



A model of collateral, investment, and adverse selection[☆]

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Abstract

This paper characterizes the relationship between entrepreneurial wealth and aggregate investment under adverse selection. Its main finding is that such a relationship need not be monotonic. In particular, three results emerge from the analysis: (i) pooling equilibria, in which investment is independent of entrepreneurial wealth, are more likely to arise when entrepreneurial wealth is relatively low; (ii) separating equilibria, in which investment is increasing in entrepreneurial wealth, are most likely to arise when entrepreneurial wealth is relatively high and; (iii) for a given interest rate, an increase in entrepreneurial wealth may generate a discontinuous fall in investment.

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1. Introduction

Consider an economy in which entrepreneurs need to borrow funds in order to take advantage of investment opportunities. In most cases, such borrowing is characterized by some degree of asymmetric information. The lender, for example, might not be able to fully assess important

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characteristics of the borrower. Or the borrower's actions might not be fully observable. In these situations, if he is to break even, the lender will need to design contracts that provide proper incentives for the borrower. In most existing models, incentives are in part provided by limiting the amount of lending in accordance with the borrower's net worth. The prevailing view that emerges from these models is that, whenever they are constrained because of informational frictions, investment and credit must be increasing in entrepreneurial wealth.

This paper shows that the prevailing view does not necessarily apply to environments of adverse selection. To this end, I develop a simple model of credit markets with adverse selection and fully characterize the relationship between aggregate investment and entrepreneurial wealth. It is shown that, even when aggregate investment is constrained due to the presence of adverse selection, it need not be monotonically increasing in entrepreneurial wealth.

In my environment, entrepreneurs need to borrow funds in order to finance their investment opportunities, on which they possess private information. Financial intermediaries seek to mitigate the asymmetry of information by offering a menu of contracts. More specifically, they try to screen entrepreneurs through the amounts of collateral that they provide and of investment that they undertake. Depending on the level of entrepreneurial wealth, it is shown that the credit market equilibrium may either entail pooling, so that all entrepreneurs borrow indistinctly at the same terms, or separation, so that different entrepreneurs borrow at different rates of collateralization, pay different rates of interest, and undertake different levels of investment. I show that the pooling equilibrium, in which investment is independent of entrepreneurial wealth, is more likely to arise when the latter is low relative to the desired level of investment. The separating equilibrium, in which investment is increasing in entrepreneurial wealth, is most likely to arise when the latter is high in relation to the desired level of investment. Moreover, I also show that for a given interest rate, increases in entrepreneurial wealth may lead to a contraction in aggregate investment by inducing the economy to switch from a pooling to a separating equilibrium.

The intuition behind these results is as follows. When entrepreneurial wealth is low relative to the desired level of investment, screening is relatively costly in my economy. Indeed, since collateral is scarce in this case, screening must be predominantly done by restricting the amount of investment undertaken by the "good" entrepreneurs. Hence, there is a strong tendency to pool all projects and have good entrepreneurs cross-subsidize their bad counterparts. Whereas cross-subsidization implies that the pooling equilibrium is costly for good entrepreneurs, it also benefits them by allowing them to expand their investment. In such a pooling equilibrium, investment is independent of entrepreneurial wealth because the marginal unit borrowed by good entrepreneurs is always fully cross-subsidized: hence, at the margin, the cost of borrowing is constant for them. As entrepreneurial wealth increases, though, the screening possibilities of intermediaries are enhanced. In particular, since screening can be increasingly done through collateralization requirements, its cost decreases. Consequently, intermediaries can eventually design profitable contracts tailored to attract the most productive entrepreneurs from the pool by asking them to provide greater quantities of collateral. The resulting equilibrium thus entails separation between different types of entrepreneurs.

As the previous analysis suggests, then, increases in entrepreneurial wealth might induce a switch from a pooling to a separating equilibrium in the credit market. What happens to aggregate investment when there is such a switch in regime? I show that, provided that the average quality of investment in the economy is above a certain threshold, a switch of regime will lead to a fall in the investment undertaken by all entrepreneurs. This must clearly be the case for bad entrepreneurs, who are cross-subsidized in the pooling equilibrium and therefore overinvest relative to their efficient level of investment. As for good entrepreneurs, the fall in their investment can be best understood

by noting that—at the switching point—they are by definition indifferent between the pooling and the separating contracts: the latter, though, entails a lower cost of funds because it does not require cross-subsidization. Good entrepreneurs can therefore only be indifferent between both contracts if the pooling entails a relatively higher level of investment. In this manner, increases in entrepreneurial wealth that induce regime switching in the credit market equilibrium will lead to a fall in aggregate investment.

This paper adopts a static approach and seeks to characterize the relationship between entrepreneurial wealth and investment in the presence of adverse selection. The finding that this relationship may be non-monotonic, though, has clear dynamic implications. In a companion paper, I explore these implications and show how, in a dynamic setting, the type of adverse selection studied here may generate fluctuations even in the absence of exogenous perturbations.¹

My environment is related to the work by Bester [2,3], de Meza and Webb [6], and Besanko and Thakor [1]. Bester analyzed the role of collateral for screening in environments of adverse selection with indivisibilities in investment. In de Meza and Webb and Besanko and Thakor, adverse selection leads to overinvestment. Between the two, the environment studied by Besanko and Thakor is closest to mine, with two important differences. The first is that, under their assumptions, good entrepreneurs are screened by being forced to overinvest relative to the efficient level of investment: as the wealth of entrepreneurs increases, then, their investment decreases towards the efficient amount. In this sense, their model also generates a decrease in investment in response to an increase in entrepreneurial wealth. In my model, though, this happens even though good entrepreneurs are constrained relative to their efficient level of investment. A second important distinction between my model and that of Besanko and Thakor refers to the existence of a pooling equilibrium. As is well known, screening environments à la Rothschild and Stiglitz [14] pose a problem for the existence of equilibrium whenever the pooling allocation Pareto dominates its separating counterparts. Whereas Besanko and Thakor deal with this problem by using the concept of reactionary equilibrium, I am able to focus on sequential Nash equilibria. I do so by following Hellwig [9] in modeling competition in the credit market as a three-stage game. Hellwig's specification always has an equilibrium: in particular, among the feasible allocations that satisfy the zero-profit condition for banks, the one that yields the highest profits to good entrepreneurs—be it pooling or separating—emerges as the competitive equilibrium of my economy.²

In Section 2, I present the basic feature of the environments. Section 3 contains a complete characterization of separating and pooling equilibria. Section 4 characterizes regime switches and discusses their impact on aggregate investment. Finally, Section 5 concludes.

2. The environment

Assume an economy that lasts for two periods, indexed by $t \in \{0, 1\}$, that I refer to as Today and Tomorrow. This economy is populated by a continuum of savers and entrepreneurs. Savers play no role in this model besides being providers of funds: in this sense, they are assumed to have a vast endowment of the economy's only consumption good Today, which they are willing to lend inelastically at the fixed gross interest factor equal to $(1 + r)$.

¹ See Martin [11].

² In a related context, Dell'Aricia and Marquez [5] use this equilibrium concept to analyze how competition among banks is affected by the degree of private information that they have on the creditworthiness of borrowers.

Entrepreneurs are central to my story. They are risk-neutral and have monotonic preferences over the economy's only consumption good Tomorrow, although their only endowment is in terms of the consumption good Today. More importantly, they are endowed with a decreasing returns to scale technology for transforming consumption goods Today into consumption goods Tomorrow. The fact that this technology can be operated solely by them, though, means that it is potentially subject to informational frictions. Assumptions on technology are as follows:

Assumption 1 (Technology). Entrepreneurs, which are uniformly distributed in the interval $[0, 1]$, may be either of type B ("Bad") or G ("Good") depending on their technology. Entrepreneurs of each type are distributed over intervals of length λ^j , $j \in \{B, G\}$, where $\lambda^G + \lambda^B = 1$. An entrepreneur of type j has a successful (unsuccessful) state tomorrow with probability p^j ($1 - p^j$), where $p^G > p^B$. If successful (unsuccessful), an entrepreneur of type j who invests I units of the consumption good Today obtains a gross return of $\alpha^j f(I)$ (zero) Tomorrow, where $\alpha^G < \alpha^B$ and $p^G \alpha^G > p^B \alpha^B$. It is assumed that $f(\cdot)$ is increasing, concave, and satisfies Inada conditions.

These technological assumptions are similar to those commonly used in the credit rationing literature, namely second-order stochastic dominance. The only difference is that, in my setup, the bad technology is not just a mean-preserving spread of its good counterpart but it actually has a lower expected return. This latter assumption allows technologies to be unambiguously ranked.

Entrepreneurs are endowed with an amount W of the consumption good Today, which they cannot use to finance their production.³ In order to do so they need to borrow funds, which they do indirectly through banks. There exists a finite number of banks that collect deposits from savers and from entrepreneurs and offer loan contracts to entrepreneurs. Banks are assumed to be risk neutral and competitive. On the deposit side, they take the gross interest factor on deposits $(1 + r)$ as given, and they Nash compete on the loan market by designing contracts that take the following form:

Assumption 2 (Loan contracts). Entrepreneurs and banks sign a contract of the form (I, R, c) , where I is the amount borrowed and invested, R is the interest factor on the loan and c is the percentage of the loan that entrepreneurs must collateralize by using their own wealth. In the event of a successful state, entrepreneurs pay back the amount borrowed adjusted by the interest factor: otherwise, they default and the bank keeps the goods put up as collateral, the interest borne by them, and the residual value of the project. Finally, and since they cannot invest it directly in the project, entrepreneurs deposit their endowment in the bank for a gross interest factor of $(1 + r)$. This implies that the expected profit that a j -type entrepreneur obtains from loan contract (I, R, c) is given by

$$\pi^j(I, R, c) = p^j[\alpha^j f(I) - RI] - (1 + r)[(1 - p^j)cI - W]. \quad (1)$$

There are two features of the contracts described in Assumption 2 that I wish to highlight. The first is that these contracts are multidimensional in their instruments. Indeed, it will be shown that both the loan size and the rate of collateralization are used to screen between good and bad

³ This assumption is introduced to simplify the exposition, but it is not restrictive. The contracts that I study, which entail collateral, could be rewritten as contracts in which entrepreneurs invest in their projects and banks provide additional funds. I adopt the former characterization because it makes contracts easier to analyze, it delivers a more tractable framework, and it encompasses the cases in which entrepreneurial wealth can or cannot be directly invested.

entrepreneurs.⁴ A second consideration is that, in principle, the rate of collateralization could either be positive or negative in this environment. Negative rates of collateralization, though, which allow for payments from banks to entrepreneurs in the event of failure, are not useful for screening purposes. On the contrary, they make contracts seem relatively more appealing to bad entrepreneurs, who have a higher probability of failure and are therefore more likely to collect any negative collateral. I therefore restrict myself, without loss of generality, to contracts in which $c \geq 0$.

I follow the adverse selection literature in making two assumptions regarding bank competition. The first is a condition of no cross-subsidization, by which banks are not allowed to offer contracts that lose money in expectation. The second assumption is that of exclusivity, by which entrepreneurs can apply to at most one of the contracts offered. This assumption implies that entrepreneurs borrow only from one bank, implicitly assuming that banks can monitor contract applications made by entrepreneurs. It is also assumed that each bank gets the same share of total deposits and, if they design the same contract, they get the same share and composition of loan applications. Given Assumption 2, a bank's expected profit of accepting an application to a contract (I, R, c) from a type- j entrepreneur are given by

$$p^j(RI) + (1 - p^j)(1 + r)cI - (1 + r)I. \quad (2)$$

3. Characterization of loan contracts

In the absence of asymmetric information, the equilibrium of my economy is trivial. Letting $\{(I^{B^*}, R^{B^*}, c^{B^*}), (I^{G^*}, R^{G^*}, c^{G^*})\}$ denote the equilibrium contracts under full information, it is straightforward to verify that they satisfy

$$\left. \begin{aligned} f'(I^{j^*}) &= \frac{1+r}{\alpha^j p^j}, \\ p^j \cdot R^{j^*} + (1 - p^j)(1+r)c^{j^*} &= 1+r \end{aligned} \right\} \text{ for } j \in \{G, B\}.$$

Hence, under full information, good entrepreneurs invest more than bad ones and banks break even in both contracts. Investment is independent of entrepreneurial wealth W : if entrepreneurs have no wealth, they simply repay everything in the event of success by setting $R^{j^*} = (1+r)/p^j$ for $j \in \{G, B\}$.⁵

Now consider the case of asymmetric information, in which banks are not able to distinguish among different types of borrowers. As in Besanko and Thakor [1] and Reichlin and Siconolfi [12], it is assumed that borrowers' types cannot be observed either directly or through realized project returns. Hence, all agents other than the owner of the project can only verify whether the latter was successful or not. In such a scenario it is known that the optimal contractual form is that of debt as assumed in Assumption 2.⁶

⁴ The remaining dimension of the contract (R) will be pinned down in equilibrium by the zero-profit condition of banks.

⁵ To be precise, the economy under full information displays many equilibria. Indeed, banks are indifferent between making entrepreneurs pay only in the event of success (i.e., setting $c^j = 0$) and making them pay partially in the event of success and partially in the event of failure (i.e., setting $c^j > 0$). All of these equilibria entail the same level of investment and are equivalent in terms of efficiency.

⁶ In a more general environment, Boyd and Smith [4] show that debt can arise as the optimal contractual form under adverse selection and costly state verification provided that verification costs are sufficiently high.

Under asymmetric information, the environment studied is analogous to the Rothschild–Stiglitz model of insurance. In the latter, an equilibrium does not always exist: in particular, it fails to do so when the pooling allocation is Pareto superior to the separating allocation. To avoid this problem, I follow Hellwig [9] and model competition in the credit market as having three stages. In the first stage, banks design contracts; in the second stage, entrepreneurs apply for these contracts and; in the third stage, banks decide whether to accept or reject these applications.

Hellwig applies the concept of sequential equilibrium to this game and shows that an equilibrium always exists: in particular, the specification allows for the existence of pooling equilibria when the values of the parameters prevent the existence of separating equilibria in Rothschild–Stiglitz games.⁷ More interestingly, an application of the Kohlberg–Mertens stability criterion selects only the allocation most preferred by good entrepreneurs as the equilibrium of the model. In other words, the most robust outcome of the aforementioned game form will be the separating contracts insofar as the latter provide good entrepreneurs with higher profits than any pooling contracts. On the contrary, if there are pooling contracts that are Pareto superior to the separating contract, the one mostly preferred by good entrepreneurs will emerge as the most robust equilibrium of the model.⁸

In what follows, I analyze the equilibrium contracts for an economy indexed by a net interest rate-entrepreneurial wealth pair (r, W) . I first characterize the separating equilibrium: as will be seen, the interesting feature of these contracts is that the size of loans and the rate of collateralization are both used as screening devices. When there is no wealth to be used as collateral, the whole weight of screening is borne by the size of loans, and the investment undertaken by good entrepreneurs is constrained with respect to the full-information benchmark. This constraint is relaxed as the relative wealth of entrepreneurs increases, making it possible to screen more through collateral and less through loan size. I then characterize pooling equilibria and show that collateral also plays an important role in determining them.

3.1. Separating equilibria

Under the assumptions of exclusivity and no cross-subsidization, a separating equilibrium is defined as follows:

Definition 1. For a given interest rate-entrepreneurial wealth pair (r, W) , a separating equilibrium is a set of contracts $C^{SEP}(r, W) = \{(I^B, R^B, c^B), (I^G, R^G, c^G)\}$ satisfying the following conditions:

1. Feasibility: contracts must respect the collateralization constraint,

$$c^j \in \left[0, \frac{W}{I^j}\right] \quad \text{for } j \in \{B, G\}. \quad (3)$$

⁷ Alternatively, the existence problem could be avoided by explicitly modeling competitive contract markets in which banks and entrepreneurs interact: such a setting always has an equilibrium (Gale [8], Geanakoplos and Dubey [7]), which in particular may entail pooling (Gale [8], Martin [10]).

⁸ The intuition is that, by adding a third stage in which applications may be rejected, banks are effectively able to anticipate the effects of their deviations. Hence, banks understand that, if they were to destroy the pooling equilibrium, pooling contracts would no longer be profitable and applications to these contracts would be rejected in equilibrium: but this, in turn, means that a deviating bank would ultimately attract bad as well as good borrowers. For a discussion of Hellwig's characterization, see Riley [13].

2. Incentive compatibility: each entrepreneur applies to the contract designed for his type,

$$\pi^j(I^j, R^j, c^j) \geq \pi^j(I^i, R^i, c^i) \quad \text{for } i \neq j, i, j \in \{B, G\}. \tag{4}$$

3. Zero-profit condition for banks: each contract must yield banks zero profits in expectation,

$$1 + r = p^j R^j + (1 - p^j)(1 + r)c^j \quad \text{for } j \in \{B, G\}. \tag{5}$$

4. No bank can profit by offering alternative contracts.

Eqs. (3)–(5) are standard: note simply that Eq. (5) stems from bank competition together with the no cross-subsidization condition. Clearly, since banks compete to attract good entrepreneurs, a separating equilibrium requires that the profits of these entrepreneurs are maximized subject to Eqs. (3)–(5). The resulting contracts are characterized in the following proposition.

Proposition 1. *Given (r, W) , the separating equilibrium is characterized by a pair of contracts $C^{SEP}(r, W) = \{(I^B, R^B, c^B), (I^G, R^G, c^G)\}$ satisfying,*

$$(I^B, R^B, c^B) = \left(I^{B*}, \frac{1+r}{p^B}, 0 \right), \tag{6}$$

$$\alpha^G p^G f'(I^G) > (1+r) \Rightarrow c^G = \frac{W}{I^G}, \tag{7}$$

and

$$c^G = \frac{[\alpha^B p^B f(I^G) - \frac{p^B}{p^G} I^G (1+r)] - [\alpha^B p^B f(I^B) - I^B (1+r)]}{(1 - \frac{p^B}{p^G}) I^G (1+r)} \leq 1. \tag{8}$$

Proof. See Section A.1.1 in Appendix A. \square

Eq. (6) implies that equilibrium contracts taken by bad entrepreneurs entail no distortions. Thus, they are lent the efficient amount at the given interest rate and they have no need to provide collateral. It is therefore on the contracts taken by good entrepreneurs that the interest of the equilibrium lies, since they must be incentive compatible. What are the properties of these contracts?

If collateral is scarce and the incentive compatibility constraint binds, Eq. (7) implies that good entrepreneurs will be rationed with respect to the full-information allocation. Hence, they will receive smaller loans than they would desire at the prevailing interest rate. The use of loan sizes to screen entrepreneurs is costly, though, whereas the use of collateral—to the extent that it is available—is not. This is because different types of entrepreneurs differ in their willingness to accept a higher interest factor in exchange for a lower collateral requirement. As long as the constraint in Eq. (3) is slack, banks can always decrease R^G and increase c^G while keeping the expected profit of the contract unchanged for good entrepreneurs: such a modification, though, increases the cost of the contract for bad entrepreneurs because they stand to lose their collateral more often.⁹

⁹ In fact, it can be easily verified that—for any level of r —the marginal rate of substitution between the interest factor R and the collateral requirement c is equal to $[-(1+r)(1-p^j)/p^j]$ for an entrepreneur of type j .

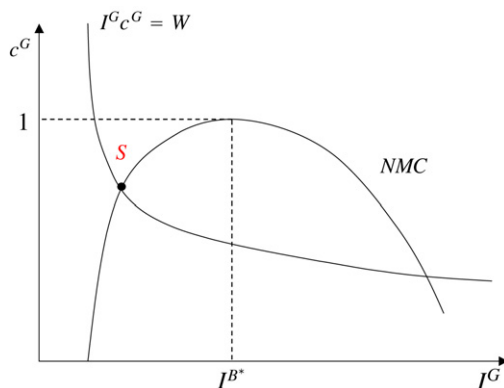


Fig. 1.

As I mentioned earlier, then, the design of the separating contracts reduces to a problem of multidimensional screening, in which the loan size and the rate of collateralization are used in order to induce separation between different technologies. In order to provide a simple graphical interpretation of Proposition 1, I define a “no mimicry constraint” (NMC) as the set of G -type contracts that satisfy Eqs. (4) and (5). Fig. 1 depicts the NMC in the (I^G, c^G) space, which is non-monotonic and is maximized when $I^G = I^{B*}$.

For low levels of c^G , it must be the case that $I^G \neq I^{B*}$: this is so because when collateral is relatively scarce, screening must be done through investment. In such a scenario, the only way to discourage bad entrepreneurs from applying to the G -type contracts is by restricting or by expanding the amount of investment that they must undertake relative to their efficient level. For higher rates of collateralization, though, there is less of a need to screen through investment and the incentive-compatible levels of I^G therefore draw closer to I^{B*} . When both loan sizes are equal, investment is no longer used for screening and the full weight of the separation must fall on the rate of collateralization. Hence, the latter is maximized at this point, at which it reaches one.

Of course, not all of the contracts on the NMC can be implemented at equilibrium: in order for this to be possible, contracts must also satisfy the collateralization constraint. Graphically, this means that the equilibrium G -type contract must lie at an intersection of the NMC and the collateralization constraint. Although there is a pair of such contracts for any given level of wealth W , competition among banks will select the one that maximizes the profits of good entrepreneurs as an equilibrium outcome. Under my assumptions, this is the contract entailing a lower loan size and a higher rate of collateralization, as depicted by point S in Fig. 1.¹⁰

This result, by which good entrepreneurs are screened in equilibrium by restricting their loan size, closely resembles the traditional Rothschild–Stiglitz result. Nonetheless, I believe that it is particularly surprising in the context of my production economy. In the complete absence of collateral, it implies that the safer, more productive technology will be constrained not only with respect to the efficient allocation, but even with respect to its riskier, less productive counterpart.

¹⁰ Intuitively, both contracts entail the same total level of collateral. Hence, bad entrepreneurs can only be indifferent between the two if the difference in output implied by them equals the difference in repayments in the event of success. However, if this is the case, good entrepreneurs prefer the one with the lower loan size: this stems from the fact that their marginal rate of substitution between I and R is lower than that of bad entrepreneurs.

3.2. Pooling equilibria

As was mentioned earlier, Hellwig’s three-stage characterization of competition in the credit market allows for the existence of a pooling equilibrium when the latter Pareto dominates the separating allocation. A pooling equilibrium is defined as follows,

Definition 2. For a given interest rate-entrepreneurial wealth pair (r, W) , a pooling equilibrium is a contract $C^{POOL}(r, W) = \{(\bar{I}, \bar{R}, \bar{c})\}$ satisfying the following conditions:

1. Feasibility: the pooling contract must respect the collateralization constraint.
2. Zero-profit condition for banks: when offered to a pool of applicants representative of the population, the contract must yield banks zero profits in expectation.
3. No bank can profit by offering alternative contracts.

In traditional models of signaling, the existence of a pooling equilibrium entails cross-subsidies from good types to bad types: this is also true in my framework, although the extent of such transfers ultimately depends on the wealth of entrepreneurs and on the way in which payments are divided between the interest factor (\bar{R}) and the rate of collateralization (\bar{c}). As the following proposition shows, any pooling equilibrium will entail a binding collateralization constraint, although the amount of investment will be independent of entrepreneurial wealth.

Proposition 2. Given (r, W) , a pooling equilibrium is given by a contract $C^{POOL}(r, W) = \{(\bar{I}, \bar{R}, \bar{c})\}$ satisfying,

$$p^G \alpha^G f'(\bar{I}) = \frac{p^G}{\bar{p}}(1 + r), \tag{9}$$

$$\bar{R} = (1 + r) \left[\frac{1 - (1 - \bar{p})\bar{c}}{\bar{p}} \right], \tag{10}$$

$$\bar{c} = \frac{W}{\bar{I}}, \tag{11}$$

where $\bar{p} = \lambda^G p^G + \lambda^B p^B$ denotes the average probability of success of all projects in the economy.

Proof. See Section A.1.2 in Appendix A. □

Eq. (11) implies that a pooling equilibrium must entail a binding collateralization constraint and, consequently, that the degree of cross-subsidization it exhibits depends on entrepreneurial wealth. Higher rates of collateralization decrease the average cost of funds for good entrepreneurs and hence increase their profits. Since the equilibrium pooling contract will be the one that maximizes the profits of these entrepreneurs while yielding zero profits to banks, Eq. (11) must hold at equilibrium.

The same reasoning applies for the level of investment in a pooling equilibrium as defined implicitly in Eq. (9). When $W = 0$, the condition must clearly be satisfied since it simply equates the marginal productivity of investment of good entrepreneurs to their marginal cost of funds. As entrepreneurial wealth increases, however, so does the rate of collateralization of the pooling contract: in doing so, the average cost of funds decreases and profits increase for good entrepreneurs.

The size of the loan, however, remains fixed, since the cost of the marginal unit borrowed by these entrepreneurs is always equal to $[p^G/\bar{p}](1+r)$ and therefore Eq. (9) must hold.

A final consideration will prove useful in order to better understand the pooling equilibrium: both the pooling loan size (\bar{I}) and the size of loans given to bad entrepreneurs in the separating equilibrium (I^{B*}) are independent of entrepreneurial wealth. In fact, their relative magnitude depends only on the value of \bar{p} , and simple arithmetic yields the following remark.

Remark 1. For any interest rate r , it is the case that

$$\bar{I}(r, \bar{p}) \leq I^{B*}(r) \iff \bar{p} \leq \frac{\alpha^B p^B}{\alpha^G} \tag{12}$$

where \bar{I} , I^{B*} and \bar{p} are defined as above.

Thus, if the ratio of good to bad entrepreneurs in the economy is high enough to make \bar{p} surpass the threshold in Eq. (12), the pooling allocation will entail overinvestment of bad entrepreneurs relative to the separating equilibrium.

4. Regime switches and investment

Let $C^{EQ}(r, W)$ denote the equilibrium contracts for an economy indexed by (r, W) : will these contracts be pooling or separating? As discussed before, the answer to this question depends on the profits that each type of contract yields good entrepreneurs.¹¹ And this, in turn, depends on the level of entrepreneurial wealth. Consider, for example, the case of pooling contracts. If they are ever to arise in equilibrium, they will do so when separating contracts yield relatively low profits: this occurs when entrepreneurial wealth and—consequently—collateral are low. It therefore seems natural to suppose that there is a critical level of entrepreneurial wealth or “switching point” that determines—for a given interest rate—a change in regime from pooling to separating or vice-versa: let $W^*(r)$ denote such switching points.

I proceed as follows: first, I restrict parameter values for which all economies indexed by $(r, 0)$ display a pooling equilibrium, i.e., for which the latter arises in the absence of collateralizable wealth. I then prove that these same restrictions guarantee that the mapping $W^*(r)$ is a function, so that there is a unique switching point for each value of the interest rate.

Lemma 1. *If $\bar{p} \geq \frac{\alpha^B p^B}{\alpha^G}$ then $C^{EQ}(r, 0) = C^{POOL}(r, 0)$ for all values of r .*

Proof. See Section A.2 in Appendix A. \square

Lemma 1 determines a threshold value of \bar{p} above which the equilibrium is always pooling when $W = 0$. From Eq. (12), this threshold makes it possible to relate loan sizes in the pooling and separating equilibria: in particular, when the pooling loan size is weakly larger than the B -type loan size in the separating contracts, an economy with no collateral will pool all loans regardless of the interest rate. The threshold value of \bar{p} contained in Lemma 1 also guarantees the existence of a unique switching point for each level of r , as the following lemma states:

¹¹ I show in Appendix A (Section A.1.2) that, whenever good entrepreneurs prefer the optimal pooling to the optimal separating contract, bad entrepreneurs do so as well.

Lemma 2. *If $\bar{p} \geq \frac{\alpha^B p^B}{\alpha^G}$, the mapping $W^*(r)$ is a function, i.e., there is a unique switching point for each value of the interest rate.*

Proof. See Section A.2 in Appendix A. \square

Therefore, whenever $\bar{p} \geq [\alpha^B p^B / \alpha^G]$, there exists a unique switching point $W^*(r)$ for each value of r : if $W < W^*(r)$, the equilibrium of the economy is pooling, whereas it is separating otherwise. I am ultimately interested in explaining the composition and level of aggregate investment, though. Hence, I now turn my attention to the impact of regime switches on investment. As I show, there are two distinct cases depending on the value of \bar{p} .

When $\bar{p} = [\alpha^B p^B / \alpha^G]$, Eq. (12) implies that $\bar{I}(r) = I^{B^*}(r)$. Additionally, I show in Section A.3 of Appendix A that in this case

$$W^*(r) = I^{B^*}(r),$$

so that the switching point also equals the size of B -type loans. This implies that, in this limiting case, there is no cross-subsidization at the switching point since the pooling loan is fully collateralized. But it must immediately follow that

$$I^G(r, W^*(r)) = W^*(r) = I^{B^*}(r). \quad (13)$$

The intuition behind Eq. (13) is simple: since the pooling loan is fully collateralized at the switching point, the only way in which good entrepreneurs can be indifferent between this contract and the separating contract is if both of them entail the same loan sizes, i.e. if $I^G(r, W^*(r)) = \bar{I}(r)$. Consequently, when $\bar{p} = [\alpha^B p^B / \alpha^G]$, aggregate investment is smooth when the equilibrium switches from pooling to separating. For $W < W^*(r)$, the economy displays a pooling equilibrium. At $W^*(r)$, there is a switch in regime and a separating equilibrium emerges: aggregate investment, however, remains constant under the new regime, since the change to separating contracts does not affect loan sizes.

When $\bar{p} > [\alpha^B p^B / \alpha^G]$, however, there is a discontinuity in aggregate investment when the regime switches from pooling to separating. In this case, $\bar{I}(r) > I^{B^*}(r)$, so that the switch from pooling to separating must entail a contraction in the amount invested by bad entrepreneurs. The same will be true of their good counterparts, for whom it must be the case that

$$I^G(r, W^*(r)) < \bar{I}(r). \quad (14)$$

What is the intuition behind Eq. (14)? At the switching point, good entrepreneurs are by definition indifferent between both kinds of contracts. The pooling contract, however, entails some degree of cross-subsidization, whereas the separating one does not. Thus, the only way in which good entrepreneurs can be indifferent between both contracts is if the separating contract entails a lower loan size. Therefore, when the economy switches from a pooling to a separating equilibrium under the assumption that $\bar{p} > \frac{\alpha^B p^B}{\alpha^G}$, there is a contraction in the amount invested by all entrepreneurs. Lemma 3 summarizes this discussion.

Lemma 3.

1. *If $\bar{p} = \frac{\alpha^B p^B}{\alpha^G}$, then*

$$\bar{I}(r) = I^G(r, W^*(r)) = W^*(r) = I^B(r).$$

2. If $\bar{p} > \frac{\alpha^B p^B}{\alpha^G}$, then

$$\bar{I}(r) > I^G(r, W^*(r)) > W^*(r) > I^B(r).$$

Proof. See Section A.3 in Appendix A. \square

5. Concluding remarks

The main results of this paper have been derived in a stylized model, thereby making it natural to inquire on their robustness to alternative settings. Here I comment on some natural directions in which the assumptions of the model could be relaxed and on their effects on my basic results:

1. In the present version of the model, debt contracts arise as the optimal arrangement due to the binary distribution of investment project outcomes. In a more general setting in which project returns were characterized by a distribution over a continuum of outcomes, Boyd and Smith (1993) have shown that debt contracts can still arise as the optimal contractual arrangements in the presence of sufficiently high verification costs.
2. I have assumed throughout that there is no cost of providing collateral, so that the latter has the same value for borrowers and investors. None of my results would change if I introduced a wedge between borrowers' and investors' valuation of collateral, as the latter would still be useful—albeit not costless—as a screening device between good and bad investors.
3. I have assumed that entrepreneurs are risk neutral. Risk aversion in the preferences of entrepreneurs would generate an additional cost of pledging their wealth as collateral, since doing so would increase the variance of consumption. Hence, this effect could restrain the amount of collateral pledged and of investment undertaken in the economy under the separating regime. On the other hand, and precisely because of this reason, collateral could be more effective as a screening device. The net impact on the level of collateralization and investment would depend on the relative importance of these two effects.
4. I have characterized the contracts under the assumption of a fixed interest rate. It could be thought that, since investment may be discontinuous, the existence of an equilibrium is not guaranteed in a general equilibrium setting in which the interest rate is determined endogenously. It can be shown, though, that any problems of this type can be dealt with through the introduction of random contracts.

Appendix A

A.1. Characterization of contracts

As explained in the main body of the paper, I follow Hellwig [9] and model competition as a three-stage game in which banks design contracts, firms apply to at most one of them and banks accept or reject applications. Hellwig shows that such a game always has a sequential Nash equilibrium. In particular, pooling contracts are Nash equilibria whenever they Pareto dominate the separating pairs. It can be shown that Nash equilibrium contracts arise from maximizing a welfare function subject to the incentive compatibility constraints under the assumptions of exclusivity and no cross-subsidization.

Throughout the appendix, I derive the family of equilibrium contracts for a given net interest rate-entrepreneurial wealth pair (r, W) . I seek for an optimum within the set of contracts that

satisfy exclusivity and no cross-subsidization. These contracts are obtained by maximizing the borrowers’ profits subject to the lenders’ participation constraints. Although there are different possible interpretations of the planner’s objective function, I make it equal to the profits of good entrepreneurs.

In the case of optimal separating contracts, these profits are maximized subject to the incentive compatibility constraints, the no cross-subsidization condition (i.e., banks’ zero-profit conditions) and the collateralization constraints. In the case of the optimal pooling contract, the optimization problem is analogous with the difference that it is not subject to an incentive compatibility constraint. Throughout the appendix, then, I use the terms “optimal separating” and “optimal pooling” contracts to denote the respective solutions of these optimization problems.

A.1.1. Separating contracts

Optimal separating contracts can be obtained as the solution to the following optimization problem:

$$\begin{aligned} & \max_{I^G, I^B, c^G, c^B, R^G, R^B} \pi^G \\ & \text{s.t.} \\ & 1 + r = p^G R^G + (1 + r)(1 - p^G)c^G = p^B R^B + (1 + r)(1 - p^B)c^B, \\ & \pi^B(I^B, R^B, c^B) \geq \pi^B(I^G, R^G, c^G), \\ & c^j \in \left[0, \frac{W}{I^j}\right] \text{ for } j \in \{G, B\}. \end{aligned}$$

I solve this problem using only one of the incentive compatibility constraints: I will later show that the other one is slack at the optimal solution. It is straightforward to see that any optimal solution will entail $p^B \alpha^B f'(I^B) = (1 + r)$. The problem can then be reinterpreted as seeking to maximize the profits of good entrepreneurs subject to a “no mimicry” (henceforth, NMC) constraint by which,

$$p^B \alpha^B f(I^G) - \left(\frac{p^B}{p^G} + \left[1 - \frac{p^B}{p^G}\right]c^G\right)(1 + r)I^G = p^B \alpha^B f(I^{B*}) - I^{B*}(1 + r) = K.$$

The previous condition specifies c^G in terms of I^G , a mapping specifying all combinations of both variables that satisfy the zero-profit condition and the incentive compatibility constraint. This mapping is given by

$$c^G = \frac{p^B \alpha^B f(I^G) - \frac{p^B}{p^G}(1 + r)I^G - K}{\left[1 - \frac{p^B}{p^G}\right](1 + r)I^G}. \tag{15}$$

Along the NMC, c^G is maximized exactly when $I^G = I^{B*}$, at which point it equals one. Moreover, simple derivation shows this mapping to be strictly concave, so that c^G is monotonically increasing in I^G whenever $I^G < I^{B*}$ and monotonically decreasing otherwise. The full-information investment of good entrepreneurs I^{G*} therefore lies on the downward sloping section of the NMC.

When wealth constraints are binding, I need to take into account an additional constraint given by $c^G I^G \leq W$. This constraint determines a hyperbola in the (c^G, I^G) space so that—when the constraint is binding—the optimal G -type contract must lie on the intersection of this hyperbola

with the NMC, as depicted in Fig. 1 in the main body of the paper. It can be shown that, of the two potential G -type contracts that satisfy both the wealth constraint and the NMC, the one with lower loan size and higher rate of collateralization is the optimal one.

Finally, I have solved the problem under the assumption that the incentive compatibility constraint for good entrepreneurs holds, so that

$$p^G \alpha^G f(I^G) - (1+r)I^G > p^G \alpha^G f(I^{B^*}) - (1+r)\frac{p^G}{p^B} I^{B^*}.$$

I know that the incentive compatibility constraint of bad entrepreneurs holds, so that I^G satisfies

$$p^B \alpha^B f(I^G) - \left[\frac{p^B}{p^G} + \left[1 - \frac{p^B}{p^G} \right] c^G \right] (1+r)I^G = p^B \alpha^B f(I^{B^*}) - (1+r)I^{B^*}.$$

Multiplying the first expression by $\frac{p^B \alpha^B}{p^G \alpha^G}$ and subtracting the second one from it, I obtain that

$$\left(1 - \frac{\alpha^B}{\alpha^G} \right) \frac{p^B}{p^G} I^G + \left[1 - \frac{p^B}{p^G} \right] W > I^{B^*} \left(1 - \frac{\alpha^B}{\alpha^G} \right). \tag{16}$$

The inequality in Eq. (16) is always satisfied for $W = 0$. Additionally, the derivative of the RHS of with respect to W is strictly positive as long as I^G is constrained below its optimal level. Hence, Eq. (16) holds for all levels of W .

A.1.2. Pooling contracts

Optimal pooling contracts are the solution to the following optimization problem,

$$\max_{I,c} p^G \alpha^G f(I) - \frac{p^G}{\bar{p}} (1+r)I - \left(1 - \frac{p^G}{\bar{p}} \right) (1+r)cI + W(1+r),$$

s.t.

$$v: c \leq \frac{W}{I},$$

$$\varphi: c \geq 0.$$

The first-order conditions of this problem are,

$$I: p^G \alpha^G f'(I) - \frac{p^G}{\bar{p}} (1+r) - \left(1 - \frac{p^G}{\bar{p}} \right) (1+r)c - v \left(\frac{W}{I^2} \right) = 0, \tag{17}$$

$$c: - \left(1 - \frac{p^G}{\bar{p}} \right) (1+r)I - v + \varphi = 0. \tag{18}$$

From Eq. (18), the collateralization constraint must bind, so that $c = \frac{W}{I}$. Replacing Eq. (18) in Eq. (17), delivers

$$p^G \alpha^G f'(\bar{I}) = \frac{p^G}{\bar{p}} (1+r),$$

which proves the result.

A.2. Characterization of regime switches

Proof of Lemma 1. The proof is by contradiction. If $\bar{p} = \frac{\alpha^B p^B}{\alpha^G}$, the profits obtained by good entrepreneurs at the contract $C^{POOL}(r, 0)$ are given by

$$\alpha^G p^G f(\bar{I}) - \frac{\alpha^G p^G}{\alpha^B p^B} (1+r)\bar{I},$$

where, from Eq. (12), $\bar{I} = I^{B*}$. Suppose that good entrepreneurs prefer the separating contract $C^{SEP}(r, 0)$ to the pooling, so that

$$\alpha^G p^G f(I^G) - (1+r)I^G > \alpha^G p^G f(\bar{I}) - \frac{\alpha^G p^G}{\alpha^B p^B} (1+r)\bar{I}.$$

The previous inequality implies that

$$\alpha^B p^B f(I^G) - \frac{\alpha^B p^B}{\alpha^G p^G} (1+r)I^G > \alpha^B p^B f(\bar{I}) - (1+r)\bar{I},$$

which is impossible since it violates the incentive compatibility constraint that bad entrepreneurs must satisfy under contracts $C^{SEP}(r, 0)$. Suppose instead the extreme case in which $\bar{p} \approx p^G$. Then, trivially, good entrepreneurs obtain higher profits from the pooling than from the separating contract. As for bad entrepreneurs, their profits would be given by

$$\alpha^B p^B f(\bar{I}) - \frac{p^B}{p^G} (1+r)\bar{I},$$

which must be higher than what they obtain under the separating contracts. This is true since $I^G > I^{B*}$ and, for $\bar{I} \approx I^G$,

$$\alpha^B p^B f'(\bar{I}) > \frac{p^B}{p^G} (1+r). \quad \square$$

Proof of Lemma 2. The proof compares the profits of good entrepreneurs under pooling and separating contracts for a given value of r . From the previous Lemma, the restriction on \bar{p} implies that

$$\pi^G(C^{POOL}(r, 0)) > \pi^G(C^{SEP}(r, 0)),$$

for all values of r . Additionally, it is easy to verify that whenever $W = \bar{I}(r)$, the following also holds:

$$\pi^G(C^{POOL}(r, \bar{I}(r))) \leq \pi^G(C^{SEP}(r, \bar{I}(r))),$$

since the profits that the pooling contract yields to good entrepreneurs are bounded from above by $\alpha^G p^G f(\bar{I})$, which are attained under full collateralization. The profits of good entrepreneurs under the separating contracts, on the other hand, are bounded from below by $\alpha^G p^G f(W)$, since the rate of collateralization is weakly smaller than one. Thus, the economy will display a pooling equilibrium when $W = 0$ regardless of the interest rate, but it will display a separating equilibrium when $W = \bar{I}(r)$. Derivation of $\pi^G(C^{POOL}(r, W))$ delivers

$$\frac{\partial \pi^G(C^{POOL}(r, W))}{\partial W} = \frac{p^G}{\bar{p}} (1+r), \tag{19}$$

which is linear and increasing in W . On the other hand, $\pi^G(C^{SEP}(r, W))$ is concave and increasing in W . Hence, both profits loci can only intersect once for a given value of r , thus proving that $W^*(r)$ is a function. \square

A.3. Investment at the switching point (Lemma 3)

Recall that the switching point is defined as a level of wealth satisfying the following equality:

$$\begin{aligned} & \alpha^G p^G f(I^G(r, W^*(r))) - (1+r)I^G(r, W^*(r)) \\ &= \alpha^G p^G f(\bar{I}(r)) - (1+r) \left[\bar{I}(r) \frac{p^G}{\bar{p}} + \left[1 - \frac{p^G}{\bar{p}} \right] W^*(r) \right]. \end{aligned} \tag{20}$$

I start by analyzing the case in which $\bar{p} = [\alpha^B p^B / \alpha^G]$. In such a scenario, $\bar{I}(r) = I^{B^*}(r)$ and Eq. (20) reduces to

$$\begin{aligned} & \alpha^B p^B f(I^G(r, W^*(r))) - \frac{p^B \alpha^B}{p^G \alpha^G} (1+r) [I^G(r, W^*(r)) - W^*(r)] \\ &= \alpha^B p^B f(I^{B^*}(r)) - (1+r) [I^{B^*}(r) - W^*(r)], \end{aligned} \tag{21}$$

which, together with the incentive compatibility constraint, implies that $I^G(r, W^*(r)) = W^*(r)$. It follows that

$$\bar{I}(r) = I^G(r, W^*(r)) = W^*(r) = I^{B^*}(r).$$

If, on the other hand, $\bar{p} > [\alpha^B p^B / \alpha^G]$, it has been shown that $\bar{I}(r) > I^{B^*}(r)$. From the incentive compatibility constraint, it then follows that

$$\begin{aligned} & \alpha^B p^B f(I^G(r, W^*(r))) - \frac{p^B}{p^G} (1+r) [I^G(r, W^*(r)) - W^*(r)] \\ & > \alpha^B p^B f(\bar{I}(r)) - (1+r) [\bar{I}(r) - W^*(r)], \end{aligned} \tag{22}$$

which, together with Eq. (20) implies:

$$\bar{I}(r) > I^G(r, W^*(r)) > W^*(r).$$

Combined with the incentive compatibility constraint, the previous inequalities also imply that $W^*(r) > I^{B^*}(r)$, which proves the result.

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