# Voting and Elections

## Political Economics: Week 1

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# A Policy Problem

- A set $\mathcal{V}$ of individuals $i = 1, ..., I$.
- Individual $i$ has utility function

$$U\left(c^i, q, p; \alpha^i\right)$$

- $\alpha^i$ denotes his exogenous idiosyncratic characteristics;
- $c^i$ is his private choice subject to constraints $H\left(c^i, q, p; \alpha^i\right) \geq 0$;
- $q$ is the policy choice from a feasible set $Q$;
- $p = P\left(c, q\right)$ describes the general equilibrium that results from the policy choice $q$ and the individual choices $c$ of all agents.

# Individual Policy Preferences

- In a well-behaved model, for each policy choice $q \in Q$ there is a unique equilibrium with actions $c(q)$ and market outcomes $p(q) = P(c(q), q)$ such that for each agent $i \in \mathcal{V}$

$$c^i(q|\alpha) = \arg\max_{\gamma} U\left(\gamma, q, p(q); \alpha^i\right) \text{ s.t. } H\left(\gamma, q, p(q); \alpha^i\right) \geq 0.$$

- With price-taking atomistic agents, the individual choice $c^i$ does not affect $p$. Having strategic market interactions makes no difference as long as there is a unique equilibrium.
- The equilibrium yields each agent's *indirect utility function*

$$W\left(q; \alpha^i\right) \equiv U\left(c^i(q), q, p(q); \alpha^i\right).$$

- The *preferred policy* or bliss point of each agent is

$$q\left(\alpha^i\right) = \arg\max_{q} W\left(q; \alpha^i\right).$$

# Preference Aggregation in General

- Let $Q$ be a set with at least three alternatives and $\mathcal{R}$ the set of all *complete* and *transitive* (i.e., "rational") preference relations on $Q$.
- Each agent $i$ has a rational preference relation $\succsim_i \in \mathcal{R}$.
- A *social welfare functional* is a function $F : \mathcal{R}^I \to \mathcal{R}$ that assigns a rational social preference relation $\succsim = F(\succsim_1, ..., \succsim_I) \in \mathcal{R}$ to any profile of rational individual preference relations $(\succsim_1, ..., \succsim_I) \in \mathcal{R}^I$.
- A social welfare functional $F$ is *Paretian* if for any pair of alternatives $\{q, q'\} \subset Q$ and any preference profile $(\succsim_1, ..., \succsim_I) \in \mathcal{R}^I$

$$\text{if } q \succsim_i q' \text{ for all } i, \text{ then } q \succsim q'.$$

- A social welfare functional $F$ satisfies *independence of irrelevant alternatives* if for any pair of alternatives $\{q, q'\} \subset Q$ and any preference profiles $(\succsim_1, ..., \succsim_I) \in \mathcal{R}^I$ and $(\succsim'_1, ..., \succsim'_I) \in \mathcal{R}^I$

$$\text{if } \succsim_i |_{\{q,q'\}} = \succsim'_i |_{\{q,q'\}} \text{ for all } i, \text{ then } \succsim |_{\{q,q'\}} = \succsim' |_{\{q,q'\}}.$$

# Arrow's Impossibility Theorem

## Theorem

*Every social welfare functional $F : \mathcal{R}^I \rightarrow \mathcal{R}$ that is Paretian and satisfies independence of irrelevant alternatives is dictatorial: there is an agent $h$ such that, for any pair of alternatives $\{q, q'\} \subset Q$ and any preference profile $(\succsim_1, ..., \succsim_I) \in \mathcal{R}^I$, if $q \succ_h q'$ then $q \succ q'$.*

- One of the conditions of the theorem is $F : \mathcal{R}^I \rightarrow \mathcal{R}$. Hence:
  - *unrestricted domain*: defined for all preference profiles;
  - *transitivity:* the social preference relation is itself rational.
- What can we do? Restrict preferences or specify institutions.
- Preference relations are *ordinal*. There is much more scope for aggregation of individual preferences into a social welfare function if we use cardinal utilities and admit their interpersonal comparison.

# Majority Rule and the Condorcet Criterion

1. *Direct democracy*. The citizens themselves make the policy choices.
2. *Open agenda.* Citizens vote over pairs of policies, such that the winning policy in one round is posed against a new alternative in the next round and the set of alternatives is the entire feasible set $Q$.
3. *Sincere voting.* Citizen $i$ votes for the alternative that maximizes his indirect utility $W\left(q;\alpha^{i}\right)$.

**Definition**

A *Condorcet winner* is a policy $q^{*}$ that beats any other feasible policy in a pairwise vote.

**Corollary**

*If a Condorcet winner $q^{*}$ exists, it is the unique equilibrium under majority rule (i.e., under the three assumptions above).*

# The Condorcet Paradox

- A Condorcet winner need not exist.
- Three agents $i \in \{1, 2, 3\}$ and three choices $\{q_A, q_B, q_C\}$. Preferences

$$q_A \succ_1 q_B \succ_1 q_C,$$
$$q_B \succ_2 q_C \succ_2 q_A,$$
$$q_C \succ_3 q_A \succ_3 q_B.$$

$\Rightarrow$ Majority-rule cycle:

$$q_A \succ q_B,\ q_B \succ q_C,\ q_C \succ q_A.$$

- Arrow's impossibility theorem for *transitive* social preference relations.

# Strategic Voting

- With sincere voting agents reveal their preferences. Why should they?
- Let $Q$ be a set with at least three alternatives and $\mathcal{P}$ the set of all rational preference relations $\succ_i$ on $Q$ having the property that no two alternatives are indifferent.
- A *social choice rule* is a function $F : \mathcal{P}^I \rightarrow Q$ that assigns a policy $q^* = F(\succ_1, ..., \succ_I) \in Q$ to any profile of individual preference relations $(\succ_1, ..., \succ_I) \in \mathcal{P}^I$.
- A social choice rule is *manipulable* if there exists a profile $(\succ_1, ..., \succ_I) \in \mathcal{P}^I$ and an agent $i$ such that

$$F(\succ_1, ..., \succ_i', ..., \succ_I) \succ_i F(\succ_1, ..., \succ_i, ..., \succ_I).$$

  If a social choice rule is not manipulable, it is *strategy-proof*.
- A social choice rule is *onto* if for each $q \in Q$ there is a $(\succ_1, ..., \succ_I) \in \mathcal{P}^I$ such that $F(\succ_1, ..., \succ_I) = q$.

# The Gibbard–Satterthwaite Theorem

**Theorem**

*Every social choice rule $F : \mathcal{P}^I \to Q$ that is onto and strategy-proof is dictatorial: there is an agent $h$ such that $F(\succ_1, ..., \succ_I) \succ_h q$ for any $q \neq F(\succ_1, ..., \succ_I)$ and for any preference profile $(\succsim_1, ..., \succsim_I) \in \mathcal{P}^I$.*

- The same result as Arrow's impossibility theorem.
- A social choice rule must be onto to be Pareto efficient, i.e., to pick a policy when all agents unanimously prefer it to all alternatives.
- With an unrestricted domain for preferences, sincere voting is typically inconsistent with strategic rationality.

# Single-Peaked Preferences

- A binary relation $\geq$ on the set $Q$ is a *linear order* if it is *reflexive*, *transitive*, and *total.*

## Definition

The rational preference relation $\succsim_i$ is *single-peaked* with respect to the linear order $\geq$ on $Q$ if there is a bliss point $q^i$ such that

$$q^i \geq q > q' \Rightarrow q \succ_i q' \text{ and } q' > q \geq q^i \Rightarrow q \succ_i q'$$

- Concretely, consider $Q \subset \mathbb{R}$ with the natural order $\geq$ and

$$W\left(q;\alpha^i\right) > W\left(q';\alpha^i\right) \text{ if } q\left(\alpha^i\right) \geq q > q' \text{ or } q' > q \geq q\left(\alpha^i\right)$$

- The indirect utility function is strictly quasiconcave with a unique maximum $q\left(\alpha^i\right)$.

# The Median-Voter Theorem

**Theorem**

*Suppose that $I$ is odd and that the preferences of every agent are single-peaked with respect to the same linear order on $Q$. Then a Condorcet winner always exists and coincides with the median-ranked bliss point $q^m$. It is the unique equilibrium under majority rule.*

- A simple separation argument.
- Since $I$ is odd, the median bliss point $q^m$ is uniquely defined.
- In a pairwise comparison of $q^m$ and any $q' > q^m$, all agents $i$ with bliss point $q^i \geq q^m > q'$ prefer $q^m$. By the assumption of sincere voting, a majority votes for $q^m$.
- Identically for any $q'' < q^m$.

# Strategy-Proofness

- Let $\geq$ be a linear order on $Q$, and $\mathcal{P}_{\geq} \subset \mathcal{P}$ the set of all strict rational preference relations on $Q$ that are single-peaked with respect to $\geq$.

**Theorem**

*If $I$ is odd there exists a social choice rule $F : \mathcal{P}_{\geq}^{I} \to Q$ that is onto and strategy-proof and always selects the median-ranked bliss point $q^m$.*

- Each agent reports a bliss point $r^i$ and the median $r^m$ is selected.
- Truthful reporting $r^i = q^i$ is a *weakly dominant strategy*.
- For any profile of reports $r^{-i}$ by all agents other than $i$:
  1. if $q^i$ is median in $\left(q^i, r^{-i}\right)$, $i$ reaches his bliss point;
  2. if the median in $\left(q^i, r^{-i}\right)$ is $r^m > q^i$, an untruthful report could make it increase but not decrease; symmetrically for $r^m < q^i$.

$\Rightarrow$ Thus $i$ can never gain by an untruthful report $r^i \neq q^i$.

# The Single-Crossing Condition

## Definition

A preference profile $(\succsim_1, ..., \succsim_I)$ satisfies the *single-crossing condition* with respect to the linear order $\geq$ on $Q$ if

$$\text{for } q > q' \text{ and } j > i,\ q \succsim_i q' \Rightarrow q \succsim_j q'.$$

- Single-peakedness is a property of an individual preference relation. Single-crossing is a property of an entire preference profile.
- A linear order of voters as well as policies is required.
- Concretely, consider $Q \subset \mathbb{R}$ and $\alpha^i \in \mathbb{R}$ with the natural order $\geq$: if $q > q'$ and $\alpha_i' > \alpha_i$, then

$$W(q; \alpha_i) \geq W(q'; \alpha_i) \Rightarrow W(q; \alpha_i') \geq W(q'; \alpha_i').$$

# The Median Voter Theorem (Second Version)

**Theorem**

*Suppose that $I$ is odd and that the agents' preference profile satisfies the single-crossing condition with respect to the linear order $\geq$ on $Q$. Then a Condorcet winner always exists and coincides with the bliss point of the median agent. It is the unique equilibrium under majority rule.*

- Proof by the same separation argument as before.
- The single-crossing property implies an ordering of bliss points $\alpha_i' > \alpha_i \Rightarrow q(\alpha_i') > q(\alpha_i)$
- Strategy-proof mechanism: each agent reports a type $\hat{\alpha}_i$ and the enacted policy is the bliss point of the median report $q(\hat{\alpha}_m)$.

# Intermediate Preferences

## Definition

Agents in the set $\mathcal{V}$ have *intermediate preferences* on $Q$ if their indirect utility function can be written as

$$W(q;\alpha_i) = J(q) + K(\alpha_i) H(q),$$

where $K(\alpha_i)$ is monotonic and $H$, $J$, and $K$ are common to all agents.

## Theorem

*Suppose that $I$ is odd and that agents have intermediate preferences on $Q$ Then a Condorcet winner always exists and coincides with the bliss point of the median agent. It is the unique equilibrium under majority rule.*

- Strong restriction, but simple and occasionally convenient.
- Project on the multidimensional policy space $Q$ the natural ordering of the one-dimensional agent type $\alpha_i$.

# Downsian Electoral Competition

Timeline:

1. *Two* candidates $A$ and $B$ simultaneously and non-cooperatively choose electoral platforms $q_A$ and $q_B$.
2. An election is held in which *all citizens vote* for either candidate.
3. The winner implements his electoral platform (*binding commitment*).

Voters' strategy:

- If $q_A \neq q_B$, sincere voting is a weakly dominant strategy.
- If $q_A = q_B$, assume that citizens vote randomly for either candidate.

# Politicians' Objectives

- Office-seeking politicians: exogenous "ego rent" from winning.
- Maximize the probability of winning $p_A$ or $p_B = 1 - p_A$.
- *Tie breaking* assumption $q_A = q_B \Rightarrow p_A = p_B = \frac{1}{2}$.
- The result is robust to the assumption that politicians have goals beyond winning, so long as they are conditional on winning: e.g., implementing a certain policy.

# Downsian Convergence

## Theorem

*Suppose that two politicians contest an election by announcing a binding policy proposal, and a set of voters $\mathcal{V}$ vote for either party following a weakly dominant strategy, and voting randomly when the two proposals are identical. Suppose that If the voters' preference profile on $Q$ is such that a Condorcet winner $q^m$ exists, there is a unique equilibrium in which both parties propose $q^m$.*

- The median-voter theorem with two-party competition.
- Competition on an ordered line à la Hotelling (1929).
- Not robust to an increase in the number of parties.

# Redistributive Taxation

- Agents have preferences over consumption $c_i$ and leisure $x_i$:

$$U(c_i, x_i) = c_i + V(x_i),$$

  where $V(x_i)$ is a well-behaved increasing and concave function.
- *Policy instrument:* a linear tax $\tau$ on earnings that is rebated via uniform lump-sum transfers $f$.
- Budget constraint: $c_i \leq (1-\tau) l_i + f$.
- Time-allocation constraint: $x_i + l_i \leq 1 + \alpha_i$.
- Individual productivity $\alpha_i$ and labour supply

$$\begin{aligned} l(\tau; \alpha_i) &= \arg\max_{l_i} \{(1-\tau) l_i + f + V(1 + \alpha_i - l_i)\} \\ &= 1 + \alpha_i - V'^{-1}(1-\tau). \end{aligned}$$

# General Equilibrium

- Unit measure of individuals.
- $\alpha_i$ has a known distribution with mean $\alpha$ and median $\alpha_m$.
- Aggregate labour supply

$$L(\tau) = 1 + \alpha - V'^{-1}(1 - \tau)$$

  such that

$$l(\tau; \alpha_i) = L(\tau) + \alpha_i - \alpha.$$

- Government budget constraint: $f \leq \tau L(\tau)$.

# Policy Preferences

- Indirect utility

$$W(\tau;\alpha_i) = (1-\tau) l(\tau;\alpha_i) + f + V(1+\alpha_i - l(\tau;\alpha_i))$$
$$= (1-\tau)(\alpha_i - \alpha) + L(\tau) + V(1+\alpha - L(\tau)).$$

- The single-crossing condition is satisfied.
- Individual preferences

$$\frac{\partial}{\partial\tau} W(\tau;\alpha_i) = -l(\tau;\alpha_i) + \frac{\partial f}{\partial\tau} = \underbrace{\alpha - \alpha_i}_{\text{redistribution}} + \underbrace{\tau L'(\tau)}_{\text{inefficiency}}.$$

- Ideal policy

$$\tau(\alpha_i) \text{ such that } \tau_i \left| L'(\tau_i) \right| = \alpha - \alpha_i$$

# Income Redistribution by the Median Voter

- A voter with average productivity $(\alpha_i = \alpha)$ wants no redistribution.

$\Rightarrow$ By definition, this is the utilitarian welfare optimum.

$\Rightarrow$ With quasilinearity, $\tau = 0$ is also clearly efficiency-maximizing.

- Voters with less than average productivity (income) desire progressive redistribution.
    - Voters with more than average income desire regressive redistribution: a production subsidy $\tau < 0$ financed by a poll tax.
- The median voter prefers $\tau_m$ such that $\tau_m \left| L'(\tau_m) \right| = \alpha - \alpha_m$.
- Redistribution increases as the gap between average and median income increases. Not any measure of income inequality.
- Empirical support for this simple model is not very strong.

# Empirical Evidence on the Downsian Model

- Gerber and Lewis (2004) focus on elections in Los Angeles County, which contains 55 (state and federal) majoritarian electoral districts.
- Voter preferences are measured from a database of 2.8m individual ballots for the 1992 election. These votes include 13 statewide ballot propositions (direct democracy) as well as candidate races.
- Ideology of the elected representative is measured from legislative voting records.
- Representatives' ideology is correlated with the median constituent's, *but* ...
- "Legislators from heterogeneous districts often take policy positions that diverge substantially from the median voter in their district."

# Beyond Condorcet Winners

- The Downsian model fails without a Condorcet winner, and is typically not applicable to multidimensional policy choices.
- The platform-choice game lacks a pure-strategy equilibrium.
- Discontinuous payoff functions and best responses are the problem.
- Introduce smoothness by making candidates uncertain about voter support.
- An *intensive margin*: a voter is the more likely to support a candidate (instead of the competitor, or instead of abstaining), the more he likes his platform.
- Exploit the cardinal dimension of preferences.

# Political Preferences

- Each candidate $P \in \{A, B\}$ is characterized by the binding platforms $q^P$ but also by exogenous non-policy characteristics.
- Voters $i$'s utility if candidate $P$ wins the election is

$$W\left(q^P, P; \alpha_i\right) = W\left(q^P; \alpha_i\right) + \xi_i^P,$$

  where $\xi_i^P$ is a stochastic ideological bias.
- Let $\xi_i^B - \xi_i^A = \sigma_i + \delta$.
- $\sigma_i$ is an idiosyncratic shock that makes $i$'s vote imperfectly predictable
- $\delta$ is a common shock that makes the election imperfectly predictable even with a continuum of voters.

# Probabilistic Voting

- Unit measure of voters.
- A fraction $\lambda_j$ belongs to group $j$ with:
  1. homogeneous policy preferences $\alpha_j$;
  2. i.i.d. ideology $\sigma_i$ with distribution $\Phi_j(\sigma_i)$.
- Given $\delta$, the fraction of group $j$ that votes for $A$ is

$$\Phi_j\left(W\left(q^A;\alpha_j\right)-W\left(q^B;\alpha_j\right)-\delta\right).$$

- Aggregate popularity $\delta$ is independently drawn from an absolutely continuous distribution $F(\delta)$.

# The Tractable Specification

- **Assumption**
  The idiosyncratic ideology parameters are uniformly distributed:

$$\sigma_i \sim U\left[-\frac{1}{2\phi_j}, \frac{1}{2\phi_j}\right] \text{ for all voters } i \text{ in group } j.$$

- Given $\delta$, candidate $A$' share of the vote is

$$\pi_A(\delta) = \frac{1}{2} + \sum_{j=1}^{J} \lambda_j \phi_j \left[ W\left(q^A; \alpha_j\right) - W\left(q^B; \alpha_j\right)\right] - \sum_{j=1}^{J} \lambda_j \phi_j \delta.$$

- Assume $\phi_j$ and $F(\delta)$ such that in each group there are voters supporting both candidates.
- Within these bounds, a group-specific mean bias is irrelevant.

# Weighted Utilitarian Welfare Function

- The probability that candidate $A$ wins the election is

$$\Pr\left(\pi_A(\delta) > \frac{1}{2}\right) = F\left(\frac{\sum_{j=1}^{J}\lambda_j\phi_j\left[W\left(q^A;\alpha_j\right) - W\left(q^B;\alpha_j\right)\right]}{\sum_{j=1}^{J}\lambda_j\phi_j}\right).$$

- Office-seeking candidates choose

$$q^* = \arg\max_q \sum_{j=1}^{J}\lambda_j\phi_j W\left(q;\alpha_j\right).$$

- Policy proposals cater to swing voters: higher $\phi_j$ means that more group members are swayed by a policy change.

# Another Specification

- **Assumption:** Candidates maximize their expected vote share.
- Candidate $A$'s expected vote share is

$$\mathbb{E}\left[\pi_A(\delta)\right] = \sum_{j=1}^{J} \lambda_j \mathbb{E}\left[\Phi_j\left(W\left(q^A;\alpha_j\right) - W\left(q^B;\alpha_j\right) - \delta\right)\right].$$

- If a symmetric pure-strategy Nash equilibrium of the policy-proposal game exists, it satisfies

$$\sum_{j=1}^{J} \lambda_j \mathbb{E}\left[\phi_j\left(-\delta\right)\right] \nabla W\left(q^*;\alpha_j\right) = 0.$$

- In particular if voters' preferences are uncorrelated ($\delta \equiv 0$):

$$q^* = \arg\max_q \sum_{j=1}^{J} \lambda_j \phi_j(0)\, W\left(q;\alpha_j\right).$$

- But a symmetric pure-strategy Nash equilibrium does not typically exist with distributions other than the uniform.

# Local Public Goods

- Individuals in group $j$ have unit income and preferences over consumption $c_j$ and a group-specific public good $g_j$:

$$U\left(c_j, g_j\right) = c_i + H\left(g_j\right),$$

  where $H\left(g_j\right)$ is a well-behaved increasing and concave function.
- *Policy instrument:* provision of public goods $\mathbf{g}$ financed by a uniform tax $\tau$.
- Government budget constraint: $\sum_{j=1}^{J} \lambda_j g_j \leq \tau$.
- Indirect utility

$$W_j\left(\mathbf{g}\right) = 1 - \sum_{i=1}^{J} \lambda_i g_i + H\left(g_j\right)$$

# Preference Aggregation

- Utilitarian welfare maximization:

$$\mathbf{g}^* = \arg\max_{\mathbf{g}} \sum_{j=1}^{J} \lambda_j W_j(\mathbf{g}) \Rightarrow H'\left(g_j^*\right) = 1 \text{ for all } j.$$

- Individual preferences

$$\frac{\partial W_j}{\partial g_j} = \underbrace{H'(g_j) - 1}_{\text{efficiency}} + \underbrace{1 - \lambda_j}_{\text{redistribution}},$$

and

$$\frac{\partial W_j}{\partial g_i} = \underbrace{-\lambda_j}_{\text{redistribution}} \text{ for all } i \neq j.$$

# Public-Goods Provision with Probabilistic Voting

- Political support:

$$\hat{\mathbf{g}} = \max_{\mathbf{g}} \sum_{j=1}^{J} \lambda_j \phi_j W_j(\mathbf{g}) \Rightarrow H'(\hat{g}_j) = \frac{\bar{\phi}}{\phi_j} \text{ for all } j,$$

  where $\bar{\phi} = \sum_{j=1}^{J} \lambda_j \phi_j$ is the average density across groups.
- If all groups are identically motivated by ideology ($\phi_j = \bar{\phi}$ for all $j$) electoral competition with probabilistic voting implements the utilitarian social optimum.
- There is no clear bias to the size of government. Some groups get more, some less than $g_j^*$.
- "Director's Law" *if* the middle class is the less ideological group.

# Group Size and Political Influence

- Persson and Tabellini (2000) highlight that $\lambda_j$ has no direct effect.
- However, $\lambda_j$ has an indirect effect through its impact on the mean $\bar{\phi}$.
- Consider a change in $\lambda_j$ that does not affect the relative sizes of the other groups.
- Then the average density across groups $i \neq j$ is a constant $\bar{\phi}_{-j}$.
- In equilibrium, $\hat{g}_j$ depends on $\lambda_j$, $\phi_j$ and $\bar{\phi}_{-j}$ according to

$$H'\left(\hat{g}_j\right) - 1 = \left(1 - \lambda_j\right)\left(\frac{\bar{\phi}_{-j}}{\phi_j} - 1\right).$$

- Smaller groups have larger policy distortions:
  - a favoured group benefits from being smaller;
  - a neglected group benefits from being larger.

# Empirical Evidence on the Power of Swing Voters

- Strömberg (2008) studies the allocation of campaign visits across states in a U.S. presidential campaign.
- A sophisticated model of the Electoral College system using probabilistic voting.
- Structural estimation of the underlying parameters based on past vote shares of the parties in each state.
- Actual campaign visits line up quite well with theoretical predictions.
  - This seems to be true for the 2008 election as well.
- However, Strömberg (2008) does not consider policy outcomes.
- Larcinese, Snyder and Testa (2008, wp) do, and find no evidence that federal funds are disproportionately allocated to states with more swing voters or more evenly matched partisan groups.

# Why Do People Vote?

- We routinely assume that all citizens vote. The assumption is strikingly counterfactual, particularly in the U.S.
- Turnout was 62.7% in the 2008 federal election, and only 39.8% for the "off-year" 2006 congressional election, which did not feature a presidential race.
- In Spain, turnout was 73.85% in the 2008 national election, and only 44.9% in the 2009 election for the European Parliament.
- Turnout also varies widely across demographic and socioeconomic groups within a country. It displays huge diversity across countries and some variation over time, notably with a downward trend in Western democracies over the past sixty years.
- Why do so many people not exercise their right to vote?
- Why do so many people bother to vote?

# The Paradox of the Rational Voter

- The fundamental rational-choice explanation is that people vote to influence the outcome of the election.
- Going to the polls require time and a modicum of effort, so it has a cost $\kappa_i$.
- A "rational voter" should vote if and only if his participation is sufficiently likely to affect the outcome:

$$\mathbb{E}\left[W_i\left(q\right)|i\right] - \mathbb{E}\left[W_i\left(q\right)|\neg i\right] \geq \kappa_i.$$

- The left-hand side is positive if the parties would implement different policies.
- The magnitude of the left-hand side equals the probability that $i$'s single vote decides the election.
- The probability is vanishingly small with millions of ballots cast.

# Pivotal-Voter Models

- Consider an election with two alternatives $q_A$ and $q_B$.
- $n_A$ voters have welfare $W_A(q)$ such that $W_A(q_A) - W_A(q_B) = 1$.
- $n_B$ have welfare $W_B(q)$ such that $W_B(q_B) - W_B(q_A) = 1$.
- The policy is determined by the toss of a fair coin if the election ends in a tie.
- Voter $i$ is pivotal and gains $\frac{1}{2}$ from voting in two cases:

1. When the number of ballots cast by other voters for either party is identical.
2. When the number of ballots cast for the opposing party is exactly one more than that of ballots cast by other voters for $i$'s own party.

# Mixed-Strategy Equilibria

- Suppose that all voters have an identical cost $\kappa > 0$ of casting a ballot.
- There cannot be an equilibrium in which all voters abstain so long as $\kappa < \frac{1}{2}$.
- Equilibria in pure strategies typically fail to exist.
- Mixed-strategy equilibria always exist, and are not unique.

1. Symmetric equilibria in which all voters in the same group vote with equal probability.
2. Asymmetric equilibria in which some voters randomize and other play either pure strategy.

# Symmetric Equilibrium

- Every $A$ supporter votes with probability $p_A$ and every $B$ supporter votes with probability $p_B$.
- In equilibrium $p_A$ and $p_B$ are such that all citizens are indifferent:

$$\Pr\left(B\left(n_B, p_B\right) - B\left(n_A - 1, p_A\right) = 0\right) + \Pr\left(B\left(n_B, p_B\right) - B\left(n_A - 1, p_A\right) = 1\right) = 2\kappa$$

$$\Pr\left(B\left(n_A, p_A\right) - B\left(n_B - 1, p_B\right) = 0\right) + \Pr\left(B\left(n_A, p_A\right) - B\left(n_B - 1, p_B\right) = 1\right) = 2\kappa,$$

  where $B\left(n, p\right)$ denotes a binomial distribution, and independence of the two binomials is implicit.
- Palfrey and Rosenthal (1983) show that as $n = n_A + n_B$ grows large, $p_A$ and $p_B$ tend to zero, and so does expected turnout.
- If $n_A = n_B$ there is also an exceptional equilibrium with $p_A = p_B \approx 1$, but the standard equilibrium with $p_A = p_B \approx 0$ continues to exist.

# An Asymmetric Equilibrium

- Let $n_A > n_B$.
- All members of the majority play a pure strategy, and exactly $n_B$ go to the polls.
- Each member of the minority votes with independent probability $p$
- Minority members are indifferent if $p^{n_B - 1} = 2\kappa$.
- Majority voters do not wish to abstain if $p^{n_B} + n_B (1 - p) p^{n_B - 1} \geq 2\kappa$.
- Majority abstainers do not wish to vote if $p^{n_B} \leq 2\kappa$.

$\Rightarrow$ The equilibrium exists for $p = (2\kappa)^{\frac{1}{n_B - 1}}$.

- Expected turnout can be arbitrarily close to one, for $n_A \approx n_B$:

$$\frac{n_B}{n_A + n_B} \left[ 1 + (2\kappa)^{\frac{1}{n_B - 1}} \right] \to \frac{2n_B}{n_A + n_B} \text{ as } n_B \to \infty.$$

# Bayesian Nash Equilibrium

- Let the cost of voting $\kappa_i$ be i.i.d. with an absolutely continuous distribution $F(\kappa_i)$ on $[\underline{\kappa}, \bar{\kappa}]$.
- There is a symmetric equilibrium in which each $A$ supporter votes if and only if $\kappa_i < \kappa_A^*$ and each $B$ supporter if and only $\kappa_i \leq \kappa_B^*$.
- In equilibrium $\kappa_A^*$ and $\kappa_B^*$ are such that all citizens are indifferent:

$$\Pr\left(B\left(n_B, F\left(\kappa_B^*\right)\right) - B\left(n_A - 1, F\left(\kappa_A^*\right)\right) = 0\right) \\ + \Pr\left(B\left(n_B, F\left(\kappa_B^*\right)\right) - B\left(n_A - 1, F\left(\kappa_A^*\right)\right) = 1\right) = 2\kappa_A^*$$

$$\Pr\left(B\left(n_A, F\left(\kappa_A^*\right)\right) - B\left(n_B - 1, F\left(\kappa_B^*\right)\right) = 0\right) \\ + \Pr\left(B\left(n_A, F\left(\kappa_A^*\right)\right) - B\left(n_B - 1, F\left(\kappa_B^*\right)\right) = 1\right) = 2\kappa_B^*.$$

- Palfrey and Rosenthal (1985) show that if $[0,1] \subset [\underline{\kappa}, \bar{\kappa}]$ then $\kappa_A^*$ and $\kappa_B^*$ tend to zero as the electorate grows large.

$\Rightarrow$ If someone certainly votes and someone certainly abstains, people only vote for non-strategic reasons, i.e., when $\kappa_i < 0$.

# Benefits of Voting

- The standard *calculus of voting* allows not only for a cost of going to the polls but also from a benefit of voting.
- The simplest assumption is that people engage in *expressive voting* and derive a psychological benefit $\psi\left(W_i\left(q_A\right) - W_i\left(q_B\right)\right)$ from supporting $A$ against $B$.
  - Turnout rises with the stakes of the election.
  - Candidates' incentives to chose platforms are analogous to those of the probabilistic-voting model, and as tractable if $\kappa_i \sim U\left[\underline{\kappa}, \bar{\kappa}\right]$.
- *Group-level* theories of turnout do not suffer from the rational-voter paradox:
  - Each voter's turnout generates positive externalities for like-minded individuals.
  - If externalities within the group are internalized, members follow he jointly optimal rule, which involves much higher turnout than the individually optimal behaviour.
  - Turnout rises in both the stakes and the closeness of the race.

# Microfoundations of Group Participation

1. Shachar and Nalebuff (1999) suggest that politicians and other *opinion leaders* invest resources in increasing their followers' benefit of voting (or reducing the cost).
2. Feddersen and Sandroni (2006) appeal to the ethics of *rule utilitarianism*.
    - Ethical agents follow the voting rule that would maximize social welfare if it were followed by all agents, while non-ethical agents abstain.
    - There are two kinds of ethical agents, who disagree over the assessment of the social-welfare consequences of the two policies.
3. Coate and Conlin (2004) give the argument a twist with *group rule utilitarianism*: each agent only cares about the welfare of members of his own group.
    - They find this model outperforms simple expressive voting in explaining participation in Texas liquor referenda.
    - Coate, Conlin and Moro (2008) in turn find that the expressive voting model outperforms the pivotal-voter model.