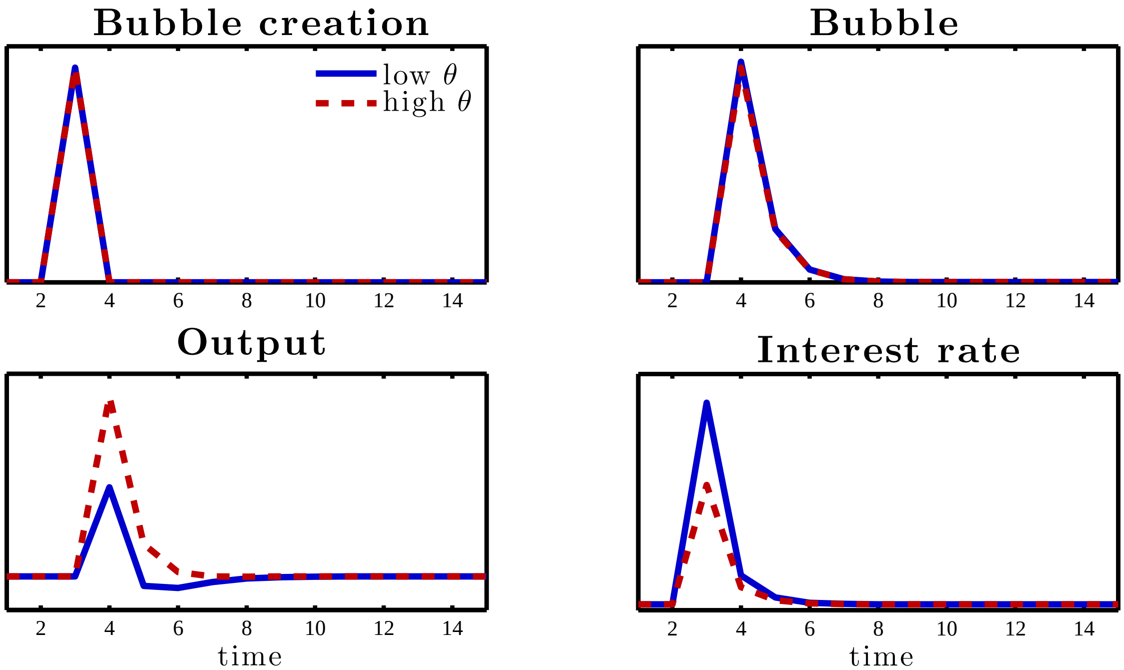


Figure 1: Response to bubble creation shock.

stock monotonically declines towards the steady state. In this case, the credit bubble generates a transitory boom.

The solid lines show the case in which the credit supply is inelastic and the bubble generates a sizable effect on the real interest rate. There is now a small initial increase in the capital stock, as the wealth effect is largely undone by the debt overhang effect. As the bubble shrinks the capital stock declines and undershoots its steady state level. In this case, the credit bubble generates a boom-bust cycle.

We can build on this simple example to develop more realistic bubble processes. The first natural extension is to recognize that there is more than one 'lucky' generation that starts credit bubbles. This means that bubble creation n_t can be positive in periods other than T. The second natural extension is to add the possibility that some generations can be 'unlucky' and cannot rollover their debts. This means that R_{t+1}^B can be random instead of always equal to R_{t+1}. We shall examine equilibria with these features in what follows. But before doing this, we want to discuss an issue that is at the heart of this paper.

2.5 Bubbles? What bubbles?

A key feature of a bubbly economy is that present credit depends on market expectations about future credit. How are these expectations formed? In particular, are these expectations anchored in terms of goods or money? To grasp the issues involved, let's continue with our simple example in which bubble creation takes place only in period $t = T$. But let us now compare two alternative market expectations regarding bubble returns:

1. *The market expects (with probability one) the credit bubble to deliver the return to a one-period non-contingent real bond:* $R^B_{t+1} = R_{t+1}$.

2. *The market expects (with probability one) the credit bubble to deliver the return to a one-period non-contingent nominal bond:* $\pi_{t+1} \cdot R^B_{t+1} = R^N_{t+1}$.

We have seen already that, if market expectations are given by Assumption 1, the bubble process is given by:

$$b_{t+1} = \frac{R_{t+1}}{\gamma} \cdot b_t \quad \text{for } t \geq T. \tag{24}$$

We analyzed this bubble in the previous subsection. Outside the liquidity trap, monetary policy cannot influence the bubble. Inside the liquidity trap, monetary policy influences the bubble since $R_{t+1} = E_t \pi^{-1}_{t+1}$. But only expected inflation matters. Realized inflation does not.

If market expectations are instead given by Assumption 2, the bubble process is given by:

$$b_{t+1} = \frac{\pi^{-1}_{t+1}}{E_t \pi^{-1}_{t+1}} \cdot \frac{R_{t+1}}{\gamma} \cdot b_t \quad \text{for } t \geq T. \tag{25}$$

Outside the liquidity trap, monetary policy now influences the bubble. In particular, surprise inflation dilutes it. Inside the liquidity trap, monetary policy also influences the bubble. But, unlike the previous case, it is only realized inflation that matters. Expected inflation does not.

Credit bubbles are implicit contracts among different generations of buyers and sellers, and their terms are determined by market expectations. Credit contracts inherit the properties of the bubbles that back them, and this gives rise to a form of nominal rigidity. If bubble expectations are set in real terms as in Assumption 1, entrepreneurs borrow today against the goods that they will receive from creditors in the future, and the return to their credit contracts is effectively indexed to inflation. If bubble expectations are instead set in nominal terms as in Assumption 2, entrepreneurs borrow today against the money that they will receive from creditors in the future, and the return to their credit contracts is not indexed to inflation. Note that this is true even though expectations are formed rationally, there is no money illusion at work and credit contracts can be made contingent to inflation at zero cost.

There are two important insights that this example reveals: (i) the mix of credit contracts traded depends on market expectations; and (ii) the effects of monetary policy depend on the mix of credit contracts traded. These two insights are crucial to understand monetary policy in a bubbly economy. We can re-phrase them as saying that nominal rigidities in credit contracts are determined by market expectations, and that the effects of monetary policy depend on the nominal rigidities in credit contracts.

What set of assumptions provides a better description of real-world credit bubbles? It seems reasonable to take the view that real-world financial markets may contain many credit bubbles that mutate over time. Let the aggregate credit bubble be the sum of many bubble types $j = 1, ..., J$; so that $B_t = \sum_j B_t^j$ and $N_t = \sum_j N_t^j$. Each of these bubbles offers a different return R_{t+1}^j. For instance, a bubble that offers a return equal to R_{t+1} backs credit contracts or debts that are indexed to inflation, safe and short-term (one period). And a bubble that offers a nominal return equal to R_{t+1}^N backs credit contracts or debts that are also safe and short-term, but not indexed to inflation. Naturally, we can (and we will) also consider bubbles that back debts that are risky and long-term, and bubbles that back stocks or equities. All bubble types must offer the same expected return in equilibrium though: $E_t R_{t+1}^j = R_{t+1}$.

How does the distribution of bubble types evolve over time? Each period, new credit bubbles start and old credit bubbles either keep their type or mutate into another type. Instead of keeping track of all the possibilities, we simply write the market share of bubble type j as λ_t^j; with $\sum_j \lambda_t^j = 1$. This implies that the return to the aggregate credit bubble is given by $R_{t+1}^B = \sum_j \lambda_t^j \cdot R_{t+1}^j$, and we can re-write Equation (20) as follows:

$$b_{t+1} = \sum_j \frac{\lambda_t^j \cdot R_{t+1}^j}{\gamma} \cdot (b_t + n_t). \tag{26}$$

Equation (26) shows the evolution of the aggregate bubble as a function of its composition, as defined by the shares λ_t^j and the returns R_{t+1}^j of each debt instrument. Thus, we define market expectations in terms of the set of debt instruments that these expectations support. This is convenient because it allows us to obtain theoretical results on the effects of monetary policy that are conditional on observables.

3 The bubble channel of monetary policy

Monetary policy plays two key roles in the bubbly economy. First, it sets the return to money as a store of value and thus determines the likelihood that the economy enters or exits a liquidity trap. Second, as long as expectations are partly set in nominal terms, it affects the evolution of the aggregate bubble and thus credit and investment. This "bubble channel" creates a role for monetary policy even in the absence of price or contractual rigidities, and it is the object of this section. To isolate this channel, we assume now that $R_{t+1} > E_t \pi_{t+1}^{-1}$, and we delay a thorough analysis of liquidity traps to Section 4.

3.1 Monetary policy and the debt overhang

We start with a simple example in which there are no bubble creation or return shocks. Our baseline economy features a mix of real and nominal bubbles - fraction $\lambda^N \in [0, 1]$ of all bubbles is

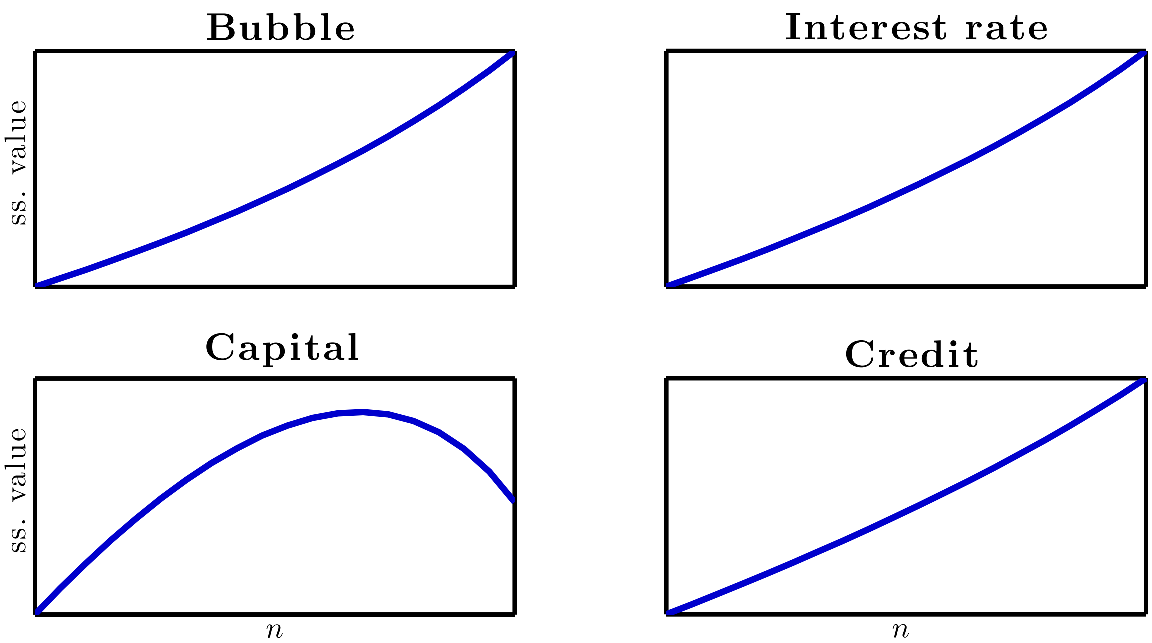


Figure 2: Steady state as a function of bubble creation.

nominal and delivers return $\pi_t^{-1} \cdot R_t^N$ while the remaining fraction is real and delivers return R_t (as described in Section 2.5). As a consequence, in equilibrium some of the credit contracts sustained by bubbly collateral are specified in nominal terms.

We first focus attention on an economy that starts from a steady state with constant bubble creation and constant inflation, i.e., $n_t = n$ and $\pi_t = \pi$ in all periods. Since inflation is constant, it makes no difference whether credit contracts, and hence bubbles, are nominal or real. Thus, regardless of the exact mix of bubbles present in the economy, the initial steady state is described by equations:[17]

$$b = \frac{R}{\gamma - R} \cdot n \tag{27}$$

$$k = \frac{1}{\gamma} \cdot \frac{R}{R - \phi} \cdot [(1-\tau) \cdot \varepsilon \cdot (1-\alpha) \cdot k^{\alpha} + n] \tag{28}$$

$$\frac{1}{\gamma} \cdot \frac{\beta^{\theta}}{\beta^{\theta} + R^{1-\theta}} \cdot (1-\tau) \cdot (1-\varepsilon) \cdot (1-\alpha) \cdot k^{\alpha} = \frac{\phi \cdot k + b}{R}. \tag{29}$$

Figure 2 shows how the steady state values b, k, and R vary with n. When $n = 0$, we are in the bubbleless economy described in Section 1.4. As n increases, both the bubble and the interest rate increase. However, the effect on the capital stock is non-monotonic. At low values of n, an increase in n raises the capital stock because the wealth effect of additional credit bubbles outweighs the debt overhang effect. But there is a threshold value of n after which the relationship reverses. At

[17] These equations follow directly from the Equations (20) - (22) describing the equilibrium dynamics.

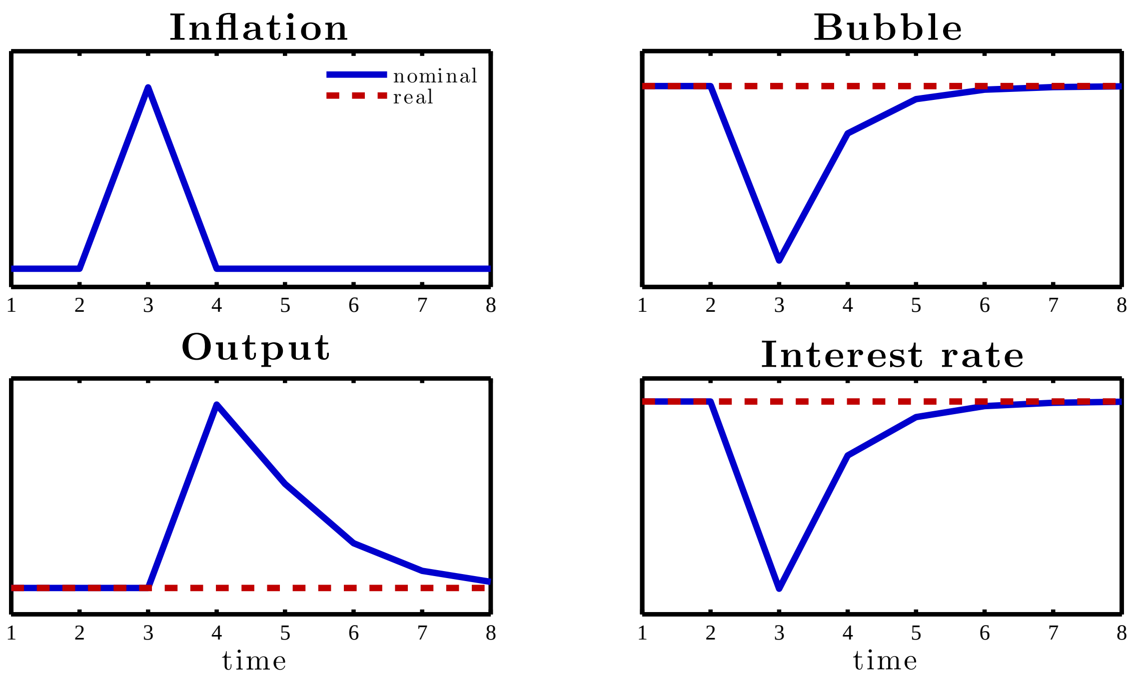


Figure 3: Response to unanticipated monetary shock.

high values of n, an increase in n lowers the capital stock because now it is the debt overhang effect that outweighs the wealth effect.

We now consider the response of the economy to different monetary shocks. To streamline the exposition, we define monetary shocks directly in terms of their impact on inflation. Specifically, we assume that inflation follows the process $\pi_t = \pi + \epsilon_t$, where ϵ_t represents a shock to monetary policy. We analyze three different specifications for ϵ_t, which correspond to three alternative scenarios regarding policy.

3.1.1 A surprise monetary shock

The first scenario is that of a one-time surprise shock to monetary policy. Specifically, we assume that there is a shock to inflation only in period T: $\epsilon_t = 0$ for all $t \neq T$ and $\epsilon_T = \epsilon$ or $\epsilon_T = -\epsilon$ with equal probability. Note that this shock has a direct effect on nominal credit bubbles, as the realized rate of inflation in period T necessarily differs from its expected value, i.e., $\pi_T \neq \pi$.

Figure 3 illustrates the response of the economy to a positive realization of ϵ_T, occurring in period 3. The solid lines refer to an economy in which a fraction of bubbles is nominal, i.e., $\lambda^N > 0$. In this case, the surprise rise in inflation leads to an expansion in investment and output. This happens because surprise inflation - by reducing the value of pre-existing nominal bubbles and, thus, the value of nominal credit that needs to be rolled over to sustain them - weakens the debt overhang effect thereby reducing the equilibrium interest rate. The lower interest rate,

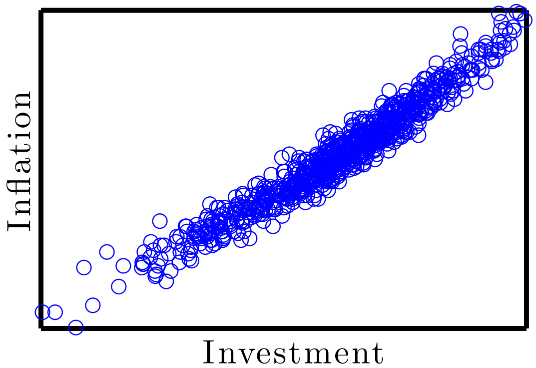


Figure 4: Bubbly Phillips curve.

in turn, boosts the financial multiplier of entrepreneurs and enables them to expand investment. Interestingly, although the shock is transitory, its effect is persistent. This happens because both state variables, the capital stock and the aggregate bubble, take time to return to their initial levels. The dashed lines in Figure 3 show an economy in which all the bubbles are real, i.e., $\lambda^N = 0$. In this case monetary policy does not affect real variables, highlighting the crucial role that nominal bubbles play in transmitting the monetary shock to the real economy.

This simple example illustrates that in presence of nominal bubbles a monetary expansion, here captured by a surprise rise in inflation, generates a drop in the real interest rate and a rise in investment. This is reminiscent of the impact of monetary expansions on the real economy in several vintages of Keynesian models. There are, however, at least two crucial differences. First, in standard Keynesian models monetary policy has real effects because of exogenous restrictions on price or wage adjustment. Instead, here nominal rigidities are the natural outcome of nominal bubbles, and do not depend on any imposed restriction on the type of contracts that agents can write. Second, in Keynesian models monetary policy affects the real interest rate because prices do not perfectly adjust to changes in nominal spending. Instead, here monetary shocks have an impact on the real interest rate by changing the value of past credit contracts, and, as a consequence, the current demand for credit.

3.1.2 The bubbly Philips curve

The second scenario is one of an i.i.d. inflation process, in which the inflation shock ϵ_t has a continuous support. In this case, when a fraction of bubbles is nominal, the bubble channel operates in each period as nominal contracts are constantly affected by inflation surprises. Since positive inflation surprises reduce the debt overhang and boost investment, this scenario naturally gives rise to an investment-based Phillips curve.

Figure 4 displays a scatter plot that illustrates the correlation between inflation and investment (left panel) and entrepreneurs' debt (right panel) emerging from a long simulation of the model.

Taken together, these two correlations highlight the similarities of the monetary transmission mechanism in our model and Fisher's debt deflation theory (Fisher, 1933). As in Fisher (1933), in our model periods of low inflation are associated with high real value of pre-existing debt and low investment. In this sense, our model can be seen as providing a possible micro foundation for nominal rigidities in credit contracts, which are at the heart of the Fisherian debt deflation channel of monetary policy.

3.1.3 An anticipated monetary shock

So far, we have considered bubbles that sustain short-term, i.e. one period, credit contracts. We now introduce the possibility that bubbly collateral might sustain long-term credit contracts, i.e. credit contracts that last longer than one period. We will see that in this case shocks to inflation expectations also affect the real economy.

To keep the analysis simple, we consider bubbles that back 'long-term' credit contracts with two period maturity. As before, we maintain that a fraction λ^N of all bubbles is nominal. But now, we also suppose that a fraction $\lambda^{LT} \in [0,1]$ of these nominal bubbles is 'long-term.' In particular, suppose that the market expects (with probability one) these bubbles to deliver the return to a non-contingent nominal two-period bond. After one period of life, the realized return to these bubbles must therefore be:

$$R_{t+1}^{2N} = \frac{\pi_{t+1}^{-1} \cdot E_{t+1}\pi_{t+2}^{-1} \cdot R_{t+2}^{-1}}{E_t\left\{\pi_{t+1}^{-1} \cdot E_{t+1}\pi_{t+2}^{-1} \cdot R_{t+2}^{-1}\right\}} \cdot R_{t+1},$$

where the superscript $2N$ indicates that these bubbles are both nominal and long-term.[18] A key observation is that these returns are affected by the arrival of new information, both regarding the realization of inflation at $t+1$ (just as with short-term bubbles) as well as regarding inflation expectations and interest rates between periods $t+1$ and $t+2$. Intuitively, a surprise increase in expected inflation and/or interest rates reduces the market value of contracts that promise to make nominal payments in the future.

Now consider a scenario in which $\epsilon_t = 0$ for all $t \leq T$ and $t \geq T+2$ but, at time T, agents learn that $\epsilon_{T+1} = \epsilon$. This process captures a situation in which a rise in inflation is perfectly anticipated one period ahead. It is straightforward to show that this change does not affect contracts originating in period T or later, which simply incorporate the new path for inflation. Nor does it affect contracts that are due in period T, since realized inflation at T has not changed. The increase in expected inflation, however, reduces the real value of long-term nominal contracts that are due in period

[18] Let $\hat{R}_{t,t+2}$ be the return promised by a non-contingent zero coupon nominal bond issued in t maturing in $t+2$. Then the real return on this bond at $t+1$ is: $R_{t+1}^{2N} \equiv \pi_{t+1}^{-1} \cdot R_{t+2}^{N-1} \cdot \hat{R}_{t+2}$, where $R_{t+2}^{N} = R_{t+2}/E_{t+1}\pi_{t+2}^{-1}$ is the nominal rate between $t+1$ and $t+2$. But no arbitrage implies $E_t R_{t+1}^{2N} = R_{t+1}$, and solving for $\hat{R}_{t+2}$ yields the desired expression.

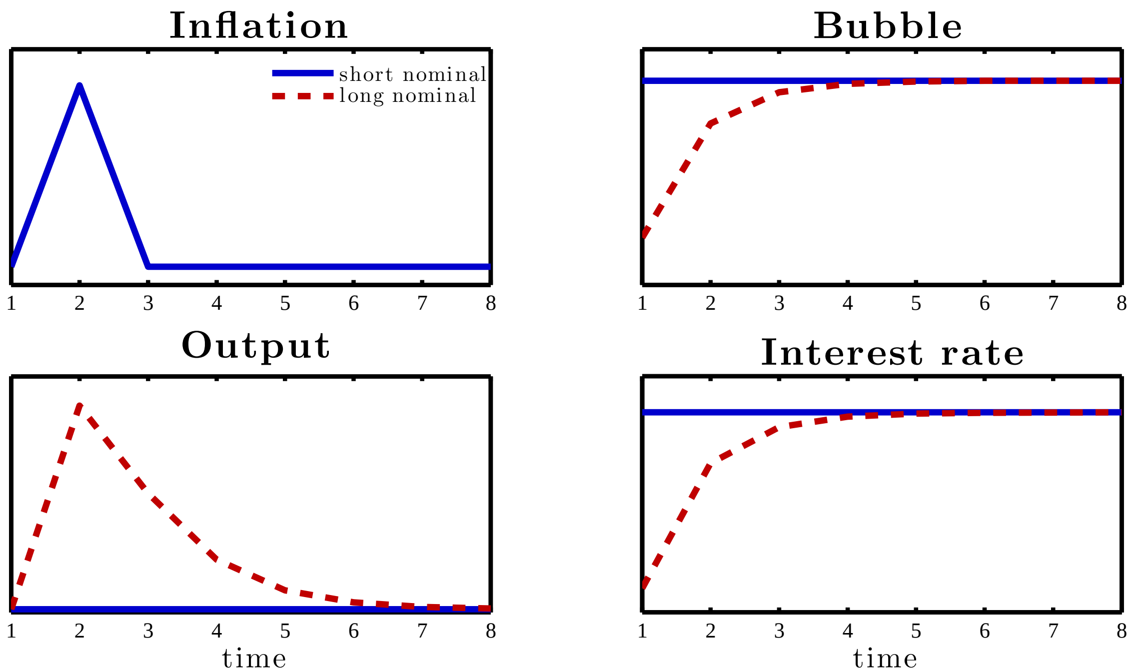


Figure 5: Response to anticipated monetary shock.

$T+1$. Through this channel, an increase in expected inflation weakens the debt overhang effect and raises investment and output. This is illustrated in Figure 5, which displays the response to a rise in period-2 inflation that is fully anticipated in period 1. As the figure shows, a shock to expected inflation has no effects when all contracts are short-term, but it boosts economic activity in the presence of long-term nominal contracts.[19]

This section has explored the basic features of the bubble channel of monetary policy. In a bubbly world, the mix of contracts that are traded depends on expectations. As long as some of these expectations are set in nominal terms, changes in inflation – both realized and expected – affect the debt overhang and thus investment and output. But what are the real-world counterparts of these bubbly contracts? In particular, the theory may seem to apply only to debt contracts, which are written in nominal terms and are naturally affected by inflation. This perception is misleading, however. The reason is that credit contracts in our framework inherit the properties of the bubbles – and thus of the expectations – that back them. Take the case of real-world equity contracts, for instance. If we interpret them through the lens of the theory, they may well be bubbly if the dividends that they are expected to deliver are ultimately backed by new credit that firms are expected to obtain in the future: moreover, they may be nominal or real depending on whether these expectations of future credit are set in nominal or real terms. The same reasoning applies to

[19]We have considered here the case of a one-time shock to inflation expectations for simplicity. A more general inflation process in which expected inflation varies continuously will give rise to a positive correlation between *expected* inflation and investment, i.e., a Philips curve relating expected inflation and investment.

bank lending or to more sophisticated financial instruments. The theory thus provides a powerful channel for monetary policy to affect financial markets, and thus macroeconomic outcomes, even in the absence of traditional price or contractual rigidities.

3.2 Managing credit booms and busts

In the bubbly economy, credit is driven by expectations and changes in expectations can be a source of credit cycles. A natural question that arises is whether monetary policy can exploit the bubble channel to stabilize these cycles. To address this question, we extend our example to allow for credit booms and busts.

To this effect, consider an economy with two aggregate states: $z_t \in \{B, F\}$. If $z_t = B$, we say that the economy is experiencing a bubbly episode. If $z_t = F$, we say that the economy is in the fundamental state. During a bubbly episode, old bubbles grow according to their rate of return and new bubbles of size n are started in every period. The end of a bubbly episode is associated to both, a bubble creation and a bubble return shock. First, the economy stops producing new bubbles, so that:

$$n_t = \begin{cases} 0 & \text{if } z_t = F \\ n & \text{if } z_t = B. \end{cases} \tag{30}$$

Second, a fraction $\omega \in (0, 1)$ of the old bubbles is destroyed, and the credit contracts that they back are defaulted upon. Naturally, lenders anticipate that they may not be repaid in full if the bubbly episode ends, and the equilibrium returns on credit contracts must reflect this as shown in Appendix D. We define φ and ψ as the probabilities that a bubbly episode starts and ends, respectively, where $\varphi < 0.5$ and $\psi < 0.5$.

To analyze the effects of monetary policy in stabilizing booms and busts we characterize the behavior of this economy under a 'bubble-blind' monetary policy, which does not react to the aggregate bubble, and under a 'bubble-conscious' monetary policy, which actively reacts to the evolution of the aggregate bubble.

3.2.1 Bubble-blind monetary policy

We analyze first policy rules in which the inflation process is independent of the aggregate bubble. The simplest such policy rule is one of strict inflation targeting, in which $\epsilon_t = 0$ and $\pi_t = \pi$ in all periods. Under constant inflation, we have already seen that there is no meaningful distinction between real and nominal contracts. Bubble shocks, in turn, give rise to repeated credit booms and busts.

Figure 6 provides an example of an economy that undergoes a bubble-driven boom-bust cycle. Start by considering the solid lines, which capture the case $\omega < 1$. The economy initially finds itself in a bubbly episode. The aggregate bubble grows continuously during the episode but its effect on

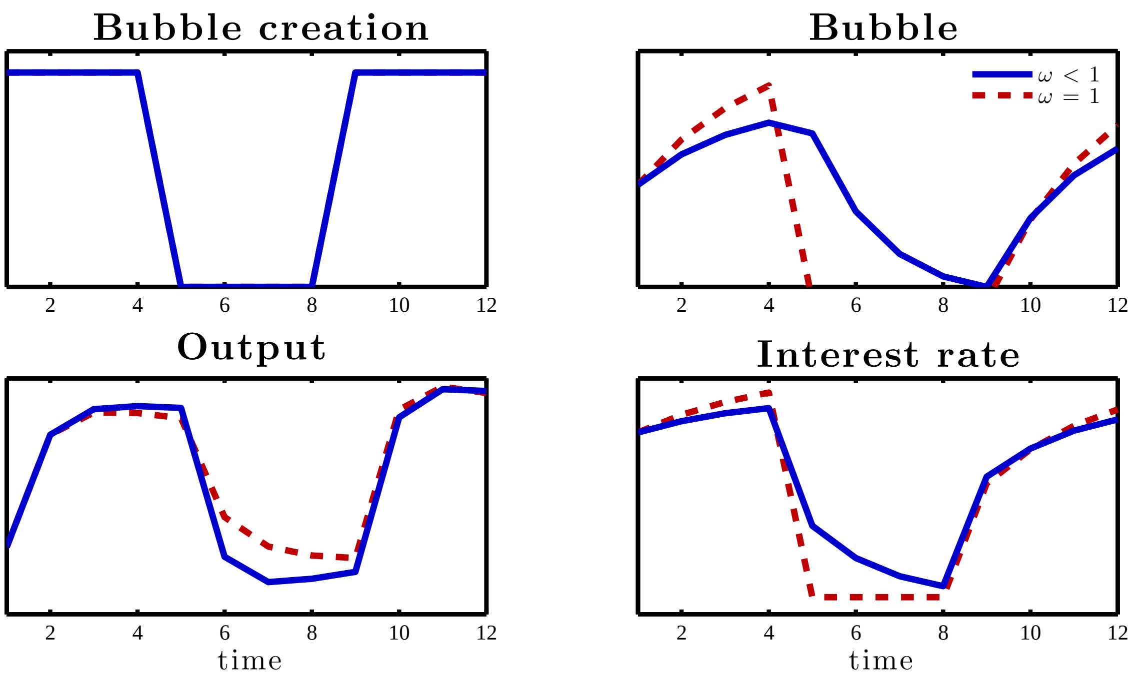


Figure 6: Bubbly business cycle.

output is non-monotonic. Initially, the wealth effect of bubble creation raises credit, investment, and output. As the episode progresses, however, the debt overhang effect becomes stronger as there is an increasing amount of credit that needs to be rolled over every period. This credit competes with investment for the economy's savings, raising the interest rate and reducing the financial multiplier of entrepreneurs. Eventually, the debt overhang effect dominates the wealth effect and investment and output begin to fall.

When the bubbly episode ends, the economy reverts to the fundamental state and investment and output collapse. Absent the wealth effect of bubble creation, entrepreneurial collateral and credit fall, bringing about a contraction in investment. If $\omega < 1$, as displayed by the solid lines, bubble creation disappears but the debt overhang effect lives on because old bubbles are not completely destroyed. Simply put, the economy is stuck with part of the credit contracts that were originated during the boom. This reinforces the collapse in investment and output that mark the end of a bubbly episode, because the need to roll-over old credit contracts prevents the interest rate from falling as much as it would otherwise do. Potentially, everyone loses form this transition to the fundamental state. Old entrepreneurs are unaffected because their assets (i.e., bubbles) decline in value but they also default on their credit contracts. Old savers lose because the credit contracts that they hold are partially defaulted upon, while young savers lose because they obtain a lower interest rate on their savings. Finally, young entrepreneurs lose because the lack of bubble creation makes them poorer. Output and investment remain low throughout the fundamental state until, eventually, a new bubbly episode begins and they boom once again.

But suppose instead that, when the economy reverts to the fundamental state, all credit contracts are defaulted upon. In this case, which corresponds to $\omega = 1$ and is illustrated by the dashed lines in Figure 6, the debt overhang effect disappears alongside the wealth effect when the bubbly episode ends. The main takeaway from this comparison is that a low value of ω stabilizes the aggregate bubble by making it safer, but it also destabilizes output. Because safer credit contracts get partially repaid in the fundamental state, they require a lower rate of return during the bubbly episode. Therefore, a low value of ω strengthens the debt overhang effect in the fundamental state, reducing output when it is low, but weakens it during bubbly episodes, raising output when it is high.

This section shows how bubble shocks drive economic fluctuations under a strict inflation targeting regime. But going back to one of the key motivating questions of this paper, could monetary policy respond to bubble shocks in a way that stabilizes output? Should it do so? We turn to these questions next.

3.2.2 Bubble-conscious monetary policy

Assume that, in the context of the economy studied in the previous section, the monetary authority follows a policy rule that is contingent on the credit bubble. For convenience, we focus on a simple policy rule that depends only on the aggregate state of the economy, reducing inflation during bubbly episodes and raising it in the fundamental state. Formally, we assume that:

$$\epsilon_{t+1} = \begin{cases} \pi_b - \pi & \text{if } z_{t+1} = B \\ \pi_f - \pi & \text{if } z_{t+1} = F, \end{cases} \tag{31}$$

where $\pi_b < \pi_f$. Under this policy, inflation equals π_b during bubbly episodes and π_f in the fundamental state.

To illustrate the effects of this policy rule relative to a constant inflation target, Figure 7 contrasts the behavior of the economy under both alternatives. The solid lines depict economic outcomes under a strict inflation target, whereas the dashed lines depict outcomes under the contingent rule specified above. There are two key takeaways from the figure. First, the bubble-conscious policy rule stabilizes output by raising it in the fundamental state and reducing it during bubbly episodes. Second, it does so by destabilizing the credit bubble. When the economy is in the fundamental state, the policy rule in Equation (31) implies a high – realized and expected – inflation rate, which dilutes the value of nominal contracts and weakens the debt overhang effect. Through this channel, the policy rule raises investment and output in the fundamental state. It does so at the cost of reducing output during bubbly episodes, however, during which the policy rule implies a low rate of inflation that raises the value of nominal contracts and strengthens the debt overhang effect.

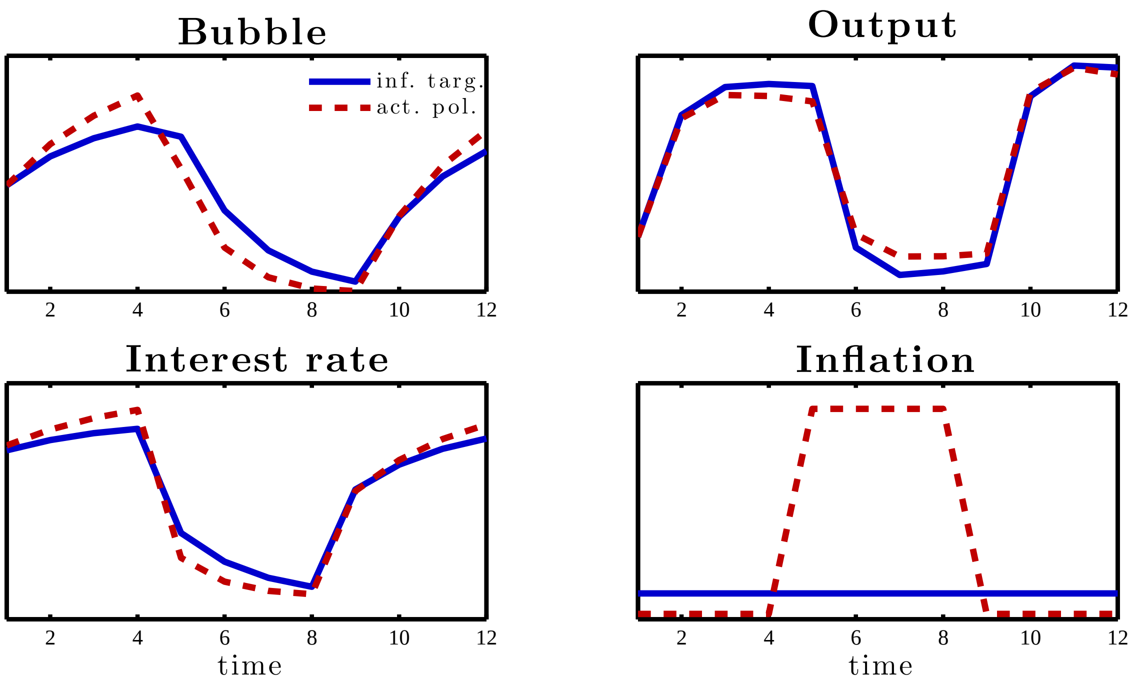


Figure 7: Countercyclical inflation policy.

Intuitively, the policy stabilizes output by making nominal contracts state-contingent, transferring the debt overhang effect from the fundamental states to bubbly episodes.

There are two key messages from this example. The first is that, being anticipated, monetary policy cannot systematically raise output by inflating away credit contracts in all states. What it can do, instead, is to stabilize output by reallocating the credit bubble – and thus the debt overhang effect that it entails – across states. The second message, which should be clear by now, is that the ability of monetary policy to perform this stabilization role depends on the mix of credit contracts that are traded in equilibrium and thus on expectations. Clearly, the power of monetary policy increases with the share of contracts that are nominal.

Even if it succeeds in stabilizing economic activity, it is worth noting that a monetary policy rule like the one outlined here may come at a cost in terms of average output. By making inflation state-contingent, monetary policy reallocates the credit bubble from fundamental states to bubbly episodes. This raises investment in the former and reduces it in the latter. The key observation, though, is that the increase in investment during fundamental states is not enough to compensate for its fall during bubbly episodes. To see why, note that the equilibrium interest rate, and thus the growth rate of the bubble, is higher during bubbly episodes. Therefore, a policy that reallocates the bubble towards these episodes must necessarily raise its average size and strengthen its debt overhang effect, which in turn reduces average investment and – possibly – average output as well. To illustrate this, Figure 8 shows how the steady-state mean and volatility of both output and the

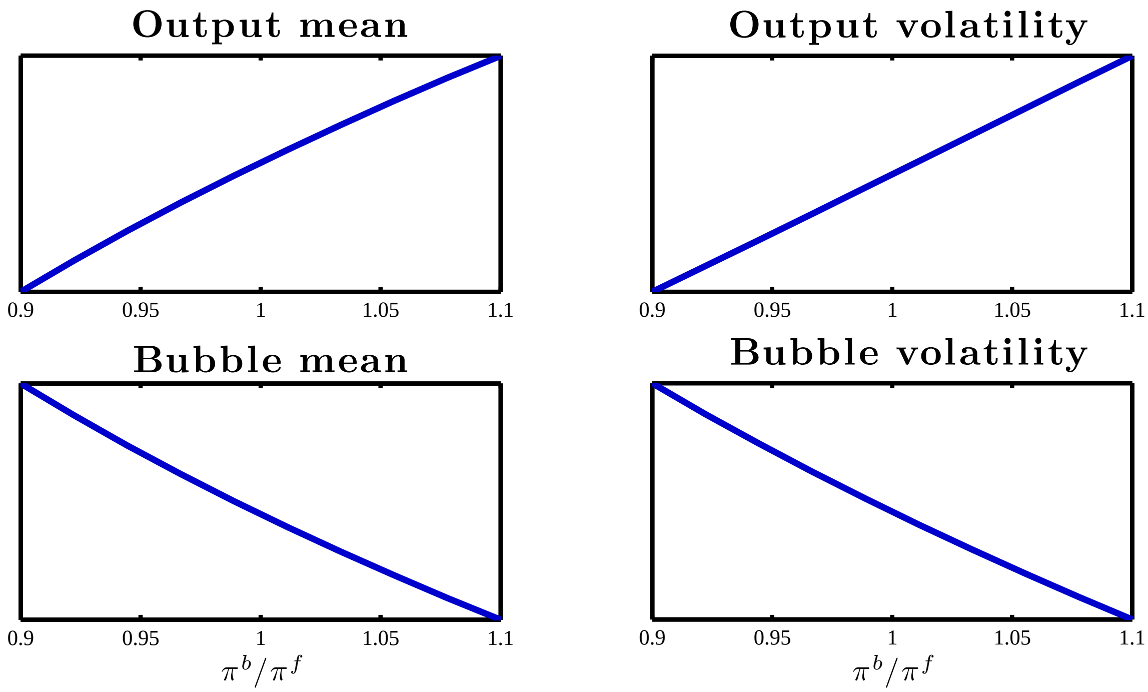


Figure 8: Inflation cyclicality: impact on means and volatilities.

credit bubble vary with the ratio of π^b to π^f. When π^b/π^f is lower than one, as we have assumed, the policy rule prescribed in Equation (31) stabilizes output and destabilizes the credit bubble: the cost of this stabilization, however, is that average output falls as the average size of the aggregate bubble rises. Naturally, the opposite is true when π^b/π^f is higher than one, in which case the same policy rule destabilizes output.

All in all, is a stabilization policy like the one outlined in Equation (31) desirable or not? The answer to this question depends on the perspective that is adopted. One possibility is to focus on the policy's effects on ex-ante or average welfare. This criterion is appropriate if we were to ask whether, from an ex-ante perspective and without knowing exactly the state into which they would be born, savers and entrepreneurs prefer to live in a world with or without a stabilizing policy rule. Here, the answer is ambiguous because the policy has potentially mixed effects on welfare. On the one hand, it may reduce fluctuations in lifetime income, which is beneficial because agents want to smooth consumption over time (even if not across states). On the other hand, though, the policy might reduce average output as in the example above.

A second possibility is to focus on the policy's ex-post effects, i.e., as it is applied on a sequential basis. This criterion is more appropriate to analyze the winners and losers generated by the policy in each history, which seems especially relevant if we think of the adherence to the policy rule as the outcome of a political economy process. Consider, for instance, an economy that transitions to the fundamental state in a given period t. At this point, the policy prescribes an increase in inflation that weakens the debt overhang effect and leads to a reduction in the interest rate. Old savers are clearly hurt by this policy, which partly inflates away the value of the credit contracts that they hold. Young savers are also hurt, because their wages are unaffected but they face a lower return on their savings as a consequence of the policy. For these same reasons, however, young entrepreneurs benefit and are able to expand their investment thereby boosting future output. In the presence of political considerations, then, whether the policy rule is followed at the end of a bubbly episode is likely to depend on the relative weights attached to the welfare of savers and entrepreneurs.

3.3 Discussion

The model developed here provides a rationale for the observed prevalence of nominal contracts in real-world financial markets, and it does so without imposing exogenous restrictions on contracting or pricing. Building on this feature, this section has shown how monetary policy can be used to manage the debt overhang effect of bubbles. As long as some expectations are set in nominal terms, changes in the rate of inflation affect the real value of bubbles and thus of the economy's total assets.

There is indeed ample evidence that, consistent with our model, asset values and returns are negatively affected by inflation. It should not be surprising that the value of assets that promise nominal payments, such as corporate bonds and bank loans, are negatively affected by inflation. But it is less obvious that inflation should affect other assets like equity. And yet, stock price booms tend to happen during periods of low inflation (Bordo and Wheelock, 2006). Bordo et al. (2008), for example, document that in post-war II US disinflation shocks are associated with market booms, while inflation shocks are associated with busts. Moreover, Galí and Gambetti (2015) show that an unexpected fall in inflation generated by a monetary policy tightening is associated with a rise in the bubbly component of stocks. Christiano et al. (2010) discuss how this negative correlation between inflation and stock prices is hard to reconcile with the standard New-Keynesian framework. In our model, this correlation arises naturally when expectations are set in nominal terms.

Before moving on, we want to briefly mention that monetary policy could do more than just shape the debt overhang effect of credit bubbles: it could also affect bubble creation and thus have wealth effects. This is an additional bubble channel of monetary policy. For this channel to operate, inflation should have an effect on credit contracts that are yet to be signed. Although this is an interesting idea, an in-depth exploration of its implications would take us too far afield. We nonetheless illustrate it through a simple example in Appendix E.

The key takeaway from the analysis of this section is that, in a bubbly world, the effects of monetary policy are ultimately shaped by expectations. It is expectations that determine the mix of contracts traded in equilibrium, namely real vs. nominal, short- vs. long-term and old vs. new. They also depend, as we now show, on whether the economy is in normal times – as we have assumed throughout this section – or inside a liquidity trap.

4 The liquidity trap

In the previous section, we assumed that the inflation target set by the monetary authority was high enough to guarantee that credit always dominates money as a store of value, i.e., $R_{t+1} > E_t \pi_{t+1}^{-1}$ in all states and periods. We now relax this assumption and consider the possibility that the economy falls into a liquidity trap, i.e., a situation in which $R_{t+1} = E_t \pi_{t+1}^{-1}$, the nominal interest rate is at the zero lower bound, and money and credit become perfect substitutes. We assume throughout that $E_t \pi_{t+1}^{-1} < \gamma$, however, which is necessary to guarantee that bubbles are possible in equilibrium.

We first explore how the behavior of the economy changes once it falls into a liquidity trap. We then analyze the fiscal impact of liquidity traps, which – as we explain below – have important implications for the authority's seigniorage revenues. Finally, we explore the effects and limitations of alternative policies that use seigniorage revenues to mitigate the consequences of a bubble crash.

4.1 Main implications of liquidity traps

We return to the economy of Section 3.2, which experiences bubble-driven fluctuations, but we now assume that inflation expectations are sufficiently low, i.e., $E_t \pi_{t+1}^{-1}$ is high, so that the economy enters a liquidity trap whenever it is in the fundamental state.

Figure 9 illustrates the behavior of the economy under a strict inflation targeting rule, which initially finds itself in a bubbly episode. The dashed line corresponds to the case of $\omega < 1$ in Figure 6, when the inflation target is high enough that the economy never enters a liquidity trap. The solid line depicts instead an economy in which the inflation target is low and the collapse of the bubble pushes the economy into a liquidity trap. During the bubbly episode, both economies behave exactly as before, displaying high levels of investment and output. When the bubbly episode ends, the disappearance of bubble creation reduces entrepreneurial collateral and thus the aggregate demand for credit. The difference between both scenarios is that, under a low inflation target, the fall in the nominal interest rate is limited by the zero lower bound. Once $R_{t+1} = \pi^{-1}$, savers demand money as a store of value and the interest rate cannot fall any further. Consequently, money balances must expand to fill the gap between the economy's savings and the demand for credit. This gives rise to the first major implication of liquidity traps: they magnify the effect of a bubble crash on investment and output.

Figure 9 also shows how entry into a liquidity trap leads to a surge in money holdings and to depressed levels of investment and output throughout the fundamental state. When a new bubbly episode begins, the rise in entrepreneurial collateral fuels the demand for credit, displacing money balances and enhancing investment until the economy exits the liquidity trap. It is worthwhile to note that we should interpret such an expansion and contraction in money balances more generally

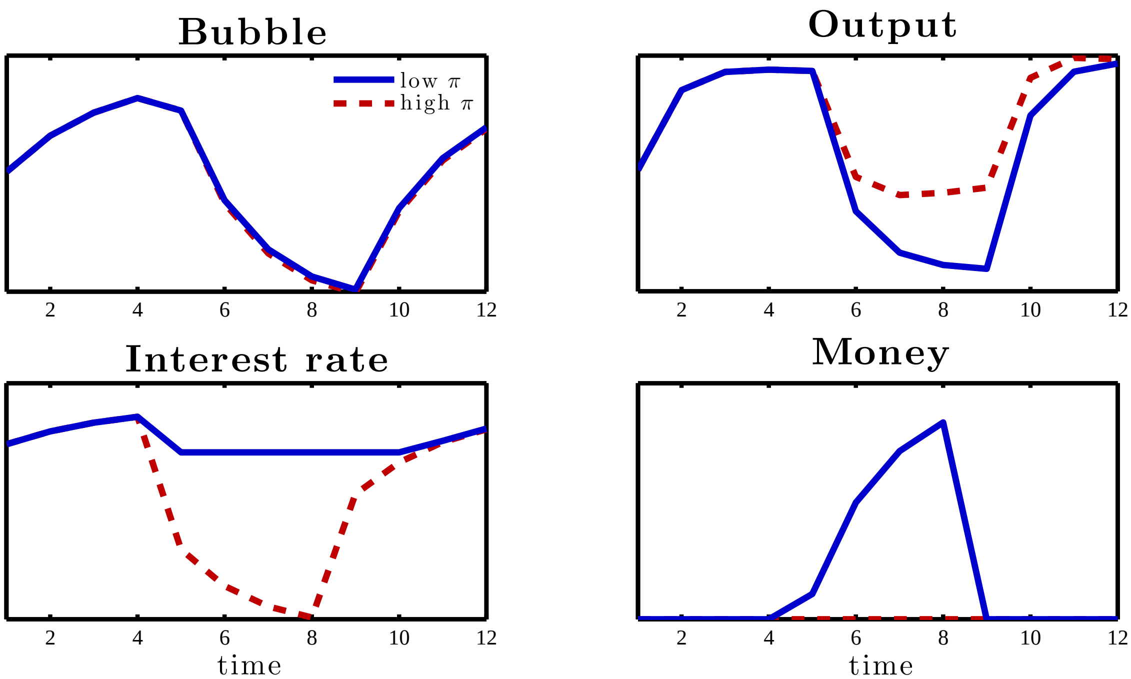


Figure 9: Liquidity trap.

as an expansion and contraction in demand for a broad class of liquid instruments, including government debt. In Appendix F, we show that our qualitative results remain unchanged with the introduction of such instruments.

A second implication of liquidity traps is that they change the effects of credit bubbles. Outside of the liquidity trap, we have seen that a key role of monetary policy is to manage the bubble induced debt overhang effect. Inside the liquidity trap, however, the debt overhang effect is irrelevant. The reason is that, in such a situation, the interest rate is not determined by the interplay of credit demand and supply but rather by inflation expectations as captured by $E_t\pi_{t+1}^{-1}$. Thus, investment is given by,

$$k_{t+1} = \frac{E_t\pi_{t+1}^{-1}}{E_t\pi_{t+1}^{-1} - \phi} \cdot (1-\tau) \cdot \varepsilon \cdot (1-\alpha) \cdot k_t^{\alpha},$$

and is independent of pre-existing credit bubbles. The intuition for this result is simple. Outside the liquidity trap, the interest rate equalizes aggregate savings and investment. Inside the liquidity trap, however, aggregate savings exceed investment at the prevailing interest rate. The difference between the two is allocated by savers to bubbles and money, so that:

$$m_t + b_t = \frac{\beta^{\theta}}{\beta^{\theta} + E_t\left\{\pi_{t+1}^{-1}\right\}^{1-\theta}} \cdot (1-\tau) \cdot (1-\varepsilon) \cdot (1-\alpha)\, k_t^{\alpha} - \frac{\gamma \cdot \phi \cdot k_{t+1}}{E_t\pi_{t+1}^{-1}}.$$

This expression implies that, given the interest rate $E_t\pi_{t+1}^{-1}$, any change in the value of the pre-existing bubble b_t is fully compensated by an opposite change in savers' demand for money. Thus, any attempt by the monetary authority to reduce the bubble overhang effect through surprise inflation will simply provoke a shift towards money in the portfolio of the savers, without having any effect on investment or output.

Monetary policy still plays a crucial role inside the liquidity trap by setting expected inflation, however. In fact, the monetary authority in our model could always escape a liquidity trap – raising investment and output – by raising inflation expectations as much as needed. But there may be costs from doing so. One of them is that an increase in the inflation target has important distributional consequences. By reducing the interest rate, such a policy hurts savers and benefits entrepreneurs by enabling them to expand their borrowing and investment. Thus, a benevolent monetary authority that puts a high weight on the welfare of the current savers may well choose to keep a low inflation target even if this entails keeping the economy inside the liquidity trap.

But there is another key implication of liquidity traps, which may well affect the monetary authority's incentives to exit it: they generate seigniorage revenues. To see this, note that money creation generates a seigniorage revenue of $V_t = \dfrac{M_t - M_{t-1}}{p_t}$ in every period, which can be expressed in efficiency units as follows:

$$v_t = m_t - \frac{m_{t-1}}{\gamma \cdot \pi_t}, \tag{32}$$

where the real balances $m_t \equiv \gamma^{-t} \cdot \dfrac{M_t}{p_t}$ satisfy:

$$m_t = \max\left\{0, \frac{\beta^\theta}{\beta^\theta + E_t\left\{\pi_{t+1}^{-1}\right\}^{1-\theta}} \cdot (1-\tau)\cdot(1-\varepsilon)\cdot(1-\alpha)\,k_t^\alpha - b_t - \frac{\gamma \cdot \phi \cdot k_{t+1}}{E_t \pi_{t+1}^{-1}}\right\}. \tag{33}$$

Equation (33) says that real balances are negligible when the economy is outside of the liquidity trap, but they become large when the economy is inside a liquidity trap. Thus, seigniorage matters only during a liquidity trap.

When the economy enters a liquidity trap, Equations (32) and (33) imply that seigniorage is positive and potentially large, since $m_{t-1} = 0$. During a liquidity trap, seigniorage also has the potential to be large, although it may be positive or negative depending on the inflation rate, the evolution of the bubble and the capital stock. When the economy exits the liquidity trap, seigniorage is negative and also potentially large since $m_t = 0$. Simply put, the monetary authority must "withdraw" the stock of real balances in circulation if it wants to maintain inflation expectations at $E_t \pi_{t+1}^{-1}$. The evolution of seignorage over the bubbly business cycle can be understood by looking at the path of money in Figure 9. During bubbly episodes, seigniorage equals zero exactly as anticipated. The crash of a bubble and subsequent entry into a liquidity trap generates a large seigniorage revenue, which remains positive as long as the economy is in the fundamental state. When a new bubbly episode begins, the economy exits the liquidity trap and seigniorage becomes large and negative for one period, as the demand for money collapses and the monetary authority is forced to re-absorb the real balances in circulation.

As long as fluctuations in seigniorage can be fully accommodated by changes in (useless) government spending, they have no effect. But fiscal policy often faces political and institutional constraints that prevent it from adjusting quickly. For this reason, we consider two ways in which seigniorage revenues might be important for policy. First, we assume that the monetary authority can keep seigniorage revenues when they are positive. This implies that, when the economy enters a liquidity trap, the authority can complement the "conventional" monetary policy of setting the inflation rate with an "unconventional" monetary policy in the form of asset purchases or credit subsidies. Second, we allow the monetary authority's fiscal backing to be limited, meaning that it has to bear some losses if seigniorage revenues become negative. This implies that, when the economy exits the liquidity trap, the authority's ability to conduct "conventional" monetary policy may be limited by the losses that it experiences. We analyze these possibilities next.

4.2 Unconventional monetary policy

We return to the example of the previous section, assuming that the monetary authority maintains a constant inflation target π and that the economy falls into a liquidity trap whenever it is in the fundamental state. In this section, we analyze the conduct of monetary policy while the economy is inside the liquidity trap. The conduct of policy when the economy exits the liquidity trap is relegated to the next section.

Let T denote the period in which the economy falls into a liquidity trap. The monetary authority's seigniorage revenues are given by 0 for $t < T$, by m_t in period $t = T$, and by $m_t - \frac{m_{t-1}}{\gamma \cdot \pi}$ in all periods $t > T$ while the economy remains in the liquidity trap. What does the monetary authority do with this revenue? We consider a type of unconventional monetary policy by which the authority uses this revenue to directly purchase new credit contracts from entrepreneurs. The effects of such an asset purchase scheme, as we now argue, depend on whether or not these purchases entail expected losses for the monetary authority.[20]

Consider first an asset purchase scheme in which the monetary authority breaks even in expectation. In this case, the intervention cannot affect total investment: regardless of who holds them, the total value of credit contracts sold by entrepreneurs equals the discounted value of their collateral. Since the interest rate is equal to π^{-1}, savings are also unaffected. Hence, the only effect of such an intervention is to rebalance the authority's own portfolio and that of private savers. To see this, suppose that the monetary authority spends one unit in purchasing a credit contract form entrepreneurs. Since the total value of contracts is unchanged, this necessarily implies that savers reallocate one unit of their savings from credit contracts to money. But this raises the seigniorage

[20] Clearly, entrepreneurs will never sell credit contracts to monetary authorities below market prices.

of the monetary authority by one unit, so that its net revenues remain constant. The policy has no effect on the economy because it cannot increase the total resources that end up in the hands of entrepreneurs. Indeed, as we illustrate in Appendix F, the logic that drives this neutrality result is more general, as it also renders neutral interventions consisting of purchases of assets such as government debt or other interest bearing instruments.

The alternative is an asset purchase scheme in which the monetary authority makes losses in expectation, i.e., in which what it pays for credit contracts exceeds the value of the collateral that backs them. Under such a policy, the monetary authority effectively transfers part or all of its seigniorage revenue to young entrepreneurs. To illustrate this point, we make two assumptions. First, the monetary authority uses all its seigniorage revenue to purchase credit contracts. Second, the credit contracts issued to the monetary authority are junior to those issued to savers.[21] Then, the law of motion for the capital stock becomes:

$$\gamma \cdot k_{t+1} = \frac{\pi^{-1}}{\pi^{-1} - \phi} \cdot \left[\varepsilon \cdot (1-\alpha) \cdot k_t^{\alpha} + v_t\right]. \tag{34}$$

Hence, when the economy enters a liquidity trap, this type of asset purchase scheme transfers $v_t = m_t$ to entrepreneurs and mitigates the fall in investment and thus in output. The monetary authority makes a loss, however, as these resources are spent on credit contracts that are not backed by collateral and will therefore not be repaid in equilibrium.

What would happen if this policy is applied in a continuous fashion? As long as the economy stays in the liquidity trap, it gradually converges to the levels of capital and money balances implicitely defined by:

$$\gamma \cdot k = \frac{\pi^{-1}}{\pi^{-1} - \phi} \cdot \left[\varepsilon \cdot (1-\alpha) \cdot k^{\alpha} + \left(1 - \frac{1}{\gamma \cdot \pi}\right) \cdot m\right], \tag{35}$$

$$m = \frac{\beta^{\theta}}{\beta^{\theta} + (\pi^{-1})^{1-\theta}} \cdot (1-\tau) \cdot (1-\varepsilon) \cdot (1-\alpha) \cdot k^{\alpha} - \frac{\gamma \cdot \phi \cdot k}{\pi^{-1}} > 0. \tag{36}$$

Equation (35) says that – if seigniorage is used to sustain investment – steady-state output inside of the liquidity trap is increasing in both the financial multiplier of entrepreneurs and in the seigniorage revenue of the monetary authority.[22]

Figure 10 illustrates the implications of this asset purchase scheme with an example. It depicts an economy that begins in a bubbly state, then transitions to the fundamental state and stays there throughout. The evolution of the economy is displayed under two alternative scenarios, depending on whether the monetary authority uses seigniorage revenues to purchase credit contracts or rebates

[21] Note that in this extreme case all of the monetary authority's lending will be defaulted upon, so that it is a pure transfer or subsidy.

[22] The same condition that insures that bubbly equilibria exists also ensures that seigniorage revenue is positive and this policy expands the capital stock.

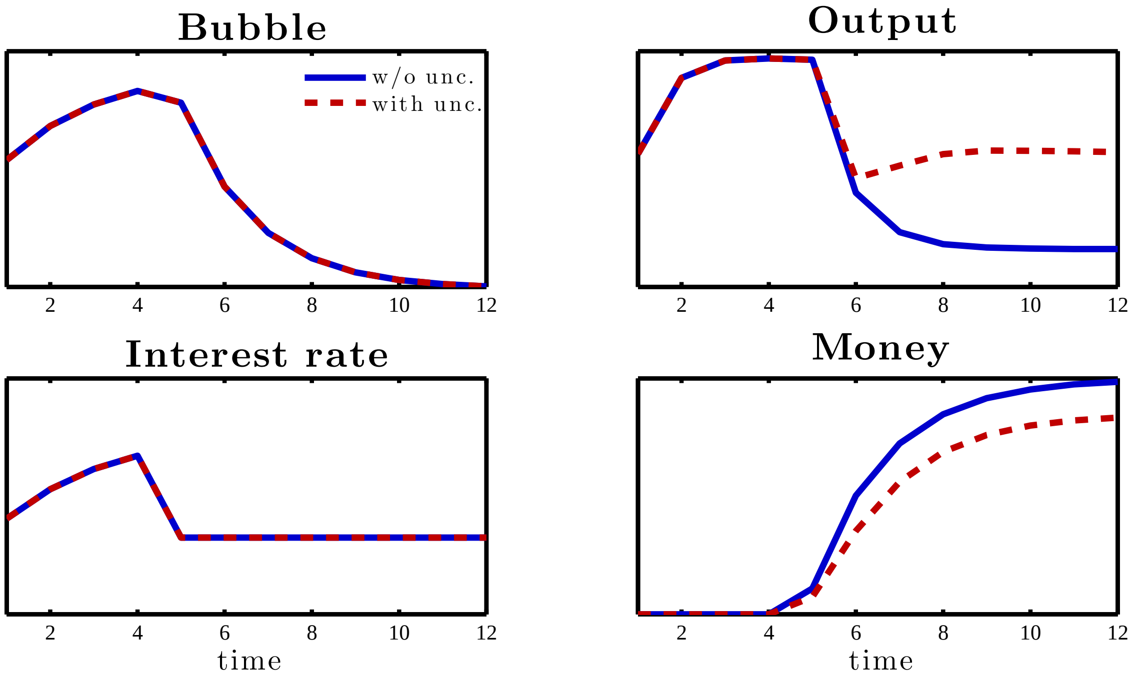


Figure 10: Liquidity trap: unconventional monetary policy.

them back to the government. As anticipated, the policy mitigates the fall of investment and output that takes place in the fundamental state. Note that money holdings are lower under the specified policy than in its absence. The reason is that the transfer of seigniorage revenues to entrepreneurs raises investment more than proportionally because these revenues are leveraged in the credit market.

The use of seigniorage revenue to sustain private investment has one crucial implication: namely, it can change the relationship between steady-state inflation and output. This is best seen through Equation (35). On the one hand, an increase in inflation boosts output by reducing the interest rate thereby raising the financial multiplier of entrepreneurs. On the other hand, an increase in inflation reduces money holdings and it may therefore lead to a decline in seigniorage revenue. To the extent that this revenue is used to sustain investment, any such decline is contractionary. Hence, if the monetary authority is interested in maximizing steady-state output, it may chose to maintain inflation low and interest rates high so as to keep the economy inside the liquidity trap. By doing so, it preserves seigniorage revenue that can be used to sustain private investment until a new bubbly episode begins.

An alternative way of interpreting this result is that, inside of the liquidity trap, money is a bubble. Like credit bubbles, it has an overhang effect because it diverts resources away from investment. Also like credit bubbles, it has a wealth effect because money creation provides seigniorage. There are two important differences with credit bubbles, though: (i) the supply of money is directly controlled by the monetary authority and, therefore (ii) the wealth effect from money creation is appropriated by the monetary authority. By transferring this wealth effect back to entrepreneurs, the policy outlined above partially recreates the effects of the credit bubble that has disappeared until a new one comes along. Could this analogy be pushed further? The answer is positive, and Appendix G shows how the monetary authority could recreate any bubbly steady-state through a combination of the right inflation target and an asset purchase scheme like the one outlined above.

This section has focused on the surge in seigniorage revenues generated by the onset of a liquidity trap. The end of a liquidity trap, in turn, generates losses because savers stop holding money as they redirect their savings to private credit. If the monetary authority is to keep inflation expectations in check, it must be ready to intervene and withdraw the stock of real balances in circulation. This is not a problem if it has access to large fiscal resources, but it may not be feasible if resources are limited. We address this question next.

4.3 Fiscal constraints on monetary policy

We now consider what happens when the economy exits the liquidity trap. To do so, we return to the example of the previous section, in which expected inflation is low enough to push the economy into a liquidity trap whenever it is in the fundamental state. In any period, there is a probability φ that a new bubbly episode begins and, when it does, we assume that bubble creation n is large enough to push the economy out of the liquidity trap.

When the bubbly episode begins, savers increase their lending to entrepreneurs and they stop holding money as a store of value. They only hold real balances to satisfy their transaction needs, which we assumed to be small and equal to υ. Hence, seigniorage revenues are given by:

$$v_t = m_t - \frac{m_{t-1}}{\gamma \cdot \pi_t} = \upsilon - \frac{m_{t-1}}{\gamma \cdot \pi_t} < 0. \tag{37}$$

How can the monetary authority deal with this negative seigniorage? There are two benchmark scenarios. In the first one, negative seigniorage is absorbed with fiscal resources, and it leads to a one-to-one decline in government spending. This corresponds to a situation in which the monetary authority receives a fiscal transfer that is large enough to "withdraw" the real balances in circulation when the economy exits the liquidity trap. In the second scenario, on which we focus for the remainder of the section, the monetary authority lacks a fiscal backing and does not receive any transfer. This imposes the restriction that seigniorage be non-negative, i.e., $v_t > 0$, so that the "excess" stock of real balances needs to be diluted through inflation. Formally, if we use T to denote the period in which the economy exits the liquidity trap, then the inflation rate conditional on exit π_T^E must be arbitrarily large: since $\upsilon \to 0$, it must be that $\pi_T^E \to \infty$. Thus, we conclude that fiscal constraints limit the monetary authority's ability to credibly maintain a low inflation target upon exit from a liquidity trap.

Paradoxically, this inability to stabilize inflation upon exit from the liquidity trap might rule out the possibility of such a trap in the first place. To see this, let π_t^S denote the inflation rate conditional on staying in the liquidity trap in period t. Then, inflation expectations throughout the liquidity trap must satisfy:

$$E_t \pi_{t+1}^{-1} = \varphi \cdot \left(\pi_{t+1}^E\right)^{-1} + (1-\varphi) \cdot \left(\pi_{t+1}^S\right)^{-1}. \tag{38}$$

The restriction that seigniorage revenues cannot be negative implies that π_{t+1}^S is bounded below. Thus, for a given φ, a larger inflation upon exit, π_{t+1}^E, implies that inflation expectations throughout the liquidity trap must be large as well (i.e., $E_t \pi_{t+1}^{-1}$ must be small). These inflation expectations can be large enough to ensure that the economy never even enters a liquidity trap.

What are the overall implications of this analysis? In the previous section, we saw how a monetary authority that keeps seigniorage revenues can use them to sustain credit and investment inside the liquidity trap. We also argued that, in such a scenario, the monetary authority may opt to set a low inflation target when the economy is in the fundamental state, keeping the economy inside the liquidity trap in order to maintain seigniorage revenues high. The analysis of this section, however, suggests that the scope for such a policy may be severely limited in the absence of fiscal backing. Simply put, insufficient fiscal backing places a lower bound on inflation expectations inside of the liquidity trap, which in turn limits the demand for real balances and thus the size of seigniorage revenues. If the probability of bubbly episodes is sufficiently high, moreover, it may rule out the possibility of liquidity traps altogether: given the high probability of exit, the admissible levels of inflation expectations inside of a liquidity trap would be too high for the liquidity trap to arise in the first place.

5 A model-based tour of recent macroeconomic history

The model developed here provides an integrated view of the interaction between bubbles, credit, money, and economic activity. In this section, we build on this view to interpret some salient facts characterizing recent macroeconomic history. In particular, we argue that the model can rationalize the large fluctuations in stock and real estate prices experienced by several advanced economies during the last two decades. Moreover, the model sheds light on the positive correlation between asset prices, investment and credit, as well as on the emergence of liquidity traps – and the associated surges in public assets – following bubble bursts.

The starting point of our analysis is the persistent decline in interest rates that has characterized the world economy since the early 1980s. This decline is illustrated in Figure 11, which depicts interest rates in the United States from 1980 onwards. This trend is not exclusive to the United States, however. As documented by Rachel and Smith (2015), the decline in interest rates has

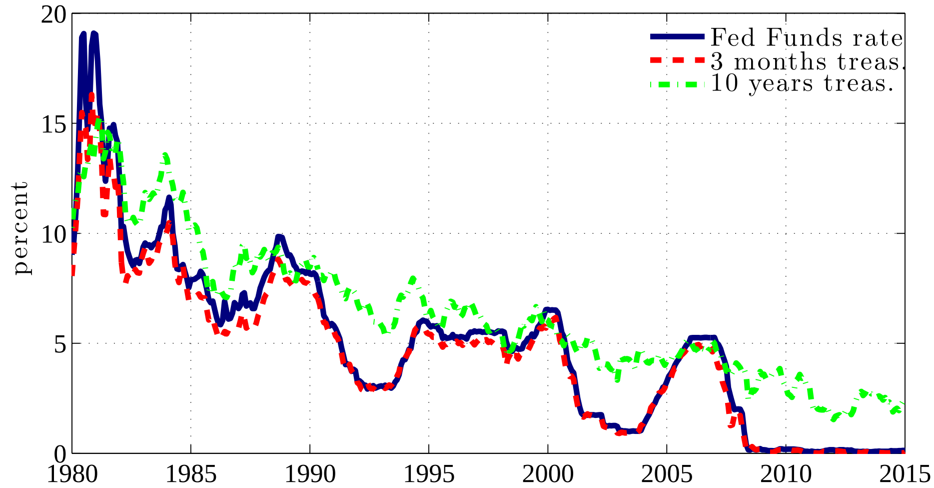


Figure 11: Selected US interest rates. Notes: see Appendix *H* for data sources.

affected both advanced and emerging economies, suggesting the existence of common factors at the global level. Although there is still debate regarding their relative contribution, some of these factors are the slowdown of productivity growth and the aging of population in industrial economies, as well as the rise in precautionary savings by emerging economies.[23] While our model is not intended to reproduce the specific characteristics of these different factors, all of them push for a higher propensity to save, which can be captured through a rise in β.

In the model, high values of β are exactly what is needed to make bubbles possible. To see this point, remember that bubbles can only be sustained if the steady state interest rate is lower than the rate of productivity growth. Moreover, equation (23) shows that – outside of the liquidity trap – the steady-state interest rate is decreasing in β. Thus, an increase in β, if it is strong enough, will reduce the real interest rate and create the conditions for bubbles to arise. Hence, our model suggests that the same structural factors that have led to the secular fall in interest rates have also opened the door to the possibility of rational bubbles.

Bubbles raise the total value of private assets in the economy, which in the model is formally given by

$$K_t + B_t,$$

at any point in time. Thus, bubbly episodes should be associated with high and fluctuating asset prices. And indeed, besides declining interest rates, the last thirty years have also been characterized by large fluctuations in asset prices – most notably equity and real estate – in industrial economies. Some notable examples have been the boom-bust cycle in asset prices experienced by Japan between the late 1980s and early 1990s, the rise and subsequent fall in equity prices experienced by the United States in the late 1990s, or the surge and subsequent fall in real estate prices that preceded

[23]Moreover, Iachan et al. (2015) argue that some of the decline in interest rates was due to financial innovation.

the 2008 financial crisis both in the United States and in the Eurozone periphery. The top-left panel of Figure 12 illustrates this fact for the case of the United States, by depicting the evolution of households net worth to GDP from 1990 onwards. Since households net worth closely tracks stock and real estate prices, the figure reflects the magnitude of asset price fluctuations.[24] These fluctuations have been remarkable in many regards. Namely, they have created and destroyed significant amounts of wealth in short periods of time, and it has been difficult to attribute them to changes in economic fundamentals.[25] Through the lens of our model, these fluctuations in asset prices can be interpreted as rational bubbles, i.e., the outcome of self-fulfilling expectations by market participants.

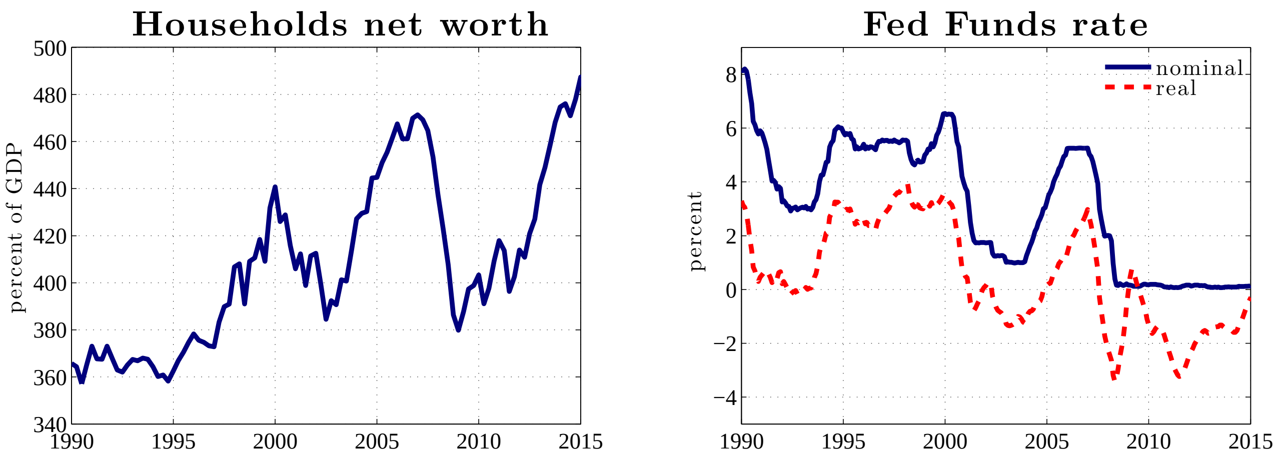
Throughout all of these boom-bust episodes, moreover, rising asset prices have been associated with high credit and investment growth, and by upward deviations of interest rates from their trend. The two top panels of Figure 12, for instance, jointly show how episodes of rapid increase in net worth have been accompanied by rising interest rates in the United States, and how interest rates have collapsed alongside net worth when these episodes have come to an end. The middle panels of Figure 12 depict investment and credit to the business sector for the United States, both of which rise during periods of rapid increase in net worth. According to our model, these are all a natural consequence of the increase in collateral generated by high asset prices, which sustains a higher demand for credit and thus a higher interest rate and investment. In this sense, our model formalizes the notion, put forward by some commentators (Summers, 2013; Krugman, 2013), that asset bubbles have partly masked the long-run decline in world rates in the US and the Euro area.

By the same token, our model also explains why interest rates and investment fall alongside credit when bubbles burst. The bursting of a bubble generates a drop in collateral, thereby leading to a decline in credit, interest rates and investment. If expected inflation is low enough, our model shows how the bursting of a bubble also pushes the economy into a liquidity trap, which in turn entails a shift from private to public assets as agents increase their holdings of money and other forms of liquid government liabilities. This is precisely what we have witnessed in the United States and the Eurozone's peripheral countries in the aftermath of the 2008 asset price crash. The bottom panels of Figure 12 depict the evolution of the monetary base and total government debt in the United States, and shows that – jointly considered – they have increased by more than 60% of GDP since the financial crisis.

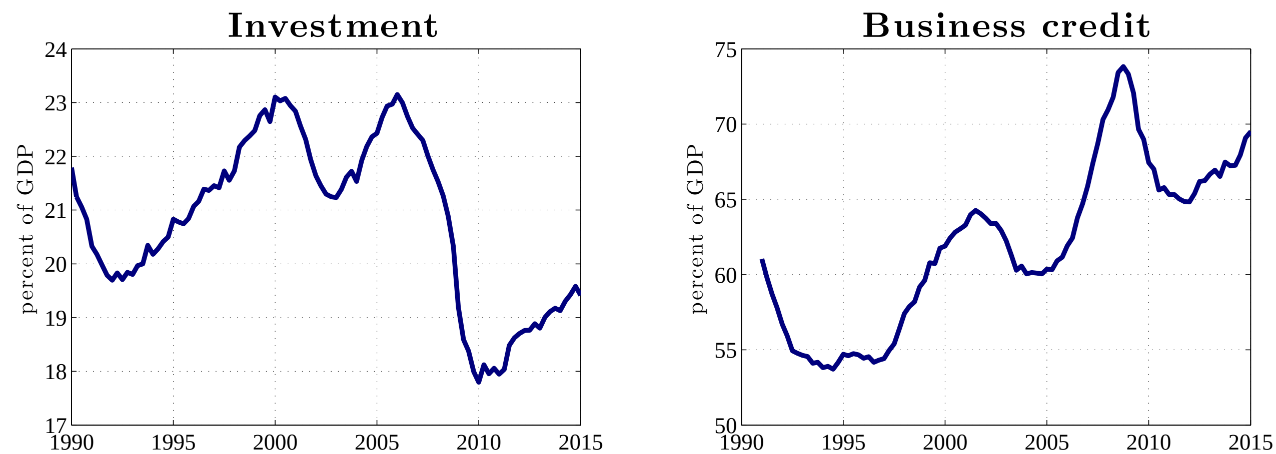
Viewed through the lens of our model, therefore, an overarching narrative of the last few decades is that we have experienced multiple bubbly episodes, made possible by low interest rates. Monetary policy, in the meantime, has been characterized in most industrial economies by inflation targeting

[24] Real estate and holdings of corporate equity (direct and indirect) make up roughly 70% of households and non-profit net worth.

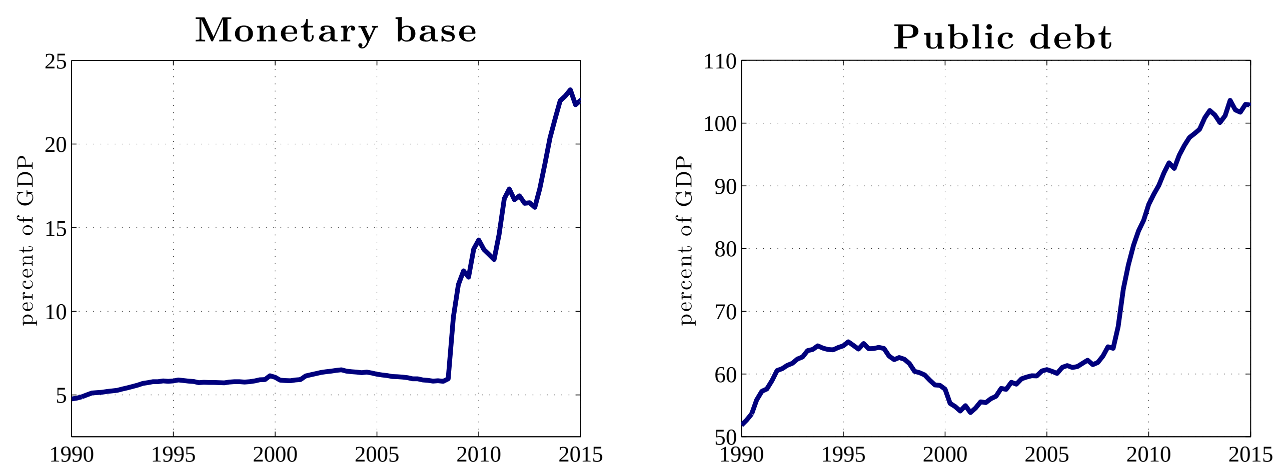
[25] See, for instance, LeRoy (2004) for an analysis of the US equity boom and bust of the late 1990s, Shiller (2007) for an analysis of the rise in housing prices in the run-up to the financial crisis of 2008, and French and Poterba (1991) for an analysis of Japanese asset prices during the late 1980s and early 1990s.



(a) Net worth of households and non-profit organizations and Federal Funds rate



(b) Investment and credit to the non-financial business sector



(c) Monetary base and Federal debt

Figure 12: Motivating facts for the United States. Notes: the real Fed Funds rate is obtained by subtracting the average CPI inflation during the previous two years, as a proxy for expected inflation, from the nominal rate. See Appendix H for data sources.

regimes, in which the objective of the central bank is to stabilize inflation around a pre-determined target. This has produced an environment of low and stable rates of inflation throughout the industrialized world.

What are the implications of our theory for the conduct of monetary policy? The first one refers to the volatility of inflation: namely, a stable inflation target is associated with high volatility of output, boosting growth during bubbly episodes but also deepening the fall in output that follows the burst of a bubble. Instead, a policy of state-contingent inflation targets, high in recessions but low during booms, could contribute to stabilize output by weakening the debt overhang effect during recessions. Our model thus provides a formal interpretation of the view, laid forth by various academics and policymakers in the aftermath of the financial crisis (Rogoff, 2008), that inflation can be an important tool to deal with the debt overhang of industrial economies. The model, however, also highlights the ex ante and ex post costs associated to such a policy. From an ex ante perspective, it shows why having a countercyclical inflation target may harm average growth. From an ex post perspective, it illustrates the redistributive implications of such a policy: during a recession, for instance, high inflation will benefit entrepreneurs at the expense of savers, and it may therefore be difficult to implement due to political economy considerations.

A second implication of the theory refers to the level of inflation. In fact, the model illustrates how the low inflation rates targeted by the world's major central banks have opened the door to the possibility that bubble bursts might lead to liquidity traps, raising the associated macroeconomic costs. According to the theory, the inflation rate that is consistent with a liquidity trap is not constant, but depends instead on the evolution of the aggregate bubble. A low inflation target may not bind during bubbly episodes, when the abundance of collateral sustains a positive nominal interest rate, but may nonetheless push the economy into a liquidity trap when the bubble disappears. Under this interpretation, the low inflation rates of the recent decades did not constitute a problem as long as the interest rate was sustained by the presence of the dot-com and the housing bubbles, but contributed to the severe recession that followed the collapse in asset prices of 2008.

A third implication of the theory refers to the policy options available to, and the risks faced by, the monetary authority inside a liquidity trap. First, it shows why it may not be desirable for the authority to exit a liquidity trap even if it can do so. Besides the aforementioned redistributive considerations, agents flock to public assets when the economy is inside the liquidity trap, generating seigniorage revenues that can be used to stimulate private investment. This implies that it is possible to sustain a higher level of economic activity by remaining in a liquidity trap until asset prices rise once again and private collateral recovers. But the theory also illustrates the dangers associated to such a policy. Namely, once the economy exits the liquidity trap, the demand for public assets collapses as agents demand private assets once again. Unless they have a proper fiscal backing, the value of these assets will then collapse, leading to a spike in inflation.

Most estimates suggest that the downward trend in world interest rates is not expected to be reversed any time soon (Rachel and Smith, 2015; Gourinchas and Rey, 2016). If that is the case, we should expect bubbly episodes to arise again in the coming years, possibly accompanied by liquidity traps when bubbles crash. It is then of crucial importance to understand the monetary policy choices that are available in a bubbly world. Our model provides a first formal treatment of these issues.

Appendix

A Entrepreneurial preferences

We assumed throughout that entrepreneurs are arbitrarily patient, $\beta^E \to \infty$, and that they therefore save all of their income. We now relax this assumption. As before, the representative entrepreneur faces budget constraints given by Equations (12) and (13), and the collateral constraints given by Equation (14). We maintain the assumption that $r_{t+1} + 1 - \delta > R_{t+1}$. Thus, Equation (15) holds. Combining these with the entrepreneur's maximization, we obtain the following law of motion for the capital stock:

$$k_{t+1} = \frac{1}{\gamma} \cdot \frac{R_{t+1}}{R_{t+1} - \phi + \beta^{E-\theta} \cdot R_{t+1} \cdot \left(\alpha \cdot k_{t+1}^{\alpha-1} + 1 - \delta - \phi\right)^{1-\theta}} \cdot \left((1-\tau) \cdot \varepsilon \cdot (1-\alpha) \cdot k_t^{\alpha} + n_t\right).$$

The remaining equations summarizing the equilibrium dynamics of the economy are as before. Note that as $\beta^E \to \infty$, the law of motion is as in Equation (21). When $\beta^E < \infty$, there are two changes. First, the financial multiplier is smaller than the case of arbitrarily high patience, which is intuitive since each unit of net worth is not fully invested in capital. Second, given the interest rate R_{t+1}, investment is declining in the marginal product of capital. The reason is that the larger is the marginal product of capital, the higher is the return on the entrepreneur's savings and, since we assume that $\theta > 1$, the entrepreneur's savings and investment must be larger as well.

B Setting the inflation rate

We have assumed that the monetary authority can achieve the inflation target it desires by appropriately adjusting the money supply. We now establish this formally. Recall that the demand schedule for money balances satisfies:

$$\frac{M_t^d(p_t)}{p_t} = \max\left\{\upsilon \cdot \gamma^t, \frac{\beta^\theta}{\beta^\theta + E_t\left\{\pi_{t+1}^{-1}\right\}^{1-\theta}} \cdot (1-\tau) \cdot (1-\varepsilon) \cdot (1-\alpha) \cdot K_t^{\alpha} - B_t - N_t - \frac{\phi \cdot K_{t+1}}{E_t \pi_{t+1}^{-1}}\right\}.$$

Letting $M_t^s(\cdot)$ denote the monetary authority's supply schedule for money, the money market clearing price level p_t is such that:

$$M_t^s(p_t) = M_t^d(p_t).$$

Suppose that the monetary authority wants to sustain a process for inflation $\{\bar{\pi}_t\}$ for all t. Let $p_{t-1} > 0$ be the equilibrium price level at $t-1$, then the following supply schedule implements inflation $\bar{\pi}_t$ at time t:

$$M_t^s(p_t) = \begin{cases} M_t^* & \text{if} \quad p_t = \bar{\pi}_t \cdot p_{t-1} \\ \frac{\upsilon}{2} & \text{otherwise,} \end{cases}$$

where M_t^* satisfies:

$$\frac{M_t^*}{\bar{\pi}_t \cdot p_{t-1}} \equiv \max \left\{ \upsilon \cdot \gamma^t, \frac{\beta^\theta}{\beta^\theta + E_t \left\{ \bar{\pi}_{t+1}^{-1} \right\}^{1-\theta}} \cdot (1-\tau) \cdot (1-\varepsilon) \cdot (1-\alpha) \cdot K_t^\alpha - B_t - N_t - \frac{\phi \cdot K_{t+1}}{E_t \bar{\pi}_{t+1}^{-1}} \right\},$$

where notice that the right hand side is evaluated at the desired (and not actual) inflation rate. The unique price level p_t that clears the money market at time t is precisely given by $p_t = \bar{\pi}_t \cdot p_{t-1}$. Thus, the associated sequence of money supply schedules $\{M_t^s(\cdot)\}$ implements the desired process $\{\bar{\pi}_t\}$ for inflation.

We have assumed here, as in most of the paper, that the monetary authority rebates all seigniorage (positive or negative) to the fiscal authority.

C Parameters used to construct the figures

- **Figure 1**: $\alpha = .3$, $\gamma = 1$, $\varepsilon = .2$, $\phi = .1$, $\tau = 0$, $n_T = .005$. Dashed lines: $\beta = 0.77$, $\theta = 1.75$. Solid lines: $\beta = 1.80$, $\theta = 3.5$.

- **Figure 2**: $\alpha = .3$, $\gamma = 1$, $\varepsilon = .2$, $\phi = .1$, $\tau = 0$, $n = .005$, $\beta = 0.77$, $\theta = 1.75$.

- **Figure 3**: $\alpha = .3$, $\gamma = 1$, $\varepsilon = .2$, $\phi = .1$, $\tau = 0$, $n = .005$, $\beta = 0.77$, $\theta = 1.75$, $\epsilon_3 = 14$. Solid lines: $\lambda^N = 1$. Dashed lines: $\lambda^N = 0$.

- **Figure 4**: $\alpha = .3$, $\gamma = 1$, $\varepsilon = .2$, $\phi = .1$, $\tau = 0$, $n = .005$, $\beta = 0.77$, $\theta = 1.75$, $\lambda^N = 1$. ϵ_t follows $\epsilon_t = \rho\epsilon_{t-1} + \kappa_t$, where $\rho = .9$ and κ_t is normally distributed with mean equal to 0 and standard deviation equal to 0.04.

- **Figure 5**: $\alpha = .3$, $\gamma = 1$, $\varepsilon = .2$, $\phi = .1$, $\tau = 0$, $n = .005$, $\beta = 0.77$, $\theta = 1.75$, $\epsilon_2 = 14$. Solid lines: $\lambda^{LT} = 0$. Dashed lines: $\lambda^{LT} = 0.5$.

- **Figure 6**: $\alpha = .3$, $\gamma = 1$, $\varepsilon = .2$, $\phi = .2$, $\tau = 0$, $n = .005$, $\beta = 0.77$, $\theta = 1.75$, $\psi = 0.1$, $\varphi = 0.1$. Solid lines: $\omega = 0$. Dashed lines: $\omega = 1$.

- **Figure 7**: $\alpha = .3$, $\gamma = 1$, $\varepsilon = .2$, $\phi = .2$, $\tau = 0$, $n = .005$, $\beta = 0.77$, $\theta = 1.75$, $\psi = 0.1$, $\varphi = 0.1$, $\omega = 0$. Solid lines: $\pi = 14$. Dashed lines: $\pi_b = 10$, $\pi_f = 50$.

- **Figure 8**: $\alpha = .3$, $\gamma = 1$, $\varepsilon = .2$, $\phi = .2$, $\tau = 0$, $n = .005$, $\beta = 0.77$, $\theta = 1.75$, $\psi = 0.1$, $\varphi = 0.1$, $\omega = 0$, $\pi = 14$.

- **Figure 9**: $\alpha = .3$, $\gamma = 1$, $\varepsilon = .2$, $\phi = .2$, $\tau = 0$, $n = .005$, $\beta = 0.77$, $\theta = 1.75$, $\psi = 0.1$, $\varphi = 0.1$, $\omega = 0$. Solide lines: $\pi = 1.65$. Dashed lines: $\pi = 14$.

- **Figure 10**: $\alpha = .3$, $\gamma = 1$, $\varepsilon = .2$, $\phi = .2$, $\tau = 0$, $n = .005$, $\beta = 0.77$, $\theta = 1.75$, $\psi = 0.1$, $\varphi = 0.1$, $\omega = 0$, $\pi = 1.65$.

D Formulas for risky debts

During a bubble episode like the one specified in Equation (30), credit contracts are subject to default risk. In particular, a fraction ω of them are defaulted upon if the bubbly episode ends. Thus, if $z_t = B$, the ex-post returns to a portfolio of short-real, short-nominal, long-real and long-nominal credit contracts are respectively given by:

$$R_{t+1}^{1R} = \begin{cases} \dfrac{1}{1-\varphi\cdot\omega}\cdot R_{t+1} & \text{if } z_{t+1} = B \\ \dfrac{1-\omega}{1-\varphi\cdot\omega}\cdot R_{t+1} & \text{if } z_{t+1} = F \end{cases},$$

$$R_{t+1}^{1N} = \begin{cases} \dfrac{\pi_{t+1}^{-1}}{E_t\pi_{t+1}^{-1}}\cdot\dfrac{1}{1-\varphi\cdot\omega}\cdot R_{t+1} & \text{if } z_{t+1} = B \\ \dfrac{\pi_{t+1}^{-1}}{E_t\pi_{t+1}^{-1}}\cdot\dfrac{1-\omega}{1-\varphi\cdot\omega}\cdot R_{t+1} & \text{if } z_{t+1} = F \end{cases},$$

$$R_{t+1}^{2R} = \begin{cases} \dfrac{R_{t+2}^{-1}}{E_t R_{t+2}^{-1}}\cdot\dfrac{1}{1-\varphi\cdot\omega}\cdot R_{t+1} & z_{t+1} = B \\ \dfrac{R_{t+2}^{-1}}{E_t R_{t+2}^{-1}}\cdot\dfrac{1-\omega}{1-\varphi\cdot\omega}\cdot R_{t+1} & \text{if } z_{t+1} = F \end{cases},$$

$$R_{t+1}^{2N} = \begin{cases} \dfrac{\pi_{t+1}^{-1}\cdot E_{t+1}\pi_{t+2}^{-1}\cdot R_{t+2}^{-1}}{E_t\left\{\pi_{t+1}^{-1}\cdot\pi_{t+2}^{-1}\cdot R_{t+2}^{-1}\right\}}\cdot\dfrac{1}{1-\varphi\cdot\omega}\cdot R_{t+1} & \text{if } z_{t+1} = B \\ \dfrac{\pi_{t+1}^{-1}\cdot E_{t+1}\pi_{t+2}^{-1}\cdot R_{t+2}^{-1}}{E_t\left\{\pi_{t+1}^{-1}\cdot\pi_{t+2}^{-1}\cdot R_{t+2}^{-1}\right\}}\cdot\dfrac{1}{1-\varphi\cdot\omega}\cdot R_{t+1} & \text{if } z_{t+1} = F \end{cases}.$$

E The wealth effect of monetary policy

This appendix illustrates how monetary policy can also have wealth effects by affecting bubble creation. To do so, we return to the ongoing example of Section 3 but allow for expectations regarding bubble creation to be set in nominal terms. A simple way to do so is to assume that the market expects bubble creation in each period to be proportional to the pre-existing credit bubble. In particular, we can modify our example by replacing Equation (30) with:

$$n_t = \begin{cases} 0+\eta\cdot b_t & \text{if } z_t = F \\ n+\eta\cdot b_t & \text{if } z_t = B, \end{cases}. \tag{E.1}$$

where η is a constant. When η is positive, bubble creation grows alongside the pre-existing bubble, i.e., the value of new credit contracts increases with the value of pre-existing credit contracts. When η is negative, bubble creation instead falls when the size of the pre-existing bubble increases, and vice-versa. This could be interpreted as a situation in which agents expect the crash of existing bubbles to 'make room' for the creation of new ones. How does this change the analysis of the

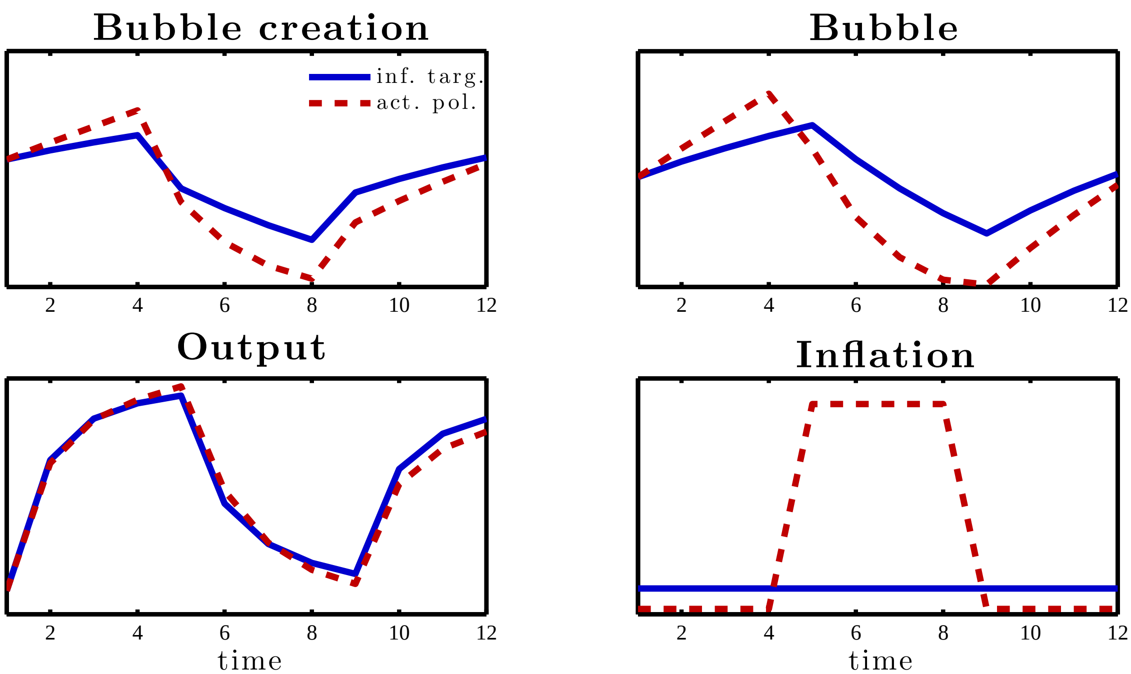


Figure 13: Wealth effects of monetary policy.

previous sections?

Assume first that $\eta > 0$. As before, a positive inflation surprise reduces the value of nominal bubbles thereby undermining the debt overhang effect. Unlike before, however, such an inflation surprise also reduces the real value of new bubbles and therefore undermines the wealth effect. The overall effect of inflation surprises on investment therefore depends on the relative strength of both effects, i.e., on the value of η. Figure 14 shows the behavior of our economy both under a constant inflation target and a stabilization policy like the one described in Equation (31). The figure assumes a relatively high value of η, so that it is the effect of monetary policy on bubble creation that ultimately prevails. This is the reason why, differently from Figure 7, the policy now destabilizes both the bubble and output. During bubbly episodes, the low rate of inflation prescribed by the policy induces yet more bubble creation, raising investment and output even further. The counterpart of this, naturally, is that the policy leads to an even lower output in the fundamental states by inflating away the value of bubble creation.

If it is assumed instead that $\eta < 0$, monetary policy affects the debt overhang and wealth effects in opposite direction, strengthening one whenever it undermines the other. This boosts the ability of policy to stabilize output. The simple policy rule outlined in Equation (31), for instance, now reduces output during bubbly episodes for two reasons. As before, the low rate of inflation raises the value of pre-existing bubbles and thus the debt overhang effect; unlike before, however, the growth of pre-existing bubbles is associated to a decline in bubble creation that reduces output even further. In the fundamental state, by contrast, the high rate of inflation prescribed by the policy has an expansionary effect on investment both because it undermines the debt overhang effect and because it strengthens the wealth effect.

As we discuss in the main body of the paper, this example underscores the complex effects of monetary policy in the bubbly world. Section 3 analyzed how these effects are shaped by the mix of real, nominal, short- and long-term contracts. This appendix explores another dimension along which the mix of credit contracts is crucial: namely, old vs. new. If the market expects pre-existing credit contracts to fuel the creation of new ones, we have seen that the effects of monetary policy through the debt overhang and wealth effects are in conflict with one another. If instead the market expects pre-existing contracts to feed on the creation of new ones, the effects of monetary policy through the debt overhang and wealth effects reinforce one another.

F Other forms of public liabilities

We have assumed that the money supply is the only form of government liability. We now show that our results extend to the more plausible setting with government debt (or other forms of interest bearing government liabilities). Indeed, the model with government debt can help us understand why in recent years we have seen a dramatic expansion in *both* the stock of the money supply and of government debt without an accompanying increase in interest rates (see Figure 12).

The government's budget constraint is now given by:

$$x_t = \tau \cdot (1-\alpha) \cdot k_t^{\alpha} + m_t - \frac{m_{t-1}}{\gamma \cdot \pi_t} + d_t - \frac{R_t^d}{\gamma} \cdot d_{t-1},$$

where $d_t \equiv \gamma^{-t} \frac{D_t}{p_t}$ is the real value of nominal bonds D_t supplied by the government and R_t^d is their gross return. These bonds are unindexed to inflation and no arbitrage implies that the nominal return on these bonds must equal the nominal interest rate, i.e., $R_t^d = \pi_t^{-1} \cdot R_t^N$ where $R_t^N = \dfrac{R_t}{E_{t-1}\left\{\pi_t^{-1}\right\}}$.

When the economy is in a liquidity trap, both money and debt pay a nominal return of 1. These assets become perfect substitutes and their total demand is:

$$m_t + d_t = \nu + \frac{\beta^{\theta}}{\beta^{\theta} + E_t\left\{\pi_{t+1}^{-1}\right\}^{1-\theta}} \cdot (1-\varepsilon) \cdot (1-\alpha)\, k_t^{\alpha} - b_t - n_t - \frac{\gamma \cdot \phi \cdot k_{t+1}}{E_t\left\{\pi_{t+1}^{-1}\right\}}.$$

In fact, the equilibrium allocations with government debt are identical to those of an economy in which real money balances are modified to $\widehat{m}_t = m_t + d_t$. Note that we also have a neutrality result for open market operations: a policy that increases (decreases) the supply of money by decreasing (increasing) the supply of government debt has no effect on the equilibrium.

When the economy is in normal times, where the demand for money balances is already at its minimum, an increase in government debt does not crowd out money at all; it crowds out credit and productive investment. In fact, the only modification to the equilibrium law of motion is that the credit market clearing interest rate is now given by:

$$R_{t+1} = \frac{\gamma \cdot (\phi \cdot k_{t+1} + E_t \{b_{t+1}\})}{\frac{\beta^\theta}{\beta^\theta + R_{t+1}^{1-\theta}} \cdot (1-\varepsilon) \cdot (1-\tau) \cdot (1-\alpha) \cdot k_t^\alpha - d_t}, \tag{F.1}$$

and therefore is increasing in the supply of government debt d_t.

G Recreating a bubble through unconventional policy

Consider the bubble process of Equation (30) in which $\psi = 0$ and bubble creation is constant and equal to n^*. Assuming that it is outside of the liquidity trap, the steady state of the economy can be summarized with the following three equations:

$$n^* = (\gamma - R^*) \cdot b^* \tag{G.1}$$

$$\gamma \cdot k^* = \frac{R^*}{R^* - \phi} \cdot (\varepsilon \cdot (1-\alpha) \cdot k^{*\alpha} + n^*) \tag{G.2}$$

$$R^* = \frac{\gamma \cdot (\phi \cdot k^* + b^*)}{\frac{\beta^\theta}{\beta^\theta + (R^*)^{1-\theta}} \cdot (1-\tau) \cdot (1-\varepsilon) \cdot (1-\alpha)\, k^{*\alpha}}. \tag{G.3}$$

Suppose however that the economy is in the fundamental state, so that it converges to the steady state given by the previous three equations assuming $n^* = b^* = 0$. The question that we ask ourselves is whether monetary authority could implement a monetary policy, accompanied by the appropriate asset purchase scheme, that would replicate the same steady-state allocation that would arise under a constant rate of bubble creation $n_t = n^*$. The answer is affirmative. All it must do is set $\pi = R^{*-1}$ and $m^* = \gamma \cdot b^* = \left(1 - \frac{1}{\gamma\pi}\right)^{-1} \cdot n^*$. It is easy to verify that, in this case, the economy is in a liquidity trap in steady state and the seigniorage revenue is exactly equal to the desired bubble creation, i.e.,

$$v^* = \left(1 - \frac{1}{\gamma\pi}\right) \cdot m^* = n^*. \tag{G.4}$$

Insofar as this seigniorage revenue is transferred to entrepreneurs via an asset purchase scheme like the one analyzed in Section 4, it follows that the steady state capital stock is exactly equal to k^* as characterized in Equation (G.2).

H Data sources

- **Federal Funds rate**: Board of Governors of the Federal Reserve System (US), Effective Federal Funds Rate, https://fred.stlouisfed.org/series/DFF.
- **3 months Treasury rate**: Board of Governors of the Federal Reserve System (US), 3-Month Treasury Constant Maturity Rate, https://fred.stlouisfed.org/series/DGS3MO
- **10 years Treasury rate**: Board of Governors of the Federal Reserve System (US), 10-Year Treasury Constant Maturity Rate, https://fred.stlouisfed.org/series/DGS10.
- **Gross domestic product**: US. Bureau of Economic Analysis, Gross Domestic Product, https://fred.stlouisfed.org/series/GDP.
- **Households net worth**: Board of Governors of the Federal Reserve System (US), Households and nonprofit organizations; net worth, Level, https://fred.stlouisfed.org/series/HNONWRA027N.
- **Consumer Price Index**: US. Bureau of Labor Statistics, Consumer Price Index for All Urban Consumers: All Items, https://fred.stlouisfed.org/series/CPIAUCSL
- **Investment**: Organization for Economic Co-operation and Development, Gross Fixed Capital Formation in United States, https://fred.stlouisfed.org/series/USAGFCFQDSMEI.
- **Business credit**: Federal Reserve Financial Accounts of the US, Nonfinancial business; debt securities and loans; liability.
- **Monetary base**: Federal Reserve Bank of St. Louis, St. Louis Adjusted Monetary Base, https://fred.stlouisfed.org/series/BASE.
- **Public debt**: US. Office of Management and Budget, Federal Debt: Total Public Debt as Percent of Gross Domestic Product, https://fred.stlouisfed.org/series/GFDEGDQ188S.

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